WHAT COUNTS IS HOW THE GAME IS SCORED
One Way to Increase Achievement In Learning Mathematics

LAYMAN E. ALLEN
GLORIA JACKSON
JOAN ROSS
STUART WHITE
University of Michigan

Prior investigation indicates that instructional gaming can be an effective tool for enhancing both motivation and achievement in the learning of mathematics. This study explores the extent to which the effectiveness of instructional gaming in facilitating the learning of specific mathematical ideas can be increased by incorporating devices that channel learners' attention upon those ideas. In particular, the effect of channeling attention by changing the method of scoring is explored.

BACKGROUND

Research on the use of RA-type (Resource Allocation) non-simulation games (Allen, 1972) began in the early 1960s with the

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first of this type of game to be published—namely, WFF 'N PROOF: The Game of Modern Logic (Allen, 1961). Initial results showed that an intensive exposure to the WFF 'N PROOF materials in a classroom tournament was accompanied by significant increases in the scores of participants on the non-language part of standard I.Q. tests (Allen, Allen, and Miller, 1966; Jeffryes, 1969); the mean increase in one study was more than 20 points (Allen, Allen, and Ross, 1970). Subsequent investigations with the game designed for facilitating the learning of mathematics—EQUATIONS: The Game of Creative Mathematics (Allen, 1963)—showed that its use in a classroom tournament setting was accompanied by significant increases in indicators of motivation and achievement. Using student absenteeism as a pervasive and pragmatic indicator of motivation in the sense of student attitudes toward the learning environment of the mathematics classroom, results of a year-long study in a Detroit inner-city school showed that the mean absentee rate in classes employing instructional gaming in a tournament structure was significantly less than (approximately one-third) the corresponding rate in control classes (Allen and Main, 1973). Using observations of student behavior, students' reports of their relationships with classmates, and students' reports of the classroom environment as indicators of various aspects of the total classroom process, DeVries and Edwards (1973) report that mathematics classes using instructional gaming tournaments had significantly more peer tutoring and that students perceived the class as significantly more satisfying, less difficult, and less competitive than students in control classes. The same investigators also found indications of greater social integration in instructional gaming classes than in other classes; there was significantly more cross-race and cross-sex interaction (DeVries and Edwards, 1972). Perhaps not surprisingly, in the same series of studies there were significantly greater gains in mathematics achievement, as indicated by Stanford Achievement Test and a specially-constructed divergent solutions test, in those classes engaged in instructional gaming that experienced these more positive social and motivational effects than in the control classes (Edwards, DeVries and Snyder, 1972). There had been earlier indications of increased gains in mathematics achievement accompanying the classroom tournament use of the EQUATIONS game; four months of instruction using gaming resulted in an average increase in arithmetic reasoning of 1.3 years, seven months more than the average gain in the control class (Egerton, 1966). Subsequent studies tend to confirm that strong effects upon achievement result when the instructional gaming experience is accompanied by attention-channeling techniques that focus attention upon specific mathematical concepts. One of these techniques uses a solitaire version of EQUATIONS with printed pamphlets called IMP (Instructional Math Play) Kits; they simulate a computer playing the game as an effective teacher of a specific concept (Allen and Ross, 1971). Students who used the IMP Kits during five class periods over a two-week period sustained significantly greater gains (doubled their performance scores) than did students who were otherwise presented the same content in control classes on tests containing extremely difficult problems (see Appendix A for examples) on concepts presented in the IMP Kits (Allen and Ross, 1975; Allen and Ross, 1974). However, no significant changes occurred in either the motivational or achievement dimension when the EQUATIONS game was used for a shorter period, without the cooperative features of the learning environment introduced with the games, and without the tournament procedure, which is designed to individualize the problems presented to each learner and to equalize the reinforcements achieved among all members of the class (Henry, 1973).

CHANNELING ATTENTION UPON SPECIFIC MATHEMATICAL CONCEPTS

The basic game of EQUATIONS allows participants virtually complete freedom in determining which of various elementary arithmetic ideas they will deal with, but it can easily be modified to focus attention upon specific ideas. The game is played by two or more persons with the objective of finding ways of expressing equations by using simple arithmetic operations and numbers. One player defines the Goal (the right side of an equation) by selection of some of the numbers and operations provided by a roll of a dozen or more special dice. The players then try to come as close as possible to supplying the left side of the equation (the Solution) without actually doing so, by moving one die at a time from the remaining dice. If a mover gets too close to a Solution or prevents all Solutions (by eliminating crucial dice from play), the other players have an opportunity to win by challenging the errant mover. All completed EQUATIONS games end with some (non-null) subset of the players having the burden of proving that there is a Solution equal to the Goal that can be built from a constrained set of digits and operators. They score points if and only if they write such a Solution. Those who do not have the burden of proof score points if and only if nobody writes a Solution. In Basic EQUATIONS the characteristics of the Solution written are unimportant with respect to the scoring. A player can write easy Solutions or ones containing difficult ideas, and neither will make any difference in his or her scores. The score of a player is independent of the degree of difficulty of the mathematics used in writing a Solution. To the extent that there is incentive for players to learn more sophisticated mathematical ideas in Basic EQUATIONS, it is provided by the game structure: those whose understanding is deeper or whose knowledge of content is broader are more likely to win. For a more detailed account of EQUATIONS, the classroom tournament setting in which it is used, and the team organization for eliciting cooperative behavior, see Allen and Main (1973).

One of the most powerful means of extending the scope of the basic game and channeling attention on specific mathematical concepts is through a variant of the game called Adventurous EQUATIONS. In this variation the players become games designers as well as continuing the three roles they perform in the basic game: learners, teachers, and diagnosticians. They create new games by adding rules to the basic game, introducing other mathematical concepts by way of the added rules. For example, an adventurous rule that emphasizes the relationship between fractions and decimals is: (a) When building a Solution, a player must specify where the decimal point(s) occur in the goal, but no decimal points are allowed in the Solution. (Thus, a goal of 2 x 3 can be interpreted as 6, .6, or 0.6.) Examples of adventurous rules that extend the mathematical content of the play are: (b) The – die shall not represent the subtraction operation; instead it shall represent the log operation. (c) The ÷ die shall not represent the division operation; instead it shall represent the imaginary number *i*.

A second method for channeling attention upon specific mathematical concepts is through variations of the Playing Mat on which the game is played. For example, instead of dealing only with equations involving the equality relationship between two expressions as in Basic EQUATIONS (i.e., $\frac{A}{\text{Solution}} = \frac{B}{\text{Goal}}$), by changing the Playing Mat, the game can be generalized to emphasize the other relations. A Playing Mat with $\underline{A} < \underline{B}$ would emphasize the "less than" relation, while a Playing Mat with $\frac{A}{B}$ is one-half of $\frac{C}{D}$ would focus attention on the concept of ratio and proportion by means of the "is one-half of" relation. A Playing Mat like the one shown would structure a variant of EQUATIONS to focus attention upon the concept of place-value by allowing players in moving the dice to require or permit other players to use numbers with various place-values.

	Forbidden	Permitted	Required
Tenths Place			
Units Place			
Tens Place			
Hundreds Place			

A third method for channeling attention upon specific mathematical concepts is through puzzles associated with various stages of play in the game. For example, the idea of dividing by a fraction would be emphasized by the following puzzles. Using just one die from the Resources, as many as you wish from Permitted, all of those that are Required, and none of those from Forbidden, construct a Solution that is equal to the Goal without using any multiple-digit numbers.

Resources	*Permitted*	*Required*	*Forbidden*
+ x ÷	3 5 7	1 4	9
− 2 8	÷ 6 2		
	______ Solution	= 1 2 Goal	

Such puzzles can be used to stimulate thought in the classroom in a variety of ways: as a cooperative activity for the teams, as remedial work for individuals, or as homework or otherwise for the entire class.

A fourth method for channeling attention upon specific mathematical ideas, one already known to be effective for increasing mathematics achievement, is play of EQUATIONS by individuals using the IMP (Instructional Math Play) Kits. The kits are pamphlet-simulations of a highly branched (usually involving thousands of alternative pathways), computer-assisted instruction program, each designed to make moves in such a way as to direct the user to consider a specific mathematical idea. (For a more detailed description of the IMP Kits, see Allen and Ross, 1975). For example, in the following situation by moving the + die to Forbidden the IMP Program would extinguish the obvious 4 + 5 Solution-Possibility and thereby ultimately lead the learner to consider a Solution involving the subtraction of a negative number (the idea it was designed to teach):

Resources	*Goal*
+ − − 2 4 5 6	9

Experience with the IMP Kits clearly suggests the importance of channeling attention on specific content as well as the frequency and immediacy of feedback as factors contributing to the effectiveness of mathematics learning.

A fifth method for channeling attention upon specific mathematical ideas—in this case upon the more complex ideas that may arise in the course of play—is a method that there is very little experience with as of yet; it is a variant of the basic game called the "Snuffing" version. In this version a player can gain a bonus point on his or her turn by (a) writing out a Solution-Possibility publicly for all the other players to see and (b) making a move that "snuffs out" (extinguishes) that Solution-Possibility. The introduction of the snuffing rule increases the feedback possibilities enormously among the players about Solutions involving different mathematical ideas. Instead of showing each other a Solution only once—at the end of the play of a game when one of the players has the burden of proof as is the case in Basic EQUATIONS—in the Snuffing version there is incentive for players to show a Solution to each other on every turn. It also should have the effect of getting rid of the Solutions that involve only relatively easy ideas early in the play of a game and gently nudge the players in the later play to explore more subtle Solutions involving ideas that are more complex. Another effect of the Snuffing version with its writing out of Solutions on every move is that it seems to slow down the pace of play, unless some compensating adjustment is made. The pace of a play can be reenlivened by having each player make two moves when it is his or her turn, instead of just one. Still another effect of the Snuffing version, unless compensated for, is to decrease the importance of the play at the end of the game; the regular scoring for achievement in sustaining the burden of proof by constructing a Solution is swamped by the bonus scoring. It was in response to this difficulty that the 4+ Scoring Method was devised; it is a sixth method for channeling attention upon specific mathematical concepts and the method that is investigated in two intact-classroom experiments in this study. In general it is a technique for providing incentive to learn more mathematical ideas by making the players' scores depend upon the quality of the solutions offered: the more complex the ideas used in a Solution, the higher the score. The 4+ Scoring Method is described in detail in the next section. There are undoubtedly many more methods for channeling attention upon specific ideas in conjunction with EQUATIONS; imaginative teachers have already devised a number of them, and many more are likely to be forthcoming.

METHOD

SUBJECTS

The samples in these two experiments consisted of 37 students from two eighth-grade classes at Clague Junior High School in Ann Arbor, Michigan (a suburban university community), and 47 students from two seventh-grade classes at Pelham Middle School in inner-city Detroit. They were all the students enrolled in the four classes for the full year for whom both pretest and posttest scores could be obtained. The Pelham classes were heterogeneous; the students were not selected in any way. At Clague the top 15% of the eighth graders are encouraged to enroll in an algebra class, rather than in one of the regular mathematics classes. Other than this, the two regular mathematics classes at Clague in the study were hetergeneous and the students unselected.

EXPERIMENTAL CONDITIONS

There was an experimental class and a control class chosen at each school before pretests were administered. At each school the same teacher taught both classes. Both had experience in using an EQUATIONS tournament in their regular mathematics classes: the Detroit teacher, three years' experience; the Ann Arbor teacher, one year. The control classes engaged in a regular

EQUATIONS tournament once a week throughout the school year and had regular classes the other four days. The experimental classes also engaged in an EQUATIONS tournament once a week and had regular classes the other four days, but they used the 4+ Scoring Method rather than the regular scoring method in their tournaments. Thus the only known difference between the two classes at each school was the method of scoring in the EQUATIONS game used in their once-a-week tournament play. The teachers were the same, the amount of tournament play was the same, the amount of regular classroom time was the same, the mode of assignment of students to the classes being compared was the same, and the average pretest scores were not significantly different.

In the regular scoring for EQUATIONS the number of points that a player receives in the play of a complete game depends upon (a) whether he or the other players sustain the burden of proof that there is a Solution and (b) whether he is the Challenger, the Mover, the Joiner, the Declarer, or an Other player. The number of points regular scoring ranges from -1 to 2. Regular scoring is summarized in Table 1.

The number of points received by a player in no way depends upon the characteristics of the Solution that he offers. The only thing that matters is that it is a Solution; it does not matter how simple or how sophisticated the mathematical ideas are that the Solution exemplifies.

The 4+ Scoring Method is just the opposite in this respect. A player's score is highly dependent upon the characteristics of the Solution that he or she offers. A player can obtain up to four bonuses, depending upon how "interesting" the Solution offered is—hence, the name: 4+ Scoring. What makes a Solution "interesting" is the set of mathematical concepts that it exemplifies. In the 4+ Scoring used in this study there are 23 different concepts for which bonuses may be obtained. For example, a player gets a bonus of 1 point if his Solution uses the concept of exponentiation; 2 points if it uses the concept of negative number; 3 points if it uses the concept of division by a fraction; 4 points if it uses the concept of a root of a fraction;

TABLE 1
Regular (R) Scoring

Points	For What
2	Win by Challenger, Mover or Mover-Joiner.
1	Win by Challenger-Joiner or any player who builds a Solution in a Force-Out.
0	Loss by any player or failure of a player other than the Force-Out Declarer (an Other player) to build a Solution in a Force-Out.
-1	Failure of the Force-Out Declarer to build a Solution in a Force-Out.

and 5 points if it uses the concept of negative exponent. Thus a Challenger who offered the Solution 4 ÷ [3*(6-7)] for a Goal of 12 would receive four bonuses (the maximum number) totaling 11 points, and his total score for that play of the game would be 13 points in accordance with the following 4+ Scoring formula:

4+ Score = (Regular Score) + (Up to 4 Bonuses)

$S_4 = R + B_4$

4 bonuses	2	Negative number (6-7)
	5	Negative exponent (3*(6-7)
	1	Exponentiation (*)
	3	Division by a fraction (4÷[3*(6-7)])

13 = 2 + 11

(In Equations the concept of exponentiation is denoted by an asterisk. For example 3*2 = 3^2 = 3 x 3 and 2*4 = 2^4 = 2 x 2 x 2 x 2.) It is possible for a player to get up to 22 points on a complete play of a game of EQUATIONS; to get the maximum score, he or she would need to build a Solution that used all four of the 5-point concepts. Of the 23 concepts for which bonus points are available, five are 1-point concepts, five are 2-point concepts, five are 3-point concepts, four are 4-point concepts, and four are 5-point concepts. All 23 concepts and their point value, as well as detailed exemplification of each, are presented in Appendix B.

It is perhaps worth emphasizing that there is nothing special about the particular 23 ideas that are presented in the version of 4+ Scoring used in this study. The number of concepts for which bonus points will be awarded, which concepts, and what point value can all be varied in accord with what the classroom teacher thinks is important and wants to focus attention upon by the scoring. Thus 4+ Scoring is a highly flexible means for channeling attention upon whatever mathematical concepts a teacher judges to be worth emphasis.

DEPENDENT VARIABLES

The effects of the learning experiences of the experimental classes and the control classes were measured by four specially constructed series of pretests and four posttests. The first pair of tests were the same type of C-tests and R-tests used in the IMP Kit studies; the latter consisted of 21 extremely difficult R-type problems involving the 21 concepts presented in the IMP Kits plus four "throw-away" items that were not counted in the scoring. There is close correspondence between these 21 concepts and what is emphasized in the 4+ Scoring used in this study; 18 of the 21 are concepts for which bonuses were given. An example of R-type problem is the following:

Column A	*Column B*	*Column C*	*YES*	*NO*
(R) - - 1 3 6	8	....	....	....

By writing an X in the YES or NO column, indicate whether or not all of the characters in Column A can be appropriately ordered and grouped (inserting parentheses wherever necessary) to form an expression equal to the number in Column B. If your answer is YES, write that expression in Column C.

An example of a C-type problem involving exactly the same concept is the following:

(C) 6-(1-3) 8 ?

All that is involved in the C-type problem is computation, i.e., evaluating an expression. However, with the R-type problem, not only is computation involved, but the problem-solver also struggles with one aspect of "applying" an idea; he or she must recognize somehow the relevance of the concept of subtracting negative numbers for solving the problem. In general, when the R-type and the C-type problems involve the same concepts, the R-type seems to be about three times as hard as the C-type; students and teachers get about one-third as many R-type problems correct as they do the corresponding C-type problems. This first pair of tests, which had as content the 21 concepts of the IMP Kits present in C- R-type tests, is referred to in this study as the *hard* tests. The second pair of tests, which had simpler content presented in C- and R-type tests, is referred to as the *easy* tests. The content of the easy tests was the set of ideas emphasized in the seventh-grade curriculum of Detroit schools: problems involving addition, subtraction, multiplication, and division of whole numbers, fractions, and decimal expressions; ordering and grouping problems; and percentage problems. There is very little correspondence between the content of the easy tests and the content emphasized in the 4+ Scoring used in this study; only three of the 23 concepts for which bonuses were given appear on the easy tests. Thus the hard tests were content-specific to the 4+ Scoring used in this study; the easy tests were not.

Two forms of each of the four tests were administered, half of each form as pretests and the other half as posttests. All of the easy tests were administered before the hard tests at both pretesting and posttesting. The order of administering the C-tests and the R-tests and the forms for both the easy tests and the hard tests was balanced by dividing the samples into eight groups and administering as follows:

	Order of Administration	*G1*	*G2*	*G3*	*G4*	*G5*	*G6*	*G7*	*G8*
Pretest	1	C1	C1	R1	R2	C2	C2	R1	R2
	2	R1	R2	C1	C1	R1	R2	C2	C2
Posttest	1	R2	R1	C2	C2	R2	R1	C1	C1
	2	C2	C2	R2	R1	C1	C1	R2	R1

Using Ca (*a*fter) to denote the score on the C-posttest and Cb (*b*efore) to denote the C-pretest score (and similarly for the R-posttests and pretests), outcome measures of six dependent variables can be specified for the easy tests and the hard tests for each pair of classes separately and for the combination of both pairs as follows:

Cb	C pretest score
Rb	R pretest score
Ca	C posttest score
Ra	R posttest score
Cg	C gain score (Cg = Ca–Cb)
Rg	R gain score (Rg = Ra–Rb)

RESULTS

The descriptive statistics for pretests, posttests, and gain scores on both the easy and hard versions of the C-tests and R-tests for the experimental classes with 4+ Scoring and the control classes with the regular scoring are summarized in Figure 1 for the seventh-grade classes, the eighth-grade classes, and for both combined. The combined results, in which the sample sizes of the experimental and control groups are about 40 subjects as opposed to half that for the individual grades, are more interesting in showing the contrast in performance gains between the 4+ Scoring classes and the regular scoring classes.

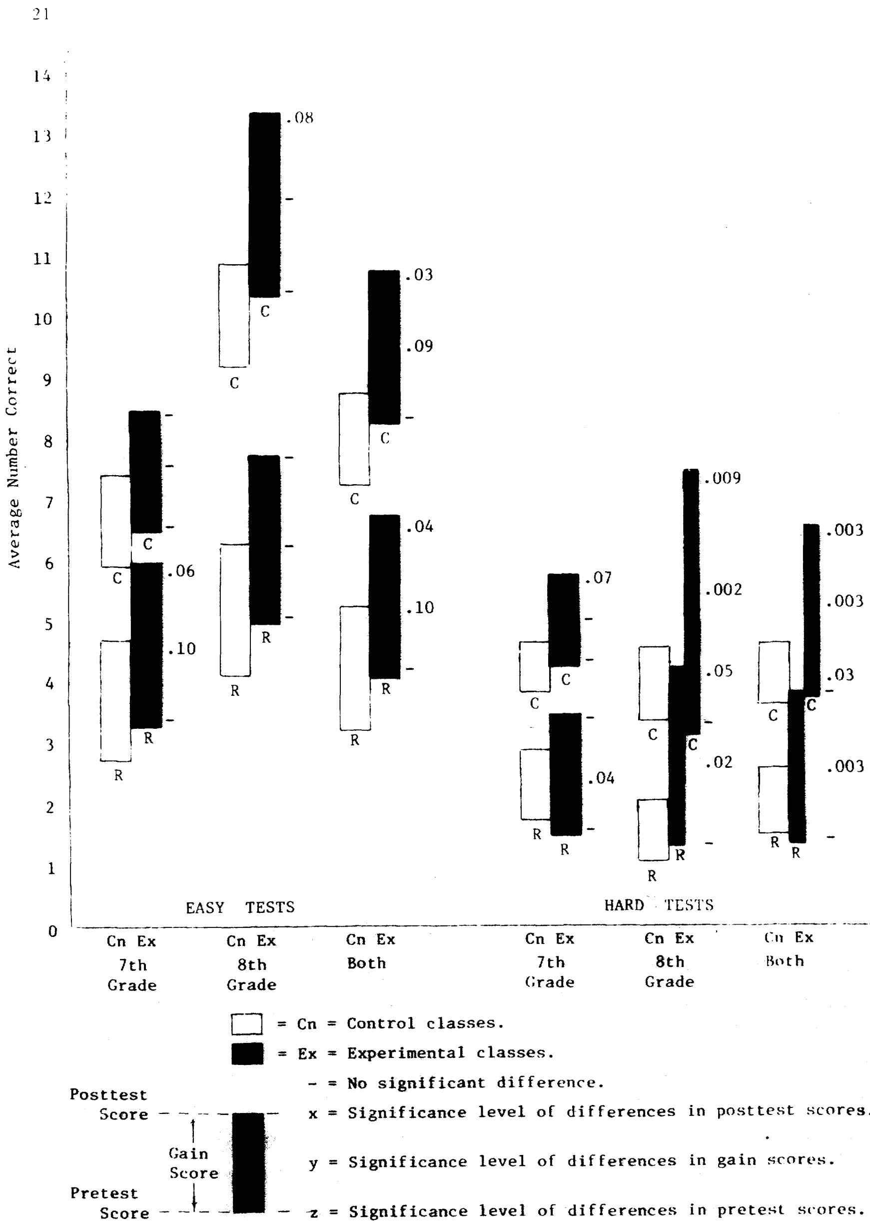

Figure 1: Differences Between Experimental (4+ Scoring) and Control (Regular Scoring) Classes on Pretest, Posttest, and Gain Scores for Both Easy and Hard Versions of C Tests and R Tests at Seventh and Eighth Grade Levels and Both Combined

A graphic representation of the mean pretest, posttest, and gain scores, as well as significant differences between the experimental and control classes on these measures, is also presented in Figure 1.

On pretest scores there were no significant differences between the experimental and control classes on either the hard tests or the easy tests for either the C-tests or the R-tests for the seventh grade, the eighth grade, or the combined classes. The Student t (one-tailed) test was used throughout to compare the performance of the 4+ Scoring classes with the control classes. Those that are significant at a confidence level of less than .01 are being characterized here as strongly significant; at less than .05, as significant; at less than .10, as weakly significant.

On posttest scores on the C-test, the scores of the eighth grade 4+ Scoring class on the easy test were weakly significantly higher (.08 level), and the scores of the 4+ Scoring classes on the easy C-posttests for both grades combined was 10.8, while for the control classes it was 8.8. On the hard C-posttests the scores of the 4+ Scoring classes were higher at each grade level and for both combined; they were weakly significantly higher (.09 level) for the eighth grade, and strongly significantly higher (.003 level) for both combined. The mean score for the 4+ Scoring classes on the hard C posttest for both grades combined was 6.6 while for the control classes it was 4.7.

On the gain scores for the C-tests the only scores of the 4+ Scoring class that were significantly higher than the control classes on the easy tests were those for the combined grades, and they were only weakly significantly higher (.09 level). However, on the hard tests, both the scores for the eighth grade and for the combined grades were strongly significantly higher (.002 and .003 levels, respectively) than the corresponding control classes. The mean gains, for which there was a significant difference on the easy tests, were 2.5 for the 4+ Scoring classes and 1.6 for the control classes. On the hard tests the mean gains for the eighth-grade classes were 4.3 and 1.3, and for the combined classes they were 2.8 and 1.0. There were no other significant differences between the experimental and control classes on the C-test scores.

On the posttest scores for the R-tests the results were similar to those for the C-tests on the easy test, except that the seventh grade had the weakly significant result rather than the eighth grade. The scores of the seventh grade 4+ Scoring class were weakly significantly higher (.06 level) than those of the control class, and the scores of the combined 4+ Scoring classes were significantly higher (.04 level) than those of the control classes. The mean scores for the seventh grade were 6.0 and 4.7, while for the combined grades they were 6.8 and 5.3. On the hard tests only two of the R-posttest differences were significant. The scores of the eighth-grade 4+ Scoring class were significantly higher (.05 level) than those of the control class, and the scores of the combined 4+ Scoring classes were significantly higher (.03) than those of the control classes.

On the gain scores for the R-tests two differences were weakly significant on the easy tests. The scores of the seventh-grade 4+ Scoring class were weakly significantly higher (.10 level) than those of the control class, and the scores of the combined 4+ Scoring classes were weakly significantly higher (.10 level) than those of the control classes. On the hard tests all three differences were significant. The scores of the seventh-grade, eighth-grade, and combined 4+ Scoring classes were significantly (.04 level), significantly (.02 level), and strongly significantly (.003 level) higher than those of the respective control classes.

The most interesting of these results, the differences in the measure of increase in performance (the gain scores), show up most clearly in the combined results for both grades summarized in Table 2. The differences in gains between the 4+ Scoring classes and the control classes on both the C-tests and the R-tests were weakly significant on the easy tests, but they were strongly significant on the hard tests that were content-specific to the 4+ Scoring.

DISCUSSION

What deserves special emphasis about these experiments

TABLE 2
Comparison on C Tests and R Tests of the Significance Levels of the Differences in Gains Between the 4+ Scoring Classes and the Control Classes on the Easy Tests and the Hard Tests

Tests	Easy	Hard
C	Weakly significant $p < .10$	Strongly significant $p < .003$
R	Weakly significant $p < .10$	Strongly significant $p < .003$

is the support they lend to the proposition that when students are involved and enthusiastic a relatively minor change can produce profound differences in the amount learned. The students in both the control and experimental classes were highly motivated in their participation in the EQUATIONS tournaments. The 4+ Scoring is a relatively minor modification of the conventional scoring method; by means of 4+ Scoring the players can obtain bonus points for an intermediate score which determines their ranking with respect to the other players in their game, which in turn determines their ultimate score for that day's play in the tournament. For the ultimate score in the game at each table, 12 points are divided among the three players: 6 to the high scorer, 2 to the low scorer, and 4 to the middle person. Thus, ranking is what is significant in determining the ultimate score, not the magnitude of the intermediate score; but where 4+ Scoring differs most from the regular scoring is merely in its effect upon the magnitude of the intermediate score, not in effect upon the ranking. Yet, this seemingly inconsequential change in the intermediate scoring (in terms of the ultimate scores of the game) produced on the tests that are of the greatest interest (the hard R-tests) gain scores in the eighth-grade 4+ Scoring classes that were nearly three times as great as those in the regular scoring classes and gains in the seventh-grade 4+ Scoring classes that were nearly double those in the regular scoring classes. The R-test provide a measure of the learner's ability to recognize the relevance of a concept for solving a problem in addition to the ability to do computations that involve the concept. This ability to discriminate what is relevant is one aspect of "applying" ideas in solving problems. That on this R-type measure the magnitude of the relative difference in achievement elicited by such seemingly minor modification of procedure is so great is indeed surprising. It seems to be grounds for optimism about the potentialities for improving learning processes by channeling attention appropriately. In the context of a recreational cognition-enhancing activity such as these EQUATIONS tournaments, attention-channeling techniques appear to be highly effective for increasing capability to discriminate what is relevant in mathematical problem-solving. They should be used more, and they deserve to be studied more.

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APPENDIX A
R1 Test

	Column A	Column B		Column C	Yes	No
1.	+ + 1 4 8	13	=			
2.	+ x 2 3 4	10	=			
3.	+ * 0 6 9	6	=			
4.	x * 1 3 7	3	=			
5.	- * 1 6 9	3	=			
6.	- - 1 3 5	7	=			
7.	÷ ÷ 1 2 4	8	=			
8.	+ * 0 5 8	6	=			
9.	+ x 1 2 3	9	=			
10.	+ √ 2 5 9	8	=			
11.	* * 2 3 7	7*6	=			
12.	+ √ 1 4 8	5	=			
13.	* √ 6 6 7	7	=			
14.	÷ √ 1 3 6	2	=			
15.	+ * 3 3 7	(3*3) x (3*7)	=			
16.	÷ * 5 5 8	(5*5) ÷ (8*5)	=			
17.	- ÷ 1 3 5	19	=			
18.	√ √ 2 5 6	$\sqrt[10]{6}$	=			
19.	+ √ 0 5 7	5	=			
20.	- * 8 9 9	1÷9	=			
21.	÷ * 2 6 6	3√6	=			
22.	- √ 5 5 6	1÷5	=			
23.	- ÷ * 2 2 3 3	6	=			
24.	÷ √ 2 8 8	8*4	=			
25.	√ √ 5 6 9	$\sqrt[(3\sqrt{3})]{5}$	=			

APPENDIX B
4 + Scoring

Score = (Regular Score) + (Up to 4 adders)

$(S = R + B_4)$

When nobody sustains the burden of proof:

S = 5R

Adder / Value	a	b	c	d	e
1	÷	*	*1	1*	More than one kind of operation
2	√	1√	√1	N/fraction	Negative number
3	0*	*0	√0	-(-)	÷(÷)
4	(÷)*	√(÷)	√*	√√	
5	*(-)	*(÷)	(-)√	(÷)√	

APPENDIX B (Continued)

Stage 4			Examples	But Not
1a.	÷	Division	6÷2, (8+4)÷4	3*(1-2)
1b.	*	Exponentiation	2*3, 3+(3*2)	2x2x2
1c.	*1	Exponent of one	4*1, 4*(3÷3)	1√6
1d.	1*	One to some power	1*3, (4÷4)*6, 1*(2÷5)	(1+4)*6
1e.	More than one kind of operation		9-(2+1), 2+(2x2)	3x2
2a.	√	Root operation	2√9, 3√(5+3)	4*(1÷2)
2b.	1√	First root	1√6, (3-2)√6	4*1
2c.	√1	Root of one	3√1, 2√(4÷4)	2√(8+1)
2d.	N/fraction (non-integer fraction)		1÷3, 5÷2, 4*(1-2)	4÷2, 0÷7
2e.	Negative number		1-5, 3*(1-3)	5-1, 2√4
3a.	0*	Zero to some power	0*6, (3-3)*5	(0+4)*3
3b.	*0	Zero exponent	3*0, 4*(2-2)	3*(2-0)
3c.	√0	Root of zero	4√0, 3√(6-6)	4√(3+0)
3d.	-(-)	Subtracting a negative number	2-(0-1)	(2-0)-1, 2-(1-0)
3e.	÷(÷)	Dividing by an N/fraction	3÷(1÷2), 5÷(5÷4)	(3÷1)÷2, 3÷(2÷1)
4a.	(÷)*	N/fraction to some power	(2÷5)*2	2÷(5*2)
4b.	√(÷)	Root of an N/fraction	2√(3÷5)	(2√3)÷5
4c.	√*	Root exponential	(4√6)*4, 4√(6*3)	(4√6)÷(3*2)
4d.	√√	Root root	2√(3√8), (2√9)√8	2√[(3√1)+5]
5a.	*(-)	Negative exponent	2*(1-3)	2*[(1-3)+4]
5b.	*(÷)	N/fractional exponent	9*(1÷2), 9*[2*(0-1)]	9*(2÷1)
5c.	(-)√	Negative root	(2-4)√9	2-(4√9)
5d.	(÷)√	N/fractional root	(1÷2)√6	1÷(2√6)

Examples of Scores

S of [2*(1-2)]√[3√(2*6)] = 2+5+5+4+4 = 20.
R a d c d

Properties 1b, 1e, 2a, 2d, and 2e are also present but a maximum of four adders can be used for the score.