RAG–PELT

Resource Allocation Games—Planned Environments for Learning and Thinking

LAYMAN E. ALLEN
University of Michigan

In any complete taxonomy of games, there should be included a category for *resource allocation games.* When the resources to be allocated in such games are symbols representing the fundamental ideas of a field of knowledge, the resulting activity can be a powerful instructional interaction. A learning environment can be designed to emphasize interacting peers creating and solving highly individualized problems for each other. In such a learning environment, when they are imbedded in games as resources to be used, ideas tend to be voraciously pursued. Players have something to do with the ideas that they are engaged in mastering; they do not merely hear them or see them expressed in print.

In this article, a priming game for one kind of resource allocation game (hereafter referred to as a RAG) will be presented. The rules that define this priming RAG will then be adapted to define a set of instructional RAGs to teach such diverse subject matters as mathematics, word structure, set theory, symbolic logic, and language structure.

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ODD 'R EVEN

The name of the priming RAG is ODD 'R EVEN, and the resources it deals with are units of space. By learning to play ODD 'R EVEN, you open the door to a way of learning that may significantly alter your ideas about learning and thinking. You will then be ready to begin learning

– mathematics by the RAG called EQUATIONS

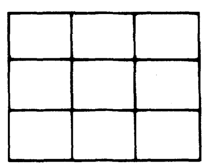
– word structure by the RAG called ON-WORDS

– set theory by the RAG called ON-SETS

– symbolic logic by the RAG called WFF 'N PROOF

– language structure by the RAG called ON-SENTS. & NON-SENTS.

The basic game of ODD 'R EVEN can be played with just pencil and paper. Two or more persons can play; three are best. First, the space units to be allocated are created by drawing a 3 x 3 matrix, just as in the game of NAUGHTS AND CROSSES. (In the United States it is called TIC TAC TOE.)



Then, in order to play ODD 'R EVEN, you need to become familiar with the following basic game ideas: goal, play, challenge, flub, solution, network, connection, joiner, burden of proof, force-out, and scoring. The first five and the last four of these ideas are used in the other five games that deal with specific subject matter.

Goal. The first player starts the game by setting the goal. He does this by writing an "O" or an "E" above the matrix. If he writes an "O," he sets an ODD goal and a solution to that goal must allow an odd number of different connections to be counted between opposite sides (top-bottom or left-right) of the matrix. If he writes an "E," he sets an EVEN goal, and then a solution must allow an even number (other than zero) of different connections.

Play. After the goal is set, the players take turns writing an "F," a "P," or an "R" in a vacant square of the matrix until someone either challenges or declares force-out.

F	P	
		R

Forbidden: An F in a square of the matrix forbids that square's inclusion in any connection.

Permitted: A P permits, but does not require, that the square appear in a connection that is counted.

Required: An R requires that the square appear in at least one connection that does not include any other R-squares.

P-squares and R-squares can be used in more than one connection that is counted.

Challenges. Although a player can write a letter or declare a force-out only when it is his turn to play, any player can at any time challenge the most recent writing of a letter. In deciding whether to challenge, declare force-out, or write a letter when it is his turn to play, a player must evaluate the previous letter written to ascertain whether or not writing it was a flub. If writing it was a flub, then the player whose turn it is to play should challenge; if he writes a letter instead, he flubs. The challenger must indicate which kind of flub he is charging.

Flubs. The writing of a letter can be a P-flub, an A-flub, or a C-flub.

P-flub. The writing of a letter is a P-flub if it [P]revents every remaining solution from being built even though P's are inserted in every vacant square.

A-flub. The writing of a letter is an A-flub if it [A]llows the building of a solution with the writing of one more letter when it was possible to write a different entry that would neither allow such a solution nor be a P-flub.

C-flub. The writing of a letter is a C-flub if the writer fails to [C]hallenge when he could have done so correctly because the previous writing of a letter was a flub. A C-flub is either a CP-flub (stemming from a prior P-flub) or a CA-flub (stemming from a prior A-flub); challengers should specify which.

Solutions. If the first player sets an ODD goal, then a network that has an odd number of counted connections is a solution. If he sets an EVEN goal, then a network that has an even number of counted connections is a solution.

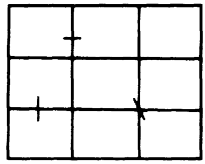
Network. A network is a matrix with some letters written in it. The following is a network with R's in squares 1 and 9, P's in squares 3 and 5, and an F in square 4.

R 1	2	P 3
F 4	P 5	6
7	8	R 9

This network can be indicated more briefly by:

R – P
F P –
– – R

Connections. A string of three touching squares filled with R's or P's stretching from the left side to the right side of a network or from the top side to the bottom side is a connection. The squares may be touching horizontally (–) or vertically (|), or diagonally (\ ).



No string that contains an F-square is a connection.

Left-right connections. A string of three touching R- or P-squares stretching from the left side to the right side of a network is a left-right connection. In the above network, 153 $(^{R}\ \mathrm{P}\ ^{P})$ and 159 $(^{R}\ \mathrm{P}\ _{R})$ are left-right connections.

Top-bottom connections. A string of three touching R- or P-squares stretching from the top to the bottom of the network is a top-bottom connection. In the above network, 159 $(^{R}\ \mathrm{P}\ _{R})$ and 359 $(\ \ \mathrm{P}\ ^{P}_{R})$ are top-bottom connections.

Required connections. Every connection that contains only one R-square is a required connection and must be counted, and every R-square must be included in at least one required connection. A solution builder who has the *burden of proof* must include all the required connections among the counted connections in the network that he offers as a solution, and he must include each R-square at least once in a required connection. In the above network, 153 $(^{R}\ \mathrm{P}\ ^{P})$ and 359 $(\ \ \mathrm{P}\ ^{P}_{R})$ are required connections, and each must be a counted connection in the solution offered.

Permitted connections. Every connection that has either 0 or 2 or more R-squares (in other words, every nonrequired connection) is a permitted connection. A solution builder may include as many of the permitted connections among the counted connections as he wishes, and he may exclude as many of them as he wishes. In the above network, 159 $(^{R}\ \mathrm{P}\ _{R})$ is a pair of permitted connections–a left-right one and a top-bottom one. The solution builder may include either one alone, or both, or neither as counted connections in the solution that he offers.

Counted connections. All the required connections must be included among the counted connections and as many of the permitted connections as the solution builder chooses. The number of counted connections must be either odd or even—as specified by the goal.

The relationship between R-squares, required connections, and counted connections can be briefly summarized by the following three rules:

(1) Required connections are those that contain one and only one R-square; all other connections are permitted connections.

(2) Each R-square must appear in at least one required connection.

(3) All required connections must be counted.

Consider these ideas in the following situations. With an even goal in this situation,

–	–	–
F	F	F
–	–	–

the writing of another F anywhere will be a P-flub, because it will be impossible to fill in the vacant squares so that a solution can be built.

With the same goal in this situation,

–	F	–
F	F	F
–	–	–

the writing of anything anywhere will be a CP-flub, because the writing of the previous letter was a P-flub.

With an odd goal in this situation,

R	–	–
–	–	–
–	–	–

the writing of a P anywhere except in 9 will be an A-flub, because after that a solution can be built—a network with one counted connection—by writing just one more letter, a P, and the writer could have avoided this by writing an F or an R.

With the same goal in this situation,

R	–	–
–	–	–
–	P	–

the writing of anything anywhere will be a CA-flub, because the writing of the previous letter was an A-flub.

With the same goal in this situation,

R	–	–
–	–	P
–	P	–

the writing of anything anywhere will be a CA-flub, because the writing of the previous letter was a CA-flub.

With an odd goal in this situation,

R	–	–
–	–	–
–	–	R

the writing of a P in either square 3, 5, or 7 is an A-flub. For example, if a P is written in 3, then by writing another P in 5 a solution with the following three counted connections can be built: $^{R}\,{}_{P}\,{}^{P}$, $_{P}\,{}^{P}_{R}$, and $^{R}\,{}_{P}\,{}_{R}$. The writer could have avoided this by writing an F or an R.

Joiners. Once a challenge is made, the role of each player in that play of the game is determined. The player making the challenge is the challenger; the player that he challenges is the writer; all other players are joiners. Each joiner must join either the challenger or the writer, whomever he believes to be correct. If a joiner joins a player who has the burden of proof, the joiner must also, independently, sustain the burden of proof.

Burden of proof. If the challenger indicates a P-flub or a

CP-flub, the writer and those who join him have the burden of proving that there is a solution by writing one. If the challenger indicates an A-flub or a CA-flub, the challenger and those who join him have the burden of proving both (1) that there is a solution when one more letter is written and (2) that the previous writer could have prevented such a solution without flubbing.

Consider the joining and sustaining of the burden of proof in this situation:

	O	
R	–	R
F	–	–
R	–	R

(The O above the matrix indicates an odd goal.) Writing an F or an R in square 8 on the next turn will be a P-flub, because it will be impossible to fill in the vacant squares so that a solution can be built. If Player A writes an R in square 8 and Player C challenges (charging a P-flub), then A is the writer, C is the challenger, and B is the joiner who must join A or C. Since the flub alleged is a P-flub, the writer has the burden of proving that there is a way of filling in vacant squares so that a solution can be built. If B joins A, then B also has the burden of proof. If B joins C, then B does not have the burden of proof. Here, B should join C in order to be correct. A will be unable to sustain his burden of proof because he will be unable to fill in the vacant squares so that the R in square 3 appears in a required connection.

In this situation,

	E	
R	–	–
–	–	–
–	–	–

the writing of a P in squares 5 or 9 on the next turn will be an A-flub, because it will be possible to build a solution by writing a P in the appropriate square, and this could have been avoided. If Player A writes a P in 5 and Player C challenges (charging an

A-flub), then C has the burden of proof. If B joins C (as he should in this case), then B also has the burden of proof. The burden of proof can be sustained by B and C by showing both

(1) that by writing a P in 9, a solution consisting of the following pair of required connections can be built: top-bottom $^{R}P_{p}$ and left-right $^{R}P_{p}$, and
(2) that A could have avoided this by writing, for example, an F in 3.

Force-out. If a player whose turn it is to play is in a situation where every possible letter that he might write would result either in a P-flub or in allowing a solution by writing one more letter, he is in what is called the force-out situation. When in this situation, he should write one of the letters whose addition will allow a solution by writing one more letter. Then the next player should declare force-out. If the next player instead challenges and declares an A-flub, he will be unable to sustain the second part of his burden of proof. When a force-out is declared, every player has the burden of proving that there is a solution with one more move, and each must prove it independently by writing a solution.

Scoring. A player scores 3 points if he is a challenger, a writer, or a joiner to the writer and he (a) has the burden of proof and sustains it, or (b) does not have the burden of proof and none of those who have it sustains it.

A player scores 2 points if
(a) he is a joiner to the challenger and

(1) he has the burden of proof and sustains it, or
(2) he does not have the burden of proof and none of those who have it sustains it, or

(b) a force-out has been declared and he sustains his burden of proof.

A player scores 1 point if he has not declared force-out and (a) he has the burden of proof and fails to sustain it, or (b) he does not have the burden of proof and some player who has it sustains it.

A player scores 0 points if he declares a force-out and is unable to sustain his burden of proof.

The winning player is either (a) the high scorer when play is for a specified period of time, or (b) the first to reach the winning score when play is to a specified winning score.

The following situation:

	E	
R	–	F
–	F	P
R	F	R

is a force-out situation. The player whose turn it is should write a P in 2 or 4, and the next player should declare a force-out. Suppose that Player A writes a P in 2 and Player B declares a force-out. Then A, B, and C each has the burden of proof. If A and B are able to sustain the burden of proof by showing that writing a P in 4 permits the solution consisting of four connections (three required connections,

R
R P P, P P, and P P, and the permitted connection, P,
R R R

and if C is unable to come up with a solution, then the scoring in this situation would be 2 points for A and B and 1 point for C.

In the following situation:

	E	
–	–	R
–	–	R
R	R	R

if A writes a P in 1, he will make an A-flub. Suppose that he does and B challenges saying that A has made an A-flub. Suppose further that C joins B and that both B and C sustain their burden of proof by showing both (1) that a six-connection solution is possible by writing a P in 5, and (2) that A could have avoided this by writing an F in 2. The scoring in this situation would be 3 points for B, 2 points for C, and 1 point for A.

The strategic possibilities in ODD 'R EVEN are enriched by adding to the goals that can be specified. The goal of B can be added, indicating a network that has both an odd number of connections that are allowed to be counted and an even number. The goals of 1, 2, 3, . . . 99 can be added, indicating a network that has *at least* (or *exactly)* that number of counted connections. In this variation, the players must specify whether they are playing the at least version or the exactly version. Further enrichment is possible by extending the size of the matrix to 4 x 4 and making the connections four squares in length. It is easy to imagine further extensions of ODD 'R EVEN, but by the time players have reached this level, they have an understanding of the games rules that make it appropriate to shift to one of the other games in which the ideas from some field of knowledge are the resources allocated in the play. Attention can now profitably turn to the RAG called EQUATIONS.

EQUATIONS

The game ideas of EQUATIONS are essentially the same as those for ODD 'R EVEN, with adaptations appropriate to the subject matter being considered. The resources of EQUATIONS are the mathematical operations and number symbols imprinted on the upward faces of a set of six-sided cubes rolled out on the table. The goal set by the first player is a number built with one to five of the resource cubes to indicate the right side of an equation. After the goal is set, play consists of the players taking turns—each allocating one of the resource cubes to one of three sections of a playing mat that are labeled "forbidden," "permitted," and "required." Flubs are defined and can be challenged, just as in ODD 'R EVEN. A solution in EQUATIONS is a number that indicates the left side of the equation and is built from the symbols available on the playing mat under the restrictions that all occurrences of symbols on the upward faces of cubes in the required section *must be used,* occurrences of symbols on cubes in the permitted section *may but need not be used,* and all occurrences of symbols on cubes in the forbidden section *must not be used.* There is no parallel in EQUATIONS to the ideas of network and connection of ODD 'R EVEN. There are, however, similar procedures for assigning the burden of proof and also rules by which the joiner joins either the challenger or the mover (the "writer" of ODD 'R EVEN) after a challenge has been made. Also, the force-out situation and the scoring are the same.

Consider the following situation in EQUATIONS in which the first player has rolled the cubes and generated these resources:

+ − x ÷ ÷ √ √ 0 1 1 2 4 4 6 8 8

Player A might then correctly perceive that he can set the goal of 16 without flubbing, and he might do so, resulting in the following situation:

Resources: + − x ÷ ÷ √ √ 0 1 2 4 4 8 8

Forbidden	Permitted	Required
Solution	=	16 (Goal)

Then it is Player B's turn. Thinking that A had in mind some relatively simple solution such as 8+8, B might try to precipitate A into making a false P-flub challenge by extinguishing the solution that B believes that A has in mind. One way that B can extinguish the hypothesized solution is by forbidding the +. Suppose that B does so, resulting in the following situation:

Resources: − x ÷ ÷ √ √ 0 1 2 4 4 8 8

Forbidden	Permitted	Required
+		
Solution =		16 Goal

Then it is Player C's turn, and he might think that B had in mind solutions such as 8 x 2 or 4 x 4. C might try to precipitate B into making a false P-flub challenge by extinguishing these solutions by forbidding the x. Alternatively, C might think that both A and B had in mind more complex solutions than these relatively simple products and that if he requires a 4 they might overlook the second of these solutions and inadvertently commit an A-flub by permitting or requiring the x or the other 4. Suppose that C's diagnosis of the other players is that they know the game rules and their multiplication tables well enough that they are not likely to fall into that trap; so he forbids the x instead, resulting in the following situation:

Resources: – ÷ ÷ $\sqrt{\ }$ $\sqrt{\ }$ 0 1 2 4 4 8 8

Forbidden	Permitted	Required
+ x		
Solution =		16 Goal

At any time during the play of the game that you as a player believe that some other player has extinguished all the solutions by some move that he has just made, you should challenge that mover for making a P-flub. Players A, B, and C should do so, also.

Now play is back to A again, and he will need to do some thinking to determine what C had in mind as a solution when he moved the x to forbidden. As he ponders it, he might hypothesize that C was thinking of the solution $8 \div (1 \div 2)$. Suppose that A extinguishes that solution by forbidding a ÷, resulting in this situation:

Resources: − ÷ √ √ 0 1 2 4 4 8 8

Forbidden	Permitted	Required
+ x ÷		
Solution	=	16 Goal

It is now B's turn, and he might be thinking that all the solutions have been prevented. But if he recalls some of his early training in algebra, he might recognize that A is a sly old devil with the solution $^{(1\div 2)}\sqrt{4}$ in mind. And if B reflects on his algebra just a little more, he just might allocate the ÷ to forbidden and extinguish what he thinks that A is thinking of as a solution. Suppose B does so, and that forbidding the other ÷ elicits a P-flub challenge from A, with C joining the challenger. B, as the mover on a P-flub challenge, of course has the burden of proof. If you were in his shoes in this situation, would you be able to sustain it?[1]

This sample game of "basic" EQUATIONS should serve to illustrate the kind of thinking about mathematics that players are engaged in when they create problems for each other and attempt to solve them in playing the game. After the basic game is mastered, they can then turn to what is called adventurous EQUATIONS. In this advanced version, the players assume a new role. In basic EQUATIONS, the players have had three roles: learners, teachers, and diagnosticians. In adventurous EQUATIONS they become games designers also. John von Neumann (and Morgenstern, 1953: 49) conceives of a game as that which is defined by its set of rules.[2] So, if players add new rules to basic EQUATIONS, they are designing a new game in this von Neumann sense. That is exactly what is done in adventurous EQUATIONS: each player adds one new game rule before the roll of the cubes, and the set of new rules added to the prior rules defines a new game.

For example, suppose that in playing adventurous EQUATIONS the players add these rules:

(a) A solution need not be equal to the goal, but if it differs then it must differ by some multiple of 7.

(b) The ÷ symbol may represent either the logarithm operation or the division operation. (Thus, (9 x 9) ÷ 3 can be interpreted either as 27 or as $\log_3 81$ (which is 4).)

(c) The $\sqrt{\ }$ symbol shall not represent the root operation. Instead, it shall represent the minimum operation.

With these three rules in the following situation, the move of the 2 to the required section is a flub.

Resources: x ÷ * $\sqrt{\ }$ 1 4

Forbidden: + – 5

Permitted: $\sqrt{\ }$ 8 ÷ 4

Required: * x 9 4 ②

Goal: 52

(The circled symbol indicates the play just made.) Do you see why? Experienced eighth-grade adventurous EQUATIONS players would.[3]

The adventurous version of EQUATIONS enables the players to steer the play into problems of enormous complexity. By their power to introduce new game rules, the players can introduce any mathematics into the game that they have imagination enough to state a rule for. That makes for a wide open game that can be challenging to any player, regardless of what he knows or how skillful he is. Like EQUATIONS, the other RAGs described here all have adventurous variations that keep them challenging to even the most knowledgeable players. One of the significant features of such RAGs is the enormous range of complexity of problems that can be generated in the course of play. The beginning versions can be mastered by elementary school youngsters, while the advanced versions can challenge the most intelligent and knowledgeable adults. This will be evident with respect to the next RAG to be considered, the ON-WORDS game.

ON-WORDS

The RAG called ON-WORDS is very similar to EQUATIONS. Since the ideas considered are different, the symbols imprinted on the cubes are different. Also, in ON-WORDS the idea of solution and its relationship to a number which is the goal are different. Otherwise, the two games are essentially the same:

> the generation of resources by rolling the cubes, the setting of the goal, the sequence of play in allocating the cubes to forbidden, permitted, or required until some player either declares force-out or challenges, the four kinds of flubs charged when challenging (P-, A-, CP- and CA-flubs), the joining, the determining of the burden of proof, and the scoring.

The symbols imprinted on the cubes in ON-WORDS are letters of the alphabet, numerals, and phonetic symbols. The numerals are used only to set the goal, the letter symbols are used to build networks of words that may be solutions, and the phonetic symbols are used to impose constraints on which networks or words qualify as solutions.

Each network of words will have a numerical value determined by the number of letters in the words of the network. If that number is equal to the goal, that network will be a solution provided that it otherwise qualifies. The numerical value of a network is the sum of the number of letters in each of the words of the network. Thus the network

```
        O
H   E   R
E
```

has the value of 7: two of its words have 2 letters, and the other has 3 (HE, OR, and HER), and 2 + 2 + 3 = 7. Each sequence of 2 or more letters from left to right and from top to bottom of a network must be a word in the language in which ON-WORDS is being played. It can, of course, be played in other languages.

In a play of basic ON-WORDS, the cubes with the phonetic symbols are not used; they are added in what is called advanced ON-WORDS. Consider the following situation in a play of basic ON-WORDS:

Resources: A I O B C F G J K N P S V W

Forbidden:

Permitted: T H X R Ⓔ

Required: M M

Goal: 9

Permitting the E is an A-flub because with just the A from the remaining resources the solution

```
H   A   M
    M   E
```

can be built. It is a network with a value of 9 (HAM, AM, ME, and ME), and it could have been avoided, for example, by forbidding the F instead of permitting the E.

In advanced ON-WORDS, the allocation of a phonetic symbol to the required section means that the solution offered must contain a word that has the sound that symbol represents; the allocation of a phonetic symbol to the forbidden section means that the solution offered must not contain a word that has the sound that symbol represents. Hence, both forbidding and requiring the same phonetic symbol would be a flub. What kind? But forbidding and requiring the same *letter* is not necessarily a flub, although it might be. In the following situation in a play of advanced ON-WORDS the allocation of the S to the forbidden section is a CA-flub:

Resources: E O U B G H J L M P T W

Forbidden: (S)

Permitted: T N A B N

Required: I N sh

Goal: 9

(Phonetic symbols are in lower-case letters. Resource letters are all upper-case. The phonetic symbol "sh" represents the sound that the letters sh express in such words as *sh*ape, wor*sh*ip, and wa*sh*. The same sound is also in the word spe*ci*al.) Instead of allocating the S, the player should have challenged the previous allocation, because it was an A-flub. With just the O from the remaining resources the solution

```
    B
N   A   T   I   O   N
    N
```

could have been built.

There is also an adventurous version of ON-WORDS in which the players introduce new rules that may compel consideration of other aspects of word structure. For example, such rules as the following might be added:

(a) Only words that are verbs are permitted in the solution.

(b) All pronouns must be in the objective case.

(c) The numerical value of a network of words may be determined by adding to its normal numerical value the numerical value of any word *within* a word in that network. For example, the word "bones" has a normal numerical value of 5. Under this rule it may have the additional values of any one of the following:

 7 (bones + on)
 8 (bones + one)
 9 (bones + ones)
 10 (bones + on + one)
 11 (bones + on + ones)
 12 (bones + one + ones)
 13 (bones + bone + ones)

14 (bones + on + one + ones)
15 (bones + on + bone + ones)
16 (bones + one + bone + ones)
18 (bones + on + one + ones + bone).

(d) An additional section called PREFIX shall be added to the playing mat. Letters may be played into this section just as they can be into other sections. Any letter or combination of letters in PREFIX may be added to a word or to the words of a network to form a solution whose numerical value is determined by the sum of all the words thus formed (with their prefixes). Any letter in PREFIX can be used as part of one or more prefixes. Any prefix can be used as part of one or more words.

The variations of ON-WORDS in the adventurous version are limited only by the imagination of the participating players. Playing any of these or even basic ON-WORDS is good preparation for the next RAG to be considered, the one called ON-SETS.

ON-SETS

ON-SETS is a RAG that deals with set theory. Hence the symbols imprinted on its cubes represent names of sets, operations on sets, and relations between sets, as well as some numeral cubes that are used in setting the goal, as in EQUATIONS and ON-WORDS. There is yet a third way of relating the solution in ON-SETS to the number that is the goal, different from the way either in EQUATIONS or in ON-WORDS. Other than that and the use of different symbols, ON-SETS is essentially the same as these other two RAGs.

There are also basic and advanced versions of ON-SETS. In basic ON-SETS, a solution is simply the name of a set that contains as members exactly as many cards as the number that has been set as the goal. The names of sets that are solutions are constructed from the symbols appearing on the cubes just as in the other games. The sets of cards named are dealt from a deck and turned up by the first player. The cards are imprinted with various combinations of colored circles (blue, red, green, and yellow). All cards with blue circles are members of the blue set (B), all cards with red circles are members of the red set (R), all cards with both red and green circles are members of the intersection of the red and green set (R∩G), all cards that have either blue or yellow circles (or both) are members of the union of the blue and yellow set (B∪Y), all cards without blue circles are members of the complement of the blue set (B′), and so on.

Basic ON-SETS employs only those symbols representing names of sets, operations on sets, and numbers. (The cubes with symbols representing relations between sets are not introduced until the advanced version.) The names of sets for the basic version are represented by colored circles on the cubes, similar to those imprinted on the cards. When the cards are turned up, defining the universe of the cards from which the solution will be offered, and the goal is set, the sequence of play proceeds as in EQUATIONS and ON-WORDS and continues until some player declares force-out or challenges. The solution in basic ON-SETS is the name of a set whose elements are equal in number to the goal. In the course of playing, the participants are compelled to think about the ideas of set theory. Consider the following situation in the play of a game of basic ON-SETS:

Universe of Cards

1	2	3	4	5	6
B				B	
R	R	R	R		R
G	G			G	G
Y		Y			Y

Resources: G B R ∩ ∪ ′

Forbidden: 4 5

Permitted: ∪ G

Required: Ⓡ

Goal: 1

The requiring of the R (red) cube is an A-flub. Do you see why? What solution, naming exactly one card from the universe of six cards, can be built with just one more symbol from the remaining resources? The needed resource is the prime sign ($'$), and the solution that can be built is R'. It names the complement of the red set, which consists of card 5 only.

In advanced ON-SETS the cubes with the relational symbols for set identity (=) and set inclusion ($\subseteq$) are added, as well as a more complex alternative that will qualify as a solution. The complex alternative consists of both the name of a set and one or more restrictive statements about sets. The relational symbols can be used to construct restriction statements about the universe cards, so that all those cards that make a restriction statement false are eliminated from the universe. Those cards that remain are the restricted universe from which a number equal to the goal must be in the set named by the other part of the solution. Because the play of advanced ON-SETS involves consideration of simple solutions, consisting of the name of a set alone as well as of complex solutions consisting of the name of a set and one or more restriction statements about sets, the players have considerably more to think about than in basic ON-SETS. The advanced version turns out to be surprisingly challenging even to players who believe themselves to be thoroughly familiar with the ideas of set theory. For those who master it, there is also an adventurous version of ON-SETS that allows players to make the play just as difficult as they can imagine rules for. On the other hand, the 33-game series of ON-SETS contains some preliminary games to basic ON-SETS that are appropriate for three- or four-year-old students who have not yet begun to read. This extremely wide range of complexity of problems that can be generated in the course of play was present in the first and most well known of all of this type of RAG, the one called WFF 'N PROOF, which will be considered next.

WFF 'N PROOF

WFF 'N PROOF is a RAG designed to teach two-valued propositional logic. There are 21 different games in the series called WFF 'N PROOF: 2 WFF games, 11 Rule-Practice games, and 8 proof games. ODD 'R EVEN, EQUATIONS, ON-WORDS, and ON-SETS are somewhat simplified versions of the proof-type games of WFF 'N PROOF. The WFF games and the rule-practice games are preliminary games meant to begin to familiarize the players with the logical ideas that they need to know in order to play the proof games competently.

The cubes used in playing WFF 'N PROOF are imprinted with six capital letters (C, A, K, E, N, and R) and six lower-case letters (p, q, r, s, i, and o) of the ordinary alphabet. These include the symbols selected by the Polish logician Jan Lukasiewiecz to represent the propositional variables and operators of two-valued logic. Popularly known as the "Polish notation," it is parentheses-free; the order of the symbols is used instead of parentheses to keep expressions syntactically unambiguous.

The goal in a proof game is a WFF (well-formed formula) that is the conclusion of a proof. A solution in the game is a pair of sets: (1) a set of WFFs that are premisses from which the goal can be deduced in a defined logical system, and (2) a set of names of rules of inference that define the logical system within which the goal can be deduced from the premisses. After the goal is set, play proceeds in the proof games of WFF 'N PROOF as in the four previous RAGs considered, players taking turns allocating resources to forbidden, permitted, and required categories until someone either declares force-out or challenges. However, the categories to which resources are allocated are slightly different and more complex in WFF 'N PROOF than in the other RAGs. Instead of forbidden and permitted sections on the playing mat, WFF 'N PROOF has corresponding sections called the permitted-premisses (PP) section and the permitted-rules (PR) section. In the previous RAGs, allocating a resource to forbidden completely forbade the use of that occurrence of the symbol in building a solution. Some occurrences of symbols cannot be completely forbidden in the play of WFF 'N PROOF–namely, those denoting the logical operators (C, A, K, E, and N). Occurrences of these symbols can only be partially forbidden. They can be forbidden as parts of the name of a rule of inference in a solution by being allocated to the PP section, and they can be forbidden as parts of premisses by being allocated to the PR section, but there is no place to allocate them in order to completely forbid their use. On the other hand, both occurrences of symbols used to denote propositional variables (p, q, r, and s) and some symbols used to denote parts of rules of inference (R, i, and o) can be completely forbidden–the former by being allocated to PR and the latter by being allocated to PP.

The third section where resources can be allocated in WFF 'N PROOF is called the essential section. Slightly stronger constraints are imposed upon resources allocated to essential than are imposed by allocations to required in the other four RAGs. Not only are essential resources required in the solution, they are also required in an essential way. What this means is that the premiss or rule name in which an essential resource appears cannot be deleted from the solution offered and the goal still be deducible from what remains. If the goal can be deduced from what remains, then the resource has not been used essentially.

Consider the following situation in a proof game in which all 11 of the rules of inference are available.

Resources:	K N
Permitted Premisses:	p i o Ⓒ
Permitted Rules:	N C K i o
Essential:	p R R o
Goal:	Arr

The allocation of the C to permitted premisses is an A-flub, because with just the N from resources the following solution can be built:

Premisses Rules

NCpp / R(Ci),R,No

The proof that Arr can be deduced from NCpp in the logical system defined by the Ci, R, and No rules is as follows:

NCpp --** Arr

1	NCpp					premiss
2	a	NArr				s
	b	1	p			s
		2	a	Np		s
			b	1	p	s
				2	p	1,R,R,
			c	Cpp		b,Ci
			d	NCpp		1,R,R,R
		3	p			2,No
	c	Cpp				b,Ci
	d	NCpp				1,R
3	Arr					2,No

The discussion above is an extremely capsulized summary of WFF 'N PROOF. It requires the better part of a school term of a class at the junior high school level for average students to work through the full 21 games and become competent in playing the final ones. Those who do, achieve a firm grasp of two-valued propositional logic developed as a formal system. WFF 'N PROOF is the fifth RAG described here so far. There is a sixth one that is currently being developed. It is called ON-SENTS. & NON-SENTS.: The Game of Sentence Structure.

ON-SENTS. & NON-SENTS.

Because ON-SENTS. & NON-SENTS. is still being developed, its description here will necessarily be tentative. Its format is more like that in ODD 'R EVEN than those in the other four RAGs: rather than allocating the resources to forbidden, permitted, or required categories, players impose these constraints on the resources. Just as the names of the other RAGs suggest their subject matters, so does ON-SENTS. & NON-SENTS. It is about sentence structure: strings of words and phrases that are sentences–and strings that are "nonsentences"–according to a given set of syntactic rules and an accompanying lexicon which constitute a small generative grammar at every level of the game.

The goal in ON-SENTS. & NON-SENTS. is a number from 1 to 99, indicating either *exactly* or *at least* the number of different sentences or nonsentences that can be constructed by appropriately constraining the resources.

The resources in ON-SENTS. & NON-SENTS. which would constitute the lexicon in a formal generative, or transformational, grammar are a list of words and phrases in categories which express grammatical relationships–noun phrase, verb phrase, determiner, pronoun, for example. Many different lists of resources are imprinted on a series of playing mats for use at different levels of the game. Each playing mat displays a set of syntactic rules, or formulas, which define sentence or nonsentence for that play of the game. Several playing mats may duplicate the same syntactic rules, but they will be matched with different sets of resources. One playing mat will result in strings that are sentences; the other will produce what we call nonsentences–recognizable but ungrammatical English. These NON-SENTS. playing mats emphasize the qualifications that must be incorporated into the evolving set of syntactic rules in order that they more accurately define grammatical English sentences. How this works should become apparent in considering the ON-SENTS. mat and the NON-SENTS. mat that follow:

ON-SENTS. Mat 3a

Summary of Grammatical Rules

Sentence ⟶ Noun Phrase + Verb Phrase

↓

{ Name / Personal Pronoun / Indefinite Pronoun }

5E
Goal

	1	2	3	4
Names				
Helen	1	2 F	3	4
Donald	1	2 R	3	4
Personal Pronouns				
he	1	2	3	4
she	1	2	3	4
Indefinite Pronouns				
someone	1	2	3	4
nobody	1	2	3	4
Verb Phrases				
is happy	1	2	3 P	4
wanted a new coat	1	2	3 (R)	4 (R)

The ⟶ indicates that what is to the left of it may be replaced by what is to the right of it and what is above it may be replaced by what is below it.

The braces { } indicate that one of the elements enclosed must be used, but you are free to choose which one.

In setting the goal of 5E, the goal setter has declared that the constraints can be so imposed on the resources that *exactly* five different sentences can be constructed.

The F under the 2 of *Helen* indicates that the word Helen is forbidden to be used twice, but that it may be used once, in constructing sentences that are part of a solution.

The R under the 2 of *Donald* indicates that the word Donald is required exactly twice in constructing the five different sentences that will make up a solution.

The P under the 3 of *is happy* indicates that the phrase 'is happy' is permitted up to three times in constructing five different sentences of a solution.

Suppose that these are the first four choices of the three players in a game of ON-SENTS. & NON-SENTS.–setting a goal of 5E, forbidding Helen-2, requiring Donald-4, and permitting is happy-3. It is again Player B's turn; if he requires the 3 of the phrase 'wanted a new coat,' he will flub. However, if he requires the 4 of the same phrase, he will not flub. Do you see why? After *wanted a new coat*-3 is required, the writing of just one more letter will allow a solution of five different sentences to be built. For example, by writing a P under 3 of the word 'someone,' a situation will result that will allow the construction of these five different sentences:

> Helen wanted a new coat.
> Donald wanted a new coat.
> Donald is happy.
> Someone is happy.
> Someone wanted a new coat.

In these five sentences 'Donald' is used exactly twice, 'wanted a new coat' is used exactly three times, and 'Helen,' 'someone,' and 'is happy' are used no more often than they are permitted to be used; so this set of five sentences is a solution.

On the other hand, writing an R under the 4 of 'wanted a new coat' is not an A-flub (or any other kind of flub), because it will still require two more letters to be written to get enough noun phrases to construct five different sentences for a solution.

After a challenge or the declaration of a force-out in ON-SENTS. & NON-SENTS., the joining, assignment of burden of proof, and scoring are exactly the same as in the other RAGs.

Now, consider the mat for the play of a NON-SENTS. version of the game. The strings of words and phrases generated by the grammar from the resources on this mat are like sentences of English, but they are ungrammatical in some respect.

NON-SENTS. Mat 3b

Summary of Gramatical Rules

Sentence ⟶ Noun Phrase + Verb Phrase

Noun Phrase ⟶ { Name / Personal Pronoun / Indefinite Pronoun }

Goal

Resources

Names				
John	1	2	3	4
Mary	1	2	3	4
Personal Pronouns				
she	1	2	3	4
it	1	2	3	4
Indefinite Pronouns				
everyone	1	2	3	4
something	1	2	3	4
Verb Phrases				
are welcome	1	2	3	4
strive vigorously	1	2	3	4

The writing of P's and R's on this mat will allow the generation of such strings of words and phrases as these:

John are welcome.
Mary strive vigorously.
She are welcome.
It strive vigorously.
Everyone are welcome.
Something strive vigorously.

The dissonance in these strings arises from lack of agreement between the noun phrases and the verb phrases: the former are singular; the latter, plural. To remedy this potentiality of generating strings of words and phrases that are not sentences, the grammar of Language Level 3 on the 3 mats will need to be qualified so that only singular verb phrases go with singular noun phrases and only plural verb phrases go with plural noun phrases. This is accomplished at Language Level 4 by introducing the bracket notation [ ] to indicate that the top element in the left bracket connects only with the top element in the right bracket, and the same with the bottom elements.

ON-SENTS. Mat 4a

Summary of Grammatical Rules

$$\text{Sentence} \longrightarrow \begin{bmatrix} \text{Noun Phrase}_{sing.} \\ \text{Noun Phrase}_{pl.} \end{bmatrix} + \begin{bmatrix} \text{Verb Phrase}_{sing.} \\ \text{Verb Phrase}_{pl.} \end{bmatrix}$$

$$\text{Noun Phrase} \downarrow \begin{Bmatrix} \text{Name} \\ \text{Personal Pronoun} \\ \text{Indefinite Pronoun} \end{Bmatrix}$$

Resources					Goal				
Singular Names					**Plural Names**				
Harry	1	2	3	4	Irishmen	1	2	3	4
Betty	1	2	3	4	Spaniards	1	2	3	4
Singular Personal Pronouns					**Plural Personal Pronouns**				
he	1	2	3	4	we	1	2	3	4
she	1	2	3	4	you	1	2	3	4
Singular Indefinite Pronouns					**Plural Indefinite Pronouns**				
everybody	1	2	3	4	–				
anyone	1	2	3	4	–				
Singular Verb Phrases					**Plural Verb Phrases**				
is sorry	1	2	3	4	are sorry	1	2	3	4
sings loudly	1	2	3	4	sing loudly	1	2	3	4

The grammar of Language Level 4 will allow the generation of such strings as the following as sentences:

Harry is sorry.
She sings loudly.
Spaniards are sorry.
We sing loudly.

But these cannot be generated as sentences:

Harry are sorry.
She sing loudly.
Irishmen is sorry.
You sings loudly.

The grammar of English will be extended and refined in the course of playing through a series of changing playing mats that gradually become more and more complex to more closely mirror the actual language. The details for ON-SENTS. & NON-SENTS. remain to be worked out, but the overall direction is clear.

The RAG called ON-SENTS. is the sixth and final one discussed here, but, before concluding, an important word of caution is appropriate about the use of games in the classroom. Sharp-edged tools can cut two ways. The aim is to arrange an environment in which learners are continuously demonstrating competence in solving problems that they regard as difficult. The feeling we want them to have is: I can do it! This will occur in the play of these games only if the players are evenly matched. The most extreme and thorough care in seeing to it that this occurs is warranted. It is not a difficult matter to arrange, and systematic procedures have been worked out to assure it does. If a teacher permits a slow student to get into active competition with two bright ones in the play of these games, what has been arranged for that student is an opportunity to display his incompetence to peers whose respect he values. More importantly, in this kind of situation he is forced to confront himself and emphasize his weaknesses and disabilities, needlessly jeopardizing his self-respect. I believe that this is an irresponsible use of competition and should be avoided. With finely honed tools like these RAGs available, it is possible to cut sharply in the other direction and to build strong feelings of confidence and competence.

CONCLUSION

In this brief survey, I have attempted to indicate that resource allocation games are an intellectually rich, diverse, and important category that should be included in any complete taxonomy of instructional games. That the resources in such games may represent the ideas of specific fields of knowledge is a fertile concept for purposes of designing instructional games. The games discussed here illustrate how such divergent subject matters as mathematics, word structure, set theory, mathematical logic, and sentence structure can be effectively embodied in resource allocation games. But these subjects do not exhaust the fields of knowledge for which effective resource allocation games can be designed. Any field which uses a set of relatively precise fundamental ideas, rich in their interrelationships, is a likely candidate to be dealt with by a RAG. To the extent that improved methods of instruction are needed in such fields, the design of an effective RAG is an alternative which deserves consideration. For many students, such RAGs can elicit an active involvement with ideas simply not achieved by more conventional methods. Such involvement may result because the learner, rather than the teacher, generates the problems to be solved. We do not fully understand all the reasons why RAGs are such compelling activities for learning; we see that they *are* compelling—without being compulsory. But whatever their complete explication may ultimately yield, resource allocation games are powerful educational media whose utilization need not and should not be delayed.

NOTES

1. All those with good memories like Player B will remember that

$$ {}^{(0-1)}\sqrt{2} = 2^{(1 \div -1)} = 2^{-1} = 1/2 $$

and will therefore come up with the solution,

$$ \left({}^{(0-1)}\sqrt{2} \right) \sqrt{4}. $$

2. He states it briefly: "The *game* is simply the totality of the rules which describe it."

3. Moving the 2 to required is a CA-flub. There has already been a prior A-flub, because only the 2 was needed to build the solution

$$\begin{aligned}(8\div 2)\sqrt{[9*(4x4)]} &= \log_2 8\sqrt{[9*(4x4)]}\\ &= \min(\log_2 8,\ [9*(4x4)])\\ &= \min(3,[9*(4x4)])\\ &= 3.\end{aligned}$$

It is because of added rule a and the fact that

$$3 \equiv 52 \atop 7$$

that the expression above is a solution.

REFERENCE

VON NEUMANN, J. and O. MORGENSTERN (1953) Theory of Games of Economic Behavior. Princeton: Princeton Univ. Press.

ERRATA

The following errors inadvertently appeared in Connie Russell's article entitled "Simulating the Adolescent Society: A Validity Study," in the June 1972 issue of *Simulation & Games.*

p. 170, line 2 should begin "sample for Coleman. . . "

p. 179, lines 28 and 29 should read "dissonant peer-value climates (23.5%). In both Coleman (1961) and . . .

p. 183, line 6 should read "was noted for subjects who played a socialite. Fifty percent of the students who play a socialite in an academic"
line 9 should read "These results may be likened to . . . "

The following equations were inadvertently omitted from Thomas H. Naylor's review essay "Up, Up and Away: World Dynamics, Prelude to Universe Dynamics," which appeared in the September 1972 issue. They should be inserted following the Definition of Variables on p. 366:

Economics: $Y_1 = A_1X_1 + B_{12}Y_2 + B_{13}Y_3 + B_{14}Y_4 + B_{15}Y_5 + U_1$

Educational: $Y_2 = A_2X_2 + B_{21}Y_1 + B_{23}Y_3 + B_{24}Y_4 + B_{25}Y_5 + U_2$

Political: $Y_3 = A_3X_3 + B_{31}Y_1 + B_{32}Y_2 + B_{34}Y_4 + B_{35}Y_5 + U_3$

Social: $Y_4 = A_4X_4 + B_{41}Y_1 + B_{42}Y_2 + B_{43}Y_3 + B_{45}Y_5 + U_4$

Environmental: $Y_5 = A_5X_5 + B_{51}Y_1 + B_{52}Y_2 + B_{53}Y_3 + B_{54}Y_4 + U_5$