This article deals with the problem of analyzing sets of ipsative variables using the common factor model. We demonstrate that the usual assumptions of the common factor model, especially the assumption of uncorrelated disturbances, are not appropriate for sets of ipsative variables. We develop a common factor model that takes into account the ipsative properties of such data and show how this model can be applied to any set of ipsative measures using the methods of confirmatory factor analysis. We then suggest that the application of this model may be useful in modeling the latent content of sets of rankings and other measures that have the ipsative property as a result of the measurement procedure. Finally, we apply the model to Kohn's measures of parental values, using sample data from the General Social Surveys.

The Factor Analysis of Ipsative Measures

DAVID J. JACKSON

National Institute of Mental Health

DUANE F. ALWIN

The University of Michigan

INTRODUCTION

Cattell (1944) initiated the use of the term "ipsative" (from the Latin *ipse*: he, himself) to refer to raw score transformations that center scores about the individual's mean. Ipsative scores were contrasted to the more common 'normative' scores, which are centered about the variable means. The ipsative transformation produces a set of scores with the property that the sum of the variables for each individual equals a constant. Perhaps the most common use of scores produced by the ipsative transformation occurs in situations where it is desirable to remove mean differences in scores among individual cases or groups of

AUTHORS' NOTE: The authors wish to thank George W. Bohrnstedt and John R. Nessleroade for helpful suggestions on an early draft of this article and Cynthia L. Sipe for computational assistance.

cases (e.g., Cunningham et al., 1977). Although, in the classical statement of the ipsative transformation, the constant is zero (because the individual's ipsative scores are deviations from his mean on the set of variables) it is now common to use the term "ipsative" to refer to any set of variables that sum to a constant for individual cases, regardless of the value of the constant (Horst, 1965: 290-291). The particular value of the constant is largely irrelevant.

It is important to note, as well, that sets of variables may occur which have the ipsative property, but not as a result of an ipsative transformation of nonipsative scores. For example, sets of variables produced from rankings and some Q-sorts have the ipsative property. The issues discussed in this article are particularly relevant to such sets of variables.

Numerous warnings exist in the psychometric literature against the application of factor analysis to sets of variables with the ipsative property (e.g. Cattell, 1952; Guilford, 1954; Tucker, 1956; Horst, 1965), but these warnings are not always heeded. In this paper we will underscore the basis for such warnings by showing that the assumptions of the common factor model, as usually stated, are inappropriate for sets of variables with ipsative properties. This is demonstrated within the framework of the development of a common factor model that can be applied to any set of variables with the ipsative property. We will show that a common factor model is defined for a set of variables that have undergone an ipsative transformation if a common factor model holds for the preipsative set, and we will show that for sets of ipsative variables that do not result from and ipsative transformation, the same model may be used under a set of assumptions regarding the response process. Finally, we will illustrate the usefulness of the model for the latter case by applying it to sample data on a set of ipsative measures of parental values

THE IPSATIVE TRANSFORMATION

Let y be a $(p \times 1)$ vector of random variables observable in some known population. The only constraints on y are that the

variables have a common metric and that the y covariance structure is positive-definite. The general form of the ipsative transformation of y is as follows:

$$x = y - 1w$$

where x is a $(p \times 1)$ vector of variables with the ipsative property, 1 is a $(p \times 1)$ unit vector and w is a scalar. In the general case, w equals $p^{-1}(1'y - c)$, where c is the constant to which the scores of each individual sum. This transformation, then, produces a set of variables that sum to c for each individual; however, the value of the constant is irrelevant.² As a matter of convenience we may set c equal to zero, so that $w = p^{-1}1'y$.

Equation 1 may be rewritten as

$$x = y - 1p^{-1}1'y$$
 [2a]

$$= y - p^{-1}11'y$$
 [2b]

$$= (I - p^{-1}U)y$$
 [2c]

where U = 11' (a $p \times p$ matrix of unities). The ipsative transformation matrix in equation 2c, $(I - p^{-1}U)$, is a symmetric idempotent matrix with a rank of p - 1 (see Horst, 1965: 288-289; Harris, 1953: 54).

Let the ipsative transformation be represented by $A = (I - p^{-1}U)$, so that x = Ay. The covariance matrix for x may be expressed using this notation as

$$\Sigma_{x} = A \Sigma_{y} A$$
 [3]

where Σ_y is the covariance matrix of the preipsative variables. As noted, A has a rank of p-1, so this constitutes a singular transformation of Σ_y . Consequently, Σ_x is singular and cannot be analyzed in the same manner that is possible for Σ_y .

The ipsative covariance matrix, Σ_x , has a number of interesting properties (see Clemans, 1956):

- (1) The rows (or columns) of Σ_x sum to zero.
- (2) The sum of the covariances of a set of ipsative variables with a criterion variable is zero.
- (3) As p increases, Σ_x approaches Σ_y .

Because the ipsative covariance matrix is not positive-definite, it may not be feasible to subject Σ_x to routine factor analysis.⁴ It is possible, however, to delete one row and column associated with an arbitrarily selected variable in order to obtain a non-singular matrix, but the routine factor analysis of this submatrix does not provide a solution to the ipsative problem.

A COMMON FACTOR MODEL FOR IPSATIVE MEASURES

In this section of the article we will develop a common factor model that can be applied to ipsative measures. As we pointed out earlier, our primary interest here is in data that have not been "ipsatized" by an ipsative transformation, but which have the ipsative property as a result of the measurement method. In order to develop a factor model that can be applied in this situation, we first must develop a factor model for variables that have undergone an ipsative transformation. To do this we assume that a factor model exists for the set of preipsative variables. Then, by transforming the coefficient matrices of this model, we may obtain the coefficient matrices of the ipsative factor model. We turn now to the development of this model.

Let

$$y = \bigwedge_{y} \xi + \epsilon$$
 [4]

be the population common factor model for the set of measured nonipsative variables, y, where \wedge_y is the factor pattern coefficient matrix, ξ is a $(k \times 1)$ vector of common factors, and ϵ is a $(p \times 1)$ vector of disturbances (see Lawley and Maxwell, 1971). For our purposes the properties of this model are as follows:

- (1) E(y) = 0
- $(2) E(\xi) = 0$
- (3) $E(\epsilon) = 0$
- (4) $E(\xi\epsilon') = 0$
- (5) $E(\xi \xi') = \Phi_y$
- (6) $E(\epsilon\epsilon') = \Psi^2_y$

For convenience we have treated all variables, latent and manifest, as centered, and we assume that ξ and ϵ are statistically

independent. The matrices Φ_y and Ψ_y^2 are the covariance matrices for the factors and disturbances respectively. Here we assume that Ψ_y^2 is a positive-definite diagonal matrix, but in general the assumption that Ψ_y^2 is diagonal is not necessary. The covariance structure of y may be written in terms of the model as:

$$E(yy') = \sum_{y} = \bigwedge_{y} \Phi_{y} \wedge \gamma' + \Psi_{y}^{2}$$
 [5]

Given the above model and knowledge of the properties of the ipsative transformation, we may obtain the ipsative common factor model by substituting equation 4 for y in equation 2b, as follows:

$$x = (\wedge_y \xi + \epsilon) - 1p^{-1}1'(\wedge_y \xi + \epsilon)$$
 [6a]

$$= (\wedge_{y}\xi + \epsilon) - 1(\bar{\lambda}'\xi + \bar{\epsilon})$$
 [6b]

where $\bar{\lambda}$ is a $(k \times 1)$ vector of the average values of the coefficients in the columns of Λ_y , and $\bar{\epsilon}$ is the average value of the residual disturbances on the model for y (see equation 4). We may rearrange equation 6b as follows:

$$x = (\wedge_y - 1\overline{\lambda}') \xi + \epsilon - 1\overline{\epsilon}$$
 [6c]

and because $(\epsilon - 1\bar{\epsilon}) = A\epsilon$, equation 6c may be rewritten as

$$x = (\wedge_y - 1\overline{\lambda}') \xi + A\epsilon$$
 [6d]

This is the common factor representation for a set of ipsative variables which have the ipsative property as a result of an ipsative transformation, given knowledge that a common factor model for the preipsative variables exists.

Note that the primary differences in the factor models for the ipsative and preipsative variables are their coefficient matrices. The factor pattern coefficient matrix for y is Λ_y , while the factor pattern matrix for x is $(\Lambda_y - 1\overline{\lambda}')$. As a result of the ipsative transformation, then, the factor pattern coefficients in the model for x are deviations of the factor pattern coefficients in Λ_y from the average factor pattern coefficients in the columns

of \wedge_{v} . The coefficient matrix relating the disturbances to the x vector is A rather than I, as is implicitly the case for y. Since A is nondiagonal, the ipsative transformation affects the disturbances by correlating them in a systematic fashion.

The common factor model for x, then, can be written as follows:

$$\mathbf{x} = [\Lambda_{\mathbf{x}} \mid \mathbf{A}] \quad \begin{bmatrix} \xi \\ \epsilon \end{bmatrix}$$
 [7]

where $\Lambda_x = (\Lambda_y - 1 \overline{\lambda}')$ and $A = (I - p^{-1} U)$. In addition to the ipsative property of x, the properties of this model are as follows:

- (1) E(x) = 0
- (2) $E(\xi) = 0$
- (3) $E(\epsilon) = 0$
- (4) $E(\xi \epsilon') = 0$
- (5) $E(\xi\xi') = \Phi_y$ (6) $E(A\epsilon\epsilon'A) = A \Psi_y^2 A = \Psi_x^2$

The covariance structure for x may then be written as:

$$E(xx') = \Sigma_x = \Lambda_x \Phi_v \Lambda_x' + A \Psi_v^2 A$$
 [8]

The above factor model for x follows from the assumptions posed from the beginning, that (a) x = Ay and (b) $y = \Lambda_v \xi + \epsilon$. Note that by the transformation of y into x, the disturbances on the factor model for x become correlated. As indicated above, the covariance structure of the disturbances in the model for x is $\Psi_x^2 = A\Psi_y^2 A$. The diagonal elements of this matrix are $\psi_{x_i}^2 = (1 - 2p^{-1}) \psi_{y_i}^2 + p^{-1} \overline{\psi}_y^2$ and the off-diagonal elements are $\psi_{x_{ij}} = -p^{-1}(\psi_{y_i}^2 + \psi_{y_i}^2 - \overline{\psi}_y^2)$, where $\psi_{y_i}^2$ and $\psi_{y_i}^2$ are the ith and jth elements in the diagonal of Ψ_{v}^{2} and $\overline{\psi}_{v}^{2}$ is the average of the diagonal elements of Ψ_{ν}^2 .

AN ILLUSTRATION

Consider the following p = 5, k = 1 factor model for some known population:

$$y = \begin{bmatrix} 1.0 \\ 1.1 \\ 1.2 \\ 1.3 \\ 1.4 \end{bmatrix} \quad \xi + \epsilon$$

$$\Phi_y = \begin{bmatrix} 2.0 \end{bmatrix}$$
Diag. $\Psi_y^2 = \begin{bmatrix} .90 & 1.2 & 1.0 & .80 & .60 \end{bmatrix}$

The population covariance structure for y given this model is:

$$\Sigma_{y} = \begin{bmatrix} 2.90 & 2.20 & 2.40 & 2.60 & 2.80 \\ 2.20 & 3.62 & 2.64 & 2.86 & 3.08 \\ 2.40 & 2.64 & 3.88 & 3.12 & 3.36 \\ 2.60 & 2.86 & 3.12 & 4.18 & 3.64 \\ 2.80 & 3.08 & 3.36 & 3.64 & 4.52 \end{bmatrix}$$

For p = 5 the ipsative transformation matrix, as defined in equation 2c, is:

$$\mathbf{A} = \begin{bmatrix} .80 & -.20 & -.20 & -.20 & -.20 \\ -.20 & .80 & -.20 & -.20 & -.20 \\ -.20 & -.20 & .80 & -.20 & -.20 \\ -.20 & -.20 & -.20 & .80 & -.20 \\ -.20 & -.20 & -.20 & -.20 & .80 \end{bmatrix}$$

The ipsative transformation of Σ_v into Σ_x is as follows:

$$\Sigma_{\mathbf{x}} = \begin{bmatrix} .80 & -.20 & -.20 & -.20 & -.20 \\ -.20 & .92 & -.26 & -.24 & -.22 \\ -.20 & -.26 & .78 & -.18 & -.14 \\ -.20 & -.24 & -.18 & .68 & -.06 \\ -.20 & -.22 & -.14 & -.06 & .62 \end{bmatrix}$$

Finally, the ipsative factor coefficent matrix is:

$$\Lambda_{x} = (\Lambda_{y} - 1\overline{\lambda}') = \begin{bmatrix} -.20 \\ -.10 \\ .00 \\ .10 \\ .20 \end{bmatrix}$$

It can be shown that $\Lambda_x \Phi_y \Lambda_x' + A \Psi_y^2 A$ exactly reproduces Σ_x .

ESTIMATION OF THE IPSATIVE COMMON FACTOR MODEL

The common factor model defined above for a set of ipsative variables bears an interesting relationship to the common factor model for the corresponding set of preipsative variables. First, it should be pointed out that the factors of the model remain the same after an ipsative transformation, and the factor covariance matrix remains unchanged. Second, under the ipsative transformation, the factor pattern coefficient matrix contains deviations from the average factor pattern coefficients in the columns of the original factor pattern matrix. Third, the ipsative transformation causes the disturbances to be correlated. These observations have implications for the factor analysis of sets of ipsative variables, and on this basis we suggest that the naive exploratory factor analysis of ipsative variables is inappropriate. Such an analysis would assume that A = I, and would therefore represent some type of an approximation. The estimated factor pattern in such an analysis would approximate $(\wedge_{y} - 1\overline{\lambda}')$, however, not \wedge_{y} , although the rank-ordering of the magnitudes of the coefficients in the two coefficient matrices would be identical. In any event, there exists a problem with the interpretation of the ipsative factor pattern coefficient matrix since the values in $\overline{\lambda}$ are unknown.

As we indicated above, Σ_x is singular, and it is not feasible to analyze this matrix without deleting a row and column associated with an arbitrarily selected variable in x. Let x^* be this set of p-1 variables. For present purposes, suppose we have deleted the p^{th} variable, although the deletion of any variable

in this vector will suffice. In fact, the parameters of the ipsative common factor model are invariant regardless of the variable deleted.

We may now write a common factor model for x* as follows:

$$x^* = [B_1 | B_2] \begin{bmatrix} \xi \\ \epsilon \end{bmatrix}$$
 [9]

where B_1 contains the first p-1 rows of the matrix $(\Lambda_y-1\overline{\lambda}')$ and B_2 contains the first p-1 rows of A. Note that B_1 is a $(p-1)\times k$ submatrix of Λ_x^* and B_2 is a $(p-1)\times p$ submatrix. In other words $\Lambda_x^*=[B_1\mid B_2]$ is a $(p-1)\times (k+p)$ coefficient matrix. The covariance matrix for $\begin{bmatrix}\xi\\\epsilon\end{bmatrix}$ in this case is as follows:

$$\Phi_{\mathbf{x}^*} = \begin{bmatrix} \Phi_{\mathbf{y}} & 0 \\ \hline 0 & \Psi_{\mathbf{y}}^2 \end{bmatrix}$$
 [10]

Recall that Ψ_y^2 is diagonal, but Φ_y may be nondiagonal.

Note that the coefficients in in B_2 are known, so they are to be constrained in the estimation of the model. Our objective, then, is to estimate the parameters in B_1 , Φ_y , and Ψ_y^2 , and they may be estimated if the model is identified. A necessary condition for the identification of the model is a minimum of k^2 independent constraints on B_1 and Φ_y (see Jöreskog, 1978; Sörbom and Joreskog, 1976). Finally, we should note that the factor pattern coefficients for the p^{th} variable may be obtained by taking the negative of the sum of the first (p-1) values in each column of B_1 .

Using the above illustration to demonstrate the manner by which the coefficients of the ipsative common factor model may be estimated, we present the k = 6 factor model that would be

estimated for x in the above 5 variable cases. The model is as follows:

$$\Lambda_{\mathbf{x}^*} = \begin{bmatrix} -.20 & .80^{\mathrm{f}} & -.20^{\mathrm{f}} & -.20^{\mathrm{f}} & -.20^{\mathrm{f}} \\ -.10 & -.20^{\mathrm{f}} & .80^{\mathrm{f}} & -.20^{\mathrm{f}} & -.20^{\mathrm{f}} & -.20^{\mathrm{f}} \\ .00 & -.20^{\mathrm{f}} & -.20^{\mathrm{f}} & .80^{\mathrm{f}} & -.20^{\mathrm{f}} & -.20^{\mathrm{f}} \\ .10 & -.20^{\mathrm{f}} & -.20^{\mathrm{f}} & .80^{\mathrm{f}} & -.20^{\mathrm{f}} & .80^{\mathrm{f}} & -.20^{\mathrm{f}} \end{bmatrix}$$

$$\Phi_{\mathbf{x}^*} = \begin{bmatrix} 2.0 & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} \\ 0.0^{\mathrm{f}} & .90 & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} \\ 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 1.2 & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} \\ 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 1.0 & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} \\ 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & .80 & 0.0^{\mathrm{f}} \\ 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & 0.0^{\mathrm{f}} & .60 \end{bmatrix}$$

Note that the coefficients in the first column of \wedge ** represent deviations of the λ_i values from the average λ_i value in the common factor model given above ($\overline{\lambda} = 1.2$). The factor pattern coefficient for the pth variable is the negative of the sum of the p-1 values in the first column of $\wedge_x *$, or .20. The remainder of the $\Lambda_x *$ matrix is constrained equal to the values there (as indicated by the "f" superscript). Note that the ϕ_{11} in the $\Phi_x *$ matrix equals Φ_y , and ϕ_{22} , ϕ_{33} , ϕ_{44} , ϕ_{55} , and ϕ_{66} in $\Phi_x *$ are the values of the residual variances for the y vector in Ψ_y^2 .

We began this illustration by defining a common factor model for y. Since the common factor model for the ipsative set of variables, x, is defined, given knowledge of the common factor model for y, the above model for x^* exactly reproduces the first (p-1) rows and columns of Σ_x^* .

AN APPLICATION OF THE IPSATIVE COMMON FACTOR MODEL

Here we will develop a set of assumptions that permit the application of the above ipsative common factor model to other

sets of variables that have the ipsative property. We make three critical assumptions: (1) that for a given set of variables with the ipsative property, e.g. a set of rankings, there exists a corresponding set of hypothetical nonipsative variables in the population; (2) that the set of ipsative variables observed in the population of interest is an ipsative transformation of the hypothetical set of nonipsative variables; and (3) that a common factor model holds in the population of interest for the hypothetical set of nonipsative variables.

Given these assumptions, we may estimate the parameters of an ipsative common factor model for any set of variables with the ipsative property. Unfortunately, there is no direct way of determining the veracity of these assumptions for the types of ipsative variables of interest, e.g. rankings, because they have not explicitly undergone an ipsative transformation. We can indirectly assess the adequacy of these assumptions using sample data from sufficiently large samples where the sample moments differ trivially from the population moments. In such cases the failure of our estimated model to reproduce the sample covariance matrix would provide some suggestion about the adequacy of the model. There is a degree of ambiguity here, however, as the failure of the model to reproduce the sample covariances may result from at least two sources: (1) the misspecification of the factor model for the hypothetical v. especially the number of factors, and (2) the misspecification of the response process: in other words, errors in the assumptions indicated above for applying the ipsative common factor model. Because there is no known exploratory factor analysis method that can be used to determine an adequate number of factors in the ipsative factor model in this case, there may be some difficulty in specifying the number of factors appropriate for the model.

In determining the goodness-of-fit of a particular application of the ipsative common factor model, the model should be compared with a baseline model that posits no common factors, as in assessing the relative fit of factor models generally. In the present case one would compare a particular model with the following model:

$$\Sigma_{x} = \Psi_{x}^{2} = A \Psi_{y}^{2} A \qquad [11]$$

The χ^2 goodness-of-fit statistic for this model can be used in the computation of a Tucker-Lewis coefficient of relative fit as a baseline model against which the fit of other models may be judged (Tucker and Lewis, 1973).

There are additional approaches to determining the adequacy of the above assumptions for modeling the latent content of sets of variables with the ipsative property which have not explicitly undergone an ipsative transformation. The first of these involves a systematic study of the response process involved in the production of such variables. What is the relation between the response process involved in ranking p objects and the response process involved in obtaining nonipsative measurements—for example, ratings of p objects along some dimension? Can the relation be described by a mathematical function such x = Av? We will not pursue this set of issues here except to note that little is known about the relationship. A second approach to assessing the above assumptions for such ipsative measures involves Monte Carlo simulation. Here it is possible to compare the factors that result from making different sets of assumptions regarding the response process.

AN ANALYSIS OF KOHN'S MEASURES OF PARENTAL VALUES

In this section of the article we apply the ipsative common factor model to Kohn's measures of parental desires for children's behavior (Kohn, 1969: 257). Kohn developed the following set of questions for measuring parental child-rearing values:

- (a) Which three qualities listed on this card would you say are the most desirable for a (boy, girl) of (child's) age to have?
- (b) Which one of these three is the most desirable of all?
- (c) All of these may be desirable, but could you tell me which three you consider least important?
- (d) And which one of these three is least important of all?
 - (1) that he has good manners
 - (2) that he tries hard to succeed
 - (3) that he is honest
 - (4) that he is neat and clean

- (5) that he has good sense and sound judgment
- (6) that he has self-control
- (7) that he acts like a boy (she acts like a girl) should
- (8) that he gets along well with other children
- (9) that he obeys his parents well
- (10) that he is responsible
- (11) that he is considerate of others
- (12) that he is interested in how and why things happen
- (13) that he is a good student.

Using this information Kohn (1969: 48) coded the responses according to the following scheme:

- 5 = The most valued of all.
- 4 = One of the three most valued, but not the most valued.
- 3 = Neither one of the three most nor one of the three least valued.
- 2 = One of the three least valued, but not the least valued.
- 1 = The least valued of all.

This measurement strategy represents a reduced-ranking procedure. Instead of ranking the 13 characteristics from most to least important, respondents are essentially asked to sort the characteristics into 5 ranked categories with a requisite number in each category. In either case, such a ranking procedure produces a set of scores with the ipsative property.

In order to use these data to relate parental values to social class, Kohn (1969: 56-58) employed the methods of exploratory factor analysis to form a weighted composite from these items. Some weighting of the item scores was obviously necessary, as the unweighted sum of the items has no variability over individuals. He reported some difficulty in inverting the correlation matrix among his measures for the purpose of computing factor scores, which would be predicted given knowledge of the ipsative property of the measures. In any event, according to Kohn (1969: 57), the exploratory factor analysis of these measures "yields one factor easily identified as self-direction versus conformity to external standards" and two additional factors. He focused his primary attention, however, on the self-direction/conformity factor.

For our purposes, we will use sample data on Kohn's measures from the General Social Surveys (GSS) of 1973, 1975, 1976 and 1978 (see Alwin and Jackson, 1980, for a complete description of the population of interest and the precise nature of the measures used in the GSS). Table 1 presents the sample covariance matrices for the subpopulations of mothers (N = 1,069) and fathers (N = 721) from the NORC-GSS surveys. The sample data from the subpopulations of mothers and fathers are analyzed separately because of the expectation of substantive differences in the coefficients of the factor model. As noted above, the rows (or columns) of the full symmetric covariance matrices for these variables sum to zero, a property of ipsative covariance matrices.

In this analysis we obtain maximum-likelihood estimates of the unconstrained parameters of the ipsative common factor model developed above (see equations 6 and 7) using confirmatory factor analysis (Sörbom and Jöreskog, 1976). The results for this model where k = 1 are given in Tables 2 and 3. We have identified the model by setting Φ_y (or ϕ_ξ) equal to unity, and because A and Φ_y contain only fixed parameters, we have estimated the 26 parameters in Λ_x and Ψ_y^2 . The estimated coefficients of Λ_y from their average, $\overline{\lambda}$. The value of $\overline{\lambda}$ is, however, unknown, but this is irrelevant, since whatever its value, the coefficients in Λ_x maintain the same relative magnitude.

The relative rankings of the pattern coefficients in the estimated Λ_x in Tables 2 and 3 conform to our substantive expectations, and these results show considerable similarity for mothers and fathers. The measures indicating a preference for self-directed behavior in children (items 12, 5, 10, 11, 6) covary in one direction with the factor, while the measures indicating conformity to external authority (items 9, 1, 4, 3, 7) covary in the opposite direction with the factor. Because the coefficients in \wedge_x are identified up to a change in sign, the direction of the latent factor is dependent on the starting values.

The pattern of coefficients reported in Tables 2 and 3 are similar, although not identical, to the results reported by Kohn for his original data and his reanalysis of the 1973 GSS data (Kohn, 1976: 540). In terms of the pattern of coefficients ob-

Sample Covariance Matrices for Kohn's Measures of Parental Values from the 1973, 1975, 1976, and 1978 General Social Surveys: mothers (N = 1069) above the diagonal and fathers (N = 721) below the diagonal TABLE 1

	13	.004	011	.003	049	061	070	083	056	.024	-,044	065	074	.479
	12	183	017	166	212	.118	058	186	034	181	.038	.011	.943	-,089
	11	074	052	034	053	035	024	059	600	085	010	.587	.013	091
	10	134	039	037	105	.025	.011	074	038	177	.583	017	003	087
	6	670.	026	071	.059	169	060	690.	033	.693	167	128	114	.080
	80	021	059	051	026	015	012	035	.388	900.	091	.011	031	091
	7	082	101	066	094	114	078	.902	035	.043	091	092	174	092
	9	051	034	024	034	012	.444	071	037	070	.007	011	031	061
	5	189	040	237	130	.858	007	130	029	218	010	.000	.056	073
	7	.061	087	.005	.665	043	058	118	033	007	037	084	193	093
١	3	007	054	.739	.039	240	019	.005	040	960	052	045	170	029
	2	038	.556	107	121	020	030	075	076	062	.039	084	034	.022
	1	.665	070	023	.062	134	084	097	058	.041	124	058	199	.041
	Variable	l. Manners	Success	Honest	Clean	Judgement	Control	Role	Gets along	Obeys	Responsible	Considerate	Interested	Studious
	Vai	1.	2.	3.	4.	.5	.9	7.	œ.	.6	10.	11.	12.	13.

NOTE: A more complete description of the variables may be found in the text.

Sample Estimates of the Ipsative Common Factor Model for Kohn's Measures of Parental Values from the 1973, 1975, 1976, and 1978 General Social Surveys: mothers (N = 1069) TABLE 2

		£13	077 ^a	.506]
		12	077 ^a	.824
		11	077 ^a	.511
		°10	077a077b077b077b077b077b077b077b077b077b077b	.555
		9	077 ^a	.516
		æ	0779 0778 0770 0773 0773 0778 0778	707
estimates	٧	۲,	077 ^a	. 983
ter est		, 9	-, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077, -, 077	.464
Parameter		5	077 ¹ 077 ² 077 ³ 077 ³ 077 ³ 077 ⁴ 077 ³ 077 ³ 077 ³ 077 ³	.780
		, 4	6,770 6,000,000,000,000,000,000,000,000,000,0	.637
		, E	077,0 077,0 077,0 077,0 077,0 077,0 077,0 077,0 077,0 077,0	.781
		εı		.600
			- 0733 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233 - 00233	[.613
	×	w	029 029 151 382 060 132 018 261 261 103 441	· ·
			7 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	(Diag.)

a. Fixed parameters χ^2 (53 df) = 174.62

Sample Estimates of the Ipsative Common Factor Model for Kohn's Measures of Parental Values from the 1973, 1975, 1976, and 1978 General Social Surveys: fathers (N = 721) TABLE 3

-.077ª i -.077ª -.077ª -.077^a -.077^a -.077^a -.077ª -.077ª .923^a -.077^a -.077ª -.077^a -.077^a -.077^a -.077ª -.077ª $-.077^{a}$.923^a -.077^a .959 -.077^a -.077^a -.077^a -.077ª .923^a -.077^a -.077ª -.077³ $-.077^{a}$ -.077^a -.077^a -.077 599 -.077^a . -.077^a -.077^a -.077^a -.077ª -.077ª -.077^a .923^a -.077^a -.077ª -.077ª -.077^a -.077 .621 -.077^a -.077ª -.077^a -.077^a -.077^d $-.077^{a}$.923 -,077ª -.077ª $-.077^{a}$ -.077^a -.077 .570 -.077ª -.077ª -.077^a -.077^a -.077ª -.077ª -,077 .923a -.077ª -.077^a -.077 -.077 -.077 . 538 -.077a -.077^a -.077ª -.077^a -.077ª -.077^a -.077" -.077 -.077 .923 -.077 -.077 -.077 Parameter estimates 1,007 -.077ª -.077^a -.077ª -.077^d -.077^a -.077ª -.077^a .482 ,423ª -.077ª -.077. -.077ª -.077ª -.077^d -.077" -,077^a -.077 -.077^a .923" -.077ª -.077^a -.077 -.077ª -.077^a -.077^a -.077 -,077 .792 -.077ª -.077 -.077" -.077" -.077^a .923" -.077" -,077 -.077 -.077^a -.077ª -,077 -.077 .728 -.077" -.9234 -.077^a -.077ª -.077° -.077^a -.077^a -.077" -.077 -.077" -.077" -.077" -.077" .833 -.077 -.077ª -.077" -.077ª .923^a -.077ª -.077ª -.077^a -.077ª -.077ª -.077'1 -.077 -.077 .659 -.077^a -.077^d .. 1077ª -.077" -.077 .280 + .923" -.077ª -.077⁴ .115 , -. 077^d -.077ª -.077 -.077 -.077 [.673 .178 - .080 .132 900. -.311 [1.000" . 340 - .169 .141 . 397 (Diag.)

a. Fixed parameters χ^2 (53 df) = 204.996

tained, the approximation to the ipsative common factor model using the methods of exploratory factor analysis in this case appears to be relatively good, although the extent to which these results can be generalized is unclear. As we pointed out in the above discussion, as p increases, the ipsative transformation matrix approaches an identity. The consequence of this is to reduce the effects of the ipsative transformation on the covariance structure. In any event, it is not clear to us how large p must be to safely ignore the effects of the ipsative property of the measures. We are reluctant to conclude from the present findings that the ipsative property of these parental values indicators may safely be ignored.

As we indicated in the preceding discussion, it is necessary to consider the overall fit of the ipsative common factor model in order to evaluate its utility. In Tables 2 and 3 we have reported the likelihood-ratio χ^2 values for evaluating the goodness-of-fit of the model presented there. Under the assumption of multivariate normality in the y vector, these χ^2 values may be used as measures of fit in large samples (see Jöreskog, 1978: 447). The χ^2 values obtained both for mothers ($\chi^2 = 174.62$) and fathers ($\chi^2 = 204.99$) with 53 degrees of freedom are statistically significant at extremely low levels of Type I error, and provide a statistical basis for the rejection of the model. It is important, however, to evaluate the goodness-of-fit of this model in light of the fit of alternative models. This is partly due to the fact that the above likelihood-ratio χ^2 statistics are partially a function of sample size and cannot therefore be interpreted unambiguously in large samples (Jöreskog, 1978: 447).

In addition, statistical measures of fit generally have meaning only within a comparative framework, and it is useful therefore propose meaningful alternatives to the present model (Jöreskog, 1978: 448). Therefore, as suggested above, we compare the results of the model presented in Tables 2 and 3 with the results of a model that posits no latent common factors for the variables; i.e., $\Sigma_x = \Psi_x^2 = A \Psi_y^2 A$. In addition, we examine a model that posits two latent factors for the observed indicators.

The model suggested here as a baseline against which to compare the present model provides a poorer fit to the sample

data for both mothers (χ^2 = 475.05) and fathers (χ^2 = 356.59), as would be expected. Using the Tucker-Lewis coefficient (TLC) to assess the relative fit of our model over this zero-factor model, a slight improvement is revealed for fathers (TLC = .361) and a moderate improvement is revealed for mothers (TLC = .636). In other words, within the set of assumptions we have made here in order to apply the ipsative common factor model to this particular set of ipsative variables, there is evidence that a self-direction/conformity factor, originally identified by Kohn (1969: 57), exists for these measures. At the same time, the evidence for this is not strong, in the sense that the model allowing for this latent factor provides only a modest improvement in fit to the sample data over a model that posits no such factor.

As noted in the above discussion, there is no direct way within the present context to evaluate the adequacy of the set of assumptions we have made in order to apply the ipsative common factor model in the present case. The relatively poor fit of our model to the sample data used here may indicate problems with this particular set of assumptions or with the complexity of the factor model assumed to hold for the v vector. In order to examine this second possibility we have estimated a 2-factor ipsative common factor model. The results for this model reflect an improved fit to the data for both mothers ($\chi^2 = 99.33$, TLC = .792) and fathers ($\chi^2 = 133.89$, TLC = .529), but the overall fit to the data in either case, as reflected in the Tucker-Lewis coefficients, is not entirely satisfactory. We suspect that the addition of factors to the model will improve the overall fit to the sample data, but it is unclear whether this would add substantially to our understanding of the applicability of the ipsative common factor model to these measures.

SUMMARY AND CONCLUSIONS

In the above presentation we have developed a common factor model for variables that have undergone an ipsative transformation, given knowledge of the factor model for the preipsative variables. The ipsative common factor model has been defined, given the existence of two facts: (1) x = Ay and (2) $y = Ay + \epsilon$. In the remainder of the article we have addressed the question of the applicability of the ipsative common factor model to sets of variables with ipsative properties which result from the measurement procedure rather than an ipsative transformation of nonipsative variables. We argued that if one is willing to assume that there exists a hypothetical nonipsative set of measures (y) corresponding to the ipsative set (x), that x = Ay, and that $y = Ay + \epsilon$, then it is possible to apply the ipsative common factor model to such measures. Unfortunately, there is no direct way to evaluate the measures' validity within this set of assumptions.

The evaluation of the applicability of the ipsative common factor model in these cases requires (1) an examination of the differences in the response processes involved in producing data with ipsative and nonipsative properties, e.g. rankings vs. ratings, and (2) the specification of plausible alternative models for sets of ipsative variables that represent both the latent content of the variables and their ipsative properties. Finally, there is also some utility in applying the ipsative common factor model to sets of ipsative variables, such as Kohn's measures of parental values, in order to obtain greater experience regarding the descriptive usefulness of the ipsative common factor model.

NOTES

- 1. Horst (1965: 287-288) refers to the ipsative transformation as the "minor transformation" or "right centering," while the normative transformation is called the "major transformation" or "left centering." When both transformations are performed, the scores are said to be "double-centered." See also Harris (1953), Tucker (1956), and Hicks (1970).
- 2. Horst (1965: 292) shows that ipsative measures that sum to the constant c for each individual will sum to zero when the x's are left-centered.
- 3. This is a function of the fact that as p increases, the ipsative transformation matrix, A, approaches an identity.
- 4. This observation may not apply to all methods of factor analysis, but methods that utilize generalized least squares (GLS) and maximum likelihood (ML) procedures to fit the model require a positive-definite sample covariance matrix (Jöreskog, 1978: 446).

REFERENCES

- ALWIN, D. F. and D. J. JACKSON (1980) "The statistical analysis of Kohn's measures of parental values." In H.O.A. Wold and K. G. Jöreskog (eds.) Systems Under Indirect Observation: Causality, Structure and Prediction. New York: Elsevier North-Holland.
- CATTELL, R. B. (1952) Factor Analysis. New York: Harper & Row.
- ——— (1944) "Psychological measurement: ipsative, normative and interactive." Psych. Rev. 51: 292-303.
- CLEMANS, W. V. (1956) "An analytical and empirical examination of some properties of ipsative measures." Ph.D. dissertation, University of Washington.
- CUNNINGHAM, W. H., I.C.M. CUNNINGHAM, and R. T. GREEN (1977) "The ipsative process to reduce response set bias." Public Opinion Q. 41: 379-384.
- GUILFORD, J. P. (1954) Psychometric Methods. New York: McGraw-Hill.
- HARRIS, C. W. (1953) "Relations among factors of raw, deviation and double-centered score matrices." J. of Experimental Education 22: 53-58.
- HICKS, L. E. (1970) "Some properties of ipsative, normative and forced choice normative measures." Psych. Bull. 74: 167-184.
- HORST, P. (1965) Factor Analysis of Data Matrices. New York: Holt, Rinehart & Winston.
- JORESKOG, K. G. (1978) "Structural analysis of covariance and correlation matrices." Psychometrika 43: 443-477.
- KOHN, M. L. (1976) "Social class and parental values: another confirmation of the relationship." Amer. Soc. Rev. 41: 538-545.
- --- (1969) Class and Conformity: A Study in Values. Homewood, IL: Irwin.
- LAWLEY, D. N. and A. E. MAXWELL (1971) Factor Analysis as a Statistical Method. New York: Elsevier North-Holland.
- SORBOM, D. and K. G. JORESKOG (1976) COFAMM—Confirmatory Factor Analysis with Model Modification. Chicago: National Educational Resources.
- TUCKER, L. R. (1956) "Factor analysis of double-centered score matrices." Research Memorandum (RM-56-3). Princeton, NJ: Educational Testing Service.
- —— and C. LEWIS (1973) "A reliability coefficient for maximum-likelihood factor analysis." Psychometrika 38: 1-10.
 - David J. Jackson is a research sociologist for the Population Research Section of the Mental Health Study Center, National Institute of Mental Health. His research interests are factorial ecology, use of census data in mental health-related research, and measurement.
 - Duane F. Alwin is Associate Research Scientist in the Survey Research Center and Associate Professor in Sociology at The University of Michigan. In addition to the quantitative analysis of measurement issues, his research interests include the sociology of education and the social psychology of inequality.