THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
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Final Report

PERFORMANCE COEFFICIENTS FOR TOWED WHEELS IN FARM SOIL

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LIST OF SYMBOLS AND TERMS

GENERAL

D = soil bed depth, in.

R = rolling resistance of wheel, lb

w = wheel load, lb

w' = actual load on wheel, lb

b = wheel width, in. (characteristic dimension of plate)

d = wheel diameter, in.

P = pressure, lb/in.^{2}

t = time, sec.

Z = sinkage, in. (axle sinkage relative to undisturbed surface)

T = torque, lb/in.

ρ = density, lb/ft^3

BEKKER SOIL VALUES

k_ϕ = sinkage parameter (frictional), lb/in.$^{n+2}$

k_c = sinkage parameter (cohesive), lb/in.$^{n+1}$

n = nondimensional sinkage parameter or index

COEFFICIENT

R/w' = nondimensional drag coefficient

Z/d = nondimensional sinkage coefficient

i = nondimensional slip coefficient

α = nondimensional shape coefficient (aspect ratio = b/d)

μ = nondimensional frictional coefficient (soil ro wheel surface)

D/d = nondimensional depth parameter

ABSTRACT

The work reported here was directed at establishing the effectiveness of performance for towed wheels in farm soil possessing both cohesion and frictional properties. The work is an extension of earlier research in which dimensionless numbers were established as performance coefficients for towed wheels in sand.

The additional dependent variables introduced by the cohesive soil increased the difficulties of establishing the dimensionless parameters. Moreover, the experiments were complicated by various conditions which caused variations between tests.

After the above problems were solved, the time remaining was insufficient for a complete series of tests to be performed. The limited data obtained, however, indicate that reproducible results can be established with the soils used in the tests.

I. INTRODUCTION

The application of dimensional analysis and similitude testing to a soil-vehicle system was outlined in previous reports (Refs. 1,2,3). In this earlier work, the tests were conducted with a typical beach sand as the soil constituent of the soil-vehicle system.

The present report covers work of a similar nature covering the same range of wheels and loads, but the soil employed was a typical Michigan farm soil having not only frictional properties as does sand but also cohesive properties.

The relationship between the model and prototype remains as developed in Ref. 1 and is given below:

$$\left(\frac{W}{d^{n+2} k_\phi}\right)_p = \left(\frac{W}{d^{n+2} k_\phi}\right)_m \qquad (1)$$

$$\left(\frac{k_c}{d\, k_\phi}\right)_p = \left(\frac{k_c}{d\, k_\phi}\right)_m \qquad (2)$$

$$\left(\frac{D}{d}\right)_p = \left(\frac{D}{d}\right)_m \qquad (3)$$

$$(\alpha)_p = (\alpha)_m \qquad (4)$$

$$(\mu)_p = (\mu)_m \qquad (5)$$

$$(n)_p = (n)_m \qquad (6)$$

$$(i)_p = (i)_m \qquad (7)$$

Also the functional relationships between dependent and independent dimensionless variables also remain as before:

$$\frac{R}{W} = f\left\{\left(\frac{W}{d^{n+2}\, k_\phi}\right), \left(\frac{k_c}{d\, k_\phi}\right), \left(\frac{D}{d}\right), (\alpha), (\mu), (n), (i)\right\} \qquad (8)$$

$$\frac{T}{Wd} = f\left\{\left(\frac{W}{d^{n+2}\, k_\phi}\right), \left(\frac{k_c}{d\, k_\phi}\right), \left(\frac{D}{d}\right), (\alpha), (\mu), (n), (i)\right\} \qquad (9)$$

$$\frac{Z}{d} = f\left\{\left(\frac{W}{d^{n+2}\, k_\phi}\right), \left(\frac{k_c}{d\, k_\phi}\right), \left(\frac{D}{d}\right), (\alpha), (\mu), (n), (i)\right\} \qquad (10)$$

The experimental work covered by this report is an attempt to conduct tests under various conditions to demonstrate, if possible, the validity of the above relations when the material on which the wheel operates is a typical farm soil having cohesion and friction.

As established in Refs. 1, 2, and 3, the dimensionless relationships were ssatisfied with the desired degree of accuracy when sand was the supporing medium.

II. PROGRAM

In order to fulfill the objectives of this investigation, the following experimental work is involved:

1. Determine with sufficient accuracy the soil value system for the material employed under conditions of various moisture contents.
2. Develop the necessary technique to restore the soil properties to a constant set of values following each wheel passage.
3. Measure the soil moisture content during each set of measurements on any one day.
4. Change the soil properties as required by the dimensionless expressions over as wide a range as possible.
5. Test wheels of various diameters, aspect ratios, loadings, and other parameters.

It is evident that the procedure first involves the establishment of a soil measuring technique to secure reproducible results and a soil treatment before the desired data for towed and powered wheels can be obtained. This report is thus divided into three further sections as follows:

Section III: Soil values, moisture content, and treatment for reproducible results.

Section IV: Additional data on soil values for various artificial soil samples such as

a. Bentonite clay
b. Mixtures of (a) with various quantities of water and glycol
c. Mixtures of (b) plus quantities of sand
d. Mixture of sand and oil
e. Farm soil treated with a Dow Chemical Company soil-fracturing agent
f. Mixtures of top soil, sand, and glass beads of varying proportions
g. Data on soil samples with plastic cover; tests for moisture retention

Section V: Obtaining the values of the dimensionless parameters.

III. MICHIGAN FARM SOIL VALUES

Considerable experimental work was involved in obtaining consistent values of the soil system. This work is outlined below.

The soil value system employed is that formulated by M. G. Bekker and expressed by the equation:

$$P = \left(\frac{k_c}{b} + k_\phi\right) Z^n \qquad (11)$$

In order to obtain the values of k_c, k_ϕ, and n, a precision bevameter described in Ref. 4 was employed.

The soil sample was contained in a box of sufficient dimensions, relative to the penetrating foot, that boundary conditions did not affect the results; similarly sufficient depth of soil (9 to 11 inches) was used to eliminate bottom effects from the tests.

Preliminary tests soon revealed that the soil, as supplied, needed a different treatment from that used for the same previously employed if reproducible results were to be obtained. At first, the difficulty was considered to be that of reproducing the same consistency, plus that of actually determining the point at which the bevameter foot first contacted the soil surface. The farm soil appeared to compact much more than sand for extremely low pressures thus introducing zero errors.

Continued work with various soil mixing techniques showed little improvement in the results. Finally, the box was completely emptied onto the floor after each test. The soil was then mixed and again placed in the box. After this process had been repeated many times, the larger soil masses were broken up, large stones were removed and a more uniform product was obtained. It was noted that the reproducibility of the test results also improved in the process until eventually excellent duplication of the load-sinkage results was achieved.

It is believed that this improvement in reproducibility was due to the elimination of fairly large stones (relative to foot size) and the breaking up of soil masses which were large relative to the measuring device. The objection can be made that the resultant soil is not typical of the type of farm soil originally postulated. The objection is valid, but one should observe that a considerable difference from the soils original state in the field already exists in the soil samples even before they have been re-worked between wheel tests. It seems that if definite results of the type desired are to be obtained for research purposes, they should be recorded for conditions that are at least reproducible with a fair degree of accuracy. Subsequently, in practice in the field, a scatter of the results must be accepted and a statistical approach must be used; that is, let the corrections and inaccuracies exist in practice, but prove that the theory is acceptable under controlled conditions.

After considerable work along the lines outlined above, it was possible to obtain P versus Z diagrams of the type shown in Fig. 1 taken with different sizes of footings on the bevameter, vis., diameters of 2", 2-1/2", and 4-1/2" for soil with a moisture content was 21.53%. Two test runs for each conditions are superimposed in each case.

Figure 1 indicates that the results have good reproducibility, an average of at least two or three such load-sinkage diagrams for each footing size were employed at all times in determining the soil values.

The moisture content of the soil was determined by heating a weighed sample of the soil in a controlled oven at 100°F for a period of at least 6 to 8 hours with continued weighing until no change of weight occurred on successive weighings.

After an acceptable procedure had been established, a series of load-sinkage diagrams was obtained from which the values of k_c, k_ϕ, and n were determined for the sample soil while the moisture content was varied from 5.8% to 22% (by weight) approximately.

A range of footing sizes was employed in these tests in an effort to ascertain the effects, if any, of different sizes upon the results obtained.

Table I records the results of tests in which three rectangular footings of the sizes indicated were used in the determination of the soil values.

Similar tests employing circular footings of 2", 2-1/2" and 4-1/2" diameter are recorded in Table II. It must be remembered that the log-log graphs plotted for P and Z from which k_c, k_ϕ, and n are determined are an average of at least two and in most cases three separate tests under similar conditions.

It will be observed that there are a number of readings where k_c became negative. This occurs mainly when the 2" and 2-1/2" diameter plates were employed and analysis of the log-log plots shows that these negative values could in some cases result from inaccuracies in the zero setting. Since the difference in diameter of the plates is small, the curves are displaced from one another by a small amount only. As a result, the zero error could easily account for negative values of k_c when that magnitude is small anyway. This explanation does not, however, meet all conditions. An example for the case of small differences is shown in Fig. 2 for a moisture content of 21.53%; here the curves for the three footing sizes 2, 2-1/2 and 4-1/2 inches are shown on one diagram. In order to achieve a positive value of k_c, the value of P for any given Z should reduce as the size of foot increases. Examination of Fig. 2 reveals that these conditions occur at the start of the curves when Z = 0.125" but by

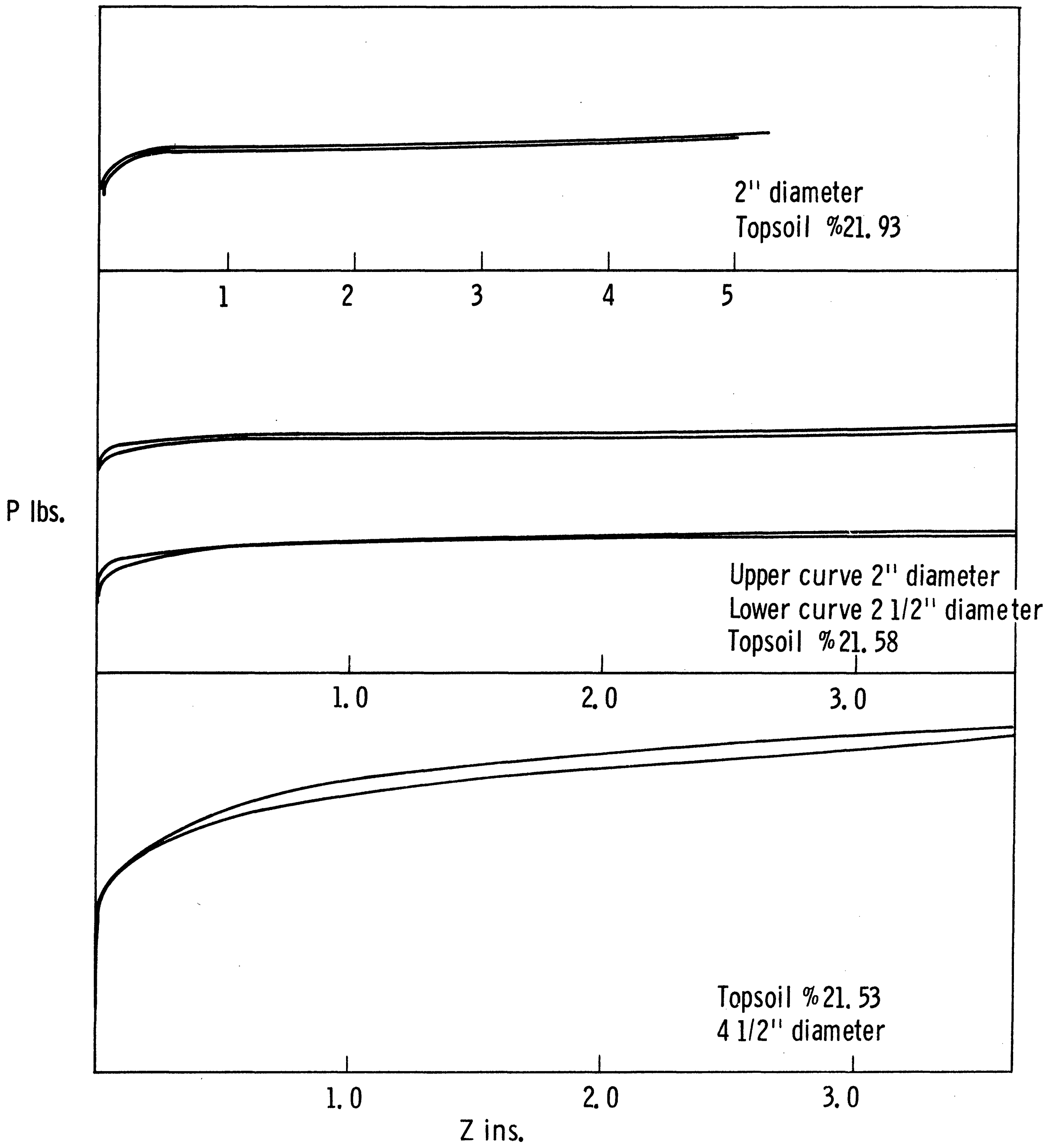

Fig. 1. Typical load-sinkage curves.

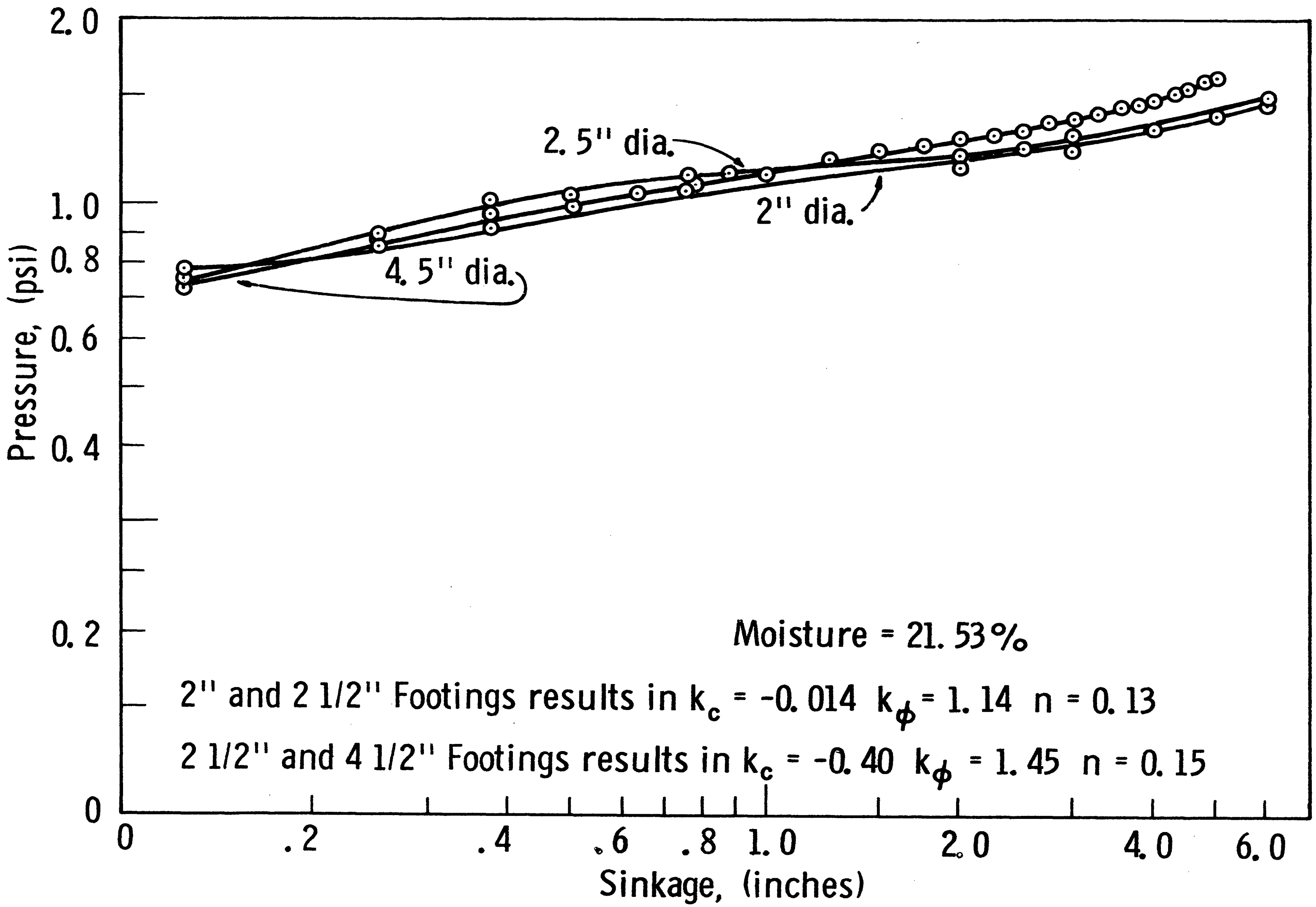

Fig. 2. Bevameter tests for moisture content of 21.53%.

TABLE I

SOIL COEFFICIENTS WITH RECTANGULAR FOOTINGS

Footing (inches)	Moisture Content (% weight)	k_c	k_ϕ	n
3/4 x 2-1/4 1-1/4 x 3-3/4	14.5	2.55	2.06	0.2
3/4 x 2-1/4 1-1/4 x 3-3/4	15.0	5.25	3.84	0.6
3/4 x 2-1/4 1-1/4 x 3-3/4	22.70	1.28	4.23	0.25
1 x 4 2 x 4	15.6	6.18	4.89	0.5
1 x 4 2 x 4	16.75	7.14	7.71	----
1 x 4 2 x 4	17.8	2.00	6.50	0.3
1 x 3 1-1/4 x 3-3/4	21.65	-1.80	9.17	0.4
1 x 3 1-1/4 x 3-3/4	24.15	1.38	6.08	0.23

the time Z = 1.0", where the values are taken to determine k_c and k_ϕ from Equation (11), it is seen that the curves have crossed one another. With the percentage of moisture in this soil (21.53%), the cohesive forces are small anyway and if k_c was zero, then theoretically the three curves should merge and become one as they do at Z = 1.0. It follows that zero and other experimental errors can easily result in plus or minus values of small magnitudes particularly when b_1 and b_2 are not greatly different. It is also seen from Fig. 2 that positive or negative values of the coefficients could be obtained by using other values of Z than 1 inch. It follows that the results obtained with the 2-1/2" and 4-1/2" footings will generally give the best results; examination of the data of Table II shows that for these two sizes of plates only one negative value was obtained and that occurred when k_c was almost zero.

TABLE II

SOIL COEFFICIENTS WITH CIRCULAR FOOTINGS

Footing Diameter (inches)	Moisture Content (% weight)	k_c	k_ϕ	n	k_c/k_ϕ
2 and 2-1/2	5.79	-0.15	1.56	0.643	0
	12.16	-0.63	2.04	0.634	---
	13.73	1.25	0.79	0.661	1.58
	14.78	2.70	0.40	0.56	6.8
	15.83	-0.60	3.22	0.574	---
	16.39	-0.55	2.62	0.533	---
	17.22	-0.20	3.95	0.372	---
	17.32	0.05	2.51	0.450	.0199
	18.44	-0.65	3.14	0.350	---
	18.60	-2.00	4.25	0.489	---
	19.55	0.05	1.91	0.157	0.0262
	19.78	2.20	1.98	0.105	1.11
	19.84	0.10	2.56	0.060	0.0374
	19.89	-0.05	1.85	0.142	---
	20.68	0.625	0.895	0.121	0.698
	20.73	0.25	1.75	0.057	0.143
	21.53	-0.40	1.45	0.134	---
	21.93	0.025	1.04	0.129	0.24
2-1/2 and 4-1/2	5.79	1.47	0.26	0.738	5.66
	12.16	1.82	0.325	0.610	5.6
	14.78	2.39	0.65	0.62	3.68
	17.32	2.39	0.64	0.593	3.74
	18.44	2.97	0.14	0.525	21.3
	18.60	2.99	0.26	0.403	11.5
	19.55	1.11	1.06	0.305	1.05
	19.89	0.58	1.34	0.200	0.433
	20.68	0.26	1.18	0.200	0.22
	21.55	-0.014	1.14	0.159	0.0

If the results with rectangular plates, given in Table I, are considered, it is seen that the k_c and k_ϕ values for any given moisture content are considerably higher than those with circular plates, but here negative k_c values exist despite a 2:1 change in the b value. It is believed that for a 2:1 change in b, a 2:1 change in l, the length of the plate, should also have been made for similarity; if this had been done, changes in k_c and k_ϕ may have resulted.

It remains to account for the difference in magnitude of the soil values between the rectangular and circular plates. Of first importance is the fact that the rectangular plates were employed in the early stages of the work before all the stones and large humps were removed or broken up; the circular plate, on the other hand, was used when the soil was in a finely divided state. It is believed that a solid lump or stone near the edge of the rectangular fooring in the early work could change k_c and k_ϕ considerably particularly since "b" is only from 3/4" to 2". Whether this provides the complete explanation has yet to be proved.

Subsequent checks with circular plates, with the soil in its final condition, gave the results recorded in Table III. It is seen that despite careful work on the maintenance of equipment, duplication of methods, etc., variation of properties still exist as foot size varies. There does not seen to be any consistent pattern in the variation. More tests under similar conditions would permit a statistical approach to the problem if this proved necessary.

TABLE III

FINAL SOIL TESTS WITH CIRCULAR FOOTINGS

Footing Size-Diameter (inches)	Moisture Content (% weight)	k_c	k_ϕ	n	k_c/k_ϕ
2 and 2-1/2	8.15	0.47	0.67	0.95	0.70
	11.85	0.40	0.83	0.95	0.48
	12.16	1.25	0.80	0.66	1.56
	12.19	1.87	1.0	--	1.87
2 and 4-1/2	12.2	1.41	1.45	--	0.94
2-1/2 and 4-1/2	12.2	1.15	1.57	--	0.732
2 and 2-1/2	9.9	1.2	2.22	--	0.54
2-1/2 and 4-1/2	9.9	2.67	1.04	--	2.56

The values of n are not subject to so wide a variation as k_c and k_ϕ; the slope of the curves are almost unaffected by the zero errors.

In Table III, the occurrence of negative values of k_c is reduced and the results are more uniform, but there is a difference between the rectangular and circular plates, with values for the latter seeming to be more consistent. With the omission of only two or three tests of Tables I, II, and III, the k_c curve of Fig. 3 can be plotted for both rectangular and circular plates. The k_c/k_ϕ curve is drawn on the basis that $k_c = 0$ when the moisture is zero. This is believed to be the approximate case since the soil then behaves and looks very similar to sand with little or no cohesion.

The most difficult curve to fit among all the points is that of k_ϕ for rectangular plates. If the results of the circular plates are plotted separately, a much better fit can be obtained, as is seen in Figs. 3, 4, 5, 6 and 7.

Fig. 8 gives the variation of the parameter k_c/k_ϕ with moisture.

The plotted points for the coefficients in Fig. 3 do not indicate that the results were very reproducible. It must be remembered, however, that there was a change in soil properties as tests proceeded due to the reasons given above. Let us look a little closer at the individual tests; take the graph for the 14.78% moisture content as an example (see Fig. 8). In this case the log-log plots for footings of 2" to 2-1/2" and 4-1/2" diameter are shown and in Table II are listed the soil values determined for the 2" and 2-1/2" diameter plates and for the 2-1/2 and 4-1/2" plates by employing the straight lines as shown in full averaging the readings from about 0.3" to 3" sinkage. Such a line conforms well for the 2" and 2-1/2" plates but the plot for the 4-1/2" plate is more curved than the others; hence the slope of the curve changes greatly and a straight line departs from the plotted points to a greater extent. This si suspected to be a boundary condition. In work reported in Ref. 5, the effect of soil depth above the bottom of the soil container was investigated and, for a 2-1/2" plate penetrating sand, a depth of sand of about 9" appeared necessary if a rapid departure from the theoretical log-log plot was not to occur until penetration reached about 4". By the same ratio for a 4-1/2" diameter foot, a soil depth of about 12" would be the minimum required for a satisfactory plot for about 3" to 4" sinkage. In the tests, however, an average depth of about 10" was employed (7-1/2" tank).

In view of this, an examination of all of the plots of P versus Z was carried out and the 4-1/2" plate was found to have the same form as that shown in Fig. 9 for all moisture contents from 6% to about 17%. Above this moisture content, the curvature was greatly reduced until at about 19% the typical straight line log-log plot existed; in fact the P-Z graph, as is seen in Fig. 10, has become almost typical of an ideal cohesive soil with $k_c = 0.10$ and P almost constant. In this case it follows that k_ϕ should be approaching zero; its actual test value was 2.56, a value arising from the small displacement of the two curves for the two plate sizes.

Under such moisture and soil conditions, which allow almost plastic deformation, it is believed that the soil depth, within reasonable limits, loses some of its effect. This was shown by the curves at high moisture content where the curvature discussed above was eliminated.

Examination of the graph in Fig. 7 indicates a fairly constant value for

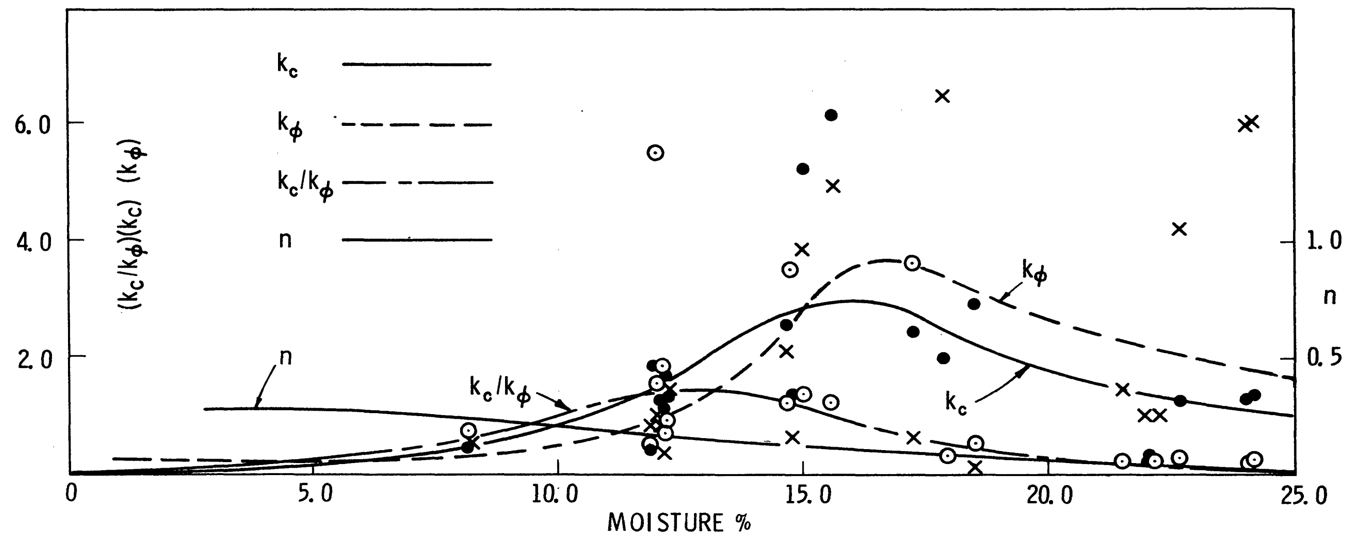

Fig. 3. Values of soil coefficients for both rectangular and circular plates.

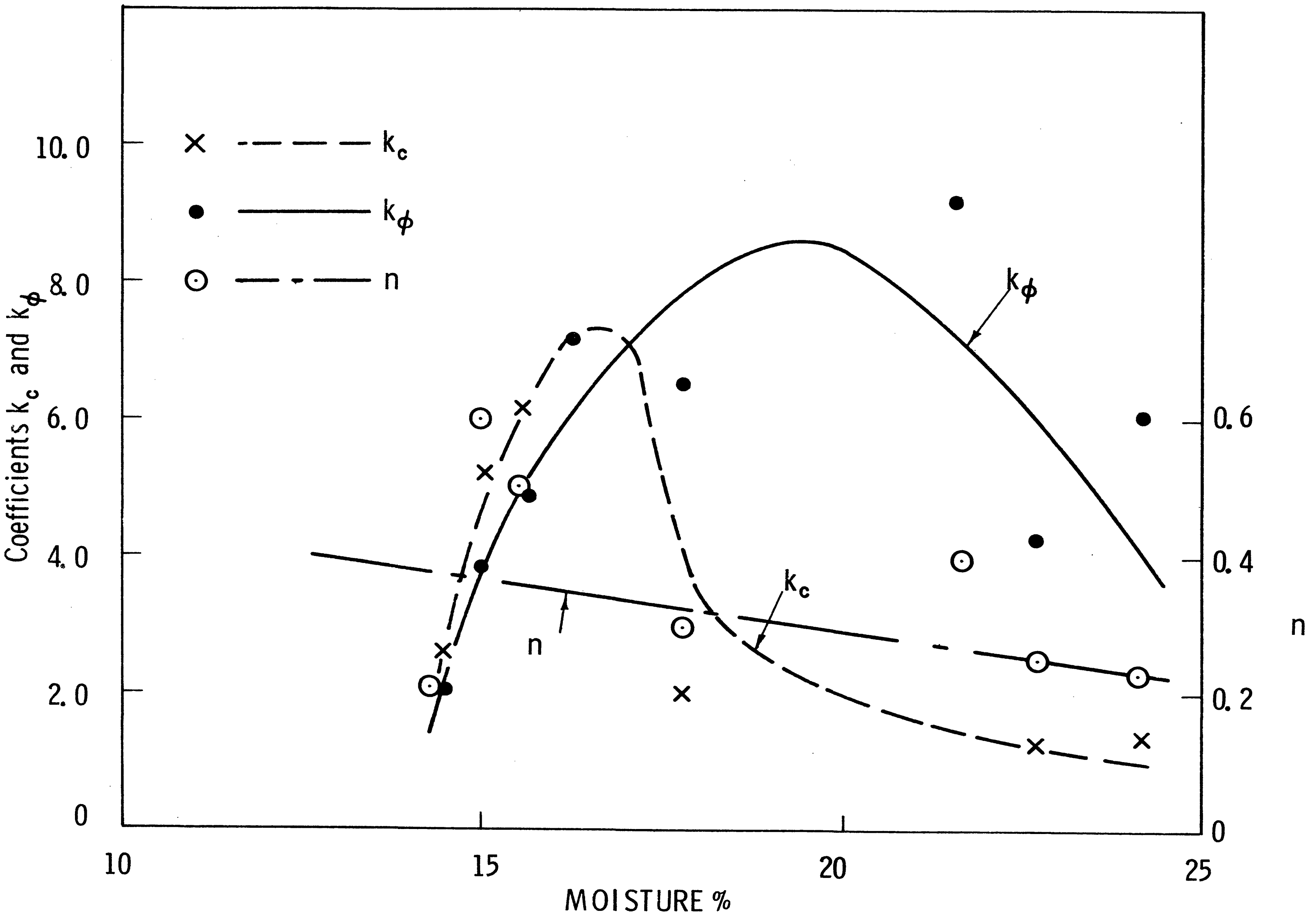

Fig. 4. Effect of moisture content on soil properties.

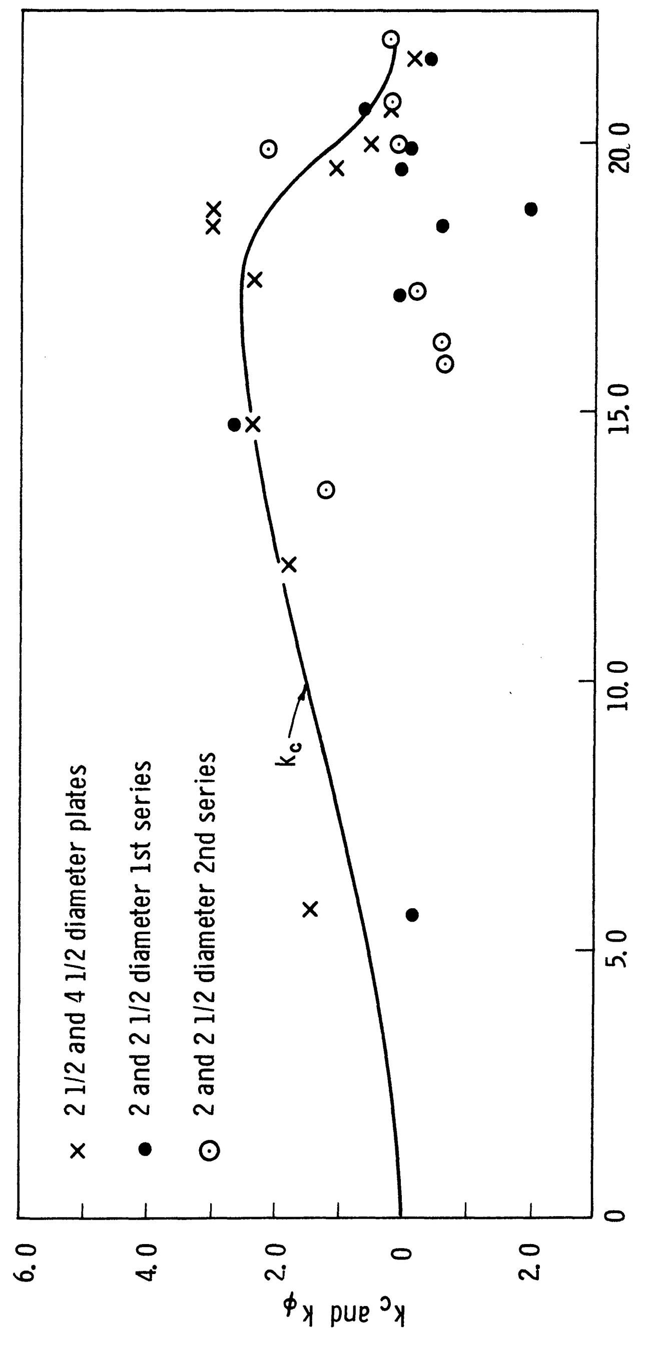

Fig. 5. Values of k_c for circular plates.

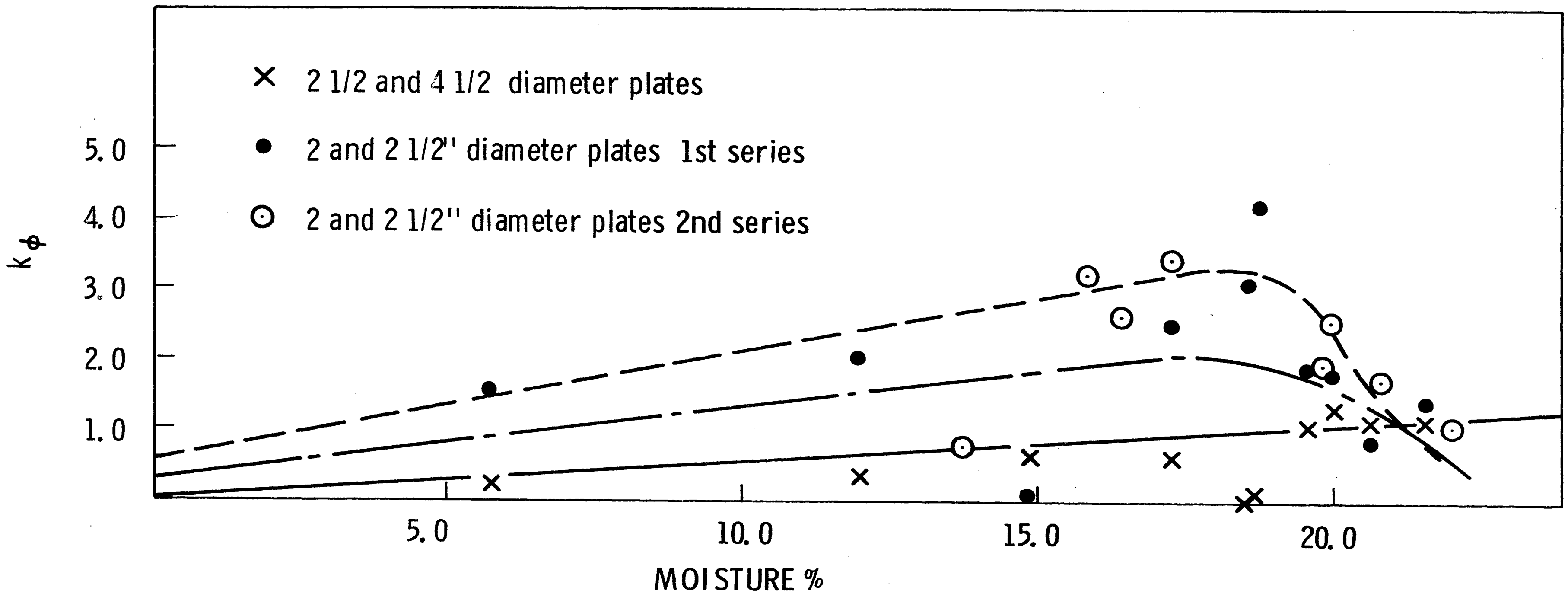

Fig. 6. Values of k_ϕ for circular plates.

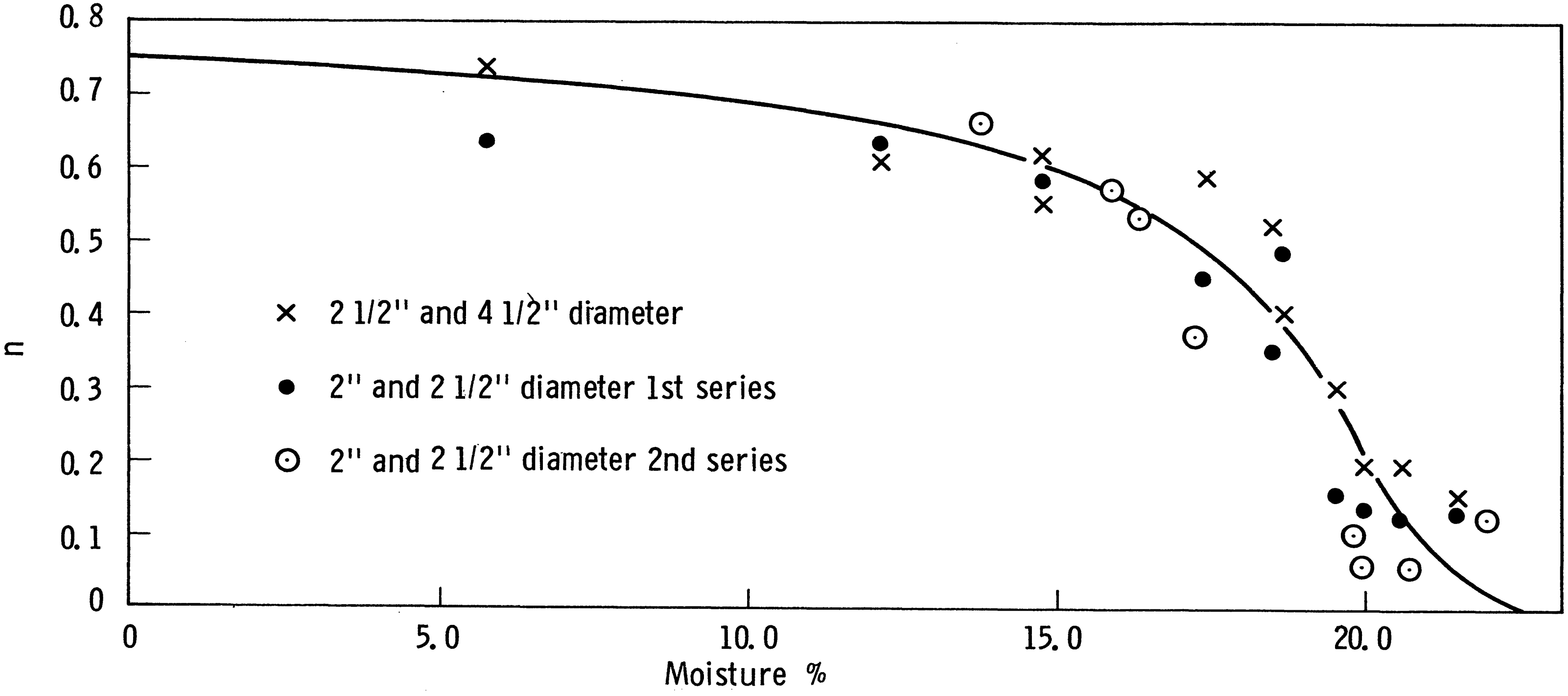

Fig. 7. Values of n for circular plates.

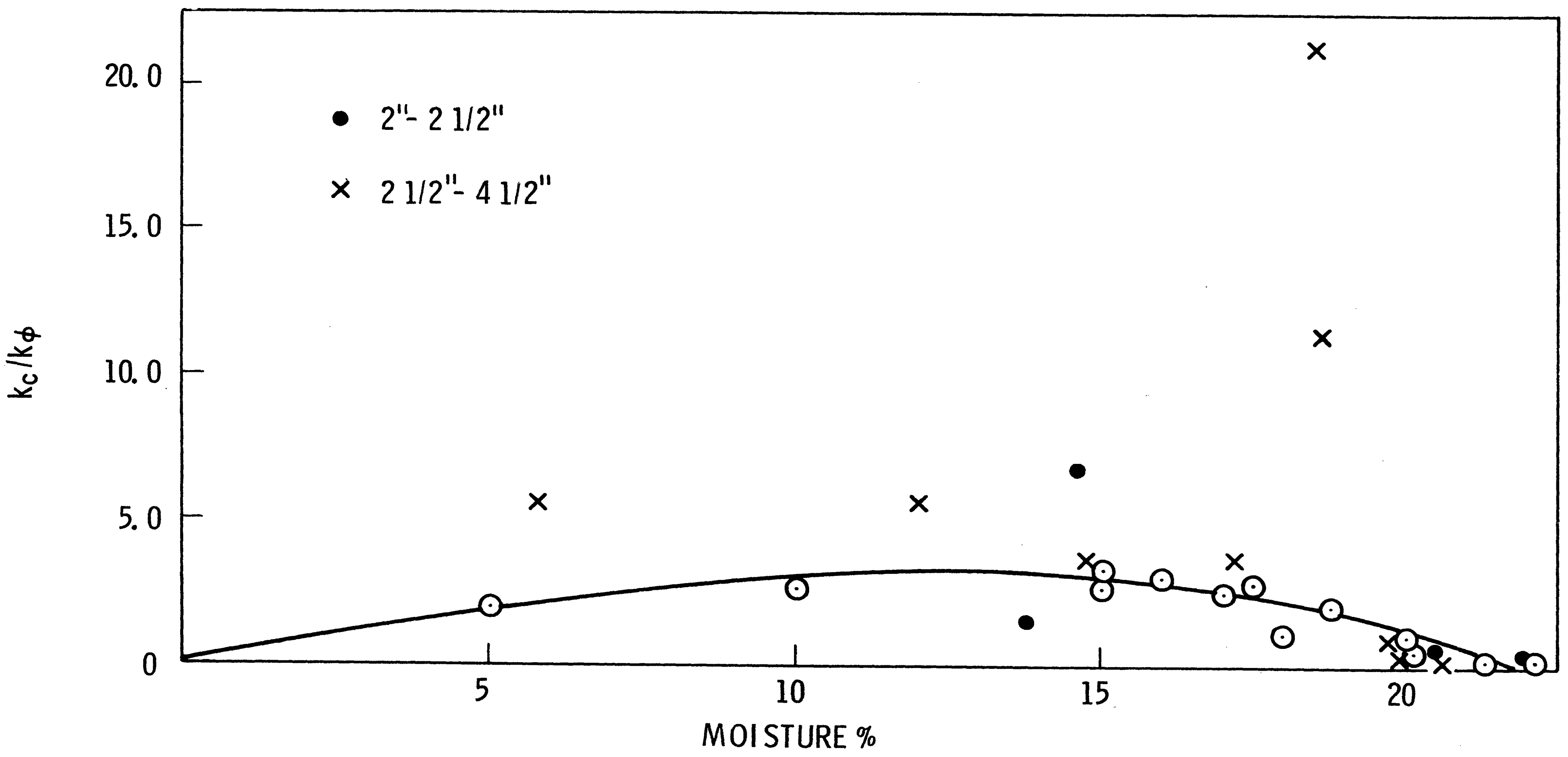

Fig. o. Values of k_c/k_ϕ for circular plates.

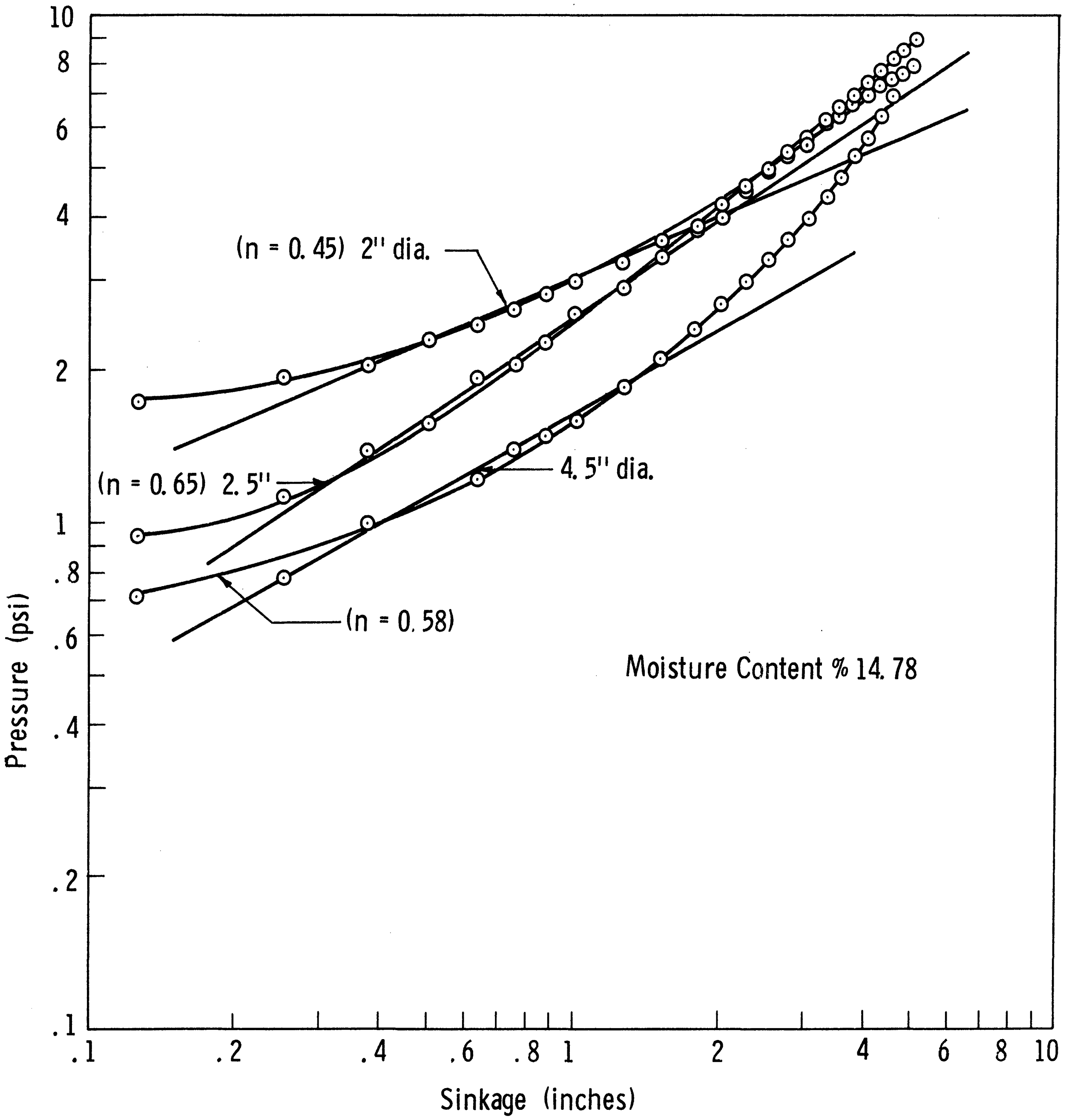

Fig. 9. Log-log plot for 14.8% moisture.

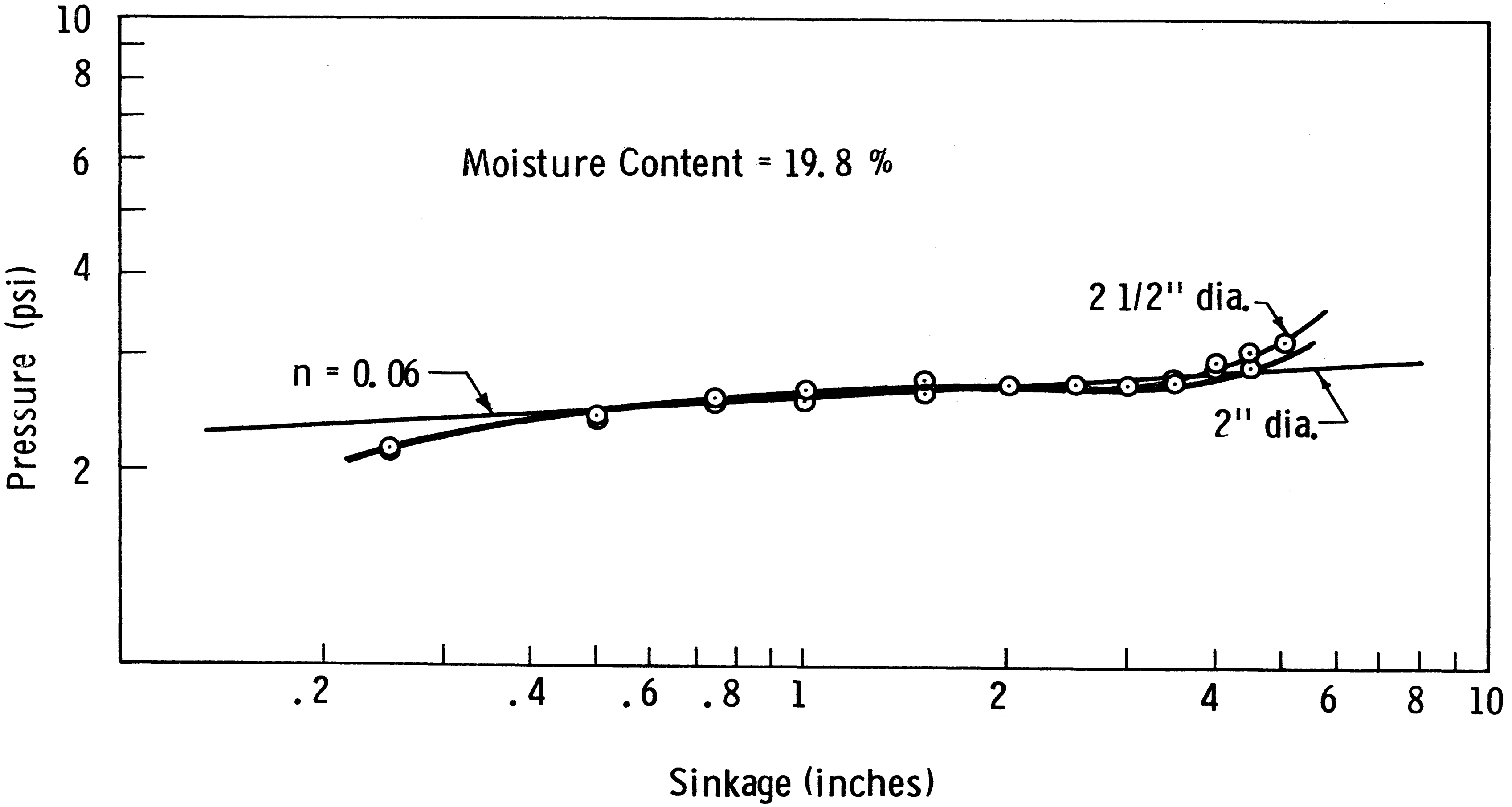

Fig. 10. Log-log plot for 19.8% moisture.

the index "n" equal to about 0.7 for moisture contents up to about 15%. Beyond this point its magnitude drops sharply reaching 0.2 approximately for 20% and almost zero at 22%. This result conforms substantially with the assumption of a plastic soil under these conditions of moisture content.

IV. MISCELLANEOUS SPECIAL SOILS AND TEST RESULTS

This section will record those experiments employing materials not commonly considered as soil, together with some special tests with the farm soil itself.

The artificial soils were originally experimented with as a result of their use at the Land Locomotion Laboratory of the Ordnance Tank-Automotive Command, Detroit (Ref. 6) and were to form the basis of the main attack on soil systems possessing both cohesive and frictional properties. The change to the Michigan farm soil was as a result of a conference with the Arsenal as to their present thinking on the problem, together with experience at The University of Michigan. The following data on the glycol combination is included in this report as a part of the record which may be of assistance to other investigations.

As was shown in Ref. 3, it is necessary that the following relation be maintained constant if its effect on the load parameter is to be eliminated from the dimensionless groups.

$$\frac{k_c}{k_\phi d} = \text{constant}$$

When only one wheel size is involved, it is of course a constant magnitude. In testing with various wheel sizes for the dimensionless expressions, it follows that the ratio $k_c/k_\phi d$ would vary; if the wheel diameter is halved, k_c/k_ϕ should double for true similarity.

Efforts to control k_c/k_ϕ for sand were reported in Ref. 5. In an attempt to obtain similar control of k_c and k_ϕ for a soil with both cohesive and frictional properties, the following tests were conducted.

A. BENTONITE CLAY

Since this material was to form the major constituent of the artificial soil, a bevameter test was conducted on the bentonite clay, in the state in which it was received, with the following results:

$k_\phi = -0.7$

$k_c = 2.44$

$n = 1.15$

The material as received is a very fine dry powder which appears to have little or no frictional properties, as judged by the manner in which it flows when poured, and at the same time little cohesion. The test figures above indicate that k_ϕ is to all intents and purposes zero, while the k_c value is of modest magnitude.

Comparing this material visually with sand, one would suspect little difference in properties. Except for some reduction in k_ϕ, the material flows much like sand and cohesion seems very low. In the tests recorded, cohesion had increased. Is this some function of the manner in which the material is contained, i.e., its boundary conditions, or is it a property of particle size, the powder being much finer than the sand? This question is of interest and should be followed up at some future time if a comprehensive soil evaluation program is undertaken, since the conclusions reached by such a study might point the way to improvements in soil values and their measurement.

B. BENTONITE, GLYCOL, AND WATER MIXTURES

In the examination and measurement of soil values of the glycol and clay mixtures, as in Ref. 6, the early work at The University of Michigan was taken into account, with the result that clay, glycol, and water combinations were employed (see Ref. 5). The data accumulated with this three-phase system are given in Table IV below.

It will be observed that there is a reasonable variation possible in the value of the ratio k_c/k_ϕ, particularly when clay and sand combinations are used as the solid ingredients.

The values of k_c and k_ϕ are the mean of a number of tests. The results seem erratic to some extent, but there were insufficient combinations tested to permit any definite conclusions to be drawn. It is apparent, however, that the equilibrium mixture that would be produced by exposure of such mixtures to an atmosphere of varying relative humidity can account for a considerable change in soil values from day to day.

C. SAND AND OIL COMBINATIONS

An examination of a mixture of sand and hydrocarbon oil as a possible means of eliminating the humidity effect was begun. At the time of the change to farm soil, only one combination had been tested; this was a mixture of 40 pounds of sand with 1.8 pounds of oil or a mixture ratio of 95.7% sand and 4.3% oil. The soil values for this combination were:

$$k_\phi = 2.53$$

$$k_c = 0.15$$

TABLE IV

CLAY, GLYCOL, AND WATER MIXTURES

Material (percent)	k_c	k_ϕ	n	k_c/k_ϕ	Relative Humidity (percent)
Bentonite Clay 50.5	0.04	0.40	0.5	0.10	53
Glycol 41.4					
Water 8.1					
Bentonite Clay 52.0	0.65	1.83	0.85	0.355	37
Glycol 43.0					
Water 5.0					
Bentonite Clay 54.0	1.50	5.10	0.27	0.294	--
Glycol 41.0					
Water 5.0					
Bentonite Clay 55.0	3.96	11.3	0.3	0.35	62
Glycol 36.3					
Water 8.7					
Bentonite Clay 48.6	0.11	2.31	0.4	0.047	37
Sand 5.4					
Glycol 41.0					
Water 5.0					

D. MICHIGAN FARM SOIL AND A DOW CHEMICAL COMPANY REAGENT

The development by the Dow Chemical Co. of Midland of a chemical to fracture soil and facilitate the entry of seeds was thought to have possibilities in the search for control of k_c and k_ϕ for the dimensionless parameters and similitude studies; tests were run with the farm soil with and without this compound with results as shown in Table V.

It is seen that the magnitude of k_c/k_ϕ is changed from 0.136 to 0.27, a two-to-one change in magnitude, which may be of considerable interest at a later stage of the dimensionless testing, despite a rather negligible change in the magnitudes of k_c and k_ϕ alone. These results are the mean of a number of tests with the above combinations.

TABLE V

FARM SOIL WITH AND WITHOUT SOIL FRACTURING AGENT

Material	k_c	k_ϕ	n	k_c/k_ϕ
Michigan Farm Soil Moisture Content 17.2%	1.1	8.1	0.18	0.136
Michigan Farm Soil Moisture Content 16.9% Soil Fracturing Agent	1.7	6.3	0.16	0.27

E. FARM SOIL, SAND, AND GLASS BEADS

In the search for control of the parameter of k_c/k_ϕ, mixtures of the soil, 5.7% moisture, with sand and glass beads of about 0.01" diameter in the dry state were tested with the following results (see Table VI and Fig. 11):

TABLE VI

SOIL, SAND, AND BEAD MIXTURES

Material (percent)			k_c	k_ϕ	n	k_c/k_ϕ	Moisture (percent)
Soil	Sand	Beads					
100	0	0	1.5	0.26	0.74	5.8	5.7
75	6.25	18.75	-3.4	6.2	1.00	-0.54	4.3
66.6	8.33	25.0	1.8	4.10	0.91	0.44	3.8
50.0	12.5	37.5	0.84	4.5	0.88	0.187	2.84
25.0	18.8	56.2	-3.2	7.5	0.79	-0.43	1.42
0	25	75	-0.5	2.5	1.0	-0.20	0.0

The plotted points show a fair agreement with the curves drawn except for two wild points; the reason for these cannot be determined.

The main feature sought, variation of k_c/k_ϕ, is seen at first sight to be of minor proportions over most of the range. The values for this factor were determined from the curves drawn for k_c and k_ϕ to mean out the errors of observation. If the range for k_c/k_ϕ is taken over that portion of the curve from soil = 100% to about 40%, a variation of this factor from 1.0 to 0.25 is seen, a 4:1 range, a very useful quantity since this would provide a 4:1 variation in wheel

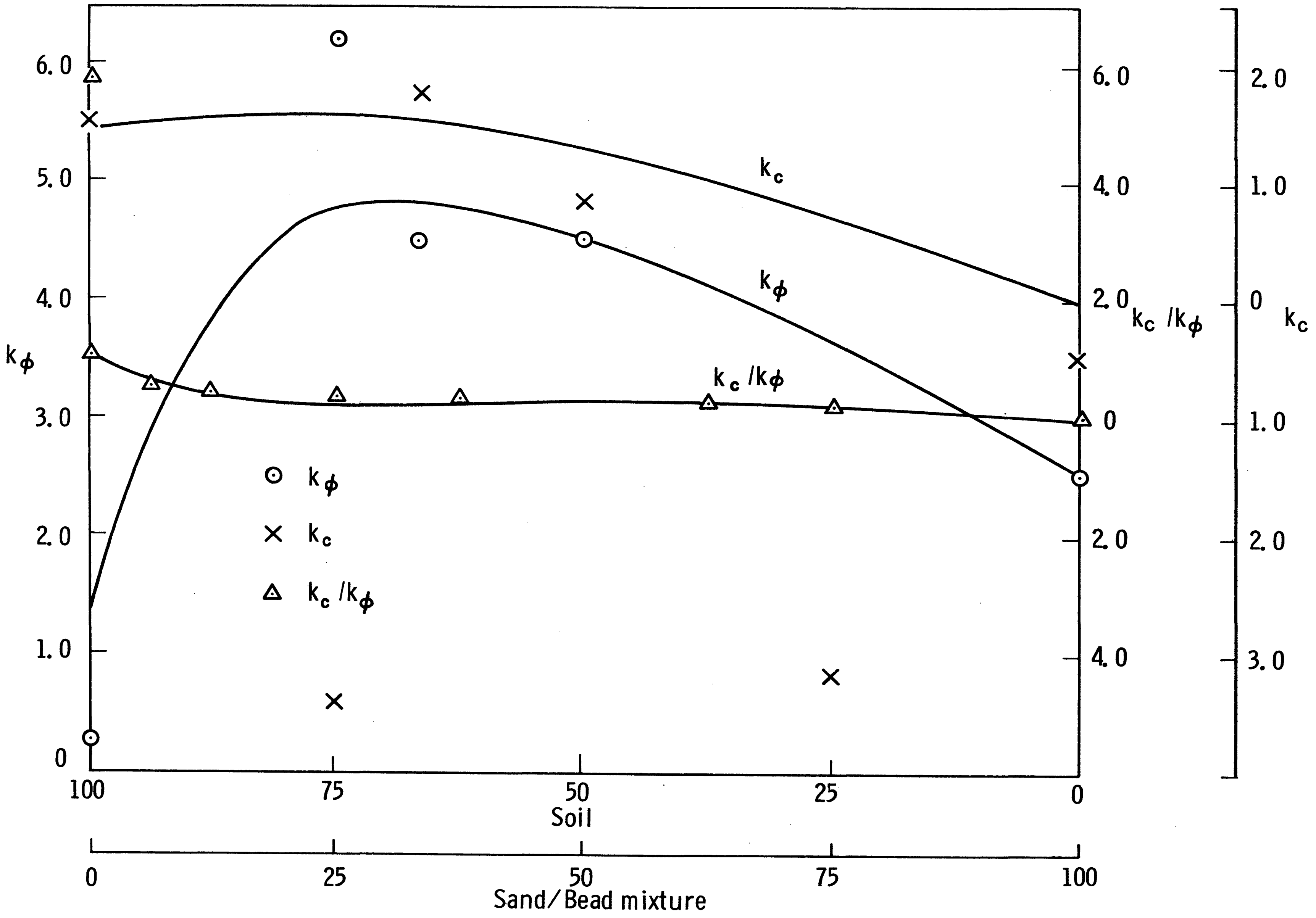

Fig. 11. Soil, sand, and bead mixtures.

size for the same value of the expression in Equation (2). Before this result can be exploited, however, it is believed that additional efforts should be made to narrow the scatter in the recorded data.

Consideration of the observed data indicated that although the results were recorded with bevameter footings of 2-1/2" and 4-1/2" diameter, considered the best combination so far used, there is still much to be desired to achieve the consistent results required for research when employing a typical soil as the main material.

Detailed examination of the curves on the log-log plots reveal one very distinct difference which is brought out by the following data in Table VII:

TABLE VII

SOIL, SAND, AND BEAD MIXTURES

Mixture (percent) Soil	Sand	Beads	k_c	k_ϕ	n	P at Z = 1"	Diameter of Plate (inches)
100	0	0	1.5	0.26	0.74	1.57	4.5
Moisture 5.7%						2.45	2.5
						2.97	2.0
75	6.25	18.75	-3.4	6.2	1.0	4.7	4.5
Moisture 4.3%						3.1	2.5
						2.35	2.0
66.6	8.33	25.0	1.8	4.5	0.91	4.5	4.5
Moisture 3.8%						5.4	2.5
						4.1	2.0
50.0	12.5	37.5	0.84	4.5	0.88	4.4	4.5
Moisture 2.84%						4.6	2.5
						3.65	2.0
25.0	18.8	56.2	-3.2	7.5	0.79	6.3	4.5
Moisture 1.42%						4.8	2.5
						4.1	2.0
0	25	75	-0.50	2.5	1.0	--	--
Moisture 0%						--	--
						--	--

The value of the pressure for one inch of sinkage with the various sizes of plates is given, the value being that at which the smooth curve through the points intersects the 1" sinkage line.

It is seen that the 100% farm soil has a different pattern from the rest of the data, the pressures being 1.57, 2.45, and 2.97, respectively, as the diameter decreases. The remaining figures show, in general, a more uniform value with the 2.5" plate having the larger value in those cases where a positive k_c and k_ϕ is determined, as it should. It should be kept in mind that there is a moisture variation throughout these tests, of a minor nature perhaps, but with some effect on the properties. This variation results from mixing the dry sand and beads with a soil of fixed moisture content.

A calibration of the bevameter is run with each use; hence the above variation is not due to an error in the instrument but could be due to dirt entering the bearings, etc. This is unlikely, however, since this possibility is carefully watched.

It is also important when comparing farm soil data to observe the moisture content of the soil for each test since rapid changes in soil value occur at certain points with moisture content.

The general conclusion to be drawn from this work with farm soil is that far more variations are possible than with sand where excellent duplication of the results were obtained with the same instruments and the same operations. The cause of such variations should be ascertained if possible, not only to improve the data regarding k_c/k_ϕ but to obtain a better understanding of the soil and its behavior.

There are a number of reasons why the farm soil is erratic; (a) the mixing process and cultivator could produce varying air pockets in the soil mass, (b) the bevameter test does not represent the actual load-sinkage relationship for a soil of the type being considered, (c) inequalities of structure exist, etc.

As far as (a) is concerned, the soil does appear to be well mixed and uniform after cultivation, despite the varying properties. No compaction of the bed is carried out after cultivation and before the tests are taken. Compaction might eliminate a varying structure caused by entrapped air pockets, etc.

The bevameter test using Equation (11) should be looked at carefully in view of the results obtained to date. Some modification of its structure might result in more uniform results with a natural soil. The question is also raised as to the previous work with sand; here the relationship expressed by Equation (11) appeared to be fulfilled with a high degree of accuracy though some small variations still existed. Perhaps this accuracy was due to the type of sand employed. Work with other varieties should be carried out to examine how universal the relationship is.

With regard to (c), there was no reason to suspect inequalities of structure as far as the sand was concerned, but this is not the case for the natural soil.

It is believed that a comprehensive project on soil, its treatment, and method of measurement, seems warranted and should be carried out, together with a theoretical examination of the equations fitting the actual results.

F. MAINTENANCE OF MOISTURE CONTENT

In some cases the complete rests required for a given condition occupied a period of time during which the soil moisture content was liable to vary by a factor sufficient to change the soil values. In order to try to control this condition, a plastic cover for the tow tank was installed.

The moisture conditions were obtained, the bed was covered, and additional tests were run at regular intervals with the following results:

Time of Test	Hours Covered	Moisture, %
4 P.M. 4/10/62	0	17.22
8:25 A.M. 4/11/62	16.25	16.74
1:15 P.M. 4/12/62	32.75	15.88

From these results, it was possible to estimate the quantity of water required over a given period to maintain a substantial constant moisture content of the soil.

V. THE DIMENSIONLESS PARAMETERS

The usefulness of the dimensionless parameters determined in Refs. 2 and 3 and repeated in this report in Equations (1) to (7) were checked for similarity with a high degree of accuracy, when sand was used as the supporting medium. In that case the parameter represented by Equation (2) was zero and dropped out of the relations, making similarity tests under these conditions comparatively simple.

With the use of a farm soil having a magnitude for both k_c and k_ϕ, this is no longer the case. For exactly similar conditions, the Drag/Load relationship is now represented by

$$\frac{R}{W} = f\left\{\frac{W}{d^{n+2}\,k_\phi} \;,\quad \frac{k_c}{k_\phi d} \;,\quad \frac{D}{d} \;,\quad \;,\quad \;,\quad n \;,\quad i\right\} \qquad (8)$$

Of the above dimensionless parameters, the following are constant or at least can be held constant during tests:

D/d	soil depth to wheel diameter
α	aspect ratio of the wheel
μ	coefficient of friction
n	nondimensional sinkage index
i	nondimensional slip coefficient

There seems to be no particular problem with these magnitudes as moisture content of the soil varies except for μ and n. A high moisture content could change μ as a result of the lubricating effect of the water while n, the index in $P = KZ^n$, is definitely not a constant (see Table II). It is believed that, as a starting point, these two magnitudes can be considered constant provided that a wide variation of the moisture is not employed. Examination of Figs. 5, 6, and 7 indicates moderate changes of n and k_c/k_ϕ for moisture contents of say 10% to 16%; hence if the tests are restricted to this range, approximately, some appraisal of the dimensionless parameters for a soil with both cohesion and friction characteristics can be made at least to a first approximation.

A. APPARATUS

The equipment employed for this work was that described in Ref. 3 with the following modifications:

1. The soil depth was maintained at a minimum of 7.5 inches in place of the 9 to 10 inches desirable. The reconditioning process in which a rototiller was used did not permit the greater depth normally desired for the tests to be used.

2. The total length of the bin was not used for measuring purposes but for the storage of the rototiller when not in use and for excess soil for mixing into the bed as desired.

   The length of run of 20 feet, approximately, was ample for satisfactory results.

3. A garden-type rototiller driven by a spark ignition engine was installed ahead of the wheel carriage, to which it could be connected at will. The tiller was mounted to the mono-rail on which the carriage moves by a set of hangers and bearings. By this means the soil was always cultivated to a constant depth (about 1-1/2" from the bottom of the tank.)

4. A water-distributing system was added to the front of the rototiller to spread a uniform quantity of water throughout the soil while it was being processed.

A photograph of the apparatus is shown in Fig. 12.

B. METHOD OF OPERATION

The soil bed was first prepared for a test run by estimating its moisture content based upon its last measurements. The rototiller was coupled to the carriage at the end of the bin, the engine started, the water flow adjusted if required, and the carriage tranversed through the soil bin. At the end of the run, the carriage and rototiller were reversed, the water turned off and the tiller returned to its standby position. This double passage of the tiller through the complete depth of soil at a slow speed was found to be ample for the breaking up of any compaction placed in the soil by the previous tests.

If any large quantity of water was added, ample time was allowed for it to distribute through the mass and a final mixing was made.

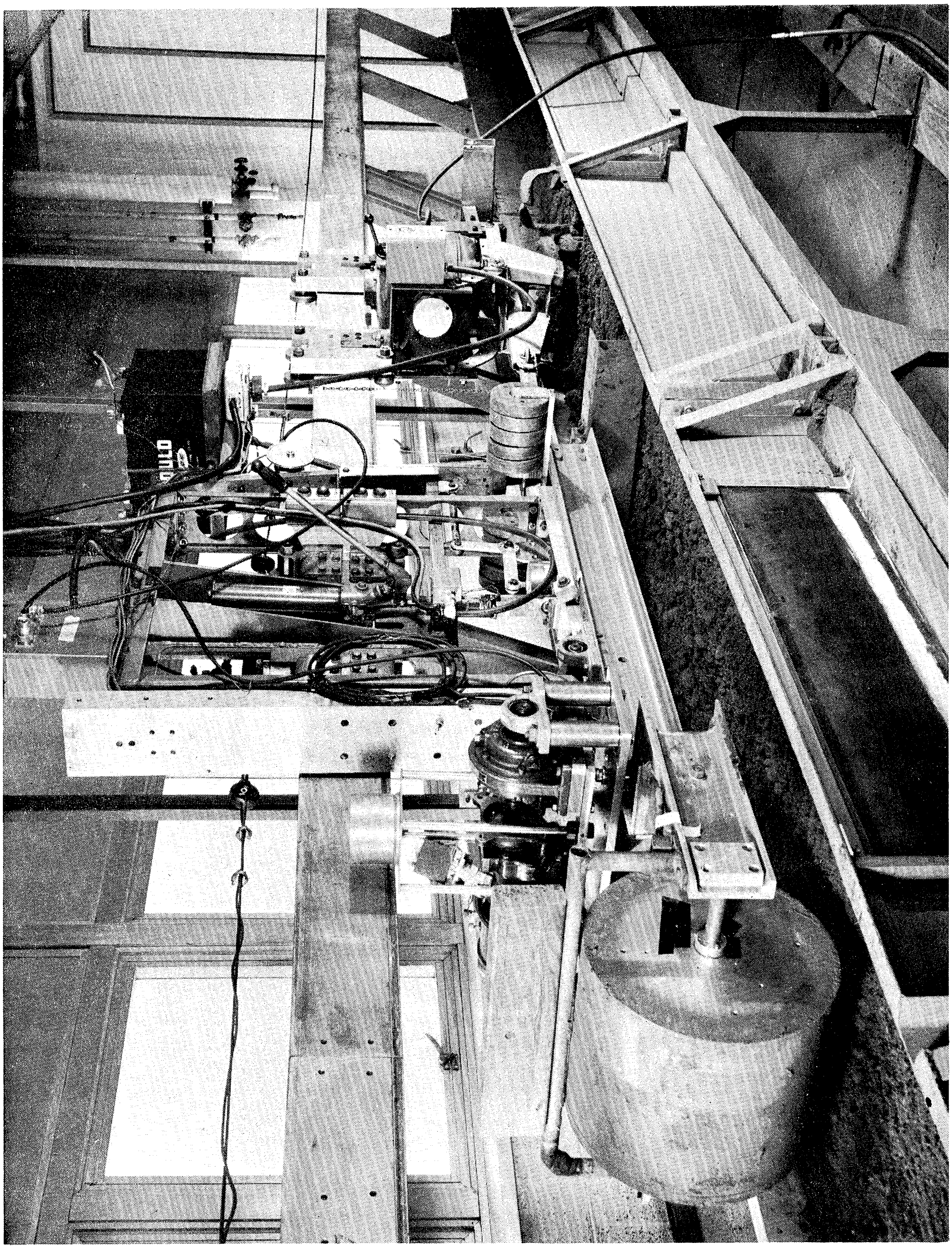

Fig. 12. Photograph of apparatus.

C. DIFFICULTIES

The apparatus performed in a reasonably satisfactory manner at the beginning.

After some tests had been run, the reversing mechanism for the blades of the rototiller failed to operate correctly. The ahead drive was satisfactory. Efforts to use just the one direction of rotation for each direction of travel of the carriage failed, since, if the blades were not reversed, the soil was thrown out of the bin.

The engine was noisy, and despite silencing measures, complaints were received from nearby classrooms. After several attempts to overcome the trouble, which was mainly the type of reversing employed (just a friction pulley), it was decided to replace the engine by a reversible electric motor. This motor overcame both mechanical and noise problems simultaneously.

With the above modifications to the rototiller, the equipment proved to be completely satisfactory for the purpose, and tests could then be run at a rapid rate.

D. TEST PROGRAM

The program of tests outlined involved the use of both towed and powered wheels of various sizes and aspect ratios for which the drag, sinkage, torque, etc., would be measured under a series of loads with the result that dimensionless plots of various parameters of interest could be made and conclusions drawn as to the validity of the methods for similitude testing of wheels.

As a result of the mechanical difficulties encountered in re-molding of the soil, there was a considerable loss of time with the result that by the expiration date of the contract, the data accumulated consisted of a limited amount relating to the towed wheel only.

The plotted results of the observations are given in Figs. 13-25.

Figure 13 is a plot of the observed values of the drag R for a wheel of 8.6" diameter for the three aspect ratios employed, viz., 0.27, 0.52 and 0.84. The points fit the curves well, with the exception of the 60 pound value at $\alpha = 0.84$, which is shown by ⓧ on the diagram. It is considered that this is an error and is neglected. From these curves were tablated the corresponding values of R and W' from which the R/W' was calculated for each load coefficient and the plot of Fig. 14 obtained.

On Fig. 14 are drawn three dotted curves which represent an attempt to separate the three aspect ratios. However, in view of the undoubted errors of observation which arises from slight moisture variation, soil treatment, etc., the straight line drawn on this diagram could represent all of the results

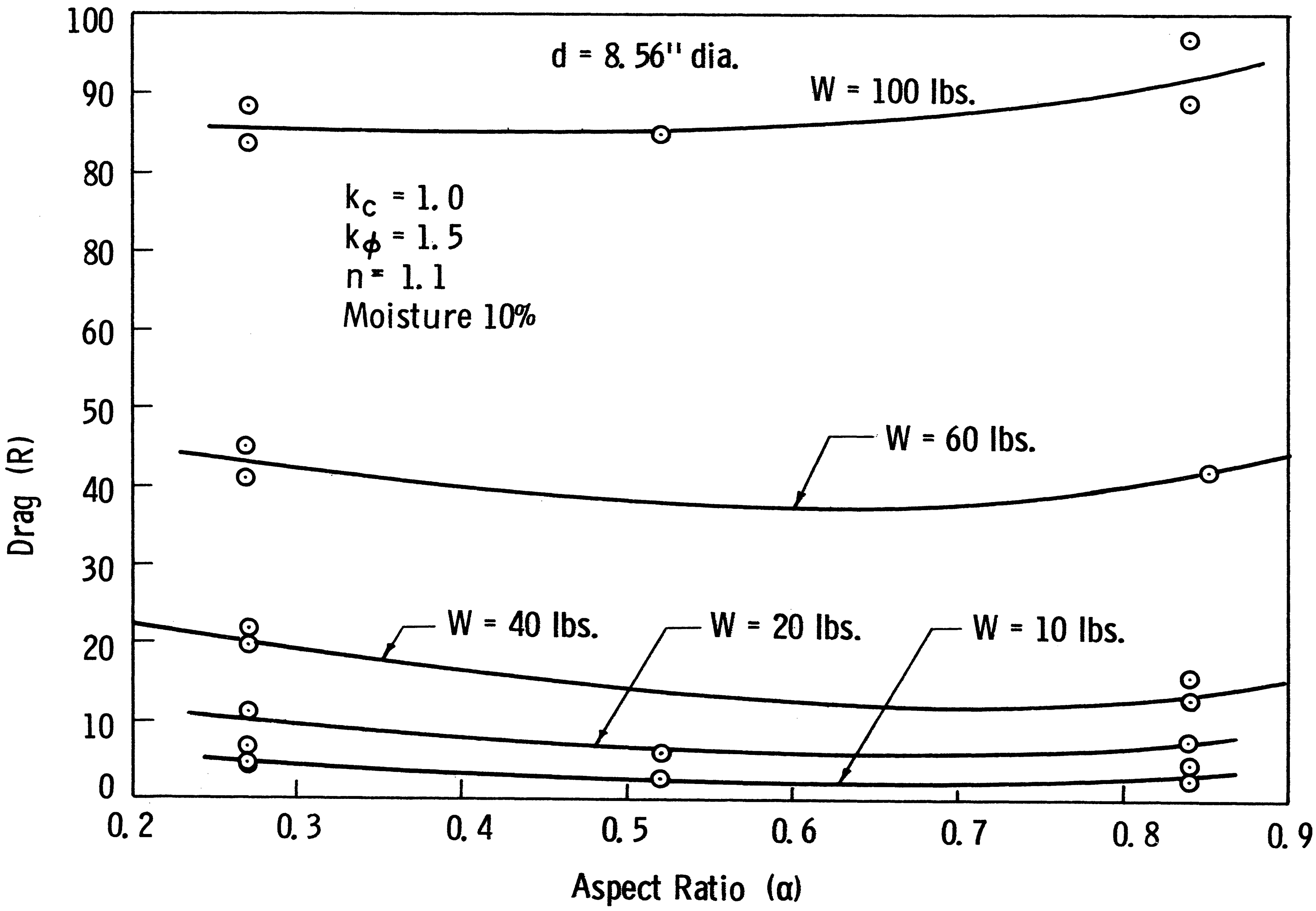

Fig. 13. Drag versus aspect ratio for 8.6-in. diam wheel.

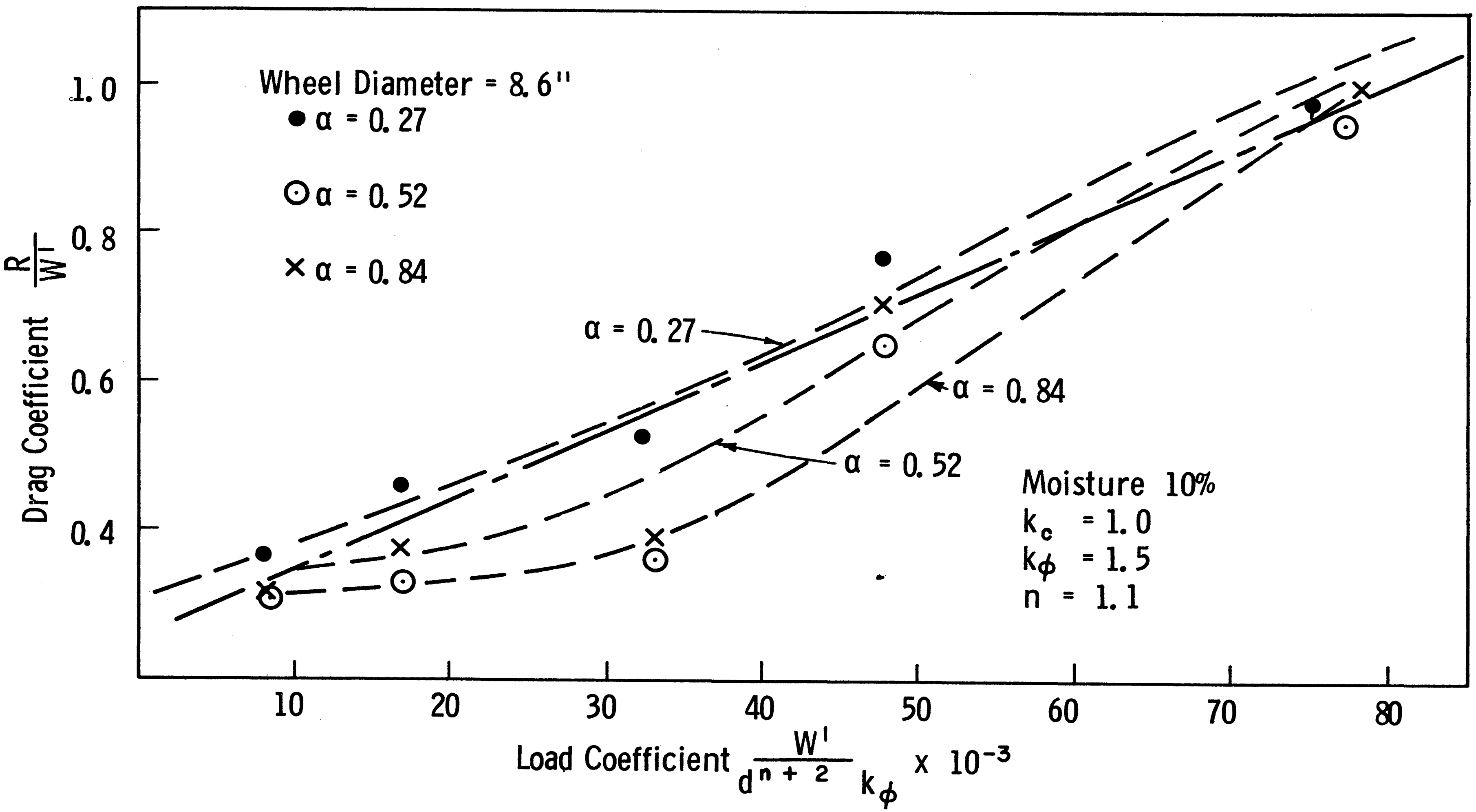

Fig. 14. Drag/load ratio and load coefficient for 8.6-in. diam wheel.

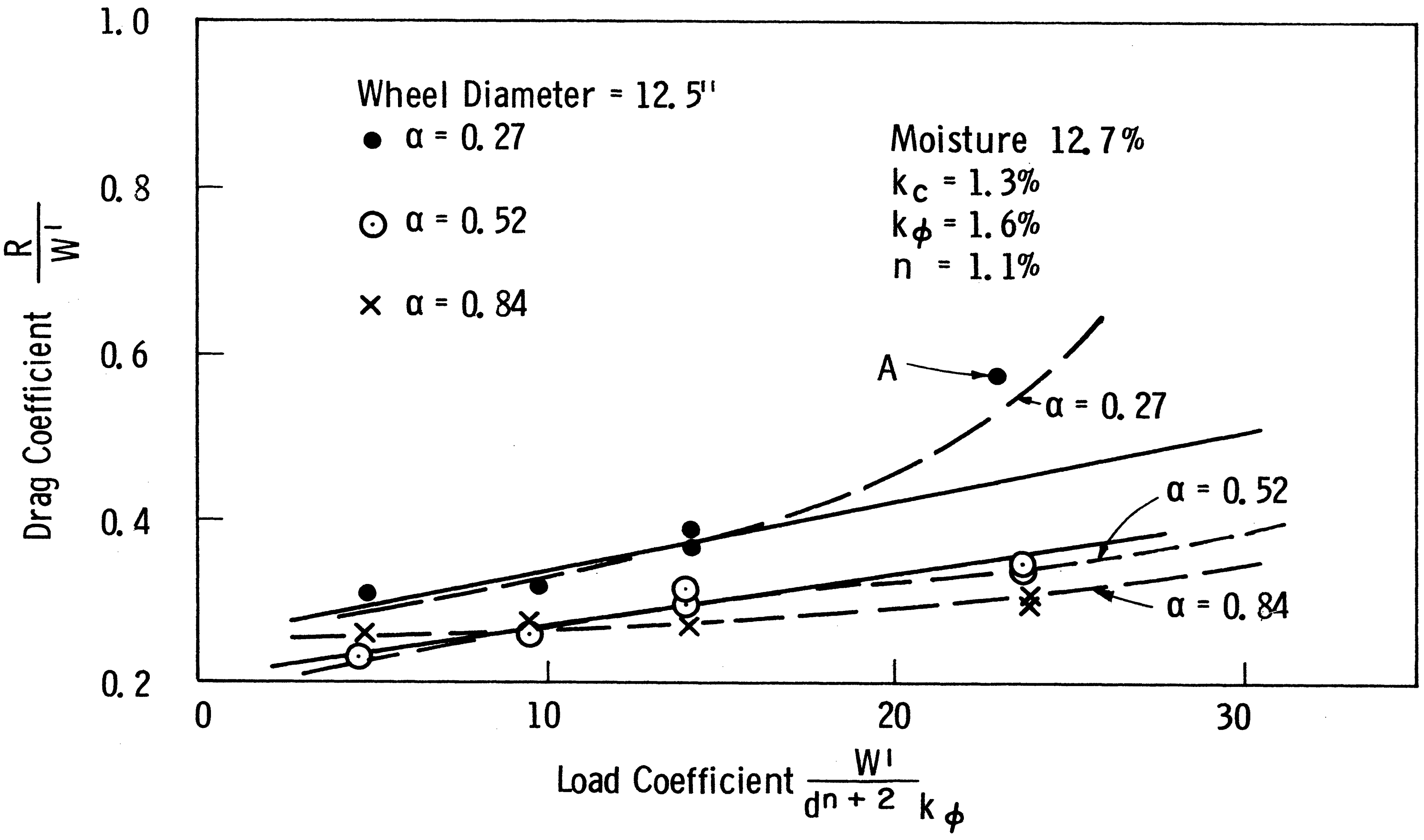

Fig. 15. Drag/load ratio and load coefficient for 12.5-in. diam wheel.

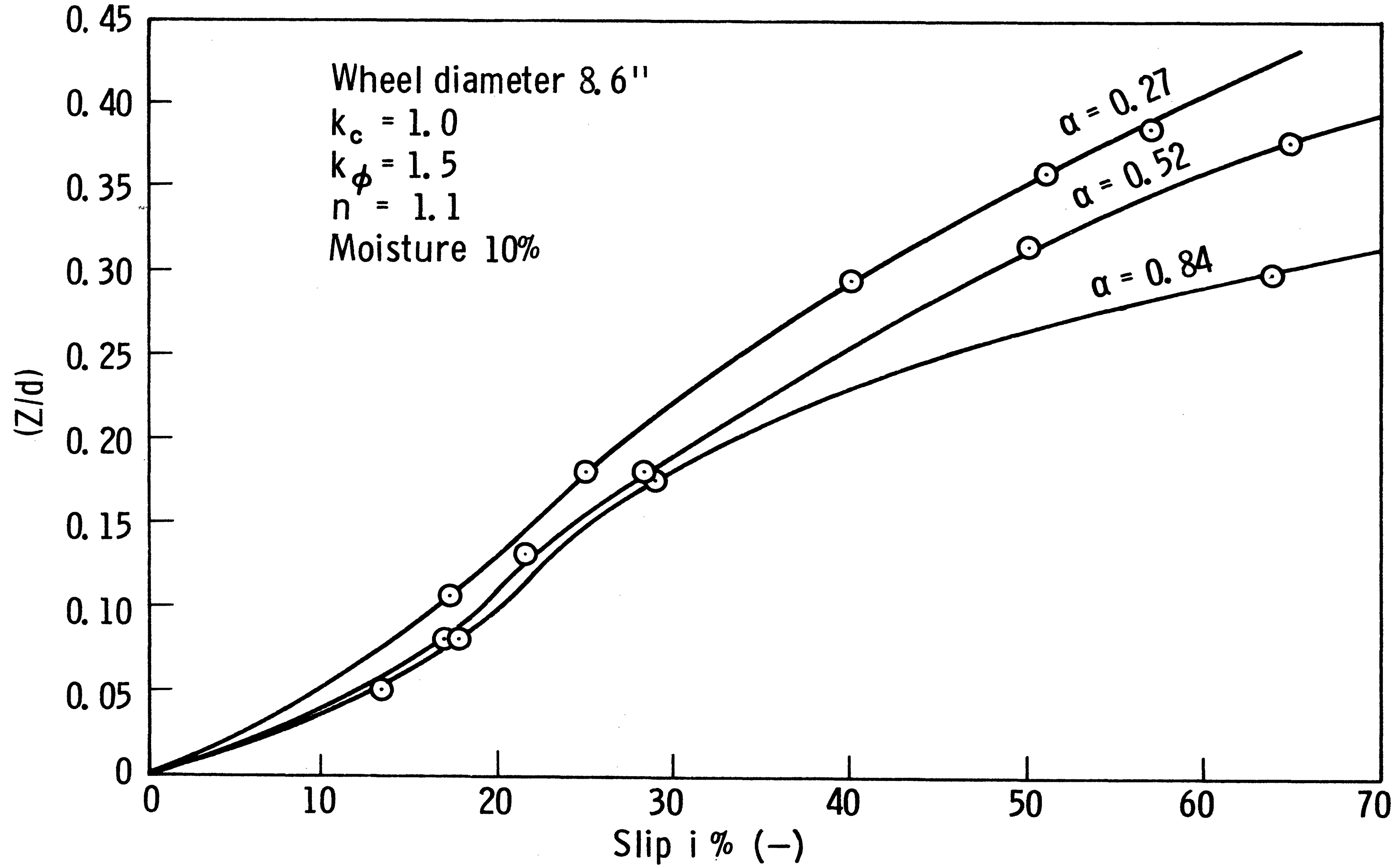

Fig. 16. Load-sinkage parameter for 8.6-in. diam wheel.

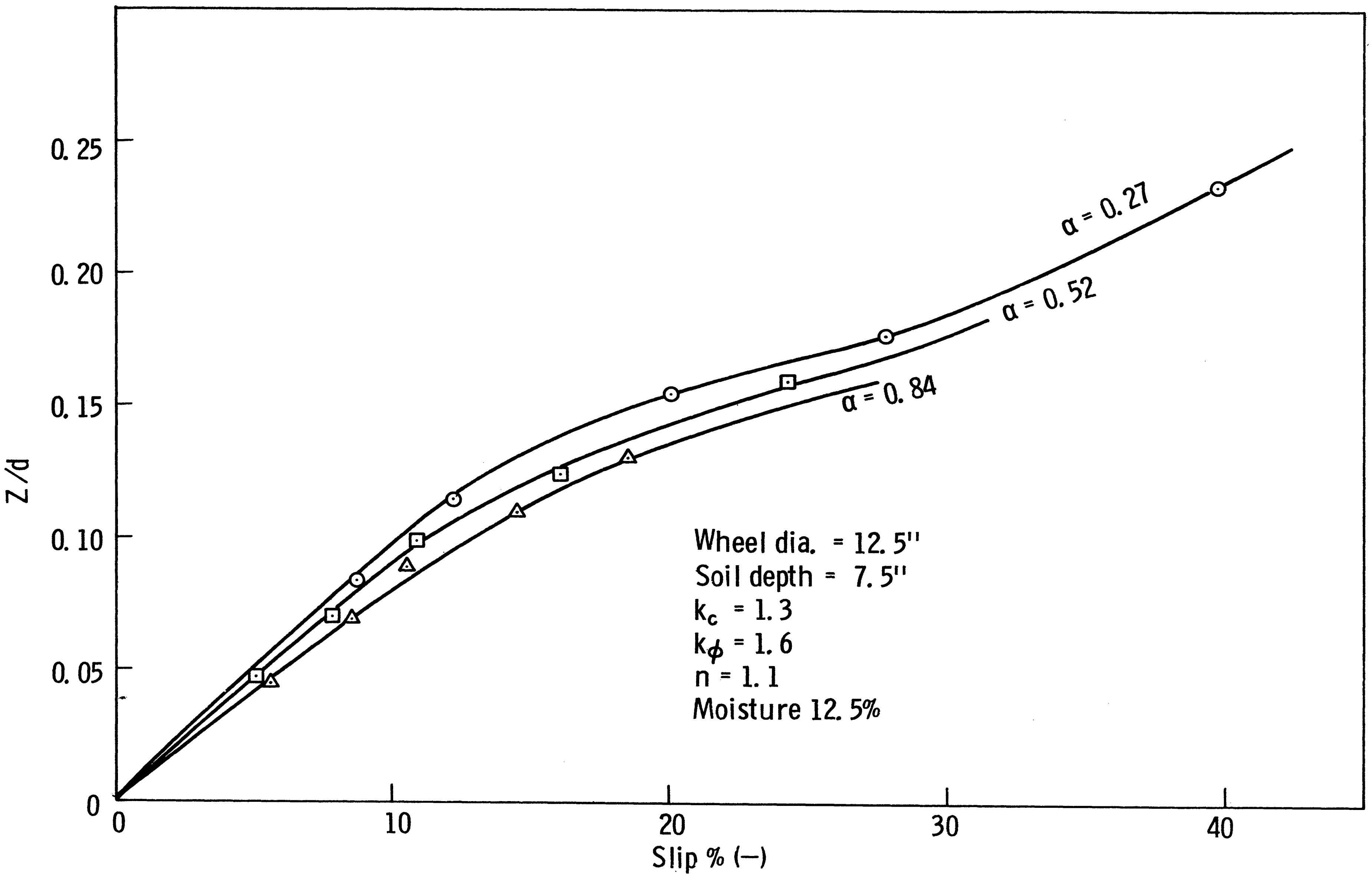

Fig. 17. Load-sinkage parameter for 12.5-in. diam wheel.

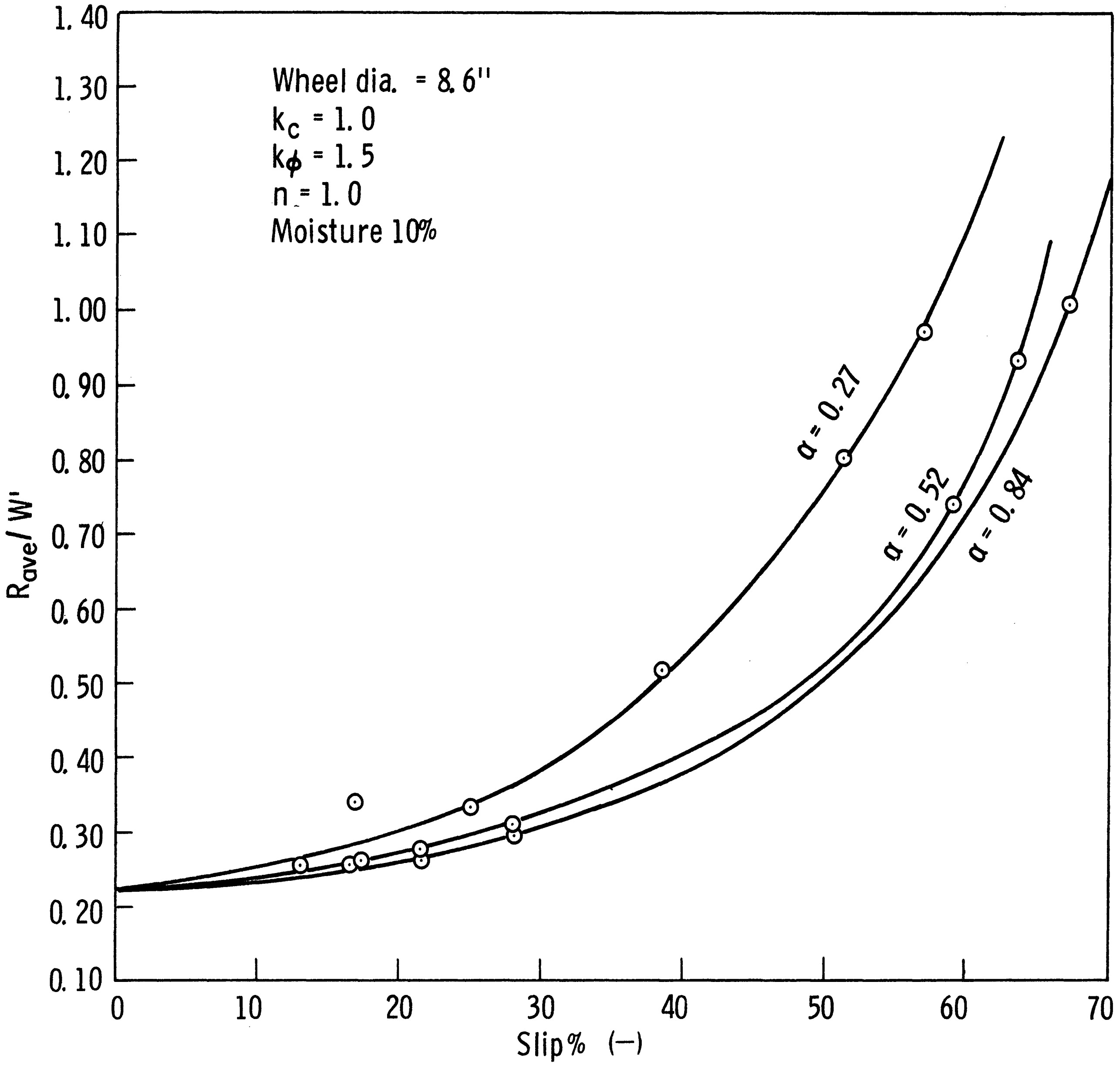

Fig. 18. Drag/load ratio and slip percent for 8.6-in. diam wheel.

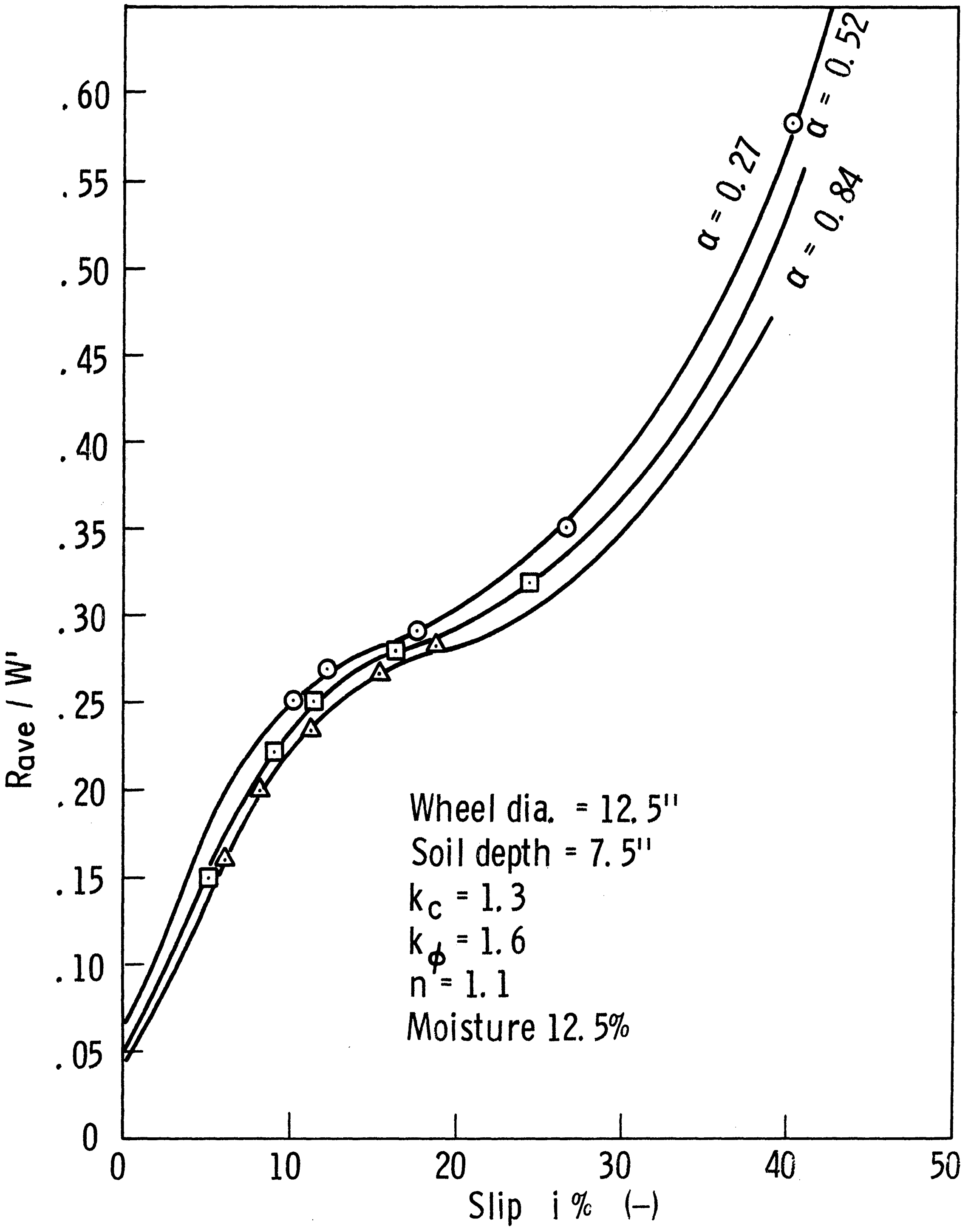

Fig. 19. Drag/load ratio and slip percent for 12.5-in. diam wheel.

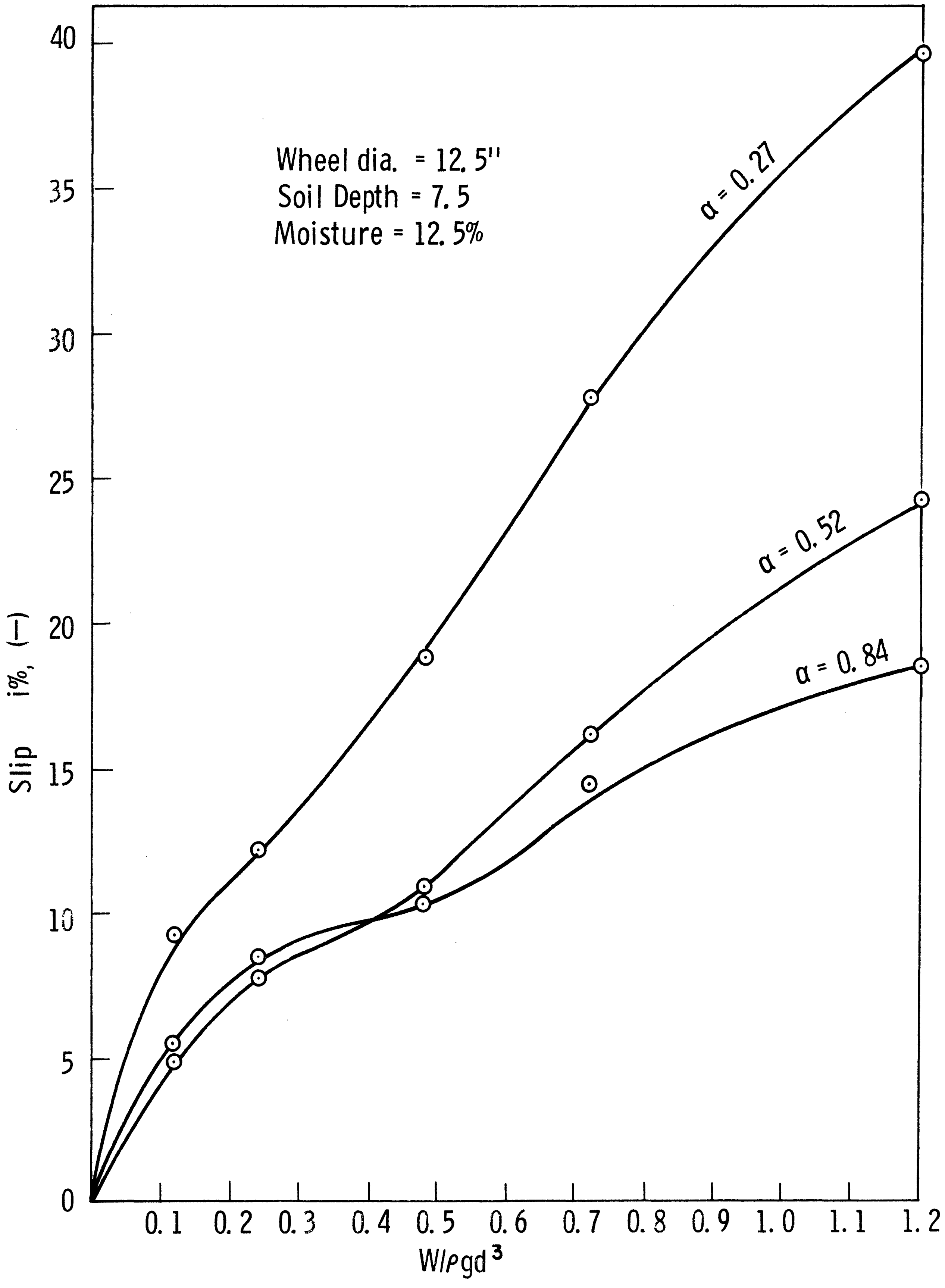

Fig. 20. Slip percent and $W/g\rho d^3$ for 12.5-in. diam wheel, 12.5% moisture.

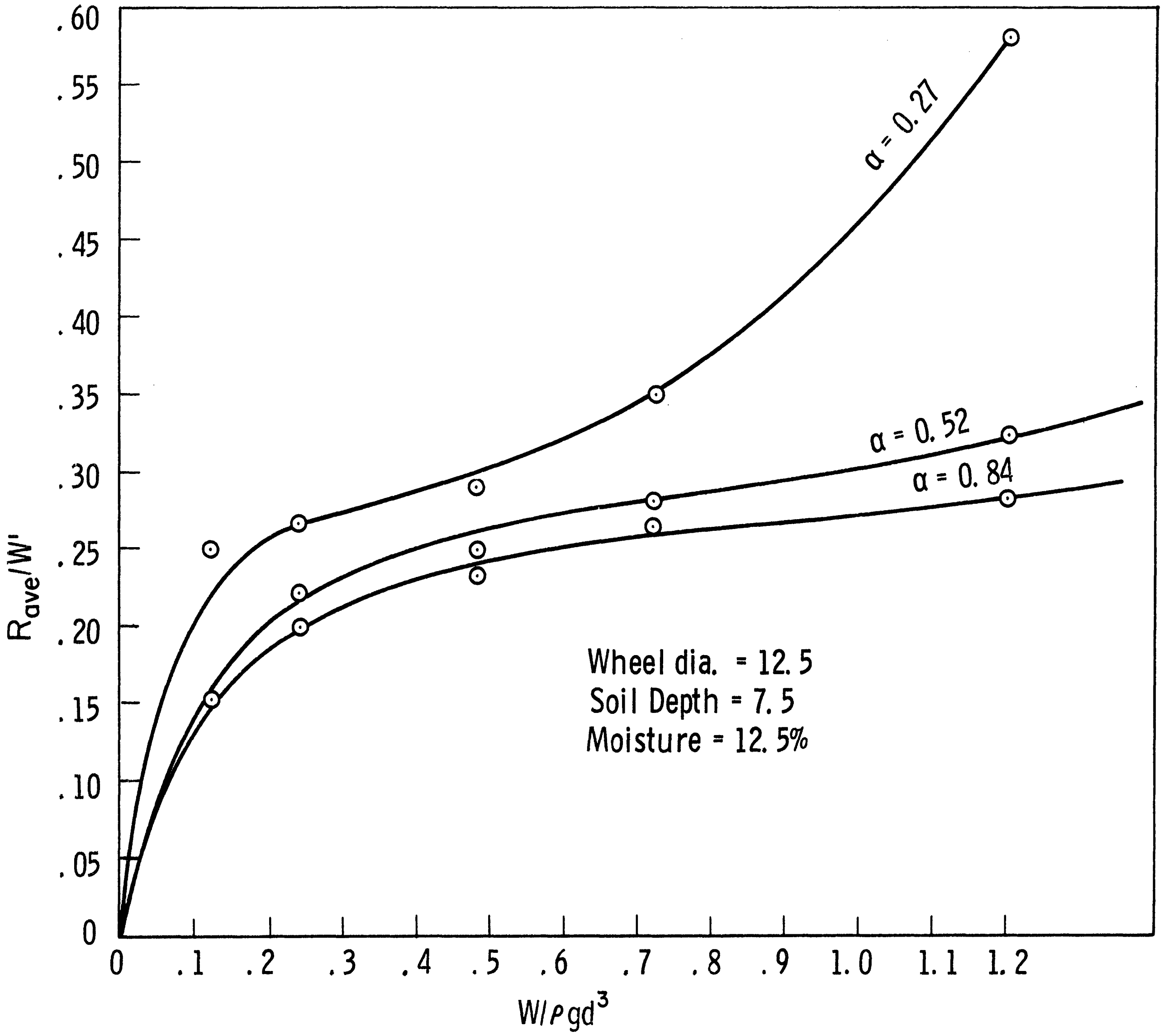

Fig. 21. Drag/load and $W/g\rho d^3$ for 12.5-in. diam wheel, 12.5% moisture.

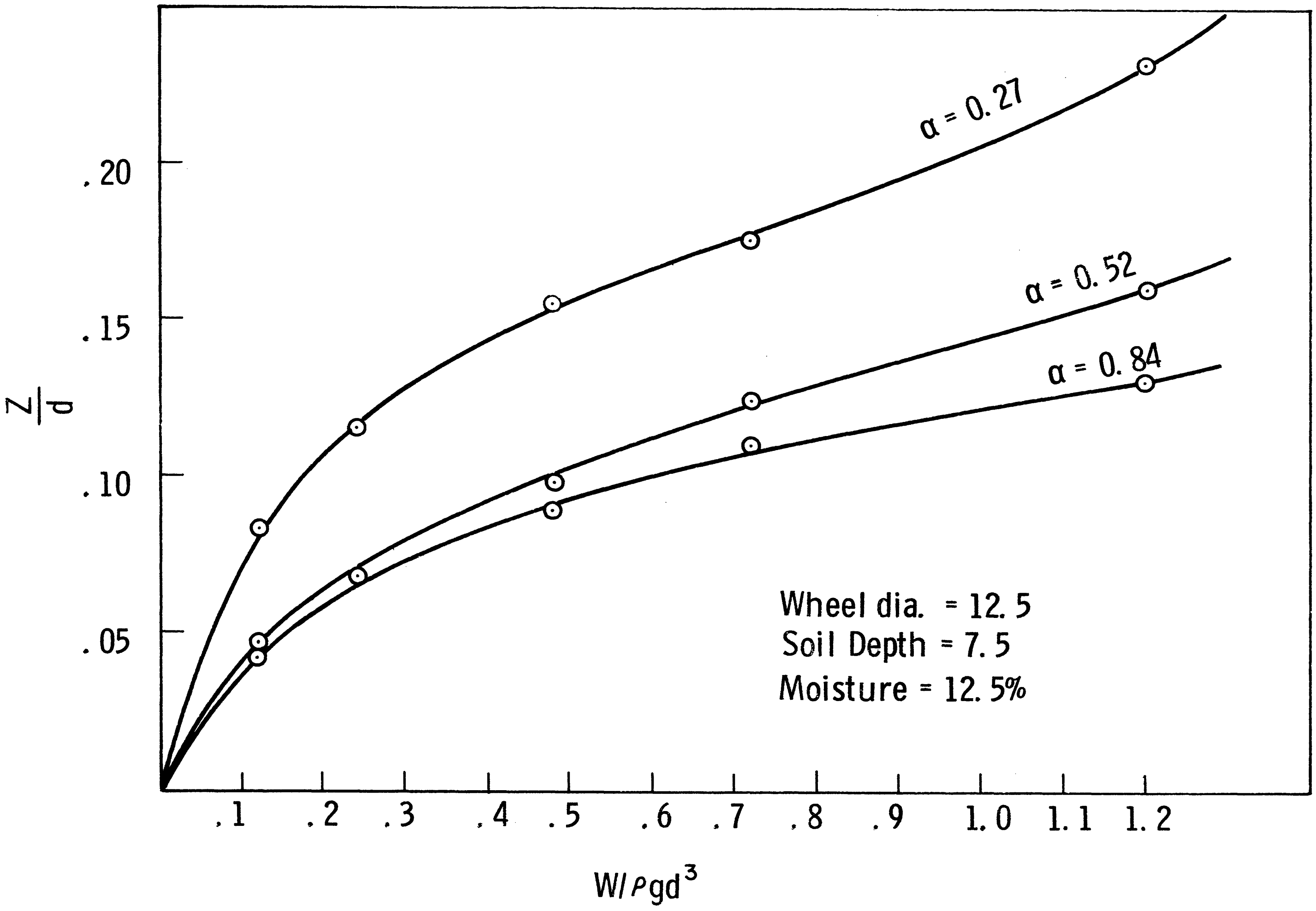

Fig. 22. A/d and W/gρd^3 for 12.5-in. diam wheel, 12.5% moisture.

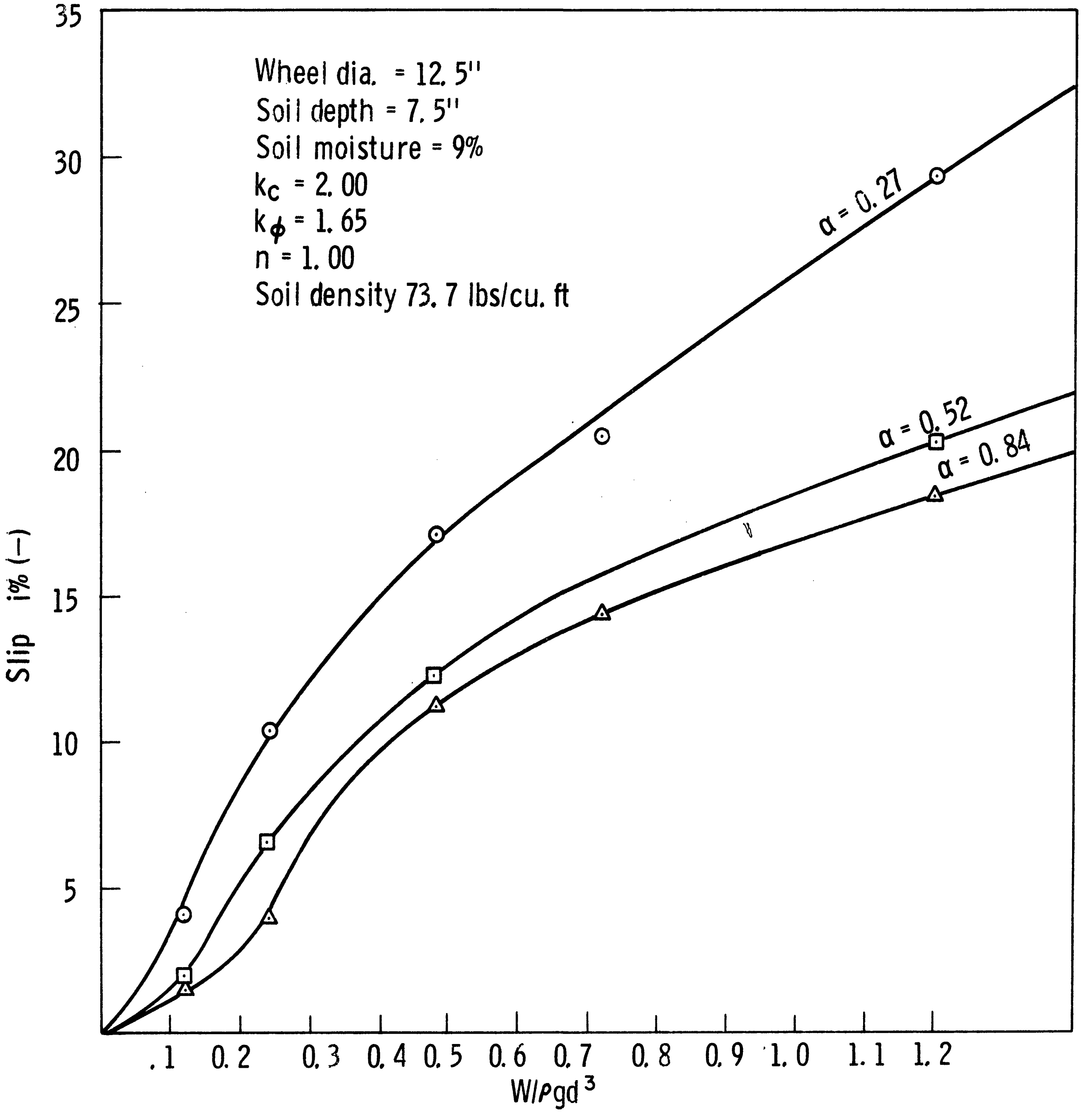

Fig. 23. Slip percent and W/gρd^3 for 12.5-in diam wheel, 9% moisture.

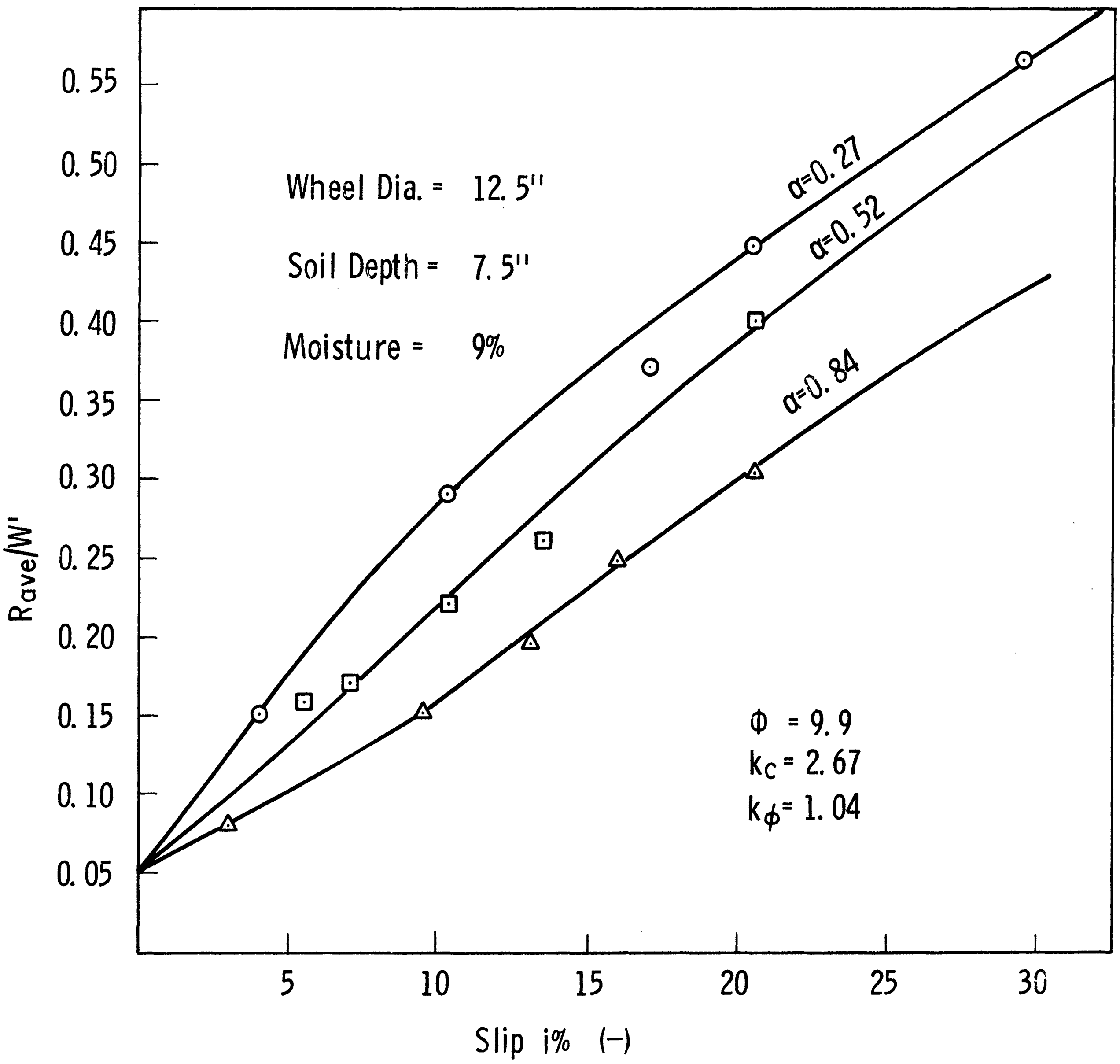

Fig. 24. Drag/load and $W/g\rho d^3$ for 12.5-in. diam wheel, 9% moisture.

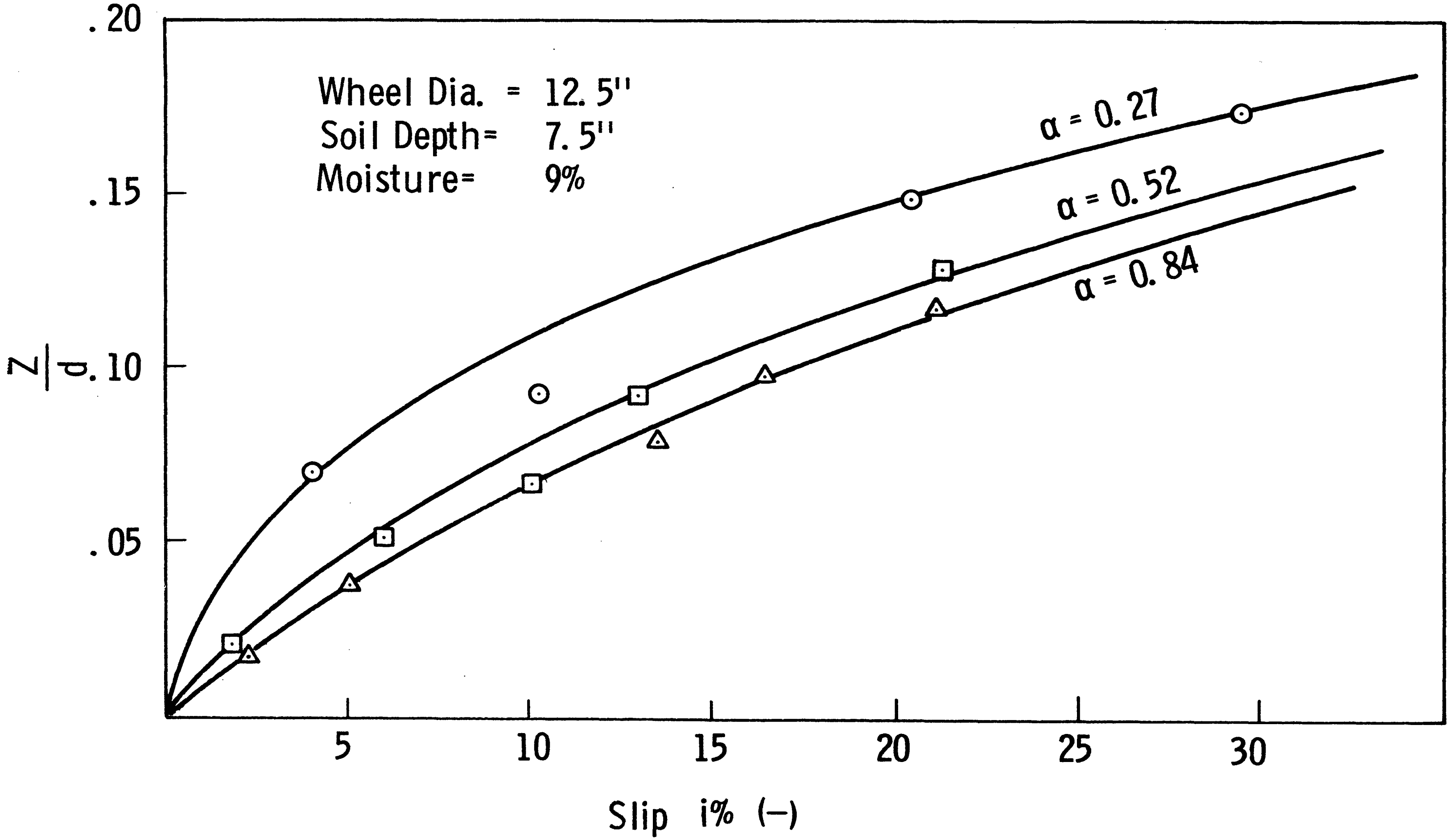

Fig. 25. Load-sinkage and $W/g\rho d^3$ for 12.5-in. diam wheel, 9% moisture.

with a good degree of accuracy except for two of the three points at the load coefficient of 33.

The process was repeated for the 12.5" diameter wheel, with the results shown in Fig. 15. The moisture content this time was 12.7%, in place of 10%. The differences in k_c, k_ϕ, and n are slight and have been taken into account in the calculations for the coefficients. Examination of this graph indicates greater difficulty in fitting a single straight line to all the points, though for the individual aspect ratios, the points line up well along straight lines with the exception of one point, indicated by Ⓐ, for $\alpha = 0.27$. This is the case of a 100-pound load being carried on a narrow wheel, with a large Z/d giving considerable bulldozing. The straight line shown may perhaps represent the change of drag only if the bulldozing had not occurred.

There are insufficient data from which to report on similarity conditions, since a greater range of moisture must be employed to control k_c/dk_ϕ. Powered wheel data must be added and definite dimensionless parameters examined.

If the two sizes of wheels tested are compared for the same load coefficient, the data of Table VIII is obtained.

TABLE VIII

DRAG/LOAD COEFFICIENTS FOR DIFFERENT ASPECT RATIOS

Wheel Diameter (inches)	Load Coefficient	R/W' Aspect Ratio		
		0.84	0.52	0.27
12.5	30	0.35	0.38	0.5
8.6		0.37	0.45	0.53
12.5	25	0.3	0.37	0.46
8.6		0.35	0.40	0.49
12.5	15	0.28	0.3	0.37
8.6		0.32	0.36	0.43
12.5	10	0.27	0.28	0.33
8.5		0.31	0.33	0.57

In view of the change of moisture content between the tests of the two wheel sizes and its effect upon the load coefficient, the agreement shown in Table VIII is considered substantial; indicating that the conditions of similarity apply to this case when using a soil with both cohesion and friction.

By the accumulation of additional data, other wheel sizes, moisture content, ratio of k_c/k_ϕ, etc.; the complete picture could be established and a workable model theory produced. The limited work to date indicated that effective results can be expected.

From the additional data recorded for the two wheel sizes employed, the load-sinkage parameter, Z/d, is shown in Figs. 16 and 17, plotted on an axis of percent slip. Comparison of these results with those of Ref. 3 for sand show that farm soil yielded considerably less slip than did sand for a given Z/d for all aspect ratios. The shape of the curves is also different. In this connection it must be considered that for complete representation on such a diagram, the magnitudes of the load coefficient and $k_c/k_\phi d$ should also be constant for each individual curve or at least a series of values or similar magnitudes should be employed if the true effect of slip only upon the sinkage is to be obtained.

The diagrams of Figs. 18 and 19 show the R/W' values on a slip base for the two sizes of wheels, for slips up to about 40%. Again the results separate out into three distinct curves.

A further set of curves have been plotted using as a base another independent variable, $W/g\rho d^3$, for the 12.5" wheel. The results are shown in Figs. 20, 21 and 22, using a density of the soil of 73.7% pounds/cubic foot when the moisture content was 13.5%. The only set of such diagrams which do not separate into three distinct curves without overlap is that of the slip in percent when plotted on a base of $W/g\rho d^3$, in which case the lines for the $\alpha = 0.52$ and 0.84 cross one another. Here it must be kept in mind that the ratio of soil depth to wheel diameter is low compared with that established for sand for the conditions, eliminating boundary layer and bottom effects. The additional soil depth required was not possible with the existing apparatus and the method of soil remolding used. This handicap undoubtedly influences the results given in this report to some extent. The results for the aspect ratio of 0.84 would be those most affected by a lack of depth, as well as by a lack of width.

The graph of Fig. 23 corresponds with that of Fig. 20 except for a change of moisture content from 12.5% to 9%. The same general arrangement exists, but the crossing over of the $\alpha = 0.52$ and 0.84 does not occur here. Additional check tests should be run to establish which of the results represent the correct picture.

Figs. 24 and 25 are curves corresponding to those of Figs. 21 and 22 for the case when the moisture was 9%. Detailed comparison reveals some changes in magnitudes, some spread of the curves, etc.

These results represent all that could be accomplished within the terms of the contract.

VI. CONCLUSIONS

The chief conclusion that can be drawn from this work is that the soil-value system and its measurement need further careful study, if reproduciable and consistent results are to be secured under operating conditions in the laboratory.

It is believed that a major effort is required in this direction if the prediction mechanism is to be perfected and is to give satisfactory results. It is true that variations exist in the field, from point to point, that exceed by far the variations in the results reported here. It must be taken into account that this report deals with controlled laboratory tests where every effort was made in processing the soil and setting up the experiment to secure the greatest reproducibility possible.

It can be concluded also that the wheel tests conducted do show a promise of signs of dimensionless relationships, but that insufficient tests are available to permit any comprehensive crossplotting to secure such results in any great detail. With additional data and time for analysis work, it is believed that a satisfactory set of similarity relationships can be proved within the framework of the initial assumptions.

It is recognized that the independent and dependent variables forming the dimensionless numbers will possibly change if the soil-value system eventually employed should change. However, it is believed that such changes would not invalidate the equations and work already completed.

VII. RECOMMENDATIONS

It is recommended that the soil-value system receive some additional attention and that the additional data required to produce a satisfactory analysis of the whole problem be acquired.

To complete the work, additional runs with other moisture contents should be made to provide some variation to the parameter k_c/k_ϕ, which has so far been almost neglected. Such runs will permit results to be selected under which various ratios of the dimensionless expressions are constant. It can be determined then how constant the variable remains.

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