THE UNIVERSITY OF MICHIGAN
College of Engineering
Department of Aerospace Engineering

Technical Report

Conversion of an Ion-Acoustic Wave by Collisions to a Neutral Sound Wave in a Nonhomogeneous Plasma

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DRDA Project 010757

Supported by:

United States Air Force
Office of Scientific Research
Grant No. AFOSR-72-2224
Arlington, Virginia

Administered through:
Division of Research Development and Administration
Ann Arbor

May 1974

This report was also a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in The University of Michigan, 1974.

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NOMENCLATURE

B	magnetic field
C_i	ion-acoustic speed, $\sqrt{(T_{i0} + T_{e0})/m_i}$
C_n	neutral sound speed, $\sqrt{T_{n0}/m_n}$
E	electric field
e	electron; charge of electron
$F_e(z)$	variable defined by $n_{e0}(z)\, v_{e1}(z)$
$\tilde{F}_e(\tilde{z})$	dimensionless F_e, $\dfrac{n_{e0}(\tilde{z})\, v_{e1}(\tilde{z})}{N_e \sqrt{T_{e0}/m_i}}$
$F_i(z)$	variable defined by $n_{i0}(z)\, v_{i1}(z)$
$F_n(z)$	variable defined by $n_{n0}(z)\, v_{n1}(z)$
$\tilde{F}_n(\tilde{z})$	dimensionless F_n, $\dfrac{n_{n0}(\tilde{z})\, v_{n1}(\tilde{z})}{N_n \sqrt{T_{n0}/m_n}}$
$G_e(z)$	variable defined by $n_{e0}(z)\, T_{e1}(z)$
$G_i(z)$	variable defined by $n_{i0}(z)\, T_{i1}(z)$
$G_n(z)$	variable defined by $n_{n0}(z)\, T_{n1}(z)$

H value defined by $\frac{12}{5}\frac{m_e}{m_n}\left(1 - \frac{2}{3}R_T\right)\frac{\Omega_{en} N_i}{R_T^{3/2} N_n}$

h_i charged particle atmosphere scale height

h_n neutral atmosphere scale height

J value defined by $-\frac{3}{5}\frac{\omega^2 m_n h_n^2 N_i}{R_T^{3/2} T_{e0} N_n}\left(\Omega_{in} + \frac{m_e}{m_n}\Omega_{en}\right)$

K parameter of Whittaker function

$$\frac{1}{2} - \frac{h_i}{2h_n}\left[\frac{3}{5} + i\,\frac{18}{25}\frac{m_e}{m_i}\left(1 - \frac{5}{3}R_T\right)\frac{\Omega_{en} N_i}{R_T N_n}\right]$$

$$-\frac{3}{5}\frac{h_i^2 \omega^2 m_n^2 R_T N_n}{T_{e0} m_e \Omega_{en} N_i}$$

$$\times \frac{-\frac{\Omega_{in} N_i}{N_n} + \frac{m_e}{m_i}\left(\frac{6}{5} - 3R_T\right)\frac{\Omega_{en} N_i}{R_T N_n}}{\left(1 - \frac{5}{3}R_T\right)\left(-\frac{6}{5} - \frac{18}{25}\frac{h_i}{h_n}\right)}$$

k wave number; parameter of Whittaker function,

$$\frac{1}{2} + \frac{5}{3}\frac{\omega^2 h_n^2 m_i}{T_{e0}(1 + R_T)} + \frac{3}{2}\frac{m_i^2}{m_n^2}\frac{\omega^2 h_n^2 \Omega_{in}}{T_{e0}\Omega\ (6-4R_T)}$$

M	parameter of Whittaker function,

$$\left\{ \frac{h_i^2}{4h_n^2}\left[\frac{3}{5} + i\,\frac{18}{25}\frac{m_e}{m_n}\left(1 - \frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right]^2 - \frac{3}{5}\,\frac{\omega^2 h_n^2 m_n}{T_{n0}}\left[1 - i\frac{\Omega_{in}N_i}{N_n} + i\,\frac{m_e}{m_n}\left(\frac{6}{5} - 3R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right] \right\}^{\frac{1}{2}}$$

m	parameter of Whittaker function,

$$- i\left[\frac{5}{3}\frac{\omega^2 h_n^2 m_i}{T_{e0}(1 + R_T)}\right]^{1/2}$$

m_e	mass of electron
m_i	mass of ion
m_n	mass of neutral
N_i	value of n_{i0} at $z = \infty$
N_n	value of n_{n0} at $z = -\infty$
n	number density
$Q(\mu)$	function as defined on page 59
$Q(is)$	function as defined on page 60
q	heat flux vector
$R(S)$	function as defined on page 49
R_n	parameter defined by n_{e0}/n_{n0}
R_T	parameter defined by T_{n0}/T_{e0}
r_1, r_2	roots of Euler's equation

S_s	effective source
t	time
v	velocity of gas
$W_{k,m}(\xi)$	Whittaker function
$W_{k,m}(\zeta)$	Whittaker function
W_r	Wronskian
z	space coordinate
$\tilde{z}$	dimensionless space coordinate, z/h_n

<u>Greek</u>

α	defined as $\dfrac{\Gamma(-2m)}{\Gamma(1/2 - k - m)}\left[\dfrac{-2m_e\,\Omega_{en}\,(6 - 4R_T)}{5m_i\,(1 + R_T)}\right]^m$
α_0	atomic polarizability in units of $10^{-24}\ cm^3$
Γ	gamma function
ϵ	defined as $\dfrac{m_e}{m_n}\dfrac{h_n}{h_i}\dfrac{\Omega_{en}}{R_T}\dfrac{N_i}{N_n}\left(1 - \dfrac{5}{3}R_T\right)\left(-\dfrac{6}{5} - \dfrac{18}{25}\dfrac{h_i}{h_n}\right)$
ζ	variable defined as $i \exp\left(-\dfrac{h_n}{h_i}\tilde{z}\right)$
λ_D	Debye length, $\sqrt{T/(4\pi n_0 e^2)}$
μ	defined as $i\dfrac{m_e}{m_n}\dfrac{\Omega_{en}}{R_T}\dfrac{N_i}{N_n}\left(1 - \dfrac{5}{3}R_T\right)\left(-\dfrac{6}{5} - \dfrac{18}{25}\dfrac{h_i}{h_n}\right)\exp\left(\dfrac{-h_n}{h_i}\tilde{z}\right)$

μ_A	reduced mass in a.m.u.
ν_{ei}	electron-ion collision frequency
ν_{en}	electron-neutral collision frequency
ν_{in}	ion-neutral collision frequency
ν_{ne}	neutral-electron collision frequency
ν_{ni}	neutral-ion collision frequency
ξ	variable defined by

$$- i \frac{2}{5} \frac{m_e}{m_i} \frac{\Omega_{en} (6 - 4R_T)}{(1 + R_T)} \exp(-\tilde{z})$$

Υ	parameter as defined on page 44
$\mathfrak{J}_1, \mathfrak{J}_2$	constants as defined on page 52
χ	defined as

$$i \frac{2}{5} \frac{m_e}{m_i} \frac{\Omega_{en} (6 - 4R_T)}{(1 + R_T)} \cdot \left[i \frac{m_e}{m_n} \frac{\Omega_{en}}{T_{0e}} \frac{N_i}{N_n} \times \left(1 - \frac{5}{3} R_T\right)\left(- \frac{6}{5} - \frac{18}{25} \frac{h_i}{h_n}\right)\right]^{-(h_i/h_n)}$$

Ψ	Kummer's function
ψ	the electrostatic potential
Ω_{en}	value of ν_{en0}/ω at $z = 0$
Ω_{in}	value of ν_{in0}/ω at $z = 0$
ω	frequency

Subscripts

e	electron
i	ion
n	neutral
0	equilibrium value
1	perturbed value
s	species "s"
r	species "r"

I. INTRODUCTION

In the search to understand the numerous emissions from the ionosphere that are recorded daily, one is faced with two separate tasks: to account for the generation mechanisms of such emissions and to explain their means of propagation through the ionosphere to the point of detection. Many diverse natural and artificial causes have been attributed to the generation of emissions from the ionosphere. Such triggering mechanisms include lightning, ionosphere electrojets, rocket engine exhausts, nuclear explosions and even disturbances originated on the surface of the distant sun. Various means are also developed to explain the propagation of emission signals through the ionospheric medium in the forms of hydromagnetic, electromagnetic, electrostatic or neutral wave modes. Elaborate schemes involving wave interactions can account for the excitation of other modes of oscillation by the emission signal, thus enabling the disturbance signal to be transmitted via more than one wave mode or via intermediary modes which cease to exist at the point of detection. Examples of such wave interactions are the generation of ionospheric Alfvén waves by parametric interaction between Whistler waves[16] and the generation of electromagnetic oscillations via wave coupling by hydromagnetic waves from nuclear explosions[25]. The purpose of the present work is to investigate the collisional coupling between low-frequency ion-acoustic waves and neutral sound waves in the ionospheric diffuse layer and the feasibility of such wave coupling as a mechanism for transmitting disturbance signals from the upper ionosphere to the ground for detection.

The low-frequency ion-acoustic waves under consideration can be excited under many situations in the ionosphere. It is well known in the equatorial and auroral electrojets, where currents are particularly strong, electron drift velocities relative to the ions often exceed the mean thermal speed of the ions, causing fluctuations to appear in the form of ion-acoustic wave[5, 14, 32, 39]. Numerous other regions exist in the ionosphere where fast-electron inhomogenities can give rise to the two-stream instability[37]. Also in the presence of the large amount of negative ions in the nocturnal ionosphere, even very modest electron drift was sufficient to cause low frequency acoustic oscillations[12]. In the present investigation, it is assumed that a triggering mechansim for the low frequency ion-acoustic wave exists. This mechanism may have arisen from any of the many natural or artificial causes. We shall however, restrict this study to cases where the disturbances are relatively weak so that shocks and other non-linear phenomena do not occur.

The propagation of the ensuing ion-acoustic wave away from the region of the disturbance is the consequence of the thermal fluctuations of the plasma particles and the inertia of the ions. The thermal fluctuations cause the plasma to resist being compressed, tending to return it to its original state, and the inertia of the ions causes the motion to "overshoot", thus providing the two requisites for wave oscillation. Although the thermal motions are random, the averaged outcome is not isotropic since particles moving in the direction of the wave vector will encounter a greater momentum change, thus a net tendency to carry the disturbance in the direction of the wave. As this ion-acoustic wave propagates downward and traverses the ionospheric diffuse layer, the collisions between the charged particles themselves become less and less frequent, due to the decreasing plasma density. Consequently this acoustic mode will begin to attenuate as its wavelength becomes comparable to the charged particles' collisional mean-free-paths. Also as the ion-acoustic wave penetrates deeper into the diffuse layer, the charged particles will encounter more and more frequently with the neutral molecules. A neutral acoustic wave may be excited as energy and momentum are transferred to the neutrals through these charged particle-neutral particle collisions. The excited wave will thus be able to continue the transmission of the disturbance signal into the neutral atmosphere. Such a wave coupling occurs if for a given frequency, the graphs of the phase velocity for the two acoustic modes cross as a function of position. In other words, somewhere in the diffuse layer these two wave modes can propagate in phase over a sufficient distance in order for the incident ion-acoustic wave to excite and reinforce the neutral oscillation.

This mechanism of transmission might possibly also apply to the propagation of "sounds" produced by meteors. It is known that the strong shock waves formed by large meteors as they enter the atmosphere decay to pressure waves capable of being detected on the ground most often in the low frequency infrasound range and more rarely in the audio frequency range[33]. But numerous smaller meteors are completely destroyed by ablation and slowed down to small velocities long before a shock is formed[30]. The chance of detecting "sound" disturbances caused by these meteors would be nil since the signal would be too weak to survive the region in the upper atmosphere where the attenuation of the neutral wave is severe. If, however, in addition to the neutral wave, the disturbance energies were stored in other forms as well (assuming that the ion-acoustic disturbances were also produced by the meteor in the present case), the "sounds" might be able to endure the region of high neutral attenuation. The neutral mode will be strongly attenuated but the ion-acoustic mode should be almost unaffected. The mechanism discussed here can be applied to explain the conversion of the survived ion-acoustic wave to the neutral mode at a lower altitude where neutral attenuation is much less. Utilizing such a scheme for wave transmission, the detection of "sounds" from the weaker meteors would probably be much more likely.

The coupling of ion and neutral acoustic waves in an inhomogeneous layer depicting the transition from weakly ionized plasma to neutral

II. THE ATMOSPHERE MODEL AND THE THREE FLUID EQUATIONS

The existence of a thick mantle of stratified ionized gas surrounding the Earth's atmosphere, has been known as early as 1901. And yet, our knowledge and understanding of the complicated processes leading to the creation and preservation of such layers and their numerous spatial, diurnal, seasonal, as well as sporadic variations are still being increased and modified as more rocket and satellite measurements are made. In this section, some of the now established features will be briefly described and be utilized in the formulation of a simplified model of the ionosphere diffuse layer. The equilibrium state will also be discussed.

We now know the ionosphere to consist of at least four different overlapping layers and numerous sub-layers. Various principal production and loss processes apply in these different strata[34] to maintain an approximate charged particle distribution as shown in Fig. 2-1. The law describing this distribution was first derived by Chapman (Refs. 9, 10, 11) and later extended and refined by Nicolet[28] and others. The simplest version of the Chapman theory gives a charged particle profile described by[7]:

$$N = N_0 \exp \frac{1}{2}\left[1 - \frac{z - z_0}{H} - \sec \chi \exp\left\{-\frac{z - z_0}{H}\right\}\right]. \tag{2.1}$$

where H = scale height,

χ = angle between the zenith and the path at which sun's radiation enters the atmosphere,

and N_o = particle density at height z_o.

Over limited regions of height, N can be accurately approximated by even simpler expressions. For example, a parabolic profile is often used near the F_2 maximum, and in the ionospheric diffuse layer (lower part of the Chapman layer) an exponential variation is quite accurate[7].

Since the presence of even a very small amount of space charge can cause enormous electrical forces, which, in the case of the ionosphere would prevent a stable steady state, the ionized gas is almost electrically neutral, i. e. there are as many positive ions as negatively charged particles per unit volume. Electrons and some negative ions formed via electron attachment to the neutral molecules make up the negative particle population.

These ions, positive and negative, are typically 60, 000 times heavier than the electrons, and as is often the case in laboratory plasmas, this large mass difference causes the electrons to have a higher temperature than the heavier ions and neutral particles. A classical result from the analysis of encounters between charged particles is that the collisional energy exchange between an electron gas with mass m and an ion gas with mass M is less efficient by a factor m/M than that between two groups of electrons at a corresponding temperature

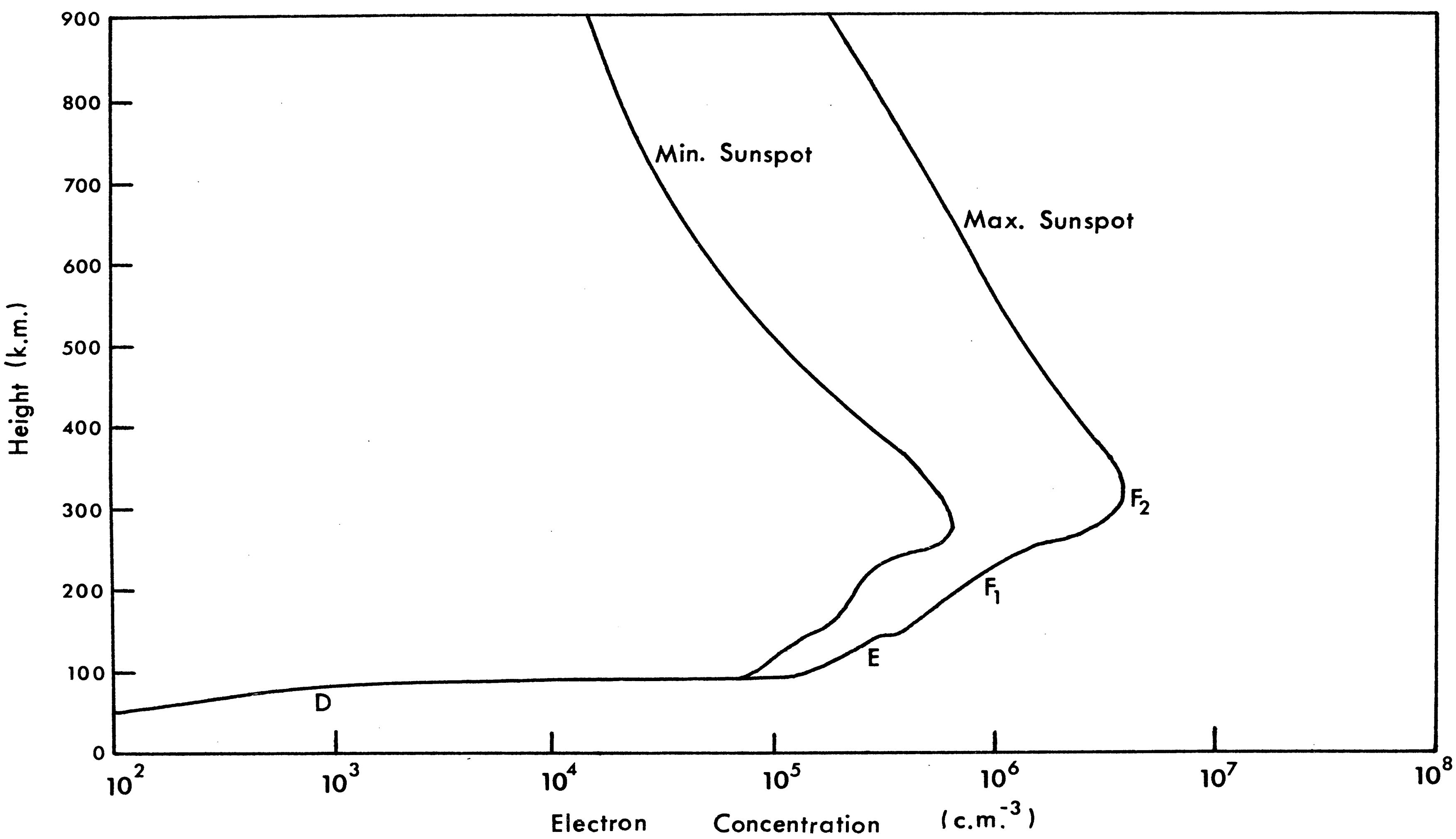

Figure 2-1. Normal Daytime Charge Particle Distribution in the Ionosphere.

differential. Thus the ambient ionosphere electron thermalization time is much shorter when they are excited by photo-electrons from solar radiation than that between the electrons and heavier particles, giving rise to the higher electron temperature. The effects of electron-ion collisions and the presence of a strong electric field can also cause a slightly higher ion temperature than that of the neutrals[27]. However, this difference in temperature is so small, one can assume that the ions and neutral particles have equal temperatures. The temperatures of different particle species in the ionosphere are presented in Fig. 2-2.

Like the charged particles, the dynamic conditions and the energy balance of the neutral atmosphere are very complicated, since many physical and chemical processes are taking place there. Nevertheless, one can draw from the data collected by balloons and rockets an averaged approximate atmospheric condition, often called the U. S. Standard Atmosphere. These distributions presented in Fig. 2-3 show that the temperature distribution is constant over a considerable range of altitudes while the neutral particle density distribution is approximately exponential.

The "effective" rates of collisions between charged particles and neutral atmospheric air molecules can also be closely described by an exponential variation with height since these rates depend upon the number density of the neutral particles[7]. In Fig. 2-4, several plots of charged particle-neutral particle collision frequencies versus height z are presented.

The model used in the present work to represent wave propagation through the ionospheric diffuse layer is shown in Fig. 2-5. The medium consists of a mixture of singly ionized plasma and neutral gas whose particle densities vary exponentially in the z-direction so that the medium is partially ionized for $z > 0$ and neutral for $z \leq 0$. A downward propagating ion-acoustic wave of unit amplitude is launched at $z = \infty$. The source of distrubance is assumed to be sufficiently above the diffuse layer, so that when the wave reaches the region of interest, the radius of curvature of the wave front is so large that a plane wave assumption is justified for the ion-acoustic wave. Encountering ever decreasing charged particle densities and increasing collisions with the neutrals this wave, as it propagates downward, will be continuously reflected and attenuated until no propagation is possible for $z \leq 0$. Instead, a neutral wave will be excited, as momentum and energy is transferred to the neutrals via ion-neutral collisions and electron-neutral collisions, and this wave will be transmitted downward into the neutral atmosphere. A measure of the amount of wave coupling via this collisional mechanism is provided by the amplitude of the resultant neutral wave as compared to the initial amplitude of the incident wave.

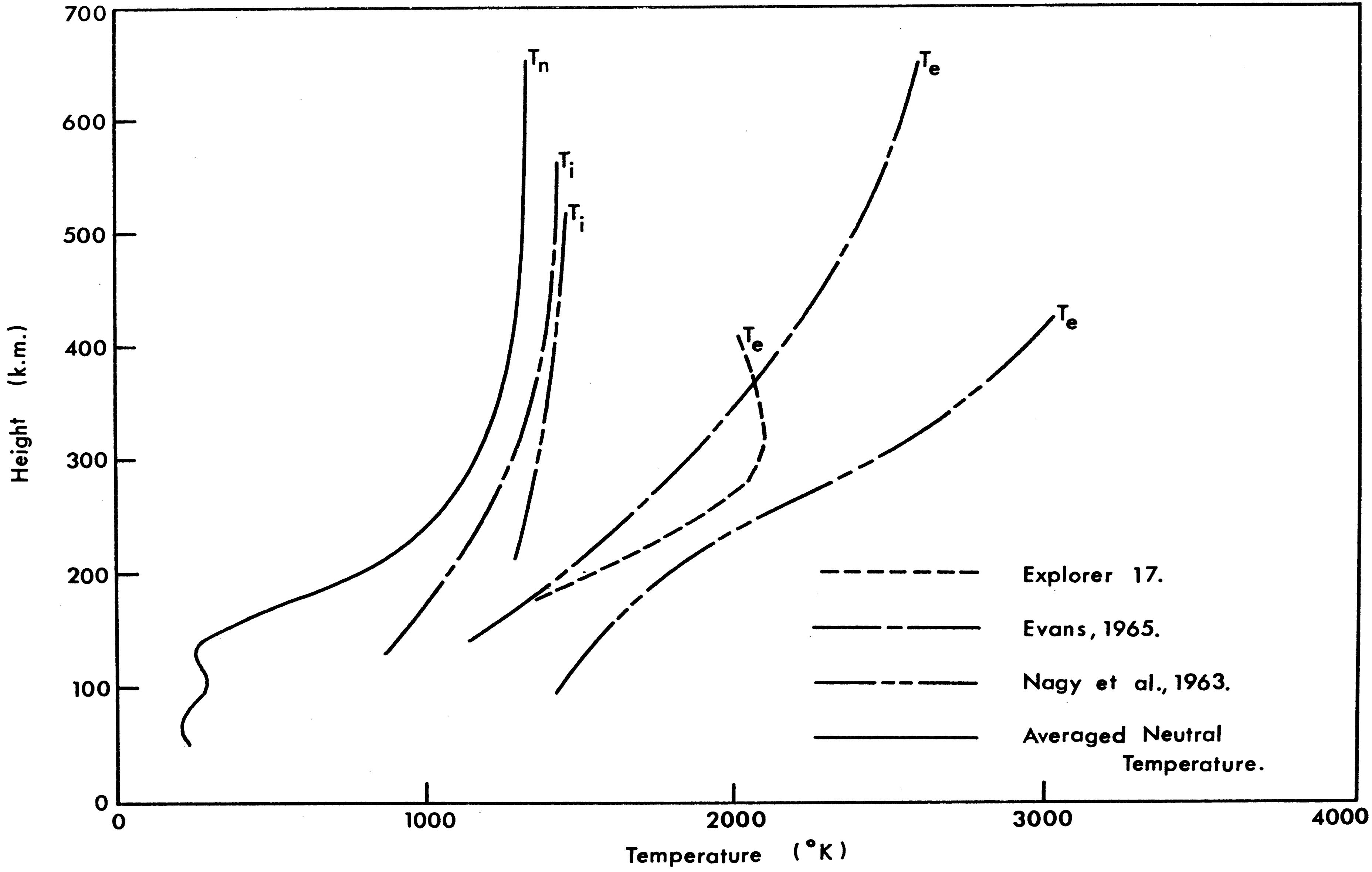

Figure 2-2. Electron, Ion, and Neutral Temperature Distributions in the Ionosphere.

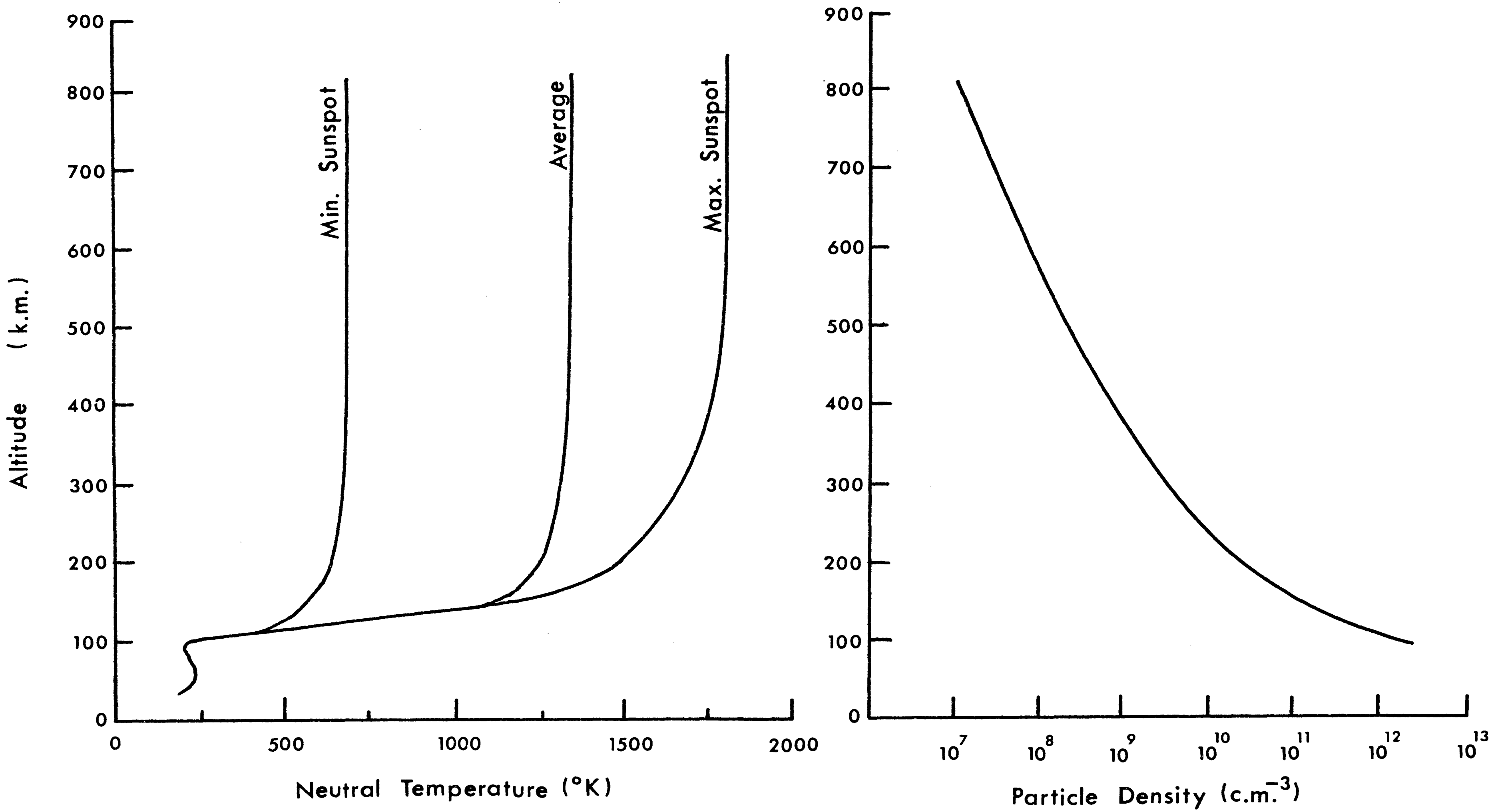

Figure 2-3. U.S. Standard Atmosphere 1962.

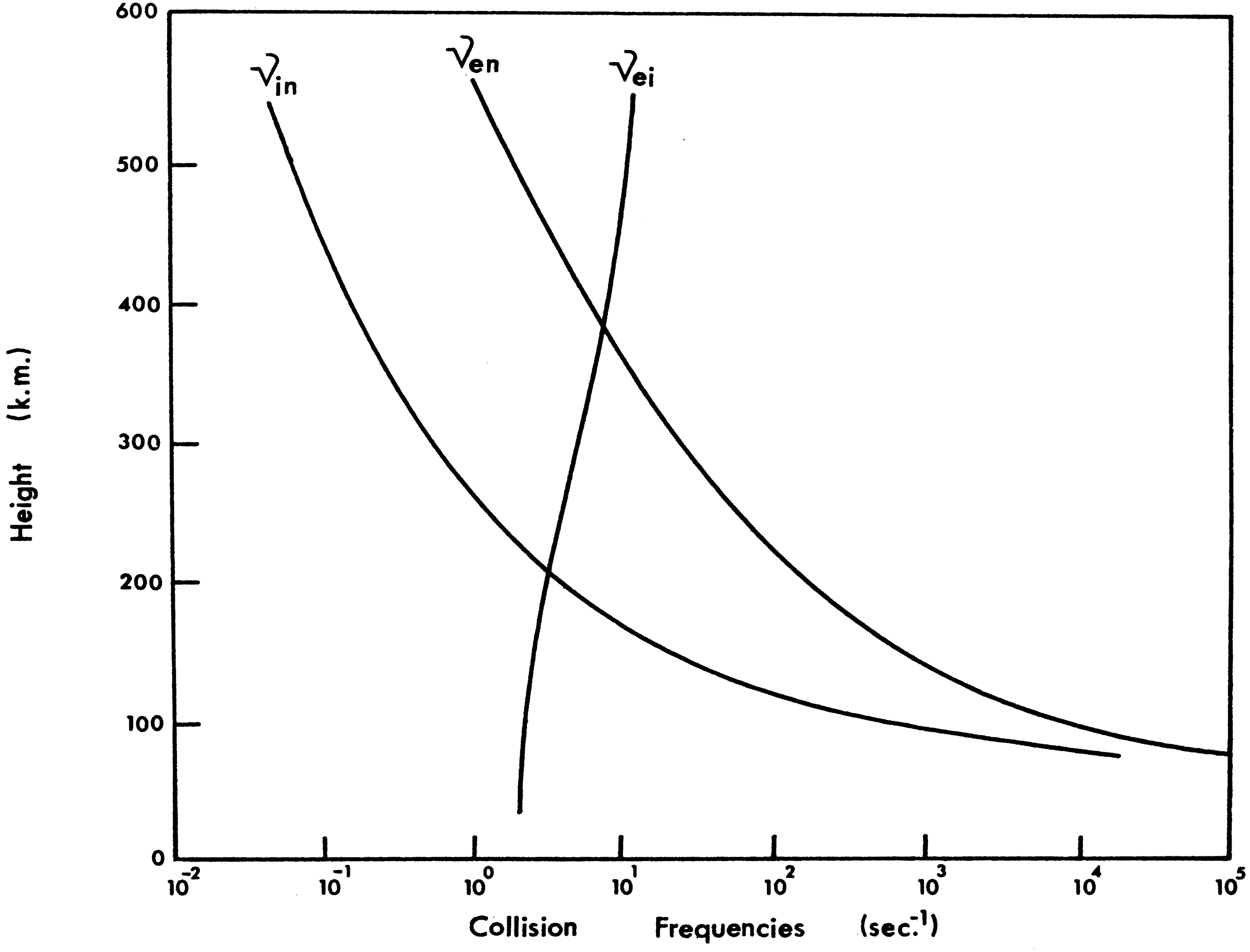

Figure 2-4. Collision Frequencies versus Altitude.

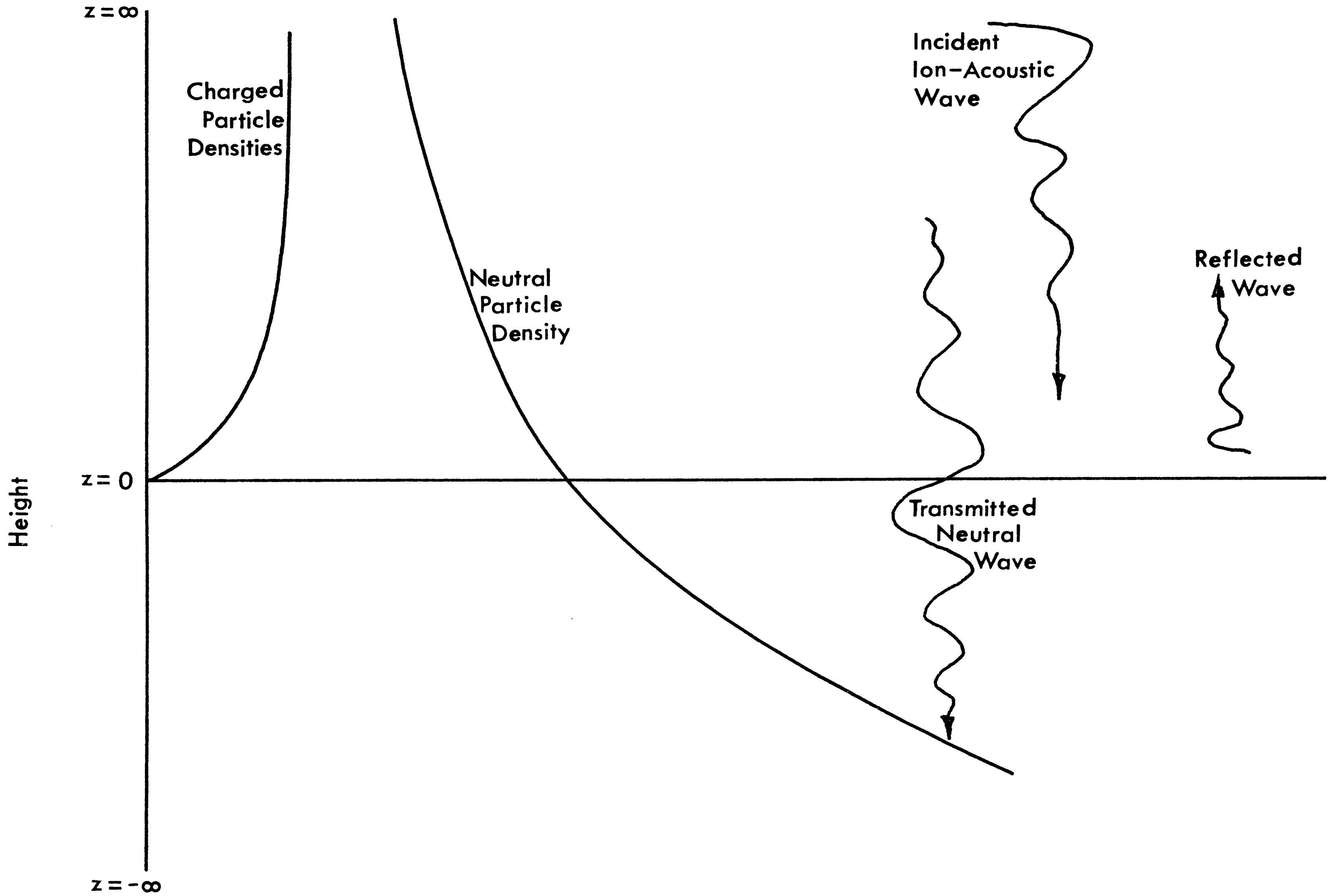

Figure 2-5. Model Representing Wave Propagation Through the Ionospheric Diffuse Layer.

The basic fluid equations describing the dynamics of a continuum in the z-direction are given by the conservation of mass:

$$\frac{\partial n_s}{\partial t} + \frac{\partial (n_s v_s)}{\partial z} = S_s \tag{2.2}$$

the conservation of momuntum:

$$n_s m_s \frac{\partial v_s}{\partial t} + n_s m_s v_s \frac{\partial v_s}{\partial z} = -\frac{\partial (n_s T_s)}{\partial z} + n_s e_s E$$
$$+ n_s m_s g + \sum_r n_s m_s \nu_{sr} (v_s - v_r) \tag{2.3}$$

and the conservation of energy:

$$\frac{\partial}{\partial t}\left[n_s\left(\tfrac{1}{2}m_s v_s^2 + \tfrac{3}{2}T_s\right)\right] + v_s\frac{\partial}{\partial z}\left[n_s\left(\tfrac{1}{2}m_s v_s^2 + \tfrac{3}{2}T_s\right)\right] =$$
$$-n_s\left(\tfrac{1}{2}m_s v_s^2 + \tfrac{3}{2}T_s\right)\frac{\partial v_s}{\partial z} - \frac{\partial}{\partial z}\left[n_s T_s v_s\right] - \frac{\partial q_s}{\partial z}$$
$$-\sum_r m_s \nu_{sr} n_s (v_s - v_r)^2 - 3\sum_r{}' \frac{m_s}{m_r}\nu_{sr} n_s (T_s - T_r) \tag{2.4}$$

where the subscript s represents ion, electrons and neutrals. T_s, n_s, m_s and v_s symbolize, respectively, temperature, particle density, mass and velocity of species s. E is the electric field intensity while e_s is the electric charge ($e_e = -e_i$ and $e_n = 0$). S_s represents the effective source that might arise from photoionization, recombination, etc. The L. H. S. of Eq. (2. 3) represents the rate of change of momentum of species S in a volume element moving with the fluid. On the R. H. S., the first term represents the net pressure force acting on the surface of the volume by species s; the second term represents the electrostatic force on species s per unit volume; the third term represents the gravitational body force per unit volume, and the fourth term represents the change in the momentum of species s due to collisions with particles of species r. The terms affecting the rate of change of energy per unit volume of species s appear on the R. H. S. of Eq. (2. 4). The first term accounts for the loss of energy due to those particles of species s flowing out of the volume element; the second term is the work done by the volume element against the pressure force of species s; the third term represents the energy carried away by thermal conduction, and the last two terms represent the energy loss during collisions between the particles of species s and those of species r. The collision frequencies ν_{sr} are assumed to be of the forms[4, 29, 35]:

$$\nu_{en} = 5.4 \times 10^{-10} \, n_n \, T_e^{1/2} \quad \text{sec.}^{-1} \tag{2. 5}$$

$$\nu_{ne} = \frac{m_e}{m_n} \frac{n_e}{n_n} \nu_{en} \tag{2. 6}$$

$$\nu_{ei} = \frac{4}{3} \sqrt{2\pi} \, n_i \frac{e^4 \ln \Lambda}{m_e^{1/2} \, T_e^{3/2}} \quad \text{sec.}^{-1} \tag{2. 7}$$

$$\nu_{ie} = \frac{m_e}{m_i} \nu_{ei} \tag{2. 8}$$

$$\nu_{in} = 2.6 \times 10^{-9} \, n_n \left(\frac{\alpha_o}{\mu_A} \right)^{1/2} \quad \text{sec.}^{-1} \tag{2. 9}$$

and

$$\nu_{ni} = \frac{n_i}{n_n} \nu_{in} \tag{2.10}$$

where α_o ≡ atomic polarizability in units of 10^{-24} cm^3

and μ_A ≡ reduced mass in a. m. u.

To complete the fluid model, the plasma is assumed to be in a quasi-neutral state; thus only low frequency, long wave-length waves are considered where $n_i \simeq n_e = n$ and $\mathbf{E} = -\nabla \psi$. ψ is the electrostatic potential. The former assumption implies that $k^2 \lambda_D{}^2 \ll 1$ where $\lambda_D = \left[T/(4\pi n e^2)\right]^{1/2}$ is the Debye length and the latter assumption implies $\nabla \times \mathbf{E} = 0$.

be linearized. This means that the electric field strength, the pressure field strengths, and the particle density variations are considered as first order perturbation quantities.

In all situations where wave propagation is considered, it is very important to first solve for the equilibrium (zeroth order) state conditions, but in the case of the ionosphere, as mentioned earlier, this is extremely difficult to accomplish due to the many complex and still unclear physical and chemical processes involved. Nevertheless, it is not difficult to establish from experimental observation of the ionosphere that such a zeroth-order state does indeed exist however complicated it may be. Retaining the most important features of the ionosphere, a simplified equilibrium state will be utilized.

The charged particles' density and temperature distributions shown in Figs. 2-1 and 2-2 result in pressure gradients downward in the direction of the gravitational force, preventing equilibrium unless zeroth order drifts and source distributions are present. Such drifts and sources do indeed exist in the ionosphere. Although they play an important role in the zeroth order solution, the effects of these drifts can generally be neglected in the first order equations when one considers wave oscillations whose wavelengths are short compared to the characteristic lengths associated with the steady-state gradients. The former are usually a few kilometers in length while the latter are of the order of 100 kilometers[18]. In neglecting the charged particle drifts in the perturbed equations, their driving forces, the charged particle density and temperature gradients, are also neglected in order to be consistent.

The effects of the ionization sources and recombination sinks are also neglected in the first order equations since the charged particle densities in the region of interest are too low to cause the type of wave instability and propagation modification as described by Aldridge[3].

Unlike the charged particles the neutral particles need not drift to maintain equilibrium. The density gradient is balanced exactly by the gravitational force while the thermal conduction term, due to the neutral temperature gradient, is balanced by the "source" term: $3(m_e/m_i)\,\nu_{en0}\,n_{e0}\,(T_{e0} - T_{n0})$.* Again since typical wavelengths are much shorter than the length associated with the neutral temperature inhomogeneity, this gradient is also neglected in the first order analysis of wave propagation. The neutral density gradient, however, is retained since it contributes greatly to the damping of the resultant neutral wave, as mentioned in Section I. The height dependences of particle densities and charged particle-neutral collision frequencies were also retained, for they cause the increase in the collisional damping of the ion-acoustic waves as they penetrate through the diffuse layer and the possible collisional excitation of neutral waves.

The magnetic field has been neglected; thus, only waves parallel to the geomagnetic field lines are considered, such as those near the magnetic poles. Modification of the incident ion-acoustic wave propagating obliquely to the B-field lines is discussed in Section IV.

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*Possible if $(dT_{n0}/dz) > 0$ and $q_n(\infty) = 0$ so that

$$\int_h^\infty 3(m_e/m_i)\,\nu_{en0}\,n_{e0}\,(T_{e0} - T_{n0})\,dz = -\,q_n(h) = K\,(dT_{n0}/dz)\ .$$

Equations (2.11) to (2.18), below, summarize the assumed forms of the zeroth order variables used in the first order equations. These equations do not represent exact solutions to the zeroth order state, but must be viewed in conjunction with wave propagation in a slowly varying inhomogeneous medium such as the diffuse layer of the ionosphere.

$$n_{no} = N_n \exp\left(-z/h_n\right) \quad , -\infty < z < \infty \tag{2.11}$$

$$n_{io} = n_{eo} = N_i \left[1 - \exp\left(-z/h_i\right)\right], \quad 0 < z < \infty$$

$$= 0, \quad -\infty < z < 0 \tag{2.12}$$

$$\frac{\nu_{eno}}{\omega} = \Omega_{en} \exp\left(-z/h_n\right), \quad 0 < z < \infty \tag{2.13}$$

$$\frac{\nu_{ino}}{\omega} = \Omega_{in} \exp\left(-z/h_n\right), \quad 0 < z < \infty \tag{2.14}$$

$$T_{eo} = \text{const}_1 \tag{2.15}$$

$$T_{no} \simeq T_{io} = \text{const}_2 < T_{eo} \tag{2.16}$$

$$\vec{B} = 0 \tag{2.17}$$

$$V_{eo} \simeq V_{io} \simeq V_{no} = 0\,. \tag{2.18}$$

III. DERIVATION OF THE COUPLING EQUATIONS

In general, three acoustic wave modes exist in a partially ionized gas. Two of the waves, the electron acoustic wave and the ion acoustic wave, are linked by the self-consistent electric field. For frequencies less than the ion plasma frequency ω_i, electrons are tied strongly to the ions, so that these two modes can be considered as one wave (hereafter referred to as the charged particle wave or the ion-acoustic wave). The propagation of the third mode, neutral wave, is nearly independent of the charged particle wave for frequencies much higher than the ion-neutral collision frequency ν_{in} since the influence of collisions between different kinds of particles is relatively unimportant. But as the frequency is lowered, to the order of ν_{in}, the effects of these interactions can no longer be ignored and the motions of the neutrals are coupled to the charged particles. The set of equations representing the collisionally coupled motions of these low-frequency ion-acoustic and neutral sound modes in an inhomogeneous, partially ionized gas mixture is presented in this section. Special consideration is given to the transfer of momentum via ion-neutral collisions and the energy transfer by electron-neutral collisions.

Let the quantities in Eqs. (2.1) to (2.10) have small oscillatory deviations which are proportional to $\mathcal{P}_1(z) \exp(i\omega t)$, where the variables with subscript 1 are small such that products of perturbed quantities can be neglected. Then, linearizing the system of equations and defining

$$F_s(z) = n_{so}(z)\, v_{s1}(z)$$

and $$G_s(z) = n_{so}(z)\, T_{s1}(z),$$

one obtains, after some algebraic manipulation using continuity equations to eliminate $n_{s1}(z)$, the sum of ion and electron momentum equations:

$$\left[\frac{iC_i^2}{\omega}\right] F_e''(z) + \left[i\omega + \nu_{ino} + \nu_{eno}\right] F_e(z) + \left[\frac{1}{m_i}\right] G_e'(z) + \left[\frac{1}{m_i}\right] G_i'(z) + \left[-R_n \nu_{ino} - R_n \nu_{eno}\right] F_n(z) = 0 \qquad (3.1)$$

the ion energy equation:

$$\frac{3}{2} i\omega G_i(z) + T_{io} F_e'(z) = 0 \qquad (3.2)$$

the electron energy equation:

$$\left[\frac{3}{2} i\omega + 3\frac{m_e}{m_i}\nu_{eno} + \frac{3}{2}\frac{m_e}{m_i}\nu_{eno}(1-R_T)\right] G_e(z) + \left[-3\frac{m_e}{m_i}\nu_{eno} R_n\right] G_n(z) + \left[T_{eo} + i3\frac{m_e}{m_i}\frac{\nu_{eno}}{\omega}(T_{eo} - T_{no})\right] F_e'(z) + \left[i3\frac{m_e}{m_i} R_n \frac{\nu_{eno}}{\omega}(T_{eo} - T_{no})\right] F_n'(z) = 0 \qquad (3.3)$$

the z-component neutral momentum equation:

$$\left[\frac{iC_n^2}{\omega}\right]F_n''(z)+\left[\frac{-iT_{no}}{m_n\omega}\frac{d\ln n_{no}}{dz}\right]F_n'(z)+\left[i\omega+\nu_{nio}+\nu_{neo}\right]F_n(z)$$
$$+\left[\frac{1}{m_n}\right]G_n'(z)-\left[\frac{\nu_{nio}}{R_n}+\frac{\nu_{neo}}{R_n}\right]F_e(z)=0 \qquad (3.4)$$

and the neutral energy equation:

$$\left[i\frac{3}{2}\omega+3\frac{m_e}{m_n}\nu_{eno}R_n\right]G_n(z)+\left[-3\frac{m_e}{m_n}\nu_{eno}-\frac{3}{2}\frac{m_e}{m_n}\nu_{eno}(1-R_T)\right]G_e(z)$$
$$+\left[T_{no}-i3\frac{m_e}{m_n}\frac{\nu_{eno}}{\omega}(T_{eo}-T_{no})R_n\right]F_n'(z)-i3\frac{m_e}{m_n}\frac{\nu_{eno}}{\omega}(T_{eo}-T_{no})F_e'(z)$$
$$=0 \qquad (3.5)$$

Here the following notation was used:

$$C_i^2\equiv\frac{T_{oi}+T_{oe}}{m_i},\quad C_n^2\equiv\frac{T_{no}}{m_n},\quad R_n\equiv\frac{n_{eo}}{n_{no}}(z)\quad\text{and}\quad R_T\equiv\frac{T_{no}}{T_{eo}}.$$

In deriving the above set of equations, it is also assumed that the effects of viscosity and heat conduction, the electron-neutral momentum transfer and the electron-ion momentum and energy exchanges can all be neglected. The dominant momentum and energy transport processes are due to ion-neutral collisions and electron-neutral collisions respectively. For plane waves, the viscosity and heat conduction terms are proportional to k^2, where k is the magnitude of the wave vector, and clearly are insignificant whenever k is small. The contributions of electron-neutral collisions and ion-neutral collisions to damping in the momentum equations are given by $\nu_{en0}\, m_e(v_{e1} - v_{n1})$ and $\nu_{in0}\, m_i(v_{i1} - v_{n1})$. Since $v_{e1} \simeq v_{i1}$ and $\nu_{in0}/\nu_{en0} \simeq (m_e/m_i)^{1/2}$, the ion contribution far exceeds that of the electrons thus justifying the omission of the electron-neutral momentum term. Finally the electron-ion momentum and energy exchanges are neglected since in the lower ionosphere such encounters are rare due to the relatively low degree of ionization there.

Combining Eqs. (3. 1), (3. 2), and (3. 3), to eliminate $G_e(z)$ and $G_i(z)$, one obtains Eq. (3. 6) (see page 27). Similarly, eliminating $G_n(z)$ between Eq. (3. 4) and (3. 5), one gets Eq. (3. 7) (see page 28).

The ion-acoustic wave is identified by the solution to Eq. (3. 6). Modification of this wave by collisions with neutrals enters through terms involving $G_n(z)$ and $F_n(z)$. Similarly the solution to Eq. (3. 7) gives the neutral sound wave, coupled to the charged particle motions through terms involving $G_e(z)$ and $F_e(z)$. To obtain these solutions analytically is extremely difficult. However it will be shown in Sections V and VI that in the case of the ionosphere diffuse layer wave coupling problem, the resulting amplitude of the neutral wave is much smaller than that of the driving incident ion-acoustic wave so that the effect of the former on the latter is small and can be neglected to the first approximation. This simplification enables one to seek the incident ion-acoustic wave solution independent of the neutral solution.

$$
\begin{aligned}
& F_e''(z)\left\{
\begin{array}{l}
\left[\dfrac{iC_i^2}{\omega}+i\dfrac{2}{3}\dfrac{T_{oi}}{m_i\omega}\right]\cdot\left[i\dfrac{3}{2}+\dfrac{m_e}{m_i}\dfrac{\nu_{eno}}{\omega}\left(\dfrac{9}{2}-\dfrac{3}{2}R_T\right)\right] \\
-\left[\dfrac{T_{eo}}{m_i\omega}+i3\dfrac{m_e}{m_i^2}\dfrac{\nu_{eno}}{\omega^2}\left(T_{eo}-T_{no}\right)\right]
\end{array}
\right\} \\
& +F_e'(z)\left\{
\begin{array}{l}
-\left[i\dfrac{3}{\omega}\dfrac{m_e}{m_i^2}\left(\dfrac{\nu_{eno}}{\omega}\right)'\left(T_{eo}-T_{no}\right)\right] \\
+\left[\dfrac{T_{eo}}{m_i\omega}+i3\dfrac{m_e}{m_i^2}\dfrac{\nu_{eno}}{\omega^2}\left(T_{eo}-T_{no}\right)\right]\cdot\left[\dfrac{m_e}{m_i}\left(\dfrac{\nu_{eno}}{\omega}\right)'\left(3-R_T\right)\right]\cdot\left[i+\dfrac{m_e}{m_i}\dfrac{\nu_{eno}}{\omega}\left(3-R_T\right)\right]^{-1}
\end{array}
\right\} \\
& +F_e(z)\left\{\left[i\omega+\nu_{ino}+\nu_{eno}\right]\cdot\left[i\dfrac{3}{2}+\dfrac{m_e}{m_i}\dfrac{\nu_{eno}}{\omega}\left(\dfrac{9}{2}-\dfrac{3}{2}R_T\right)\right]\right\} \\
& = G_n'(z)\left\{-3\dfrac{m_e}{m_i^2}\dfrac{\nu_{eno}}{\omega}R_n\right\}+F_n''(z)\left\{i3\dfrac{m_e}{m_i^2}R_n\dfrac{\nu_{eno}}{\omega^2}\left(T_{eo}-T_{no}\right)\right\} \\
& +G_n(z)\left\{
\begin{array}{l}
-3\dfrac{m_e}{m_i^2}\left(\dfrac{\nu_{eno}}{\omega}R_n\right)' \\
-\left[3\dfrac{m_e}{m_i^2}\dfrac{\nu_{eno}}{\omega}R_n\right]\cdot\left[\dfrac{m_e}{m_i}\left(\dfrac{\nu_{eno}}{\omega}\right)'\left(\dfrac{9}{2}-\dfrac{3}{2}R_T\right)\right]\cdot\left[i\dfrac{3}{2}+\dfrac{m_e}{m_i}\dfrac{\nu_{eno}}{\omega}\left(\dfrac{9}{2}-\dfrac{3}{2}R_T\right)\right]^{-1}
\end{array}
\right\} \\
& +F_n'(z)\left\{
\begin{array}{l}
-\left[i\dfrac{3}{\omega}\dfrac{m_e}{m_i^2}\left(\dfrac{\nu_{eno}}{\omega}R_n\right)'\left(T_{eo}-T_{no}\right)\right] \\
-\left[3\dfrac{m_e}{m_i^2}\dfrac{\nu_{eno}}{\omega^2}R_n\left(T_{eo}-T_{no}\right)\right]\cdot\left[\dfrac{m_e}{m_i}\left(\dfrac{\nu_{eno}}{\omega}\right)'\left(\dfrac{9}{2}-\dfrac{3}{2}R_T\right)\right]\cdot\left[i\dfrac{3}{2}+\dfrac{m_e}{m_i}\dfrac{\nu_{eno}}{\omega}\left(\dfrac{9}{2}-\dfrac{3}{2}R_T\right)\right]^{-1}
\end{array}
\right\} \\
& +F_n(z)\left\{R_n\left[\nu_{ino}-\nu_{eno}\right]\cdot\left[i\dfrac{3}{2}+\dfrac{m_e}{m_i}\dfrac{\nu_{eno}}{\omega}\left(\dfrac{9}{2}-\dfrac{3}{2}R_T\right)\right]\right\} \qquad (3.6)
\end{aligned}
$$

$$F_n''(z)\left\{\begin{array}{l}\dfrac{iC_n^2}{\omega}\left[i\dfrac{3}{2}+3\dfrac{m_e}{m_i}\dfrac{\nu_{eno}}{\omega}R_n\right]\\ -\left[\dfrac{T_{no}}{m_i\omega}-i\dfrac{3}{\omega}\dfrac{m_e}{m_n^2}\dfrac{\nu_e}{\omega}R_n(T_{eo}-T_{no})\right]\end{array}\right\}$$

$$+F_n'(z)\left\{\begin{array}{l}\dfrac{-iT_{no}}{m_n\omega}\dfrac{d\ln n_{no}}{dZ}\left[i\dfrac{3}{2}+3\dfrac{m_e}{m_i}\dfrac{\nu_{eno}}{\omega}R_n\right]\\ +\left[\dfrac{i3}{\omega}\dfrac{m_e}{m_i^2}\left(\dfrac{\nu_{eno}}{\omega}\right)'(T_{eo}-T_{no})\right]\\ -\left[\dfrac{T_{no}}{m_i\omega}-i3\dfrac{m_e}{m_i^2}\dfrac{\nu_{eno}}{\omega^2}R_n(T_{eo}-T_{no})\right]\cdot\left[3\dfrac{m_e}{m_i}\left(\dfrac{\nu_{eno}}{\omega}R_n\right)'\right]\cdot\left[i\dfrac{3}{2}+3\dfrac{m_e}{m_n}\dfrac{\nu_{eno}}{\omega}R_n\right]^{-1}\end{array}\right\}$$

$$+F_n(z)\left\{\left[i\omega+\nu_{nio}-\nu_{neo}\right]\cdot\left[i\frac{3}{2}+3\frac{m_e}{m_i}\frac{\nu_{eno}}{\omega}R_n\right]\right\}$$

$$=G_e'(z)\left\{-\frac{m_e}{m_i^2}\frac{\nu_{eno}}{\omega}\left(\frac{9}{2}-\frac{3}{2}R_T\right)\right\}+F_e''(z)\left\{-i3\frac{m_e}{m_i^2}\frac{\nu_{eno}}{\omega^2}(T_{eo}-T_{no})\right\}$$

$$+G_e(z)\left\{\begin{array}{l}\dfrac{-m_e}{\omega m_i^2}\left(\dfrac{\nu_{eno}}{\omega}\right)'\left(\dfrac{9}{2}-\dfrac{3}{2}R_T\right)\\ -\left[\dfrac{m_e}{m_i^2}\dfrac{\nu_{eno}}{\omega}\left(\dfrac{9}{2}-\dfrac{3}{2}R_T\right)\right]\cdot\left[3\dfrac{m_e}{m_i}\left(\dfrac{\nu_{eno}}{\omega}R_n\right)'\right]\cdot\left[i\dfrac{3}{2}+3\dfrac{m_e}{m_i}\dfrac{\nu_{eno}}{\omega}R_n\right]^{-1}\end{array}\right\}$$

$$+F_e'(z)\left\{\begin{array}{l}-i\dfrac{3m_e}{\omega m_i^2}\left(\dfrac{\nu_{eno}}{\omega}\right)'(T_{eo}-T_{no})\\ -\left[i3\dfrac{m_e}{m_i^2}\dfrac{\nu_{eno}}{\omega}(T_{eo}-T_{no})\right]\cdot\left[3\dfrac{m_e}{m_i}\left(\dfrac{\nu_{eno}}{\omega}R_n\right)'\right]\cdot\left[i\dfrac{3}{2}+3\dfrac{m_e}{m_i}\dfrac{\nu_{eno}}{\omega}R_n\right]^{-1}\end{array}\right\}$$

$$+F_e(z)\left\{\left[\frac{\nu_{nio}}{R_n}+\frac{\nu_{neo}}{R_n}\right]\cdot\left[i\frac{3}{2}+3\frac{m_e}{m_i}\frac{\nu_{eno}}{\omega}R_n\right]\right\}. \tag{3.7}$$

IV. THE INCIDENT ION-ACOUSTIC WAVE

Collisionally coupled low-frequency ion-acoustic and neutral sound waves in an inhomogeneous, partially ionized plasma are given by the solutions to the set of Eqs. (3.6) and (3.7). As mentioned earlier, the exact solutions to this type of coupled wave equations are extremely difficult to obtain analytically. However, if the dependent variables are such that the "coupling terms" are small over certain ranges of altitude, the method of successive approximations may be utilized. As a first approximation, the small coupling terms are neglected, and the ensuing uncoupled equations are solved independently. A better approximation to the original wave equations can then be obtained by substituting the first approximation solutions into the "coupling" terms and solving the resulting set of nonhomogeneous differential equations. This procedure can be repeated until desired accuracy is achieved. Such a method was used by Försterling[7,15] to obtain solutions for the vertical incidence of electromagnetic waves.

A variation of this method shall be used to get an estimate of the ion-acoustic and neutral wave coupling in the region of ionosphere-neutral atmosphere overlap. Since the amplitude of the neutral wave is initially much smaller than that of the driving ion-acoustic wave, one can, to a good approximation, neglect the effects of the neutral motion on the ion-acoustic wave. Dropping the terms on the right hand side of Eq. (3.6), one can solve for the charged particle motion without considering the feedback coupling of the neutral wave. The solution of this incident ion-acoustic wave can then be substituted into the "coupling" terms of Eq. (3.7) to obtain the resultant neutral oscillations.

By the time the amplitude of the neutral wave has grown sufficiently to invalidate the above assumption, the ion-acoustic wave will have damped so severely it will no longer have much influence on the outcome of the coupling nor on the final strength of the transmitted wave. From Fig. 4-1 showing the variation of wave amplitudes with height, one can also see that the range of altitudes over which these two wave modes interact is much larger than the range over which the assumption is invalid. Thus by neglecting the neutral wave motion feedback on the charged particle wave, one can still obtain a fairly accurate estimate of the final neutral wave amplitude.

In this section, the solution of the incident ion-acoustic wave is derived using the forementioned scheme.

Neglecting the effects of neutral motions, Eqs. (3.1), (3.2), and (3.3) reduce to

the momentum equation:

$$C_i^2 F_e''(z) - i\frac{\omega}{m_i} G_i'(z) - i\frac{\omega}{m_i} G_e'(z) + \left[\omega^2 - i\omega \nu_{ino}\right] F_e(z) = 0 \qquad (4.1)$$

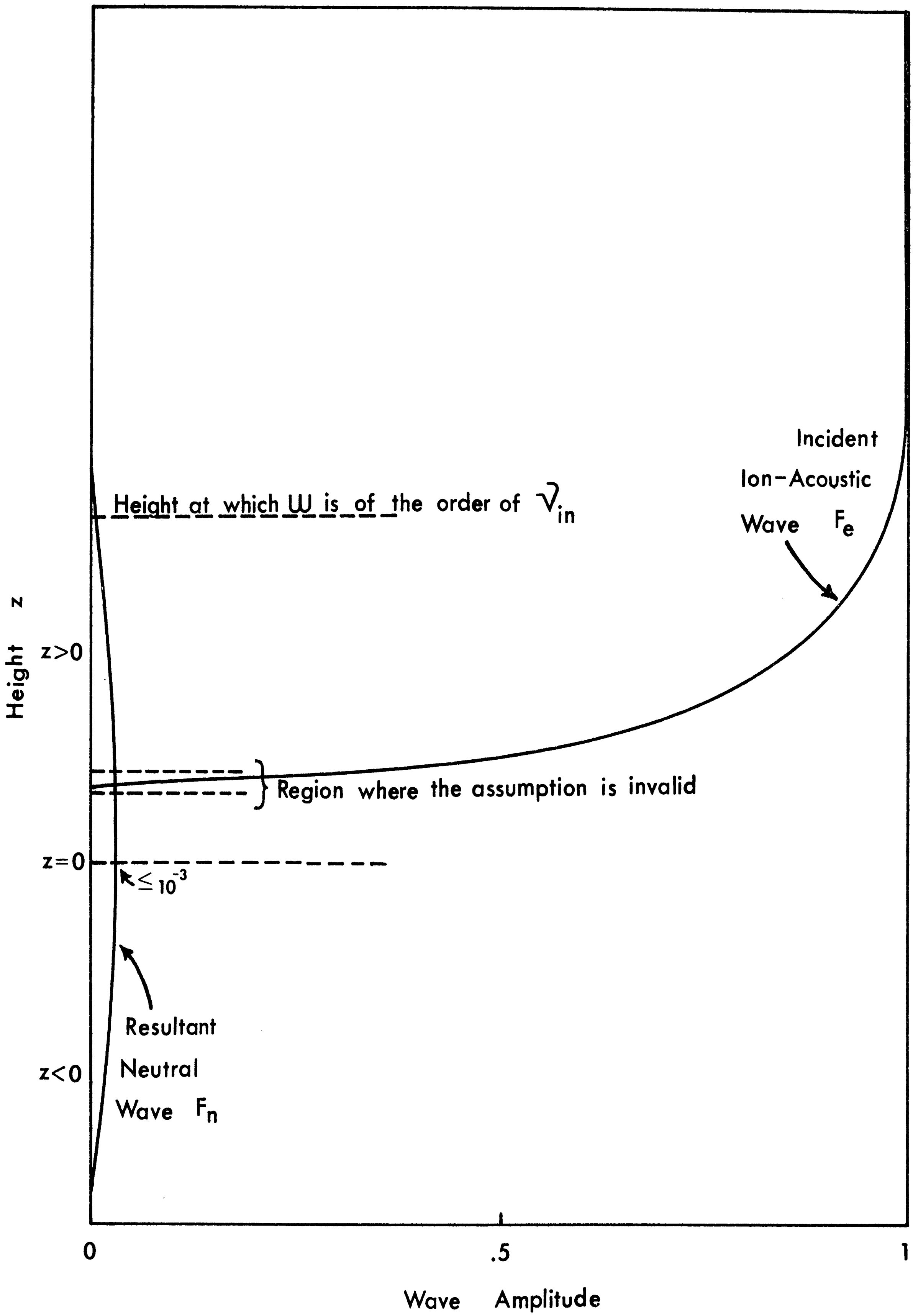

Figure 4-1. Variations of Wave Amplitudes as Function of Altitude.

the ion energy equation

$$G_{Ti}'(z) = i\frac{2}{3}\frac{T_{oi}}{\omega}F_e''(z) \tag{4.2}$$

and the electron energy equation

$$G_{Te}(z) = \frac{-\left[T_{oe} + i\,3\frac{m_e}{m_i}\frac{\nu_{eno}}{\omega}(T_{eo} - T_{no})\right]F_e'(z)}{\left[\frac{3}{2}i\omega + 3\frac{m_e}{m_i}\nu_{eno} + \frac{3}{2}\frac{m_e}{m_i}\nu_{eno}(1 - R_T)\right]} \,. \tag{4.3}$$

Assuming $(m_e^2/m_i^2)(\nu_{en0}^2/\omega^2)$ is much smaller than 1, the above set of equations can be combined to give

$$\begin{aligned} &\left[C_i^2 + \frac{2}{3}\frac{T_{oi}}{m_i} + \frac{2}{3}\frac{T_{oe}}{m_i} + i\frac{2}{3}\frac{m_e}{m_i}\frac{\nu_{eno}}{\omega}\left(6\frac{T_{oe}}{m_i} - 4\frac{T_{on}}{m_i}\right)\right]F_e''(z) \\ &+ \left[i\frac{2}{3}\frac{m_e}{m_i}\left(6\frac{T_{oe}}{m_i} - 4\frac{T_{on}}{m_i}\right)\left(\frac{\nu_{eno}}{\omega}\right)'\right]F_e'(z) \\ &+ \left[\omega^2 - i\omega\nu_{ino}\right]F_e(z) \simeq 0 \end{aligned} \tag{4.4}$$

Justification for neglecting $(m_e^2/m_i^2)(\nu_{en0}^2/\omega^2)$ compared to 1 in the incident ion-acoustic wave propagation problem can be shown by the following analysis. From the phase velocity diagram of low frequency acoustic waves in Fig. 4-2, one can see that a downward propagating ion-acoustic wave, with frequency ω, will be essentially non-dispersive until reaching an altitude where $\nu_{in0}(z)$ is of the order of ω, at which point the interaction with the neutrals will strongly attenuate the charged particle wave until no propagation is possible for lower altitude (where $\omega \ll \nu_{in0}$). Thus an ion-acoustic wave of frequency

$\simeq 1$ radian/sec can propagate only above an altitude $\simeq 300$ kilometers while a wave of frequency $\simeq .01$ radians/sec will exist only above 600 kilometers altitude. From Fig. 4-3, one can see that the value of $(m_e/m_i)\nu_{en0}$ is less than ω for altitudes at and above the points where ion-acoustic waves cease to exist. Thus $\left[(m_e/m_i)(\nu_{en0}/\omega)\right]^2 \ll 1$ is justified in the range of altitudes where the charged particle waves propagate. Terms of the order $(m_e/m_i)(\nu_{en0}/\omega)$ are not neglected since the important transfer of heat via electron-neutral collisions is of this order of magnitude.

Substituting the specified steady-state neutral and plasma density distributions and effective collision frequency relations given by Eqs. (2.11) through (2.14) into Eq. (4.4), one gets:

$$F_e''(z) + \left\{ -i\frac{2}{5}\frac{m_e}{m_i}\frac{\left(6\frac{T_{e0}}{m_i} - 4\frac{T_{n0}}{m_i}\right)\Omega_{en}}{C_i^2 h_n} \exp\left(\frac{-z}{h_n}\right)\right\} F_e'(z)$$
$$+\frac{3\omega^2}{5C_i^2}\left\{1 - i\left[\frac{2}{5}\frac{m_e}{m_i}\frac{\left(6\frac{T_{e0}}{m_i} - 4\frac{T_{n0}}{m_i}\right)\Omega_{en}}{C_i^2} + \frac{3}{5}\frac{\Omega_{in}}{C_i^2}\right]\exp\left(\frac{-z}{h_n}\right)\right\} F_e(z)$$
$$= 0 \qquad (4.5)$$

or in the non-dimensionalized form:

$$\tilde{F}_e''(\tilde{z}) + \left\{ -i\frac{2}{5}\frac{m_e}{m_i}\frac{\Omega_{en}(6-4R_T)}{(1+R_T)}\exp(-\tilde{z})\right\}\tilde{F}_e'(\tilde{z})$$
$$+\frac{3}{5}\frac{\omega^2 h_n^2 m_i}{T_{e0}(1+R_T)}\left\{1 - i\left[\frac{2}{5}\frac{m_e}{m_i}\frac{\Omega_{en}(6-4R_T)}{(1+R_T)} + \Omega_{in}\right]\exp(-\tilde{z})\right\}\tilde{F}_e(\tilde{z})$$
$$= 0 \qquad (4.6)$$

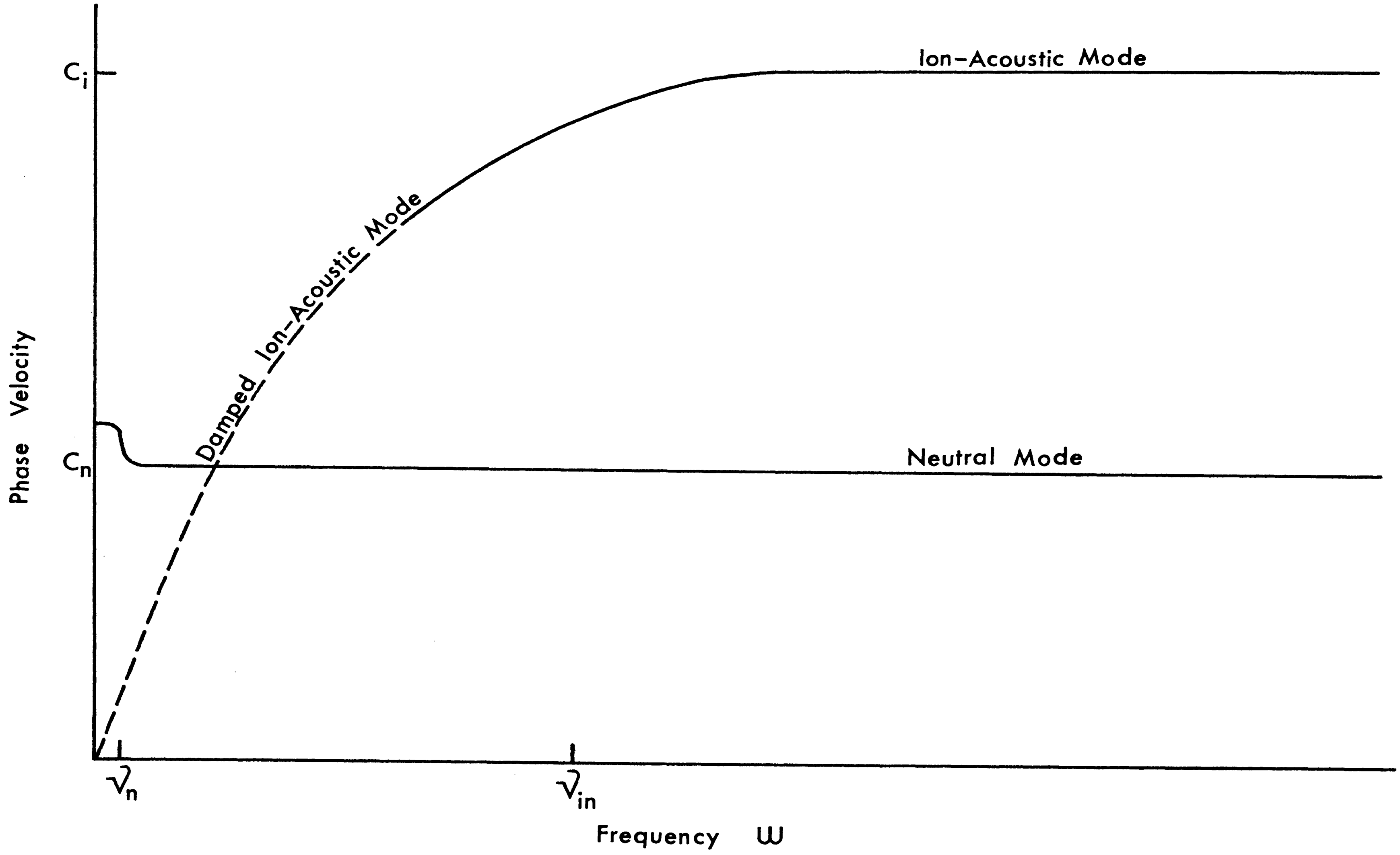

Figure 4-2. Phase Velocity vs. Frequency of Low-Frequency Acoustic Modes.

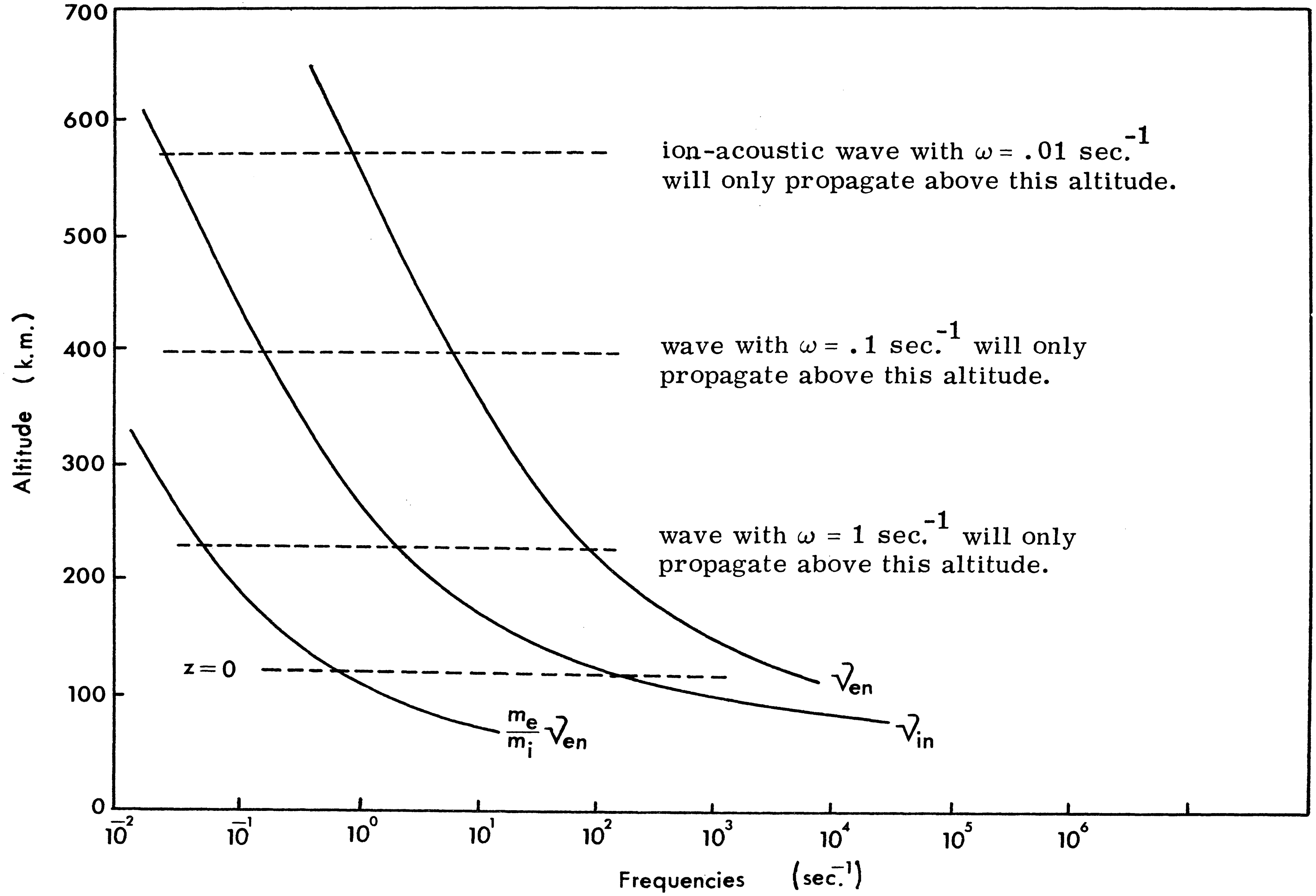

Figure 4-3. Incident Ion-Acoustic Wave Propagation Regions vs. Altitude.

where

$$\tilde{F}_e(\tilde{z}) \equiv \frac{n_{eo}(\tilde{z})\, \nu_{ei}(\tilde{z})}{N_e \sqrt{T_{eo}/m_i}}$$

and $\tilde{z} = z/h_n$. The transformation

$$\tilde{F}_e(\tilde{z}) = \xi^{-1/2} \exp(\xi/2)\, W(\xi)$$

and
$$\xi = -i\frac{2}{5}\frac{m_e}{m_i}\frac{\Omega_{en}(6-4R_T)}{(1+R_T)} \exp(-\tilde{z}) \tag{4.7}$$

converts the above equation into the Whittaker's equation (also commonly known as the normal form of the Kummer equation or the confluent hypergeometric equation)

$$\xi^2 W''(\xi) + \left\{ -\frac{\xi^2}{4} + \xi\left[\frac{1}{2} + \frac{5}{3}\frac{\omega^2 h_n^2 m_i}{T_{oe}(1+R_T)} + \frac{3}{2}\frac{m_i^2}{m_e^2}\frac{\omega^2 h_n^2 \Omega_{in}}{T_{eo}\Omega_{en}(6-4R_T)}\right] + \left[\frac{1}{4} + \frac{5}{3}\frac{\omega^2 h_n^2 m_i}{T_{eo}(1+R_T)}\right]\right\} W(\xi) = 0 \tag{4.8}$$

In general, the Whittaker's equation has eight solutions but only two of these form a fundamental system of solutions under all circumstances[13], these being:

$$W_{k,m}(\xi) = \exp(-\xi/2)\, \xi^{m+\frac{1}{2}}\, \Psi\left[\frac{1}{2} - k + m, 2m+1, \xi\right]$$

$$\Psi[a,b,\nu] \xrightarrow[|\nu|\to 0]{} \frac{\Gamma(1-b)}{\Gamma(1+a-b)} + \frac{\Gamma(b-1)}{\Gamma(a)}\nu^{1-b}$$

where Re $[b] = 1$ and $b \neq 1$, $\tilde{F}_{e5}(\tilde{z})$ can be decomposed at $\tilde{z} \to \infty$ into an unit amplitude incident wave and a reflected wave with small amplitude γ:

$$\frac{\tilde{F}_{e5}(\tilde{z})}{\alpha} \to \exp(-m\tilde{z}) + \gamma \exp(m\tilde{z})$$

with

$$\alpha \equiv \frac{\Gamma(-2m)}{\Gamma(\frac{1}{2}-k+m)}\left[\frac{-2m_e\Omega_{en}(6-4R_T)}{5m_i(1+R_T)}\right]^m$$

and

$$\gamma \equiv \frac{\Gamma(2m)\Gamma(\frac{1}{2}-k-m)}{\Gamma(-2m)\Gamma(\frac{1}{2}+m-k)}\left[\frac{-2m_e\Omega_{en}(6-4R_T)}{5m_i(1+R_T)}\right]^{2m}$$

The incident ion-acoustic wave solution is thus completely determined and given by the formula

$$\tilde{F}_e(\tilde{z}) = \frac{\Gamma(\frac{1}{2}-m-k)}{\Gamma(-2m)}\exp(-m\tilde{z})\cdot$$

$$\cdot\Psi\left[\frac{1}{2}-k+m, 1+2m, -i\frac{2}{5}\frac{m_e}{m_i}\frac{\Omega_{en}(6-4R_T)}{(1+R_T)}\exp(-\tilde{z})\right]$$

where $$k = \frac{1}{2} + \frac{5}{3}\frac{\omega^2 h_n^2 m_i}{T_{oe}(1+R_T)} + \frac{3}{2}\frac{m_i^2}{m_e}\frac{\omega^2 h_n^2 \Omega_{in}}{T_{oe}\Omega_{en}(6-4R_T)}$$

and

$$m = -i\left(\frac{5}{3}\frac{\omega^2 \ell_{in}^2 m_i}{T_{eo}(1+R_T)}\right)^{1/2}.$$

This solution can now be substituted into the "coupling" terms in Eq. (3. 7) to obtain the resultant neutral acoustic oscillations.

The effects of transverse geomagnetic field lines on the propagation of the incident ion-acoustic wave were not included in the above analysis. Had such effects been included, the problem would become extremely complex due to the modification of the sound mode by the presence of other hydromagnetic waves, namely, the Alfvén and the modified Alfvén waves. Only in certain limited frequency ranges and geometric field arrangements can approximate expressions be found for the complicated dispersion relation. In general, only numerical methods are feasible. To anticipate how the inclusion of a transverse magnetic field in the problem would modify the incident ion-acoustic wave, numerical results are presented in Table I showing increased damping with decreasing height and with increasing angle between the wave vectors and the geomagnetic field lines.

Table I. Attenuation and Phase Velocity for Ion-Acoustic Waves with Wave Vectors at Various Angles with Magnetic Field. $\omega = .1$ c/s. (Ref. 1)

Angle	Height (km.)	Attenuation (db/km.)	Phase Velocity (km./sec)
0°	140	0.264×10^2	0.199
40°	140	0.342×10^2	0.154
60°	140	0.513×10^2	0.103
80°	140	0.118×10^3	0.459×10^{-1}
0°	360	0.310	0.128×10
40°	360	0.169	0.984
60°	360	0.259	0.642
80°	360	0.748	0.223

V. THE APPROXIMATE SOLUTION OF THE NEUTRAL WAVE

The solution of the resultant neutral wave in the ionosphere diffuse layer wave coupling problem will be obtained in this and the following section. In this section a small parameter contained in the differential equation describing the neutral wave is utilized to simplify the equation thus facilitating the acquisition of an approximate solution. A more rigorous solution is given in Section VI.

Via collisions, the downward propagating ion-acoustic wave will drag the neutral particles along with the charged particle oscillations, causing a neutral wave to be initiated. For $z > 0$ the motion of this driven neutral wave is governed by Eqs. (3.4) and (3.5). From Fig. 4-3, one sees $(m_e/m_i)\,\nu_{en0} < \omega$ is true for the region $z > 0$, therefore, $[(m_e/m_i)(\nu_{en0}/\omega)]^2 \ll 1$ can be assumed in combining Eqs. (3.4) and (3.5) to eliminate $G_n(z)$. After some algebraic manipulation, one gets in non-dimensional form:

$$\tilde{F}_n''(\tilde{z}) + \frac{3}{5}\frac{\omega^2 m_n h_n^2}{R_T T_{oe}}\left\{1 - i\frac{\nu_{nio}}{\omega} + i\frac{m_e}{m_i}\frac{\nu_{eno} R_n}{\omega R_T}\left(\frac{6}{5} - 3R_T\right)\right\}\tilde{F}_n(\tilde{z})$$

$$+\tilde{F}_n'(\tilde{z})\left\{\begin{array}{l} -\frac{3}{5}\frac{d\ln n_{no}}{d\tilde{z}} - i\frac{6}{5}\frac{m_e}{R_T m_n}\left(\frac{\nu_{eno}}{\omega}R_n\right)'\left(1-\frac{5}{3}R_T\right) \\ -i\frac{18}{25}\frac{m_e}{m_n}\frac{\nu_{eno}}{\omega}\frac{R_n}{R_T}\left(1-\frac{5}{3}R_T\right)\frac{d\ln n_{no}}{d\tilde{z}} \end{array}\right\}$$

$$= i\frac{m_e}{m_n}\frac{3}{5R_T^{3/2}}\left(\frac{\nu_{eno}}{\omega}\frac{N_i}{N_n}\right)\left(4-\frac{8}{3}R_T\right)\tilde{F}_e''(\tilde{z})$$

$$+ i\frac{m_e}{m_n}\frac{3}{5R_T^{3/2}}\left(\frac{\nu_{eno}}{\omega}\right)'\frac{N_i}{N_n}\left(4-\frac{8}{3}R_T\right)\tilde{F}_e'(\tilde{z})$$

$$+\frac{3}{5}\frac{N_i}{R_T^{3/2}N_n}\frac{\omega^2 m_n h_n^2}{T_{oe}}\left\{-i\frac{\nu_{nio}}{\omega R_n} - i\frac{\nu_{neo}}{\omega R_n}\right\}\tilde{F}_e(\tilde{z}) \qquad (5.1)$$

where

$$\tilde{F}_n(\tilde{z}) = \frac{n_{no}(\tilde{z})\, v_{n1}(\tilde{z})}{N_n\sqrt{T_{on}/m_n}}, \qquad \tilde{F}_e(\tilde{z}) = \frac{n_{eo}(\tilde{z})\, v_{e1}(\tilde{z})}{N_e\sqrt{T_{eo}/m_i}}$$

and $\tilde{z} \equiv z/h_n$. Substituting the specified steady-state neutral and plasma density distributions and the effective collision frequency relations given by Eqs. (2.11) through (2.14) into the above and utilizing the transformation

$$W(\mu) = \mu^{\left[\frac{1}{2} - \frac{h_i}{2h_n}\left(\frac{3}{5} + i\frac{18}{25}\frac{m_e}{m_i}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]} \cdot \exp\left[\frac{-\mu}{2}\right] \cdot$$

$$\cdot\left[i\frac{m_e}{m_n}\frac{h_n}{h_i}\frac{\Omega_{en}N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5} - \frac{18}{25}\frac{h_i}{h_n}\right)\right]^{-\Upsilon} \cdot \tilde{F}_n(\tilde{z})$$

and
$$\mu = i\,\frac{m_e}{m_n}\,\frac{\Omega_{en} N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\exp\left[\frac{-\hbar_n}{\hbar_i}\tilde{z}\right]$$

where
$$\Upsilon = \frac{\hbar_i}{2\hbar_n}\left[\frac{3}{5}+i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right]. + \frac{\hbar_i}{\hbar_n}\left\{\begin{matrix}\left[\frac{3}{5}+i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right] \\ -\frac{12}{5}\frac{\omega^2 m_n \hbar_n^2}{T_{on}}\left[1-\frac{i\Omega_{in}N_i}{N_n}+i\frac{m_e}{m_i}\left(\frac{6}{5}-3R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right]\end{matrix}\right\}^{1/2}$$

one can reduce **Eq.** (5. 1) to the normal form:

$$W''(\mu) + W(\mu)\cdot$$

$$\cdot\left\{\begin{matrix} -\frac{1}{4}+\frac{1}{\mu^2}\left[\begin{matrix}\frac{1}{4}-\frac{\hbar_i^2}{4\hbar_n^2}\left(\frac{3}{5}+i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)^2 \\ -\frac{3}{5}\frac{\omega^2\hbar_n^2 m_n}{T_{on}}\left[1-i\frac{\Omega_{in}N_i}{N_n}+i\frac{m_e}{m_n}\left(\frac{6}{5}-3R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right]\end{matrix}\right] \\ +\frac{1}{\mu}\left[\begin{matrix}\frac{1}{2}-\frac{\hbar_i}{2\hbar_n}\left(\frac{3}{5}+i\frac{18}{25}\frac{m_e}{m_i}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right) \\ -\frac{\hbar_i^2\, 3\omega^2 m_n^2 R_T N_n}{5 T_{on} m_e \Omega_{en} N_i}\left\{\frac{-\frac{\Omega_{in}N_i}{N_n}+\frac{m_e}{m_n}\left(\frac{6}{5}-3R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}}{\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)}\right\}\end{matrix}\right]\end{matrix}\right\}$$

$$= \left(\frac{\hbar_i}{\hbar_n}\right)^{2-\frac{\hbar_i}{\hbar_n}}\left[\frac{i m_e}{m_n}\frac{\hbar_n}{\hbar_i}\frac{\Omega_{en}}{R_T}\frac{N_i}{N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]^{-\Upsilon-\frac{\hbar_i}{\hbar_n}}\cdot\exp\left[\frac{-\mu}{2}\right]\cdot$$

$$\cdot\,\mu^{\left[-\frac{3}{4}+\frac{\hbar_i}{\hbar_n}-\frac{\hbar_i}{\hbar_n}\left(\frac{3}{5}+i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]}$$

$$\cdot\left\{iH\mu^2\frac{\hbar_n^2}{\hbar_i^2}\tilde{F}_e''(\mu)+iH\mu\left(\frac{\hbar_n^2}{\hbar_i^2}+\frac{\hbar_n}{\hbar_i}\right)\tilde{F}_e'(\mu)+iJ\,\tilde{F}_e(\mu)\right\}$$

where

$$H = \frac{12}{5} \frac{m_e}{m_n R_T^{3/2}} \left(1 - \frac{2}{3} R_T\right) \frac{\Omega_{en} N_i}{N_n}$$

and

$$J = -\frac{3}{5} \frac{\omega^2 m_n h_n^2 N_i}{R_T^{3/2} T_{oe} N_n} \left(\Omega_{in} + \frac{m_e}{m_n} \Omega_{en}\right).$$

Since

$$\left| \frac{m_e}{m_n} \frac{h_n}{h_i} \frac{\Omega_{en}}{R_T} \frac{N_i}{N_n} \left(1 - \frac{5}{3} R_T\right)\left(-\frac{6}{5} - \frac{18}{25} \frac{h_i}{h_n}\right) \right| \ll 1$$

one can change the independent variable, writing

$$\mu = \epsilon \varphi$$

where

$$\epsilon = \frac{m_e}{m_n} \frac{h_n}{h_i} \frac{\Omega_{en} N_i}{R_T N_n} \left(1 - \frac{5}{3} R_T\right)\left(-\frac{6}{5} - \frac{18}{25} \frac{h_i}{h_n}\right)$$

and

$$\varphi = i \exp\left[-\frac{h_n}{h_i} \tilde{Z}\right],$$

then Eq. (5. 2) becomes

$$
\begin{aligned}
& W''(\xi) + W(\xi) \cdot \\
& \cdot \left\{ -\frac{\epsilon^2}{4} + \frac{1}{\xi^2} \left[\frac{1}{4} - \frac{h_i^2}{4h_n^2} \left(\frac{3}{5} + i \frac{18}{25} \frac{m_e}{m_i} \left(1 - \frac{5}{3} R_T\right) \frac{\Omega_{en} N_i}{R_T N_n} \right)^2 \right. \right. \\
& \qquad \left. - \frac{3}{5} \frac{\omega^2 h_n^2 m_n}{T_{on}} \left\{ 1 - i \frac{\Omega_{in} N_i}{N_n} + i \frac{m_e}{m_i} \left(\frac{6}{5} - 3R_T \right) \frac{\Omega_{en} N_i}{R_T N_n} \right\} \right] \\
& \quad + \frac{\epsilon}{\xi} \left[\frac{1}{2} - \frac{h_i}{2h_n} \left(\frac{3}{5} + i \frac{18}{25} \frac{m_e}{m_i} \left(1 - \frac{5}{3} R_T\right) \frac{\Omega_{en} N_i}{R_T N_n} \right) \right. \\
& \qquad \left. \left. - \frac{h_i^2 3\omega^2 R_T N_n}{5 T_{on} m_e \Omega_{en} N_i} \left\{ \frac{ -\frac{\Omega_{in} N_i}{N_n} + \frac{m_e}{m_i} \left(\frac{6}{5} - 3R_T \right) \frac{\Omega_{en} N_i}{R_T N_n} }{ \left(1 - \frac{5}{3} R_T\right) \left(-\frac{6}{5} - \frac{18}{25} \frac{h_i}{h_n} \right) } \right\} \right] \right\} \\
& = [i]^{-\Upsilon - \frac{h_i}{h_n}} \exp\left[-\frac{\epsilon \xi}{2} \right] \cdot \xi^{\left[\frac{3}{2} - \frac{h_i}{2h_n} \left(\frac{3}{5} + i \frac{18}{25} \frac{m_e}{m_n} \left(1 - \frac{5}{3} R_T\right) \frac{\Omega_{en} N_i}{R_T N_n} \right) + \frac{h_i}{h_n} \right]} \\
& \quad \cdot \epsilon^{\left[-\Upsilon + \frac{1}{2} - \frac{h_i}{2h_n} \left(\frac{3}{5} + i \frac{18}{25} \frac{m_e}{m_n} \left(1 - \frac{5}{3} R_T\right) \frac{\Omega_{en} N_i}{R_T N_n} \right) \right]} \cdot \left(\frac{h_i}{h_n} \right)^{\left[2 - \frac{h_i}{h_n} \right]} \cdot \\
& \quad \cdot \left\{ iH \xi^2 \frac{h_n^2}{h_i^2} \tilde{F}_e''(\xi) + iH \xi \left(\frac{h_n^2}{h_i^2} + \frac{h_n}{h_i} \right) \tilde{F}_e'(\xi) + iJ \tilde{F}_e(\xi) \right\}. \qquad (5.3)
\end{aligned}
$$

Using a perturbation technique, one can assume

$$W(\xi) = W_0(\xi) + \epsilon W_1(\xi) + \epsilon^2 W_2(\xi) + \text{-------},$$

then $W_0(\xi)$ satisfies the equation

$$\rho^2 W_0''(\rho) + W_0(\rho) \cdot \left\{ \begin{array}{l} \frac{1}{4} - \frac{\hbar_i^2}{4\hbar_n^2}\left(\frac{3}{5} + i\frac{18}{25}\frac{m_e}{m_i}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)^2 \\ -\frac{3}{5}\frac{\omega^2 \hbar_n^2 m_n}{T_{on}}\left[1 - i\frac{\Omega_{in}N_i}{N_n} + i\frac{m_e}{m_n}\left(\frac{6}{5}-3R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right] \end{array} \right\}$$

$$= [i]^{-\Upsilon+1-\frac{\hbar_i}{\hbar_n}} \cdot \exp\left[\frac{-\epsilon\rho}{2}\right] \cdot \rho^{\left[\frac{1}{2} - \frac{\hbar_i}{2\hbar_n}\left(\frac{3}{5}+i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right) + \frac{\hbar_i}{\hbar_n}\right]} \cdot$$

$$\cdot\, \epsilon^{\left[-\Upsilon + \frac{1}{2} - \frac{\hbar_i}{2\hbar_n}\left(\frac{3}{5}+i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]} \cdot \left(\frac{\hbar_i}{\hbar_n}\right)^{\Upsilon} \cdot$$

$$\cdot \left\{ H\rho^2\frac{\hbar_n^2}{\hbar_i^2}\tilde{F}_e''(\rho) + H\rho\left(\frac{\hbar_n^2}{\hbar_i^2}+\frac{\hbar_n}{\hbar_i}\right)\tilde{F}_e'(\rho) + J\tilde{F}_e(\rho) \right\} \qquad (5.4)$$

and represents an approximation to $W(\zeta)$ while the parameter ϵ can be throught of as a measure of the error of this approximation. Although better approximations can be obtained by solving for the higher order coefficients $W_1(\zeta)$, $W_2(\zeta)$, etc., for the present analysis $W_0(\zeta)$ will be assumed satisfactory. The nonhomogeneous equation (5.4) will now be solved by the method of variation of parameters.

Characteristic solutions of the simplified neutral equation (5.4) are the solutions to the Euler's equation

$$W_0''(\zeta) + \frac{W_0(\zeta)}{\zeta^2} \cdot \left\{ \begin{aligned} &\frac{1}{4} - \frac{\hbar_i^2}{4\hbar_n^2}\left(\frac{3}{5} + i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)^2 \\ &-\frac{3}{5}\frac{\omega^2\hbar_n^2 m_n}{T_{on}}\left[1 - i\frac{\Omega_{in}N_i}{N_n} + i\frac{m_e}{m_n}\left(\frac{6}{5}-3R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right] \end{aligned} \right\} = 0 \qquad (5.5)$$

namely ζ^{r_1} and ζ^{r_2} where

$$r_1, r_2 \simeq \frac{1}{2} \pm \frac{\hbar_i}{2\hbar_n}\left\{\frac{9}{25} - \left[\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right]^2 - \frac{12}{5}\frac{\omega^2\hbar_n^2 m_n}{T_{on}}\right\}^{1/2} \cdot$$

$$\cdot \left\{ 1 + i \left[\frac{\frac{54}{125}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n} + \frac{6}{5}\frac{\omega^2\hbar_n^2 m_n}{T_{on}}\left(-\frac{\Omega_{in}N_i}{N_n} + \frac{m_e}{m_n}\left(\frac{6}{5}-3R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)}{\frac{9}{25} - \left[\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right]^2 - \frac{12}{5}\frac{\omega^2 m_n \hbar_n^2}{T_{on}}} \right] \right\}.$$

The solution of the inhomogeneous equation for $W_0(\zeta)$ may then be written in the form:

$$W_0(\zeta) = \frac{\zeta^{r_1}}{W_r}\int_a^{c_p} s^{r_2} R(s)\,ds + \frac{\zeta^{r_1}}{W_r}\int_{c_p}^{\zeta} s^{r_2} R(s)\,ds$$

$$+ \frac{\zeta^{r_2}}{W_r}\int_{\zeta}^{b} s^{r_1} R(s)\,ds\,, \qquad (5.6)$$

where

$$R(s) = \left[i\right]^{\left[-\Upsilon + r_2 + \frac{1}{2} - \frac{\hbar_i}{2\hbar_n}\left(\frac{3}{5} + i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]}$$

$$\cdot \exp\left[\frac{-i\epsilon s}{2}\right] \cdot s^{\left[-\frac{3}{2} - \frac{\hbar_i}{2\hbar_n}\left(\frac{3}{5} + i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right) + \frac{\hbar_i}{\hbar_n}\right]}$$

$$\cdot\, \epsilon^{\left[-\Upsilon + \frac{1}{2} - \frac{\hbar_i}{2\hbar_n}\left(\frac{3}{5} + i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]} \cdot \left(\frac{\hbar_i}{\hbar_n}\right)^{\Upsilon} \cdot$$

$$\cdot \left\{H s^2 \frac{\hbar_n^2}{\hbar_i^2}\tilde{F}_e''(s) + Hs\left(\frac{\hbar_n^2}{\hbar_i^2} + \frac{\hbar_n}{\hbar_i}\right)\tilde{F}_e'(s) + J\tilde{F}_e(s)\right\},$$

$$W_r = \text{Wronskian} = (r_2 - r_1)\left[\frac{m_e}{m_n}\frac{\hbar_n}{\hbar_i}\frac{\Omega_{en}N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5} - \frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]^{-1}$$

and C_p is the value of ζ at the "coupling point". The "coupling point" denotes the altitude at which the wave frequency is of the order of ν_{in} and below which the charged-particle wave is strongly damped. The values of a and b are chosen such that at $z = -\infty$ only a forward neutral wave exists while only a backward wave exists at $z = \infty$. A forward wave is that wave propagating in the direction of decreasing z. The contributions from the last two integrals in expression (5.6) can be assumed negligible because the second integral represents the small reinforcement of the forward neutral wave by the highly damped ion-acoustic wave below the coupling region and the third integral represents the weak backward wave generated by the coupling. Assuming that the charged-particle wave is not damped appreciably until it reaches the region near the "coupling point", the complicated expression for $F_e(s)$ in the first integral of Eq. (5.6) can be approximated by its asymptotic representation for small arguments. Thus for $z \leq C_p$

$$W_0(\rho) \simeq \frac{\rho^{r_1}}{W_r}\left[i\right]^{\left[-\Upsilon + r_2 + \frac{1}{2} - \frac{\hbar_i}{2\hbar_n}\left(\frac{3}{5} + i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]} \cdot$$

$$\cdot \left[H\left(m_e^2 + m_e\right) + J\right] \cdot \left[\frac{\hbar_i}{\hbar_n}\right]^{\Upsilon} \cdot$$

$$\cdot \int_0^{\exp\left[-\frac{\hbar_n}{\hbar_i}C_p\right]} \exp\left[\frac{-i\epsilon s}{2}\right] s^{\left[-\frac{3}{2} + r_2 + (m_e+1)\frac{\hbar_i}{\hbar_n} - \frac{\hbar_i}{2\hbar_n}\left(\frac{3}{5} + i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]} ds \cdot$$

$$\cdot \epsilon^{\left[-\Upsilon + \frac{1}{2} - \frac{\hbar_i}{2\hbar_n}\left(\frac{3}{5} + i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]} \tag{5.7}$$

or

$$\tilde{F}_n(\tilde{z}) \simeq \left[\frac{m_e}{m_n}\frac{\hbar_n}{\hbar_i}\frac{\Omega_{en}N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(\frac{6}{5} - \frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right] \cdot \left(r_2 - r_1\right)^{-1} \cdot$$

$$\cdot \exp\left[\frac{i m_e}{2 m_n}\frac{\hbar_n}{\hbar_i}\frac{\Omega_{en}N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(\frac{6}{5} - \frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\exp\left(-\frac{\hbar_n}{\hbar_i}\tilde{z}\right)\right] \cdot$$

$$\cdot \exp\left\{\left[r_1 - \frac{1}{2} + \frac{\hbar_i}{2\hbar_n}\left(\frac{3}{5} + i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right] \cdot \left[-\frac{\hbar_n}{\hbar_i}\tilde{z}\right]\right\} \cdot$$

$$\cdot \int_0^{\exp\left[-\frac{\hbar_n}{\hbar_i}C_p\right]} \exp\left[\frac{-i\epsilon s}{2}\right] s^{\left[-\frac{3}{2} + r_2 + (m_e+1)\frac{\hbar_i}{\hbar_n} - \frac{\hbar_i}{2\hbar_n}\left(\frac{3}{5} + i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]} ds \cdot$$

$$\cdot \left(\frac{\hbar_i}{\hbar_n}\right)^{\Upsilon} \cdot \left[i\right]^{r_1 + r_2} \cdot \left[H\left(m_e^2 + m_e\right) + J\right] \tag{5.8}$$

In the region $z \leq 0$ the forcing ion-acoustic wave ceases to exist and the transmitted neutral wave becomes a damped oscillation satisfying the non-dimensional equation

$$\frac{5}{3} R_T \tilde{F}_n''(\tilde{z}) - R_T \frac{d \ln n_{no}}{d\tilde{z}} \tilde{F}_n'(\tilde{z}) + \frac{\omega^2 m_n h_n^2}{T_{oe}} \tilde{F}_n(\tilde{z}) = 0 \qquad (5.9)$$

with the solution

$$\tilde{F}_n(\tilde{z}) = \varphi_1 \exp\left\{\left[\frac{1}{2} + i\omega h_n \left(\frac{9}{100\,\omega^2 h_n^2} - \frac{3}{5}\frac{m_n}{T_{on}}\right)^{1/2}\right]\tilde{z}\right\}$$

$$+ \varphi_2 \exp\left\{\left[\frac{1}{2} - i\omega h_n \left(\frac{9}{100\,\omega^2 h_n^2} - \frac{3}{5}\frac{m_n}{T_{on}}\right)^{1/2}\right]\tilde{z}\right\} \qquad (5.10)$$

for $\tilde{z} \leq 0$.

$\tilde{F}_n(\tilde{z})$ and $\tilde{F}_n'(\tilde{z})$ from expression (5. 8) are matched at $z = 0$ with those of the exponentially damped neutral solution (5. 10), in order to determine the constants φ_1 and φ_2 with the result:

$$\varphi_1 = i\epsilon\,(r_2-r_1)^{-1}\cdot \exp\left(\frac{i\epsilon}{2}\right)\cdot\left[H\left(m_e^2+m_e\right)+J\right]\cdot\left(\frac{h_i}{h_n}\right)^{r}\cdot$$

$$\cdot\int_0^{\exp\left[-\frac{h_n}{h_i}C_p\right]} \exp\left[\frac{-i\epsilon S}{2}\right]\cdot S^{\left[-\frac{3}{2}+r_2+(m_e+1)\frac{h_i}{h_n}-\frac{h_i}{2h_n}\left(\frac{3}{5}+i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]}\,dS\cdot$$

$$\cdot\left\{\begin{array}{l}\left[\begin{array}{l} i\frac{h_n}{h_i}\frac{\epsilon}{2}+\frac{h_n}{h_i}\left(r_1-\frac{1}{2}+\frac{h_i}{2h_n}\left(\frac{3}{5}+i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right) \\ \quad -\frac{1}{2}-i\left(\frac{9}{100}-\frac{3}{5}\frac{m_n}{T_{on}}\omega^2 h_n^2\right)^{1/2}\end{array}\right]\cdot \\ \cdot\left[2i\left(\frac{9}{100}-\frac{3}{5}\frac{m_n}{T_{on}}\omega^2 h_n^2\right)^{1/2}\right]^{-1} + 1\end{array}\right\}$$

and

$$\varphi_2 = i\epsilon\,(r_2-r_1)^{-1}\cdot \exp\left(\frac{i\epsilon}{2}\right)\left[H\left(m_e^2+m_e\right)+J\right]\cdot\left(\frac{h_i}{h_n}\right)^{r}\cdot$$

$$\cdot\left[2i\left(\frac{9}{100}-\frac{3}{5}\frac{m_n}{T_{on}}\omega^2 h_n^2\right)^{1/2}\right]^{-1}\cdot$$

$$\cdot\int_0^{\exp\left[-\frac{h_n}{h_i}C_p\right]} \exp\left[\frac{-i\epsilon S}{2}\right] S^{\left[-\frac{3}{2}+r_2+(m_e+1)\frac{h_i}{h_n}-\frac{h_i}{2h_n}\left(\frac{3}{5}+i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]}\,dS\cdot$$

$$\cdot\left\{\begin{array}{l}\frac{1}{2}+i\left(\frac{9}{100}-\frac{3}{5}\frac{m_n}{T_{on}}\omega^2 h_n^2\right)^{1/2}-i\frac{h_n}{h_i}\frac{\epsilon}{2} \\ \quad -\frac{h_n}{h_i}\left(r_2-\frac{1}{2}+\frac{h_i}{h_n}\left(\frac{3}{5}+i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right)\end{array}\right\}.$$

Thus the neutral wave solution for $z \leq 0$ is now completely specified. The behavior of the neutral wave as a function of z and ω for various values of h_i/h_n and R_T are presented in Figs. 5-1 and 5-3.

Figure 5-1 shows that the neutral acoustic wave is severely damped for $R_T = 1$ case. This increased damping is attributed to the neutral density gradient which has been neglected in Kahalas and Parker's analysis. As the value of R_T is decreased, the effects of electron-neutral energy transfer can overcome the damping to allow a resultant neutral wave of significant amplitude to be transmitted with wave frequency $\simeq .005$ rad/sec.

Figure 5-3 shows that by decreasing the ratio h_i/h_n marked increase in coupling is observed. Decreasing h_i while holding h_n fixed corresponds to increased overlapping of the ionosphere and the neutral atmosphere. This enables the two wave modes to interact over greater distances hence the greater amount of coupling. In the limit $h_i \to 0$ (for finite z's), one gets a uniform ionosphere, i.e. constant charged particle density for all altitudes.

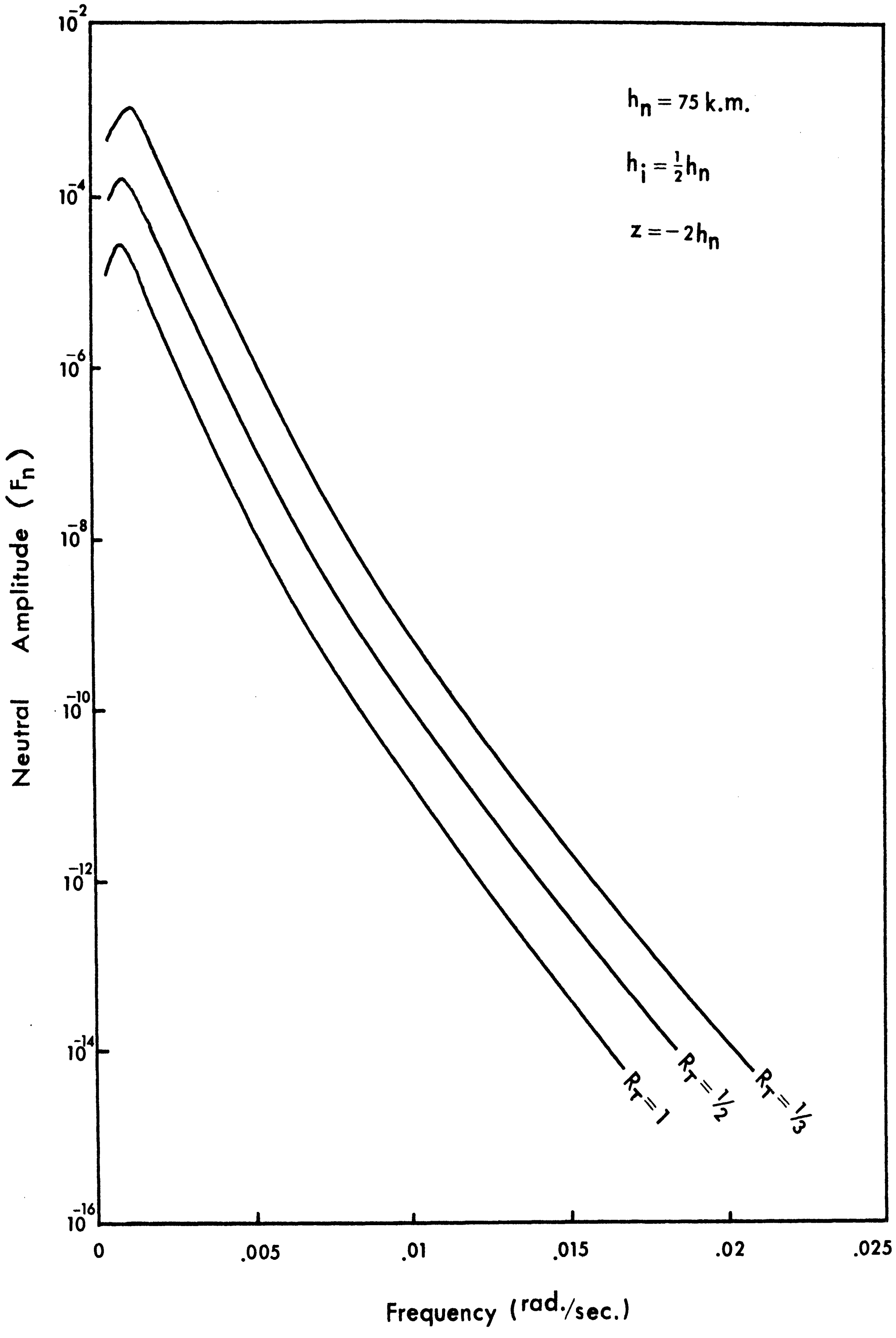

Figure 5-1. Neutral Amplitude vs. Frequency for Various Values of R_T.

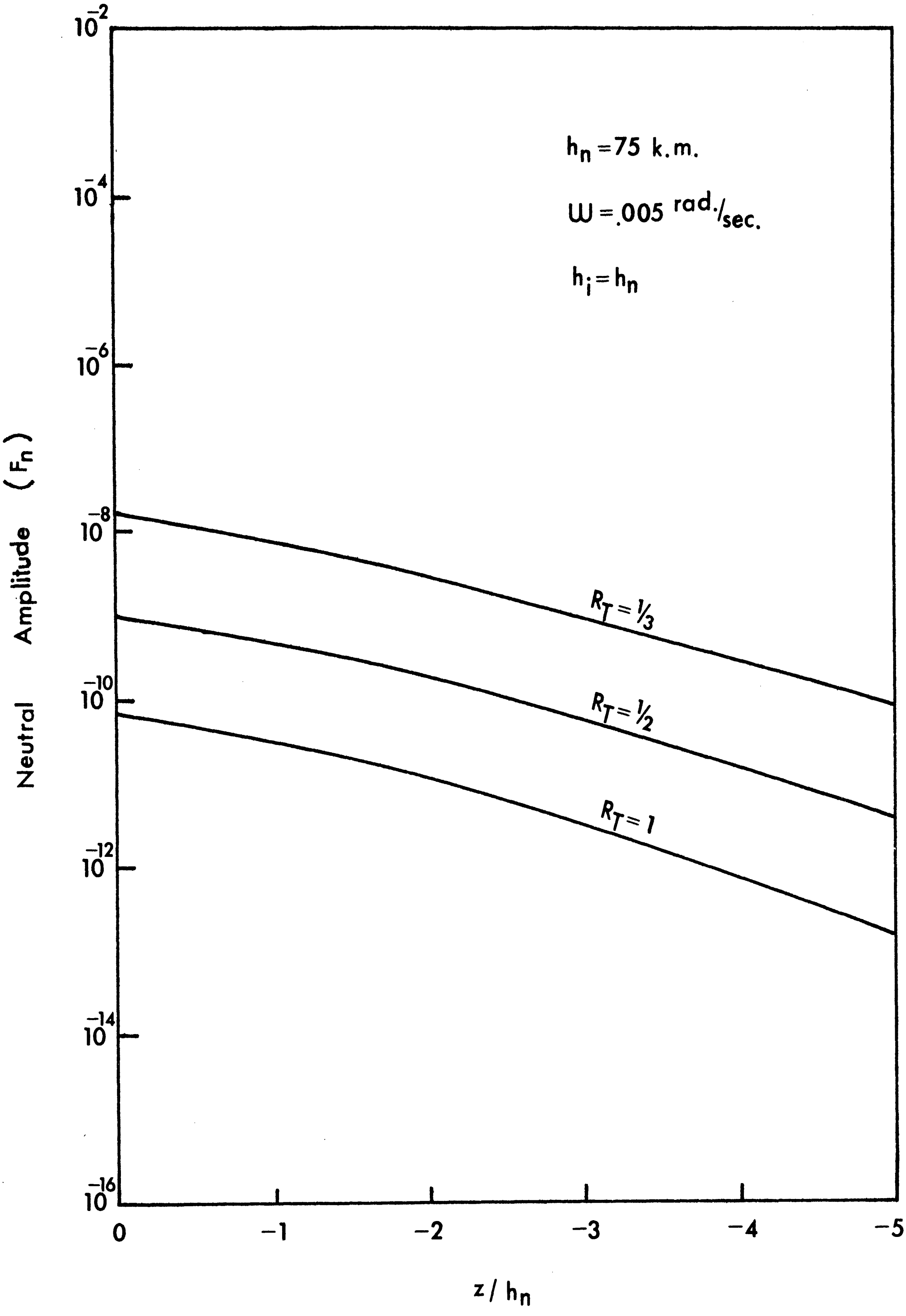

Figure 5-2. Damping of Neutral Waves as a Function of Distance.

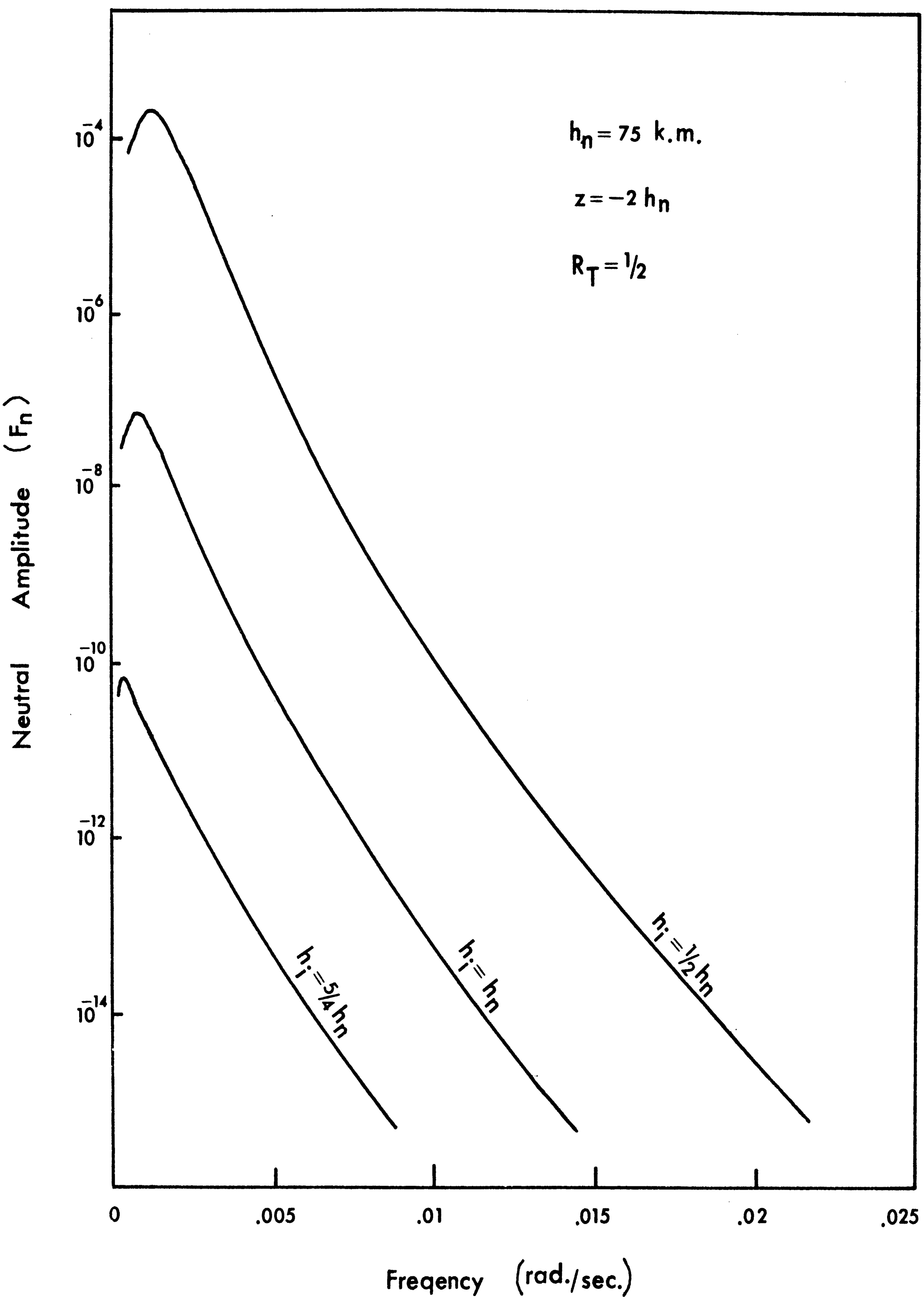

Figure 5-3. Neutral Amplitude vs. Frequency for Various Values of h_i.

VI. SOLUTION OF THE RESULTANT NEUTRAL WAVE

The transmitted neutral sound wave given by the solution to Eq. (5. 2) is evaluated in a more rigorous manner in this section. The solution $W(\mu)$ is obtained using the method of variation of parameters and expressed in integral form. Evaluation of the complicated integrals is done numerically using a digital computer and the results are then presented and compared to the approximate solution.

The characteristic solutions to Eq. (5. 2) are solutions to the Whittaker equation

$$
\begin{aligned}
&W''(\mu) + W(\mu) \cdot \\
&\cdot \left\{ -\frac{1}{4} + \frac{1}{\mu^2} \left[\frac{1}{4} - \frac{h_i^2}{4 h_n^2} \left(\frac{3}{5} + i \frac{18}{25} \frac{m_e}{m_n} \left(1 - \frac{5}{3} R_T\right) \frac{\Omega_{en} N_i}{R_T N_n} \right)^2 - \frac{3}{5} \frac{\omega^2 h_n^2 m_n}{T_{on}} \left\{ 1 - i \frac{\Omega_{in} N_i}{N_n} + i \frac{m_e}{m_n} \left(\frac{6}{5} - 3 R_T \right) \frac{\Omega_{en} N_i}{R_T N_n} \right\} \right] \right. \\
&\left. + \frac{1}{\mu} \left[\frac{1}{2} - \frac{h_i}{2 h_n} \left(\frac{3}{5} + i \frac{18}{25} \frac{m_e}{m_n} \left(1 - \frac{5}{3} R_T\right) \frac{\Omega_{en} N_i}{R_T N_n} \right) - \frac{3}{5} \frac{h_i^2 \omega^2 m_n^2 R_T N_n}{T_{on} m_e \Omega_{en} N_i} \left\{ \frac{ -\frac{\Omega_{in} N_i}{N_n} + \frac{m_e}{m_n} \left(\frac{6}{5} - 3 R_T \right) \frac{\Omega_{en} N_i}{R_T N_n} }{ \left(1 - \frac{5}{3} R_T\right) \left(-\frac{6}{5} - \frac{18}{25} \frac{h_i}{h_n} \right) } \right\} \right] \right\} \\
&= 0 \qquad (6.1)
\end{aligned}
$$

namely,

$$W_{K,M}(\mu) = \exp\left[\frac{-\mu}{2}\right] \left[\mu\right]^{M+\frac{1}{2}} \Psi\left[\frac{1}{2} - K + M, 2M+1, \mu\right]$$

and

$$W_{-K,M}(-\mu) = \exp\left[\frac{\mu}{2}\right]\left[-\mu\right]^{M+\frac{1}{2}} \Psi\left[\frac{1}{2}+K+M, 2M+1, -\mu\right]$$

where

$$K = \frac{1}{2} - \frac{\hbar_i}{2\hbar_n}\left[\frac{3}{5} + i\,\frac{18}{25}\frac{m_e}{m_i}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right]$$

$$- \frac{3}{5}\frac{\hbar_i^2\omega^2 m_n^2 R_T N_n}{T_{oe}\, m_e\, \Omega_{en} N_i}\left\{\frac{\dfrac{-\Omega_{in}N_i}{N_n} + \dfrac{m_e}{m_n}\left(\dfrac{6}{5}-3R_T\right)\dfrac{\Omega_{en}N_i}{R_T N_n}}{\left(1-\dfrac{5}{3}R_T\right)\left(-\dfrac{6}{5}-\dfrac{18}{25}\dfrac{\hbar_i}{\hbar_n}\right)}\right\}$$

and

$$M = \left\{\begin{array}{l} \dfrac{\hbar_i^2}{4\hbar_n^2}\left[\dfrac{3}{5} + i\,\dfrac{18}{25}\dfrac{m_e}{m_n}\left(1-\dfrac{5}{3}R_T\right)\dfrac{\Omega_{en}N_i}{R_T N_n}\right]^2 \\ -\dfrac{3}{5}\dfrac{\omega^2\hbar_n^2 m_n}{T_{on}}\left[1-\dfrac{i\Omega_{in}N_i}{N_n} + i\,\dfrac{m_e}{m_n}\left(\dfrac{6}{5}-3R_T\right)\dfrac{\Omega_{en}N_i}{R_T N_n}\right] \end{array}\right\}^{1/2}.$$

Hence the general solution $W(\mu)$ is given by

$$W(\mu) = \frac{W_{K,M}(\mu)}{W_r}\int_a^{\mu} W_{-K,M}(s)\,Q(s)\,ds$$

$$+ \frac{W_{-K,M}(-\mu)}{W_r}\int_{\mu}^{b} W_{K,M}(s)\,Q(s)\,ds \qquad (6.2)$$

where

$$W_r = \text{Wronskian} = \exp\left[-i\pi K\right]$$

$$Q(\mu) = \left(\frac{\hbar_i}{\hbar_n}\right)^{2-\frac{\hbar_i}{\hbar_n}} \left[\frac{i m_e}{m_n}\frac{\hbar_n}{\hbar_i}\frac{\Omega_{en} N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]^{-\Upsilon-\frac{\hbar_i}{\hbar_n}}$$

$$\cdot \exp\left[\frac{-\mu}{2}\right] \cdot \mu^{\left[-\frac{3}{4}+\frac{\hbar_i}{\hbar_n}\left(\frac{2}{5}-i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en} N_i}{R_T N_n}\right)\right]}$$

$$\cdot \left\{ iH\mu^2 \frac{\hbar_n^2}{\hbar_i^2}\tilde{F}_e''(\mu) + iH\mu\left(\frac{\hbar_n^2}{\hbar_i^2}+\frac{\hbar_n}{\hbar_i}\right)\tilde{F}_e'(\mu) + iJ\tilde{F}_e(\mu)\right\}$$

and the limits of integration are chosen such that there is only a forward propagating neutral wave at z = - ∞ and a backward propagating wave at z = ∞, or a = 0 and b = ∞. Substituting the solution for the driving ion-acoustic wave $F_e(z)$ from Eq. (4.9) into the above expression and noting that $F_e(z) = 0$ for $z \leq 0$, one gets:

$$\left\{W(\mu)\right\}_{z=0} = W\left[\frac{i m_e}{m_n}\frac{\Omega_{en} N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]$$

$$= \exp\left[\frac{i\pi}{2}(1+2K)\right] \int_0^{\left[\frac{m_e}{m_n}\frac{\Omega_{en} N_i}{N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]} W_{-K,M}(-is)\,\overline{Q}(is)\,ds \,.$$

$$W_{K,M}\left[\frac{i m_e}{m_n}\frac{\Omega_{en} N_i}{R_T N_n}\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\left(1-\frac{5}{3}R_T\right)\right] \qquad (6.3)$$

where

$$\bar{Q}(t) = \left(\frac{h_i}{h_n}\right)^{2-\frac{h_i}{h_n}} \left[\frac{i m_e}{m_n}\frac{h_n}{h_i}\frac{\Omega_{en} N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{h_i}{h_n}\right)\right]^{-\Upsilon+\frac{1}{2}-\frac{h_i}{h_n}} \cdot$$

$$\cdot \exp\left[\frac{-t}{2}\right] \cdot \left[t\right]^{\left[-\frac{3}{4}+\frac{h_i}{h_n}\left(\frac{3}{5}-i\frac{18}{25}\frac{m_e}{m_n}\left(1-\frac{5}{3}R_T\right)\frac{\Omega_{en}N_i}{R_T N_n}\right)\right]} \cdot$$

$$\exp\left[\frac{-\chi}{2}t^{\frac{h_i}{h_n}}\right] \cdot \frac{\Gamma\left(\frac{1}{2}+m-k\right)}{\Gamma(-2m)} \cdot \left[i\frac{2}{5}\frac{m_e}{m_n}\frac{\Omega_{en}(6-4R_T)}{(1+R_T)}\right]^{-\frac{1}{2}-m} \cdot$$

$$\cdot \left\{ \begin{aligned} &\left[\left(k^2-\frac{1}{4}\right)iH + iJ\right] W_{-k,m}\left[\chi t^{h_i/h_n}\right] \\ &+ \left[-(1+2k)iH\right] W_{-k+1,m}\left[\chi t^{h_i/h_n}\right] \\ &+ iH\, W_{-k+2,m}\left[\chi t^{h_i/h_n}\right] \end{aligned} \right\}$$

and

$$\chi = \left[i\frac{2m_e}{5m_n}\frac{\Omega_{en}(6-4R_T)}{(1+R_T)}\right]\left[i\frac{m_e \Omega_{en} N_i}{m_n T_{oe} N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{h_i}{h_n}\right)\right]^{-\frac{h_i}{h_n}} .$$

via the Leibnitz rule for integrals, one also gets from Eq. (6.2):

$$
\left\{W'(\mu)\right\}_{z=0} = \left\{
\begin{array}{l}
W_{K,M}\left[\frac{i m_e}{m_n}\frac{\Omega_{en} N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]\cdot \\
\cdot W_{-K,M}\left[\frac{i m_e}{m_n}\frac{\Omega_{en} N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]\cdot \\
\cdot \overline{Q}\left[i\frac{m_e}{m_n}\frac{\Omega_{en} N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]\cdot \exp\left[\frac{i\pi}{2}(1+2K)\right]
\end{array}
\right\}
$$

$$
+\left\{
\begin{array}{l}
\left\{\frac{1}{2}-K\left[\frac{i m_e}{m_n}\frac{\Omega_{en} N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{1}{25}\frac{\hbar_i}{\hbar_n}\right)\right]^{-1}\right\}\cdot \exp\left[\frac{i\pi}{2}(1+2K)\right]\cdot \\
\cdot W_{K,M}\left[\frac{i m_e}{m_n}\frac{\Omega_{en} N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]\cdot \\
\int_0^{\left[\frac{m_e}{m_n}\frac{\Omega_{en} N_i}{N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]} W_{-K,M}(-is)\,\overline{Q}(is)\,ds
\end{array}
\right\}
$$

$$
+\left\{
\begin{array}{l}
W_{K+1,M}\left[\frac{i m_e}{m_n}\frac{\Omega_{en} N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]\cdot \\
\cdot\left[i\frac{m_e}{m_n}\frac{\Omega_{en} N_i}{R_T N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]^{-1}\cdot \\
\cdot \exp\left[\frac{i\pi}{2}(1+2K)\right]\cdot\int_0^{\left[\frac{m_e}{m_n}\frac{\Omega_{en} N_i}{N_n}\left(1-\frac{5}{3}R_T\right)\left(-\frac{6}{5}-\frac{18}{25}\frac{\hbar_i}{\hbar_n}\right)\right]} W_{-K,M}(-is)\,\overline{Q}(is)\,ds
\end{array}
\right\}
\qquad (6.4)
$$

Using the expansion

$$W_{\alpha,\beta}(xe^{\pm i\frac{\pi}{2}}) = \frac{\pi}{\sin(2\pi\beta)} \cdot$$

$$\cdot \left\{ \begin{aligned} &\frac{\exp\left[\pm \frac{i\pi}{2}(\frac{1}{2}+\beta)\right]}{\Gamma(\frac{1}{2}-\beta+\alpha)\Gamma(1+2\beta)} 2^{2\beta\mp 2\alpha} x^{\frac{1}{2}\pm\alpha}\, \Gamma(\beta\mp\alpha) \cdot \\ &\qquad \cdot \sum_{n=0}^{\infty} \frac{(2\beta\mp 2\alpha)_n (\mp 2\alpha)_n\, i^n}{n!\,(1+2\beta)_n} J_{\beta\mp\alpha+n}(\tfrac{1}{2}x) \\ &+ \frac{\exp\left[\pm i\pi(\frac{1}{2}-\beta)\right]}{\Gamma(\frac{1}{2}+\beta-\alpha)\Gamma(1-2\beta)} 2^{-2\beta\mp 2\alpha} x^{\frac{1}{2}\pm\alpha}\, \Gamma(-\beta\mp\alpha) \cdot \\ &\qquad \sum_{n=0}^{\infty} \frac{(-2\beta\mp 2\alpha)_n (\mp 2\alpha)_n\, i^n}{n!\,(1-2\beta)_n} J_{-\beta\mp\alpha+n}(\tfrac{1}{2}x) \end{aligned} \right\}$$

$$(a)_n \equiv a(a+1)(a+2)\ldots(a+n-1) \quad , \quad a_0 = 1$$

to represent the Whittaker function with imaginary variables, the integrals in Eqs. (6.3) and (6.4) are evaluated numerically by subroutine QATR in the IBM Scientific Subroutine Package. The resulting values for $F_n(z)$ and $F_n'(z)$ at $z = 0$ are matched with the exponentially damped solutions of the neutral wave in the region $z \leq 0$. Since the sign of the variable in the Whittaker functions changes for various values of R_T, for example, sign $(\mu) > 0$ for $R_T = 1$, while sign $(\mu) < 0$

for $R_T = 1/2$, extreme care is needed to ensure that the correct combinations of and appropriate expressions for the Whittaker functions are used in order to obtain sensible results. In Figs. 6-1 and 6-2 the amplitude of the neutral oscillations as a function of wave frequency and distance from the ionosphere-neutral atmosphere interface for different temperature ratios are shown. $R_T = 1$ corresponds to no thermal energy transfer from the electrons to the neutrals thus reducing to Kahalas and Parker's case except that now the damping effects of the density gradient on the neutral waves has been included. As T_{e0} is raised relative to T_{n0} and T_{i0}, the amplitude of the resultant neutral oscillation increases markedly.

Figure 6-1 shows that as ω is increased, coupling drops. The physical reason behind this trend can be explained as follows. An ion-acoustic wave with higher frequency can penetrate lower in altitude before coupling occurs. Since the rate of increase in the collision rates goes up with decreasing altitude, the damping of a higher frequency ion-acoustic wave (and therefore the interaction) is more abrupt than that of a wave with a lower frequency. This means a smaller range of altitudes over which the higher frequency charged-particle wave can reinforce the excited oscillation thus resulting in a weaker transmitted neutral wave.

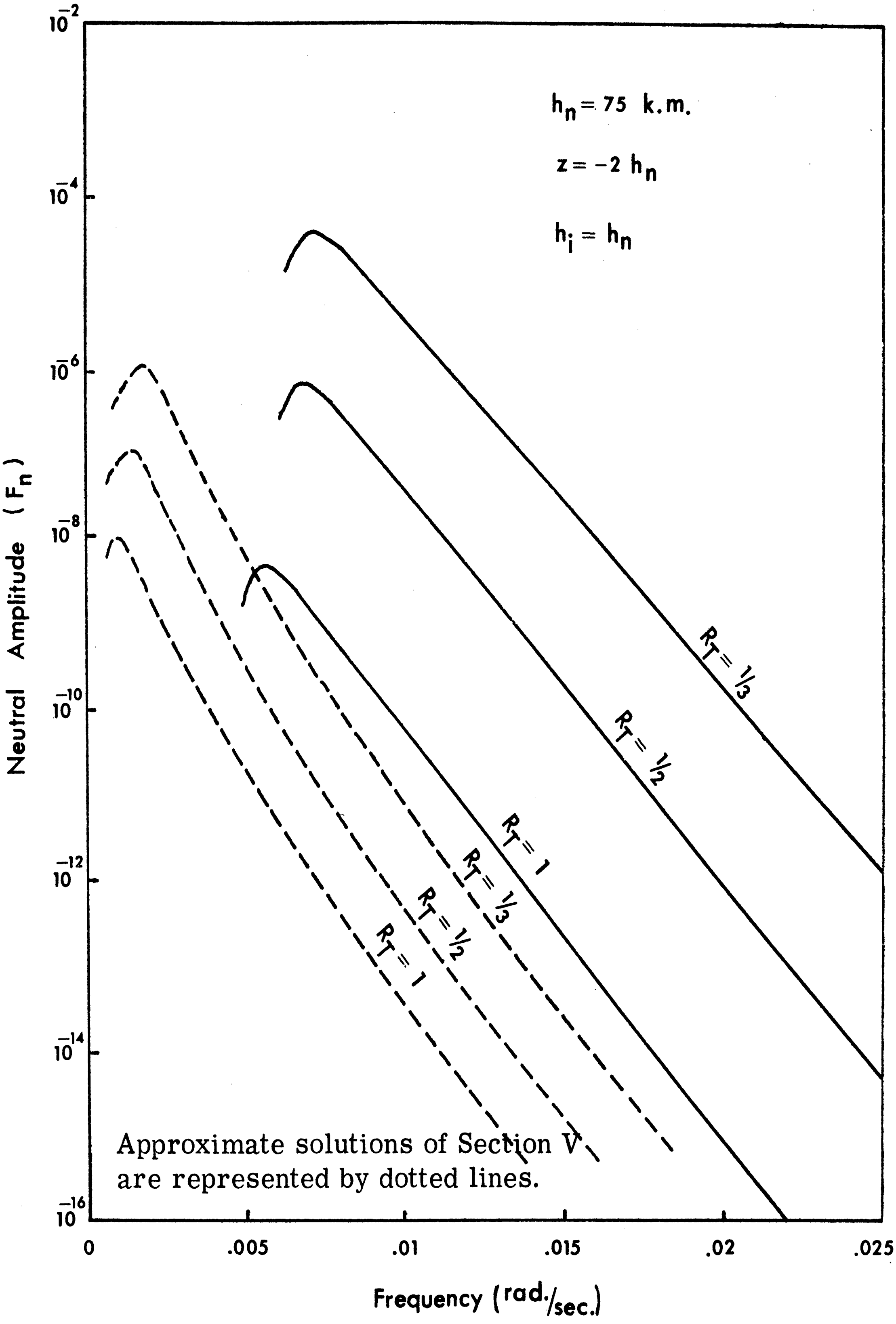

Figure 6-1. Neutral Amplitude vs. Frequency for Various Values of R_T.

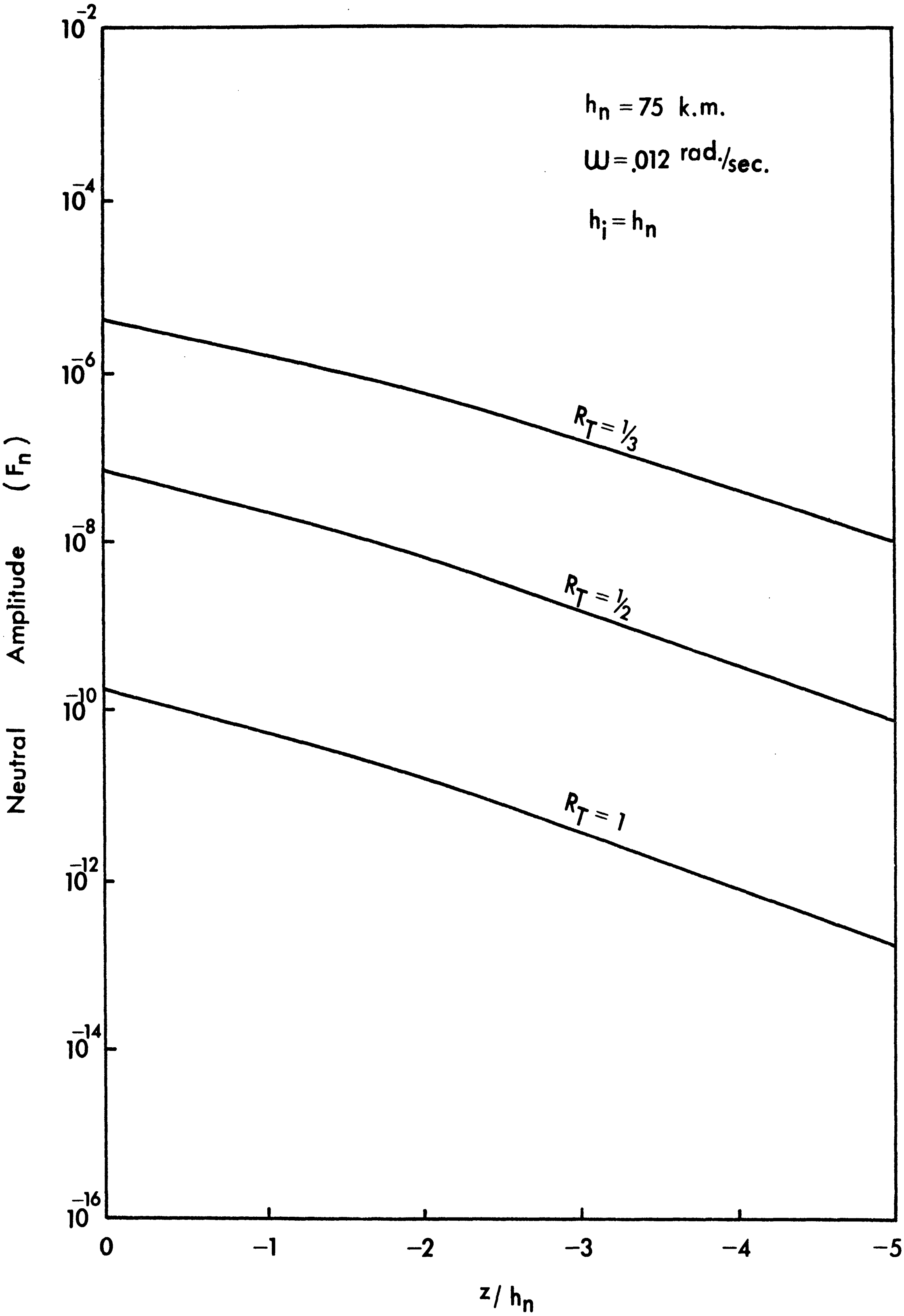

Figure 6-2. Damping of Neutral Wave as Function of Distance.

On the other hand, as the frequency is lowered, the resulting neutral signal can again be weaker. As the coupling point moves higher and higher in altitude the ion-acoustic wave tends to damp out before a neutral oscillation that is strong enough to survive the neutral density gradient damping effects can be generated.

Qualitative agreement between those results obtained via the extremely simplified method of Section V and those of the more rigorous approach of the present section can be seen in Fig. 6-2. The results of Section V are represented by the dashed curves.

Due to the nature of the simplification used in Section V, one cannot expect quantitative agreement between the two methods. The approximate results are "shifted" in frequency from the results of the present section. Also, at the ground, the magnitude of the transmitted wave derived from the simplified method is less than that derived from the precise approach. In the method of Section V, the incident ion-acoustic wave (which is expressed in terms of a complicated Whittaker function) is replaced by a simplified function (a function where the wave amplitude is unity for $z >$ the "coupling" point and zero for $z \leq$ the "coupling" point). Thus the resultant neutral wave will cease to be "forced" at a higher altitude than in the more exact approach. The decrease in the range over which the two wave modes interact results in the weaker signal at the ground for the simplified approach.

In a traveling plane wave the pressure variation is related to the velocity perturbation by

$$P_{n1} = C_n \zeta_{n0} v_{n1}$$

where ζ_{n0} ≡ undisturbed density $\simeq 1.4 \times 10^{-3}$ g/cm^3 at earth surface, and C_n ≡ phase velocity of wave $\simeq 3 \times 10^4$ cm/sec at earth's surface. For wave frequency $\simeq .005$ rad/sec, $h_i = 1/2\ h_n$ and $R_T = 1/3$ one finds

$$v_{n1} \simeq 10^{-4} v_{i1} \frac{N_{i0}}{N_{n0}} \simeq 10^{-8} v_{i1}$$

Thus the resultant neutral pressure fluctuation is related to initial ion-acoustic velocity perturbation by

$$P_{n1} \simeq 4.2 \times 10^{-7} \cdot v_{i1} \quad \text{dynes/cm}^2$$

An initial ion-acoustic velocity perturbation of 2,000 cm/sec would register a pressure fluctuation of approximately 10^{-3} dynes/cm^2. Under quiet wind conditions, such pressure fluctuations can be detectable. For smaller initial velocity perturbations, the resulting neutral pressure fluctuations would be too weak to be distinguishable from wind noises when detected at the ground.

SUMMARY AND CONCLUSION

The collisional coupling of low-frequency ion-acoustic and neutral sound waves in the ionosphere-neutral atmosphere diffuse layer has been investigated. The assumed medium consists of a mixture of singly ionized plasma and neutral gas whose particle densities varied exponentially with height in such a way that the medium is neutral in the lower region and becomes progressively more ionized as height is increased. The lower region depicted the neutral atmosphere while the weakly ionized upper region represented the ionosphere. A downward propagating ion-acoustic wave launched at sufficiently great height will eventually cease to exist due to the collisional damping arising from the increasing neutral encounters. At the same time, a coupled neutral sound wave can be excited from the momentum and energy transferred to the neutrals. These resultant neutral oscillations can then continue the wave transmission down into the neutral atmosphere.

Kahalas and Parker (26) have forwarded a theory for the conversion of low-frequency ion-acoustic wave to neutral sound wave in the ionospheric diffuse layer where the transfer of momentum occurred via ion-neutral collisions. The present investigation extended the work of Kahalas and Parker to include the dominant energy effects and to consider the modification of the resultant neutral wave by the neutral density gradient.

To determine the degree of coupling between the ion-acoustic and the neutral-acoustic waves in the inhomogeneous plasma layer, the coupled wave equations (3.6) and (3.7) were solved approximately. A first approximation to the incident ion-acoustic wave was obtained by neglecting the effects of the small neutral wave motions in Eq. (3.6). The subsequent damped solution for the ion-acoustic mode was then substituted into Eq. (3.7) as a known "forcing function" which drives the neutral mode. Indication of how much wave coupling occurred through the diffuse layer is given by the magnitude of the resultant neutral wave amplitude as compared to the unit amplitude of the ion-acoustic wave at the point of incidence.

The solution of the neutral mode was obtained via two approaches. In Section V a small parameter contained in the differential equation describing the neutral wave was utilized to simplify the equation thus facilitating the acquisition of an approximate solution. A much more rigorous solution involving integrals of Whittaker functions was given in Section VI.

It is shown that for the plasma profiles and operating frequencies considered, the neutral-acoustic wave is severely damped by the neutral particle density gradient for the $R_T = 1$ case. However, noticing that the ionospheric electrons are typically two to three times hotter than the neutral particles and taking into account the resulting heat transfer via the electron-neutral collisions, the coupling became significant for frequencies $\simeq .005$ rad/sec.

It is also shown that significant increase in coupling is observed when the ratio h_i/h_n is decreased. Decreasing h_i while holding h_n fixed corresponded to increased overlapping of the ionosphere and the neutral atmosphere. This enabled the two wave modes to interact over greater distances hence the increased coupling.

It was found that under certain conditions (for example, $R_T = 1/3$, $h_i = 1/2\, h_n$, $\omega \simeq .005$ rad/sec, and $v_{i1} \geq 2{,}000$ cm/sec) the neutral pressure fluctuations were strong enough to be detected under quiet wind conditions. Thus under these limited conditions the transmission of emissions from the upper ionosphere via the present mechanism to the ground for detection is indeed possible.

A number of extensions to this analysis may prove profitable. Extensions to include temperature variations in altitude, horizontal wind movements, and geomagnetic field are some interesting possibilities.

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Collisional wave coupling, Low frequency waves, Ion-acoustic wave, Neutral acoustic wave, Inhomogeneous plasma, Weakly ionized plasma, Collisional energy transfer, Ionosphere

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The collisional coupling of low frequency ion-acoustic waves and neutral sound waves propagating downward in the ionosphere is investigated. The ionosphere is considered as a nonhomogeneous weakly ionized plasma consisting of electrons, ions and neutrals. The equations of conservation of mass, momentum, and energy of the three species constitute the basic equations for the analysis. The plasma is assumed to be electrically neutral. Collisional energy transfer between the hot electrons and the colder neutrals are taken

20. into account. The coupled differential equations for the wave amplitudes are solved analytically. The solution yields an expression for the amplitude of the neutral sound wave at low altitudes. Results showing the behavior of the neutral amplitude as a function of wave frequency and distance from the ionosphere-neutral atmosphere interface for different temperature ratios and different scale hights are presented.