Active Vibration Isolation of Multi-Degree-of-Freedom Systems

WASSIM M. HADDAD
School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150 USA

ALI RAZAVI
George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0405 USA

DAVID C. HYLAND
Department of Aerospace Engineering, The University of Michigan, Ann Arbor, MI 48109-2118 USA

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Abstract: One of the principal objectives of vibration isolation technology is to isolate sensitive equipment from a vibrating structure or to isolate the structure from an uncertain exogenous disturbance source. In this paper, a dynamic observer-based active isolator is proposed that guarantees closed-loop asymptotic stability and disturbance decoupling between the vibrating structure and isolated structure. The proposed active isolator is applied to a uniaxial vibrational system and compared to an optimal linear-quadratic design.

Key Words: Disturbance rejection, active isolation, vibration suppression, multi-degree-of-freedom systems

1. INTRODUCTION

One of the principal objectives of vibration isolators is to either isolate sensitive equipment from a vibrating structure or to isolate the structure from an uncertain exogenous disturbance source. Vibration suppression between a base body (containing the disturbance source) and an isolated body can be achieved by intrastructural damping approaches or active isolation. In intrastructural damping approaches to isolation, a damping energy dissipation mechanism is inserted between the two bodies, which can be implemented passively (e.g., viscoelastic dampers) or actively (e.g., piezoelectric actuators). However, since such isolation members transmit vibrational energy in the process of dissipating energy, they simply reduce the resonance peaks of the isolated body response but do not reduce the broadband nonresonant response. Alternatively, active isolation approaches that combine intrastructural actuation and inertial sensing can prevent vibration transmission into the isolated body and hence suppress the resonant and nonresonant responses over a broad frequency band (Hyland and Phillips, 1996). In this paper, we extend the single-input/single-output active vibration

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isolation framework proposed by Hyland and Phillips (1996) to multi-input/multi-output, multi-degree-of-freedom systems. Specifically, a dynamic observer-based active isolator is proposed that guarantees closed-loop asymptotic stability and disturbance decoupling between the base body and the isolated body. A practically useful feature of the proposed active isolator is that only rate measurements are required. The proposed active isolator is applied to a uniaxial flexible structure and compared to an optimal linear-quadratic controller, which requires measurements of both position and velocity.

2. PRELIMINARIES

In this section, we establish notation and definitions used in the paper. Let $\mathbf{R}$ denote the set of real numbers, let $\mathbf{R}^{n\times m}$ denote the set of real $n \times m$ matrices, let $(\ )^T$ and $(\ )^*$ denote transpose and complex conjugate transpose, respectively, and let I_n denote the $n \times n$ identity matrix. Furthermore, we write $M \geq 0$ $(M > 0)$ to denote the fact that the Hermitian matrix M is nonnegative (positive) definite.

A *Lyapunov stable transfer function* is a transfer function each of whose poles is in the closed-left half-plane with semisimple poles on the $\jmath\omega$ axis. An *asymptotically stable* transfer function is a transfer function each of whose poles is in the open-left half-plane. A square transfer function $G(s)$ is called *positive real* (Anderson and Vongpanitlerd, 1973) if $G(s)$ is Lyapunov stable and $G(s) + G^*(s)$ is nonnegative definite for $\mathrm{Re}[s] > 0$. A square transfer function $G(s)$ is called *strictly positive real* (Wen, 1988) if $G(s)$ is asymptotically stable and $G(\jmath\omega) + G^*(\jmath\omega)$ is positive definite for all real ω. Note that a minimal realization of a positive real transfer function is stable in the sense of Lyapunov, while a minimal realization of a strictly positive real transfer function is asymptotically stable.

3. ACTIVE ISOLATOR MODEL

Consider the linear time-invariant controllable and observable system

$$\dot{x}(t) = Ax(t) + Dw(t), \quad x(0) = x_0, \quad t \in [0, \infty), \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

where $x \in \mathbb{R}^n$, $w \in \mathbb{R}^d$, $y \in \mathbb{R}^d$, $A \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{n\times d}$, and $C \in \mathbb{R}^{d\times n}$. To reject the uncertain exogenous disturbance w, we introduce a linear time-invariant dynamic filter between the disturbance source w and the plant as shown in Figure 1. In particular, to ensure disturbance rejection, we construct a feedback control signal $u(t)$ predicated on the output signal of the plant $y(t)$ and the filter output signal $y_f(t)$. This feedback interconnection is characterized by

$$\dot{x}(t) = Ax(t) + D[-u(t)], \quad x(0) = x_0, \quad t \in [0, \infty), \tag{3}$$

$$y(t) = Cx(t), \tag{4}$$

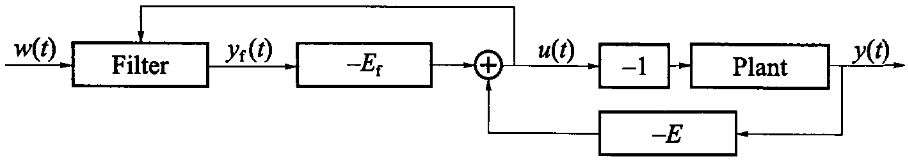

Figure 1. Block diagram of active isolator model.

$$\dot{x}_f(t) = A_f x_f(t) + B_f u(t) + D_f w(t), \tag{5}$$

$$y(t) = C_f x_f(t), \tag{6}$$

$$u(t) = -\begin{bmatrix} E & E_f \end{bmatrix} \begin{bmatrix} y(t) \\ y_f(t) \end{bmatrix}, \tag{7}$$

where (5) and (6) represent the dynamic filter and $x_f \in \mathbb{R}^{n_f}$, $u \in \mathbb{R}^d$, $y_f \in \mathbb{R}^d$, $A_f \in \mathbb{R}^{n_f \times n_f}$, $B_f, D_f \in \mathbb{R}^{n_f \times d}$, $C_f \in \mathbb{R}^{d \times n_f}$, and $E, E_f \in \mathbb{R}^{d \times d}$. The filter matrices (A_f, B_f, C_f, D_f) and the feedback gains (E, E_f) will be designed to guarantee disturbance rejection and closed-loop stability. We assume that (A_f, B_f, C_f) is minimal. Note that the -1 block in Figure 1 is added to ensure that the signal $u(t)$ driving the plant is equal in magnitude but opposite in sign to the signal fed back to the filter. Now, it follows from (3) and (7) that if $E_f \to 0$, then disturbance decoupling between $y(t)$ and $w(t)$ is achieved.

Next, with $x(0) = 0$ and $x_f(0) = 0$, (3) through (7) have the frequency-domain representations given by

$$y(s) = -C(sI_n - A)^{-1} D u(s), \tag{8}$$

$$y_f(s) = -C_f(sI_{n_f} - A_f)^{-1}\left[B_f u(s) + D_f w(s)\right], \tag{9}$$

$$u(s) = -\left[E y(s) + E_f y_f(s)\right]. \tag{10}$$

Defining the system transfer function from $-u$ to y and the filter transfer functions from u to y_f and w to y_f, respectively, by

$$G(s) \triangleq C(sI_n - A)^{-1} D, \tag{11}$$

$$G_f(s) \triangleq C_f(sI_{n_f} - A_f)^{-1} B_f, \tag{12}$$

$$H_f(s) \triangleq C_f(sI_{n_f} - A_f)^{-1} D_f, \tag{13}$$

equations (8) and (9) can be written as

$$y(s) = -G(s)u(s), \tag{14}$$

$$y_f(s) = G_f(s)u(s) + H_f(s)w(s). \tag{15}$$

Substituting (10) into (14) yields

$$y(s) = G(s)[Ey(s) + E_f y_f(s)], \tag{16}$$

or, equivalently,

$$y(s) = [I_d - G(s)E]^{-1}G(s)E_f y_f(s). \tag{17}$$

Hence, letting $E_f \to 0$ guarantees that the system output y is decoupled from the uncertain exogenous disturbance w.

Next, we give conditions on the filter gains (A_f, B_f, C_f, D_f) and the feedback gains (E, E_f) such that the closed-loop system (3)-(7) is asymptotically stable with $w \equiv 0$. For this result, we assume that $G(s)$ is feedback positive real, that is, there exists $F \in \mathbb{R}^{d\times d}$ such that $[I - G(s)F]^{-1}G(s)$ is positive real.

Theorem 3.1. *Consider the closed-loop system (3)-(7) with frequency-domain representation*

$$y(s) = -G(s)u(s), \tag{18}$$

$$y_f(s) = G_f(s)u(s) + H_f(s)w(s), \tag{19}$$

$$u(s) = -\left[E_f y_f(s) + Ey(s)\right]. \tag{20}$$

Let $G_f(s)$ be strictly positive real and assume $G(s)$ is feedback positive real, that is, there exists $E \in \mathbb{R}^{d\times d}$ such that $H(s) \triangleq [I_d - G(s)E]^{-1}G(s)$ is positive real. If $E_f \neq 0$ is positive real, that is, $E_f + E_f^T \geq 0$, and there exists $\alpha \geq 0$ such that $E = \alpha E_f^T$, then the closed-loop system (18)-(20) is asymptotically stable with $w \equiv 0$. Furthermore, letting $E_f \to 0$ the system output y is decoupled from the disturbance w.

Proof. Disturbance decoupling is immediate. To show asymptotic stability, note that, using (17) and the definition of $H(s)$, (20) can be written as

$$\begin{aligned} u(s) &= -\left[E_f y_f(s) + EH(s)E_f y_f(s)\right] \\ &= -\left[I_d + EH(s)\right] E_f y_f(s) \\ &= -\tilde{G}(s)y_f(s), \end{aligned} \tag{21}$$

where

$$\tilde{G}(s) \triangleq [I_d + EH(s)]E_f. \tag{22}$$

Next, since $H(s)$ is positive real, it follows that $\tilde{G}(s)$ is Lyapunov stable. Furthermore,

$$\tilde{G}(s) + \tilde{G}^*(s) = [I_d + EH(s)]E_f + E_f^T[I_d + H^*(s)E^T],$$

which, using $E = \alpha E_f^T$ where $\alpha \geq 0$, yields

$$\tilde{G}(s) + \tilde{G}^*(s) = E_f + E_f^T + \alpha E_f^T[H(s) + H^*(s)]E_f \geq 0, \quad \text{Re}[s] > 0.$$

Hence, $\tilde{G}(s)$ is positive real.

Now, to show asymptotic stability of the closed-loop system (18)-(20) (with $w \equiv 0$), it need only be noted that (18)-(20) or, equivalently, (19), (21) corresponds to a negative feedback interconnection of $G_f(s)$ with $\tilde{G}(s)$. Hence, since by assumption, $G_f(s)$ is strictly positive real and $\tilde{G}(s)$ is positive real, it follows that the negative feedback interconnection of $G_f(s)$ and $\tilde{G}(s)$ is asymptotically stable (Joshi and Gupta, 1996). □

Remark 3.1. In Theorem 3.1, we assumed that $G(s)$ is feedback positive real. A necessary and sufficient condition that guarantees $G(s)$ is feedback positive real is if the transmission zeros of $G(s)$ are in the closed-left half-plane with semisimple zeros on the $\jmath\omega$ axis and $G(s)$ has relative degree $\{r_1, r_2, ..., r_d\} = \{1, 1, ..., 1\}$ so that $\det(CD) \neq 0$ (see Abdallah et al., 1991).

Remark 3.2. Note that the feedback gain E_f can be used to trade off asymptotic stability with disturbance rejection. Specifically, asymptotic stability requires $E_f \neq 0$ while complete disturbance rejection requires $E_f \to 0$. Furthermore, the filter parameters (A_f, B_f, C_f, D_f) can be used to shape the force/displacement transmissibility frequency response.

Remark 3.3. It is important to note that the closed-loop control scheme proposed in this section assumes knowledge of the exogenous disturbance. Hence, alternatively one can design an open-loop feedforward scheme to cancel out the disturbance. However, such a scheme does not allow the designer to obtain desired closed-loop performance since it is predicated on open-loop control. Furthermore, using a two-stage feedback and feedforward scheme results in controllers that need frequent readjustment as disturbance conditions change. No such adjustment is necessary with the proposed method. Finally, for certain exogenous disturbances, the complexity involved in data processing and control decision algorithms can render two-stage control schemes impractical.

Remark 3.4. As shown by Hyland and Phillips (1996), the feedback controller proposed in this section is physically realizable using active isolation fittings. Such devices have been built and applied on actual vibrational systems. The reader is referred to U.S. Patent No. 5,626,332 by Phillips et al. (1997) for the physical realizability of the controller proposed in this paper.

4. APPLICATION TO VIBRATION ISOLATION OF MATRIX SECOND-ORDER SYSTEMS

In this section, we specialize the results of Section 3 to matrix second-order vibrational systems controlled by a dynamic filter whose structure is constrained to have the same form as the vibrational system. Specifically, consider the matrix second-order vibrational system with proportional damping and rate measurements given by

$$\hat{M}\ddot{q}(t) + \hat{C}\dot{q}(t) + \hat{K}q(t) = -\hat{B}u(t), \quad q(0) = q_0, \quad \dot{q}(0) = \dot{q}_0, \quad t \in [0, \infty), \tag{23}$$

$$y(t) = \hat{B}^{\mathrm{T}}\dot{q}(t), \tag{24}$$

where $q \in \mathbb{R}^r$ represents the generalized position coordinates, $u \in \mathbb{R}^d$ is the control signal, $y \in \mathbb{R}^d$ represents the rate measurements, $\hat{M} \in \mathbb{R}^{r\times r}$ is the positive definite inertia matrix, $\hat{C} \in \mathbb{R}^{r\times r}$ is the nonnegative definite energy dissipation matrix, $\hat{K} \in \mathbb{R}^{r\times r}$ is the nonnegative definite stiffness matrix, and $\hat{B} \in \mathbb{R}^{r\times d}$. Next, consider the dynamic filter

$$\hat{M}_{\mathrm{f}}\ddot{q}_{\mathrm{f}}(t) + \hat{C}_{\mathrm{f}}\dot{q}_{\mathrm{f}}(t) + \hat{K}_{\mathrm{f}}q_{\mathrm{f}}(t) = \hat{B}_{\mathrm{f}}u(t) + \hat{D}_{\mathrm{f}}w(t), \quad q_{\mathrm{f}}(0) = q_{\mathrm{f}0}, \quad \dot{q}_{\mathrm{f}}(0) = \dot{q}_{\mathrm{f}0}, \tag{25}$$

$$y_{\mathrm{f}}(t) = \hat{B}_{\mathrm{f}}^{\mathrm{T}}[\dot{q}_{\mathrm{f}}(t) + \epsilon q_{\mathrm{f}}(t)], \tag{26}$$

$$u(t) = \hat{K}_u[y(t) - y_{\mathrm{f}}(t) - \mu_{\mathrm{v}}v(t)], \tag{27}$$

where $q_{\mathrm{f}} \in \mathbb{R}^{r_{\mathrm{f}}}$ represents the filter states, $w \in \mathbb{R}^d$ is an uncertain exogenous disturbance, $y_{\mathrm{f}} \in \mathbb{R}^d$ represents the filter outputs, $v \in \mathbb{R}^d$ is the drive voltage of the actuator, $\mu_{\mathrm{v}} \in \mathbb{R}$ represents the strain/voltage constant of the actuator, $\epsilon > 0$, and $\hat{K}_u \in \mathbb{R}^{d\times d}$, $\hat{M}_{\mathrm{f}}$, $\hat{C}_{\mathrm{f}}$, $\hat{K}_{\mathrm{f}} \in \mathbb{R}^{r_{\mathrm{f}}\times r_{\mathrm{f}}}$, $\hat{B}_{\mathrm{f}}, \hat{D}_{\mathrm{f}} \in \mathbb{R}^{r_{\mathrm{f}}\times d}$ are constant gain matrices such that $\hat{K}_u$, $\hat{M}_{\mathrm{f}}$, $\hat{C}_{\mathrm{f}}$, and $\hat{K}_{\mathrm{f}}$ are positive definite. Furthermore, assume that the piezo actuator $\mu_{\mathrm{v}}v$ has the input-output representation

$$\mu_{\mathrm{v}}v(t) = \mu y(t) - \mu_{\mathrm{f}}y_{\mathrm{f}}(t), \quad \mu_{\mathrm{f}}, \mu \in \mathbb{R}, \tag{28}$$

$\hat{C} - (\mu - 1)\hat{B}\hat{K}_u\hat{B}^{\mathrm{T}} \geq 0$, and $\hat{B}^{\mathrm{T}}\{s^2\hat{M} + s[\hat{C} - (\mu - 1)\hat{B}\hat{K}_u\hat{B}^{\mathrm{T}}] + \hat{K}\}^{-1}\hat{B}$ is Lyapunov stable with no poles at the origin.

Now, substituting (28) into (27) yields

$$\begin{aligned} u(t) &= \hat{K}_u[y(t) - y_{\mathrm{f}}(t) - \mu y(t) + \mu_{\mathrm{f}}y_{\mathrm{f}}(t)] \\ &= -[(1-\mu_{\mathrm{f}})\hat{K}_u \quad (\mu - 1)\hat{K}_u]\begin{bmatrix} y_{\mathrm{f}}(t) \\ y(t) \end{bmatrix}. \end{aligned} \tag{29}$$

In the frequency-domain, with $q(0) = 0$, $\dot{q}(0) = 0$, $q_f(0) = 0$, and $\dot{q}_f(0) = 0$, the closed-loop system (23)-(26), (29) can be represented by (18)-(20) where

$$G(s) = s\hat{B}^T(s^2\hat{M} + s\hat{C} + \hat{K})^{-1}\hat{B}, \quad G_f(s) = (s+\epsilon)\hat{B}_f^T(s^2\hat{M}_f + s\hat{C}_f + \hat{K}_f)^{-1}\hat{B}_f \ ,$$

$$H_f(s) = (s+\epsilon)\hat{B}_f^T(s^2\hat{M}_f + s\hat{C}_f + \hat{K}_f)^{-1}\hat{D}_f, \quad E_f = (1-\mu_f)\hat{K}_u, \quad E = (\mu - 1)\hat{K}_u.$$

Furthermore, note that $G_f(s)$ is asymptotically stable since $\hat{C}_f, \hat{K}_f > 0$. Hence,

$$\begin{aligned} G_f(\jmath\omega) + G_f^*(\jmath\omega) &= \hat{B}_f^T(\hat{K}_f - \omega^2\hat{M}_f - \jmath\omega\hat{C}_f)^{-1}\Big[2\omega^2(\hat{C}_f - \epsilon\hat{M}_f) \\ &+ 2\epsilon\hat{K}_f\Big](\hat{K}_f - \omega^2\hat{M}_f + \jmath\omega\hat{C}_f)^{-1}\hat{B}_f, \end{aligned}$$

is positive definite for all real ω if $\epsilon > 0$ such that $\hat{C}_f - \epsilon\hat{M}_f \geq 0$. Hence, $G_f(s)$ is strictly positive real for sufficiently small $\epsilon > 0$. Next, note that

$$H(s) = [I_d - G(s)E]^{-1}G(s) = s\hat{B}^T\{s^2\hat{M} + s[\hat{C} - (\mu - 1)\hat{B}\hat{K}_u\hat{B}^T] + \hat{K}\}^{-1}\hat{B},$$

is Lyapunov stable by assumption. Furthermore,

$$\begin{aligned} H(s) + H^*(s) &= T^*(s)\{2|s|^2\mathrm{Re}[s]\hat{M} + 2|s|^2\,[\hat{C} - (\mu - 1)\hat{B}\hat{K}_u\hat{B}^T] + 2\mathrm{Re}[s]\hat{K}\}T(s) \geq 0, \\ \mathrm{Re}[s] &> 0, \end{aligned}$$

where

$$T(s) \triangleq \{s^2\hat{M} + s[\hat{C} - (\mu - 1)\hat{B}\hat{K}\hat{B}^T] + \hat{K}\}^{-1}\hat{B},$$

and hence $H(s)$ is positive real. Now, using Theorem 3.1, the closed-loop system (23)-(26), (29) is asymptotically stable for $\mu_f < 1$ and $\mu \geq 1$. Furthermore, letting $\mu_f \to 1$, y is decoupled from the disturbance w.

Remark 4.1. Note that even though (24) restricts sensor measurements to rate measurements, in the single-input/single-output case, that is, $d = 1$, our results also apply to the case where

$$y(t) = \hat{B}^T q(t). \tag{30}$$

In this case, it can be shown that, with

$$y_f(t) = \hat{B}_f^T q_f(t), \tag{31}$$

$(\mu-1)(1-\mu_f)<0$, and $\{1+(\mu-1)\hat{K}_u\hat{B}^T[\hat{K}-(\mu-1)\hat{K}_u\hat{B}\hat{B}^T]^{-1}\hat{B}\}(1-\mu_f)\hat{K}_u \geq 0$, the feedback interconnection (23), (25), (29), (30)-(31) is asymptotically stable. Furthermore, letting $\mu_f \to 1$, y is decoupled from the disturbance w. This special case corresponds to the result reported by Hyland and Phillips (1996).

5. ILLUSTRATIVE NUMERICAL EXAMPLE

Consider the vibrational system shown in Figure 2(a). Here we are interested in isolating the mass m from the base body by reducing the displacement transmissibility from the exogenous displacement disturbance w to the mass displacement q using the active vibration isolation model shown in Figure 2(b). This system is characterized by the dynamic equations

$$m\ddot{q}(t)+c\dot{q}(t)+kq(t)=-r(t),\quad q(0)=q_0,\quad \dot{q}(0)=\dot{q}_0,\quad t\in[0,\infty), \tag{32}$$

$$m_f\ddot{q}_f(t)+c_f\dot{q}_f(t)+k_fq_f(t)=r(t)+c_f\dot{w}(t)+k_fw(t),\quad q_f(0)=q_{f0},\quad \dot{q}_f(0)=\dot{q}_{f0}, \tag{33}$$

$$r(t)=-f(t)-c[\dot{q}_f(t)-\dot{q}(t)]-k[q_f(t)-q(t)], \tag{34}$$

where we assume $m = 240$ kg, $k = 1.6\times 10^4\frac{\text{N}}{\text{m}}$, $c = 10^3\frac{\text{N}\cdot\text{s}}{\text{m}}$, $m_f = 30$ kg, $k_f = 1.5\times 10^3\frac{\text{N}}{\text{m}}$, and $c_f = 150\frac{\text{N}\cdot\text{s}}{\text{m}}$. Next, defining η, η_f, and u by

$$\begin{aligned}\eta(t) &\triangleq \frac{1}{c_f}e^{-\frac{k_f}{c_f}t}\int_0^t e^{\frac{k_f}{c_f}\tau}q(\tau)\mathrm{d}\tau,\\ \eta_f(t) &\triangleq \frac{1}{c_f}e^{-\frac{k_f}{c_f}t}\int_0^t e^{\frac{k_f}{c_f}\tau}q_f(\tau)\mathrm{d}\tau,\\ u(t) &\triangleq \frac{1}{c_f}e^{-\frac{k_f}{c_f}t}\int_0^t e^{\frac{k_f}{c_f}\tau}r(\tau)\mathrm{d}\tau,\end{aligned} \tag{35}$$

and using the relationships

$$q(t)=c_f\dot{\eta}(t)+k_f\eta(t),\quad \eta(0)=0, \tag{36}$$

$$q_f(t)=c_f\dot{\eta}_f(t)+k_f\eta_f(t),\quad \eta_f(0)=0, \tag{37}$$

$$r(t)=c_f\dot{u}(t)+k_fu(t),\quad u(0)=0, \tag{38}$$

(32)-(34) can be characterized by

$$m\ddot{\eta}(t) + c\dot{\eta}(t) + k\eta(t) = -u(t), \quad \eta(0) = 0, \quad \dot{\eta}(0) = \frac{q_0}{c_f}, \tag{39}$$

$$m_f\ddot{\eta}_f(t) + c_f\dot{\eta}_f(t) + k_f\eta_f(t) = u(t) + w(t), \quad \eta_f(0) = 0, \quad \dot{\eta}_f(0) = \frac{q_{f0}}{c_f}. \tag{40}$$

Now, using

$$y(t) = \dot{\eta}(t), \tag{41}$$

$$y_f(t) = \dot{\eta}_f(t) + \frac{c_f}{m_f}\eta_f, \tag{42}$$

$$u(t) = -[(1-\mu_f)k_u \quad (\mu-1)k_u]\begin{bmatrix} y_f(t) \\ y(t) \end{bmatrix}, \tag{43}$$

where $k_u > 0$, (39)-(43) is equivalent in form to (23)-(26), (29). Finally, note that, using (38), (41), and (43), yields

$$\begin{aligned} r(t) &= -[(1-\mu_f)k_u \quad (\mu-1)k_u]\begin{bmatrix} c_f\dot{y}_f(t) + k_f y_f(t) \\ c_f\dot{y}_f(t) + k_f y(t) \end{bmatrix} \\ &= -[(1-\mu_f)k_u \quad (\mu-1)k_u]\begin{bmatrix} \frac{d}{dt}\left[c_f\dot{\eta}_f(t) + k_f\eta_f(t)\right] + \frac{c_f}{m_f}\left[c_f\dot{\eta}_f(t) + k_f\eta_f(t)\right] \\ \frac{d}{dt}\left[c_f\dot{\eta}(t) + k_f\eta(t)\right] \end{bmatrix} \\ &= -[(1-\mu_f)k_u \quad (\mu-1)k_u]\begin{bmatrix} \dot{q}_f(t) + \frac{c_f}{m_f}q_f(t) \\ \dot{q}(t) \end{bmatrix}, \end{aligned}$$

which, using (34), implies

$$f = kq + [c + k_u(\mu-1)]\dot{q} + \left[k_u(1-\mu_f)\frac{c_f}{m_f} - k\right]q_f + [k_u(1-\mu_f) - c]\dot{q}_f. \tag{44}$$

Hence, the system shown in Figure 2(b) with f given by (44) is asymptotically stable for $1 \le \mu \le 1 + \frac{c}{k_u}$, $\mu_f < 1$, and $k_u > 0$. Furthermore, letting $\mu_f \to 1$, $\dot{\eta}$ is decoupled from the disturbance w where η is defined by (35). Fitted active vibration isolator.

Figures 3 through 5 compare the displacement response $q(t)$ and the actuator force $f(t)$ for a unit impulse input disturbance of an optimal linear-quadratic regulator (LQR) design based on the performance measure

$$J = \int_0^\infty [q^2(t) + 0.001u^2(t)]\,dt$$

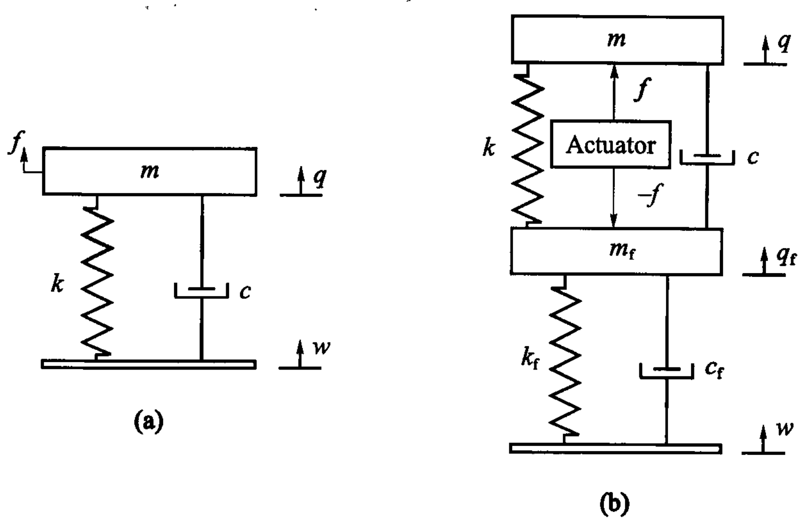

Figure 2(a) Dynamical system (b) Fitted active vibration isolator.

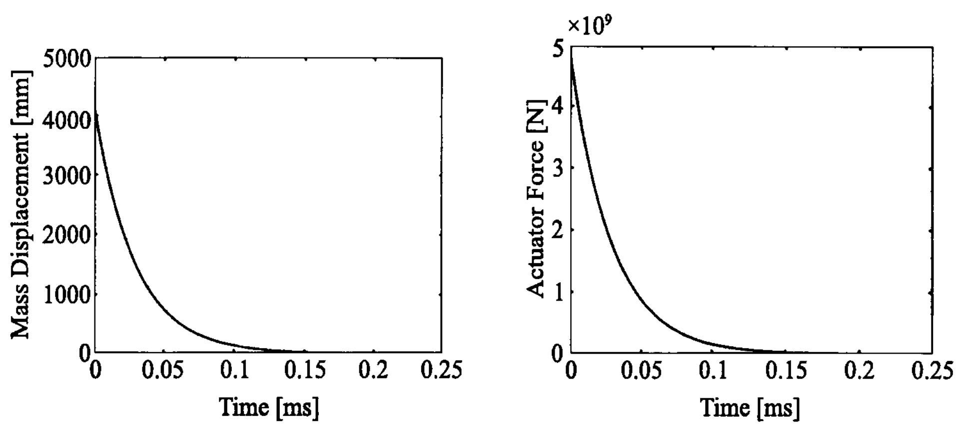

Figure 3. LQR design.

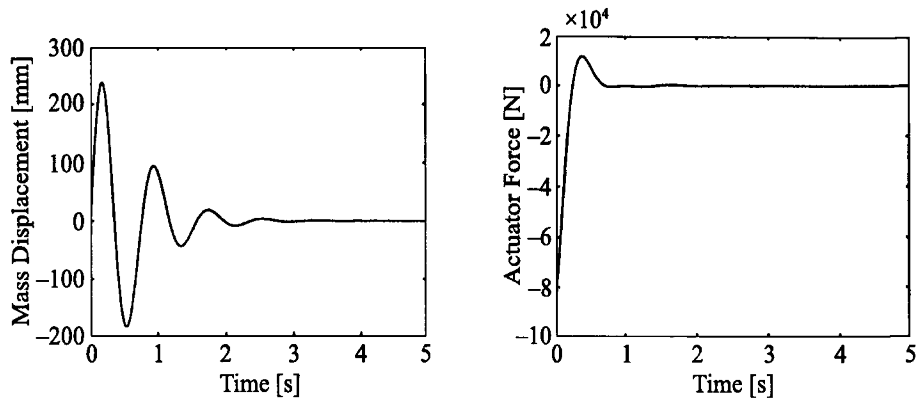

Figure 4. Active vibration isolation design with $\mu = 1$ and $(1 - \mu_f)k_u = 100$.

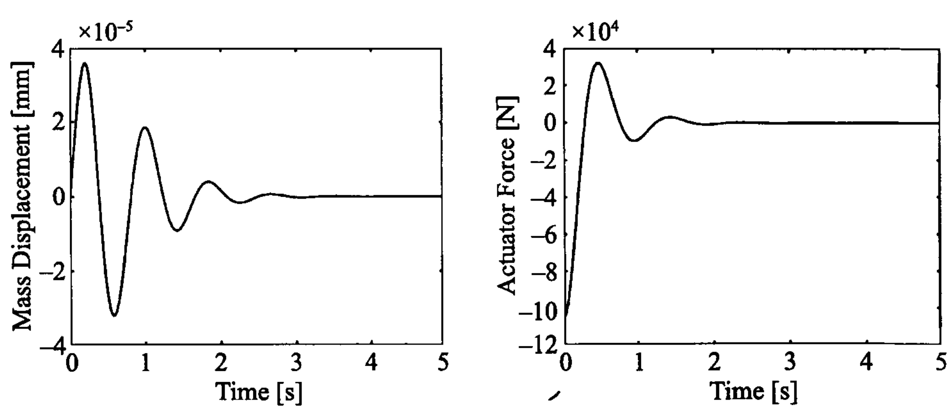

Figure 5. Active vibration isolation design with $\mu = 1$ and $(1 - \mu_f)k_u = 1.1111 \times 10^{-5}$.

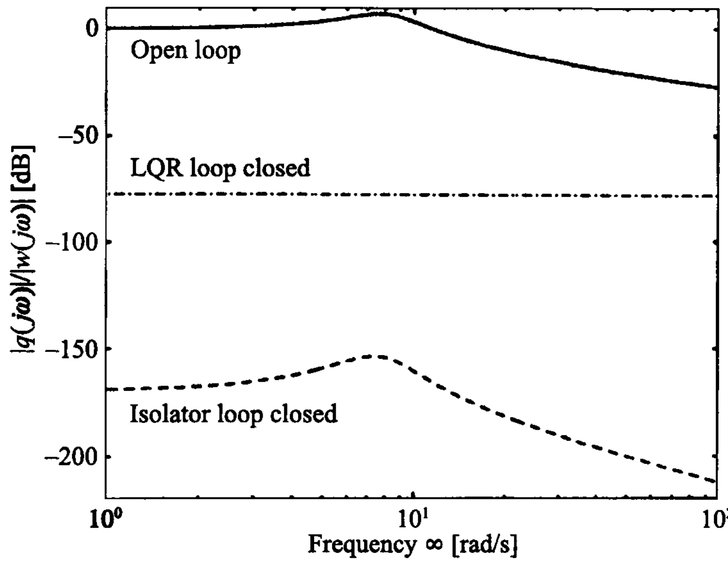

Figure 6. Transmissibility transfer function.

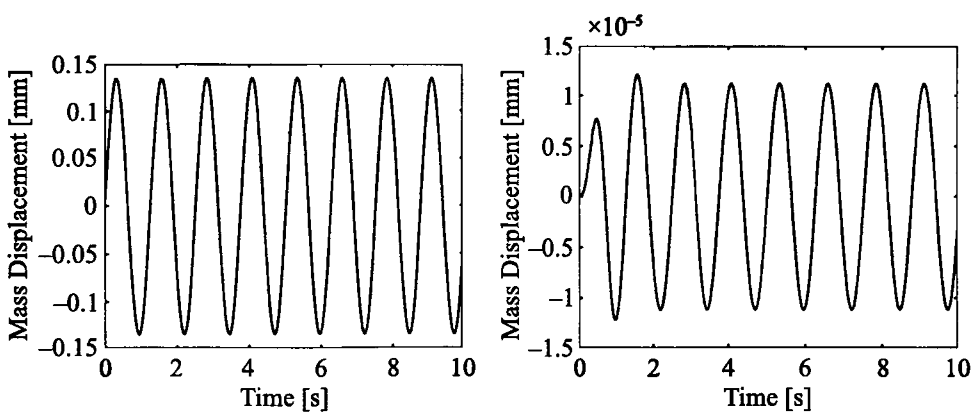

Figure 7. Comparison of sinusoidal response of LQR design (left) and active vibration isolation design with $\mu = 1$ and $(1 - \mu_f)k_u = 1.1111 \times 10^{-5}$ (right).

and the active vibration isolation design given by (44). As can be seen from these figures, the active vibration isolation design given by (44) yields considerable improvement in reducing the mass displacement $q(t)$. Note that as shown in Figures 4 and 5, E_f or, equivalently, $(1 - \mu_f)k_u$ can be used to enhance disturbance rejection. Figure 6 shows the frequency response of the transmissibility transfer function of the open-loop and closed-loop systems. Note that the active vibration isolation design achieves over 150 dB attenuation over the broadband frequency range of 1 to 100 rad/s. Finally, Figure 7 compares the displacement response $q(t)$ to a sinusoidal disturbance $w(t) = \sin(10^{0.7}t)$ for the two designs.

6. CONCLUSION

A dynamic observer-based active isolator for multi-degree-of-freedom structural systems is proposed that guarantees closed-loop asymptotic stability and disturbance decoupling between the base body and the isolated body. While the proposed active isolator is not based on an optimality criterion, numerical calculations indicate that its performance is superior (state response and broadband disturbance rejection) to optimal linear-quadratic controllers. An additional benefit of the active vibration isolator as compared to the optimal linear-quadratic controller is the fact that the active vibration isolator requires only rate measurements, whereas the optimal linear-quadratic controller requires both position and velocity measurements.

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