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Technical Report

THE EFFECT OF FINITE-AMPLITUDE PROPAGATION IN THE ROCKET-GRENADE EXPERIMENT FOR UPPER-ATMOSPHERE TEMPERATURE AND WINDS

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ABSTRACT

This report presents an estimate of the effect of the finite-amplitude propagation on the travel times of pressure waves from the explosions to the ground and a method for taking this effect into account in the data reduction of the rocket-grenade experiment for upper-atmosphere temperature and winds.

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1. INTRODUCTION

The rocket-grenade experiment for upper-atmosphere temperature and winds is based on measuring the time of travel and the direction of arrival at the ground of waves set by charges exploding at high altitudes. It has been assumed in previous calculations that the waves travel with the velocity of sound, i.e., the velocity of an infinitesimally small disturbance. In reality, the wave from the explosion travels at a higher velocity, and the departure from the velocity of sound increases with increased amplitude of the wave. This effect of the actual propagation can thus be called the finite-amplitude-propagation effect.

As part of the assessment of the systematic accuracy of the rocket-grenade experiment in determining the temperature and winds, an attempt was made to estimate the error due to the finite-amplitude propagation of the pressure waves from the explosion. The problem is rather complicated since it is necessary to estimate the difference in finite-amplitude propagation for two successive explosions. To the first approximation, the finite-amplitude-propagation effect on temperatures and winds computed for a layer between two explosions cancels out for two successive explosions if equal charges are used. In other words, it is necessary to assess how the differences in ambient conditions between the bottom and the top of the layer between two explosions affect the shock-wave propagation. No experimental data could be found on this subject within the range of air densities involved in the experiment.

The examination of the problem shows that, other conditions (primarily the explosive charge and the distance from the explosion) being equal, the wave travels faster in a more rarefied medium. This is so because for the same energy content, the wave in a rarefied medium has to exhibit a higher relative pressure (ratio of the wave pressure to the ambient pressure). With increased relative pressure, increased velocity results. Thus the travel time from a higher explosion is shortened more than the travel time from the lower explosion. If this effect is not corrected, the calculated temperatures will be too high.

The calculations which have been carried out indicate that the effect of finite-amplitude propagation is rather small, but not negligible. The effect increases rapidly with increasing altitude.

It is suggested that the following correction for the finite-amplitude-propagation effect be introduced into the data reduction: a certain interval of time t_g should be added to the measured time of propagation. That is to say, the measured time of travel from the explosion to the ground microphones should be increased (for the purpose of calculating the temperatures and winds) by an amount which is believed equal to the shortening of the travel time because of the faster-than-sound propagation. This shortening of the travel time has been computed for different altitudes for 4-lb and 2-lb explosive charges of the type used in the current experiments.

It should be pointed out that such a correction is especially important in the layer between the explosion of a 2-lb grenade and an explosion of a 4-lb grenade.

NOTATION

C	velocity of sound
E	total energy of the explosion
E_w	energy of the shock wave
F_o	finite-amplitude-propagation effect (distance) at R_o
F_g	total finite-amplitude-propagation effect (distance) from the explosion to the ground
H_o	altitude of the explosion
H_s	atmospheric scale height
L	length of the positive phase of the wave
L_o	length of the positive phase of the wave at distance R_o
ΔL	finite-amplitude-propagation effect (distance) from R_o to ground
P	ambient pressure on the path of propagation
P_o	ambient pressure at the explosion, assumed unchanging to R_o
R	distance from the explosion
R_o	distance from the explosion at which $\Pi = .075$
t_g	total finite-amplitude-propagation effect (time) from the explosion to the ground
t	time
t_o	time from the moment of the explosion to the arrival at R_o
T	absolute temperature
ΔT	error in temperature determination
V	shock velocity
x	dimensionless distance from the explosion $R/2H_s$
x_o	dimensionless distance from the explosion at R_o, $R_o/2H_s$
x_g	dimensionless distance from the explosion at the ground, $H_o/2H_s$
γ	ratio of the specific heats
Π	relative overpressure (ratio of excess pressure to ambient pressure)
Π_o	relative overpressure at R_o, equal to .075

2. CALCULATION OF THE FINITE-AMPLITUDE-PROPAGATION EFFECT

The calculations are based on the approach that has been outlined in the quarterly report of November 15, 1955, Report 2387-6-P.

The velocity of a shock wave V is related to the relative overpressure Π by the following equation:

$$V = C\left(1 + \frac{\gamma + 1}{2\gamma}\Pi\right)^{1/2} , \quad (1)$$

and for small overpressures:

$$V \cong C\left(1 + \frac{\gamma + 1}{4\gamma}\Pi\right) , \quad (2)$$

where γ is the ratio of the specific heats. These equations[1] are based on the Rankine-Hugoniot relations and the equation of state of the perfect gas.

In the calculations, symmetrical propagation at the explosion was assumed: the effect of the velocity of the grenade at the moment of the explosion (equal approximately to the velocity of the rocket) has been neglected. This velocity will usually be of the same order of magnitude as the mean velocity of the molecules of the explosive gases. For grenades exploded on the up-leg of the rocket trajectory, this directed velocity of the sphere of the explosion products causes a pressure build-up in the upward direction, and a decrease in the amplitude of the wave propagating downwards. Thus, the directed velocity tends to lessen the effect of finite-amplitude propagation for a single explosion; but, on the other hand, it introduces a new element in the correction, i.e., the correction depends on the difference in the velocities of the grenades at the times of the successive explosions relative to the path from the explosion to the microphones. Since the grenades are ejected on the up-leg, this velocity decreases with successive explosions. This tends to increase the finite-amplitude-propagation effect, since the difference in the finite-amplitude propagation between two successive grenades increases.

The calculation of the propagation for each explosion has been broken down into two phases: (1) propagation up to the radius R_0, up to which point the change in ambient density relative to the point of the explosion has been neglected; and (2) propagation from R_0 to the ground, where the attenuation of energy of the wave has been neglected.

The calculations are largely based on a dimensionless solution of spherical blast waves by H. L. Brode.[2] A particular wave form, shown in Fig. 1, has

1. For the derivation, see, for instance, J.W.M. DuMond, et al., "A Determination of the Wave Forms and Laws of Propagation and Dissipation of Ballistic Shock Waves," J. Acoustical Soc. of America, 18, 97-118 (July, 1946). Note a misprint in Eq. (10) on p. 104; instead of ρ_0, read p_0.

2. H. L. Brode, "Numerical Solutions of Spherical Blast Waves," J. Appl. Phys, 26, 766 (June, 1955). The numerical values describing the wave form of Fig. 1 were obtained from Brode in private correspondence.

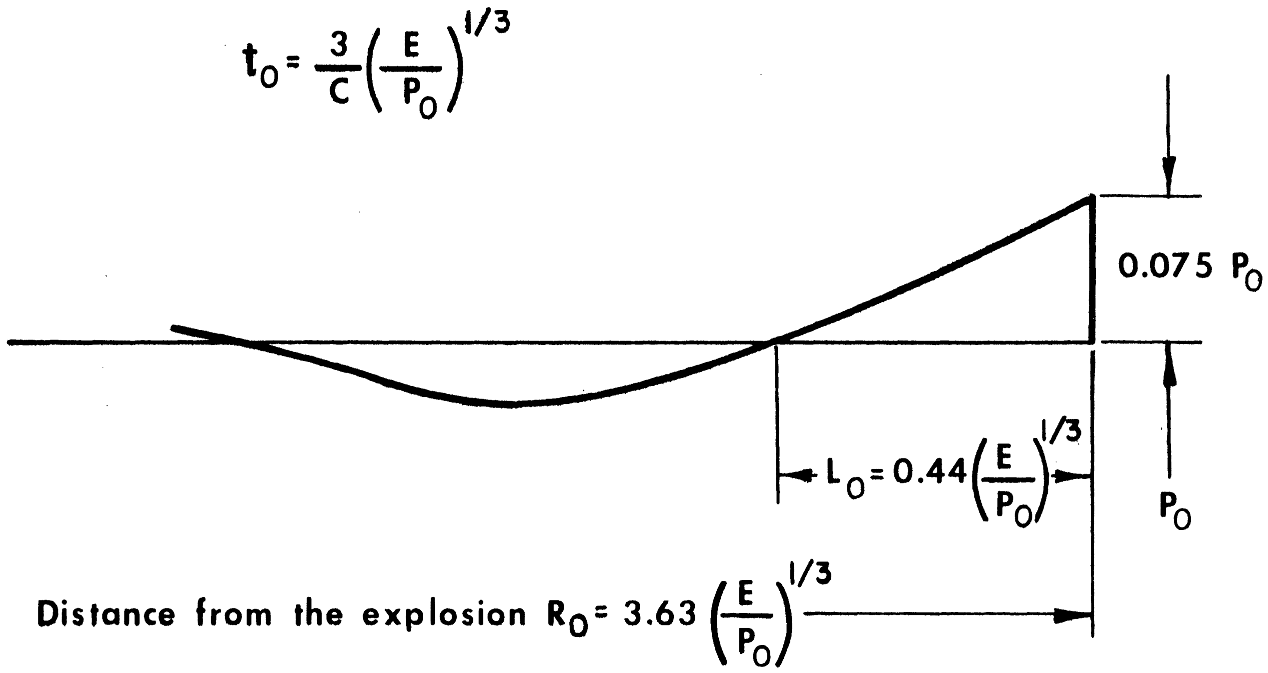

Fig. 1. The wave form at the distance R_0 from the explosion.

been used as the starting point of the calculations. This wave form with an overpressure of about .075 occurs at the distance R_0

$$R_0 = 3.63\ (E/P_0)^{1/3} \quad , \tag{3}$$

and at the time t_0

$$t_0 = (3/C)(E/P_0)^{1/3} \quad , \tag{4}$$

where E is the energy of detonation, and P_0 is the ambient pressure. The finite-amplitude-propagation effect (the distance from the center of the explosion minus the distance that a sound wave would cover in the same interval of time) at this stage, i.e., up to R_0, amounts to

$$F_0 = R_0 - t_0C = (3.63 - 3)\ (E/P_0)^{1/3} = .63(E/P_0)^{1/3} \quad . \tag{5}$$

The length of the region of positive overpressure L_0 is

$$L_0 = 0.44(E/P_0)^{1/3} \quad . \tag{6}$$

Brode's solution takes into account the viscosity of the air. Thus, up to the distance R_0, the attenuation of energy is not neglected.

The energy of the explosion of a 4-lb grenade was taken as 1.472×10^6 kgm and of a 2-lb grenade as $.736 \times 10^6$ kgm, corresponding to the assumed specific energy of 1900 cal/g for the explosive. The value of $(E/P_0)^{1/3}$ is thus 5.22 m.

for the sea-level pressure P_o = 10332 kg/m². Different values of $(E/P_o)^{1/3}$ with P_o changing by a ratio of 2 are compiled in column 2 of Table I for 4-lb grenades and of Table II for 2-lb grenades. In columns 3, 4, and 5 the lengths F_o, L_o, and R_o are tabulated; the calculations are based on Eqs. (5), (6), and (3), respectively. It will be noted here again that up to the radius R_o, the change in ambient pressure with the distance from the explosion is neglected. Since R_o is at most of the order of 1 km, the effect of changing pressure is very small. In column 6 the altitude H_o corresponding to the pressure in column 1 is tabulated, using the ARDC 1956 Model Atmosphere.[3]

For propagation from R_o on, the total energy of the wave is assumed constant and equal to

$$E_w = \pi \frac{R^2 \Pi^2 (R)\ L(R) P(R)}{\gamma} , \tag{7}$$

where R is the distance from the explosion, L(R) the length of the positive overpressure region, $\Pi(R)$ the relative overpressure (the ratio of excess pressure to ambient pressure), and P(R) the ambient pressure.[4]

The finite-amplitude-propagation effect (lengthening in L), is computed by means of Eq. (2) for the shock velocity as a function of the relative overpressure. The relative overpressure is .075 at R_o and decreases in accordance with Eq. (7) with increasing radius. For such small overpressures, the use of Eq. (2) as compared with more exact Eq. (1) involves only a very slight inaccuracy.

Considering the fact that the zero-overpressure point moves almost exactly with the velocity of sound, and using Eq. (2), we express the rate of increase in the length L of the positive phase of the wave as the function of the relative overpressure:

$$\frac{dL(R)}{dR} = \frac{dL(R)}{dt} \cdot \frac{dt}{dR} \cong \frac{dL(R)}{dt} \cdot \frac{1}{C(R)} , \tag{8}$$

$$\frac{dL(R)}{dt} = V(R) - C(R) \cong C(R) \frac{\gamma+1}{4\gamma} \Pi(R) , \tag{9}$$

$$\frac{dL(R)}{dR} = \frac{\gamma+1}{4\gamma} \Pi(R) . \tag{10}$$

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3. R. A. Minzner and W. S. Ripley, "The ARDC Model Atmosphere, 1956," AFCRC TN-56-204, ASTIA Document 110233, Geophysics Research Directorate, Air Force Cambridge Research Center, December, 1956.

4. H. A. Bethe, et al., "Shock Hydrodynamics and Blast Waves," AECD-2860, October, 1944. Equation (7) of this report can be obtained from Eq. (367) by Bethe by substituting the value of C.

TABLE I

4-LB EXPLOSIONS

	1	2	3	4	5	6	7	8	9	10
	P_o µbars	$(E/P_o)^{1/3}$ meters	F_o meters	L_o meters	R_o meters	H_o km	H_s km	x_o	$-Ei(-x_o)$	x_g
1	144,000	10.00	6.30	4.40	36.3	13.9	6.37	0.00285	5.29	1.09
2	72,000	12.60	7.94	5.54	45.7	18.32	6.38	0.00358	5.06	1.43
3	36,000	15.88	10.00	6.98	57.6	22.76	6.39	0.00451	4.83	1.78
4	18,000	20.00	12.60	8.79	72.6	27.21	6.58	0.00550	4.63	2.07
5	9,000	25.20	15.88	11.1	91.5	31.86	7.00	0.00650	4.47	2.27
6	4,500	31.75	20.0	14.0	115.3	36.92	7.45	0.00774	4.29	2.48
7	2,250	40.00	25.2	17.6	145.2	42.24	7.94	0.00914	4.13	2.66
8	1,125	50.40	31.7	22.2	183.0	47.72	8.40	0.0109	3.95	2.84
9	562.5	63.50	40.0	27.9	230.6	53.76	8.38	0.0138	3.72	3.31
10	281.2	80.00	50.4	35.2	290.5	59.36	7.75	0.0187	3.42	3.83
11	140.6	100.8	63.5	44.3	366.0	64.50	7.18	0.0255	3.12	
12	70.3	127.0	80.0	55.8	461.2	69.34	6.63	0.0348	2.82	
13	35.15	160.0	100.8	70.4	581.1	73.74	6.14	0.0473	2.52	
14	17.57	201.6	127.0	88.7	732.2	77.86	5.90	0.0620	2.26	
15	8.78	254.0	160.0	111.7	922.6	81.97	5.91	0.0780	2.05	
16	4.39	320.0	201.6	140.7	1162	86.06	5.92	0.0981	1.84	
17	2.20	403.2	254.0	177.3	1465	90.13	5.93	0.1235	1.63	
18	1.10	508.0	320.0	223.4	1846	94.33	6.24	0.1479	1.48	

TABLE I (Concluded)

	11	12	13	14	15	16	17	18	19	20	21
	$-Ei(-x_g)$	$-Ei(-x)\Big\|_{x_o}^{x_g}$	* meters	$L_g^{3/2}/L_o^{1/2}$ meters	$L_g^{3/2}$ meters	L_g meters	ΔL meters	F_g meters	C m/sec	t_g msec	ΔT °C
1	0.19	5.10	8.92	13.32	28.0	9.22	4.82	11.1	295.1	37.6	0.27
2	0.11	4.95	10.9	16.4	38.6	11.4	5.86	13.8	295.1	46.8	0.33
3	0.07	4.76	13.2	20.2	53.4	14.2	7.22	17.2	295.1	58.3	0.37
4	0.04	4.59	16.1	24.9	73.8	17.6	8.81	21.4	301.4	71.0	0.46
5	0.03	4.44	19.6	30.7	102	21.8	10.7	26.6	307.9	86.4	0.53
6	0.03	4.26	23.7	37.7	141	27.1	13.1	33.1	318.1	104	0.69
7	0.02	4.11	28.8	46.4	195	33.6	16.0	41.2	327.9	126	0.83
8	0.02	3.93	34.7	56.9	268	41.5	19.3	51.0	337.0	151	1.16
9	0.01	3.71	41.2	69.1	365	51.1	23.2	63.2	336.4	188	1.66
10		3.42	47.9	83.1	493	62.4	27.2	77.6	323.3	240	2.02
11		3.12	55.0	99.3	660	75.9	31.6	95.1	310.8	306	2.43
12		2.82	62.7	118.5	885	92.1	36.3	116	298.5	389	2.9
13		2.52	70.6	141.0	1182	112	41.6	142	286.7	495	3.4
14		2.26	79.8	168.5	1586	136	47.3	174	281.3	619	3.8
15		2.05	91.2	202.9	2140	166	54.3	214	281.3	761	4.8
16		1.84	103.1	243.8	2890	203	62.3	264	281.3	938	5.8
17		1.63	115.1	292.4	3890	247	69.7	324	281.3	1152	6.5
18		1.48	131.7	355.1	5300	304	80.6	401	288	1392	

*In column 13 we have $\frac{L_g^{3/2}}{L_o^{1/2}} - L_o = \frac{3(\gamma+1)}{2 \cdot 4\gamma} R_o \Pi_o \left[-Ei(-x)\right]\Big|_{x_o}^{x_g} = 0.0482 \cdot R_o \left[-Ei(-x)\right]\Big|_{x_o}^{x_g}$.

TABLE II

2-LB EXPLOSIONS

	1	2	3	4	5	6	7	8	9	10
	P_o μbars	$(E/P_o)^{1/3}$ meters	F_o meters	L_o meters	R_o meters	H_o km	H_s km	x_o	$-Ei(-x_o)$	x_g
1	72,000	10.00	6.30	4.40	36.3	18.32	6.38	0.00285	5.29	1.43
2	36,000	12.60	7.94	5.54	45.7	22.76	6.39	0.00358	5.06	1.78
3	18,000	15.88	10.00	6.98	57.6	27.21	6.58	0.00438	4.86	2.07
4	9,000	20.00	12.60	8.79	72.6	31.86	7.00	0.00518	4.69	2.27
5	4,500	25.20	15.88	11.1	91.5	36.92	7.45	0.00614	4.52	2.48
6	2,250	31.75	20.0	14.0	115.3	42.24	7.94	0.00726	4.36	2.66
7	1,125	40.00	25.2	17.6	145.2	47.72	8.40	0.00864	4.18	2.84
8	562.5	50.40	31.7	22.2	183.0	53.76	8.38	0.0109	3.95	3.31
9	281.2	63.50	40.0	27.9	230.6	59.36	7.75	0.0148	3.65	3.83
10	140.6	80.00	50.4	35.2	290.5	64.50	7.18	0.0202	3.34	
11	70.3	100.8	63.5	44.3	366.0	69.34	6.63	0.0276	3.04	
12	45.15	127.0	80.0	55.8	461.2	73.74	6.14	0.0376	2.74	
13	22.57	160.0	100.8	70.4	581.1	77.86	5.90	0.0492	2.48	
14	11.28	201.6	127.0	88.7	732.2	81.97	5.91	0.0619	2.27	
15	5.64	254.0	160.0	111.7	922.6	86.06	5.92	0.0779	2.05	
16	2.82	320.0	201.6	140.7	1162	90.13	5.93	0.0980	1.84	
17	1.41	403.2	254.0	177.3	1465	94.33	6.24	0.1174	1.68	

TABLE II (Concluded)

	11	12	13	14	15	16	17	18	19	20	21
	$-Ei(-x_g)$	$-Ei(-x)\Big\|_{x_o}^{x_g}$	* meters	$L_g^{3/2}/L_o^{1/2}$ meters	$L_g^{3/2}$ meters	L_g meters	ΔL meters	F_g meters	C m/sec	t_g msec	ΔT °C
1	0.11	5.18	8.94	13.34	28.0	9.22	4.82	11.1	295.1	37.6	0.27
2	0.07	4.99	11.0	16.5	38.8	11.5	5.96	13.9	295.1	47.1	0.31
3	0.04	4.82	13.4	20.4	53.9	14.3	7.32	17.3	301.4	57.4	0.38
4	0.03	4.66	16.3	25.1	74.5	17.7	8.91	21.5	307.9	69.8	0.44
5	0.03	4.49	19.8	30.9	103	22.0	10.9	26.8	318.1	84.3	0.52
6	0.02	4.34	24.1	38.1	142	27.2	13.2	33.2	327.9	101	0.73
7	0.02	4.16	29.1	46.7	196	33.8	16.2	41.4	337.0	123	0.92
8	0.01	3.94	34.8	57.0	268	41.6	19.4	51.1	336.4	152	1.37
9		3.65	40.6	68.5	362	50.9	23.0	63.0	323.3	195	1.62
10		3.34	46.8	82.0	486	62.0	26.8	77.2	310.8	248	2.0
11		3.04	53.6	97.9	651	75.3	31.0	94.5	298.5	317	2.4
12		2.74	60.9	116.7	870	91.2	35.4	115	286.7	401	2.8
13		2.48	69.5	139.9	1173	111	40.6	141	281.3	501	3.2
14		2.27	80.1	168.8	1590	136	47.3	174	281.3	619	3.8
15		2.05	91.2	202.9	2140	166	54.3	214	281.3	761	4.8
16		1.84	103.1	243.8	2890	203	62.3	264	281.3	938	5.2
17		1.68	118.6	295.9	3940	249	71.7	326	288	1132	

*In column 13 we have $\frac{L_g^{3/2}}{L_o^{1/2}} - L_o = \frac{3(\gamma+1)}{2 \cdot 4\gamma} R_o \Pi_o \left[-Ei(-x)\right]\Big|_{x_o}^{x_g} = 0.0482 \cdot R_o \left[-Ei(-x)\right]\Big|_{x_o}^{x_g}$.

Introducing the value of $\Pi(R)$ from Eq. (7), we have

$$\frac{dL(R)}{dR} = \frac{\gamma+1}{4\gamma}\left(\frac{\gamma E_W}{\pi}\right)^{1/2}\frac{1}{R[P(R)L(R)]^{1/2}} \quad . \tag{11}$$

It will now be assumed that the propagation takes place vertically downward and that the pressure on the path explosion-to-ground changes exponentially with altitude, the scale height H_s being constant and equal to the scale height at the altitude of the explosion; therefore

$$P(R) = P_o e^{R/H_s} \quad . \tag{12}$$

Introducing this value of pressure into Eq. (11), we obtain

$$\frac{dL(R)}{dR} = \frac{\gamma+1}{4\gamma}\left(\frac{\gamma E_W}{\pi}\right)^{1/2}\frac{e^{-R/2H_s}}{R\,P_o^{1/2}[L(R)]^{1/2}} \quad , \tag{13}$$

$$[L(R)]^{1/2}dL(R) = \frac{\gamma+1}{4\gamma}\left(\frac{\gamma E_W}{\pi P_o}\right)^{1/2}\frac{e^{-R/2H_s}}{R}\,dR$$

$$= K\,\frac{e^{-R/2H_s}}{R/2H_s}\,d\,\frac{R}{2H_s} = K\,\frac{e^{-x}}{x}\,dx \quad , \tag{14}$$

where the dimensionless variable x is defined as

$$x = \frac{R}{2H_s} \quad , \tag{15}$$

and the value of the constant K is

$$K = \frac{\gamma+1}{4\gamma}\left(\frac{\gamma E_W}{\pi P_o}\right)^{1/2} = \frac{\gamma+1}{4\gamma}\,R_o L_o^{1/2}\,\Pi_o \quad . \tag{16}$$

The second equality in Eq. (16) follows from Eq. (7). Integrating Eq. (14) between R_o and the ground, we obtain:

$$\frac{2}{3}\left(L_g^{3/2} - L_o^{3/2}\right) = K\int_{x_o}^{x_g}\frac{e^{-x}}{x}\,dx$$

$$= K\left\{[-Ei(-x_o)] - [-Ei(-x_g)]\right\}$$

$$= \frac{\gamma+1}{4\gamma}\,R_o L_o^{1/2}\,\Pi_o\left\{[-Ei(-x_o)] - [-Ei(-x_g)]\right\} \quad , \tag{17}$$

where L_g is the length of the positive phase at the ground, and where x_o and x_g are given by

$$x_o = \frac{R_o}{2H_s} , \tag{18}$$

$$x_g = \frac{H_o}{2H_s} . \tag{19}$$

The exponential integrals $-Ei(-x)$ have been tabulated.[5]

Columns 7-16 of Tables I and II represent steps in computing L_g by means of Eq. (17). In column 17, the finite-amplitude-propagation effect from R_o to the ground

$$\Delta L = L_g - L_o \tag{20}$$

is tabulated. In column 18 we have the finite-amplitude-propagation effect F_g in meters, from the explosion point to the ground:

$$F_g = F_o + \Delta L . \tag{21}$$

In column 19 the velocity of sound C at the altitude of the explosion is tabulated from the ARDC Model Atmosphere, 1956. In column 20, the time t_g, the shortening of travel time, is computed by the formula

$$t_g = \frac{F_g}{C} . \tag{22}$$

The results, F_g and t_g, are plotted, respectively, in Figs. 2 and 3 both for 4-lb and 2-lb grenades as a function of altitude H_o of the explosion.

The effect of neglecting the problem of the finite-amplitude propagation on the temperatures derived from the rocket-grenade experiment can be computed readily under certain simplifying assumptions. Two examples, one involving 4-lb grenades and one involving a 2-lb grenade at the lower explosion and a 4-lb grenade at the upper explosion, will be presented in detail.

Assume that 4-lb-grenade explosions occur at 82 km and 86 km. In this altitude interval the speed of sound is approximately 281 m/sec. The difference in travel times from the explosions to the ground, for close to vertical propagation, will be of the order of (86,000 - 82,000)/281 = 14.2 sec. The difference in the finite-amplitude propagation for the two explosions will be

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5. A. N. Lowan, Tech. Director, Tables of Sine, Cosine, and Exponential Integrals, Vol. I, 1940, prepared by the Federal Works Agency, Work Projects Administration for the City of New York.

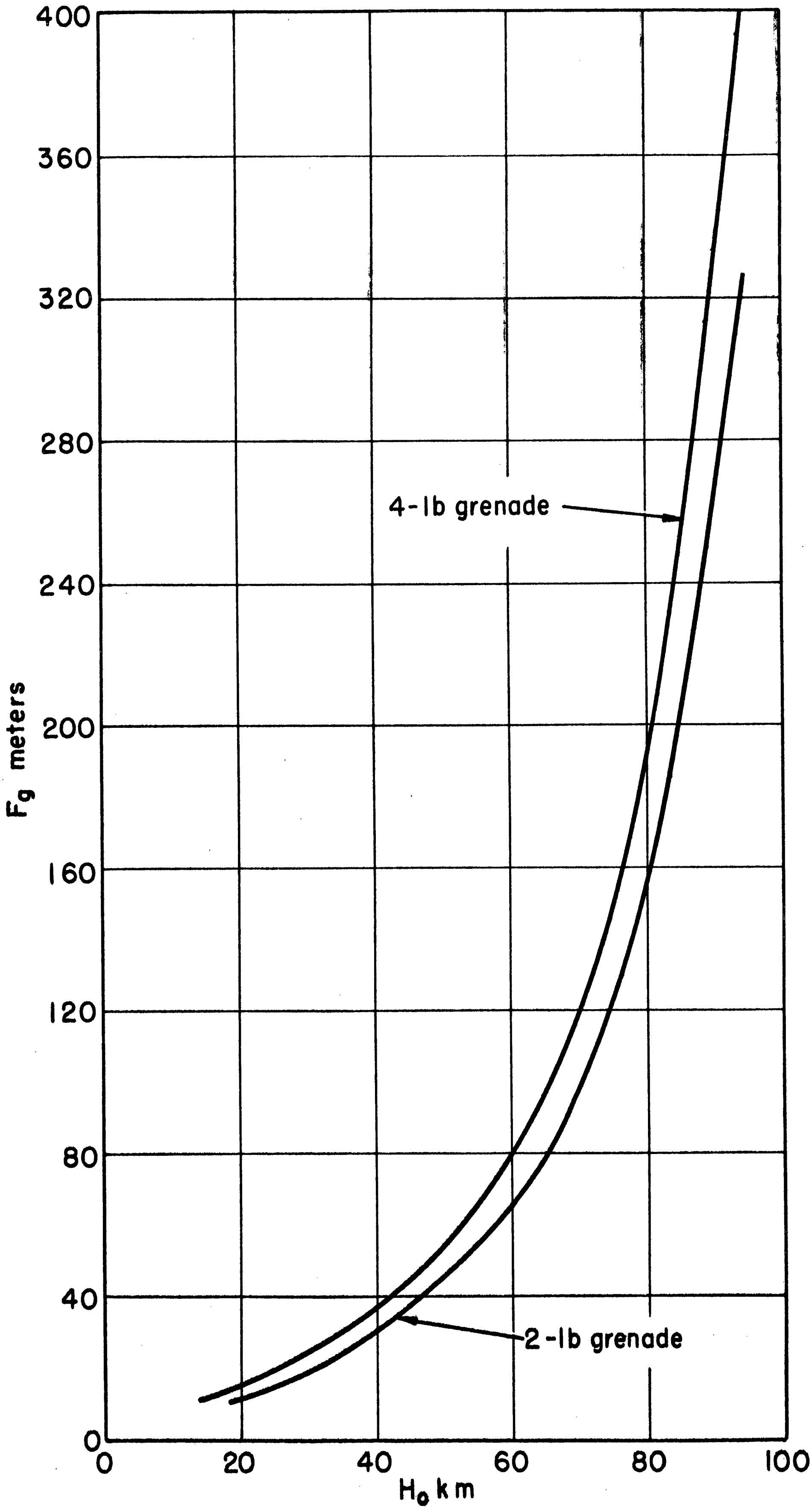

Fig. 2. The finite-amplitude-propagation effect (distance) as a function of the altitude of the explosion.

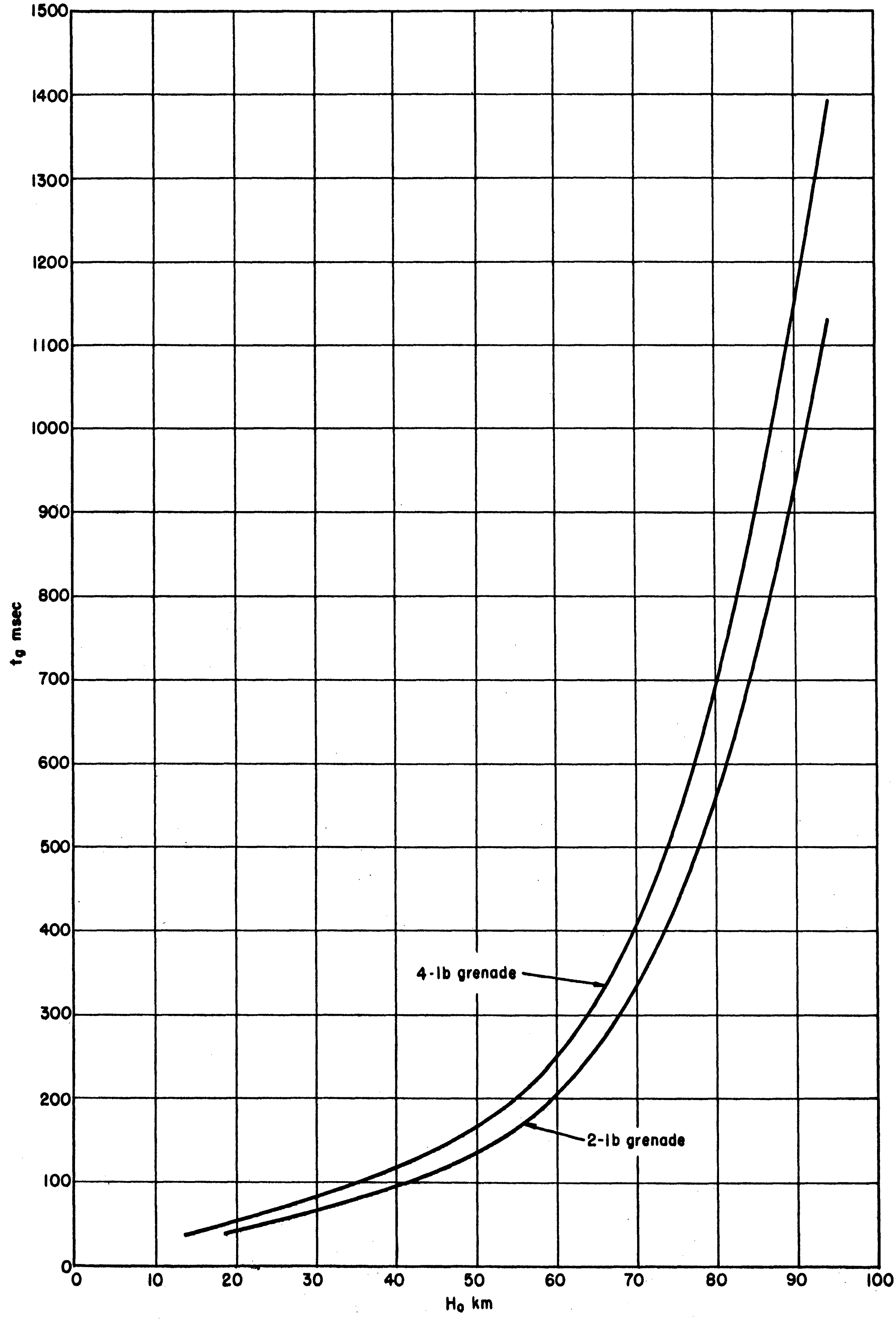

Fig. 3. The finite-amplitude-propagation effect (time) as a function of the altitude of the explosion.

(from Table I, column 20) $t_{g\ upper} - t_{g\ lower} = 938 - 761 = 177$ msec. Thus, the error in the determination of the velocity of sound in the layer between the explosions will be of the order of (.177/14.2) = 1.25%. The error ΔT in the determination of the temperature T will be of the order of +2.5%T, or about +4.8°C. In columns 21 of Table I and Table II the error ΔT has been tabulated for 4-lb and 2-lb grenades, respectively. This error has been calculated as in the above example, and applies to standard atmosphere and to explosion intervals corresponding to altitudes tabulated in columns 6 of the tables.

Assume that an explosion of a 2-lb grenade occurs at 64.5 km and an explosion of a 4-lb grenade occurs at 69.3 km. The difference in the travel times will be of the order of (69,300 - 64,500)/304 = 15.8 sec, assuming the average velocity of sound to be 304 m/sec. The difference in the finite-amplitude propagation will be (from Tables I and II, column 20) $t_{g\ upper} - t_{g\ lower} = 389 - 248 = 141$ msec. The error in the velocity of sound will be of the order of (.141/15.8) = .9%; and the error ΔT in the temperature will be about +1.8%T, or +4.2°C.

Certain comments will be now made regarding the calculations, reflecting on the expected accuracy of the results. The calculations by Brode apply to the point-source solution in an ideal gas. This solution resulted in a higher overpressure at the scaled distance $3.63(E/P_0)^{1/3}$ than the other calculations by Brode, carried out for real air and for TNT explosions. The solution resulting in the highest overpressure was chosen because it corresponded more closely with the experimental data reported by Schardin[6] and by Granström.[7] For the same scaled distance, Schardin reports a relative overpressure of about 0.10 and Granström, of 0.08, as compared with an overpressure of 0.075 for the point source in an ideal gas solution by Brode. The numerical values by Schardin and by Granström would, if used in calculating the finite-amplitude-propagation effect, produce results higher by about 33% and 7%, respectively, than the current calculations.

The pressure and temperature profile, as stated, has been assumed according to the ARDC Model Atmosphere, 1956. It can be shown by evaluating the expression $[d(E/P_0)^{1/3}]/dH_0 \cdot \Delta H_0$ in terms of average atmospheric parameters in the layer, that the effective correction term for the distance ($F_{g\ upper} - F_{g\ lower}$) between explosions of equal charges is approximately inversely proportional to the average absolute temperature, and inversely proportional to the 1/3 power of the average ambient pressure in the layer. The effective correction term for the time ($t_{g\ upper} - t_{g\ lower}$) is approximately inversely proportional to the 3/2 power of the absolute temperature, and inversely proportional to the 1/3 power of the ambient pressure. The temperature is not expected to depart by more than 10% from the Model in the range of altitudes

6. Hubert Schardin, "Measurement of Shock Waves," Comm. Pure Appl. Math., 7, 223-243 (1954).

7. S. A. Granström, "Loading Characteristics of Air Blasts from Detonating Charges," Trans. Roy. Inst. of Tech., Stockholm, Sweden, No. 100, 1956.

considered, but the actual ambient pressure can be different by a factor of 3 or more. These possible very large departures in actual pressure from the assumed pressure may cause considerable changes in the finite-amplitude propagation, even though they are reflected through the 1/3 power only.

It should be pointed out that recalculation of the results for a given atmospheric profile could be done in approximately one working day. This use can be made without changing the first 6 columns of Tables I and II.

The computation of ΔL depends on H_s. The exponent $1/2\ H_s$ is assumed to be constant on the propagation path from R_o to the ground, where H_s is taken equal to the scale height at the altitude H_o; ΔL is only about 1/4 of F_g; and H_s varies rather slowly. Most of ΔL is due to finite-amplitude propagation at the distance from R_o to $5R_o$, that is, not far from the explosion. Because of these three points, it is thought that a constant H_s is a good approximation.

Vertical propagation has been assumed. For propagation at an elevation angle α with the vertical, instead of Eq. (12) the formula for the variation of pressure with the distance from the explosion will be

$$P = P_o e^{R \cos \alpha / H_s} \quad . \tag{23}$$

For small angles of elevation, and considering again that ΔL is only a quarter of the total effect F_g, $\cos \alpha = 1$ is a reasonable approximation.

The divergence or convergence of the wave, other than the spherical spread of the wave, has been assumed negligible for close-to-vertical propagation.

3. CONCLUSION

The effect of finite-amplitude-propagation has been calculated for travel times from explosions at various altitudes of 4-lb and 2-lb grenades. The results are presented in the forms of tables and graphs. The effect, if not taken into account, can cause errors in the determination in upper-atmosphere temperatures of the order of 5°C. The appropriate correction consists of adding to the measured travel times the calculated shortening of the travel times from the explosions to the ground due to faster-than-sound propagation.