Effects of Turbulence on Laminar Skin Friction and Heat Transfer

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Experiments were performed in two low-turbulence wind tunnels ($u_\infty' < 0.1\%$) to determine the effects of turbulence on heat transfer from plates and circular cylinders in incompressible flow. Grid turbulence up to 6% was imposed. Heat transfer was increased about 30% in the laminar region of a flat plate and up to 70% on a circular cylinder; smaller though still significant increases in shearing stress at the wall were measured by hot wires near the surface. A phenomenological theory is given which shows good agreement with the experiment.

1. INTRODUCTION

THIS paper contains salient results of an investigation[1] comprising, (1) measurements of heat transfer and skin friction made on a flat plate and on circular cylinders located at various distances downstream of a grid in two low-turbulence wind tunnels, and (2) a phenomenological theory which correlates with the results of the measurements.

The present study was undertaken primarily because of the wide disparity among the results of previous investigations of the local heat transfer from circular cylinders normal to the flow, as shown and referenced in Fig. 12; the coordinates used in the plot follow from the theoretical considerations described in Sec. 4. Review of the previous studies indicated that much of the disparity might be attributable to wide variations in the free-stream turbulence. Under conditions of high free-stream turbulence the structure of the turbulent field could be expected to be quite different with and without the imposed turbulence. Since the structure of the turbulence as well as its magnitude would affect the properties of the laminar layer, the present investigation was undertaken in wind streams in which free-stream turbulence of less than 0.1% (rms streamwise component relative to free-stream velocity) could be achieved. In addition, results of previous measurements on flat plates by Edwards and Furber[2] and by Kestin, Maeder, and Wang[3] were reviewed since it appeared that the background turbulence might have been relatively high in these experiments also. Further experiments on the flat plate were performed; the results are given herein.

The present results on a flat plate are self-consistent and are in general agreement with some of the previous results. The measured laminar heat transfer and skin friction on a flat plate at a Reynolds number of 10^5 were increased by about 30% over the low turbulence values when the imposed turbulence at the leading edge was 6%. On the circular cylinder the measured values near the stagnation point were increased by about 70% for heat transfer and 50% for skin friction with a turbulence level of about 6% and a Reynolds number of 240 000.

A phenomenological theory for the heat transfer and skin friction near the stagnation point of a circular cylinder is presented. The theory, which postulates a Reynolds stress proportional to the imposed turbulence level in the ambient flow and to the distance from the wall, agrees satisfactorily with the experimental results.

In general, previous theoretical studies represented by Lighthill,[4] by Lin,[5] and by Glauert[6] resulted in predictions of little or no effect of free-stream oscillations on the mean laminar skin friction or heat transfer of flat plates or circular cylinders. The previous experiments on flat plates agree in general with the theories indicating little or no effect of turbulence, provided the Reynolds number and free-stream turbulence level were low enough (less than 100 000 and 5%, respectively). However, the experiments on circular cylinders of Giedt,[7] of Kestin, Maeder, and Sogin,[8] of Seban,[9] of Zapp,[10] and of Schnautz[11] indicated quantitative disagree-

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[1] M. C. Smith, Ph.D. thesis, University of Michigan (1964).

[2] A. Edwards and B. N. Furber, Proc. Inst. Mech. Engrs. **170**, 941 (1956).

[3] J. Kestin, P. F. Meader, and H. E. Wang, Intern. J. Heat Mass Transfer **3**, 133 (1961).

[4] M. J. Lighthill, Proc. Roy. Soc. (London) **A224**, 1 (1954).

[5] C. C. Lin, in *Proceedings of the Ninth International Congress on Applied Mechanics* (University of Brussels, Brussels, Belgium, 1957).

[6] M. B. Glauert, J. Fluid Mech. **1**, 97 (1956).

[7] W. H. Giedt, Trans. ASME **71**, 375 (1949).

[8] J. Kestin, P. F. Maeder, and H. H. Sogin, Z. Angew. Math. Phys. **12**, 115 (1961).

[9] R. A. Seban, J. Heat Transfer **82**, 101 (1960).

[10] G. M. Zapp, MSE thesis, Oregon State University (1950).

[11] J. A. Schnautz, Ph.D. thesis, Oregon State University (1958).

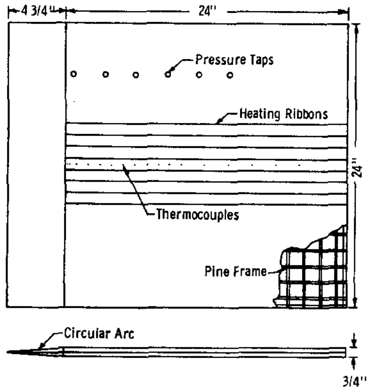

FIG. 1. The test plate.

ment with the theories, in that large increases (10 to 70%) in heat transfer near the forward stagnation point of circular cylinders were observed. The experiments did not provide firm evidence of increased skin friction near the forward stagnation region, nor was a consistent relation evident among cylinder Reynolds number, free-stream turbulence level, and heat transfer increase.

The recent theoretical studies by Sutera, Maeder, and Kestin[12] and by Sutera[13] show that, in steady rotational flow near a stagnation point, the stretching of the vorticity component parallel to the wall causes a substantial increase in the heat transfer at the wall, accompanied by a much smaller percentage increase in the wall friction. These findings agree qualitatively with the analytical results described in Sec. 4.

2. APPARATUS

The flat plate, shown in Fig. 1, was built of hardwood and balsa. Heating ribbon of 0.002-in. Nichrome extended lengthwise over the downstream 24 in. of both upper and lower surface. The theormocouples shown were embedded in both surfaces. The gaps between the lengths of ribbon were 0.01 in. and were filled flush with the bonding material. The heating ribbons provided a constant rate of heat generation per unit area at the surface of the plate. Velocities close to the plate were obtained with hot wires and pitot probes.

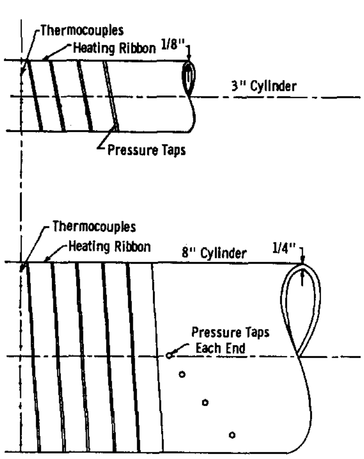

FIG. 2. The test cylinders.

The circular cylinders were commercially available cast acrylic plastic with diameters of 3 and 8 in. and wall thicknesses of $\frac{1}{8}$ and $\frac{1}{4}$ in., respectively. They extended from floor to ceiling to the tunnel. The cylinders used for the heat transfer measurements were wrapped with heating ribbon; thermocouples and pressure taps were installed as indicated in Fig. 2. Analyses and test at two heating currrents (see Fig. 8) showed that corrections for radiation and conduction losses were satisfactory; the surface temperature was about 20°F above ambient for all tests, except for the one noted in Fig. 8.

Two wind tunnels were used in the experiments; an open circuit tunnel with 2- × 2-ft test section, and a closed circuit tunnel with 5- × 7-ft test section. Both tunnels had large settling chambers, contraction ratios around 16:1 and turbulence reducing screens so that the turbulence level in the free stream was less than 0.1%. Both tunnels could be fitted with round rod, square mesh, grids at the entrance to the test section.

The results reported here were obtained in the free streams and at various distances behind grids comprising $\frac{1}{4}$-in.-diam rods with 1-in. mesh. The turbulence levels downstream of the grids are shown in Fig. 3 at various air speeds compared with measurements reported by Dryden *et al.*[14] The increase in turbulence level with increasing air speed in the 5- × 7-ft tunnel is attributed to vibration of the grid at the higher wind speeds. In this figure, u_∞'

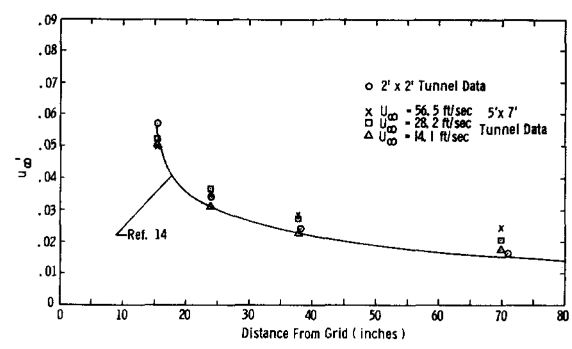

FIG. 3. Free-stream turbulence downstream of 1- × 1-in. grid.

[12] S. P. Sutera, P. F. Maeder, and J. Kestin, J. Fluid Mech. 16, 497 (1963).
[13] S. P. Sutera, J. Fluid Mech. 21, 513 (1965).

[14] H. L. Dryden, G. B. Schubauer, W. C. Mock, Jr., and H. K. Skramstad, NACA Report 581 (1937).

is the ratio of the rms u component of the free-stream velocity fluctuations to the free-stream mean speed U_∞, and Re_D is the cylinder Reynolds number based on cylinder diameter and free-stream conditions.

Turbulence measurements were made with Thiele–Wright and Shapiro–Edwards constant-current equipment. Standard equipment was used for measuring thermocouple voltages, pressures, etc.

The effect of turbulence on wall friction was measured by means of a hot-wire bug, consisting of a silver-coated platinum hot wire cemented to the wall and etched in place. This instrument, developed by W. W. Willmarth and B. J. Tu of the University of Michigan, permitted measurements of the velocity at a distance of about 0.001 in. from the wall.

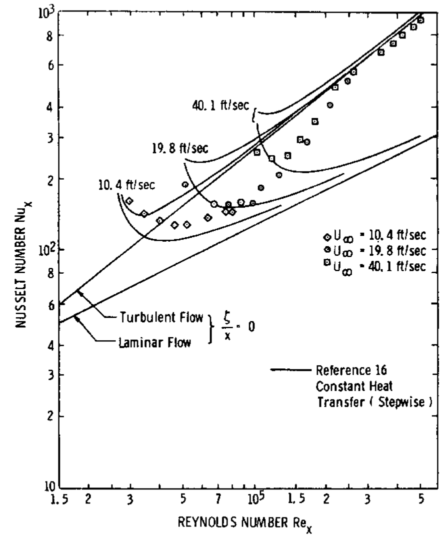

FIG. 4. Heat transfer from flat plate with leading edge 15½ in. downstream of grid.

3. RESULTS, ANALYSIS, AND DISCUSSION

In this section representative results are presented on heat transfer from the flat plate in the 2- × 2-ft tunnel and from the cylinders in the 2- × 2-ft and 5- × 7-ft tunnels at various distances behind the ¼-in.-diam rod, 1-in. mesh, grid over a range of wind speeds. The effect of turbulence on wall friction on the cylinder was determined. The results are in satisfactory agreement with predictions of a phenomenological theory.

The plate and cylinder were maintained at about 20°F above the ambient temperature. Repeat measurements at 40°F rise showed no measurable change in Nusselt number.

3.1 Flat Plate

The flat plate heat transfer data in the form Nu_x vs Re_x are plotted in Figs. 4 and 5 for the clear tunnel and for the plate leading edge 15.5 in. downstream of the grid: Reference to Fig. 3 shows that in this configuration the turbulence level was 6% at the leading edge and decreased with distance downstream. The Nusselt number and the Reynolds are based on distance from the leading edge x and free-stream conditions. The experimental condition approximates closely that of a stepwise constant heat transfer,[15] treated theoretically by Eckert, Hartnett, and Birkebak[16] for both laminar and turbulent boundary layers. The theoretical curves are shown in the figure, in which, the distance from the leading edge to the start of the heating is 4.75 in.; the values of Re_x corresponding to this distance are the starting points of the curves for the three air speeds under the average conditions of the tests. The results shown in Fig. 4, transformed to the equivalent constant temperature flat plate by the method of Ref. 16 are shown in Fig. 6. The data are in good agreement with the classical isothermal flat plate theory of Pohlhausen, described in Goldstein.[17]

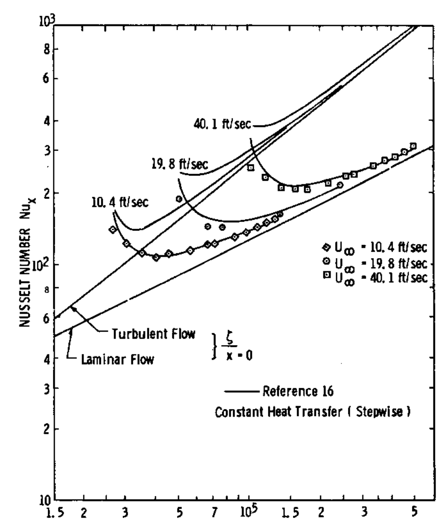

FIG. 5. Heat transfer from flat plate in clear tunnel.

[15] The resistance of the ribbon remains constant so that the rate of heat generation is constant over the heated portion of the plate. Calculations of radiation and convection[1] indicate that the deviation of the heat transfer rate from constancy is about 0.5% per in. Thus the heat transfer rate is about 12% lower at the trailing edge than at the leading edge of the heated section.

[16] J. Hartnett, E. Eckert, and R. Birkebak, in *Proceedings of the Sixth Midwestern Conference on Fluid Mechanics* (Austin University Press, Austin, Texas, 1959), pp. 47–70.

[17] S. Goldstein, *Modern Developments in Fluid Dynamics* (Oxford University Press, New York, 1938).

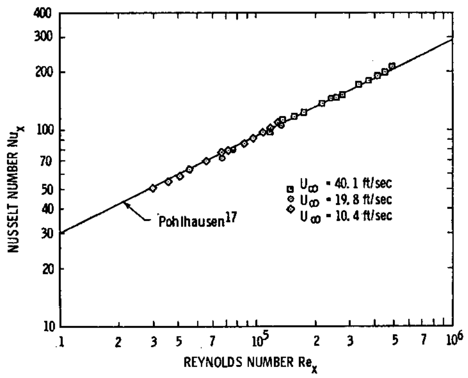

FIG. 6. Comparison of heat transfer measurements on flat plate with theory.

Comparison of Figs. 4 and 5 indicates that at x Reynolds numbers near that for the start of the heating the effect of turbulence on the Nusselt number is small, except at the lowest air speed. The turbulence causes transition to a turbulent boundary layer to be complete near $Re_x = 2 \times 10^5$.

In another test the heating ribbon was extended over the leading edge and the heat transfer was determined at $x = 0.5$ in. ($Re_x = 5000$) in the clear tunnel and with a turbulence level of 6%. The difference between the two measured values of heat transfer was within the experimental accuracy of around 2%.

Figure 7 is a plot of the ratios of Nusselt numbers in the clear tunnel to those behind the grid, as functions of Reynolds number and free-stream speed. The data for the clear tunnel and that for $x = 15.5$ in. were the same as the data shown in Figs. 4 and 5. The arrows below the abscissa indicate approximately the leading edge of the heated portion of the plate for the three air speeds. The curves show the initial gradual increase in Nusselt number ratio with Reynolds number, followed by more rapid increases. Factors which might affect the rate of increase of the ratio are the Reynolds numbers based on leading edge radius and on grid mesh; since no systematic study was made of the effects of either of these variables on the flat plate flow we cannot on the basis of the present data assess their relative importance.

To interpret the results in Fig. 7 we adopt the usual criterion that the appearance of turbulent bursts is indicative of the onset of transition to a turbulent layer. Hot-wire records showed that, with 6% free-stream turbulence, the first turbulent bursts occurred at $Re_x = 125\,000$ and the turbulent field appeared to be fully developed at $Re_x = 250\,000$. Thus, the increases in Nusselt number shown in Fig. 7 at Reynolds numbers below 125 000 can be attributed to an eddy heat transfer generated within the laminar layer by the free-stream turbulence. In this range oscillograms and measured spectra showed low-frequency velocity fluctuations within the boundary layer, similar to those reported by Dryden.[18]

With the above criterion Fig. 7 shows an increase of the Nusselt number for the laminar boundary layer of about 30% at $Re_x = 125\,000$ at wind speeds of 10 and 20 ft/sec at a grid to plate leading edge distance of 15.5 in. ($u'_\infty = 6\%$).

3.2 Circular Cylinder

The cylinder wrapped with resistance strip, as for the heated portion of the flat plates, approximated closely a constant local heat transfer condition over the entire cylinder. As with the flat plate the method of Eckert, Hartnett, and Birkebak[16] was used to transform the results to those for isothermal surface.

Measurements are reported in Ref. 1 for four turbulence levels; those for the maximum and minimum levels are plotted in Figs. 8[19] and 9.

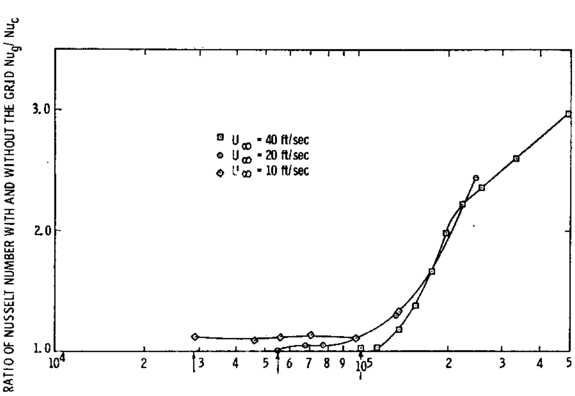

FIG. 7. Comparison of heat transfer with grid at $15\frac{1}{2}$ in. with measurements in clear tunnel.

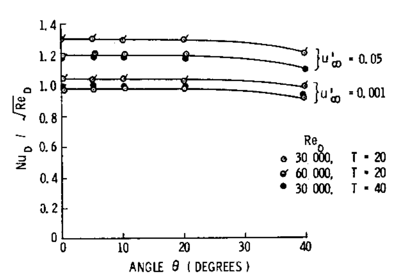

FIG. 8. Measured heat transfer from 3-in.-diam cylinder in clear tunnel and with grid at $15\frac{1}{2}$ in. T is the temperature of cylinder surface relative to ambient.

[18] H. L. Dryden, NACA Report 562 (1936).

[19] Figure 8 also shows that when the temperature rise was doubled to 40°F the change in Nusselt number was within the experimental error.

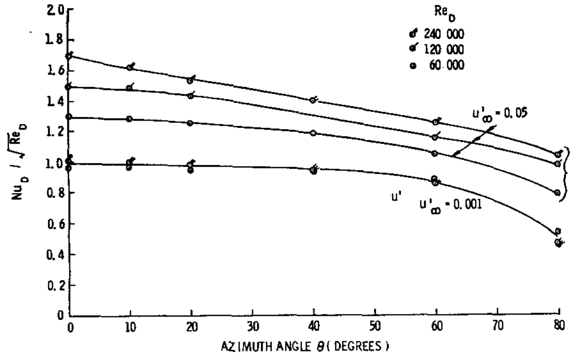

FIG. 9. Measured heat transfer from 8-in. cylinder in 5-× 7-ft tunnel.

A cross-plot of the data for the stagnation line, $\theta = 0°$, is shown in Fig. 10. The angle θ is measured around the cylinder from the forward stagnation point. This figure shows an approximately linear relation between the stagnation point Nusselt number and the free-stream turbulence level for a constant Reynolds number. The slopes of the lines drawn through the data increase with Reynolds number, showing that the influence of the free-stream turbulence on the heat transfer increases with Reynolds number. Not all of the data were taken at fixed free-stream turbulence levels, due to noise and vibration of the grid at some air speeds in the 5- × 7-ft tunnel. Thus, a cross plot was obtained from the lines faired through the data of Fig. 10, rather than from the data points themselves. This cross plot is given in Fig. 11 and shows the Nusselt number versus Reynolds number for constant free-stream turbulence levels. The faired relation shown for $u'_\infty \approx 0$ is

$$\mathrm{Nu}_D = (\mathrm{Re}_D)^{\frac{1}{2}}, \tag{1}$$

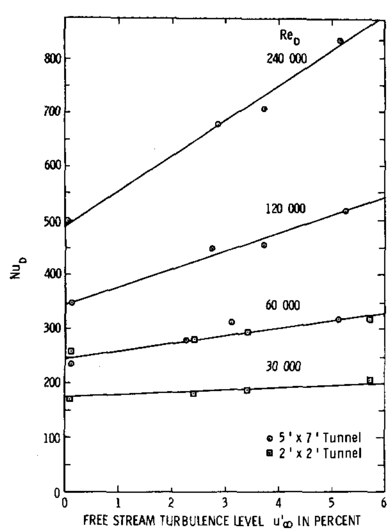

FIG. 10. Heat transfer data at stagnation point of cylinders.

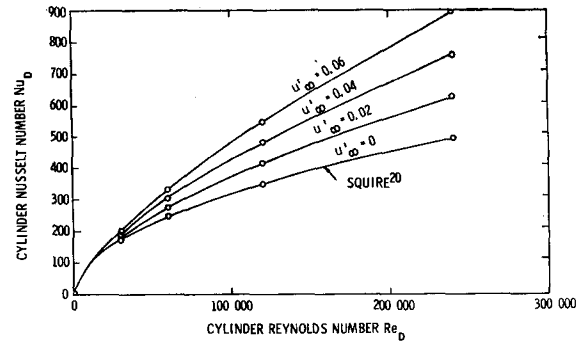

FIG. 11. Faired heat transfer data from Fig. 10.

which correlates well with the data and with the theory of Squire.[20,21]

The stagnation point heat transfer data are plotted in Fig. 12; the coordinates of the plot are those indicated by the phenomenological theory given below which is represented by the dashed curve. Experimental points from other investigations are also shown. The scatter among these data indicates that in some of the experiments one or more uncontrolled variables, such as the background turbulence, must have significantly affected the observations. While the agreement between the present data agrees well with the theory, the plots of the faired data, Figs. 13 and 14, indicate that the stagnation point data at low Reynolds numbers are not adequately represented by the single curve on Fig. 12. These data show in fact that

$$\{[\mathrm{Nu}_D/(\mathrm{Re}_D)^{\frac{1}{2}}] - 1\}/u'_\infty(\mathrm{Re}_D)^{\frac{1}{2}} = f(\mathrm{Re}_D), \tag{2}$$

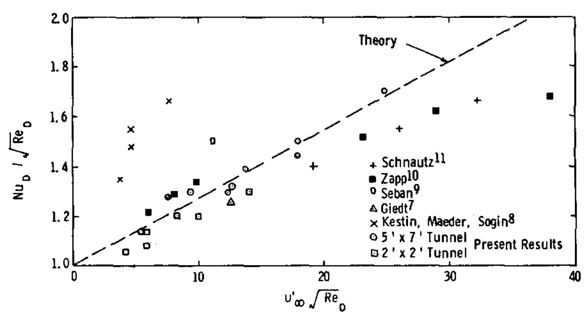

FIG. 12. Heat transfer data at stagnation point of cylinders compared with data from other sources and with theory of Sec. 3.3.

[20] See Ref. 17, Vol. II, p. 631.

[21] One of the questions to be answered is whether effects, similar to those found as a function of free-stream turbulence, can be traced to velocity fluctuations induced over the forward portion of the cylinder by the unsteady flow in the wake. As a result of auxiliary tests, employing a streamlined afterbody which effectively eliminated the oscillations due to the wake, it was concluded that the effect is negligible. The agreement of the measured results in the clear tunnel with Squire's theory,[20] as shown in Fig. 11, supports this conclusion.

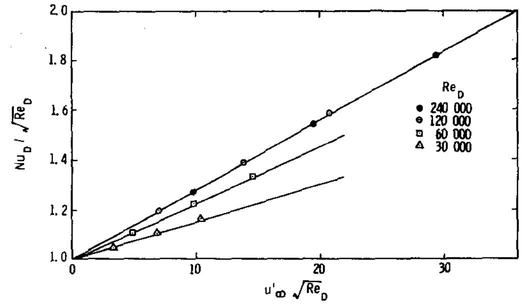

FIG. 13. Data of Fig. 10 plotted to show the Reynolds number effect not predicted by theory.

with

$$f(\mathrm{Re}_D) = 0.277(1 - e^{-2.9\times10^{-5}\mathrm{Re}_D}). \tag{3}$$

Figure 14 indicates that $f(\mathrm{Re}_D)$ is approximately constant at Reynolds numbers over 10^5. The data points of Fig. 14 are taken from the faired data, Fig. 11, for $u'_\infty = 0.02$, 0.04, and 0.06.

3.3 Analysis

The phenomenological theory leading to the theoretical curve in Fig. 12 is described below by analysis of the flow at the stagnation point on a circular cylinder. The eddy viscosity is assumed to be proportional to the turbulence in the free stream, the proportionality constant being determined from the experimental data. The approximate boundary layer equations for incompressible flow near the stagnation point, including a term to represent a turbulent stress, are

$$U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[(\kappa + \epsilon)\frac{\partial T}{\partial y}\right], \tag{4}$$

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left[(\nu + \epsilon)\frac{\partial U}{\partial y}\right], \tag{5}$$

$$(\partial U/\partial x) + (\partial V/\partial y) = 0, \tag{6}$$

where ρ, κ, and ν are the fluid density, thermometric conductivity, and kinematic viscosity, respectively. In these equations ϵ is the eddy viscosity and eddy conductivity, thus implying a turbulent Prandtl number of unity. The symbols U and V refer to mean values of velocity components in the tangential and normal directions, respectively, corresponding to the coordinates x and y, while T and P refer to mean values of temperature and pressure.

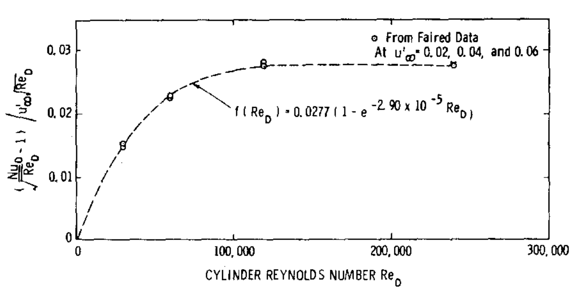

FIG. 14. Data of Fig. 10 plotted to show asymptotic behavior.

It is assumed that the eddy viscosity is proportional to the turbulence in the free stream and to the distance from the wall, that is,

$$\epsilon = Ku'_\infty U_\infty y. \tag{7}$$

The method of analysis of Squire[20] is used with the subscripts w referring to the wall and ∞ to the free stream. Then, with U_e the mean velocity at the edge of the boundary layer.

$$\begin{aligned} U_e &= \beta x, \\ U &= U_e f'(\eta), \\ V &= -(\nu\beta)^{\frac{1}{2}} f(\eta), \\ \eta &= (\beta/\nu)^{\frac{1}{2}} y, \\ a &= \tfrac{1}{2} K u'_\infty \mathrm{Re}_D^{\frac{1}{2}}, \\ \theta(\eta) &= [T_w - T(\eta)]/(T_w - T_\infty), \\ \sigma &= \nu/\kappa, \end{aligned} \tag{8}$$

where σ is the molecular Prandtl number, $\beta = 4U_\infty/D$ for flow near the forward stagnation point of a circular cylinder, Eqs. (4)–(7) become

$$f'^2 - f''(f + a) = 1 + (1 + a\eta)f''', \tag{9}$$

$$\theta'' + \theta'(f + a)/(a\eta + 1/\sigma) = 0. \tag{10}$$

In Eq. (8), K is a constant to be determined from the data. The solution of Eq. (10) which satisfies the boundary conditions $\theta(0) = 0$, $\theta(\infty) = 1$ gives

$$\mathrm{Nu}_D = \alpha(\sigma, a)(\beta D^2/\nu)^{\frac{1}{2}}, \tag{11}$$

where

$$\alpha(\sigma, a) = \left[\int_0^\infty \exp\left(-\int_0^\eta \frac{f + a}{a\eta + 1/\sigma}\,d\eta\right)d\eta\right]^{-1}. \tag{12}$$

The boundary conditions on the solution of Eq. (9) are $f(0) = f'(0) = 0$ and $f'(\infty) = 1$.

Equation (9) was solved on a digital computer with the boundary conditions at infinity applied at $\eta = 6$.

A value

$$K = 0.164 \tag{13}$$

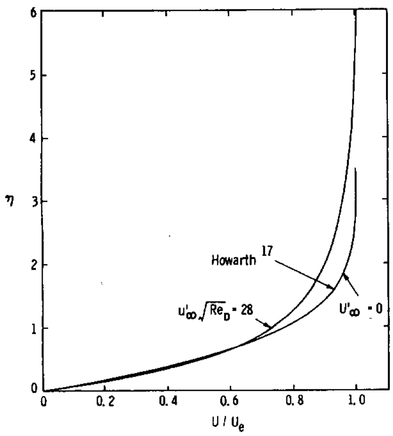

FIG. 15. Theoretical velocity profile ($K = 0.164$) compared with laminar theory.

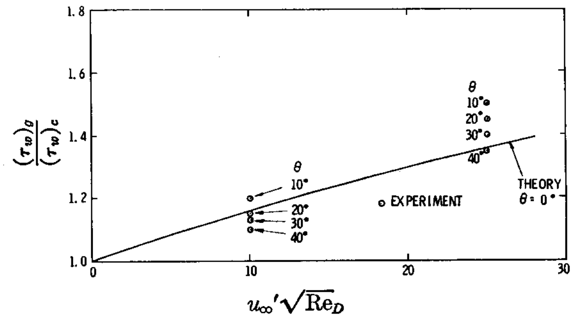

FIG. 17. Comparison of measured shearing stress at surface of cylinder at various azimuth angles with theory.

was obtained using the faired data point at $u_\infty' (\mathrm{Re}_D)^{\frac{1}{2}} = 10$ in Fig. 12 and $\alpha = 0.643$. In this calculation the values of K and $f''(0)$ were varied until $f'(6) = 1$ and the calculated value of α was that given by Eq. (11) for the chosen data point. With the value of K thus determined, values of α were determined for various values of $u_\infty' (\mathrm{Re}_D)^{\frac{1}{2}}$; in each case $f''(0)$ was varied until the condition $f'(6) = 1$ was satisfied. In this way the theoretical curve of Fig. 12 was obtained. In addition, velocity and temperature distributions were determined; representative calculated profiles are shown in Figs. 15 and 16. These indicate a somewhat greater effect of turbulence on heat transfer than on skin friction.

3.4 Comparison with Experiment

Comparison of the theoretical and experimental effects of turbulence on skin friction is shown in Fig. 17. The experimental effect was taken as the measured ratio between the velocities at about 0.001 in. from the surface in the clear tunnel and with the grid at 15.5 in. upstream of the stagnation point at two Reynolds numbers. The measurements were made by the hot-wire instrument described in Sec. 2; the indicated velocity was corrected for the proximity of the wall by the method of Piercy, Richardson, and Winny.[22] Since the theory applies to stagnation point flow it cannot show dependence on the angular position, as do the experiments. A summary plot of the ratios of $\partial U/\partial y$ at 0.001 in. and of Nusselt number in the clear tunnel and with the grid 15.5 in. upstream are shown in Fig. 18 as functions of the azimuth angle. Velocity ratio fluctuations near the stagnation point were strongly influenced by the unsteady wake. The mean velocity is near zero in this region and motion of the stagnation point causes alternate intervals of flow reversal at the hot-wire position; the measured fluctuation amplitudes are for this reason not significant and these ratios are not shown in Fig. 18.

The increase in skin friction with increasing free-stream turbulence as shown in Figs. 17 and 18, is within 70% of that of the heat transfer. All of the available theories, however, except the one given here, predict insignificant increases in skin friction. In the somewhat related analysis of flow around a cylinder in a steady rotational flow by Sutera[13] the stretching of the vorticity component parallel to the surface near a stagnation point causes a

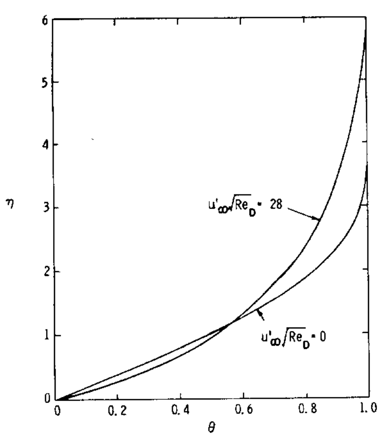

FIG. 16. Theoretical temperature profile ($K = 0.164$) compared with laminar theory.

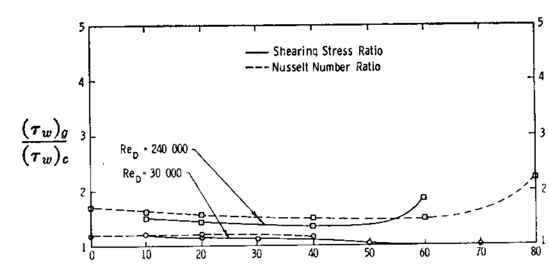

FIG. 18. Measured shearing stress and Nusselt number ratios versus azimuth angle.

[22] N. Piercy, E. Richardson, and H. Winny, Proc. Phys. Soc. (London) **B69**, 731 (1956).

large increase in the heat transfer in that region, but has a much smaller effect on the skin friction. In the experimental study of Ref. 23 the stretching of vortex filaments in the flow near the stagnation point of a body of revolution was also given as the cause of the amplification of the free-stream turbulence in that region. Conceptually, one might consider the cross-section variation of vorticity, as postulated by Sutera[13] as qualitatively analogous to free-stream turbulence; qualitative agreement between the measurements shown in Figs. 12 and 13 and Sutera's theoretical results support the analogy. That the analogy has serious limitations, however, is indicated by the appreciable (though smaller) effects of turbulence on the heat transfer at a flat plate shown in Figs. 4, 5, and 7; Sutera's theory would predict no effect of turbulence on this flow.

The results for the flat plate as well as for the circular cylinder indicate that a Reynolds shear stress is generated through the effect of the gradient in mean velocity on the turbulence entering the laminar boundary layer.[24] The comparisons between theory and experiment for the cylinder flow, shown in Figs. 12, 13, and 14, indicate that the dependence of Reynolds stress on turbulence in the free stream and distance from the wall is described to a good approximation by Eq. (7), with a constant proportionality factor K, at Reynolds numbers over 10^5. For smaller Reynolds numbers, the turbulent Prandtl number and the proportionality factor might well be functions of Reynolds number, resulting in the departure from constancy shown in Fig. 14. The magnitude of the eddy viscosity for the boundary layer on the 3-in. cylinder, as given by Eqs. (7) and (13), is roughly equal to the laminar kinematic viscosity for $u'_\infty = 0.05$, $U_\infty = 40$ ft/sec, $y = 0.0005$ ft; this value of y corresponds to $y/\delta \approx 0.4$.

[23] A. M. Kuethe, W. W. Willmarth, and G. H. Crocker, in *Proceedings of the 1961 Fluid Mechanics and Heat Transfer Institute* (Stanford University Press, Stanford, California, 1961), pp. 10–22.

[24] Details of the growth of the Reynolds stress and other statistical properties were calculated by R. G. Deissler [Phys. Fluids 4, 1187 (1961)] for a homogeneous turbulent field in a uniform velocity gradient.

CONCLUSIONS

(1) In the laminar boundary layer on a flat plate, a turbulence level of 6% at the leading edge causes increases in heat transfer of around 30% at an x Reynolds number of 125 000 and a free-stream speed of 20 ft/sec. This Reynolds number corresponds to a point just upsteam of the first appearance of turbulent bursts, that is, just before the start of transition. Thus, the laminar boundary layer develops an increased heat transfer under the influence of the free-stream turbulence. This conclusion is in general agreement with the results of other investigators.

(2) Turbulence causes large increases in laminar heat transfer and skin friction from the forward portion of a circular cylinder. At the stagnation point an approximately linear relation was found between $\mathrm{Nu}_D\ (\mathrm{Re}_D)^{\frac{1}{2}}$ and $u'_\infty\ (\mathrm{Re}_D)^{\frac{1}{2}}$. At values of $\mathrm{Re}_D < 10^5$, however, the measurements deviate systematically from this relation; the deviation is expressed by Eq. (3)

(3) The measurements given here of the heat transfer at the stagnation point are self-consistent. The degree of agreement with previous investigations is evident in Fig. 12. The measurements of Zapp[10] and Schnautz[11] show the best agreement over the entire range of $u'_\infty\ (\mathrm{Re}_D)^{\frac{1}{2}}$ with the present measurements.

(4) A phenomenological theory for the effect of turbulence on flow in the boundary layer near the stagnation point is presented. The eddy viscosity is assumed to be proportional to the free-stream turbulence and to distance from the wall; the proportionality factor, determined from the measurements, is found to be constant at Reynolds numbers greater than 10^5. The ratio of the skin friction to the heat transfer under the influence of the free-stream turbulence is approximately constant up to angles of about 40° from the forward stagnation point. The increase in the heat transfer and skin friction due to the free stream turbulence also are nearly constant up to angles of about 40°; thus the stagnation point theory predicts the increases well over the entire region.