to minimize diffuse reflection at the sample interfaces and to give support to the sample at higher temperatures.

Measurements cannot be made when the incident light is reflected from the front surface of the sample directly into the photometer. This will occur if $\theta=180°-2\phi$ and measurements at θ's close to this should be avoided.

At positive values of ϕ the incident light will be reflected from the front surface of the sample onto the rear wall of the chamber. Measurements at values of θ close to 2ϕ must be avoided, for then the photometer will receive light which is reflected diffusely from this illuminated spot on the rear wall (see Fig. 3).

The effects of stray light, diffuse reflection, and scattering by glass microscope slides may be determined by making a run in which the two microscope slides are separated by the same amount as in an actual run (by a polythene spacer) and in which a low scattering liquid (e.g., benzene) having a refractive index somewhat similar to that of the polymer is placed between the slides. This was done, and the points are included in Fig. 2. It is seen that in this case the scattering is negligible as compared with that of the polymer.

THE REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 25, NUMBER 9 SEPTEMBER, 1954

Wide-Band Amplitude Distribution Analysis of Voltage Sources

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(Received January 14, 1954)

A method of wide-band amplitude distribution analysis is described which employs two commercial oscilloscopes and a phototube. The minimum set-up can be assembled quickly in the laboratory from standard equipment, and gives a rapid analysis accurate to better than 5 percent. It is free from the severe band-width restrictions of other methods (N. Knudtzon, Research Laboratory for Electronics, M. I. T. Tech. Rept. No. 115, July 15, 1949), and can often be employed up to 500 mc. Results on two noise sources are included for illustration, and a modification for improving the resolution and increasing the accuracy is presented.

1. INTRODUCTION

A RAPID method of wide-band amplitude distribution analysis is very useful in studies of certain voltage sources, such as random noise. Desirable properties of such a method include the use only of equipment which is readily available in the laboratory, simplicity of the setup, speed of analysis, reasonable accuracy, and wide band width. For the method described in this paper, only two oscilloscopes, one phototube, and a few small parts are required for the minimum setup, which can be assembled in two hours. The amplitude distribution density function is displayed as an oscillogram which can be photographed in five seconds. The accuracy of analysis is better than 5 percent under most conditions and may be improved by modifying the apparatus. The band width of the system is determined by the vertical amplifier of the *A* scope, but under certain conditions it may be extended to 500 mc.

2. INCREMENTAL PROBABILITY DENSITY $p_\Delta(y)$

Consider a voltage source whose amplitude may be expressed as a function of time

$$y=f(t). \tag{1}$$

The incremental probability density of this function is a measure of the probability of finding the function between the limits $y-\frac{1}{2}\Delta y$ and $y+\frac{1}{2}\Delta y$ for any given y. It is defined by the relation

$$p_\Delta(y)=[P(y-\tfrac{1}{2}\Delta y)-P(y+\tfrac{1}{2}\Delta y)]/\Delta y, \tag{2}$$

where $P(y)$ is the probability of finding an amplitude larger than y, and Δy is the *unit resolution.* As Δy decreases toward zero, the function $p_\Delta(y)$ approaches the probability density $p(y)$, which is the negative derivative of $P(y)$.

In analyzing the source, the observed amplitude distribution density may differ slightly from the probability density for short sampling periods. However, as the sampling period is increased to an order of magnitude larger than the period of the lowest frequency component present in $y=f(t)$, the observed amplitude distribution density rapidly approaches the probability density. No confusion should arise, therefore, if these two terms are used interchangeably for the remainder of this paper.

For convenience, the system resolution may be specified by means of a resolution index. The *peak resolution index* is defined by the relation

$$R_p=Y/\Delta y, \tag{3}$$

where $Y=y_{\max}-y_{\min}$, the peak-to-peak value of the source. For noise sources, which have large but infrequent peak values, an rms index is often more convenient. The *rms resolution index* is defined by the relation

$$R_{\mathrm{rms}} = Y_{\mathrm{rms}}/\Delta y, \qquad (4)$$

where Y_{rms} is the rms value of $y=f(t)$.

As a simple example, let us consider a normalized sine voltage, $y=\sin\omega t$. The incremental probability density $p_\Delta(y)$ may be calculated for a resolution index $R_p=20$, and a plot of the result is shown in Fig. 1A. The experimental results of sinusoid analysis with equipment resolution index of 20 are shown in the oscillograms. Figure 1B shows the analysis of a 10-mc sine wave, and Fig. 1C is that of a 10-kc sine wave showing the expected identical distribution. An indication of the accuracy of the method may be had by comparing the oscillograms with Fig. 1A. The width of the central pulse in Fig. 1B may be taken as a measure of the unit resolution. These oscillograms were made with only the minimum setup described below. Even greater accuracy may be obtained by modifying the apparatus.

3. METHOD OF ANALYSIS

The method of analysis is based on the conversion into light of the average electron density falling on a small area of the phosphor screen of a cathode-ray tube, and the further conversion of this light into a voltage representing the average electron density. The required voltage is obtained with a phototube having an *RC* load circuit. To obtain linearity, it is necessary to use a short persistence phosphor, and observe certain other conditions so that the phosphor does not saturate. These conditions are easily realizable, and very good linearity is obtained.

The voltage to be analyzed is displayed in the usual manner on a short persistence CRO called the *A* scope (analyzing oscilloscope). A fixed sampling slit and phototube are mounted in front of this, and scanning is accomplished by moving the display electrically.

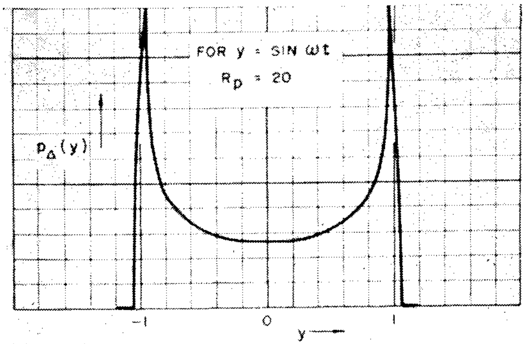
The minimum setup is shown in Fig. 2. In this setup the sampling slit is formed on the face of the *A* scope with Minnesota No. 33 electric tape. The slit is 1 mm high and 8 mm wide. Its horizontal edges are lined up parallel with the horizontal trace. The 1P42 phototube is shielded by an enlarged section of the coaxial cable and supported so that its window is centered on and touching the slit.

Integrating capacitor C_y is chosen large enough to smooth the 60-cycle beam flicker, and was found adequate for the satisfactory integration of all signals analyzed. The dc input resistance R_y of the *D* scope (display oscilloscope) is employed as the phototube load, and voltage is furnished from a small 67-volt battery B_1. The integrated phototube output voltage is thus displayed on the *Y* axis of the *D* scope.

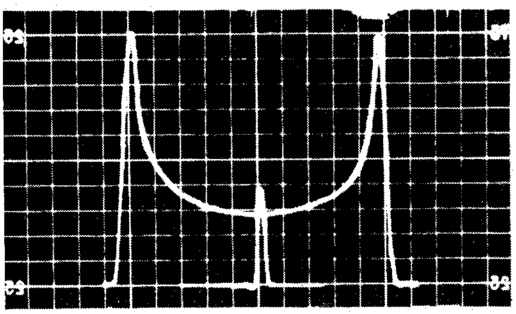
The *X* input of the *D* scope is furnished with a voltage V_2. This voltage is caused to be in synchronism with V_1 (the average dc potential of the lower deflection plate) during scanning by means of the network R_1, C_1, R_3, R_4, and C_4. The requirement for synchronism may be stated by the relation

$$R_1C_1(R_3+R_4) = R_3R_4C_4. \qquad (5)$$

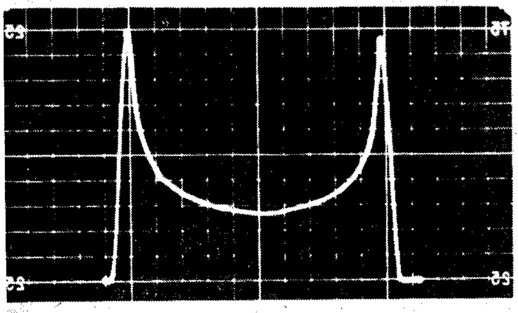
Battery B_2 cancels a large portion of the dc component, so that the horizontal centering control of the *D* scope may operate normally.



A. CALCULATED



B: OBSERVED ON 10 MC SINE WAVE.



C. OBSERVED ON 10 KC SINE WAVE.

FIG. 1. Incremental probability density for a sinusoid.

4. COMPONENT REQUIREMENTS

The requirements for the *A* scope are as follows. It must have a P5, P11 or equivalent short-persistence phosphor. The vertical amplifier must be free of drift, and of adequate band width for the required analysis. However, if there is sufficient voltage available from the source, it may be capacitively coupled direct to the deflection plates. In this case no vertical amplifier is required, and the system band width is limited only

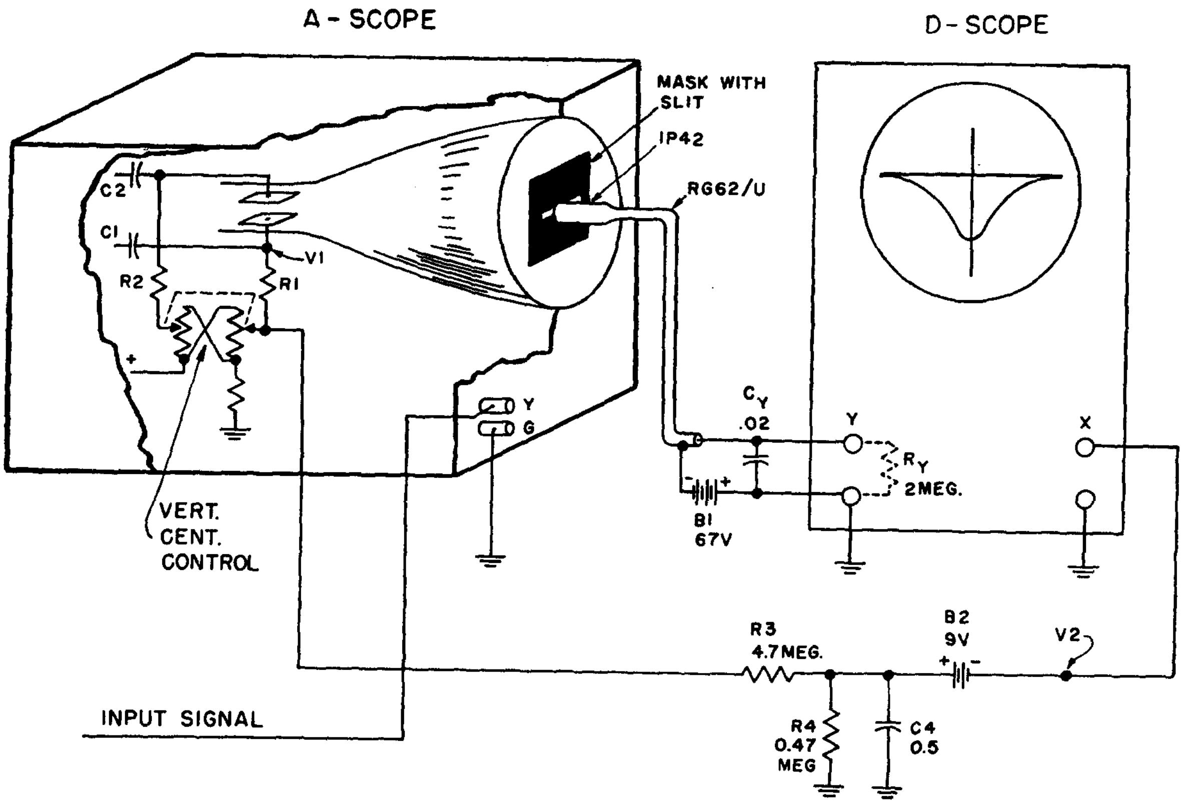


FIG. 2. Minimum setup for analyzer.

by the transit time of the electron beam during deflection. If the deflection region has an axial length of 1 in. and the beam energy at deflection is 2 kilovolts, the transit time is 10^{-9} sec, giving a system band width of 500 mc.

For the *D* scope we require a large vertical gain, large input resistance for the *Y* input, and dc amplifiers for both *X* and *Y* inputs which are free from drift. A Dumont 304-H is suitable.

The 1P42 vacuum phototube was chosen for its end illumination feature, small physical size, and negligible dark current (less than 10^{-9} amp). The normal maximum illumination used was 10^{-3} lumen. This gave a 2-in. signal on the *D* scope.

5. OPERATION

To prepare the equipment for analysis, the following adjustments are made. First the horizontal sweep of the *A* scope is adjusted near 100 kc at some frequency asynchronous with any major frequency component in the signal to be analyzed. The horizontal synchronism control is set at zero. A sweep length of about 1 in. is used so that an ample portion of the sweep extends beyond each side of the 8-mm-wide slit. Before applying the signal, both oscilloscopes are centered in the usual way. The contour of the resolution pulse may be observed by moving the vertical centering control of the *A* scope slowly through zero with the *Y* input of this scope shorted. A trace similar to the central pulse in Fig. 1B is obtained.

To plot the $p_\Delta(y)$ function, the input signal is applied to the *Y* input of the *A* scope and appropriate vertical gain adjustment is made so that the signal nearly fills the face of the *A* scope, but without overloading the amplifier. As mentioned above, the source may be coupled direct to the vertical deflection plates if adequate voltage is available.

To perform scanning, the vertical centering control of the *A* scope is turned slowly (in 5 sec) from one extreme to the other. During scanning an oscillogram may be made by time exposure on the *D* scope.

Manually controlled scanning has the distinct advantage that the spot on the *D* scope may be made to move with approximately constant velocity regardless of the shape of the curve. The result is a clear, even oscillogram.

Resolution is determined by the slit width and optics of the system. When a 1-mm slit and 1-mm spot diameter are used on the *A* scope, the unit resolution is approximately 3 mm, taken as the distance between the two positions of 50 percent phototube response. Considerable improvement in resolution may be obtained by modifying the optics. This is discussed in a later section.

6. RESULTS

The results of analyzing two noise sources with the minimum setup are presented for illustration in Figs. 3 and 4. In both figures *A* is the unclipped source, *B* is the source with positive clipping, *C* is the source with negative clipping, and *D* is the source with both positive and negative clipping. Figure 3 shows the results on Noise Source No. 1, which used a 931-A multiplier phototube as the noise generator. The efficiency of clipping is manifest by the sharpness in fall-off of the curve outside the "horn." When both positive and negative clipping are used as in Fig. 3D, a bimodal curve is produced. Figure 4 shows the results on Noise Source No. 2, which used a 6D4 gas triode as the noise generator. Although the same clipper circuits were used here, the distributions are quite different. In particular, Fig. 4D is monomodal instead of bimodal. These differences are accounted for in part by the fact that with Source No. 2 the energy level at the clipper stage was considerably different than with Source No. 1.

7. MODIFIED EQUIPMENT

By forming a sharp image of the *A*-scope display upon a slit, so that slit and image fall in the same plane, a great improvement in resolution may be obtained. A method for doing this is shown in Fig. 5. A Land Polaroid camera is used to form the image, and the film is replaced by an accurately made slit. A multiplier phototube giving increased sensitivity is mounted above the slit in a light-tight housing. This setup has the advantage of permitting the entire *A*-scope display to be observed through the camera viewing tube before and during analysis. The sensitivity of the system may conveniently be controlled by adjusting the regulated voltage used to excite the phototube. Because of the great increase in sensitivity with the multiplier phototube, a lower intensity may be used on the *A* scope, permitting a smaller spot at sharp focus. A much smaller slit width may be employed so that a unit resolution of less than 1 mm can be obtained, giving greater accuracy and resolution to the analysis.

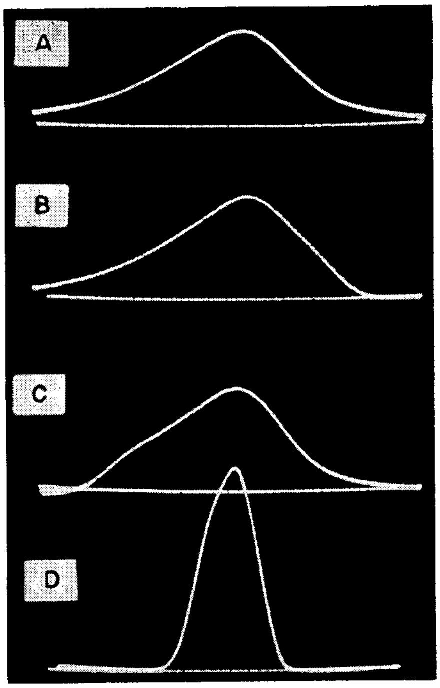


FIG. 4. Analysis of Noise Source No. 2. *A*, no clipping; *B*, positive clipping; *C*, negative clipping; *D*, both positive and negative clipping.

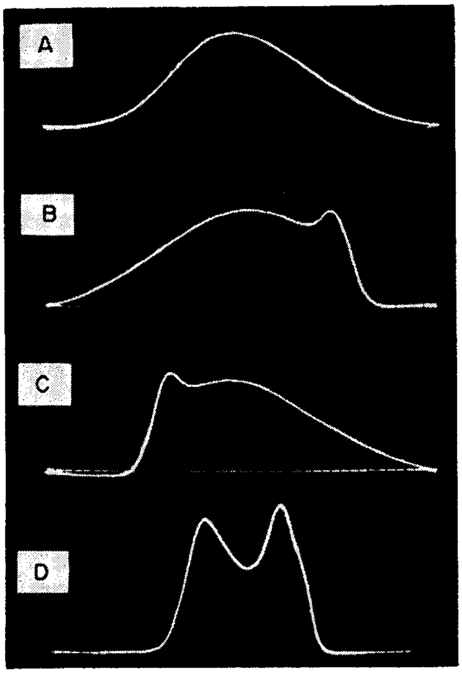


FIG. 3. Analysis of Noise Source No. 1. *A*, no clipping; *B*, positive clipping; *C*, negative clipping; *D*, both positive and negative clipping.

As a different approach to the problem, it has been suggested that the *A* scope and phototube might be replaced by a special electron beam deflection tube. In place of a phosphor screen, this tube would contain an internal slit and collector anode, the anode current signal being fed directly to the *D* scope. Alternatively a split-anode beam deflection tube might be similarly employed to plot the $P(y)$ instead of the $p_\Delta(y)$ distribution.

8. CONCLUSION

The accuracy of the method may be judged by comparing the calculated and observed distribution density of a known source, as in Fig. 1 for the sinusoid. The sinusoid is a good test case because of the large variation in amplitude distribution density, particularly at the "horns." When enlargements of Figs. 1A and 1B are superimposed, the two curves are found to be almost

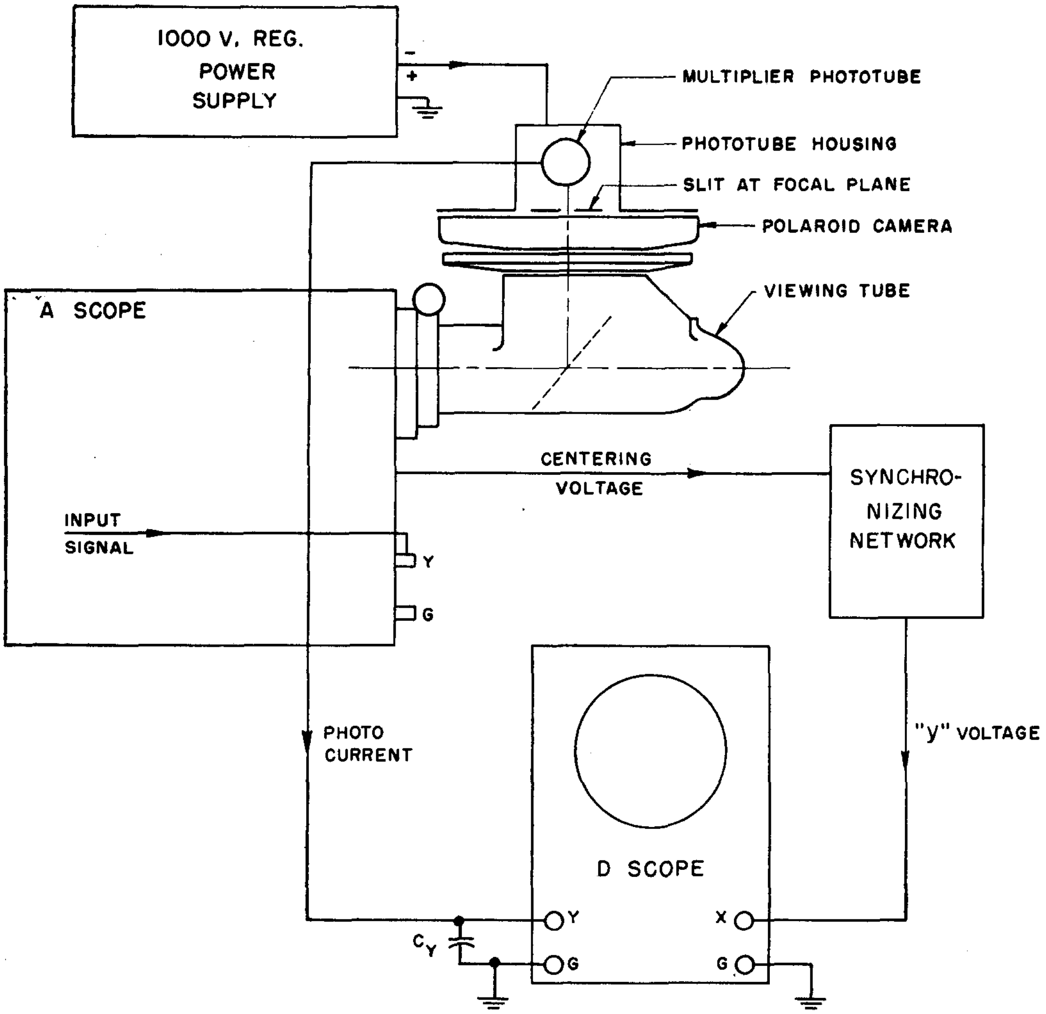


Fig. 5. Modified analyzer.

identical except at the horns. The observed horns are neither as sharp nor as high as the calculated ones. This is due to the shape of the resolution pulse (center of Fig. 1B), which is clearly not rectangular, as was implicitly assumed in the curve calculated from Eq. (2).

Considering the simplicity of the setup, its wide-band capabilities, low cost, and speed of analysis, the method is surprisingly accurate and versatile, and its performance at this laboratory has been most gratifying.

ACKNOWLEDGMENT

This work has been done under U. S. Army Signal Corps Contract DA-36-039 sc-15358. Permission for the publication of this paper is gratefully acknowledged.