# Reflection of Plane-Polarized, Electromagnetic Radiation from an Echelette Diffraction Grating*

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A variational method is used to calculate the distribution of energy reflected from an echelette diffraction grating. Experiments performed using a klystron generating 3.2-cm electromagnetic waves which impinge upon a large scale grating agree with the calculations to within a few percent, the error being that expected from the variational formulation.

## I. INTRODUCTION

THE problem of predicting the distribution of radiation diffracted from a periodic surface has been the subject of a considerable amount of experimental and theoretical work in the past few years.[1] Recently one of the authors reported a variational method for the treatment of the problem.[2] It is the purpose of this paper to present the results of some experiments performed to check the theoretical predictions. The results agree with the calculations to within a few percent, the error being about the same as that assigned to the calculations.

## II. THEORY

The problem will be stated with reference to Fig. 1. Details of the theoretical treatment may be found in reference 2; the method will be merely sketched here for the two-dimensional problem. It is supposed that a plane electromagnetic wave is incident upon a periodic surface with the wave propagation vector lying in a plane normal to the grooves of the reflecting surface. The propagation vector of the incident wave makes an angle $\theta_i$ with the $z$ axis. Calculations have been made for the parallel-polarized case, that is where the electric field vector is parallel to the grooves. It is supposed that the surface is perfectly conducting. Then one has the problem of finding the function $E_y(x,z)$ satisfying the equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+k^2\right)E_y(x,z)=0, \quad (1)$$

in the region $z<\zeta(x)$ where $\zeta(x)$ is the function describing the reflecting surface. The function $E_y$ represents the electric field, where the $y$ axis is chosen perpendicular to the page in Fig. 1. In Eq. (1), $k=2\pi/\lambda$ with $\lambda$ the wavelength of the radiation. The boundary condition on $E_y$ for a perfectly conducting surface is given by

$$E_y[x,\zeta(x)]=0. \quad (2)$$

These conditions together with the radiation condition at infinity form the statement of the problem.

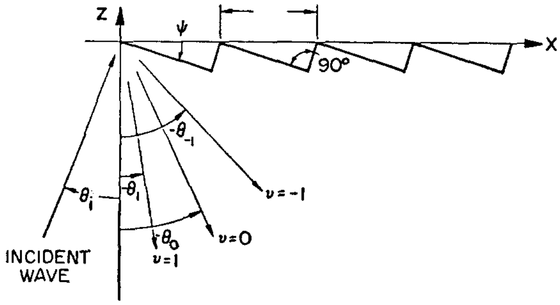


FIG. 1. The diagram shows the symbols used in the discussion of the reflection problem.

The reflected field in the region removed from the surface is made up of a system of diffracted plane waves, both homogeneous and inhomogeneous. In Fig. 1 is shown a representative set of diffracted waves. The problem consists of finding the weighting coefficients in this representation of the reflected field. The squares of these coefficients represent the energy reflection coefficients for the given diffracting surface. The choice of the coefficients was made in the present calculation through the use of a variational technique which minimized the error in the boundary condition [Eq. (2)].

The problem of the reflection of radiation from surfaces of the type shown in Fig. 1 has been programed for MIDAC, the University of Michigan digital computer. The storage capacity of the machine limited the number of diffracted waves used in the calculation to ten.

## III. EXPERIMENTAL ARRANGEMENT

A surface of the type shown in Fig. 1 was built up by overlapping aluminum ST24 bar stock of dimension $\frac{1}{4}$ by 2 by 36 in. The bars were overlapped so as to form a grating with a surface wavelength $1\frac{3}{4}$ in.; the grating was about 36 in. wide.

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[1] C. H. Palmer, Jr., J. Opt. Soc. Am. 42, 268 (1952); K. Artmann, Z. Physik 119, 529 (1942); V. Twersky, J. Acoust. Soc. Am. 22, 539 (1950); L. M. Brekhovskikh, Zhur. Eksptl. i Teort. Fiz. 23, 275 (1952).
[2] W. C. Meecham, J. Appl. Phys. 27, 361 (1956).

A klystron generated 3.2-cm waves which were brought to the focus of a collimating mirror by a bent section of wave guide. This mirror was a paraboloidal searchlight mirror of 90-cm diameter and 35-cm focal length. The parallel beam of rays from this mirror was inclined downward at 8° to the horizontal and fell upon the diffraction grating 9 m away. The grating could be rotated about a vertical axis and its angular position measured. A second searchlight mirror similar to the first served as the telescope and collected the radiation reflected downward from the grating. This mirror was mounted on an arm that could be turned about the same vertical axis as the grating so as to scan through the energy in the several spectral orders emerging at different angles from the grating. A crystal detector was placed at the focus of this second mirror.

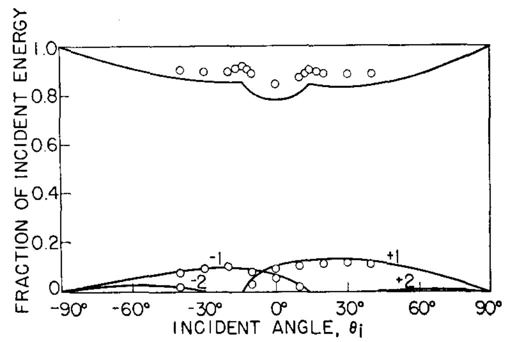


FIG. 2. The curves show the calculated values for the energy reflection coefficients using the variational method. The points are the measured values. For the calculation $\lambda=0.76D$, $\psi=8°13'$; the incident radiation is parallel polarized. The $\theta_i$ is the angle of the incident plane wave. No energy was detected in the $+2$ order. The top curve is the zeroth order.

## IV. COMPARISON OF THEORY AND EXPERIMENT

The particular experiment being reported here was run with $D=1.75$ in., $\lambda=0.76D$, and $\psi=8°13'$, when $\lambda$ is the radiation wavelength and $D$ the grating spacing. In the experiment, the incident angle (with the normal) was varied from $-40°$ to $+40°$ in 10° intervals. The energy within a given order was obtained by rotating the receiving mirror through the diffraction pattern of that order and integrating the received energy. The half-angle for these patterns was $\approx 2°$ which agreed with the diffraction pattern expected for a 36-in. aperture at this wavelength. As a check on the experiment, the total energy reflected from the grating was compared with the energy reflected from a plane surface. These quantities had an average deviation of a few percent.

The experimental results shown in Fig. 2 have been normalized; that is, for every incident angle each measurement has been divided by the sum of the measurements at that angle. The calculated values made using the variational method are also shown in Fig. 2.

Using the reciprocity theorem and energy conservation, it can be shown that the calculation is in error at most by about 5%.[2] This would account for the discrepancy remaining between the measured and the calculated values. The irregularities in the experimental and the theoretical values at $|\theta_i|\approx 14°$ are the Wood anomalies.[3] The irregularities appear for those incident angles at which orders first appear (at grazing angles).

It may be interesting to note that even though the incident beam in the experiment uniformly covers only about 15 grating elements, nevertheless the measured values are in fairly good agreement with the values calculated for an infinite grating radiated by a plane wave of infinite extent.

In conclusion, it appears that experiment and theory are in good agreement for reflecting surfaces of the type shown in Fig. 1 when the angle $\psi$ is small, 10° or less.

[3] R. W. Wood, Phil. Mag. 4, 396 (1902).