in the conventional ultrasonic machining process; i.e., one in which a tapered cone of circular cross section is caused to oscillate longitudinally in near-contact with a rigid boundary (see Fig. 1). The basic apparatus consists of a 25-kc magnetostrictive transducer unit from a Raytheon Ultrasonic Impact Grinder loaned to us by the manufacturer, modified for use with an external driving voltage. Typical peak-to-peak vibration amplitudes used in our experiments are of the order of 10^{-5} cm, much lower than those typically encountered in ultrasonic machining processes. The transducer is mounted on a specially constructed framework as shown in Fig. 1.

Acoustic streaming theory indicates that in this arrangement vibration of the tip will generate eddying in the liquid localized to a small ring-shaped region near the tip periphery. The existence of such eddying has been verified experimentally, by a technique in which small polystyrene spheres (6–15 μ in diameter) are used as indicators of the motions. Observations are carried out by placing a square glass chamber filled with a glycerine-water mixture beneath the transducer so that the tip is immersed in the liquid. The liquid, in which the indicator particles are suspended, is degassed in order to reduce the possibility of cavitation beneath the vibrating tip. By use of side illumination with a narrow pencil of light, motions of the indicator particles can be viewed through a microscope (60× magnification) in some detail.

Following the motion of single indicator particles as they move through the liquid, one observes small steady circulations occurring in that region of the liquid which is immediately beneath and adjacent to the tip periphery. The form of these circulations or vortices for a given situation[2] is shown in Fig. 2. (Symmetry exists about the axis of the vibrating cone.) It can be seen from Fig. 2 that there are actually two distinct pairs of vortices present in the liquid in the region near the cone tip: an upper pair denoted in the figure by A, A' and a lower pair denoted in the figure by B, B'. The former pair appears in the liquid whenever the cone is set into oscillation, even when it is far removed from the lower boundary. The latter pair, however, appears only when the separation h between the lower boundary and the oscillating cone tip is quite small. (Typically, $h \sim 400$ microns for the situation described here. The tip diameter is approximately 3.2 mm.) It has been observed that the shape and speed of the A, A' circulations does not undergo much change when h is altered, but depends mainly upon the vibration amplitude of the tip. The shape and speed of the B, B' circulations is, however, strongly dependent on h; their speed falls off very rapidly as h increases. At the larger values of h it becomes difficult to follow the relatively slow B, B' circulations since the indicator particles tend to be caught up with the faster A, A' vortices or by thermal convection currents in the liquid (the latter being difficult to avoid). It is found that for both the A, A' and B, B' circulations, the speed of an indicator particle is greatest at that part of its path when it is immediately adjacent to or beneath the tip periphery.

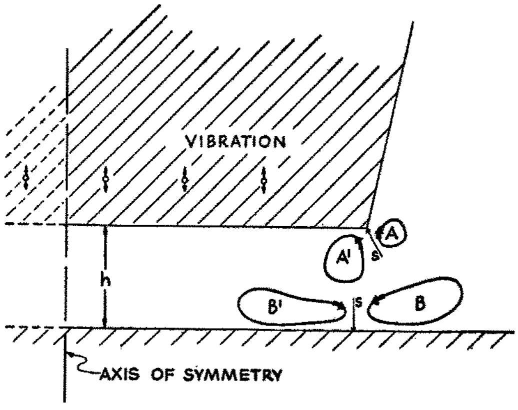

FIG. 2. Cross-sectional view of sonically induced eddying near an oscillating tip. Symmetry exists about the axis of the cone. The solid arrows, besides indicating the flow directions, are so placed that their positions are just the points at which the speed is greatest in the respective particle orbits. The lines s indicate stagnation regions in the liquid between the A and A' vortices, etc. The figure is drawn approximately to scale; a typical tip diameter is 3.2 mm; a typical spacing h is 0.4 mm.

Preliminary experiments on sonic effects at surfaces using the arrangement of Fig. 1 bear out the expectation that, under appropriate circumstances, reactions are preferentially accelerated at sites of the eddies. (Results on these effects are to be published elsewhere.) Whether or not the eddies are significant in respect to the ultrasonic machining process is not known. However, Miller[3] has reported that the abrasion of the workpiece in ultrasonic machining commences in a ring-shaped area about the tool-tip periphery. This might be expected if the initial machining action were in some way affected by sonically induced eddies in the abrasive slurry surrounding the tool tip.

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* Supported in part by the Air Force Office of Scientific Research, Contract AF18(603)-54.

[1] F. J. Jackson and W. L. Nyborg, J. Acoust. Soc. Am. 30, 614 (1958).

[2] Observations now in progress, using a large-scale low-frequency model, show that the detailed nature of the eddying depends on a number of parameters. These include, especially, (1) frequency, (2) viscosity of the liquid, (3) tip diameter, (4) tip-to-plane spacing, and (5) "roundness" of the corner at the tip edge. The streamlines sketched in Fig. 2, observed with the 25-kc setup, are indicative of significant features of the patterns obtained generally.

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On Thermal-Diffusion Effect in the Surface Treatment of Metals by Diffusion out of a Streaming Gas*

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IN a previous note[1] it was pointed out that when a mixture of gases of different molecular weights flows along a hot plate, the thermal-diffusion separation of the gas mixture can be so significant that this phenomenon can be properly utilized to separate isotopes. A similar situation arises in the following diffusion problem of some technical importance. For surface treatment of metals, Wagner[2] suggested passing over the surface of the metal at elevated temperature a stream of gas containing a component which is to diffuse into the metal (see Fig. 1). For instance, a surface layer of chromium can be produced by passing a mixture of $CrCl_2+H_2+HCl$ at temperature T_1 over iron which is kept at a higher temperature T_0. In determining the distribution of the diffusing component in the metal as a function of the space coordinates x, y and of time t by solving the diffusion equation, Wagner assumed that in the gas phase the concentration is independent of y (see Fig. 1). This assumption may possibly lead to serious error on account of the thermal-diffusion effect previously pointed out.[1] Since the problem of diffusion out of a streaming gas mixture into a metal at an elevated temperature is of some general technical importance, and the thermal-diffusion effect is inherent to the problem, it is felt that this finding is worth a brief note.

A simple example is used here to illustrate the thermal-diffusion effect that is involved in Wagner's problem. Under the influence of the temperature gradient along y, the thermal-diffusion flux will be set up so that the lighter gas tends to move to the hot region and the heavier gas to the cold region. The large temperature gradient in the thermal boundary layer, which is developed

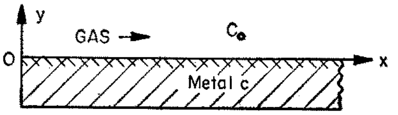

FIG. 1.

when gas flows along a hot plate, tends to establish a concentration gradient[1] normal to the plate. By neglecting the convection flow effect on the temperature gradient, we can obtain a close approximation to the equilibrium concentration in the gas phase at the metal-gas interface, namely

$$C_0' = (T_1/T_0)^\alpha C_0,$$

where α is called the thermal-diffusion factor,[3] provided $C_0 \ll 1$.

Like Wagner, we are concerned with the calculation of the concentration c of the diffusion component in the metal as a function of the space coordinates and of time. We assume, however, for simplicity, that the streaming gas mixture is so abundant that C_0 is independent of x. Note that the depth of the diffusion layer in the metal is small compared with the thickness of the metal which implies that diffusion in the x direction is negligible compared with that in the y direction. We are thus interested in a problem of one-dimensional diffusion in metal.

The equation governing diffusion in metal can be written

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial y}\left(D\frac{\partial c}{\partial y}\right), \tag{1}$$

where the diffusion coefficient for chromium in iron can be approximated[2] as

$$D = D_0(1+ac),$$

in which a is an empirical constant. For simplicity we assume, following Wagner, that equilibrium has been established at the metal-gas interface, the ratio of the concentration in the metal and in the gas at the interface, being given by a distribution coefficient κ, i.e.,

$$c_0 = \kappa C_0 (T_1/T_0)^\alpha$$

at $y=0$ for all values of t. Also at $t=0$, $c=0$ for all values of y.

With the boundary conditions prescribed as above, Eq. (1) can be reduced to an ordinary differential equation by the introduction of a new variable, $\eta = (1/2)y/(D_0 t)^{\frac{1}{2}}$. The ordinary nonlinear equation thus obtained can be solved by iteration.[4] The solution illustrated in Fig. 2 corresponds to the case when $T_0/T_1 = 3$ and $\alpha = 0.6$. For comparison purposes, the case in which the thermal-diffusion effect is ignored is also shown in Fig. 2. The thermal-diffusion effect depends upon the factor $(T_1/T_0)^\alpha$. The results presented in Fig. 2 can be extrapolated to other values of T_0/T_1 and α. Experimental measurement data of α for a mixture of $CrCl_2$ and H_2 are not available. The value of α used in the illustrative example might have been too high.

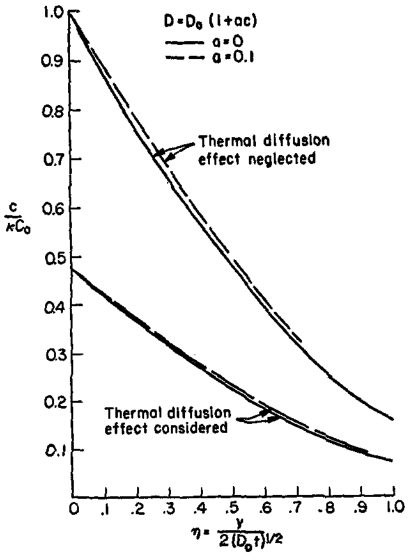

FIG. 2.

The technical significance of the thermal-diffusion effect in Wagner's process of surface treatment of metals is thus made obvious. It is hardly necessary to mention that the present discussion also applies to a liquid instead of a solid medium for diffusion.

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Free-Carrier Faraday Effect in *n*-Type Germanium

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THE infrared Faraday effect due to free carriers was first observed in InSb.[1,2] Similar measurements are now reported for *n*-type germanium.

The Faraday effect is the rotation of the plane of polarization of a beam of radiation passing through a material when a magnetic field is applied parallel to the beam. At relatively long wavelengths the rotations measured are due to free carrier dispersion only. In effect the experiment determines the dispersion due to cyclotron resonance absorption.

The theoretical expression for the Faraday rotation per unit length due to free carriers when the energy surfaces are ellipsoidal, as in *n*-type germanium, has been given by Lidiard and Stephens.[3] In rationalized units this becomes

$$\theta = \frac{re^2B}{2A_H cn\epsilon_0\omega^2}\,\frac{K^2(K+2)^2}{(2K+1)^2 m_l^2}, \text{ with } K = m_l/m_t, \tag{1}$$

where B is the magnetic induction, n the refractive index, $2\pi\omega$ the frequency, A_H the Hall coefficient, and m_l and m_t are the longitudinal and transverse effective masses. The parameter r is the scattering factor involved in the relation between the low-field Hall coefficient and carrier concentration N so that

$$A_H = (r/Ne)[3K(K+2)/(2K+1)^2]. \tag{2}$$

Hence

$$\theta = e^3BN\lambda^2/8\pi^2c^3\epsilon_0 nm^{*2}, \tag{3}$$

where

$$m^* = [3/K(K+2)]^{\frac{1}{2}} m_l. \tag{4}$$

Measurement of the Faraday rotation may therefore be used to determine m^*. For the special case of spherical energy surfaces (i.e., $K=1$) this is the isotropic mass,[1,2] while for $K \ll 1$ it gives approximately $\sqrt{3}m_t$.

The experiments were carried out on plane parallel specimens of single crystal *n*-type Ge, 2 to 5 mm thick. The radiation traversed the specimen normally, making an angle of 50° with the magnetic field direction, so that only the resolved part of this field (~8000 oe) was used. The wavelength was selected by a double monochromator with rock-salt prisms and the radiation was polarized and analyzed by piles of polythene plates.[4]

The carrier concentrations were determined from low-field Hall effect and resistivity measurements. For the highly doped specimens, assuming $r=1$ and using the cyclotron resonance value of K, approximate values of concentration and mobility μ were obtained. Now the lattice mobility μ_L is known from drift mobility measurements on intrinsic material[5] to be 3800 cm²/volt sec. The ratio μ/μ_L is thus known and μ/μ_I may be determined to be 0.100 from the theoretical curve of Conwell.[6] Jones[7] has treated the problem of the effect of mixed lattice and impurity scattering on the constant r and has presented his results in the form of a