Computer-assisted magnet shimming

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An iron electromagnet with a gap width of 23.5 cm and diameter of 30 cm was shimmed so that the field was uniform within ± 4.2 parts in 10^4 over the central 19 cm of the axis. A description is given of the shim configuration used, and of the algorithm by which the measured field changes (produced by perturbing a shim) are used to predict an improved set of shim dimensions. The method is not limited to any specific geometry of either the shims or the region over which field homogeneity is desired.

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INTRODUCTION

Previous attempts to shim the field of large iron electromagnets have been limited to the case where a uniform field is required over a relatively small region at the center of a gap which is much smaller than the pole diameter. The shim geometry, and, more importantly, the method of adjustment described below, made possible correction of the magnetic field along the axis of a cylindrical gap of width greater than its radius. The method is not limited to any specific geometry of either the shims or the region over which field homogeneity is desired. It is particularly useful for iron magnets, for which *a priori* calculation is considerably less satisfactory than for air-cored solenoids.

The development of this method was occasioned by an experiment[1] to measure the lifetime of positronium formed in a magnetic field. A Varian V4012-3B electromagnet was used, which, after removal of the pole pieces and pole spacers, produced a maximum field of about 0.45 T (4.5 kG) in a cylindrical gap of diameter 30 cm (12 in.) and width 23.5 cm (9.25 in.). The field homogeneity required was better than one part in 10^3 (i.e., comparable to the other sources of error in this particular experiment) along the symmetry axis of the gap, to which the positrons would be closely confined by the field.

A Bjorken and Bitter[2] thick-ring shim was tried, but was found to overcorrect the field away from the gap center, because the actual pole and shim violated Bjorken and Bitter's simplifying assumption that the magnetization be both uniform and axially directed. The semiempirical method described below eliminates this assumption by using the measured field from shims in the neighborhood of the existing one to calculate an improved set of shim dimensions.

I. METHOD

Let f be a scalar-valued function of position s and of a vector of n parameters x. In the present application, f will be either the magnitude of the magnetic field, or its axial component, and the components of x will be the dimensions of the shim. We discretize the position dependence by restricting attention to the values f_i of f at a set of points $s^{(i)}$, $i = 1, \ldots, m$. Grouping the values f_i into a column vector f, we have for parameter values x sufficiently close to $x^{(0)}$:

$$f(x) = f^{(0)} + J \cdot (x - x^{(0)}), \tag{1}$$

where the Jacobian J has elements $J_{i,j} = \partial f_i / \partial x_j$, and $f^{(0)} = f(x^{(0)})$. In this linear approximation, the parameter values that cause f to assume the value h are obtained by solving

$$J \cdot (x - x^{(0)}) = h - f^{(0)}. \tag{2}$$

It is convenient to allow $m > n$ and then to interpret solving (2) as minimizing some norm of the residual vector

$$r = J \cdot (x - x^{(0)}) + f^{(0)} - h. \tag{3}$$

For the present application the use of the Chebyshev ("maximum" or "infinity") norm is appropriate, and an algorithm due to Madsen and Powell[(3)] was employed.

We now approximate J by differences. Let $f^{(k)} = f(x^{(k)})$ for any n vectors $x^{(k)}$, $k = 1, \ldots, n$ such that the vectors $x^{(k)} - x^{(0)}$ span the x space. Defining $F = (f^{(1)}, \ldots, f^{(n)})$, $X = (x^{(1)}, \ldots, x^{(n)})$ and 1_n as a column of n ones, we have from (1)

$$F = f^{(0)} \cdot 1_n^T + J \cdot (X - x^{(0)} \cdot 1_n^T), \tag{4}$$

where superscript T denotes transpose. Substituting (4) in (2), we obtain

$$(F - f^{(0)} \cdot 1_n^T)(X - x^{(0)} \cdot 1_n^T)^{-1}(x - x^{(0)}) = h - f^{(0)} \tag{5}$$

from which x is most readily obtained by solving

$$(F - f^{(0)} \cdot 1_n^T) y = h - f^{(0)}, \tag{6}$$

and then calculating

$$x = x^{(0)} + (X - x^{(0)} \cdot 1_n^T) y. \tag{7}$$

The stipulated conditions on the choice of the $x^{(k)}$ are sufficient for $(X - x^{(0)} \cdot 1_n^T)$ to be nonsingular, and are necessary for $(F - f^{(0)} \cdot 1_n^T)$ to be nonsingular.

If the components of f are required only to be (approximately) equal to each other, rather than to have pre-

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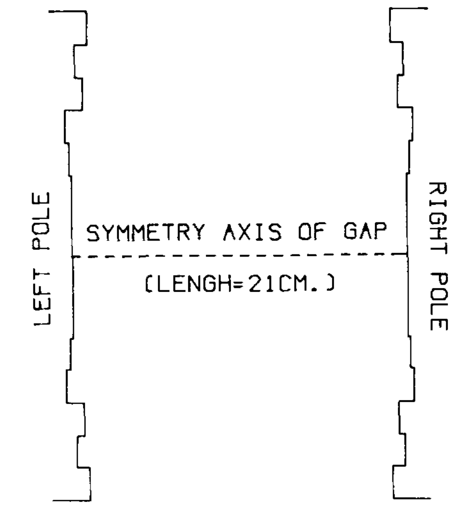

FIG. 1. Cross section of the final shim configuration in a plane containing the axis of rotational symmetry.

determined values (i.e., if the field is required only to be uniform), one can proceed as follows. Set $h = 0$ and append to F an extra column $f^{(n+1)}$ which is a multiple of 1_m. To avoid unnecessary rounding error in the solution process, a good choice is $f^{(n+1)} = 1_m(f^{(0)T} \cdot 1_m)/m$. The solution vector y of (6) will then have $n + 1$ components, only the first n of which are used in (7).

To the extent that the linear approximation (1) is adequate (which can be controlled by careful parametrization of the shim), one solution of (5) will suffice. In general, iteration will be required. The values $f^{(k)}$ and $x^{(k)}$ which define the Jacobian approximation may be remeasured after each iteration, or only when the predicted solution fails to improve the norm of the residual vector. It is expedient to set an upper limit to $|x - x^{(0)}|$ and $|x^{(k)} - x^{(0)}|$ to ensure at least local adequacy of the linear approximation, and finite precision in measuring f will set a lower limit to the usable value of $|x^{(k)} - x^{(0)}|$.

II. APPLICATION AND RESULTS

A computer program was written to perform a single iteration of the algorithm of Sec. I, imposing optional user-supplied constraints on the solution. For most of the shimming process, the magnitude of the magnetic field was measured with an Alpha model 3193 Digital NMR Gaussmeter, which has a resolution of 1 part in 10^6. Initially, however, the field gradients were so large that the NMR gaussmeter would not function with its probe more than 0.5 cm from the gap center, so a Bell model 640 (Hall-effect) incremental gaussmeter, which measured the axial component of the field, was used. In both cases, a special jig was employed to ensure repeatable positioning of the probe.

It was conjectured that a shim consisting of five concentric rings would suffice. This conjecture was supported by the results of a series of six numerical simulations of the entire magnet, unshimmed and with a unit-thickness single ring of each of the five sizes. These simulations were carried out using the computer program TRIM,[4] and processing of the results by the algorithm of the previous section showed that the required degree of field correction should be obtainable. The necessarily approximate and *a priori* nature of the TRIM results made it essential that the actual shim parameters be readily adjustable, and rings were accordingly machined in a binary series of thicknesses for each set of radial dimensions. A cylindrical center plate, of diameter equal to the inner diameter of the smallest ring, was used to shift the origin of ring thickness so as to allow an effectively negative thickness for some rings. One would expect the linear approximation (1) to be reasonably good for this shim geometry, since the magnitude of the magnetic moment of (part of) a ring will be approximately proportional to its thickness, and although the direction of the moment may not be known, it will be approximately constant.

At the time that the method of Sec. I was introduced, the region over which the field was to be corrected was the central 19 cm of the symmetry axis of the gap, the remainder of the axis being inaccessible to the gaussmeter probe because of the presence of the shims. The relative field inhomogeneity had been reduced to $\pm 1.4 \times 10^{-2}$ by ad hoc experimentation with the shims. One iteration of Eq. (5) with $n = 5$ and $m = 29$ yielded an inhomogeneity of $\pm 7 \times 10^{-4}$. At this stage it was found necessary to allow the outermost ring thickness to be different for the two poles, in order to correct a field tilt of $\pm 2 \times 10^{-4}$ resulting from slight asymmetry of the magnet. The resulting field was spatially very nearly equal ripple, and was corrected to its final inhomogeneity of $\pm 4.2 \times 10^{-4}$ manually, since the expected improvement was small and the corresponding changes in shim thickness were comparable to the minimum available change, which for some rings was 0.05 mm (2×10^{-3} in.). The final shim configuration is shown in Fig. 1, and the on-axis field variation is shown in Fig. 2, which, as expected, resembles a Chebyshev polynomial.

III. DISCUSSION

It is not easy to determine *a priori* how many shim parameters will suffice for a specified uniformity. For many shim geometries, the fact that the shims must be

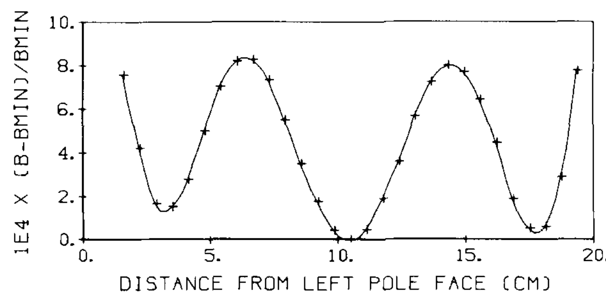

FIG. 2. Relative field deviation as a function of position on the symmetry axis of the magnet gap. B_{min} is 0.45 T and error bars are too small to be shown.

distant from the region where uniformity is required produces, via the well-known averaging property of Laplace's equation, a strong tendency toward linear dependence of the columns of $(F - f^{(0)} \cdot 1_n^T)$. This tends to result in calculated shim dimensions which are unnecessarily large (compared to other solutions which are almost as good) and disparate in sign. A similar situation occurs in the design of compensated solenoids.[5] With five-parameter shims, one could in principle obtain more extrema, and hence a smaller inhomogeneity, than are shown in Fig. 2, but the calculated shim dimensions are then nonrealizable. Use of constraints as mentioned earlier leads to the solution shown.

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