Fluid Mechanics of Meteor Trails

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(Received 15 May 1969; final manuscript received 12 September 1969)

The initial dispersion of a meteor trail which consists of atoms, ions, etc., evaporated from a hot meteoric body has an important bearing on its detectability via electromagnetic wave scattering. In the contemporary literature, the classical diffusion equation, in one form or another, is often used for an approximate description. This leads to much uncertainty and conceptual difficulty. From the Maxwell–Boltzmann viewpoint of kinetic theory, it is found that a new relation in the form of the telegraph equation can be derived which removes the present difficulties and gives closer agreement with observations.

I. INTRODUCTION

In the study of the upper atmosphere by observations of electromagnetic waves scattered from the ionized trails of meteors, various scattering theories have been developed that relate the amplitude and duration of a wave echo to the electron line density of the meteor trail in question. In such scattering theories uncertainities and inaccuracies have been attributed to, among other factors, the finite initial radius of the meteor trail[1] which is composed of evaporated meteor atoms, ionized atmospheric particles, and free electrons. The cause of a meteor trail having a finite radius during an echo observation is at least twofold: (i) the initial expansion and diffusion of the evaporated meteor atoms which dissociate and ionize the air molecules through collision processes and (ii) fragmentation of the meteor body. It is in the study of the initial expansion pertinent to factor (i) that the present work is motivated. In other words, the purpose of the present investigation is to apprehend the kinetic process of interactions between the evaporated meteor atoms and the ambient atmosphere which is basic to the initial radius problem of interest.

The immense significance of the problem has led to much earlier discussion[2,3] where a variable mean-free-path approach was proposed to take into account the dependence of mean free path upon the meteor velocity and collision cross section. Such an almost-free molecular approach to treat a rarefied flow problem is well taken for a meaningful analysis[4] provided the "persistence of velocity" of Jeans[5] is appropriately taken into account. Mounting observational evidence[6,7] apparently suggests that the initial spreading of a meteor tail is "explosively" fast and certainly not diffusionlike. Appropriate physical insight into the initial expansion of a meteor trail is still lacking. Here it is intended to pursue a more rigorous kinetic approach from the viewpoint of the Maxwell–Boltzmann equation.[8]

II. AN IDEALIZED MODEL

In order to sharpen our focus on the inherent kinetic process of meteor trail expansion, we remove the geometrical complications by using a point source to simulate the outlet of the evaporated meteor atoms which spread in a homogeneous atmosphere. It is further observed that the meteor body velocity (V) $\gg$ mean lateral speed of evaporated atoms (A) relative to the meteor body, a stationary source can be used to discuss the distribution of the evaporated meteor atoms which is related to that of a moving source with a simple Galilean transformation of coordinates.

From the viewpoint of kinetic theory, meteor atoms start as a free molecular radial beam until they collide with the air molecules after traveling a distance, on the average, of a mean free path; after collisions, the slow classical diffusion process takes over. The meteor atoms of interest undergo stages of free molecular, transitional, and continuum flows. The distribution of meteor atoms in question $f(\mathbf{r}, \mathbf{c}, t)$ in a six-dimensional phase space $(\mathbf{r}, \mathbf{c})$ is governed by the Maxwell–Boltzmann equation.[8]

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Although a numerically iterated solution for the thermalization of a joint source flux is within the reach of contemporary computational capability, it does not usually provide physical insight into the problem of interest. In view of the incomplete understanding of the initial velocity distribution, the collision and the charge transfer of the evaporated meteor atoms, it appears fruitful to use the original transfer equation of Maxwell,[5] which can be derived as a moment form of the Maxwell–Boltzmann equation, to evaluate the average value of any quantity $Q(\mathbf{c})$ without knowing much about the distribution function $f(\mathbf{r}, \mathbf{c}, t)$.

III. EQUATIONS OF METEOR TRAIL EXPANSION

Consider a point source of evaporated meteor atoms at $r = 0$. The distribution of meteor atoms with velocity $\mathbf{c}$ at time t and position $\mathbf{r}$ is denoted by $f(\mathbf{r}, \mathbf{c}, t)$. They interact with ambient atmospheric molecules which are prescribed by the distribution function $F(\mathbf{c}_1)$. Neglecting the geogravitational force field and ignoring collisions among meteor atoms, the Maxwell–Boltzmann equation for $f(\mathbf{r}, \mathbf{c}, t)$ can be written[8]

$$\frac{\partial f}{\partial t} + \mathbf{c}\cdot\frac{\partial f}{\partial \mathbf{r}} = \iint [f(\mathbf{c}')F(\mathbf{c}_1') - f(\mathbf{c})F(\mathbf{c}_1)] \cdot |\mathbf{c}_1 - \mathbf{c}|\, G\, d^2\Omega\, d^3c_1, \qquad (1)$$

where G denotes the differential cross section for collisions of particles whose velocities change from $(\mathbf{c}, \mathbf{c}_1)$ to $(\mathbf{c}', \mathbf{c}_1')$, respectively. Multiplying both sides of Eq. (1) by $Q(\mathbf{c})$, which is a function of velocity $\mathbf{c}$, and integrating over the velocities of the meteor atoms $(\mathbf{c})$ gives[5]

$$\frac{\partial}{\partial t}(n\bar{Q}) + \operatorname{div}(n\overline{\mathbf{c}Q}) = \iiint [Q(\mathbf{c}') - Q(\mathbf{c})] \cdot [F(\mathbf{c}_1)f(\mathbf{c})]\, |\mathbf{c}_1 - \mathbf{c}|\, G\, d^2\Omega\, d^3c_1\, d^3c, \qquad (2)$$

where the bar indicates an average over $\mathbf{c}$ and $n(\mathbf{r})$ the number density of meteor atoms at $\mathbf{r}$. The terms on the left-hand side of Eq. (2) determine the rate of change of $n\bar{Q}$ due to flow in the phase space. The term on the right-hand side gives the average rate of change of Q due to collisions.

Let $Q = 1$, $\mathbf{c}Q = \mathbf{u}$, the resultant drift velocity of meteor atoms. On the right, $Q(\mathbf{c}') = Q(\mathbf{c})$ for an elastic collision. Approximation for inelastic collisions of ionized meteor atoms can be taken into account by letting $Q(\mathbf{c}') = 2$ in an ionizing collision or $Q(\mathbf{c}') = 0$ for an attachment or recombination. The transfer equation (2) gives

$$\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{u}) = n(\bar{\nu}_i - \bar{\nu}_a), \qquad (3)$$

where $\bar{\nu}_i$ and $\bar{\nu}_a$ denote frequencies of ionization and attachment, respectively.

Let $Q = \mathbf{c}$ and note that this does not lead to zero for the term on the right-hand side of Eq. (2) because collisions with ambient molecules change the total momentum of the meteor atoms. Equation (2) for $Q = \mathbf{c}$ becomes

$$\frac{\partial}{\partial t}(mn\mathbf{u}) + \operatorname{div}\mathbf{p} = -mn\bar{\nu}_c\mathbf{u}, \qquad (4)$$

where the pressure tensor $\mathbf{p}$ is defined as

$$p_{ij} = nm\overline{c_i c_j}$$

and is assumed to be a symmetric tensor, whose inference will be discussed later

$$\mathbf{p} = \mathbf{U}p(n, T),$$

thereby ignoring the nondiagonal elements due to transport effects; m and $\bar{\nu}_c$ denote the mass and the collision frequency of a meteor atom, respectively; $\mathbf{U}$ is a unit tensor.

In the present discussion we ignore the inelastic collisions, hence $\bar{\nu}_i = \bar{\nu}_a = 0$. Equations (3) and (4), when operated by $\partial/\partial t$ and div, respectively, can be combined into a single equation

$$\nabla^2 n = \frac{1}{A^2}\frac{\partial^2 n}{\partial t^2} + \frac{1}{D}\frac{\partial n}{\partial t}, \qquad (5)$$

where $A^2 = \kappa T/m$ and $D = \kappa T/(m\bar{\nu}_c)$ with T, assumed constant, denoting the effective kinetic temperature of the evaporated meteor atoms. The quantity D approximately equals the diffusion coefficient of meteor atoms in the atmosphere. The quantity A represents the propagation speed of a small pressure impulse of meteor atoms assuming isothermal thermodynamic process in the propagation.

IV. INITIAL EXPANSION OF METEOR TRAIL

Notice that Eq. (5) is in the form of telegraph equation where quantity A denotes the dissipationless propagation speed of a telegraph signal; D is the coefficient of diffusion of the signal during propagation. It is anticipated that the solution of Eq. (5) would show the phenomenon of retardation, i.e., the solution would have a well-defined wave front, as the beam of the evaporated meteor atoms

behaves prior to their collisions, on the average, with the ambient atmospheric molecules, in addition to residual disturbances which persist at all points traversed by the wave front. The telegraph equation thus lies between the simple wave equation, whose solutions have a wave front but no residual disturbances, and the classical diffusion equation, whose solutions have a residual disturbance but no wave front. The difference between them becomes indistinguishable after a few mean free times from the initial instant at $t = 0$.

The resulting Eq. (5) for the representation of spreading of the evaporated meteor atoms which originate from a point source thus removes the conceptual difficulty of using a classical diffusion equation for the spreading phenomenon near $t = 0$. For instance, it is well known that the fundamental solution to diffusion equation

$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2} \tag{6}$$

is

$$n(x, t) = (4\pi Dt)^{-1/2} \exp [-x^2/(4Dt)] \tag{7}$$

which behaves like a Dirac delta function $\delta(x)$ at $t = 0$ and acquires finite values everywhere at $t = 0^+$, an infinitesmally small time interval. This implies an infinitely large speed of propagation which is nonphysical. Equation (5) also appears to give a relatively fast initial expansion of the meteor trail which agrees with observations.[6,7]

The Green's function of Eq. (5) which describes the concentration at position $\mathbf{r}$ and time t due to a unit point source at the origin $r = 0$ is[9]

$$n(\mathbf{r}, t) = \frac{A}{r} \exp\left(-\frac{A^2}{2D} t\right) \cdot \left[\delta(At - r) + \frac{Ar}{2D(r^2 - A^2t^2)^{1/2}} J_1 \cdot \left(\frac{A}{2D}(r^2 - A^2t^2)^{1/2}\right) H(At - r)\right], \tag{8}$$

where the step function $H(x) = 0$ when $x < 0$ and $H(x) = 1$ when $x > 0$; J_1, the first-order Bessel function of the first kind.

To illustrate the physical process, the source solution to the one-dimensional form of Eq. (5) is used[9]

$$n(x, t) = 2\pi A \exp(-A^2t/2D) J_0 \cdot [A(x^2 - A^2t^2)^{1/2}(2D)^{-1}] H(At - x), \tag{9}$$

where J_0 denotes the zeroth-order Bessel function of the first kind. It is observed from solution (9) that the concentration disturbances that give the initial discontinuity are confined within the signal zone, $x \leq At$. The concentration at a point x_1 remains undisturbed until the instant when $x_1 = At_1$. At this time, which is that required for a disturbance to travel from the origin ($x = 0$) to x_1, a concentration wave with intensity $\sim \exp(-A^2t_1/2D)$ passes through x_1. After this initial wave the slower diffusion process takes over.

V. DISCUSSION AND CONCLUSIONS

It has been qualitatively established in the above discussion that the concentration of meteor atoms in question at displacement $\mathbf{r}$ and time t comparable to mean free path and mean free time, respectively, of the spreading meteor atoms behaves according to the solution of the telegraphlike equation (5) which is undistinguishable from that of the diffusion equation (6) at larger values of r and t. Prior to their collisions, the meteor atoms, which are evaporated from the meteoric body with mean speed A, move with $r/t = A$. Quantitative comparisons of the two solutions of Eqs. (5) and (6) can be made by constructing solutions, under various initial conditions, from the use of the fundamental solutions (8). The result of this mathematical exercise will not be presented here since solutions to the telegraph equation are abundantly available.[10,11]

The significance of the present finding pertaining to the initial expansion of meteor trail can be interesting in the study of the initial radius of an ionized meteor trail—an important factor in determining the detectability of meteor via radio echo at an altitude, say above 100 km, where the mean free path of a meteor atom in the ambient atmosphere is not very small.[11]

The simplicity of Eq. (5) is owed in no small measure to the assumption of a symmetric tensorial form for the stress $\mathbf{p}$. This is obviously an oversimplification considering the nonisotropic nature of the beam of evaporated meteor atoms. Presumably a more accurate result can be obtained

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by extending the present zeroth-order approximation.

The accommodation of the present stationary source solution to a moving one in order to simulate an actual meteor is also omitted here because it is simply a straightforward computation using a Galilean transformation very similar to that for the diffusion of a heat source.[12]

It is hoped that the results of the present study may help in the new approaches[13,14] to the kinetic problems of meteors and comets using the discipline of fluid physics.

ACKNOWLEDGMENT

The present work has been partly supported by a National Aeronautics and Space Administration Grant, NGR 23-005-094.

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THE PHYSICS OF FLUIDS VOLUME 13, NUMBER 1 JANUARY 1970

Intermolecular Forces: Thermal Diffusion and Diffusion in Ar–Kr

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(Received 18 July 1969)

Thermal diffusion and diffusion measurements on Ar–^{85}Kr, made by the two-bulb method, are reported for the temperature range 77–600°K. In part of the temperature range there is an unexplained discrepancy between our thermal diffusion data and those of Grew and Mundy, making it no longer clear that the thermal diffusion factor has a positive minimum. However, it is confirmed, by the use of a specially constructed thermal diffusion column, that the thermal diffusion factor is positive between 77 and 90°K. The ordinary diffusion coefficients are in excellent agreement with other work. Notwithstanding the thermal diffusion discrepancy, it is clear that the exp-6, Kihara core, and Morse potential models are not entirely satisfactory.

I. INTRODUCTION

Measurement of thermal diffusion in binary mixtures of simple gases is potentially a powerful technique for the investigation of unlike molecular interactions, and recent advances in the machine computation of molecular collision integrals[1–4] promise to help realize its full potentiality. Unfortunately, however, it is difficult to measure thermal diffusion in the system Ar–Kr(trace), which has that its literature abounds with inconsistent data.[5–7] It is, therefore, important to check any data that appear in the least unusual or suspicious. This paper primarily reports a reinvestigation of thermal diffusion in the system Ar–Kr (trace), which has previously been reported to have an unusual temperature dependence.

Some time ago Grew and Mundy[8] reported measurements on Ar–Kr thermal diffusion at lower temperatures than had hitherto been reached.[9] Their most noteworthy observation was that the thermal diffusion factor α_T exhibited a positive minimum at about 150°K; furthermore, while still positive, α_T was increasing rapidly at the lowest temperatures obtainable with liquid oxygen and nitrogen. Such behavior, while theoretically possible,[10,11] is quite unusual. Any minimum would

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