APPLICATIONS OF LINEAR INTEGER PROGRAMMING TO PROBLEMS OF LAND USE ALLOCATION

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CONTENTS

CONSTRAINED OPTIMIZATION AND LAND USE

Constraints in the designation of terrain for uses can appear in the form of budget limitations for development, rules, and regulations which limit the uses of sites, minimum space needs for certain types of uses such as parks and housing, locational requirements for certain land uses, physical-chemical-biological dynamics that must be preserved, standards of site suitability for particular uses, certain measures of user satisfaction, regulations on ownership or controllership, standards on environmental quality, and total amount of land available. From the variety of ways and means of limiting land use, it is obvious that no single model is adequate to cover all combinations of possible constraints.

Optimization implies the existence of a preferred pattern or patterns of land use selected on the basis of a comparison of alternatives from a set of possibilities. Criteria for optimization may be expressed in objective terms such as dollar costs, number of users, dollar income, amounts of wastes discharged, user-miles or user-days, site productivity, or number of user demands that are satisfied. Subjective criteria may also be used. For example, a quantified measure of site "suitability" consisting of a weighted linear numerical combination of physical and aesthetic characteristics is subjective. Whenever one individual states a preferred choice between two uses on a given site, he is exercising a subjective but *nonquantified* criterion. The use of quantitative criteria forces one to be specific about those factors that dominate his preference behavior. Much more disagreement is to be expected over the basis for rating one plan or site use over another when the criteria are subjective rather than objective.

Constrained optimization means a choice of the most preferred pattern of site uses from a limited set of alternatives. Limitations can occur for any of the reasons just mentioned, the most basic of which is the limited supply of land. Constraints are not independent of criteria. For example, a short supply of land drives up the price that certain groups are willing to pay for user rights. An "optimized" allocation of uses to sites is a decision, choice, or recommendation, whether hypothetical or actual, that corresponds to selecting the top-ranked alternative from those available under the given limitations. A form of constrained optimization is practiced in matters of land use decision making, although not with the aid of computers and mathematics. Developers or other entrepreneurs, public and private, determine their own preferences for land use, which cover a limited set of sites. Each acts so as to gain his maximum advantage so that the set of land use decisions is optimized from the collective points of view of this set of users. Regulatory agencies enter the picture by placing limitations on permissible alternatives because conflicts arise among competing users, or between competing users and other potential users (public or private). Conflicts arise over differences in criteria adopted by user groups, the most publicized example being economics or job versus preservation of resources or "ecology." Conflicts also arise because one user group uses a site for a purpose that interferes with the purpose of another user group on the same site or a different site. Expression of user preferences may exist in the form of requests for zoning variances, public opinion stated through hearings, requests for permits to undertake development, or outright development of a site. Development always occurs over time in a sequential pattern, so that no single agency can identify at any particular time the set of interested users, their set of preferred alternatives, or their criteria for choice. Therefore, planning or regulatory agencies cannot "optimize" land use decisions for other independent users. They can advise or present suggested plans according to a set of criteria, or they can act as arbitrators by setting constraints on permissible land use decisions. It is very difficult, if not impossible, in actual practice for an agency to measure the degree to which a pattern of existing land uses is optimum whenever several independent user groups are involved in the decisions. It is sometimes possible for a *single* user or a *single* agency acting on behalf of a group of users to optimize land use "plans" (on a limited scale). A realistic example occurs in the case of a state agency whose task it is to purchase and develop acreages for public recreation. A set of possible sites are first selected on the basis of a "site suitability" analysis. Next, the sites are ranked according to their natural and cultural features and according to their accessibility to the region's population. Finally, a subset of sites must be selected, each with a set of acreages such that budget constraints are met and some measure of user satisfaction is maximized. The resulting solution can be called an "open-space plan." It is a set of hypothetical decisions that would be optimum if executed simultaneously under the conditions assumed in the statement of the problem. It should be noted that a "site suitability" analysis may precede the site selection process in order to provide an information base for the preliminary selection process. Site suitability analyses may consider many factors, and the results may be displayed in graphical format, but this analysis does not in itself represent an optimized land use plan.

A second type of "optimized" land use planning arises from the need to control undesirable consequences of land uses. As opposed to the previous case, where a set of hypothetical decisions are made to purchase or otherwise develop certain sites (so as to maximize user satisfaction or minimize development costs), the present problem is to limit undesirable side effects of land use. Waste discharges either in total amount of distribution, disruption of plant and animal communities, or a lowering of aesthetic values are examples of undesirable effects that may need to be controlled. This can be accomplished in part by zoning regulations on land use or by laws regulating waste discharges. In the case of zoning, a public agency may act to protect its constituency against excessive waste discharges or other "infringements" by developing a zoning ordinance that maximizes user access to land use and development subject to standards of "environmental protection." An important input to this type of plan is the "site capacity analysis," which is an analysis of the degree of deterioration that a site undergoes if subjected to different uses.

Constraints occur in either hypothetical or real decision problems regarding the use of land whenever an interested party is not free to attain his preferred alternative. When several such constraints occur and several decisions are to be made, constrained optimization implies a procedure for finding a solution that is superior to most others which meet the limitations imposed by the constraints. Most constrained optimization problems in land use decisions involve methods that are not based upon a mathematical model. Persons involved can state their preferences and value judgments without committing themselves to numbers. The disadvantage is that users may overlook a feasible alternative that they would have preferred had they discovered it. The other main advantage to using a mathematical model of land use decision making programmed for exercise on a computer is the vast amount of bookkeeping that can be done and alternatives that can be quickly and systematically checked.

A SIMPLIFIED LAND USE ASSIGNMENT PROBLEM

A number of parcels of land are specified, each of which is capable of sustaining any one of several alternative uses. The problem is to assign exactly one use to each parcel in such a way that some measure of user satisfaction is maximized. There may be side conditions to be satisfied such as a minimum, maximum, or exact limit on the number of sites that may be assigned any particular use. This problem is illustrated by the following example.

Mission Peninsula, in Michigan, divides the lower part of Grand Traverse Bay into east and west arms. By superimposing a square grid on the peninsula, as shown in Figure 1, the land area is subdivided into

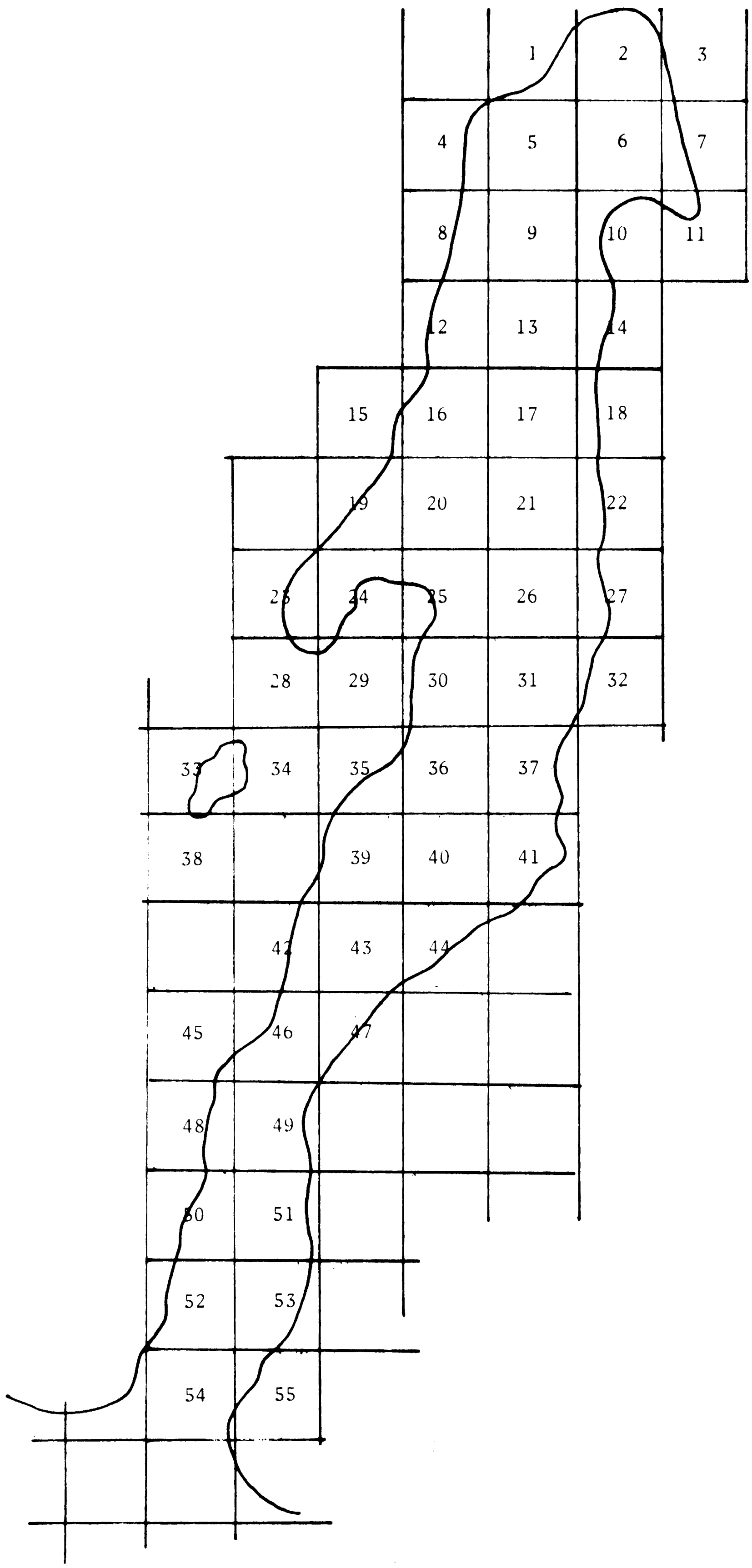

Figure 1. Parcel Identification

parcels, each of which is 1 sq mi in size. The grid lines follow the official section lines that define the land units by township and range designation. For purposes of illustration, each of 55 cells are assumed to require an assignment of a single "land use." That is, any grid cell that overlaps any part of the peninsula is to be assigned exactly one use uniformly applied over the land within that cell. Six different uses are defined as recreational (R), residential (RS), industrial (I), recreational-residential (R-RS), recreational-industrial (R-I), and residential-industrial (RS-I). The "value" of each type of land use in a particular cell is assumed to be specified by a single number. In Table 1 a value for each use is specified for each of the 55 parcels. It is also assumed that the following land use requirements are given (Table 2). Each parcel is to be assigned to exactly one use in such a way that the total value is maximized. That is, the sum of the individual contributions by each parcel to total "value" is to be maximized. The statement of the problem is summarized in Table 3.

The identification of the parcels by numbers from 1 through 55 is given in Figure 1 and the solution is given in Figure 2. The meaning of the "solution" is that no other assignment of land uses to parcels subject to the specified requirements gives a higher cumulative value, obtained by adding the values contributed by each parcel with its assigned use.

The Assignment Model

The problem as stated above fits the standard format of the "assignment" problem, which is an integrer linear program given by equations 1-4.

Table 1. Land Use Type

Cost and Requirements

Parcel #	R DEST...1	RS DEST...2	I DEST...3	R-RS DEST...4	R-I DEST...5	RS-I DEST...6
1	-100	-90	-80	-95	-90	-85
2	-100	-90	-80	-95	-90	-85
3	-100	-90	-80	-95	-90	-85
4	-100	-90	-80	-95	-90	-85
5	-100	-90	-80	-95	-90	-85
6	-50	-90	-80	-70	-65	-85
7	-100	-90	-80	-95	-90	-85
8	-100	-80	-80	-90	-90	-80
9	-50	-90	-80	-70	-65	-85
10	-100	-90	-80	-95	-90	-85
11	-100	-90	-80	-95	-90	-85
12	-100	-90	-80	-95	-90	-85
13	-50	-90	-80	-70	-65	-85
14	-100	-90	-80	-95	-90	-85
15	-100	-90	-80	-95	-90	-85
16	-100	-90	-80	-95	-90	-85
17	-50	-90	-60	-70	-55	-75
18	-100	-80	-60	-90	-80	-70
19	-50	-80	-60	-65	-55	-70
20	-50	-80	-60	-65	-55	-70
21	-50	-80	-60	-65	-55	-70
22	-100	-80	-60	-90	-80	-70
23	-100	-90	-80	-95	-90	-85
24	-100	-90	-80	-95	-90	-85
25	-100	-90	-60	-95	-80	-75

Table 1. Continued

Parcel #	R DEST...1	RS DEST...2	I DEST...3	R-RS DEST...4	R-I DEST...5	RS-I DEST...6
26	-50	-80	-60	-65	-55	-70
27	-100	-80	-60	-90	-80	-70
28	-100	-90	-80	-95	-90	-85
29	-100	-90	-80	-95	-90	-85
30	-100	-90	-60	-95	-80	-75
31	-100	-80	-60	-90	-80	-70
32	-100	-90	-80	-95	-90	-85
33	-100	-60	-60	-80	-80	-60
34	-100	-60	-60	-80	-80	-60
35	-100	-80	-60	-90	-80	-70
36	-100	-80	-60	-90	-80	-70
37	-100	-80	-60	-90	-80	-70
38	-100	-60	-60	-80	-80	-60
39	-100	-80	-60	-90	-80	-70
40	-50	-80	-60	-65	-55	-70
41	-100	-80	-60	-90	-80	-70
42	-100	-90	-60	-95	-80	-75
43	-100	-90	-60	-95	-80	-75
44	-100	-90	-60	-95	-80	-75
45	-100	-100	-60	-100	-80	-80
46	-100	-100	-60	-100	-80	-80
47	-100	-90	-60	-95	-80	-75
48	-100	-90	-80	-95	-90	-85
49	-100	-100	-80	-100	-90	-90
50	-100	-90	-80	-95	-90	-85
51	-100	-90	-80	-95	-90	-85
52	-100	-90	-80	-95	-90	-85
53	-100	-90	-80	-95	-90	-85
54	-100	-90	-100	-95	-100	-95
55	-100	-90	-100	-95	-100	-95

Table 2. Hypothetical Land Use Requirements

Use	R	RS	I	R-RS	R-I	RS-I
Parcels Required	19	4	5	19	4	4

Table 3. Maximize Total Value of Parcel--Use Assignment

<table>
<tr><th colspan="8">LAND USE TYPE</th></tr>
<tr><th>Parcel #</th><th>R</th><th>RS</th><th>I</th><th>R-RS</th><th>R-I</th><th>RS-I</th><th>Amount Available</th></tr>
<tr><td>1
2
.
.
.
.
.
.
.
.
54
55</td><td colspan="6">Value
(Table 1)</td><td>1
1
.
.
.
.
.
.
.
.
1
1</td></tr>
<tr><td rowspan="2"></td><td>19</td><td>4</td><td>5</td><td>19</td><td>4</td><td>4</td><td>55</td></tr>
<tr><td colspan="6"># Parcels Required</td><td>Total</td></tr>
</table>

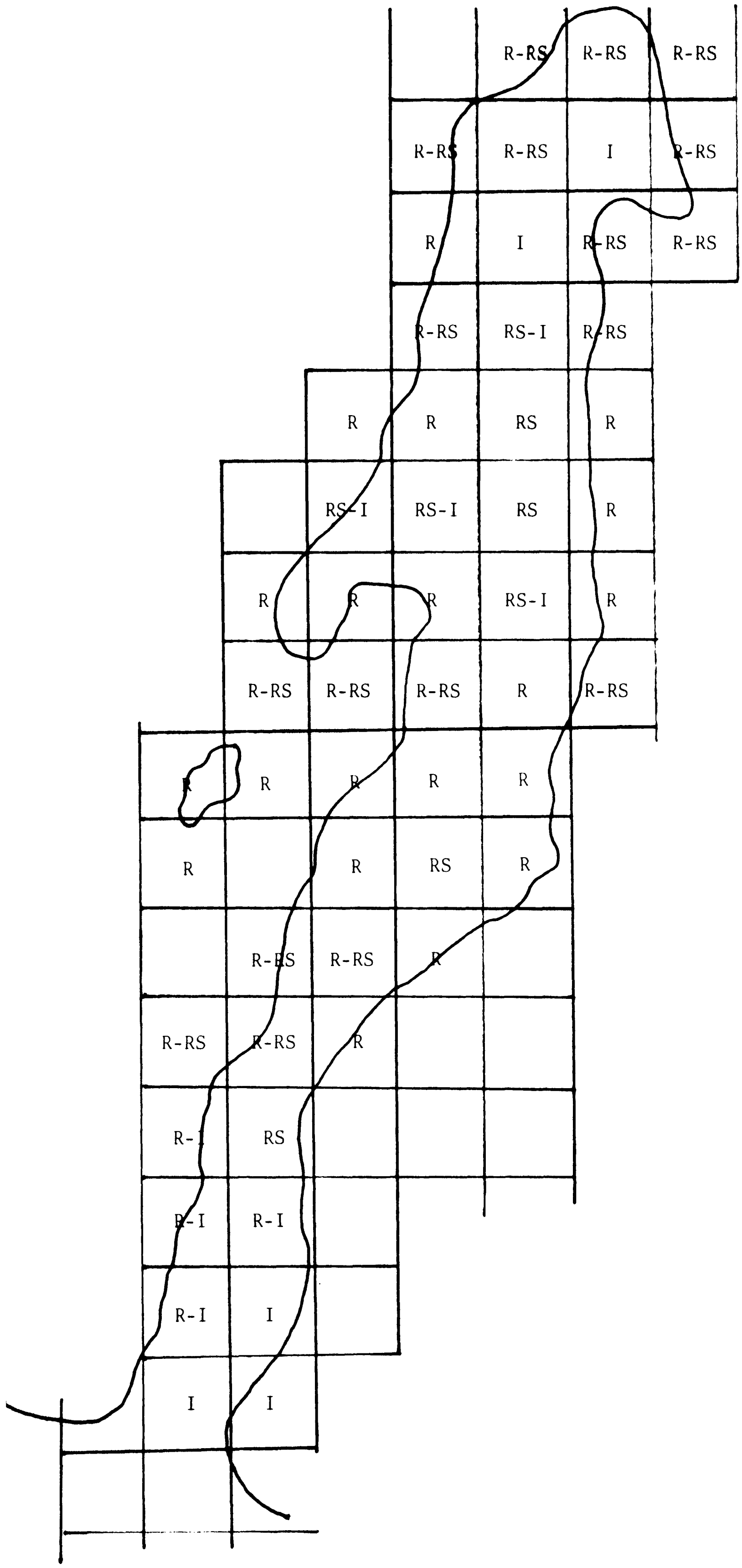

Figure 2. Solution Assignment

$$\max \sum_{i=1}^{M} \sum_{j=1}^{N} x_{ij} v_{ij} , \tag{1}$$

where M = feasible sites and N = needs, subject to

$$\sum_{j=1}^{N} x_{ij} = 1 \qquad (i = 1, \ldots, M) \tag{2}$$

$$\sum_{i=1}^{M} x_{ij} = r_j \qquad (j = 1, \ldots, N) \tag{3}$$

$$x_{ij} \geq 0 \tag{4}$$

This problem can be solved by a variety of algorithms. In particular, the Graves-Thrall algorithm (Spivey and Thrall, 1970) was used to solve the preceding problem, in which M = 55 and N = 6. The algorithm coded as outlined by Hall et al. (1971) and stored under the Michigan Terminal System (MTS) is an efficient computational tool. A problem of this magnitude can be solved at a cost of $1-2.

A MULTIPLE LAND USE ASSIGNMENT PROBLEM

If any parcel is to be permitted one or more land uses simultaneously, and if the amount of usable land varies with the parcel, then the model using equations 1-4 is inadequate. For example, Table 4 shows a set of

Table 4. Percentages of Each Available Land Use Type

Parcel	R	RS	I	R-RS	R-I	RS-I	Percent Available
1	*	6	0	8	5	3	10
2	*	64	40	72	60	52	80
3	*	3	0	4	3	2	05
4	*	*	*	*	*	*	20
5	86	52	9	69	47	31	90
6	90	56	20	73	55	38	100
7	*	18	14	19	17	16	20
8	*	29	18	32	27	23	35
9	*	60	0	80	50	30	100
10	*	*	*	*	*	*	50
11	*	12	0	16	10	6	20
12	*	36	0	48	30	18	60
13	*	60	0	80	50	30	100
14	*	*	*	*	*	*	35
15	*	2	0	4	2	1	05
16	78	46	0	62	39	23	80
17	*	60	0	40	50	30	100
18	12	0	16	10	6	*	20
19	*	14	16	35	28	23	40
20	90	52	10	71	50	31	100
21	*	60	0	80	50	30	100
22	*	15	0	20	13	8	25
23	*	*	*	*	*	*	25
24	*	38	36	39	38	37	40
25	53	28	23	40	38	25	75

Table 4. Continued

Parcel	R	RS	I	R-RS	R-I	RS-I	Percent Available
26	*	64	10	82	55	37	100
27	*	42	0	56	35	21	20
28	*	*	*	*	*	*	05
29	*	*	*	*	*	*	05
30	*	14	4	17	12	9	20
31	90	51	0	71	45	26	95
32	*	*	*	*	*	*	10
33	*	15	0	20	13	8	25
34	*	14	12	14	14	13	15
35	*	12	0	16	10	6	20
36	*	54	0	72	45	27	90
37	*	48	0	64	40	24	80
38	*	*	*	*	*	*	05
39	*	54	16	67	48	35	80
40	*	68	20	84	60	44	100
41	*	27	0	36	23	14	45
42	*	12	0	16	10	6	20
43	*	54	0	72	45	27	90
44	*	24	0	32	20	12	40
45	*	*	*	*	*	*	05
46	*	44	18	52	39	31	60
47	*	24	0	32	20	12	40
48	*	12	0	16	10	60	20
49	*	52	32	70	56	46	80

Table 4. Continued

Parcel	R	RS	I	R-RS	R-I	RS-I	Percent Available
50	*	34	10	42	30	22	50
51	*	48	0	64	40	24	80
52	*	48	8	61	42	28	75
53	*	36	0	48	30	18	60
54	*	87	76	91	86	82	95
55	*	10	9	10	10	9	10

percentages of area available in each parcel for each land use together with the total area percentage available in each parcel for assignment. Note that the acreages available for each use in a parcel are upper limits rather than exact requirements, and therefore can sum to more than the total percentage of land available in the cell. (Percentages are converted to acreages by multiplying each percentage by 640.) An asterisk means that it is permissible to assign a particular parcel to a given use in its entirety.

Interpreting the entries in Table 1 as value gained for each additional percentage of land area assigned to a given use, and imposing percentage requirements shown in Table 5,

$$\text{acres required} = \text{percentage required}(640) ,$$

one can define the problem of assigning mixes of land uses to each parcel in such a way that total cumulative value is maximized, the availability constraints on each parcel are not violated, and the requirements of the total amount of land assigned to each use are satisfied. The optimum

Table 5. Hypothetical Land Use Requirements

Land Use Type	R	RS	I	R-RS	R-I	RS-I
Total % Required	463	463	463	462	462	462

assignment is presented in Table 6, while Figure 3 shows a possible placement of the optimum assignments.

The format of this problem matches that of the standard transportation or distribution problem well known in linear programming literature:

$$\max \sum_{i=1}^{M} \sum_{j=1}^{N} x_{ij} v_{ij} \qquad (5)$$

subject to

$$\sum_{j=1}^{N} x_{ij} = a_i \qquad (6)$$

$$(i = 1, \ldots, M)$$

$$\sum_{i=1}^{M} x_{ij} = r_j \qquad (7)$$

$$(j = 1, \ldots, N)$$

$$0 \leq x_{ij} \leq c_{ij} \qquad (8)$$

$$(i = 1, \ldots, M)$$
$$(j = 1, \ldots, N)$$

Equation 6 states the total amount of land available for assignment in each parcel, equation 7 states the overall requirements for each land use type, and equation 8 states the limitations that may exist on assigning particular uses to particular parcels.

Table 6. Optimum Assignment of Uses to Parcels

	463		463		463		462		462		462			
	Rec		Res		Ind		Rec Res		Rec Ind		Res Ind		% Land Avail.	
Cell #	% Cell	Acres	% Cell	Acres	% Cell	Acres	% Cell	Acres	% Cell	Acres	% Cell	Acres	in Cell	Acres
1							2	12.8	5	32	3	19.2	10	64
2					40	256			40	256			80	512
3									3	19.2	2	12.8	5	32
4					15	96			5	32			20	128
5					9	57.6	3	19.2	47	300.8	31	198.4	90	576
6			42	268.8	20	128					38	243.2	100	640
7					14	89.6			6	38.4			20	128
8					18	115.2			17	108.8			35	224
9			60	384					10	64	30	192	100	640
10					50	320							50	320
11							4	2.56	10	64	6	38.4	20	128
12							12	76.8	30	192	18	115.2	60	384
13			60	384					10	64	30	192	100	640
14					35	224							35	224
15							2	12.8	2	12.8	1	6.4	5	32
16							18	115.2	39	249.6	23	147.2	80	512
17			60	384			10	64			30	192	100	640
18	10	64					10	64					20	128

Table 6. Continued

Cell #	Rec		Res		Ind		Rec Res		Rec Ind		Res Ind		% Land Avail.	
	% Cell	Acres	% Cell	Acres	% Cell	Acres	% Cell	Acres	% Cell	Acres	% Cell	Acres	in Cell	Acres
19			1	6.4	16	102.4					23	147.2	40	256
20			52	332.8	10	64	7	44.8			31	198.4	100	640
21			60	384			10	64			30	192	100	640
22	5	32					20	128					25	160
23					25	160							25	160
24					36	230.4			4	25.6			40	256
25	19	121.6	16	102.4			40	256					75	480
26			63	403.2							37	236.8	100	640
27	16	102.4					4	25.6					20	128
28					5	32							5	32
29					5	32							5	32
30	3	19.2					17	108.8					20	128
31	90	576					5	32					95	608
32					10	64							10	64
33	25	160											25	160
34	15	96											15	96
35	4	25.6					16	102.4					20	128
36	90	576											90	576
37	80	512											80	512
38	5	32											5	32
39	41	262.4							39	249.6			80	512
40			36	230.4	20	128					44	281.6	100	640
41	22	140.8							23	147.2			45	288

Table 6. Continued

Cell #	Rec		Res		Ind		Rec Res		Rec Ind		Res Ind		% Land Avail.	
	% Cell	Acres	% Cell	Acres	% Cell	Acres	% Cell	Acres	% Cell	Acres	% Cell	Acres	in Cell	Acres
42	4	25.6					16	102.4					20	128
43	18	115.2					72	460.8					90	576
44	8	51.2					32	204.8					40	256
45			5	32									5	32
46			8	51.2			52	332.8					60	384
47	8	51.2					32	204.8					40	256
48									10	64	10	64	20	128
49					32	204.8	2	12.8			46	294.4	80	512
50					10	64			30	192	10	64	50	320
51							21	134.4	40	256	19	121.6	80	512
52					8	51.2	25	160	42	268.8			75	480
53							30	192	30	192			60	384
54					76	486.4			19	121.6			95	608
55					9	57.6			1	6.4			10	64
Total	463	2963.2	463	2963.2	463	2963.2	462	2956.8	462	2956.8	462	2956.8	2775	

Total Acres Available = 17,760

Total % in Cells = 2,775

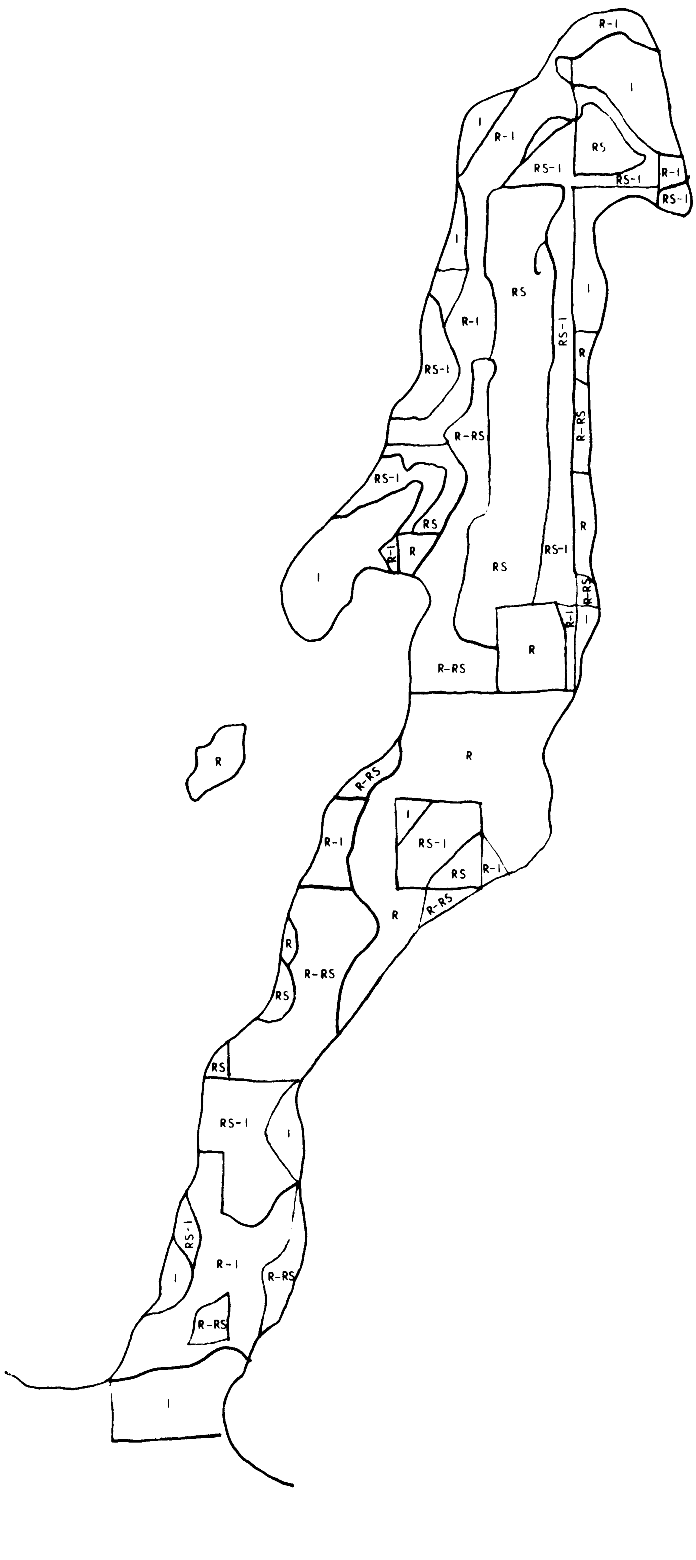

Figure 3. Hypothetical Optimized Land Use Pattern

The two numerical examples given illustrate the type of answers obtained for the "land use assignment" problem as defined. One finds the term "land use planning" frequently used, but it is difficult to determine what the user really intends to convey by the words. Similarly, it is not clear that the land use assignment problems as defined here can be interpreted in a practical sense so that various constraints are realistic and the criterion for optimization is objective. As interpreted here, *land use* means activity by man upon the surface of terrain which may involve a reconfiguration of the terrain or may result in subsurface changes to soil, water balances, and mineral inventories, or may result in changes to the plant and animal populations living on and above the surface. *Planning* as used here means a systematic analysis of the impacts of human activities upon terrain, an assessment of future user demands, and a recommendation of permissible terrain uses.

DIFFICULTIES IN FORMULATING LINEAR OBJECTIVE FUNCTIONS

Objective functions (1) and (5) assume that the degree of preference of one assignment of uses to sites is the sum of the individual "values" derived from assigning uses to each site separately. No value is placed upon the *pattern* per se. This assumption is valid in cases where an objective criterion such as monetary cost is used and for which the overall pattern effect is nonexistent. For example, the case of choosing a subset of preselected recreational sites so as to minimize development cost would not involve the pattern effect. Pattern effects introduce nonlinearities into the objective function. If one uses subjective values in the criterion for numerically ranking alternative assignments, the same difficulty arises. If two sites are each rated a value of two when assigned a given use, is this particular alternative equivalent to assigning the same use to a third site, which receives a rating of four? In the objective functions (1) and (5) the answer is assumed to be affirmative, which implies no pattern effect. The existence of a pattern effect means that sites must be considered in groups of two or more as well as individually. Various possibilities occur for incorporating pattern effects into a linear objective function, one of which is to enlarge the size of some or all of the sites, thereby reducing the total number of sites. The disadvantage of this procedure is that the ratings assigned to particular uses on a given site will not apply over the entire site. Another way to account for pattern effects is to remove any uses or combinations of uses that create pattern difficulties. If zoning a given site "industrial" lowers the value of an adjacent site zoned "residential," then one or the other uses can be removed from the problem. An alternative is to define an industrial use to include a buffer zone so that devaluation of adjacent sites does not arise.

DIFFICULTIES IN SPECIFYING CELL SIZES

The smaller the land unit area, the more homogeneous its characteristics and hence the more uniform its numerical description. Steinitz

(1970) has found 2.5 acres to represent an adequate balance between numbers of cells and uniformity of cell characteristics when performing suitability and capability analyses over an area of several thousand acres. In the integer programming model examples just given, each cell requires one equation so that doubling cell size reduces the number of equations by 50 percent. The limitations of the code described by Hall et al. (1971) restrict the product of the number of cells and the number of uses to be less than 70,000 for a "capacitated" problem and less than 90,000 for an "uncapacitated" problem. From the viewpoint of computational efficiency, it would be preferable to reduce the number of equations and hence increase cell size. Another difficulty created by small cell sizes is that uses tend to be sprinkled over an area in a "salt and pepper" fashion, more so than in the case of larger cell sizes. This tendency focuses upon a basic element of unrealism in the two models specified, which place no minimum requirement on the size of local areas assigned to a single use. For example, a shopping center that requires 1,000 acres cannot be designed on 10 separate locations of 100 acres each. This shows the capacitated transportation and assignment models are inadequate to solve a *location-allocation* problem which implies decision making on where to reserve a number of land parcels, each of a specified size, for specific uses in an "optimal" manner, subject to possible land use constraints of the types previously listed. In the location-allocation problem, the area required for each of a fixed number of uses is specified in advance. The possible sites for each use are identified in advance, and the problem is to locate the uses in an optimal manner. The problem of a "patchwork quilt" type of assignment pattern never arises. The sizes of the cells do not affect the area distribution of the uses.

A MODIFIED SITE ASSIGNMENT PROBLEM

The assignment model specified by (1) through (4) is more realistically applied when sites are redefined as "feasible" sites for specific land use needs such as shopping centers, industrial parks, recreational parks, or residential subdivisions. Sites are now variable in area and shape and are located specifically in terms of identified "needs." The site "use" now becomes a need which is defined in terms of cost, required land characteristics, area, and other identifying features. A set of needs, N, is defined, which leads to a selection of feasible sites, M, upon which to satisfy some or all of the needs. The parameters of the problem are summarized in Table 7. The assignment problem is easily solved using the code defined by Hall et al. (1971). It makes no difference whether $M \geq N$ or $M < N$ as far as the numerical solution is concerned. The algorithm simply defines fictitious feasible sites or needs as required to "balance" the problem. The feasible sites can be chosen with the aid of site suitability and site capacity maps developed during or preceding the site feasibility study. The algorithm assigns needs to feasible sites in such a way that total cost is minimized. If the total minimum cost exceeds a budget limitation, then needs can be reduced until budget constraints are met.

Note that the assignment can be made using dollar costs and remade using some other objective criterion such as user convenience, number of users, or damage to the environment. The solutions can be compared before a final choice is determined.

If a given site is infeasible for a specific need, the corresponding cost is assigned a high value so that the need is never assigned to that site. Even if feasible sites exceed the number of needs, there may still exist competition between needs for a particular set of sites. If the number of needs and feasible sites is sufficiently small, a computer analysis is unnecessary. In a sense there always exists "competition" of a proposed need with the existing use of a site. The "loss" associated with converting the site from one use to another can be considered when defining the cost coefficients shown in Table 7.

It is important to note that two "needs" may be two different acreage requirements for the same basic use, for example, a large residential subdivision and a small residential subdivision. If the costs are nonlinear with the size of the site required, the appropriate costs can be stated without difficulty in the cost table. Needs arise in a sequential fashion. If needs can be anticipated to a time horizon of T years, then all anticipated needs can be simultaneously considered over this period. The longer the time horizon for anticipating needs, the more difficult it is to propose alternative sites since the uses of adjacent lands may have changed before a decision can be implemented, thus making the "costs" unrealistic.

Table 7. Tableau for Assignment Problem

Feasible Sites	Needs				Amount Available
	1	2	. . . .	N	
1	C_{11}	C_{12}	. . . .	C_{1N}	1
2	C_{21}	C_{22}			1
.	.		(Cost)		
.	.				
.	.				
M	C_{M1}				1
Amount Required	1	1		1	

APPLICATIONS OF MIXED INTEGER PROGRAMMING

Mixed integer programming applications are introduced by IBM (1972). Conditions in land use decision making which lead to mixed integer programming problems are

(1) either use A or use B, but not both, is required on a given site;

(2) a given development is to be carried out on a scale of either A, B, or C acres;

(3) several recreational sites are to be located so as to satisfy user demands, and each site has an overhead cost and a variable operating cost.

Mixed integer programming requires algorithms and codes different from integer programming and is not pursued here.

GOALS PROGRAMMING

"Goals programming" is defined by Spivey (1970) as a linear program, a special case of which is described and illustrated in this report. The problem considered here is the following: A manager has M geographic zones under his control, to each of which he must budget N different resources. He has established "goals" based upon past performance and future expectations which specify the amount of each resource he prefers to assign to each zone. There are penalties for overbudgeting or underbudgeting resources in any given zone which may differ by zone and type of resource.

There also exist minimum resource requirements necessary for each zone that are less than those specified in the goals. There may exist, in addition, upper limits on the amount of each resource available for distribution over all zones. As in the case of a total dollar budget that can be broken down by zone and by resources within zones, there may exist an overall budget limit. Finally, there may exist limitations called "zone capacities" that place restrictions on "total equivalent resources" that can be assigned to each zone. The problem is to allocate resources to zones in such a way that the total cost of deviation from the goals is minimized subject to the given limitations.

The problem is formulated as a linear program by first defining the following quantities. Let

$$R = \underset{M \times N}{\begin{bmatrix} r_{11} & - - - - & r_{1N} \\ \vdots & & \\ r_{M1} & - - - - & r_{MN} \end{bmatrix}} = \underset{M \times 1}{\begin{bmatrix} R_1 \\ \vdots \\ R_M \end{bmatrix}}, \qquad (9)$$

where r_{ij} = preferred level of resource j established for zone i

$(j = 1, 2, ..., N)$

$(i = 1, 2, ..., M)$.

$$\underline{L} = \underset{N \times 1}{\begin{bmatrix} \underline{\ell}_1 \\ \vdots \\ \underline{\ell}_N \end{bmatrix}}, \qquad (10)$$

where $\underline{\ell}_i$ = minimum required amount of resource i over all zones

(i = 1, ..., N).

$$\bar{L} = \begin{bmatrix} \bar{\ell}_1 \\ \vdots \\ \vdots \\ \bar{\ell}_N \end{bmatrix}, \qquad (11)$$

$$N \times 1$$

where $\bar{\ell}_i$ = maximum permissible amount of resource i assignable to all zones.

$$\underset{M \times N}{\Theta} = \underset{M \times N}{\begin{bmatrix} \Theta_{11} & - - - - & \Theta_{1N} \\ \vdots & & \\ \vdots & & \\ \Theta_{M1} & - - - - & \Theta_{MN} \end{bmatrix}} = \underset{M \times 1}{\begin{bmatrix} \Theta_1 \\ \vdots \\ \vdots \\ \Theta_M \end{bmatrix}}, \qquad (12)$$

where Θ_{ij} = coefficient used to convert resource j in zone i into a standard or equivalent unit, for example, number of vehicles → dollars or → number of vehicles → square feet of land surface.

$$U = \underset{M \times N}{\begin{bmatrix} u_{11} & - - - - & u_{1N} \\ \vdots & & \\ \vdots & & \\ u_{M1} & - - - - & u_{MN} \end{bmatrix}} = \underset{M \times 1}{\begin{bmatrix} U_1 \\ \vdots \\ \vdots \\ U_M \end{bmatrix}}, \qquad (13)$$

where u_{ij} = units of resource j overbudgeted for zone i

$(i = 1, \ldots, M)$

$(j = 1, \ldots, N)$.

$$V = \begin{bmatrix} v_{11} & - - - - & v_{1N} \\ \vdots & & \\ v_{M1} & - - - - & v_{MN} \end{bmatrix} = \begin{bmatrix} V_1 \\ \vdots \\ V_M \end{bmatrix}, \tag{14}$$

where v_{ij} = units of resource j underbudgeted in zone i

$(j = 1, \ldots, N)$

$(i = 1, \ldots, M)$.

$$C = \underset{M \times N}{\begin{bmatrix} c_{11} & - - - - & c_{1N} \\ \vdots & & \\ c_{M1} & - - - - & c_{MN} \end{bmatrix}} = \underset{M \times 1}{\begin{bmatrix} C_1 \\ \vdots \\ C_M \end{bmatrix}}, \tag{15}$$

where c_{ij} = unit cost of overbudgeting resource j in zone i converted to a common or equivalent unit.

$$C^* = \underset{M \times N}{\begin{bmatrix} c_{11}^* & - - - - & c_{1n}^* \\ \vdots & & \\ c_{M1}^* & - - - - & c_{MN}^* \end{bmatrix}} = \underset{M \times 1}{\begin{bmatrix} C_1^* \\ \vdots \\ C_M^* \end{bmatrix}}, \tag{16}$$

where c_{ij}^* = unit cost of underbudgeting resource j in zone i converted to a common or equivalent unit.

The problem is to minimize

$$\sum_{i=1}^{M} U_i C_i^T + \sum_{i=1}^{M} V_i C_i^{*T} \quad , \tag{17}$$

subject to

$$\sum_{i=1}^{M} U_i^T - \sum_{i=1}^{M} V_i^T \geq \underline{L}^T - \sum_{i=1}^{M} R_i^T \tag{18}$$

(minimum requirements on resources)

$$\sum_{i=1}^{M} U_i^T - \sum_{i=1}^{M} V_i^T \leq \bar{L}^T - \sum_{i=1}^{M} R_i^T \tag{19}$$

(maximum permissible assignments of resources)

$$\sum_{j=1}^{N} \sum_{i=1}^{M} (r_{ij} + u_{ij} + v_{ij}) d_{ij} \leq K \quad , \tag{20}$$

(overall budget unit)

where d_{ij} = unit budget equivalent for resource j on zone i.

$$\left.\begin{array}{l} \Theta_1(U_1 - V_1) \leq a_1 - \Theta_1 R_1^T \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ \Theta_M(U_M - V_M) \leq a_M - \Theta_M R_M^T \end{array}\right\} \tag{21}$$

$$U,V \geq 0 \ , \tag{22}$$

(limitations on total equivalent value on resources assignable to each zone)

where a_i = maximum equivalent value assignable to zone i.

In certain problems some subset of restrictions (18) - (22) may be missing.

Example Problem

A university planner requires acreage for four uses (resources) which are (1) open space, (2) recreation, (3) parking, and (4) housing, each of which must be distributed over three campuses. His goal is specified as follows:

R =

Zone	Open Space (sq. ft.)	Recreation Parks (# people served)	Parking (# Vehicles)	Housing (# Family Units)
1	500,000	500	100	100
2	500,000	500	100	100
3	300,000	500	75	50

The common unit for allocation of acreage is square feet, and conversion units are specified by

$$\Theta = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} \underline{\text{o.s.}} & \underline{\text{rec. pks.}} & \underline{\text{park.}} & \underline{\text{hous.}} \\ 1 & 1{,}000 & 150 & 1{,}000 \\ 1 & 1{,}000 & 150 & 1{,}000 \\ 1 & 2{,}000 & 150 & 1{,}000 \end{bmatrix}$$

It is mandatory that the following minimum requirements be met for open space, recreational parks, vehicle parking, and family housing:

$$\underline{L} = \begin{bmatrix} 1{,}000{,}000 \\ 1{,}000 \\ 200 \\ 100 \end{bmatrix} \begin{array}{l} \text{(sq. feet)} \\ \text{(people)} \\ \text{(vehicles)} \\ \text{(family units)} \end{array}$$

The zones have maxima 1,500,000 (zone 1), 1,000,000 (zone 2), and 1,000,000 (zone 3) sq ft of area available for assignment.

The planned goal will not be met immediately, so a relative cost of over- and underbudgeting acreage for use is specified:

$$\underset{\text{(over-budget)}}{C} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} \underline{\text{open space}} & \underline{\text{rec.}} & \underline{\text{parking}} & \underline{\text{housing}} \\ 1 & 2 & 5 & 10 \\ 1 & 2 & 5 & 10 \\ 1 & 2 & 20 & 50 \end{bmatrix}$$

		open space	rec.	parking	housing
	1	1	2	10	20
C* = (under-budget)	2	1	2	10	20
	3	1	3	5	4

When programmed for solution using a simplex code, there are 24 decision variables (12 u_{ij}'s and 12 v_{ij}'s). As explained by Hall (1971), u_{ij} and v_{ij} (for all i,j) cannot both be simultaneously strictly positive. That is, one cannot simultaneously overbudget and underbudget acreage to the same use in the same zone. The solution is specified in terms of u_{ij} and v_{ij}. The actual allocation is therefore $r_{ij} + u_{ij} + v_{ij}$.

This example is a small version of an actual problem that exists for university planners. At The University of Michigan in Ann Arbor, three campuses are North, Central, and Athletic. Nine site uses are organic gardening, crafts centers, day-care centers, married-student housing, high-rise apartments, classrooms and office space, parking, single-student apartments, and industrial research. Goals have been specified for each use type (in thousands of square feet of land surface).

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