

which interacts only with the surface, and it disturbs that surface very little since the beam has both a negligible heat capacity and a negligible thermal conductivity.

Thus, in spite of the fact that it is an elaborate and difficult technique, the molecular beam method of surface temperature measurement could be useful as a check on pyrometer calibrations at high temperatures (say above 2000°K). Also the method may be useful for precise determination of temperatures of nonmetallic surfaces below 1000°K. Pyrometer measurements are difficult in this range because of the low radiation intensity, and thermocouple measurements are unreliable because of the poor thermal conductivity of the specimen.

The following results give an idea of the precision of the molecular beam method. For potassium beams reflected from copper surfaces, temperatures determined from the half-maximum positions on the high velocity sides of the measured distributions agreed with the surface temperatures as measured by a thermocouple to within  $\pm 2\%$ . The measurements made at higher temperatures using the tungsten surface are less precise because the resolution of the selector decreases with increasing beam temperature. For surface temperatures in the neighborhood of 2100°K the "beam temperatures" calculated from the measured "high velocity" half-maximum positions average about 4% higher than surface temperatures measured with an optical pyrometer and corrected by use of spectral emissivity tables. Somewhat better agreement between the beam and pyrometer temperatures was obtained by choosing as beam temperature the temperature of that Maxwell curve which was judged to give the best over-all agreement with the measured curve. Beam temperatures measured in this way were, on the average, about 1% higher than the pyrometer temperatures.

A considerable improvement in the precision of the beam temperature measurements is possible. For example, a threefold increase in the selector rotation speed is probably feasible. This increase, combined with an increase in the mass of the beam atoms (e.g., changing from a potassium to a rubidium beam), would increase the resolution of the selector by a factor of four. It is estimated that the increased resolution would make it possible to measure beam temperatures to a precision of at least 1% in the range 500° to 3000°K.

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<sup>1</sup> P. M. Marcus and J. H. McFee, *Recent Research in Molecular Beams* (Academic Press, Inc., New York, 1959), Chap. 3.

<sup>2</sup> The results are contained in Technical Rept. No. 1 to the U. S. Office of Naval Research, Contract Nonr 760 (11), January 1960, "Velocity distributions in direct and reflected atomic beams." (To be published.)

<sup>3</sup> The only previous careful measurements of the velocity distributions in scattered molecular beams appear to be those of A. Ellett and V. W. Cohen, *Phys. Rev.* **52**, 509 (1937).

<sup>4</sup> One exceptional case was found: The velocity distribution of potassium atoms scattered from a cleaved LiF surface was definitely non-Maxwellian at high surface temperatures. However, LiF surfaces seem to be rather exceptional in this respect. Other reflected-beam experiments on LiF, such as diffraction of He atoms, and reflection of Hg atoms indicate this. The inertness of the surface to attack by water vapor and the exceptionally smooth cleavage planes suggest that the LiF surface would be different from most other surfaces. In any case, if lack of complete accommodation exists, it can easily be detected by the observations; however, we expect such incomplete accommodation for potassium atoms to be very rare.

## A Simple Method for Improving the Accuracy of Cross-Spectral Density Measurements

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LET the cross correlation of two random functions of time,  $e_1(t)$  and  $e_2(t)$ , be given by

$$R_{12}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T e_1(t)e_2(t+\tau)dt.$$

Then the real and imaginary parts of the cross-spectral density, respectively,  $E_{12}(\omega)$  and  $F_{12}(\omega)$ , are defined by

$$E_{12} + iF_{12} = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{12}(\tau)e^{i\omega\tau}d\tau.$$

In a recent paper Gilbert and Uberoi<sup>1</sup> have described an instrument for direct measurement of  $E_{12}$  and  $F_{12}$ . Experience has shown that the major source of error is inconstancy of phase shift in the two selective filters used. This note presents a method for eliminating the effect of the error without continually adjusting the phase calibration of the instrument.

The instrument produces a measurement

$$M_{12} = aE_{12} + bF_{12}.$$

Ideally, for measurement of  $E_{12}$ ,  $a=1$ ,  $b=0$ . Actually,  $a \approx 1$ ,  $0 < |b| \ll 1$ , and an error results which may be relatively large if  $|F_{12}| \gg |E_{12}|$ . To correct the error a second measurement  $M_{21}$  is made with the inputs  $e_1(t)$  and  $e_2(t)$  interchanged. Since  $E_{21} = E_{12}$  and  $F_{21} = -F_{12}$ ,

$$M_{21} = aE_{21} + bF_{21} = aE_{12} - bF_{12}.$$

Thus,

$$E_{12} = (M_{12} + M_{21}) / (2a) \cong (M_{12} + M_{21}) / 2.$$

Similarly, when the instrument is set up to measure  $F_{12}$ ,  $0 < |a| \ll 1$ ,  $b \cong 1$ , and the corrected reading is

$$F_{12} = (M_{12} - M_{21}) / (2b) \cong (M_{12} - M_{21}) / 2.$$

When the cross-spectral density has line components or

peaks which are much smaller in bandwidth than the bandwidth of the analyzer, care should be taken to keep the center frequency of the analyzer exactly the same when  $M_{12}$  and  $M_{21}$  are obtained. Otherwise, phase differentials at the different center frequencies may cause  $a$  and  $b$  to be different for  $M_{12}$  and  $M_{21}$ .

<sup>1</sup> M. S. Uberoi and E. G. Gilbert, Rev. Sci. Instr. **30**, 176 (1959).