

Experiments on the Design of Synchrotron Magnets*

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(Received July 1, 1947)

An important aspect of the problem of synchrotron magnet design is that of obtaining a given magnetic energy in the guide field with the minimum expenditure of iron, copper, and condensers. Tests were made on d.c. magnet models of several configurations, using as criteria (1) the ratio of the peak energy in the useful portion of the magnet air gap to the peak energy which must be stored in the condensers, and (2), the ratio of magnetic flux in the gap to the flux in the return path of the magnetic circuit. Comparative performance data are tabulated. The method of shaping the pole faces to obtain the desired fall-off of B with r over the greatest range in r is discussed, and a shape worked out by trial and error is given.

IN the course of designing the synchrotron which is being constructed at the University of Michigan,¹ a comparative study of several basic types of magnets was made. Since the results obtained apply quite generally to the problem of synchrotron design, it is believed that their publication will be of value.

As is often the case in the design of large pieces of apparatus, the criteria which serve to distinguish a good design from a poor one are mainly economic in nature. A synchrotron magnet is an alternating current magnet, and it is connected to a capacitor bank in either a series-fed or a parallel-fed resonant circuit. The cost of the magnet and coils (the iron must, of course, be laminated) is the same order of magnitude as the cost of the capacitor bank. Therefore, in considering variations in design, economy in the magnet and in the capacitor bank should be given about equal importance. The problem, stated briefly, is to produce a usable magnetic field in a volume which has the shape of a ring, of inside and outside radii r_1 and r_2 , and height h . By a usable magnetic field we mean one in which B , the magnetic induction, falls off with radius according to the relation $B = B_0 r^{-n}$ where $0 < n < 1$. The field can easily be made to satisfy this condition throughout the entire height, from one pole face to the other, but in the radial direction only the center portion can be made to have the desired fall-off, because of fringing.

The energy in the oscillating magnet circuit must appear entirely in the capacitors at one instant in each cycle, and entirely in the magnetic field at another instant ($\frac{1}{4}$ cycle later). The energy of the magnetic field at any instant, in joules, is $[(10^{-7}/8\pi) \int \mathbf{B} \cdot \mathbf{H} dv]$ integrated over all space. Since $\mathbf{H} = \mathbf{B}/\mu$, the energy in the iron is small in comparison to that outside the iron. The energy is also given by $\frac{1}{2}LI^2$. The maximum energy which the capacitor bank is able to store, $\frac{1}{2}CV_{Pk}^2$, must be equal to $[(10^{-7}/8\pi) \int \mathbf{B} \cdot \mathbf{H} dv]_{Pk}$ or $(\frac{1}{2}LI^2)_{Pk}$, where the subscript Pk indicates that the expressions are to be evaluated at the peak of the cycle. The field strength in the useful part of the gap (volume V) is practically uniform (it falls off only as $1/r^n$) so an approximate expression for the useful energy, in joules, is $(10^{-7}B^2V/8\pi\mu_a)$ where B is the average magnetic induction in the gap and μ_a is μ for air ($=1$). The non-useful energy is that which resides mainly in the fringing field and in the leakage field through the windings, and every joule of this has to be paid for in extra capacitors. Thus, one of the important quantities which must be maximized in the design of a magnet is the ratio of the energy in the useful part of the magnetic field to the total energy.

The length of the return path in the iron largely determines the weight of the magnet for a given size of pole face. The length of the return path depends in turn upon the cross-sectional area of copper which has to be enclosed. Fortunately, the number of ampere turns in the coil can be calculated, to a good approximation, with no more information than the height of the gap, since $NI = 10hB/4\pi$, where either peak value or r.m.s.

* This work was supported by the U. S. Navy, Bureau of Ordnance, under contract NOrd 7924.

¹ H. R. Crane, *Phys. Rev.* **69**, 542 (1946); **70**, 800 (1946).
D. M. Dennison and T. H. Berlin, *Phys. Rev.* **69**, 542 (1946); **70**, 764 (1946).

value is used for both I and B . This formula neglects the reluctance of the iron return path which is usually small compared to the reluctance of the air gap. With this, and a decision as to how many watts copper loss will be permitted, the cross-sectional area of copper is determined. There will then be several ways in which the return path in the iron can be constructed, and these form the main subject for the experimental tests to be described.

MAGNET MODELS

Magnet models of the basic types shown in Fig. 1 were constructed for use in the measurements. All the models were made of cold-rolled steel, and were operated with direct current in the coils. The measurements were made at flux densities well below the saturation value of the steel, but sufficiently high to insure iron permeabilities such that the reluctance of the iron path was small compared to the reluctance of the air gap. The reluctance of the iron path was not neglected, however, in the computations.

The sketches of the magnets shown in Fig. 1 are to scale, and the dimensions are given. In the series of measurements in which the various types of magnets were compared, all of the pole faces were flat and parallel, with the exception of one, in which the pole faces were shaped as in Fig. 2.

MEASUREMENT OF INDUCTANCE

In order to measure the self-inductance of the models, all windings were made with a double conductor. One of the conductors carried the magnetizing current and the other was connected to a ballistic galvanometer. The mutual inductance of the coils was measured. Since the magnetic linkage is the same for both conductors, the self-inductance of either winding equals the mutual inductance. The C -type models were full circles, but the H -type models were not. It was desired to compare all the models on the same basis; for that reason, corrections to the self-inductances were made for the effect of the ends of the windings in the H -type models. The corrections were estimated theoretically, and they amounted to about 4 percent.

ABSOLUTE MEASUREMENT OF FLUX

The total flux through a given area, usually the entire pole face, was measured by the usual method employing a pick-up loop, a ballistic galvanometer, and a standard mutual inductance. The current through the magnet model was reversed for each measurement, to minimize the effect of residual magnetization. The absolute value of the field strength at the center of the gap was measured with a pick-up coil of small area.

MEASUREMENT OF THE SHAPE OF THE FIELD IN THE GAP

The determination of the shape of the field requires the measurement of small changes, from

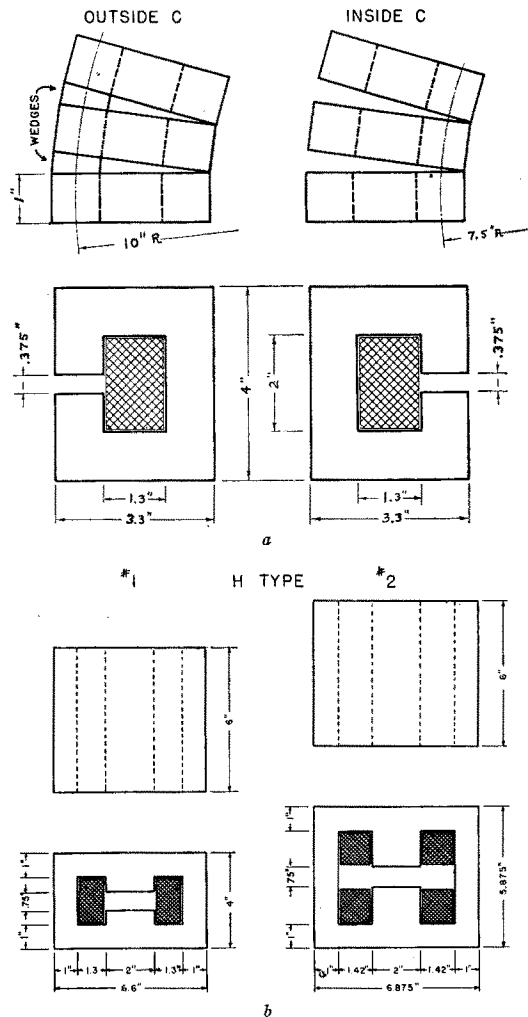


FIG. 1. Dimensioned drawings of the models tested. Only sectors of the inside and outside C magnets are shown. Full circles were used in the tests. The H -type magnets had lengths as shown in the drawing.

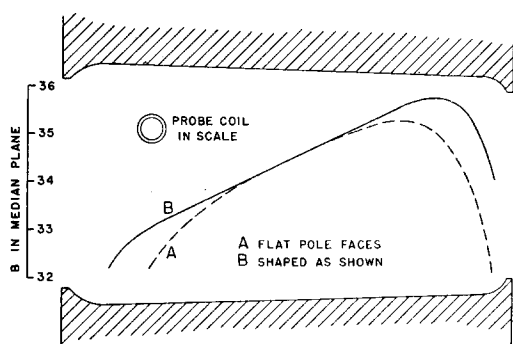


FIG. 2. Cross section of the shaped pole gap. The model on which the magnetic measurements were made was identical to *H*-type No. 2 (Fig. 1) except that in this case the gap height at the center was 1 inch instead of $\frac{3}{4}$ inch, and the pole faces were shaped as shown.

one point to another. To accomplish this a generating voltmeter was constructed as follows: The probe coil, which was 3 mm in diameter, was mounted on the end of a long Lucite spindle which rotated at 1750 r.p.m. Another coil, connected in series opposition with the probe coil, was mounted near the other end of the spindle and between the poles of an auxiliary electromagnet. The auxiliary magnet was mounted on the framework which carried the spindle and was fixed in position with respect to it, except that a micrometer screw arrangement was provided for rotating the magnet around the spindle axis, for the purpose of making the phases of the voltages generated by the two coils the same. The winding of the auxiliary magnet was connected across a variable shunting resistance and in series with the coil of the magnet model. This method of connection served to balance out the first-order effects of variations in the d.c. line. In plotting the field across the gap, the shunt and the phase adjustment were set so that the output from the two coils was zero at the starting position. The shunt was thereafter left fixed, but the phase adjustment was trimmed at each location. The a.c. output thus gave the change in the field strength, and the amount by which the phase had to be adjusted at each location gave the change in direction of the field. The a.c. output of the pair of coils was amplified and read on a vacuum-tube voltmeter. The changes observed with the differential connection were compared with the total field strength measured (on the same relative scale) by temporarily removing the auxiliary

magnet. Differential changes which were equal to 0.1 percent of the total field strength could be measured reliably.

SHAPING OF POLE FACES

For stability of the orbits in the synchrotron, it is necessary to have the magnetic induction fall off with radius according to $B = B_0 r^{-n}$ where $0 < n < 1$. If we call the radius to the center of the gap a , and $r - a = \rho$, then to a good approximation $B = B_0/a^n(1 + n\rho/a)$. When the two pole faces are not parallel, the value of B_0 , to a first approximation, varies across the gap inversely as the height of the gap. The height of the gap as a function of ρ should therefore be, to a first approximation, $h = h_0(1 + (n/a)\rho)$ where h_0 is the height at $\rho = 0$.

A second requirement is that as great a fraction as possible of the width of the gap should be useful; that is, should have the prescribed rate of fall-off with radius. To strengthen the field near the edges where fringing occurs, and thereby make a greater portion of the gap useful, the pole faces are made with cusps at the edges. We have found experimentally that, in a gap of given height and width, the fraction of the width that can be made useful depends largely upon the contour given the pole faces; only slightly upon the magnetic return path. Consequently, all the models considered have practically the same possibilities in this respect.

A remark should be made about saturation in a pole piece which has cusps at its edges. In a synchrotron it is necessary that the field have the correct fall-off over the greatest possible width at the time of injection when the magnetic field is very small and during the early part of the acceleration cycle. As the field grows, the cross-sectional area of the beam shrinks, so that at the time when the cusps of the pole pieces begin to

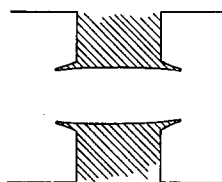


FIG. 3. "Ubangi" pole pieces. This sketch shows how cusps which saturate early in the magnetic cycle can be used to give a larger volume of useful magnetic field at low field strength than at high, effecting a saving in the condenser energy required.

saturate, it is permissible to have a reduction in the usable width of the field. Thus economy, both in iron and condenser kva can be effected by designing the pole pieces with cusps which saturate at high field. Carried to an extreme, this argument would lead to the design of pole pieces as illustrated in Fig. 3. This is a possibility to keep in mind in regard to the design of very large machines.

The contour for the pole faces for the University of Michigan synchrotron, arrived at by trial and error, is shown in the scale drawing in Fig. 2. The plot of the field obtained using this contour, and the plot of the field obtained with flat pole faces before shaping, are shown in the same figure. The useful width of the field is approximately 0.6 the width of the pole face. Since the amount of width lost due to fringing bears a practically constant ratio to the gap height, h , a rough empirical formula for the useful width is $w' = w - 0.8h$, where w and w' are the full width and the useful width, respectively. This holds well up to a height to width ratio of $\frac{1}{2}$.

RESULTS AND DISCUSSION

We have seen that the fraction of the gap width which is useful depends primarily upon the local shaping of the pole tips. Therefore, the only criterion we can use for comparing the efficiencies of the different models is the amount of iron and copper required and the amount of capacitor energy storage which must be used to create a useful field of given dimensions. Table I gives the essential information on the models tested.

ϕ_1 is the flux measured by a wire loop which lies in the median plane of the gap and just encloses the projections of the pole faces on the median plane. It includes the flux in the wedge-shaped area between neighboring pole faces. Thus, in the outside C model it includes the flux

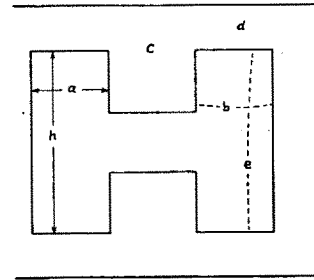


FIG. 4. Sketch showing the important components of leakage flux through the winding space.

through the iron wedges when they are in place and includes the flux through the same area when they are not in place.

ϕ_2 is the flux through the iron return path, measured by means of a wire loop around the back leg of the magnet, in the median plane. In the H -type magnet the values of ϕ_2 for the two return paths are lumped together.

W_1 is a quantity very nearly equal to the energy in joules, of the magnetic field in the gap. It is obtained from the following relation:

$$W_1 = \frac{10^{-7} B^2 V}{8\pi\mu_a} = \frac{10^{-7} \phi_1^2 h}{8\pi\mu_a A}$$

Where ϕ_1 is the flux in maxwells measured as prescribed above, h and A are the height and area of the gap in cm and cm², and μ_a is unity in the system of units used. It is evident that W_1 is slightly in error because the square of the mean rather than the mean square of ϕ_1 is used. However, that this error is small has been verified by measurement of the flux distribution, which is essentially the same for all the models with flat pole faces.

W_2 is $\frac{1}{2}LI^2$, which is the total energy stored in the magnet, and also the energy which must be stored in the capacitor bank.

The ratio W_1/W_2 is the energy efficiency of the magnet. This value is of interest because the cost of a condenser bank is proportional to the energy it will store, assuming that the frequency remains constant.

The ratio ϕ_1/ϕ_2 is the "flux efficiency", and in general will determine the cross-sectional area of the back leg, for a given height and area of gap. In some cases, however, this ratio may not provide sufficient information for the design of

TABLE I.

Magnet	ϕ_2/ϕ_1	W_2/W_1	$\mu\phi$	μL	ϕ_2/ϕ_1 Extrapolated to $\mu = \infty$	W_2/W_1 Extrapolated to $\mu = \infty$
Outside C without wedges	.52	.46	500	400	.54	.49
Outside C with wedges	.55	.54	500	400	.56	.57
Inside C	.63	.52	500	400	.64	.55
H -type No. 2	.52	.52	250	330	.54	.53
H -type No. 1	.71	.62	250	230	.72	.66
H -type No. 2 with concave pole faces (see Fig. 3)	.45	.45	320	330	.45	.48

the return path. If the width of the window which contains the windings is small (a in Fig. 4) the component of flux b may be so strong that the maximum flux no longer will be found in the back leg in the median plane, but at c or d . This is an argument for making the winding window wide. On the other hand, if it is made too wide in relation to its height, the amount of vertical flux becomes large, necessitating an increase in the cross section of the back leg and also wasting condenser kva because of the large energy storage in this vertical flux. The weight of iron is still another quantity which should be minimized, so, clearly, a compromise is necessary. If there are no external restrictions on the design, the best compromise in relation to width and height of the window can be arrived at by trial and error with the model. Experience shows that roughly a square window, $a = h$, will be the most efficient. In designing the racetrack a width smaller than optimum was chosen, because it was desirable to have the windings extend as little as possible into the gap between quadrants of the magnet.

The corrected values of W_2/W_1 and ϕ_2/ϕ_1 represent extrapolations to the case of iron of infinite permeability. The actual value of the permeability under the conditions of each measurement of ϕ and L is recorded in the table, in columns headed μ_ϕ and μ_L . In extrapolating the values it is assumed that if the current in the coils is held constant and the permeability of the iron increased, the field strength in the gap (and ϕ_2) will increase as the reciprocal of the reluctance of the magnetic circuit which follows through the air gap and around through the back leg. It is assumed that the leakage flux will not increase appreciably. The leakage paths in general contain such a great length of air that a small change in the reluctance of the iron part of the path will have a negligible effect. Therefore ϕ_1 will increase only by approximately the amount of the increase in ϕ_2 , not the same percentagewise. The extrapolation of the inductance is done in the

same way as that of ϕ_1 , that is, it is assumed that the part of the inductance attributable to ϕ_2 will increase as ϕ_2 , discussed above, but the part caused by the leakage flux will not increase appreciably. The corrections arrived at by the use of the foregoing assumptions are not capable of high accuracy, but fortunately they are small, amounting in most of the cases to only a few percent. The values obtained for $\mu = \infty$ will represent very closely the behavior to be expected when high permeability transformer iron is used.

It should be pointed out that the flux efficiency and the energy efficiency are strongly dependent upon the height of the air gap. As the gap height is increased, the leakage flux which passes through the windings (for example e in Fig. 4) increases. Therefore, from the viewpoint of magnet design, it is advantageous to use a vacuum "doughnut" of a rather flat cross section.

APPLICATION OF THE RESULTS OF D.C. MEASUREMENTS TO A.C. MAGNETS

The results of the studies made on d.c. models are of course expected to serve as a guide in the design of a.c. magnets, which will be constructed of laminae of transformer steel. The outside C with wedges, and both of the H models, are applicable to the a.c. case without correction because the pole faces are continuous. If the inside C were to be built as an a.c. magnet, it would be desirable to fill in the small triangular wedge-shaped spaces between neighboring poles to eliminate cross flux in this region of high flux density, since cross flux near the pole tips would give rise to quadrature components in the magnetic field, which are undesirable in a synchrotron. Filling these spaces in the d.c. models, however, raises the energy and flux ratios by only 1 to 2 percent. The outside C model without the wedges which fill the spaces between the sections would not be used in an a.c. application because of cross flux. The measurements are only included as an interesting comparison.