Dynamic instabilities in the power spectrum of deeply modulated semiconductor lasers

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Semiconductor lasers under large-signal direct modulation by a square waveform are found to exhibit a transition from a power spectrum characterized by a fundamental frequency and FM sidebands to a continuous spectrum with a catastrophically broadened linewidth of the order of several GHz. The interesting feature of the phenomenon is that the photon output remains periodic apart from noise-induced fluctuations, and the broadening of the power spectrum is attributed to the sensitivity of the phase of the optical field to a large difference in the relaxation oscillation frequencies in the ON and OFF states as well as the coupling between motions at the intrinsic resonance frequency of the system and the externally-imposed modulation frequency. It is shown that under deep modulation by a periodic injection current, the optical phase becomes aperiodic generating a wide range of new frequencies in the power spectrum. It is also demonstrated that by confining the excursions of the injection current to the region of almost-linear optical response, linewidth broadening may be avoided. Quantitative criteria for determining the boundary of the broadened-linewidth region are presented for several modulation frequencies.

INTRODUCTION

The distinct characteristic of semiconductor lasers is that their optical output may be modulated directly by varying the current injected into the laser diode. The application of semiconductor lasers as sources in optical communication systems has relied on this property of semiconductor lasers extensively and has led to ever more stringent requirements on their operational parameters, in particular, noise characteristics and spectral purity.[1] Spectrally pure semiconductor lasers are highly desirable in direct-detection systems since the dispersion of the laser pulse in an optical fiber is proportional to its spectral width and are critical in coherent-detection systems where a stable phase is necessary to maintain the coherence of the detected signal. On the other hand, efficient optical communication systems require a very high ON-OFF (extinction) ratio to avoid power penalties at the receiver. The latter requirement entails the need to modulate the laser diode with a large amplitude swing in the injected current and with the OFF state sufficiently close to the lasing threshold. GHz modulation frequencies are necessary to achieve gigabit data transmission rates.

In the light of these requirements on semiconductor lasers defined by the demands of modern communication systems, it is important to understand the aspects of semiconductor laser physics relevant to large-signal modulated operation. While the small-signal response of semiconductor lasers has been studied at length and is at present well understood,[2] the predictions of the small-signal analysis do not necessarily apply to modulation with the amplitude swing close to the lasing threshold. Under high-speed deep pulse modulation, the transient chirping effect may be significantly enhanced[3] in addition to the degradation of the optical response.[4] Considerable broadening in the time-averaged power spectrum of the deeply modulated laser diode has also been observed.[5] Although these effects are known experimentally to set limits on the maximum bit transmission rate and/or to impose tangible power penalties in lightwave communication systems, no detailed theoretical understanding of the modifications in the power spectrum under large-signal modulation exists, to the best of our knowledge. In this communication, we perform a numerical analysis necessary to develop such an understanding identifying the appropriate limits on the width of the optical emission from a single-mode semiconductor laser modulated by a square wave at GHz frequencies.

MODEL

The quantum-mechanical equations of motion for the interacting light-semiconductor system in the limit relevant to single-mode laser diode operation reduce to coupled nonlinear equations for the single-mode electric field and the charge density. In the absence of modulation, external injection, or feedback, this set of equations is autonomous, i.e., the time derivatives of the field and the charge density do not contain an explicit time dependence. Autonomous systems are distinguished by the fact that *asymptotic* limits of their time evolution can be readily analyzed. In the case of semiconductor lasers, small perturbations from equilibrium are found to decay exponentially with oscillations at the resonant frequency of the system. The equation for the electric field density can be better appreciated by separating it into the equations for its amplitude or the photon density and its phase. The resulting set of rate equations for a single-mode semiconductor laser becomes:[2]

$$\frac{dn}{dt}=\frac{J}{ed}-R_{\rm sp}(n)-\left(\frac{c}{n_g}\right)\Gamma g(n)(1-\epsilon S)S+F_n(t), \quad (1)$$

$$\frac{dS}{dt}=[\Gamma g(n)(1-\epsilon S)-\alpha_c]\left(\frac{c}{n_g}\right)S+\beta R_{\rm sp}(n)+F_S(t), \quad (2)$$

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$$\frac{d\varphi}{dt}=\frac{\alpha}{2}\left(\frac{c}{n_g}\right)[\Gamma g(n)(1-\epsilon S)-\alpha_c]+(\Omega-\nu)\left(\frac{\bar{n}}{n_g}\right)+F_\varphi(t), \tag{3}$$

where n is the carrier density, S is the photon density, φ is the optical phase, J is the injected current, d is the active region width (taken to be 100 nm), R_{sp} is the spontaneous emission rate, g is the optical gain, n_g is the group index, $\bar{n}$ is the modal index, α_c is the total cavity loss (taken to be 50 cm^{-1}), β is the fraction of spontaneous emission coupled into the lasing mode (taken to be 10^{-5}), α is the linewidth enhancement factor at the carrier threshold density (taken to be 5.0), Ω is the cavity mode ("passive") frequency, and ν is the ("active") frequency of the optical field. Phenomenological terms $(1-\epsilon S)$ have been introduced to account for gain saturation at finite photon densities, where ϵ is the gain compression coefficient which has been independently calculated[6] to equal 5×10^{-18} cm^3. The spontaneous emission rate and the optical gain are calculated for a bulk GaAs/AlGaAs active region by solving the Schrödinger equation in the effective mass approximation and diagonalizing the four-band Kohn–Luttinger Hamiltonian to obtain the conduction and valence band structures, respectively, and applying the Fermi golden rule to optical transitions.[7] The gain dependence on the carrier density near the transparency point can be well approximated by a straight line with the slope $dg/dn=1.18\times10^{-15}$ cm^2. It is assumed that the laser is forced to oscillate in a single mode, e.g., by an internal feedback scheme. The laser threshold current density has been estimated to be 958 A/cm^2. The shot noise term $F_n(t)$ is neglected, while the spontaneous emission noise terms are taken to have vanishing cross correlation and the following autocorrelations derivable from the Langevin treatment of quantum noise:

$$\langle F_S(t)F_S(t')\rangle=2\beta R_{sp}(\langle n\rangle)\langle S\rangle\delta(t-t'), \tag{4}$$

$$\langle F_\varphi(t)F_\varphi(t')\rangle=\frac{\beta R_{sp}(\langle n\rangle)}{2\langle S\rangle}\delta(t-t'), \tag{5}$$

where triangular brackets denote ensemble averages. The equation for the optical phase can be further simplified:

$$\frac{d\varphi}{dt}=\frac{\alpha}{2}\left(\frac{c}{n_g}\right)\Gamma[g(n)-g(\langle n\rangle)](1-\epsilon S)+F_\varphi(t). \tag{6}$$

If a square waveform is imposed in place of the current density term, the rate equations become nonautonomous and may acquire fairly complex nonlinear dynamical behavior. However, owing to the damping provided by the nonlinear gain and other terms, the observation of intensity instabilities of directly modulated semiconductor lasers is difficult.[8] Nevertheless, it is interesting to note from Eq. (6) that the optical phase is coupled to the photon density with a coupling strength proportional to the linewidth enhancement factor. Therefore, amplitude modulation in semiconductor lasers with a finite value of the enhancement factor results in phase modulation.[9] The effect of phase modulation on the dynamics of semiconductor lasers is most easily characterized in terms of the power spectrum of the optical field defined as the Fourier transform of the autocorrelation function:

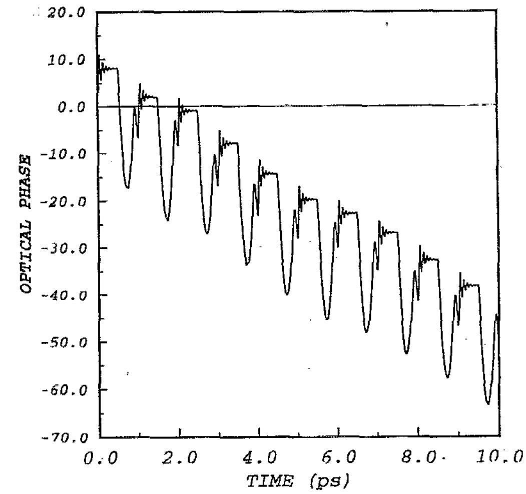

FIG. 1. The optical phase as a function of time for a modulation frequency of 1 GHz and an amplitude swing from 1045 A/cm^2 ($\approx1.09\ J_{th}$) to 2875 A/cm^2 ($\approx3.0\ J_{th}$).

$$\Gamma(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_0^T E^*(t+\tau)E(t)dt$$

$$=\lim_{T\to\infty}\frac{1}{T}\int_0^T e^{i\omega_0\tau}e^{i[\varphi(t+\tau)-\varphi(t)]}\times S(t+\tau)^{1/2}S(t)^{1/2}\,dt, \tag{7}$$

where ω_0 is the lasing frequency.

RESULTS

Equations (1), (2), and (6) have been solved by a fourth-order Runge–Kutta method in the time domain for a current density represented by a square wave. The noise terms have been treated as Gaussian random variables by a Monte Carlo numerical technique in Ref. 10. While for a wide range of modulation amplitudes and bias points, it has been found that the photon density and phase response are periodic with characteristic ringing indicative of relaxation oscillations, for a sufficiently large amplitude swing the optical phase becomes aperiodic acquiring a finite shift during each modulation period. This behavior, occurring for a large difference between the relaxation oscillation frequencies during the ON and OFF parts of the pulse, may be understood in terms of the equation for the derivative of the phase, in which the optical phase itself does not figure. In a steady state, this is equivalent to an arbitrary initial phase, while for a sufficiently deep modulation, it translates into a rotation of the field by a certain amount during each period, while the optical output of the laser remains periodic (apart from noise-induced fluctuations) with a high ratio between the photon densities in the ON and OFF states. The behavior of the phase for a modulation frequency of 1 GHz and an amplitude swing from 1045 to 2875 A/cm^2 is shown in Fig. 1.

An examination of the power spectrum during the transition from the periodic to shifting phase response demonstrates the introduction of new frequencies in the transition.

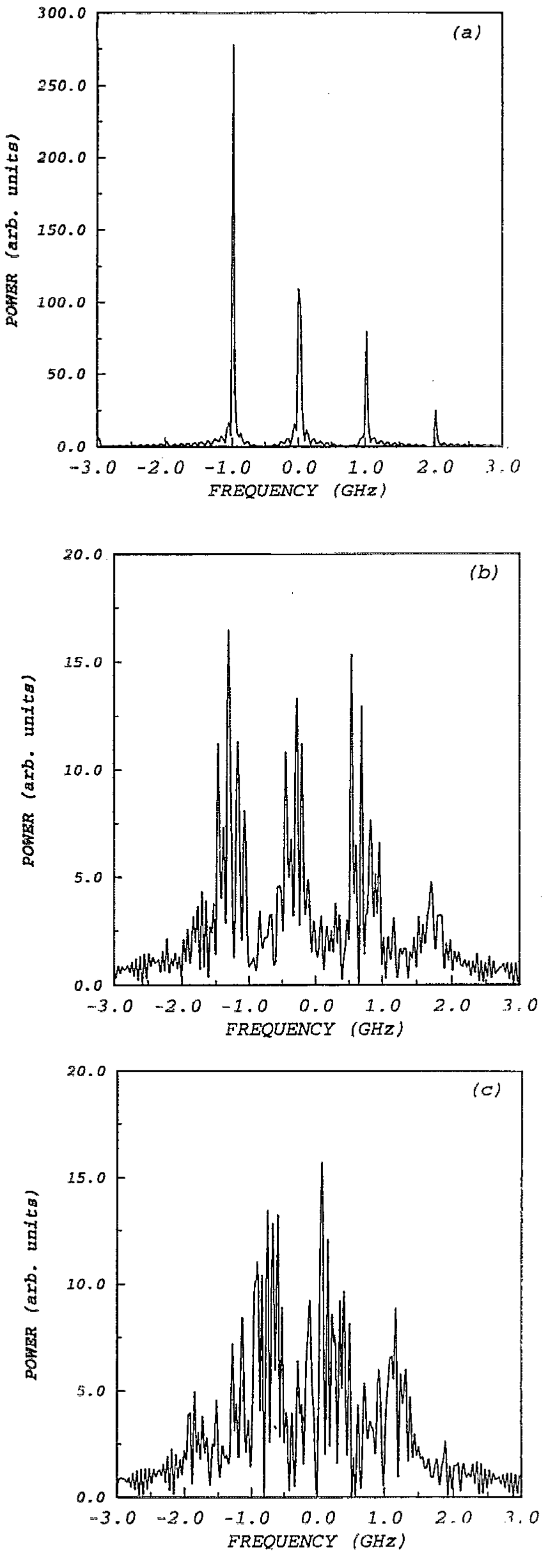

FIG. 2. The power spectrum of a laser diode modulated at 1 GHz with an amplitude swing (a) from 1306 A/cm^2 ($\approx 1.36J_{\mathrm{th}}$) to 2875 A/cm^2, (b) from 1077 A/cm^2 ($\approx 1.12J_{\mathrm{th}}$) to 2875 A/cm^2, (c) from 1045 A/cm^2 to 2875 A/cm^2.

For a small amplitude swing, the power spectrum consists of the fundamental peak and dynamically generated FM sidebands. As the optical phase becomes aperiodic, the frequency intervals between the fundamental and the sidebands start to

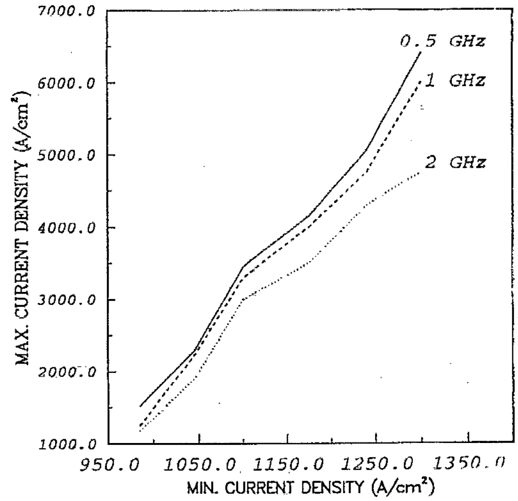

FIG. 3. Contour plots of the maximum current density for the ON state vs the current density for the OFF state allowable for the emission linewidth given by the width of the fundamental peak for several modulation frequencies.

become filled, finally resulting in a dramatically broadened spectrum in which neither the fundamental nor the FM sidebands are clearly distinguishable, as shown in Fig. 2. A finite cutoff frequency of several GHz can be observed, resulting in the GHz linewidth of the broadened power spectrum. Taking into account the role linewidth broadening is expected to play in optical communications, it is important to identify the boundaries of the linewidth-broadened region. It is desirable to characterize them in terms of the dc characteristic of the laser diode even though the transition to the broadened spectrum is governed by the dynamically varying quantities such as the ratio of the relaxation oscillation frequencies during the ON and OFF parts of the pulse which are in general different from the oscillation frequencies at the same photon output levels in the absence of modulation. The results of numerical calculations for the boundary of the linewidth-broadened region are shown in Fig. 3. The current density during the OFF part of the pulse is measured along the horizontal axis, while the current density during the ON part of the pulse is measured along the vertical axis. The desired boundary is represented in terms of a contour plot for several modulation frequencies close to 1 GHz. The region in the lower part of the plane corresponds to amplitude swings at corresponding frequencies for which the power spectrum is well characterized by the fundamental and the FM sidebands. The transition occurs when the line separating the upper and lower parts of the plane is crossed. Therefore, the figure supplies the maximum allowable amplitude swing for a given current density during the OFF part of the pulse and a given modulation frequency, for which the emission linewidth remains narrow. The conclusion to be drawn from the figure is that for a given minimum current density close to the lasing threshold, linewidth broadening may be avoided by confining the excursions of the injected current to a sufficiently small range. This range decreases as the modulation frequency grows due to the coupling between the internally and externally generated frequencies, and increases for the minimum current density farther away from the threshold. The importance of the positioning of the OFF state may be appreciated by noting that the relaxation oscillation frequency exhibits the fastest variation near the threshold as predicted by the small-signal analysis, where $f_{\text{rel.osc.}}$ is proportional to $S^{1/2}$. Note also that for a modulation frequency larger than 4 GHz, the bias photon density must be quite large for a reasonably faithful optical response to be obtained; therefore, linewidth broadening becomes unavoidable.

CONCLUSIONS

In conclusion, we have examined numerical solutions of the rate equations for the carrier density, photon density, and optical phase for a representative semiconductor laser deeply modulated by a square wave. The results indicate the presence of a transition to a catastrophically broadened linewidth region for a large amplitude swing with the OFF state close to the lasing threshold at GHz modulation frequencies. The boundaries of the region have been identified quantitatively.

ACKNOWLEDGMENT

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