THE REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 24, NUMBER 10 OCTOBER, 1953

A Low-Frequency Oscillator

R. M. Howe and R. J. Leite
Department of Aeronautical Engineering, University of Michigan, Ann Arbor, Michigan

(Received January 14, 1953)

A low-frequency simple-harmonic oscillator, employing a relatively new method of approach, is described. An electronic analog computer of the differential analyzer type is used to solve the differential equation of a mass-spring system. Three dc operational amplifiers perform the computation and a sinusoidal output is obtained as the solution of the differential equation. Frequency is variable from 0.01 to 100 radians per second in steps of 0.01 radian per second. Distortion is less than 0.1 percent. Maximum output is 90 volts peak into a load of 100 000 ohms. The oscillator is particularly useful in the design and testing of servomechanisms and electronic simulators.

I. INTRODUCTION

THE need for a low-frequency, simple-harmonic oscillator often arises in instrumentation, particularly in the design and testing of servomechanisms. The oscillator described here is actually a small, compact electronic analog computer which solves the differential equation of a mass-spring system. This method is obvious to anyone who has worked with analog computers of the differential-analyzer type. However, not all those interested in low-frequency oscillators may be familiar with this type of computer, so the authors have felt it worth while to point out this specific application. The computer described produces sinusoidal oscillations over a frequency range of 0.01 to 100 radians per sec (0.0016 to 16 cylces per sec) with a distortion of less then 0.1 percent. The maximum output is 90 v peak into a 100 000-ohm load.

II. THEORY OF OPERATION

The theory of electronic differential analyzers has been well described in the literature.[1–3] The basic component of this type of analog computer is the operational amplifier, which consists of a high-gain, dc amplifier with external input and feedback impedances. If the amplifier gain is very large compared with the ratio of feedback to input impedance, then it can perform the operation of voltage integration and sign inversion with high accuracy. For example, the three operational amplifiers shown in Fig. 1 are connected to solve the equation of simple-harmonic motion. Amplifiers A_2 and A_3 each have an input resistor R and a feedback capacitor C; the output voltage of each amplifier is equal to $-(1/RC)$ times the time integral of the input voltage. Amplifier A_1 has equal input and feedback resistors; its output voltage is the negative of the input voltage. Evidently the circuit shown obeys the differential equation

$$(RC)^2\frac{d^2e_0}{dt^2}+e_0=0,$$

where e_0 is the output voltage of amplifier A_3. The solution of this equation is simple-harmonic motion of angular frequency $1/RC$. When we wish to generate such a solution, initial conditions $de_0/dt=0$ and $e_0=V$ at $t=0$ are imposed by short-circuiting the feedback capacitor of amplifier A_2 and applying V volts across the feedback capacitor of amplifier A_3. The initial condition switch is the opened, and the circuit proceeds to generate

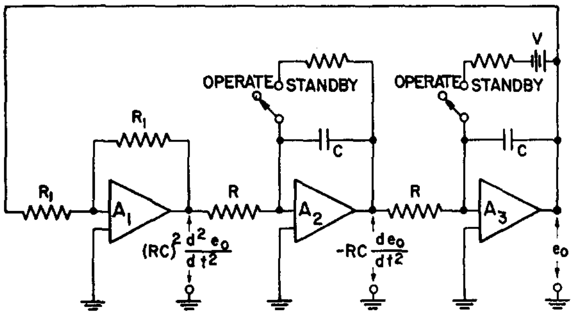

FIG. 1. Electronic differential analyzer circuit for generating simple harmonic motion.

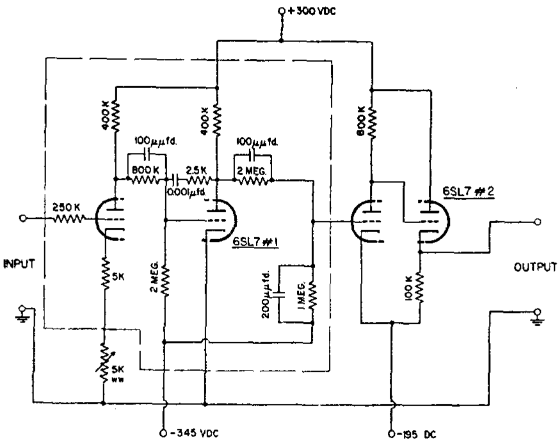

FIG. 2. High gain dc operational amplifier. Circuitry within dotted line is enclosed within Vector turret can No. C12-OK.

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[1] Raggazzini, Randall, and Russell, Proc. Inst.Radio Engrs. 35, 444 (1947).

[2] Hagelbarger, Howe, and Howe, "Investigation of the Utility of an Electronic Analog Computer in Engineering Problems," UMM 28, April, 1949, Engineering Research Institute, University of Michigan.

[3] G. A. Korn and T. M. Korn, *Electronic Analog Computers* (McGraw-Hill Book Company, Inc., New York, 1952).

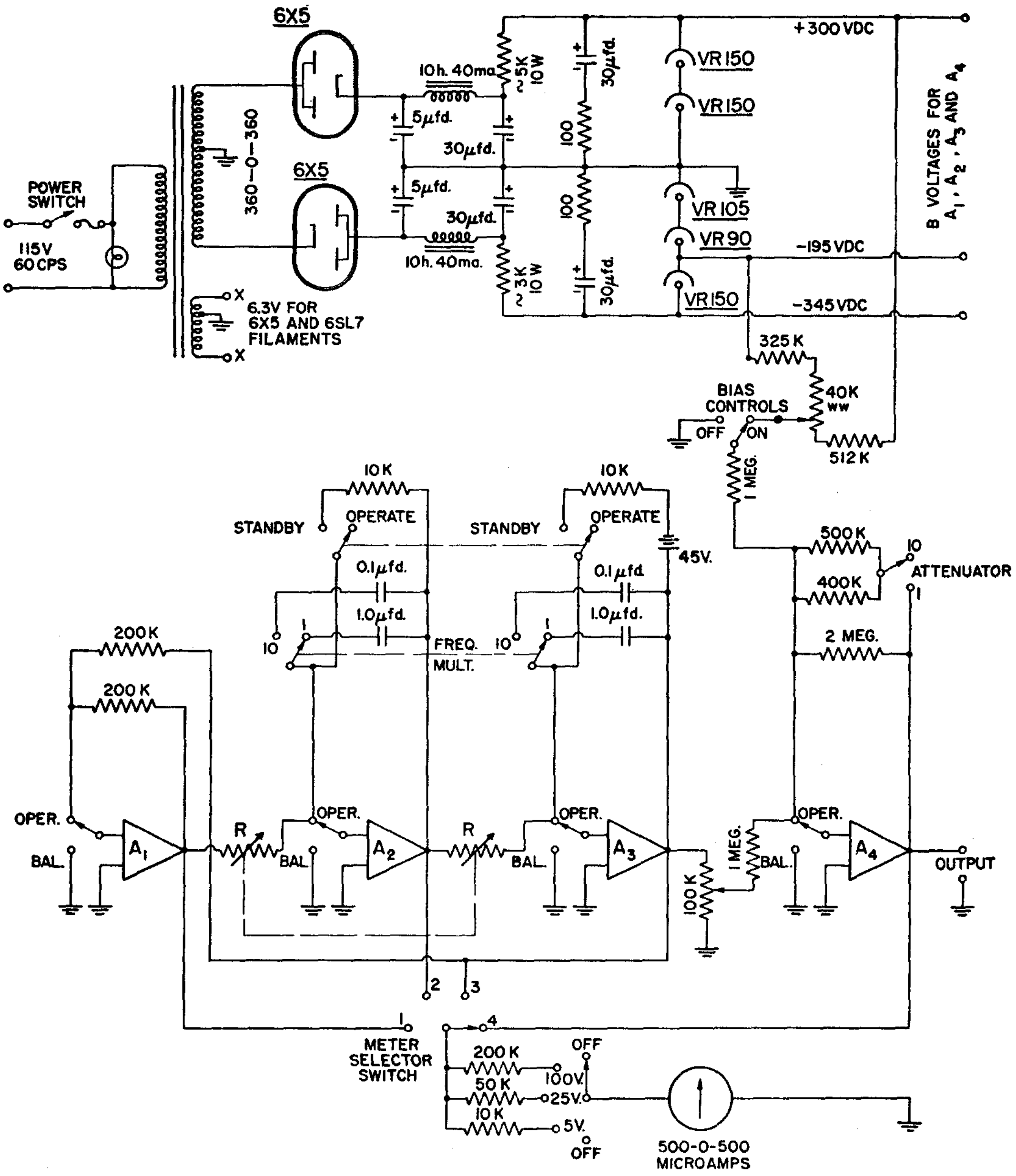

FIG. 3. Complete circuit diagram for low-frequency osillator.

an output voltage

$$e_0 = V\cos\frac{t}{RC}.$$

The accuracy with which the circuit in Fig. 1 generates a pure sine wave of constant amplitude depends on a number of factors, including the leakage resistance across the integrating capacitors, and the gain, band width, and zero-drift stability of the dc amplifiers. Integrating capacitors with polystyrene or polyethelene dielectric[4] have a leakage resistance which causes negligible effect. One can show[5,6] that the finite amplifier gain causes the sinusoidal oscillation to decay with a time constant equal to RC times the amplifier gain. On the other hand, the fall off in amplifier gain at high frequency causes the sinusoidal oscillation to build up exponentially.[6] Finally, with a *VR*-regulated power supply and the amplifier circuit shown in Fig. 2, zero-drift will seldom exceed 0.02 v over a period of several hours. Better regulated power supplies[5] or chopper-

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[4] For example, Western Electric 1μfd, 200 VCD, No. D168233; also Condensor Products 1μf, 200 VDC, Plasticon, Cat. No. LAC2C1.

[5] C. E. Howe, R. M. Howe, and L. L. Rauch, "Application of the Electronic Differential Analyzer to the Oscillation of Beams, Including Shear and Rotary Inertia," UMM 67, Appendix 5, January 1951, Engineering Research Institute, University of Michigan.

[6] A. B. Macnee, Proc. Inst. Radio Engrs. **40**, 303 (1952).

stabilized dc amplifiers[3,7] can improve this drift by several orders of magnitude.

III. DESCRIPTION OF OSCILLATOR

The dc amplifier curcuit used for the operational amplifiers is shown in Fig. 2. It has a voltage gain of about 8000 and a frequency response which falls off at −6 db/octave until the gain is well below unity. This is necessary to insure stable operation under all feedback conditions. With a one to one ratio of feedback to input impedances, the amplifier frequency response is flat to 40 kc. The first 6SL7 twin triode and all but two of the resistors and capacitors are contained in a Vector turret can (No. C12-OK). Thus, virtually the entire amplifier is a simple plug-in unit.

The power supply and computer circuitry external to the dc amplifiers proper are shown in Fig. 3. Note that an additional operational amplifier A_4 is used to provide an attenuation and bias control for the output. The two input resistors R for the integrating amplifiers A_2 and A_3 are switched in gang to provide different frequencies of oscillation.[8] The frequency can be increased by a factor of 10 by switching from 1.0 to 0.1 microfarad feedback capacitors on amplifiers A_2 and A_3. A 500–0–500 microammeter connected as a voltmeter monitors the sinusoidal output at amplifier A_4.

The dc balance of each operational amplifier is adjusted by placing the balance switch in the balance position. This provides maximum amplification by opening the feedback loop and prevents any signal from entering the amplifier by grounding the input. Then the balance potentiometer can be positioned such that the output is zero volts. For convenience in adjusting the zero-balance, a selector switch is provided to connect the meter to the output of any one of the four amplifiers. The actual unbalance of the operational amplifier is approximately the voltage indicated on the meter divided by the open-loop gain of the amplifier. If the amplifiers are not exactly balanced, the only effect would be the addition of a small dc component to the sinusoidal output of the oscillator. This dc component can be eliminated or given any desired value by means of the bias control.

A simple damping control could be added to the oscillator circuit to maintain the amplitude of oscillation constant at all frequencies. However, growth or decay of sinusoidal outputs is so slight with the circuit shown (amplitude increase of about 0.1 percent per cycle at 16 cps, amplitude decrease of about 0.1 percent per cycle at 0.16 cps) that the authors decided not to complicate the operation by the addition of a damping control.

Because of the effect of this exponential decrease or increase in the amplitude of oscillation, the output is not a single frequency but instead consists of a continuous spectrum of frequencies which falls off rapidly on either side of the frequency, f_0, given by $1/RC$. This continuous spectrum is attenuated by a factor of two when the frequency is about 0.02 percent from f_0. To generate a sinusoidal output voltage then, it is only necessary to set in the desired frequency with the decade switches and release the initial conditions with the start switch.

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[7] Williams, Amey, and McAdam, Elec. Eng. 68, 934 (1949).

[8] The frequency is given by $1/RC$, and any particular value of $1/R$ is obtained by the proper parallel combination of resistors. For example, 1 radian per second is obtained with $R=1$ megohm, $C=1$ microfarad, while 1.2 radians per second is obtained by adding 5 megohms in parallel with the previous 1 megohm, making $R=5/6$ as required.