Ultralinear Cathode Follower*

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The design of a modified cathode follower circuit which possesses an extremely linear response and a voltage gain of essentially unity is presented. Analysis of the circuit shows that, in principle, the harmonic distortion may be made arbitrarily small, and at the same time the voltage gain may be made arbitrarily close to unity. The modification is applied to a White cathode follower where the increase in linearity and gain is accompanied by a decrease in the output impedance. A practical modified White follower having a measured intermodulation distortion of $2 \times 10^{-5}\%$, a calculated voltage gain of $(1-10^{-6})$, and a calculated output impedance of 2×10^{-3} ohms is given.

I. INTRODUCTION

A CATHODE follower is sometimes used to transform the impedance of a signal containing components of two or more frequencies of about the same amplitude. Certain critical applications, such as ac cross-modulation Hall effect measurements, require that the amplitude of the sum and difference frequencies produced by inter-modulation distortion in the cathode follower input stage be less than a small fraction of the amplitude of the components of the input signal. The modified cathode follower circuit described here was developed to meet these extreme linearity requirements.

The modified cathode follower circuit consists of a number of tubes placed in series across the plate supply and interconnected by feedback loops. The analysis will show that, at least in principle, any desired degree of linearity can be obtained by using a sufficient number of tubes and their associated feedback loops. The analysis will also show that as the linearity of the circuit is increased, the voltage gain approaches unity.

This modification may be applied to most types of cathode followers. Since a particularly attractive form of the modified circuit results from the modification of the White cathode follower,[1] we shall treat this case in detail. The modification of other types of cathode followers to ultralinear operation will be clear from the analysis. The attractiveness of the modified White follower (MWF) lies mainly in the fact that as the linearity of the circuit is increased, the output impedance falls to even lower values than in the ordinary White follower, which itself exhibits a much lower output impedance than the simple cathode follower.

In the analysis we shall, for simplicity, consider only circuits using triodes, although the analysis for pentode operation is in essence quite similar. Also we shall consider only the low frequency operation of the MWF, i.e., operation at frequencies low enough so that the effects of the

interelectrode capacitances may be neglected.[2] Lastly, we shall always assume that the grid current in all tubes is zero.

II. DESCRIPTION OF THE CIRCUIT

The major source of harmonic distortion in a cathode follower circuit is the active element, the tube, especially if the associated passive circuit elements are essentially noninductive. The fact that tube characteristics are always slightly curved causes the tube to operate upon an input signal in a nonlinear fashion. Nevertheless, a cathode follower is a more linear device than the ordinary amplifier because of the negative feedback which forces the cathode voltage to "follow" the grid excursions.

If *both* the cathode and the plate voltages in a triode cathode follower could be constrained to follow closely the grid voltage both in amplitude and in phase, an even more linear operation is possible. The triode would then remain almost at its quiescent operating point under all signal conditions, and the curvature of the dynamic characteristic would have only a very small effect on the response. The ultralinear cathode follower presented here is an ordinary cathode follower modified so that both the plate and the cathode closely follow the grid. The circuit is given in a form which allows the "follow" to be as close as desired.

Examples of circuits in which both the cathode and the plate follow the grid are the electrometer circuit of Krakauer[3] and the high input impedance cathode follower of Macdonald.[4] The present circuit is a logical extention of Macdonald's follower with the emphasis, however, on attaining linear response. The recommendations made by Macdonald may also be applied to the present circuit to give a high input impedance.

Let us now consider the modification of the White follower to ultralinear operation. The circuit of the ordinary White follower is shown in Fig. 1. The input signal is inserted on the grid of the upper tube T_b, which corresponds to the usual cathode follower stage. The operation

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[1] E. L. C. White, U. S. Patent No. 2,358,428, September 19, 1944. See also: C. M. Hammack, MIT Radiation Lab. Rept. No. 469 (1943).

[2] For an analysis of the operation of the White follower at high frequencies see: M. Brown, Rev. Sci. Instr. **31**, 403 (1960).
[3] S. Krakauer, Rev. Sci. Instr. **24**, 496 (1953).
[4] J. R. Macdonald, Rev. Sci. Instr. **25**, 144 (1954).
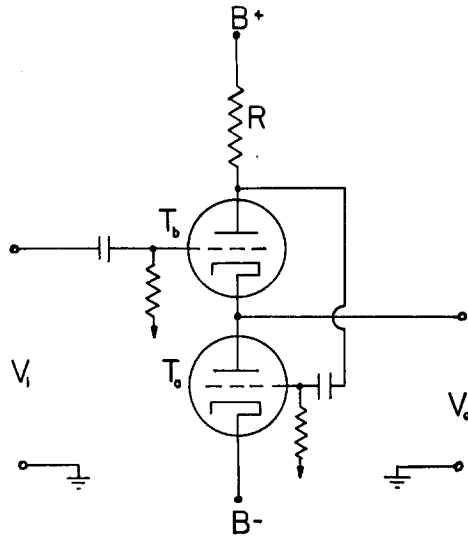
FIG. 1. Ordinary White follower.

of the lower tube T_a, which has its grid coupled to the lower end of the load resistance R, makes possible a low output impedance.

The circuit of the MWF of order n, where $n = 1, 2, 3, \ldots$, is shown in Fig. 2. The $(2n+2)$ triodes $T_1, \ldots, T_{2n+2}$ and the load resistance R are arranged in series across the plate supply. The input signal V_i is applied to the grid of the input tube T_2, and the output signal V_o is taken from the cathode of this same tube. The lowest tube T_1 has the same function as in the ordinary White follower. The grids of the n tubes immediately above T_2 are coupled together and tied to the cathode of T_2, and likewise the grids of the n tubes above R are coupled together and tied to the lower end of R. The additional $2n$ tubes and the associated feedback loops may be regarded as elements inserted into the ordinary White follower circuit in order to linearize the operation of the input tube.

III. ANALYSIS OF THE CIRCUIT

Let us first consider the ordinary two-tube White follower circuit of Fig. 1. The cathode-to-grid voltage gain of this circuit is

$$\frac{V_0}{V_i} = \frac{1}{1 + \mu_b^{-1}[1 + (R + r_{pb})(\mu_a R + r_{pa})^{-1}]}, \quad (1)$$

and the output impedance is

$$Z_0 = \frac{r_{pa}(R + r_{pb})}{(1 + \mu_b)(\mu_a R + r_{pa}) + (R + r_{pb})}, \quad (2)$$

where r_{pk} and μ_k are the plate resistance and the amplification factor of the kth tube. These relations are derived under the assumption that the grid coupling networks have essentially a zero phase shift at all frequencies of interest.

This assumption will be used throughout the analysis and is essential to the results obtained.

The cathode-to-grid voltage gain and the output impedance of the MWF of order n may be expressed in the same form as the expressions for the ordinary White follower with the substitutions

$$\mu_a = \mu_1, \quad r_{pa} = r_{p1}, \quad (3)$$
$$\mu_b = (\mu_b^*)_n, \quad r_{pb} = (r_{pb}^*)_n, \quad R = (R^*)_n,$$

where the starred quantities are given by[5]

$$(\mu_b^*)_n = \mu_2 \left[\prod_{j=3}^{n+2} (1 + \mu_j) \right], \quad (4)$$

$$(r_{pb}^*)_n = \sum_{i=3}^{n+2} \left\{ r_{p(i-1)} \left[\prod_{j=i}^{n+2} (1 + \mu_j) \right] \right\} + r_{p(n+2)}, \quad (5)$$

$$(R^*)_n = R \left[\prod_{j=n+3}^{2n+2} (1 + \mu_j) \right]$$
$$+ \sum_{i=n+4}^{2n+2} \left\{ r_{p(i-1)} \left[\prod_{j=i}^{2n+2} (1 + \mu_j) \right] \right\} + r_{p(2n+2)}. \quad (6)$$

The MWF may thus be pictured as having the same configuration as the ordinary White follower, but where the load resistance is $(R^*)_n$ and the amplification factor and the plate resistance of the effective upper tube are $(\mu_b^*)_n$ and $(r_{pb}^*)_n$. Since the starred parameters are much larger than their unstarred counterparts, the cathode-to-grid voltage gain of the modified circuit is closer to unity and the output impedance is lower than in the ordinary circuit.

The extention of the ultralinear modification to any cathode follower circuit is now clear. One merely inserts n triode stages above the input tube and n triode stages above the load resistance. The voltage gain and the output impedance will have the same form as in the unmodified circuit if the substitution of the starred quantities Eqs. (4)–(6) in place of their unstarred counterparts is made in these relations.[6]

Thus far, we have considered only the cathode-to-grid gain of the MWF circuit. In order to demonstrate that the circuit is both a cathode and a plate follower, we must also consider the plate-to-grid voltage gain of T_2. Unfortunately, this plate-to-grid gain cannot be related in a simple way to a corresponding expression for the ordinary White follower, for there is no point in the ordinary circuit which corresponds to the plate of T_2 in the modified circuit. However, from a straightforward analysis of the circuit of

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[5] We use the definition here that

$$\prod_{j=m}^{m} x_j = x_m.$$

[6] This same type of modification may also be applied to an amplifier circuit to increase its gain.

Fig. 2, we find that

$$\left(\frac{V_0'}{V_i}\right)_n = \cfrac{1}{1+\cfrac{(1+\mu_2)S-r_{p2}(\mu_b{}^*)_n}{(\mu_b{}^*)_n[\mu_1(R^*)_n+r_{p1}+r_{p2}]-\mu_2 S}}, \quad (7)$$

where

$$S=(\mu_1+1)(R^*)_n+(r_{pb}{}^*)_n+r_{p1}.$$

From Eq. (7) we see that the plate-to-grid gain is close to unity, so that the plate of the input tube in the MWF does in fact "follow" the grid.

In order to visualize the capabilities of the MWF, it will be convenient to employ some approximations which may be easily attained in practice and which will simplify the foregoing results. However, the use of the MWF is by no means restricted to situations where these approximations are valid.

Let us assume that all $2n+2$ tubes are identical and are operated so that $\mu_i=\mu_k=\mu$, $r_{pi}=r_{pk}=r_p$, for $i,k=1,2,\ldots,$ $2n+2$. Then, if we further assume that $\mu\gg1$, Eqs. (4)–(6) reduce to

$$(\mu_b{}^*)_n\approx\mu(\mu^n), \quad (r_{pb}{}^*)_n\approx r_p(\mu^n),$$
$$(R^*)_n\approx R(\mu^n)+r_p(\mu^{n-1}). \quad (8)$$

Inserting Eqs. (8) into the expressions for the cathode-to-grid gain, the output impedance, and the plate-to-grid gain of the MWF, we have

$$\left(\frac{V_0}{V_i}\right)_n \approx \frac{1}{1+\mu^{-(n+1)}[1+(R+r_p)(\mu R+r_p)^{-1}]}, \quad (9)$$

$$(Z_0)_n \approx \frac{r_p(R+r_p)}{\mu^{(n+1)}(\mu R+r_p)}, \quad (10)$$

$$\left(\frac{V_0'}{V_i}\right)_n \approx \frac{1}{1+(\mu^2 R+r_p)[\mu^{(n+1)}(\mu R+r_p)]^{-1}}. \quad (11)$$

Now, in the case where $R\approx r_p$, Eqs. (9)–(11) take on the simple form

$$(V_0/V_i)_n \approx 1-\mu^{-(n+1)}, \quad (12)$$

$$(Z_0)_n \approx 2r_p\mu^{-(n+2)}, \quad (13)$$

$$(V_0'/V_i)_n \approx 1-\mu^{-n}. \quad (14)$$

In this approximation, both the plate and the cathode of the input tube in the MWF of order n follow the grid to within at least one part in μ^n, and the output impedance of the circuit is about μ^{-n} times that of the ordinary White follower and about $\mu^{-(n+1)}$ times that of the simple cathode follower.

IV. DISCUSSION

The expressions for the cathode-to-grid and plate-to-grid voltage gains of the MWF of order n show that, by
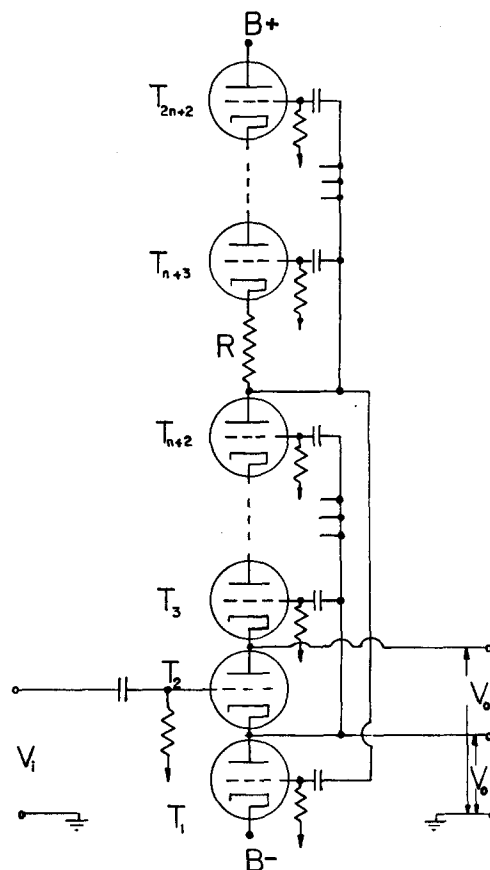

FIG. 2. Modified White follower of order n.

using a suitable value of n, the cathode and plate of the input tube may be made to follow its grid as closely as desired. The closeness of the follow of the plate will always be somewhat less than that of the cathode. Thus, the input tube may be made to operate on an arbitrarily small portion of its dynamic characteristic by making a proper choice of n. This means that the harmonic and intermodulation distortion due to the operation of the input tube along a nonlinear characteristic may be made, in principle, vanishingly small. Similar conclusions hold for the application of the ultralinear modification to other cathode follower circuits.

These conclusions hold for the operation of only the input tube, since the other tubes in the circuit are not all cathode-and-plate followers. However, the tubes other than the input tube do not operate directly on the input signal. Their function is to make the input tube a nearly perfect cathode-and-plate follower, and their ability to perform this task depends on their amplification factors, which for triodes are almost constant over a wide range of tube operation. Thus, the nonlinearity in the operation of these tubes will, in practice, not affect the linearity of the operation of the input tube.

The expressions for the output impedance of the MWF show that, in principle, the circuit can have an arbitrarily

low value of output impedance by making a suitable choice of n. It should be noted that in any practical circuit there will be some negative grid current in the input tube, and as a result there will be a lower limit to the attainable output impedance. This lower limit will be a function of the source impedance[7] and will usually be higher than the values indicated by Eqs. (10) or (13). Thus, just as in more conventional circuits, the output impedances calculated under the assumption of zero grid current must not be taken too seriously. Nevertheless, the MWF will in practice exhibit very low output impedances.

To make more concrete the capabilities of the MWF, let us consider the case where n is 2; i.e., a six-tube circuit. The analysis indicates that it is advantageous to use high μ triodes, so let us consider each of the six triodes to be one-half a 12AX7. Let us set the operating conditions at 100 v/tube and 300-μa dc plate current, in which case $\mu \approx 100$ and $r_p \approx 10^5$ ohms. For simplicity let R also be 10^5 ohms. Now, it is extremely difficult to calculate the expected distortion in this case, for the input tube operates over such a small portion of the dynamic characteristic that the curvature of this portion is immeasurable. However, from Eqs. (12)–(14) we find that

$$(V_0/V_i)_{n=2} = 1 - 10^{-6}, \quad (Z_0)_{n=2} = 2 \times 10^{-3} \text{ ohms},$$
$$(V_0'/V_i)_{n=2} = 1 - 10^{-4}.$$

These values represent a vast improvement over the calculated values for the ordinary (i.e., with $n=0$) White follower and represent the type of results to be expected from any modified cathode follower circuit.

This six-tube MWF was constructed and has now been operating successfully for several months as the input stage of the Hall voltage detector in an ac cross-modulation Hall effect apparatus. The intermodulation distortion of the circuit was measured by inserting 60 and 85 cps at 1:1 on the input grid and measuring the amount of 25-cps difference frequency produced. (These are the frequencies used in our apparatus.) In order to eliminate 60-cps contamination, battery plate and filament supplies were used,

and the apparatus was carefully shielded. For an input level of 2 v for each of the primary frequencies, less than 4×10^{-7} v of the difference frequency was detected. This means that the intermodulation distortion of the six-tube circuit is less than $2 \times 10^{-5}\%$. This compares with a measured value of $5 \times 10^{-2}\%$ for the unmodified White follower.

The measurements of the very high gains and of the very low output impedances were rather difficult. Using a 2-v input, we found that within the limits of our apparatus (0.1%) the gains were unity. These measurements confirm in a rough way the results of the analysis. More precise measurements would be needed for true confirmation, but these were not attempted since the linearity of the circuit was our prime concern. Also, with a source impedance of 10^3 ohms we measured an output impedance at 25 cps of the order of 0.5 ohm. The discrepancy between this value and the calculated value of 2×10^{-3} ohms is appreciable and is probably due to grid current in the input tube.

Aside from the low intermodulation distortion, the gain of nearly unity, and the low output impedance, the experimental circuit exhibited the virtues of very low noise, low dc plate current, high input impedance, and excellent decoupling[8] from the plate supply. These advantages were gained, however, at the price of a threefold increase in the complexity of the circuit. Another slight drawback is that the circuit has a long recovery time when seriously overdriven.

The analysis and experimental results as given indicate that the proposed modification of cathode follower circuits does indeed give unusually low values of nonlinear distortion. In applications where excellent linearity is required, the advantages of the modified circuit may well outweigh the additional complexity and cost which it involves.

V. ACKNOWLEDGMENTS

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[7] See reference 4 for a discussion of this point.

[8] See discussion in R. G. Wylie, J. Sci. Instr. **31**, 382 (1954).