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TECHNICAL REPORT NO. 2

L. GOLDBERG

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INTRODUCTION

The first Technical Report[1] described a redetermination of the solar curve of growth based on all lines of Fe, Ti, and V for which measured f-values were available. An attempt to fit the derived curve with theoretical curves calculated on the basis of the Milne-Eddington theory led to a kinetic temperature of 19,000°, ±4000°K for the solar reversing layer. Alternatively, the observations could be represented by a kinetic temperature of 5700°K with the addition of 2.0 km/sec turbulent motion.

On the assumption that faint Fraunhofer lines are broadened solely by temperature or by the turbulence of overlying layers, we investigated the question of the relative importance of turbulence versus kinetic temperature as a source of line broadening. By comparing line widths for atoms of various atomic weights, we concluded, on the foregoing assumption, that the width of the lines is caused primarily by the high kinetic temperature in the reversing layer and that turbulence does not play an important role. The widths correspond to a temperature of 15,000°K.

In this paper, we remove the restriction that weak lines are broadened solely by temperature and/or turbulence, and examine the possible effects

1. Technical Report No. 1, ONR Project M720-5, Ann Arbor, Oct. 1947.

of collisional damping on the line broadening. The damping effects are shown to be of great importance for many lines, particularly for those of high excitation potential. It is clear that if the damping is sufficiently large, it must be taken into account in the determination of kinetic temperatures from the measured half-widths, even of relatively weak lines. In addition, kinetic temperatures derived from the curve of growth depend on the value of the damping constant employed, since, for large values of the damping constant, the saturation portion of the curve of growth, above the Doppler section, is affected both by a change in position and by a decrease in its length.

STUDIES OF Fe I LINES

An unpublished investigation by Goldberg provides a clue to the existence of large damping effects in the solar atmosphere. K. O. Wright[2] has derived a solar curve of growth, employing solar and laboratory intensities of low level Fe I lines (E.P. 0-2 volts). This curve differs very little from that given by the writers in Technical Report No. 1. For each of a number of excitation potentials, ranging from 0 to about 5 volts, Goldberg selected the three strongest multiplets of Fe I and used Wright's curve to determine relative values of log Nf. The values of log Nf were then reduced to infinite temperature by correction with a Boltzmann factor for T = 4700°K. The reduced values of log Nf indicated that, on the average, the number of absorbing atoms was 20 to 30 times greater for transitions from

2. Ap. J. 99, 249, 1944.

the odd terms of Fe I (E.P. 3-5 volts) than for lines arising from the low, even terms (E.P. 0-2 volts).

At first, this result was attributed either to deviations from local thermodynamic equilibrium or to a systematic increase of <u>f</u> with excitation potential. The latter possibility seemed remote, since it would probably require a serious violation of the f-sum rule. Therefore, an attempt was made to investigate possible departures from thermodynamic equilibrium through a determination of the excitation temperature from the high-excitation lines of Fe I. Because of the absence of accurate laboratory gf-values for iron lines with lower excitation potentials greater than 1.6 volts, the calculation was based on theoretical line strengths. The latter can be determined on the basis of L-S coupling. A considerably more accurate determination of temperature might be expected, however, from application of the J-file sum rule, which may be employed when solar intensities are available for all important lines arising from a given level.

The J-file Sum Rule

According to the J-file sum rule, the sum of the strengths of all the lines of one or more transition arrays which originate in (or terminate on) a given level is proportional to 2J + 1. The sum is independent of configuration interaction and type of coupling, provided that the intersystem combinations and transitions to levels of both overlapping configurations are included in the sums. The mechanics of the application of the J-file sum rule to the determination of excitation temperature have been given in Detail by Menzel, Baker, and Goldberg[3] in connection with Ti I.

3. Ap. J. <u>87</u>, 81, 1938.

Values of X_o, the optical depth at the center of the observed line, and proportional to the strength of the line, are read from a curve of growth for all lines originating from a given lower level of a low configuration. The quantity, ΣX_o, for each level is then taken as proportional to 2J + 1 and to the Boltzmann factor. In the absence of configuration interaction, a single upper configuration may be employed. When two upper configurations overlap, both should be included. For Fe I, the low configuration chosen was $3d^6 4s4p$; the upper configurations selected were $3d^6 4s4d$ and $3d^6 4s5s$. A total of 816 lines was included in the determination.

The strengths of blended lines in permitted multiplets were estimated from the theoretical strengths and from the observed strengths, X_o, for the unblended lines. For blended lines of multiplets that do not follow L-S coupling, a strength equal to the mean of the other lines of the multiplet was taken. It is permissible to neglect them completely, since their contribution to the ΣX_o is very small. Values of log X_o were taken from the standard curve of growth.[4]

Table I gives a summary of the results. The columns list the term, the J value, the number of lines entering into the sum for each level, log ΣX_o, log (2J + 1), the ratio log ΣX_o/2J + 1, the excitation potential E_p of each lower level, and the approximate mean wave length of the multiplet. In Figure 1, log ΣX_o/2J + 1 is plotted against E_p. In this figure, the portion of the curved line in the region of E_p = 3 volts corresponds to an

4. The curve of growth obtained from the low excitation lines of Ti, V, and Fe, given in the first Technical Report, corresponds closely to that obtained by other investigators. This curve will be designated as the standard solar curve of growth" in our discussion.

TABLE I

Term	J	No. of Lines	Log ΣX_o	Log(2J+1)	Log $\frac{\Sigma X_o}{2J+1}$	E_p	λ Approx.
z^7F^o	6	10	6.10	1.11	4.99	2.80	3600
	5	19	6.06	1.04	5.02	2.82	
	4	23	5.85	0.95	4.90	2.84	
	3	25	5.63	0.85	4.78	2.85	
	2	24	5.50	0.70	4.80	2.86	
	1	18	5.40	0.48	4.92	2.87	
	0	7	5.18	0.00	5.18	2.87	
z^7P^o	4	26	5.69	0.95	4.74	2.93	3700
	3	31	5.61	0.85	4.76	2.99	
	2	26	5.19	0.70	4.49	3.03	
z^5D^o	4	29	5.27	0.95	4.32	3.20	4000
	3	36	4.97	0.85	4.12	3.23	
	2	35	4.89	0.70	4.19	3.25	
	1	26	4.78	0.48	4.30	3.27	
	0	9	4.00	0.00	4.00	3.28	
z^5F^o	5	23	5.31	1.04	4.27	3.32	5000
	4	28	5.02	0.95	4.07	3.35	
	3	32	5.04	0.85	4.19	3.38	
	2	33	4.78	0.70	4.08	3.40	
	1	18	4.36	0.48	3.88	3.42	
z^5P^o	3	37	4.62	0.85	3.77	3.59	5000
	2	35	4.45	0.70	3.75	3.64	
	1	19	4.03	0.48	3.55	3.67	
y^5P^o	3	31	3.60	0.85	2.75	4.54	6000
	2	27	3.34	0.70	2.64	4.59	
	1	13	3.01	0.48	2.53	4.62	
x^5D^o	4	20	3.55	0.95	2.60	4.89	8000
	3	21	3.30	0.85	2.45	4.93	
	2	24	3.26	0.70	2.56	4.97	
	1	17	2.89	0.48	2.41	4.99	
	0	5	2.53	0.00	2.53	5.00	
x^5F^o	5	17	3.31	1.04	2.27	4.97	8000
	4	24	3.26	0.95	2.31	5.01	
	3	20	3.10	0.85	2.25	5.04	
	2	16	2.97	0.70	2.27	5.06	
	1	12	2.68	0.48	2.20	5.08	

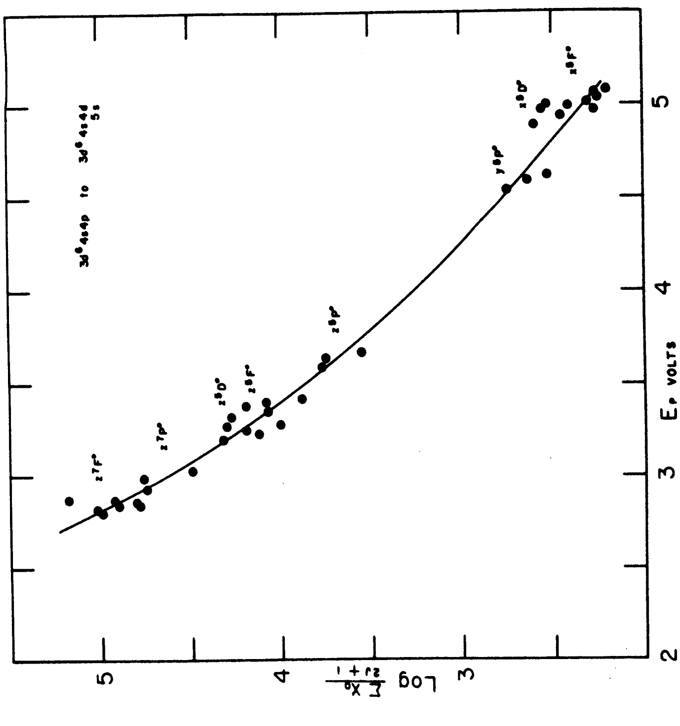


FIG. 1

excitation temperature of 3400°K; the lower part of the curve, near $E_p = 5$ volts, corresponds to 6000°K. In Technical Report No. 1, a similar study for lines with excitation potentials between 0 and 2.0 volts gave excitation temperatures in the neighborhood of 4700°K. Although a difference in excitation temperature between lines of low and high excitation potential might occur as the result of departures from thermodynamic equilibrium, it seems unlikely that a minimum of 3400°K at 3.0 volts should occur between the values of 4700°K at 0 to 1.0 volts and perhaps 6000°K at 5 volts. A possible interpretation of the foregoing result is that the Fe I lines of intermediate and high excitation potentials do not fit the standard curve of growth. An attempt was made, therefore, to construct separate curves of growth for Fe I lines of different excitation potentials.

Curves of Growth for High Excitation Potential Iron Lines

For permitted multiplets in L-S coupling, a curve of growth can be obtained for each multiplet. For high excitation Fe lines, the validity of the theoretical strengths was tested by using a selected transition array $3d^6 4s4p-3d^6 4s5s$ to obtain the solar excitation temperature. Since there was excellent agreement between the result obtained from a single transition array and the result obtained from the more complete J-file sum rule, we concluded that each multiplet is in good L-S coupling.

Equivalent widths divided by λ were obtained by measurement of line areas in the Utrecht Photometric Atlas of the Solar Spectrum. Log $\frac{W}{\lambda} 10^6$ was plotted against the log of the theoretical line strength taken from the tables of H. E. White[5] (see Figure 2). For each multiplet, the fit to the

5. Introduction to Atomic Spectra, McGraw-Hill, 1934.

standard curve has been made for the weakest lines of the set falling near or on the Doppler portion of the curve. The remaining points fall systematically higher than the standard curve, which is determined in this region from vanadium and titanium. The deviations are in the direction of larger damping and indicate damping factors of the order of 50 to 400 times the classical values.

Table II gives values of $a = \frac{\Delta\lambda_N}{\Delta\lambda_D}$, the ratio of damping width to Doppler width. For each multiplet these values have been interpolated from a series of curves of growth drawn for several different values of a. Multiplet $z^5F^o - e^5D$ is the subject of a careful investigation by Ten Bruggencate and Houtgast.[6] They find $\gamma = 0.83 \times 10^{10}$ sec^{-1} for observations at the center of the sun's disk. This corresponds to a = 0.23 (T = 5740°K) and is in good agreement with the value given in Table II. The results given here are in disagreement with those reported for vanadium by R. B. King and K. O. Wright,[7] who found that the high excitation potential curves fall below those of low excitation potential.

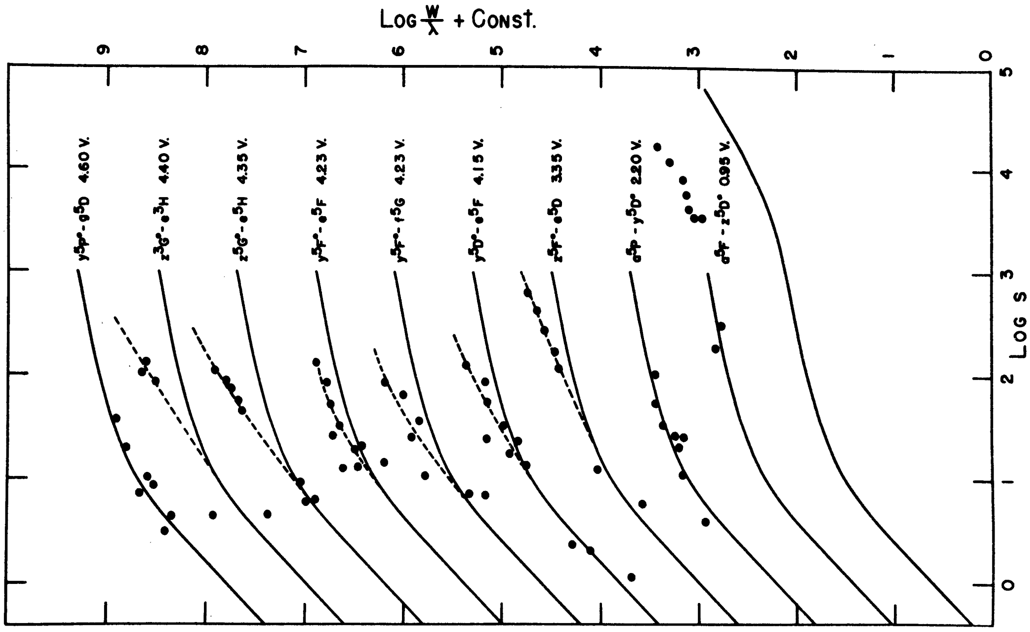
We should like to stress at this time the need for laboratory determinations both of gf-values and of damping factors for high level Fe lines.

The effect of the adoption of larger damping factors on the determination of excitation temperature may now be investigated. For this purpose, the transition array $3d^64s4p-3d^64a5s$, covering the range 2.5 to 5.0 volts, was employed. Twelve permitted multiplets in this array were used to

6. Zeit. f. Astrop. 20, 149, 1940.
7. Ap. J. 106, 224, 1947.

TABLE II

Multiplet	Mean E_p	a
$a^5F-z^5D°$	0.95	0.01
$a^5P-y^5D°$	2.20	0.01
$z^5F°-e^5D$	3.35	0.20
$y^5D°-e^5F$	4.14	0.40
$y^5F°-f^5G$	4.23	0.70
$y^5F°-e^5F$	4.23	0.50
$z^5G°-e^5H$	4.35	1.50
$z^3G°-e^3H$	4.40	1.50
$y^5P°-g^5D$	4.60	--



Log W/λ + Const.
Log S
y5P° - e5D 4.60 V.
z3G° - e3H 4.40 V.
z5G° - e5H 4.35 V.
y5F° - e5F 4.23 V.
y5F° - f5G 4.23 V.
y5D° - e5F 4.15 V.
z5F° - e5D 3.35 V.
a5P - y5D° 2.20 V.
a5F - z5D° 0.95 V.

FIG. 2

determine the excitation temperature. Table III and Figure 3 give the results. In the table the columns give the multiplet designation, the theoretical strengths of the multiplet, the mean wave length of the lines, $\log X_o'$, E_p, the number of lines used in the determination, and the value of Y, to be defined. The ordinate Y is defined by the equations

$$Y = \log X_o - \log X_o' = L - \frac{5040}{T} E_p, \qquad (1)$$

where

$$\log X_o = L + \log S - \log \frac{\Sigma s}{s} - \frac{5040}{T} E_p \qquad (2)$$

and

$$\log X_o' = \log S - \log \frac{\Sigma s}{s}, \qquad (3)$$

where $\log X_o'$ is the observed strength obtained from the observed value of $\log \frac{W}{\lambda}$ and the curve of growth. Log X_o' may be obtained from tables by Goldberg[8] and by White.[9] For the present purpose, L may be regarded as an arbitrary constant, so that the shift in ordinate of the two curves in Figure 3 has no significance. The upper set of points (open circles) was obtained from the standard curve of growth. This plot gives $T_{Ep} = 3400°$ for E_p = 3.0 volts, and $T_{Ep} = 5100°K$ for E_p = 5.0 volts, which is in excellent agreement with the determination of temperature by the J-file sum rule. (The J-file sum rule gave $T_{Ep} = 3400°K$ for E_p = 3.0 volts and $T_{Ep} = 6000°K$, for lines whose excitation potential is 5.0 volts.) We feel confident, therefore, that the use of this transition array will give results in good accord with those obtained by using the exact analysis provided by the sum rule.

The lower set of points (filled circles, Figure 3) was obtained from the semi-empirical curves of growth based on the observed partial curves

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8. Ap. J. 82, 1, 1935.
9. Loc. cit.

TABLE III

Designation	S	λ	Log X_0'	E_p	No. of Lines	y Open Circles	y Filled Circles
$z^7D°-e^7D$	35	4229	0.39	2.44	12	4.93	3.84
$z^7F°-e^7D$	49	4926	0.29	2.85	15	4.43	3.81
$z^7P°-e^7D$	21	5193	0.31	2.98	9	4.36	3.24
$z^5D°-e^5D$	25	5275	0.20	3.24	12	3.72	3.19
$z^5F°-e^5D$	35	5639	0.11	3.38	12	3.74	3.12
$z^5P°-e^5D$	15	6309	0.05	3.63	9	3.48	2.91
$z^3F°-e^3D$	21	5037	0.43	3.93	5	2.89	2.36
$z^3D°-e^3D$	15	4988	0.20	3.91	7	2.88	2.45
$z^3P°-e^3D$	9	5672	0.14	4.22	6	2.80	2.38
$y^5F°-g^5D$	15	6759	0.13	4.59	7	2.41	2.15
$x^5D°-g^5D$	25	8487	0.19	4.96	10	1.76	1.73
$x^5F°-g^5D$	35	9051	0.39	5.06	6	1.96	1.73

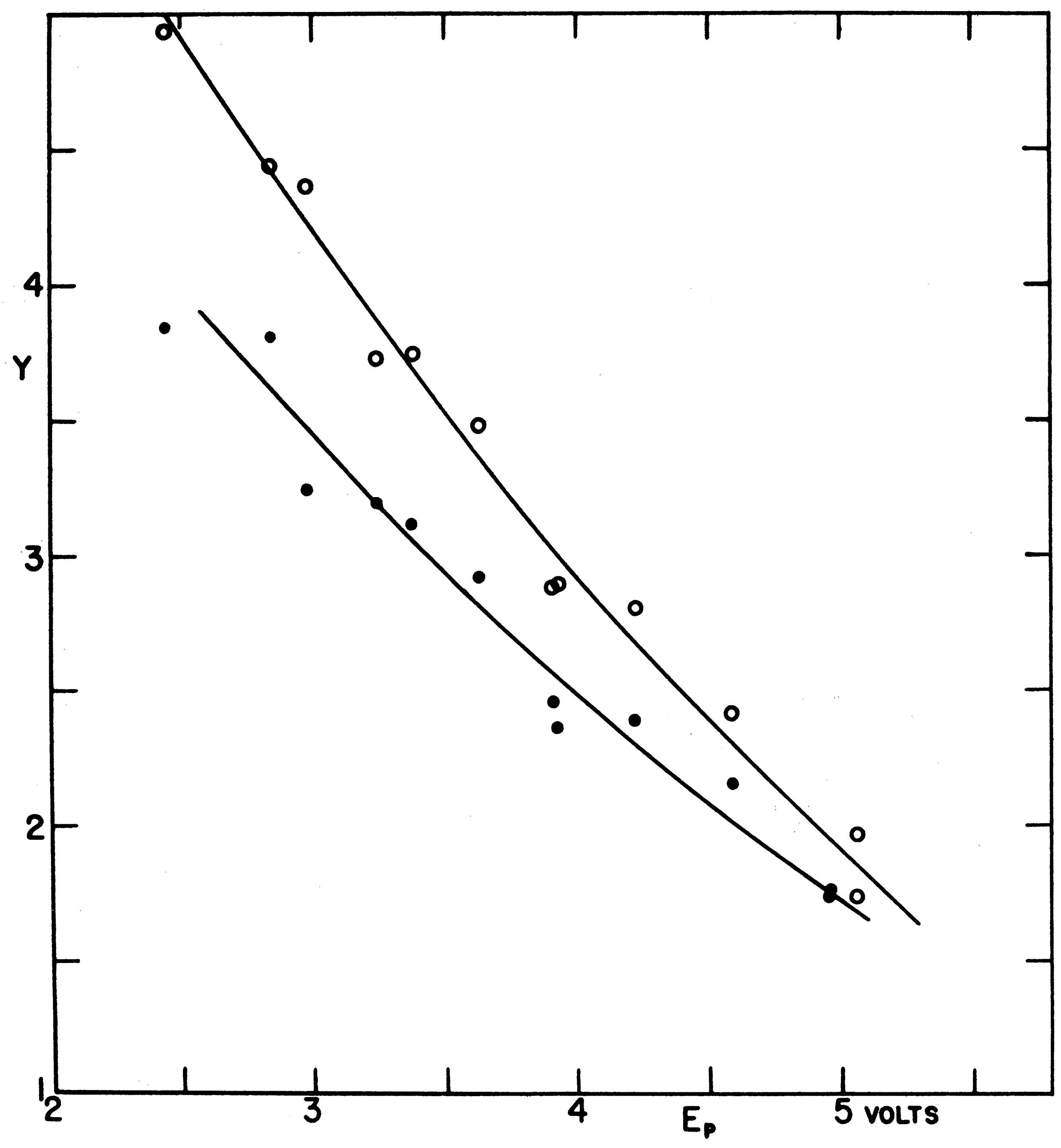

FIG. 3

of growth of Figure 2 for the high-level iron lines. The damping constants, from $\underline{a}$ = 0.1 to 1.0, were made a linear function of excitation potential in the range 2.5 to 5.0 volts. The excitation potential of each line then determined the curve of growth from which log X_o was to be read. As before, Y versus the excitation potential was plotted. The curved line through the points gave T_{Ep} = 4700°K at 3.0 volts and T_{Ep} = 7100°K for lines where excitation potential is 5.0 volts.

Pressure Displacements of Spectral Lines

As pointed out by Unsold,[10] if the damping constant is sufficiently large, there is an interesting consequence: investigations of the collision process by quantum mechanics show that a line is broadened, becomes asymmetrical, and is shifted in position.

By the use of the interferometer and an arc, in air and vacuum, H. D. Babcock[11] has obtained the relative pressure shifts of the energy levels of iron as a function of their term values. These observations together with the damping constants, derived above, permit the calculation of "effective solar pressures" from the theory of pressure broadening and of pressure shifts. Lenz[12] has obtained an analytic expression for the ratio of the shift of a line $\Delta\nu_s$, to the half-breadth, $\Delta\nu_L$:

$$\frac{\Delta\nu_s}{\Delta\nu_L} = -\frac{1}{2}\tan\frac{\pi}{p-1} \qquad (4)$$

10. Viert. Astron. Gesell. 78, 213, 1943.
11. Ap. J. 67, 240, 1928.
12. Zeit. f. Phys. 80, 423, 1933.

p is defined from the equation for the mutual potential energy of the quasi-molecule formed by the colliding partners; namely,

$$V'(r) - V(r) = ar^{-p} \qquad (5)$$

Though the experimental values of p vary from 5 to 10, it is interesting to compute $\frac{\Delta\nu_s}{\Delta\nu_L}$ for p = 6, (van der Waals' forces). For this value of p, the shift is very nearly one-third the half-breadth.

For iron lines with lower excitation potential equal to 4.0 volts, the solar value of $\underline{a} = \frac{\Delta\lambda N}{\Delta\lambda D}$, the ratio of collisional width to Doppler width, is approximately equal to unity. Evaluating $\Delta\lambda D$ for T = 5730°K and using $\underline{a}$ = 1.0, gives $\Delta\lambda N$ = 0.024A or 0.080 cm^{-1}. One third of $\Delta\nu_n$ gives $\Delta\nu_s$ = 0.026 cm^{-1}. On the basis of the wave number corresponding to each of the levels producing the lines and the derived value of $\Delta\nu_s$, Babcock's curve indicates that the pressure of the solar layers, that are effective in producing these lines, is of the order of 1.0 atmosphere. This result is approximate because of the approximate value of $\underline{a}$ and because 5730°K is used as a mean temperature in computing the Doppler width.

If the pressure in the region of the solar atmosphere that is effective in producing the high excitation lines is 1 atmosphere, then we should expect line displacements amounting to 0.026 cm^{-1}, as compared to low excitation lines when we compare solar and vacuum wave numbers. A search was made for wave-length shifts for the high excitation potential lines. If the pressure in the solar atmosphere is 1 atmosphere, then laboratory wave lengths determined in air at 1 atmosphere and solar wave lengths for low and high excitation level lines should agree except for the relativity shift of $2.12 \times 10^{6} \times \lambda$ to the red and possible Doppler effects.

The laboratory wave lengths for two multiplets arising from 0.95-volt and 4.35-volt levels, are given in the second column of Table IV. The third column gives the wave length from the Revised Rowland, and the fourth gives the difference, sun-lab. The relativity shift is +0.011 Angstrom units. The approximate equality of the differences for the two multiplets might be indicative of a solar pressure of 1 atmosphere for the high level multiplet, but the abundance of unknown and systematic errors (Doppler effects, pole shift, different observers, etc.) makes the results very uncertain. The same conclusion was reached in treating a much larger body of material.

SOLAR CURVES OF GROWTH FOR INDIVIDUAL ELEMENTS

An examination was made of solar curves of growth of individual elements to study the possible effects of assumed damping constants on the determination of kinetic temperature and turbulence.

It was pointed out in the introduction that the position (in the ordinate log W/λ) of the flat or saturation portion of the curve, depends on the particular value of $\underline{a}$ that is adopted. Since the kinetic temperature and turbulence are determined by fitting to the theoretical curves, the observational points, which in many cases define only a small portion of the complete curve of growth, it is apparent that the resulting temperatures depend on the adopted value of $\underline{a}$. For many elements there are not enough available lines, for which the laboratory or theoretical constants are known, to determine accurately the damping constant from the shape of the curve of growth. Since the damping constant is uncertain, the temperature cannot be given with certainty.

TABLE IV

Term	Lab. λ	Solar λ	Sun-Lab.	Observer
$z^5G°-e^5H$	5424.072	0.082	+0.010	I
4.35V-6.60V	5383.374	0.382	0.008	I
	5369.965	0.976	0.011	I
	5367.470	0.478	0.008	I
	5364.874	0.882	0.008	I
	5295.316	0.315	-0.001	u
Mean			+0.008	
$a^5F-z^5D°$	526[illegible].541	0.552	+0.011	I
0.95V-3.20V	5328.042	0.053	0.011	I
	5371.493	0.503	0.010	B
	5405.778	0.787	0.009	B
	5434.527	0.536	0.009	B
	5397.131	0.143	0.012	B
	5429.699	0.708	0.009	B
	5446.920	0.926	0.006	B
	5455.613	0.626	0.013	B
	5501.469	0.479	0.010	B
	5506.782	0.793	0.011	B
	5497.519	0.528	0.009	B
Mean			+0.010	

I H. D. Babcock ApJ 66 256 1927
u MIT, Unpublished
B International Secondary Standards

Iron, Vanadium, and Titanium

The curves of growth for low excitation potential lines. (0 to 2.0 volts), which have been obtained for iron, vanadium, and titanium, are given in the first Technical Report. The curves of growth for the separate elements should be fairly reliable. The observed curve for iron, in the damping portion, has a greater slope than the theoretical value of $\tan^{-1} 1/2$. For the weaker lines, the best fit was obtained for $a = 0.01$. The increased slope causes the observed curve to cross over the parallel theoretical damping curves to $a = 0.05$. The observed points, together with the theoretical curves for the two values of a, gave 19,000°K and 15,500°K for the kinetic temperatures.

It should be pointed out that the value $a = 0.01$ depends on fitting a partial Fe curve to the partial mean curves of Ti and V, and not to a series of points for any single element covering the whole range of the curve of growth. Hence, a definitive value of a was not determined. We must admit, therefore, the possibility that the value of a lies outside the range 0.01 to 0.05 with a corresponding change in the temperatures.

Carbon

The accessible lines of carbon arise from lower levels in the range 7.4 to 8.8 volts and thus should be very suitable for the examination of high excitation effects. The recent table by H. D. Babcock and C. E. Moore,[13] extending the work of the Revised Rowland Atlas from λ6800 to λ13495, assisted greatly in the identification and measurement of equivalent widths

13. The Solar Spectrum, λ6600 to λ13495.

of the infrared carbon lines. A partial curve of growth for two transition arrays at λ = 10,000 and λ = 11,700 was obtained. (See Figure 4.) The theoretical line strengths were taken from the tables prepared by H. E. White[14] and L. Goldberg.[15] The line intensities are a weighted mean of Allen's[16] measures, when available, and those obtained from direct-intensity tracings made at the McMath-Hulbert Observatory.[17] When the observational curve was fitted to a theoretical curve, the displacement in ordinate corresponded to T_{kin} = 10,000°K and $\underline{a}$ = 0.01. The determination of $\underline{a}$, which is smaller by a factor of two at λ10,000 than at λ5,000, is not definitive due to the small range of intensities.

We must also take into account the effect on the curve of growth of finite central intensities. This effect is particularly large for the lines in the infrared. It seems likely that the carbon lines are formed in a manner satisfying the conditions of line formation by pure absorption. The solution of the equation of radiative transfer for pure absorption (M. E. model) gives

$$\frac{I(o,\theta)}{I_o(\theta)} = \frac{1 + \beta_o \frac{1}{1 + K\nu/K} \cos\theta}{1 + \beta_o \cos\theta} \qquad (6)$$

where β_o is the coefficient of limb darkening at wave length, λ, and $\frac{I(o,\theta)}{I_o(\theta)}$ is the ratio of line intensity compared to the continuous background. If $\beta_o(\lambda = 10,000) = 1.0$ is substituted in Equation 6, and if it is assumed that the ratio of line absorption to continuous absorption approaches infinity in

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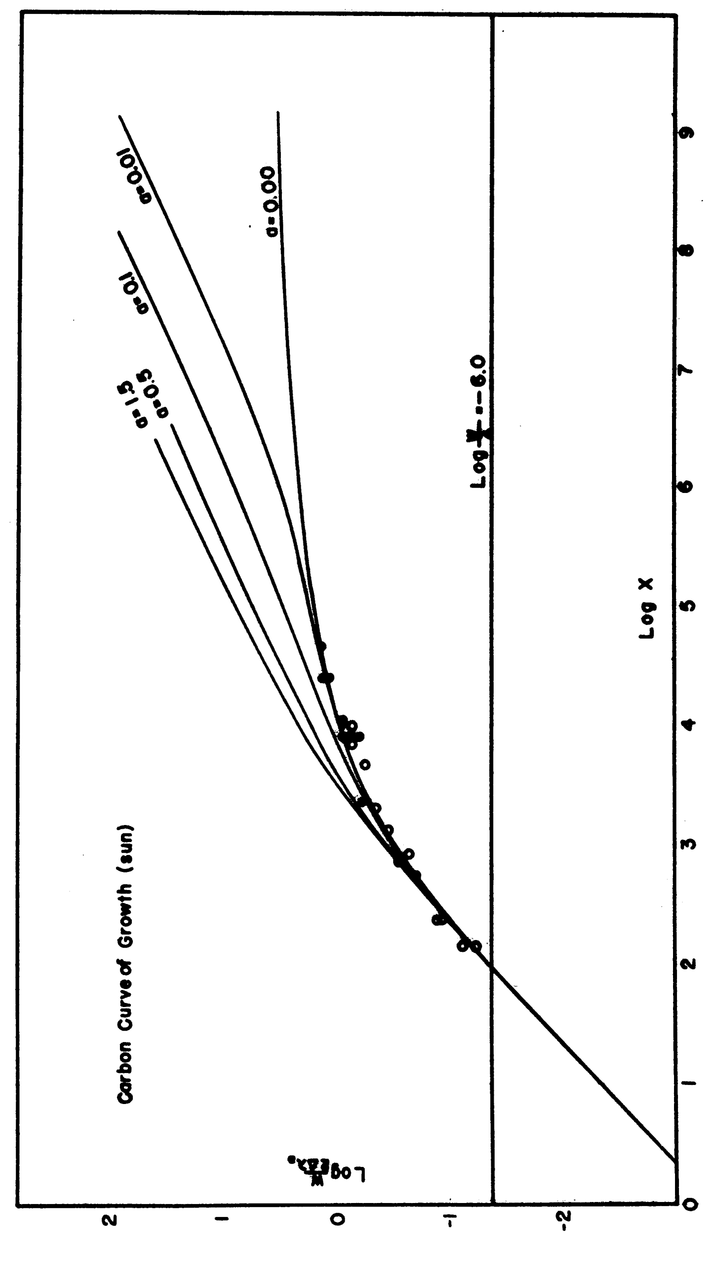
14. Loc. cit.
15. Loc. cit.
16. Ap. J. 88, 125, 1938.
17. A description with illustrations of the solar infrared reflecting spectrometer has been given by McMath and Mohler in "Sky and Telescope" 7, 143, 1948.



Carbon Curve of Growth (sun)
a=1.5
a=0.5
a=0.1
a=0.01
a=0.00
Log X
0
1
2
3
4
5
6
7
8
9
2
1
0
-1
-2

FIG. 4

the line center, then $\frac{I(o,\theta)}{I_o(\theta)} = 0.50$. Unsold[18] has indicated how the curve of growth must be modified in the case of finite central intensities. For $T = 10,000°$, the introduction of $R_c = 0.50$, increases a from 0.01 to 0.1. The increase is greater if a lower temperature is adopted. It should be remarked that T and a cannot be determined accurately at the present time for carbon because the observational data are insufficient to define more than a portion of the curve.

Silicon

Because of the high solar abundance of silicon and the high excitation potential of the many accessible solar lines, a curve of growth for this element was undertaken. In the Utrecht Atlas of the sun, the individual lines appear broad and most distinctive. Unfortunately, the curve of growth based on theoretical line strengths exhibited so much scatter among the points that a satisfactory curve could not be obtained. Apparently, silicon is already so far from L-S coupling as to make the analysis impractical until better oscillator strengths are available. Theoretical calculations of the f-values are being undertaken **at Harvard College Observatory.**

Calcium

A basic curve of growth for Ca can be drawn from the laboratory intensity measures of Schuttevaer, Bont, and van den Broek.[19] To this curve we added multiplets by sliding them horizontally until the best fit with the initial curve was obtained. The position of the resonance portion was

18. Phys. d. Sternatmospharen, Springer, 1938.
19. Physica 10, 544, 1943.

determined solely from the two lines λ6573 and λ4227. The former line lies on the Doppler section, and the latter on the resonance section of the curve of growth. These are indicated by filled circles in Figure 5.

To the resonance curve we have added the H and K lines, and the Ca triplet at 8000A. These latter points serve to define the slope but not the position of the damping portion of the curve. The solid line drawn through the observations indicates an a of 0.08 and a kinetic temperature of 70,000°K. Because the position of the damping portion depends on a single laboratory determination of the relative intensity of λ6573 and λ4227, it is possible that the position of the damping portion is in error. The dotted line of Figure 5 shows an attempt to fit the observations with a reasonable kinetic temperature. The dotted line is drawn for T_{kin} = 10,000°K and a = 0.5.

This value of a for lines having a mean excitation potential of 2.5 volts seems large in comparison with values derived for corresponding Fe lines. However, the outer electron structure of Ca is similar to that of Mg and, for the latter, Unsold[20] has shown that the quadratic Stark effect is an important source of line broadening. The 1p-n'D series which Unsold used in Mg yields only three observable lines for Ca: n = 4, 5, 6. These lines do not provide sufficient data for an unequivocal proof of Stark widening.

Observations of the laboratory intensities of λ4227 and λ6573 in absorption with the King electric furnace are urgently needed for a definitive determination of the damping constant.

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20. Loc. cit.

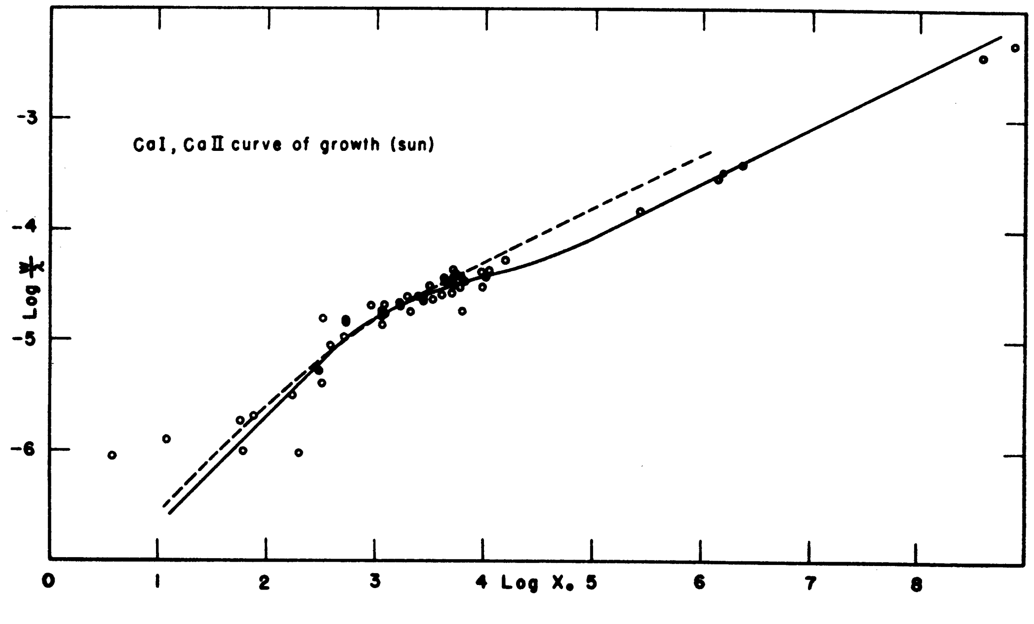

FIG. 5

Sodium

Righini's[21] curve of growth for sodium has been re-examined on the basis of all known wave-length corrections. (See Figure 6.) The detailed theory of the curve of growth shows that a line is weakened with increasing continuous absorption, a smaller depth in the atmosphere being reached at those wave lengths for which the general absorption coefficient is larger than the mean (Menzel[22]).

The theoretical treatment of Chandrasekhar,[23] which follows the suggestion of Wildt[24] that the general solar absorption is due to the negative hydrogen ion, is in excellent agreement with observation to the red of $\lambda 4000$. However, an analysis by Münch,[25] based on limb darkening and on monochromatic energy radiated at different wave lengths, indicates a pronounced increase in the value of the solar absorption coefficient above that given by the H^- atom, in the region below $\lambda 4000$. We have adopted the observed variation of the absorption coefficient derived by Münch for correcting the values of log X_o. A small additional wave-length correction, which arises from the existence of a solar temperature gradient, has been included.

In Figure 7, log $W/\lambda\ 10^6$ is plotted versus the log of the theoretical strengths. The ordinate for each point is a weighted mean of log $W/\lambda\ 10^6$ for Righini's,[26] Allen's,[27] and our measures from the Utrecht Atlas.[28]

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21. Zeit. f. Astrop. 10, 344, 1935.
22. Ap. J. 84, 462, 1936.
23. Ap. J. 104, 430, 1946.
24. Ap. J. 89, 295, 1939; 93, 47, 1941.
25. Ap. J. 102, 385, 1945.
26. Loc. cit.
27. Mem. Commonwealth Solar Obs. Canberra, 1, No. 5, 1934; MN 96, 843, 1936; Ap. J. 88, 125, 1938.
28. Photometric Atlas of the Solar Spectrum, Utrecht, 1940.

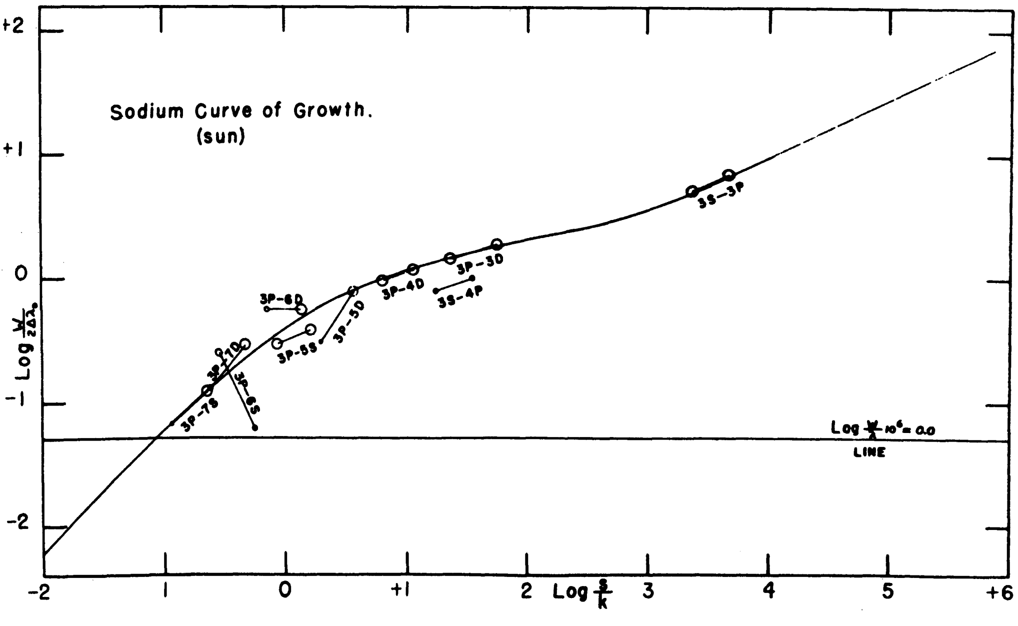
Sodium Curve of Growth.
(sun)
+2
+1
0
-1
-2
-2
1
0
+1
2
3
4
5
+6
3P-7S
3P-7D
3P-6S
3P-6D
3P-5S
3P-5D
3P-4D
3P-3D
3S-4P
3S-3P
LINE

FIG. 6

An excitation temperature of 5040°K was arbitrarily assumed. The large circles represent the unblended lines of better quality and for these the curve is very well delineated. It fits a theoretical curve for which a = 0.01, or three times the classical value, and T_{kin} = 9400°K.

This is in disagreement with Shane's[29] results obtained from profile measurements of the D lines. His observations for the center of the disk may be represented by a Doppler widening in the core corresponding to 5700°K. The wings give a resonance widening of 110 times the classical value, (a = 0.316). Stromgren[30] has used a value of a approximately thirty times the classical figure, based on the observed and theoretical collisional cross sections of Na coupled with the solar pressures derived from model atmospheres. The discrepancy between our value of a, determined from the curve of growth, and those of Shane and Stromgren, might be explained in part by our assumption of 5040°K for the excitation temperature. A considerable increase in a results from an increase in excitation temperature. For example, by increasing T_{Ep} from 5040°K to 10,000°K the value of a is changed from a = 0.01 to a = 0.10. Stratification in the sun's atmosphere may introduce further variations in the results. As Professor Shane stressed, his determination of a was based on the M-E approximation and the center-to-limb observations indicated some deviations from the simplified case.

The results of the present investigation suggest that a single curve of growth does not suffice for all elements. Furthermore, the value of the damping constant may differ from one multiplet to another and its value, to a first approximation, may be a function of excitation potential.

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29. LOB 19, 119, 1941.
30. Festschrift für E. Stromgren, p. 218, 1940.

We adopted a kinetic temperature of 10,000°, for those cases in which the determination is uncertain.

These considerations must be borne in mind in the application of solar f-values to stellar problems. The acquisition of more laboratory and theoretical oscillator strengths is of prime importance for future analysis of solar and stellar spectra.

LINE PROFILES

A further test for large damping factors was made from a study of the profile of the carbon line λ8335, whose excitation potential is 7.65 volts. By application of the Saha and Boltzmann equations, it can be shown that the number of atoms of carbon in the 7.65-volt level is nearly constant with depth in the sun's atmosphere. For this reason, the M-E model for the case of pure absorption is a very good approximation. Profiles were computed from the relationship

$$\frac{I_\lambda}{I_o} = (1 + \beta_o \cos \theta)^{-1} \left(1 + \beta_o \frac{1}{1+\frac{\kappa_\lambda}{\kappa}} \cos \theta\right) \tag{7}$$

where $\frac{\kappa_\lambda}{\kappa} = \frac{f(v,a)}{\kappa}$ and $v = \frac{\lambda - \lambda_o}{b^\lambda}$. v is the wave-length interval in terms of the Doppler width b^λ. Hjerting[31] and Mitchell and Zemansky[32] give f(v,a) for a series of values of the parameter a. The fictitious absorption coefficient at the line center was adjusted until a fit with corrected central intensity was obtained. Then, for each value of the damping constant, the rest of the line was fitted to the observed line by adopting

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31. Ap. J. 88, 508, 1938.
32. Resonance Radiation and Excited Atoms, Cambridge, 1934.

an effective Doppler width or temperature, T_e. This procedure neglects the variation in Doppler width and damping, with depth in the sun's atmosphere.

Table V gives the joint values of a and T_e computed to give the best fit to the observed profile in the Utrecht Atlas, the latter having been corrected for instrumental distortion.

TABLE V

a	T_e
0.1	56,500°K
0.5	27,000°
1.0	13,400°
1.5	7,150°

A kinetic temperature in the reversing layers greater than 20,000°K seems unlikely from the analysis of curves of growth and line profiles of heavier elements. Therefore, if our basic assumptions are correct, or nearly so, the damping constant, a, of this line is 0.7 or larger.

The possibility was investigated that a line profile calculated by the more exact theory of Chandrasekhar[33] might be broader than one calculated by the M-E theory. The latter is an approximation while the former is a solution for the equation of transfer that exactly satisfies the boundary conditions. If the more exact theory introduces a broader line profile, values of a and T_{kin}, derived from the curves of growth and from line profiles, might change by significant amounts. Comparison of the two values showed differences of only 0.5 per cent for cos θ = 1.0 and 1.25 per cent for cos θ = 0.10, in per cent of the continuous background. θ is the angle from the normal to the sun's surface to the lines of sight. A similar calculation

33. Ap. J. 106, 145, 1947.

by Greenstein[34] has been published recently. He concludes that an error of about 0.02 in $\log_{10} W/\lambda$ may occur. An error of this size is not significant in the present work as it changes the computed temperature for Fe, for example, by only 10 per cent. The effect on the damping constant is also expected to be small.

Solar Kinetic Temperatures

Finally, we wish to collect the observational evidence for a high kinetic temperature in the solar reversing layer. The authors would like to quote the remarks of H. H. Plaskett[35] in his 1947 presidential address. He is commenting on the large turbulent velocities for certain stars found by the curve-of-growth method: "Its interpretation by 'turbulence' is more open to question. In granulation, as observed on the sun, we are familiar with a bodily motion of masses of gas in vertical streams, the motion being maintained by the energy of ionization of the abundant hydrogen, but the phenomenon of granulation does not lead to an increase in the equivalent widths of the spectrum lines. Each granule over the disk of the sun contributes its own slightly displaced absorption line, and the sum of these displaced lines results in a washed-out profile of unchanged equivalent width. Only in the event of the moving masses of gas overlying each other and each making its own contribution to the observed absorption line will there be an increase of equivalent width. Under these conditions, the masses of gas will collide with each other, but these collisions will not be, as Milne has pointed out, the elastic collisions contemplated in the kinetic

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34. Ap. J. 107, 151, 1947.
35. M.N. 107, 117, 1947.

theory. The collisions will be between the individual atoms of the gas-masses, not of the gas-masses as units, and the result will be a dissipation of kinetic energy into energy of thermal agitation of the atoms. 'Turbulence', therefore, affords no escape from kinetic temperatures of the order of millions of degrees."

Van de Hulst[36] and joint workers have measured six titanium lines near $\lambda 5900$ in a study based on the original tracings of the Utrecht Atlas, and on a redetermination of the instrumental corrections. They derive a Doppler width which corresponds to T = 13,700°K.

Shane's[37] interferometer observations of two lines of iron and one of nickel gave a temperature of 16,000°K. This result was obtained by fitting a Doppler profile to the curves.

Table VI gives kinetic temperatures for the center and for the limb of the solar disk based on the observed Doppler widths of six titanium lines. The values of $\Delta\lambda_D$ are those obtained by E. J. Prouse from high precision interferometer observations.[38] The first column gives the wave length of the lines; the second and fourth give $\Delta\lambda_D'$, which is the Doppler width after the effect of broadening due to damping is removed. Kinetic temperatures are given in columns three and five. The calculations have been carried out for $\underline{a}$ = .316 and $\underline{a}$ = .016. The former is larger than might be expected in view of the rather low excitation potentials of these lines; nevertheless, its use will serve to define an upper limit to the pressure widening. Even if we adopt an $\underline{a}$ of 0.316, we get T = 10,000° for the sun's center, and at the limb [illegible]0,000°! This may be interpreted as an increase in kinetic temperature with height in the solar reversing layers.

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36. B.A.N. 10, 79, 1946.
37. L.O.B. 16, 76, 1932.
38. Thesis (unpub.), Univ. of Calif., 1940.

TABLE VI

$\Delta\lambda_D'$ = Doppler + Turbulence or Doppler Width from Observation.

	λ	Center of Solar Disk $\Delta\lambda_D'$	Center of Solar Disk T°K	Limb of Solar Disk $\Delta\lambda_D'$	Limb of Solar Disk T°K
$a = 3.16 \times 10^{-1}$					
	5866.4	0.0363	10,000	0.0513	20,000
	5899.3	0.0389	11,000	0.0501	18,800
	5922.1	0.0372	10,300	0.0501	18,600
	5953.2	0.0407	12,200	0.0513	19,400
	5965.8	0.0308	6,950	0.0490	17,600
	5978.5	0.0355	9,200	0.0501	18,500
$a = 1 \times 10^{-2}$					
	5866.4	0.0417	13,200	0.0575	25,200
	5899.3	0.0447	14,900	0.0589	26,800
	5922.1	0.0417	13,200	0.0569	24,200
	5953.2	0.0468	16,100	0.0589	26,500
	5965.8	0.0437	14,000	0.0575	24,200
	5978.5	0.0498	18,100	0.0575	24,200