The effect of alpha particles on the stability of the ELMO Bumpy Torus (EBT) reactor

D. Wojtowicz and T. Kammash
University of Michigan, Ann Arbor, Michigan 48109

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The macroscopic stability of an ignited ELMO Bumpy Torus (EBT) reactor is investigated by studying the effects of the alpha particles generated by the deuterium–tritium (D–T) fusion reaction on the background interchange mode, the interacting interchange mode, and the high-frequency compressional Alfvén and coupled modes. A fluid description is used for the background plasma while a kinetic treatment is utilized for the hot-electron species and the alpha particles. It is shown that the alphas tend to mildly destabilize the interacting interchange while stabilizing the background interchange because of their sizable Larmor radii. The destabilization is most pronounced when the beta of the alpha particles is highest, i.e., at birth, and recovery of stabilization takes place as these particles slow down toward thermalization. It is also shown that the alphas completely stabilize the high-frequency modes, so that it can safely be concluded that fusion alphas present no detrimental effects on the stability of an EBT reactor that possesses an appropriate hot-electron ring for macroscopic stability.

I. INTRODUCTION

The ELMO Bumpy Torus (EBT) device has been advanced as a possible fusion reactor because of a unique combination of toroidal and mirror plasma confinement properties. Its reactor potential, however, depends critically on its ability to support a high beta (ratio of plasma pressure to magnetic field pressure) since the power density of such a reactor is proportional to this quantity. The maximum achievable beta is determined by the macroscopic stability of the system, and many papers have recently been written addressing this question.[1–7] The major element in the magnetohydrodynamics (MHD) stability of EBT is the ring of hot electrons which provides a magnetic well for the stabilization of the interchange mode of the background plasma, even though this hot species could give rise to other instabilities such as the interacting interchange[3,7] mode. In order to provide the needed magnetic well the hot electrons must have sufficiently high energies, which raises the question concerning radiative losses[8] as well as proper relativistic treatment of their dynamics, especially in conjunction with the stability question. It has been shown[9] that sensitivity of the results to relativistic effects goes hand in hand with the model used (e.g., slab, local, etc.) in describing the system, and that relativistic effects can be ignored if the "deep well" approximation is invoked.

For an EBT reactor buring a deuterium–tritium (D–T) mixture, the question imediately arises as to the effect of the reaction product, namely the alpha particle, on the stability of the system. These particles are born at a very high energy ($\sim$3.5 MeV) and slow down rather quickly to energies comparable to those of the background ions. At birth the distribution function of the alphas can be taken to be a delta function in energy, and as they begin to slow down on the target plasma their distribution begins to spread, eventually assuming a shape that can be approximated by a Maxwellian in speed. Once they are totally thermalized their distribution will be no different from the background ions at a comparable temperature, and as a result the so-called "thermal alphas" will present no new effects and will not be addressed in this analysis. Rather, the focal point of this paper will be a study of the effect of the "fast alphas" on the macroscopic stability of an ignited EBT plasma. Specifically, we will examine the stability of low- (e.g., background and interacting interchange) and high- (Alfvén) frequency modes in the presence of these high-energy particles and establish their effect relative to an unignited system. In so doing, one can in principle also assess the effect of externally injected energetic particles, e.g., neutral beam heating on the stability of the system. The analysis shows that alpha particles tend to destabilize the interacting interchange mode while modestly stabilizing the background interchange and that the destabilization is more pronounced at short (compared to the alpha gyroradius) wave length oscillations. Moreover, the largest destabilizing effect seems to occur at the instant of birth of these particles when their beta is largest, and recovery toward the original stability boundary takes place as the alphas slow down. At high frequency when an unignited plasma shows instabilities associated with coupling to the Alfvén waves, the presence of alphas seems to provide total stabilization.

II. BASIC EQUATIONS AND ANALYSIS

We begin by examining the dynamics of alpha particles generated by the (D–T) reaction. For a 50%–50% mixture of (D–T) the rate of production of alphas particles is given by $(n_i^2/4)\langle\sigma v\rangle$, where n_i is the ion density, σ is the fusion reaction cross section, and $\langle\sigma v\rangle$ is the reaction rate. At birth these particles are born at 3.5 MeV energy and could be represented by a delta function distribution function. If we assume that they are isotropic then a function like $f_\alpha \sim \delta(v^2 - v_\alpha^2)$ would be appropriate. If, on the other hand, we wish to simulate a distribution that is more appropriate for a mirror-confined plasma, then we could employ

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$f_\alpha \sim \delta(v_\perp^2 - v_{\perp\alpha}^2)_x \delta(v_\parallel^2 - v_{\parallel\alpha}^2)$, where the thermal velocities are expressible in temperatures $T_{\perp\alpha}$ and $T_{\parallel\alpha}$ perpendicular and parallel to the magnetic field, respectively.

Soon after their birth the alpha particles slow down on the background plasma. In this paper we shall ignore the interaction of the alphas with the hot electrons and also ignore the interaction of the fast alphas with those that have already thermalized, or simply the "thermal" alphas. At energies sufficiently higher than the average energy of the background plasma the slowing down power can be written as[10]

$$\frac{dE}{dt} = -\left(C_1 E + \frac{C_2}{\sqrt{E}}\right), \tag{1}$$

where C_1 and C_2 are constants that depend, among other things, on the masses of the interacting particles and the background electron temperature. The first term in Eq. (1) reflects the slowing down on the plasma electrons, while the second represents the interaction with the ions. In order to deduce an expression for the velocity (or the energy) distribution of the fast alphas, we consider an energy interval ΔE. The alphas enter this ΔE by slowing down from above and exit by slowing down from below; they can also escape because of confinement considerations characterized by a confinement time $\tau(E)$. If we call the number of alphas per unit energy $n_\alpha(E)$, then assuming a steady-state situation we can write

$$\frac{d}{dt}[n_\alpha(E)\Delta E] = 0, \tag{2}$$

which upon expansion yields a differential equation of the form

$$\frac{\partial n_\alpha(E)}{\partial t}\left(\frac{dE}{dt}\right) + n_\alpha(E)\frac{\partial}{\partial E}\left(\frac{dE}{dt}\right) + \frac{n_\alpha(E)}{\tau(E)} = 0. \tag{3}$$

This equation can be readily integrated to give

$$n_\alpha(E) = n_\alpha(E_0)\exp\left[\int_E^{E_0}\left(\frac{1}{\tau(E)(dE/dt)} + \frac{(\partial/\partial E)(dE/dt)}{dE/dt}\right)dE\right], \tag{4}$$

where $E_0(\sim 3.5$ MeV) is the birth energy of the alpha particles. At $E = E_0$, $n_\alpha(E) = n_\alpha(E_0)$, given by

$$n_\alpha(E_0) = \frac{(n_i^2/4)\langle\sigma v\rangle}{(dE/dt)_{E_0}}, \tag{5}$$

which when combined with Eqs. (4) and (1) yields

$$n_\alpha(E) = \frac{(n_i^2/4)\langle\sigma v\rangle}{C_1 E + C_2/\sqrt{E}} \times \exp\left(\int_E^{E_0}\frac{\sqrt{E}\,dE}{(C_1 E + C_2/\sqrt{E})\tau(E)}\right). \tag{6}$$

In a reactor grade plasma it is expected that the slowing down time be much smaller than the confinement time, so if we let $\tau \to \infty$ in the above expression, it becomes

$$n_\alpha(E) = \frac{(n_i^2/4)\langle\sigma v\rangle}{C_1}\frac{\sqrt{E}}{E^{3/2} + E_c^{3/2}}, \tag{7}$$

where

$$E_c^{3/2} = C_2/C_1.$$

It should be noted at this time that the assumption of perfect confinement for the alphas represents an extreme case so far as the stability is concerned since all the alpha particles are assumed to participate in the wave–particle interaction that underlies the instability. Moreover, the distribution function given by (7), except for minor angular effects, has been used to study the role of alphas in tandem mirrors.[11]

We turn now to the derivation of the dispersion equation for the modes of interest, and in this regard a standard procedure is followed. It has been pointed out[3,12] that at frequencies well below the drift frequency of the hot electrons the interchange modes dominate and as the frequencies approach the curvature drift frequency the compressional Alfvén waves and the coupled modes begin to come into play. Studies of these modes have been carried out either in the "slab" model[1,13] where the magnetic field curvature is simulated by a gravitational force, or in the "local" model[4] in which curvature effects enter in a natural way. We shall use the latter in this analysis and derive the necessary equations from the basic equations, namely the quasineutrality and quasistatic conditions given by

$$\sum_j q_j n_j' = 0; \quad n_j' = \int f_{ij}\, d^3v,$$

$$BB_\parallel^1 + 4\pi P_\perp' = 0; \quad P_{\perp j}' = \frac{m_j}{2}\int v_\perp^2 f_{ij}\, d^3v, \tag{8}$$

where q_j is the charge of the jth species, n_j' is the perturbed density of the jth species, $p_\perp'$ is the perturbed pressure perpendicular to the magnetic field, f_1 is the perturbed distribution function and $B_\parallel'$ is the perturbed axial magnetic field. The equilibrium magnetic field is taken to be

$$\mathbf{B} = B_0(1 + \epsilon x)\hat{\mathbf{z}}, \tag{9}$$

where ϵ^{-1} is assumed to be larger than the particles gyroradii so that their trajectories can be expanded about the guiding center x_g. With the perturbed fields expressed in terms of the scalar and vector potentials φ, $\mathbf{A}$, the linearized Vlasov equation can be solved for f_1 to give

$$f_1 = q\frac{\partial f_0}{\partial H}\varphi + \frac{1}{B}\frac{\partial f_0}{\partial x_g}A_y + iq\frac{\omega}{m}\left(m\frac{\partial f_0}{\partial H} + \frac{k_y}{\omega\Omega}\frac{\partial f_0}{\partial x_g}\right)\int_{-\infty}^{t}\left(\varphi - \frac{\mathbf{v}\cdot\mathbf{A}}{c}\right)dt', \tag{10}$$

where the integral is over the unperturbed particle orbits, and Ω is the gyrofrequency. In writing this result, perturbations at frequency ω of the plane wave type have been utilized, and an equilibrium distribution function f_0 as a function of the constants of motion x_g, $H = (1/2)mv^2$ has also been used. In a straightforward but lengthy manner the integral in Eq. (10) can be carried out, and the result becomes

$$f_1 = q\frac{\partial f_0}{\partial H}\varphi + \frac{q\omega}{m}D\left(\varphi\sum_{m,n}\frac{J_n(k_\perp v_\perp/\Omega)J_m e^{i(m-n)\theta}}{\omega_D - \omega - n\Omega} - \frac{iv_\perp A_x}{c}\sum_{m,n}\frac{J_n J_m' e^{i(m-n)\theta}}{\omega_D - \omega - n\Omega}\right), \tag{11}$$

where

$$\omega_D = \frac{k_y v_\perp^2}{2\Omega}\left(-\frac{\beta_t}{2\Delta} + \frac{\sigma}{R_c}\right) + \frac{k_y v_\parallel^2}{\Omega R_c},$$

$$D = m\frac{\partial f_0}{\partial H} + \frac{k_y}{\omega\Omega}\frac{\partial f_0}{\partial x_g}, \tag{12}$$

$$\sigma = 1 + \tfrac{1}{2}(\beta_\perp - \beta_\parallel),$$

$$\Delta^{-1} = |\ln \nabla n|.$$

Here R_c is the radius of curvature, J_n is the ordinary Bessel function, and β_t is the total beta.

For low-frequency modes it is reasonable to take $\omega \ll \omega_{*\alpha}, \omega_{D\alpha}, \Omega_\alpha$, where $\omega_{*\alpha} = k_y T_\alpha / m\Delta\Omega_\alpha$ is the diamagnetic drift frequency, and the other frequencies as defined earlier, and in so doing the perturbed distribution function for the alpha particles can be put in the form

$$f_{1\alpha} = -\frac{q_\alpha}{T_\alpha} f_{0\alpha}\varphi + \frac{q_\alpha}{T_\alpha}\omega_{*\alpha} f_{0\alpha}\left(\varphi\frac{J_0^2}{\omega_{D\alpha}} - \frac{iv_\perp A_x}{c}\frac{J_0 J_0'}{\omega_{D\alpha}}\right). \tag{13}$$

A similar approximation is invoked for the hot electrons, while for the core species which are taken to be relatively cold the approximation $\omega \gg \omega_{*c}, \omega_{Dc}$ is adopted; this in turn yields for these species

$$f_{1c} = -\frac{q_c}{T_c} f_{0c}\varphi + \frac{q_c}{T_c} f_{0c}\left\{\varphi\left[1 - \frac{k_\perp^2\rho_c^2}{2}\left(\frac{v_\perp}{v_c}\right)^2\right] + \frac{iv_\perp A_x}{c}\frac{k_\perp \rho_c v_\perp}{v_c}\left[\frac{1}{2} - \frac{3k_\perp^2\rho_c^2}{16}\left(\frac{v_\perp}{v_c}\right)^2\right]\right\},$$

$$f_{1h} = -\frac{q_h}{T_h} f_{0h}\,\varphi + \frac{q_h}{T_h} f_{0h}\left\{\varphi\left(\frac{1 - k_\perp^2\rho_h^2(v_\perp/v_h)^2}{\omega_{0h}}\right) + \frac{iv_\perp A_x}{c}\frac{k_\perp\rho_h v_\perp}{v_h}\left[1 - \frac{3}{16}k_\perp^2\rho_h^2\left(\frac{v_\perp}{v_h}\right)^2\right]\right\}. \tag{14}$$

It is evident that in obtaining these results the small argument expansion of the Bessel functions has been utilized. The velocity integrals shown in Eq. (8) can now be carried out and the resulting equations can be expressed in a matrix form as

$$\begin{pmatrix} D_{es} & -(\beta_i/2)D_{ct} \\ D_{ct} & D_{em} \end{pmatrix}\begin{pmatrix} \varphi \\ BB_\parallel' \end{pmatrix} = 0,$$

which in turn gives rise to the dispersion relation

$$D_{es}D_{em} + (\beta_i/2)D_{ct}^2 = 0, \tag{15}$$

where D_{es} represents the electrostatic term, D_{em} the electromagnetic term, and D_{ct} the cross product term. For low-frequency modes Eq. (15) is quadratic in ω, i.e., it can be expressed as

$$a\omega^2 + b\omega + c = 0,$$

for which the stability condition is given in the usual form, i.e., $b^2 - 4ac > 0$ with

$$a = \frac{k^2\rho_i^2}{2} + \frac{n_{0h}}{n_{0i}}\frac{T_i}{T_h}\left[1 - \frac{\omega_{*h}}{\omega_{Dh}}\left(1 - \frac{k^2\rho_h^3}{2}\right)\right] + \frac{4n_{0\alpha}}{n_{0i}}\frac{T_i}{T_\alpha}\left(1 - \frac{R_c}{2\Delta}I_1\right) + \frac{\beta_i}{2}D_{em}^{-1}\left\{k^2\rho_i^2 - \frac{n_{0h}}{n_{0i}}\left[1 - \frac{\omega_{*h}}{\omega_{Dh}}\left(1 - \frac{k^2\rho_h^2}{2}\right)\right] + 2\frac{n_{0\alpha}}{n_{0i}}\left(1 - \frac{R_c}{2\Delta}I_3\right)\right\} \times\left[\frac{3}{4}k^2P_i^2 - 1 + \frac{n_{0h}}{n_{0i}}\frac{\omega_{*h}}{\omega_{Dh}}\left(\frac{1}{2} - 3\frac{k^2\rho_h^2}{16}\right) - \frac{2n_{0\alpha}}{n_{0i}}I_2\right], \tag{16}$$

$$b = (\omega_{*i} - \overline{\omega}_{Di})\left(1 - \frac{n_{0e}}{n_{0i}}\right) - \frac{k^2\rho_i^2}{2}(\omega_{*i} - \overline{\omega}_{Di}) \times\left(1 - \frac{n_{0e}}{n_{0i}}\right) + \frac{\beta_i}{2}D_{em}^{-1}(\omega_{*i} - \overline{\omega}_{Di})\left\{\frac{7}{4}k^2\rho_i^2 - \frac{n_{0h}}{n_{0i}}\left[1 - \frac{2\omega_{*h}}{\omega_{Dh}}\left(1 - \frac{7}{16}k^2\rho_i^2\right)\right] + \frac{n_{0e}}{n_{0i}} - 1 + \frac{2n_{0\alpha}}{n_{0i}}\left(1 - \frac{R_c}{2\Delta}I_3 - \frac{R_c}{\Delta}I_2\right)\right\}\left(1 + \frac{n_{0e}T_e}{n_{0i}T_i}\right), \tag{17}$$

$$C = (\omega_{*i} - \overline{\omega}_{Di})\omega_{Di}\left(1 + \frac{n_{0e}T_e}{n_{0i}T_i}\right) - \frac{k^2\rho_i^2}{2}\omega_{*i}\overline{\omega}_{Di}\left(1 + \frac{n_{0e}T_e}{n_{0i}T_i}\right) + \frac{\beta_i}{2}D_{em}^{-1}\left[(\omega_{*i} - \overline{\omega}_{Di})\left(1 + \frac{n_{0e}T_e}{n_{0i}T_i}\right)\right]^2, \tag{18}$$

$$D_{em} = 1 + \beta_i + \beta_e + \beta_h\frac{\omega_{*h}}{\omega_{Dh}}\left(\frac{1}{2} - \frac{3k^2\rho_h^2}{16}\right) + \frac{\beta_\alpha}{2\Delta}R_c I_4. \tag{19}$$

In these expressions we have introduced the following terms:

$$\overline{\omega}_{Di} = \int \omega_{Di}\, d^3v, \quad \overline{\omega}_{Di} = \int \frac{v_\perp^2}{v_{tni}}\omega_{Di}\, d^3v, \tag{20}$$

and the following integrals which pertain to the alpha particles:

$$I_1 = \int \frac{J_0^2(k_\perp v_\perp/\Omega) f_{0\alpha}}{\omega_{0\alpha}}\, d^3v,$$

$$I_2 = -\int \frac{J_0 J_0' v_\perp f_{0\alpha}}{\omega_{D\alpha}}\, d^3v, \tag{21}$$

$$I_3=\int \frac{J_0^2 v_\perp^2 f_{0\alpha}}{\omega_{D\alpha}}\, d^3v,$$

$$I_4=-\int \frac{J_0 J_0' v_\perp^3 f_{0\alpha}}{\omega_{D\alpha}}\, d^3v,$$

with the drift velocity ω_D defined in Eq. (12). The beta of the alpha particles can be expressed in terms of the ion beta since the ratio of their densities is fixed and the alpha temperature is chosen consistent with the assumptions made.

In the high-frequency regime where the frequency of the oscillation can be comparable to the ion gyrofrequency, i.e., $\omega \lesssim \Omega_i \ll \Omega_e$, the quasistatic condition employed in deriving the previous dispersion relation will no longer be valid; instead, the perpendicular form of Ampere's law and the quasineutrality condition constitute the basic equations, i.e.,

$$\sum_j q_j n_j' = 0,$$

$$k_\perp^3 A_\perp = -\frac{4\pi}{c}\sum_j q_j \int k_\perp v_\perp \sin(\theta+\psi) f_{ij}\, d^3v, \tag{22}$$

where the angles are defined appropriately in velocity space.

Moreover, the difference between the actual position of the particle and its guiding center position takes on added importance in the high-frequency regime, so that in this case the perturbed distribution function for a species s becomes

$$f_{1s} = -\frac{q_s}{T_s} f_{0s}\varphi + \frac{(\hat{\mathbf{b}}\times\nabla f_{0s})}{cB}\cdot \mathbf{A}_\perp + q_s\omega D \times\left(\varphi \sum_n \frac{J_n^2}{\omega_{Ds}-\omega-n\Omega_s} - \frac{i v_\perp A_\perp}{c}\sum_n \frac{J_n J_n'}{\omega_{Ds}-\omega-n\Omega_s}\right) \times\left(1+n\frac{\mathbf{k}\times\hat{\mathbf{b}}\cdot\nabla \ln n_{0s}}{k_\perp^2}\right). \tag{23}$$

In order to make the analysis tractable we retain only the $n=0, \pm 1$ terms in this expression for all the species. Once again we assume that $\omega \gg \omega_{DE}, \omega_{*c}$ for the cold species but keep the full frequency range for the alphas and the hot electrons. With the aid of Eqs. (22) the matrix equation for the perturbed fields assumes the usual form, namely,

$$\begin{pmatrix} D_{es} & -(\beta_i/2)D_{ct} \\ D_{ct} & D_{em}\end{pmatrix}\begin{pmatrix}\varphi \\ BB_\parallel'\end{pmatrix}=0, \tag{24}$$

but with the new elements given by

$$D_{es} = \left(\frac{n_{0i}}{N}-\frac{n_{0e}}{N}\right)\left(\frac{\omega_{*i}-\overline{\omega}_{Di}}{\omega}\right) + \left(\frac{n_{0i}}{N}+\frac{n_{0e}T_e}{NT_i}\right)\frac{(\omega_{*i}-\overline{\omega}_{Di})\overline{\omega}_{Di}}{\omega^2} - \frac{n_{0i}}{N}\frac{b_i\Omega_i^2}{\omega^2-\Omega_i^2} - \frac{n_{0i}}{N}\frac{\omega\omega_{*i}}{\omega^2-\Omega_i^2} + \frac{n_{0h}}{N}\frac{T_i}{T_h}\left(\frac{\omega_{*h}-\overline{\omega}_{Dh}}{\omega-\overline{\omega}_{Dh}}\right) + \frac{4n_{0\alpha}T_i}{NT_\alpha}\left\{1+\frac{\omega-\omega_{*\alpha}}{\omega_{*\alpha}}\frac{R_c}{\Delta}\left[I_5+\left(1-\frac{\omega_{*\alpha}}{b_\alpha\Omega_\alpha}\right)I_6+\left(1+\frac{\omega_{*\alpha}}{b_\alpha\Omega_\alpha}\right)I_7\right]\right\}, \tag{25}$$

$$D_{ct} = \frac{n_{0i}}{N}\frac{\Omega_i^2}{\omega^2-\Omega_i^2} + \frac{n_{0e}}{N} + \frac{n_{0i}}{N}\frac{\omega_{*i}\omega}{b_i(\omega^2-\Omega_i^2)} + \frac{n_{0h}}{N}\left(\frac{\omega-\omega_{*h}}{\omega-\omega_{Dh}}\right) - \frac{4n_{0\alpha}}{N}\frac{1}{k_\perp\rho_\alpha}\frac{R_c}{\Delta}\frac{\omega-\omega_{*\alpha}}{\omega_{*\alpha}} \times\left[I_8+\left(1-\frac{\omega_{*\alpha}}{b_\alpha\Omega_\alpha}\right)I_9+\left(1+\frac{\omega_{*\alpha}}{b_\alpha\Omega_\alpha}\right)I_{10}\right], \tag{26}$$

$$D_{em} = 1+\beta_i+\beta_c+\beta_i\left(\frac{\omega_{*i}\omega(1-3b_i)}{2b_i^2(\omega^2-\Omega_i^2)}+\frac{n_{0e}}{n_{0i}}\frac{\omega_{*i}\omega}{2b_i^2\Omega_i^2}\right)+\frac{\beta_h}{2}\left(\frac{\omega-\omega_{*h}}{\omega-\overline{\omega}_{Dh}}\right) \times\left[1-\frac{\omega_{*h}(\omega-\overline{\omega}_{Dh})}{b_h^2\Omega_i^2}\left(\frac{m_e}{m_i}\right)^2\right]-\frac{\beta_\alpha}{2}\left(\frac{\omega-\omega_{*\alpha}}{\omega_{*\alpha}}\right)\frac{R_c}{\Delta}\left[I_{11}+\left(1-\frac{\omega_{*\alpha}}{b_\alpha\Omega_\alpha}\right)I_{12}+\left(1+\frac{\omega_{*\alpha}}{b_\alpha\Omega_\alpha}\right)I_{13}\right]. \tag{27}$$

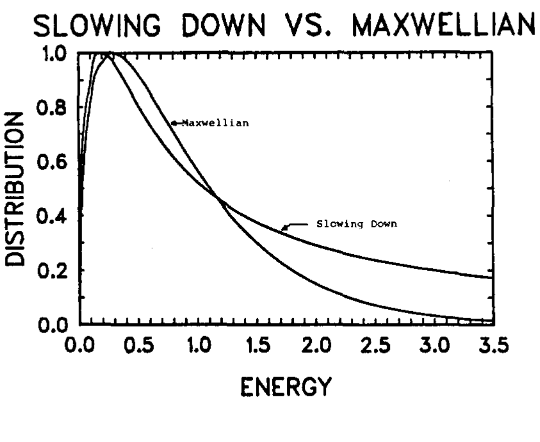

FIG. 1. Comparison of slowing down and Maxwellian distributions.

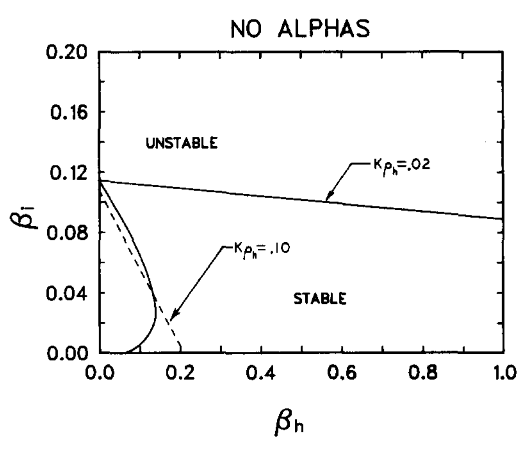

FIG. 2. Stability boundary in the absence of alphas.

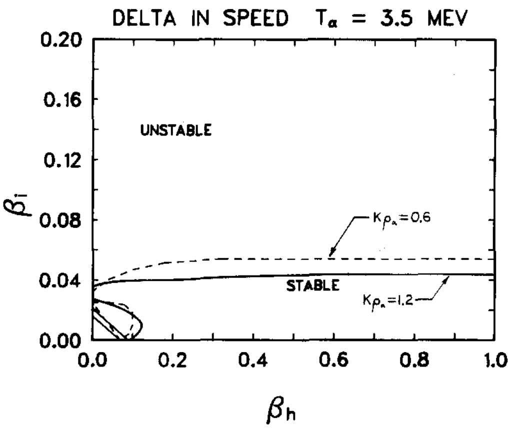

FIG. 3. The effect of newly born alphas on stability.

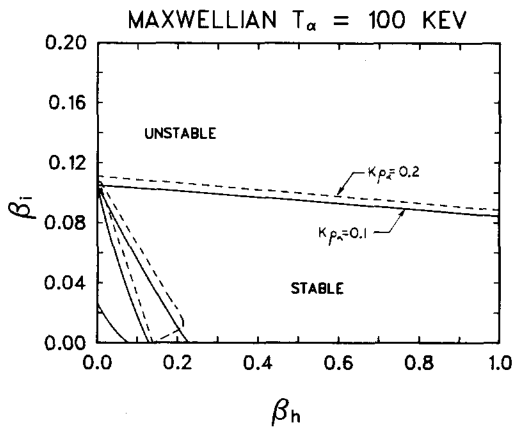

FIG. 5. Stability regions for alpha temperature at ten times the ion temperature.

In these expressions the average drift frequency $\overline{\omega}_D$ has already been defined in Eq. (20), while the quantity b_j is defined by

$$b_j = k_\perp^2 T_j / m_j \Omega_j, \tag{28}$$

and the integrals I_5 through I_{13} pertaining to the alpha particle are given by

$$I_5 = \xi \int f_{0\alpha} \frac{J_0^2(k_\perp v_\perp/\Omega) v_\perp \, dv_\perp \, dv_\parallel}{\omega - \omega_{0\alpha}},$$

$$I_6 = \xi \int \frac{f_{0\alpha} J_1^2 v_\perp \, dv_\perp \, dv_\parallel}{\omega - \omega_{0\alpha} + \Omega_\alpha}, \quad I_7 = \xi \int \frac{f_{0\alpha} J_1^2 v_\perp \, dv_\perp \, dv_\parallel}{\omega - \omega_{D\alpha} + \Omega_\alpha},$$

$$I_8 = \xi \int \frac{f_{0\alpha} J_0 J_0^{\prime} v_\perp^2 \, dv_\perp \, dv_\parallel}{\omega - \omega_{D\alpha}}, \quad I_9 = \xi \int \frac{f_{0\alpha} J_1 J_1^{\prime} v_\perp^2 \, dv_\perp \, dv_\parallel}{\omega - \omega_{D\alpha} + \Omega_\alpha}, \tag{29}$$

$$I_{10} = \xi \int \frac{f_{0\alpha} J_1 J_1^{\prime} v_\perp^2 \, dv_\perp \, dv_\parallel}{\omega - \omega_{D\alpha} - \Omega_\alpha}, \quad I_{11} = \xi \int \frac{f_{0\alpha} (J_0^{\prime})^2 v_\perp^3 \, dv_\perp \, dv_\parallel}{\omega - \omega_{D\alpha}},$$

$$I_{12} = \xi \int \frac{f_{0\alpha} (J_1^{\prime})^2 v_\perp^3 \, dv_\perp \, dv_\parallel}{\omega - \omega_{D\alpha} + \Omega_\alpha}, \quad I_{13} = \xi \int \frac{f_{0\alpha} (J_1^{\prime})^2 v_\perp^3 \, dv_\perp \, dv_\parallel}{\omega - \omega_{D\alpha} - \Omega_\alpha},$$

$$\xi = 2\pi[-(\Delta / R_c)\omega_{*\alpha}].$$

Since in the high-frequency case the dispersion relation is not quadratic in ω, though algebraic, the solution is obtained by plotting ω/Ω_i vs β_i, and an instability would occur when two real roots merge to form a double root.[7] That information is then utilized in plotting the stability boundary for the various conditions to be discussed below.

III. DISCUSSION OF RESULTS

As we noted earlier, the slowing down distribution function for the fast alphas represented by Eq. (7) does not lend itself readily to the needed analytical manipulations and as a result we replace it with the Maxwellian shown in Fig. 1. An alpha particle temperature of about 600 keV allowed for the good agreement displayed in the figure and for comparable first and second moments of both distributions. In order to provide a comparison with the stability results in the absence of alphas, we include Fig. 2 which shows the stability boundary for the background and interacting interchange modes obtained by several authors.[1,3,12] These results were obtained from the dispersion equation (15) when the low-frequency approximation was indeed invoked and a hot-electron temperature of T_h = 2.5 MeV was used. At the instant of birth the alpha particles are assumed to have a delta

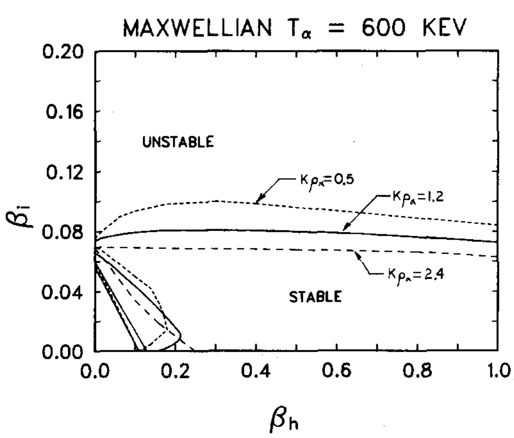

FIG. 4. Stability boundary in the presence of alphas.

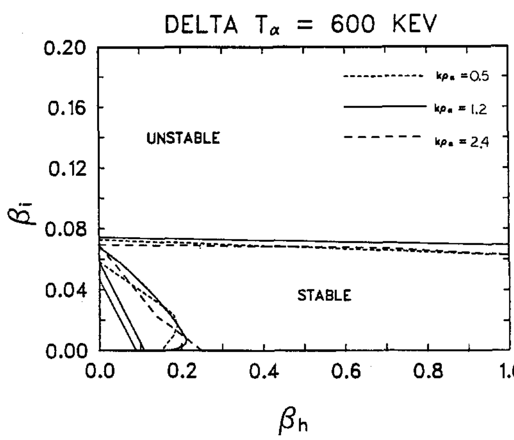

FIG. 6. Stability domain for a peaked distribution of hot alphas.

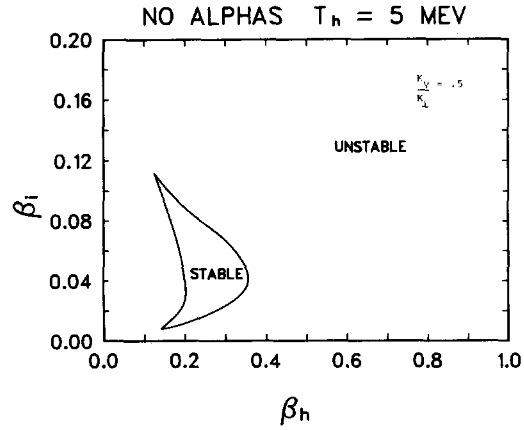

FIG. 7. Stability boundary as obtained in the "exact" calculation.

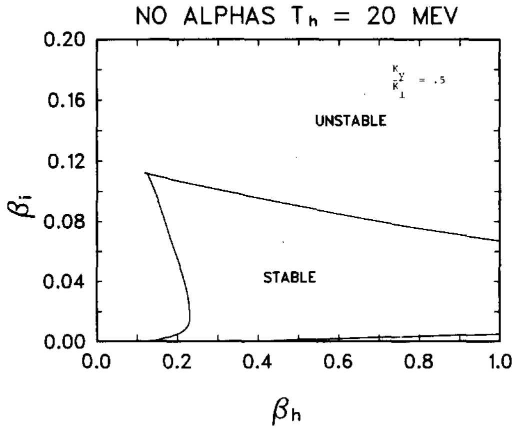

FIG. 9. Region of stability for a very hot electron ring in the absence of alphas.

function distribution in speed corresponding to T_α of 3.5 MeV, and the stability boundary in this case is shown in Fig. 3. We note immediately the destabilizing effect of the alphas on the interacting interchange mode with the concomitant stabilizing effect on the background interchange as reflected by the reduction in the hot-electron beta (β_h). Recovery of some of the stability of these modes is obtained when the alphas slow down, and this is vividly illustrated in Fig. 4 which gives the stability boundary for Maxwellian alphas at $T_\alpha \simeq 600$ keV. In all of these results the background plasma was assumed to have a temperature of 10 keV. Further recovery of stability is achieved as the alphas slow down further. This is depicted in Fig. 5 where T_α is taken to be 100 keV. It should be noted that in choosing this temperature no attempt was made to satisfy the steady-state condition invoked in arriving at the slowing down distribution and its concomitant Maxwellian; rather, the view is adopted that a Maxwellian distribution at 600 keV has slowed down to a temperature of 100 keV and the resulting stability boundary has changed from that in Fig. 4 to that in Fig. 5. On this basis complete recovery of the original stable regime is in principle achievable if the alphas are allowed to totally thermalize with the background ions, but this analysis does not include this case since Eq. (7) breaks down in this limit. It is clear that so long as the alphas remain reasonably hot relative to the background plasma, they will have a net destabilizing effect since they act much like the hot electrons in destabilizing the interacting interchange. Figure 6 is included to provide a simulation for a neutral beam heated EBT since the alphas in this instance were taken to have delta function distributions in both the perpendicular and parallel energies.

The results shown in the figures mentioned are somewhat inaccurate since they are based on a quadratic equation in ω which one obtains when the low-frequency approximation is invoked. If such an approximation were not made and the frequency was allowed to assume its value up to and including the ion gyrofrequency, then the "exact" value for the stability boundary would be obtained and the results might be significantly different from those of the low-frequency approximation. Similar observations were noted in Ref. 7. For example, in the absence of alphas the exact treatment shows that no stable region exists when $T_h = 2.5$ MeV and comparable conditions to those of Fig. 2 are used. When T_h is increased to 5 MeV, a small stable region appears as demonstrated in Fig. 7, which in turn would also be smaller had the full coupling condition between the wave and the drifts, i.e., $k_y/k_\perp = 1$, been used as was done in the case of Fig. 2. This trend can be seen in Fig. 8 where the stability boundary for two values of the coupling parameter is shown. Moreover, it can be seen from Figs. 7, 8, and 9 that the stabil-

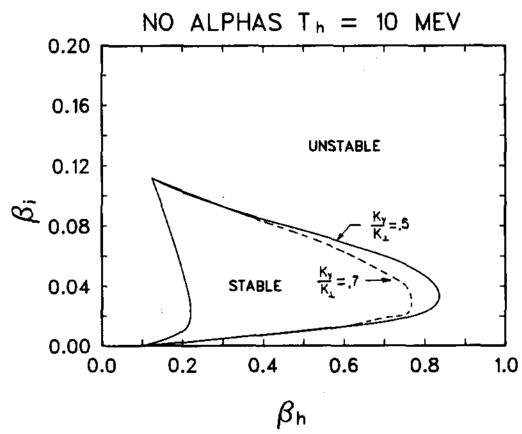

FIG. 8. Stability domain for two values of the coupling parameter.

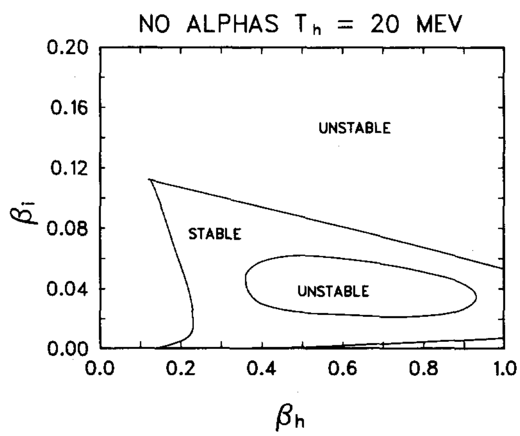

FIG. 10. Stability boundary for the high-frequency Alfvén waves.

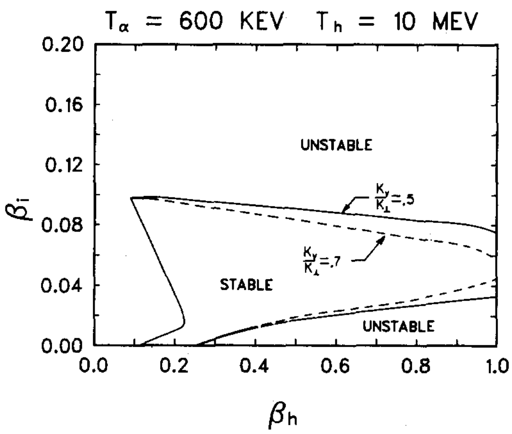

FIG. 11. The effect of fast alphas on stability for $T_h = 10$ MeV.

ity region is enhanced when the hot-electron temperature is increased from 5 to 20 MeV. The latter value of T_h is chosen in order to make T_h/T_i equal to 2000 and thus provide for comparison with other works[1,7] even though such a temperature may not be desirable from a reactor standpoint.[8] Comparison of Figs. 9 and 10 reveals that if all other parameters are kept the same and only the coupling parameter is increased from 0.5 to 0.7, a new unstable region (island) appears which is associated with the high-frequency Alfvén waves.

When the effects of alpha are included in the exact treatment, it can be seen from Figs. 8 and 11 on one hand, and Figs. 9 and 12 on the other, that the alphas tend to be slightly destabilizing of the (low-frequency) interacting interchange (as reflected in the region near small values of β_h) while stabilizing of the high-frequency modes. In fact, the presence of alphas completely stabilizes the Alfvén mode, as can be seen from Figs. 10, 12, and 13, where also the coupling parameter was increased from 0.5 to its full value of unity in order to dramatize the most unstable case.

IV. CONCLUSIONS

The stability of an ignited EBT reactor burning a mixture of deuterium and tritium was examined by assessing the effects of the alpha particles generated by such a fuel on the

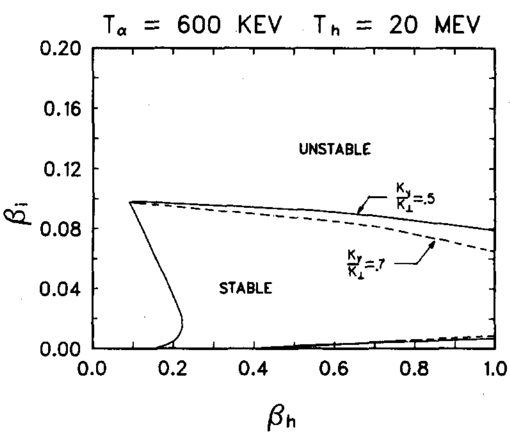

FIG. 12. The effect of alphas at a higher electron ring temperature.

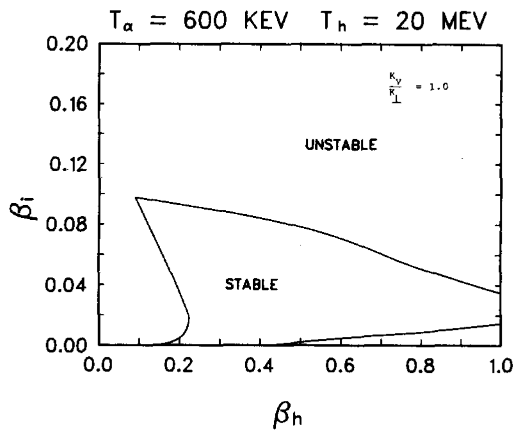

FIG. 13. Stabilization of Alfvén waves by the alpha particles.

stability of the background interchange modes, the interacting interchange modes, and the high-frequency compressional Alfvén and coupled modes. It is shown that in the absence of alphas a certain minimum value of the hot-electron temperature is required for stability. It is also shown that the alpha particles tend to stabilize the background interchange mode (through its sizable finite Larmor radius) while mildly destabilizing the interacting interchange mode. In addition, the alphas tend to completely stabilize the high-frequency Alfvén and coupled modes. The most pronounced destabilization by the alphas comes about when their energy is highest (at or near birth energy of 3.5 MeV), but significant recovery of stability is obtained as these particles slow down towards thermalization. In short, there appear to be no serious detrimental effects on stability that would arise from the presence of alpha particles in an EBT reactor that contains a hot-electron ring appropriately chosen to provide macroscopic stability.

ACKNOWLEDGMENT

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[1]D. B. Nelson, Phys. Fluids **23**, 1850 (1980).
[2]D. B. Nelson and C. L. Hedrick, Nucl. Fusion **19**, 283 (1979).
[3]J. W. VanDam and Y. C. Lee, in Proceedings of the Workshop on EBT Ring Physics, Oak Ridge, December 1979, Paper No. CONF-7911228, p. 471.
[4]H. L. Berk, J. W. VanDam, M. N. Rosenbluth, and D. A. Spong, Phys. Fluids **26**, 201 (1983).
[5]K. Nguyen and T. Kammash, Phys. Fluids **27**, 1483 (1984).
[6]K. T. Tsang and C. Z. Cheng, in *Proceedings of the Workshop on EBT Stability Theory*, Oak Ridge, May 1981 (ORNL, Oak Ridge, TN, 1981), p. 141.
[7]C. Z. Cheng and K. T. Tsang, Ref. 6, p. 161.
[8]S. K. Borowski, N. A. Uckan, E. F. Jeager, and T. Kammash, in Proceedings of the Workshop on EBT Ring Physics, Oak Ridge, December 1979, Paper No. CONF-7911228, p. 319.
[9]K. Nguyen and T. Kammash, Phys. Fluids **26**, 3085 (1983).
[10]T. Kammash, *Fusion Reactor Physics, Principles and Technology* (Ann Arbor Science Publishers, Ann Arbor, MI, 1975), Chap. 4.
[11]J. D. Hanson and E. Ott, Phys. Fluids **27**, 150 (1984).
[12]D. A. Spong (private communication).
[13]K. Nguyen, T. Kammash, and R. Kashuba, in *Proceedings of the Workshop in Hot Electron Ring Physics*, San Diego, December 1981 (ORNL, Oak Ridge, TN, 1982), p. 289.