New carrier-heated electron-hole instability in semiconductor plasmas*

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A study of the quasistatic hybrid mode in high-electron-mobility semiconductor plasmas subject to perpendicular static electric and magnetic fields shows that when carrier-heating effects are fully taken into account an entirely new mode arises as a result of the carrier heating. This mode has properties suitable for readily generating the two-stream instability and the interaction of this mode with hole-cyclotron harmonics is much stronger than that of any unheated mode.

I. INTRODUCTION

It is shown that in a semiconductor plasma of high electron mobility subject to crossed electric and magnetic fields, when carrier heating is taken into account, a new quasistatic hybrid carrier mode arises which is peculiar to the heated state. Moreover, this mode has significant attributes for generating the two-stream instability in materials such as InSb.

The distribution function which includes carrier-heating effects in the presence of applied static fields is derived in Sec. II assuming an equilibrium Maxwellian distribution function. The distribution function obtained is then used in full form to obtain the rf number density associated with the quasistatic hybrid mode in Sec. III. Examination of this mode in Sec. IV reveals a harmonic structure and in particular the presence of the zeroth harmonic component is directly related to carrier heating. This latter mode has a collisional damping which decreases substantially as the magnetic field is increased. In addition, those carriers of the velocity distribution function whose velocities are less than the carrier thermal velocity in the plane perpendicular to the static magnetic field are found (Sec. V) to contribute negatively to the electrokinetic energy density. Conversely, those carriers with velocities greater than the thermal velocity contribute positively to the electrokinetic energy density. If the magnetic field is sufficiently strong the low-velocity carriers dominate and the mode possesses a negative electrokinetic energy density.

In Sec. VI a study of the interaction of this carrier-heated electron mode with unheated holes reveals that an instability readily occurs near the harmonics of the hole-cyclotron frequency. It is important to note that this interaction is much stronger than any noncarrier-heated-mode interactions largely because the latter occur at much larger wave numbers.

II. DERIVATION OF THE HEATED CARRIER VELOCITY DISTRIBUTION FUNCTION

It is desired to determine the velocity distribution function for a drifting stream of charge carriers in a static magnetic field $\mathbf{B}_0 = B_0\hat{z}$ and static electric field $\mathbf{E}_0 = E_{0y}\hat{y}$. This distribution function is given as

$$f_0 = f_{0L} + f_{1L}, \tag{1}$$

where f_{0L} is the known equilibrium distribution function in the absence of any external fields and f_{1L} is the perturbation due to the applied static fields ($\mathbf{E}_0$, $\mathbf{B}_0$). Boltzmann's equation is written in the well-known form, with an assumed collision frequency ν,

$$\frac{\partial f_0}{\partial t} + \mathbf{v}\cdot\frac{\partial f_0}{\partial \mathbf{r}} + \frac{1}{m^*}\mathbf{F}\cdot\frac{\partial f_0}{\partial \mathbf{v}} = -\nu(f_0 - f_{0L}), \tag{2}$$

where $\mathbf{F}$ is the external force and m^* is the carrier effective mass. The function f_0 then satisfies the following relationship in the steady state, where η is the carrier charge to effective mass ratio:

$$\eta\left(\mathbf{E}_0\cdot\frac{\partial f_{0L}}{\partial \mathbf{v}} + \mathbf{E}_0\cdot\frac{\partial f_{1L}}{\partial \mathbf{v}} + (\mathbf{v}\times\mathbf{B}_0)\cdot\frac{\partial f_{1L}}{\partial \mathbf{v}}\right) = -\nu f_{1L}, \tag{3}$$

where

$$(\mathbf{v}\times\mathbf{B}_0)\cdot\frac{\partial f_{0L}}{\partial \mathbf{v}} = 0 \tag{4}$$

has been used corresponding to isotropic media, and the term $\eta\mathbf{E}_0\cdot(\partial f_{1L}/\partial\mathbf{v})$ is retained to account for the carrier heating. For the assumed field directions Eq. (3) becomes

$$v_y\frac{\partial f_{1L}}{\partial v_x} + \left(\frac{E_{0y}}{B_0} - v_x\right)\frac{\partial f_{1L}}{\partial v_y} + \frac{\nu}{\eta B_0}f_{1L} = -\frac{E_{0y}}{B_0}\frac{\partial f_{0L}}{\partial v_y}. \tag{5}$$

The velocity transformation

$$v_x \triangleq v_H + u\cos\theta;\quad v_H \triangleq E_{0y}/B_0,$$
$$v_y \triangleq u\sin\theta, \tag{6}$$

is introduced so that Eq. (5) may be written

$$\frac{\partial f_{1L}}{\partial\theta} - \frac{\nu}{\omega_c}f_{1L} = v_H\frac{\partial f_{0L}}{\partial v_y};\quad \omega_c \triangleq \eta B_0. \tag{7}$$

Equation (7) is readily solved[1] by employing an integrating factor, with the result

$$f_{1L} = v_H\exp\left(\frac{\nu}{\omega_c}\theta\right)\int_{c_1}^{\theta}\exp\left(-\frac{\nu}{\omega_c}\theta'\right)\frac{\partial f_{0L}}{\partial v_y}\,d\theta'. \tag{8}$$

where c_1 is a constant to be determined.

The equilibrium distribution function is now chosen to be Maxwellian, viz.,

$$f_{0L} = \frac{N_0}{(2\pi v_T^2)^{3/2}}\exp\left(-\frac{v_x^2+v_y^2+v_z^2}{2v_T^2}\right), \tag{9}$$

where N_0 is the equilibrium carrier number density and v_T is the carrier thermal velocity in the absence of any electric fields.

From Eqs. (6) and (9) it can be shown that

$$\frac{\partial f_{0L}}{\partial v_y} = -\frac{v_y}{v_T^2}f_{0L}$$

$$= -\frac{N_0}{(2\pi v_T^2)^{3/2}}\frac{u\sin\theta}{v_T^2}\exp\left(-\frac{v_H^2}{2v_T^2}\right)\exp\left(-\frac{u^2}{2v_T^2}\right)\times\exp\left(-\frac{v_z^2}{2v_T^2}\right)\exp\left(-\frac{uv_H}{v_T^2}\cos\theta\right), \quad (10)$$

and since

$$\exp\left(-\frac{uv_H}{v_T^2}\cos\theta\right) = \sum_{n=-\infty}^{\infty} I_n\left(-\frac{uv_H}{v_T^2}\right)\exp(jn\theta), \quad (11)$$

the following is true:

$$\frac{\partial f_{0L}}{\partial v_y} = g_0(u)\sin\theta \sum_{n=-\infty}^{\infty} I_n\left(-\frac{uv_H}{v_T^2}\right)\exp(jn\theta), \quad (12)$$

where

$$g_0(u) = -\frac{N_0}{(2\pi v_T^2)^{3/2}}\frac{u}{v_T^2}\exp\left(-\frac{v_H^2}{2v_T^2}\right)\times\exp\left(-\frac{u^2}{2v_T^2}\right)\exp\left(-\frac{v_z^2}{2v_T^2}\right). \quad (13)$$

Equation (8) is then readily solved to obtain

$$f_{1L}(\theta) = v_H g_0(u)\sum_{n=-\infty}^{\infty} I_n\left(-\frac{uv_H}{v_T^2}\right)\times\frac{(jn-\nu/\omega_c)\sin\theta - \cos\theta}{1+(jn-\nu/\omega_c)^2}\exp(jn\theta), \quad (14)$$

where the constant c_1 has been taken into account via the requirement that $f_{1L}(\theta) = f_{1L}(\theta + 2\pi)$.

The carrier distribution function in the presence of the applied static fields is then given from Eqs. (1), (9), and (14) as

$$f_0 = \frac{N_0}{(2\pi v_T^2)^{3/2}}\exp\left(-\frac{u^2+v_H^2+v_z^2}{2v_T^2}\right)\sum_{n=-\infty}^{\infty} I_n\left(-\frac{uv_H}{v_T^2}\right)\exp(jn\theta)\times\left(1-\frac{uv_H}{v_T^2}\frac{(jn-\nu/\omega_c)\sin\theta-\cos\theta}{1+(jn-\nu/\omega_c)^2}\right), \quad (15)$$

which, if desired, can be transformed back to the original (v_x, v_y, v_z) reference frame using Eq. (6). Inspection of Eq. (15) shows that to conserve particles properly it is necessary that

$$0 = \int\int_{-\infty}^{\infty}\int f_{1L}\,dv_x\,dv_y\,dv_z = -\frac{N_0 v_H}{2\pi v_T^4}\exp\left(-\frac{v_H^2}{2v_T^2}\right)\times\int_0^{2\pi}\int_0^{\infty}\frac{1}{1+(jn-\nu/\omega_c)^2}u^2\exp\left(-\frac{u^2}{2v_T^2}\right)I_n\left(-\frac{uv_H}{v_T^2}\right)\times[(jn-\nu/\omega_c)\sin\theta-\cos\theta]\exp(jn\theta)\,du\,d\theta, \quad (16)$$

and the integral on the right-hand side is found as

$$\sum_{n=-\infty}^{\infty}\frac{1}{1+(jn-\nu/\omega_c)^2}\int_0^{2\pi}\int_0^{\infty}u^2\exp\left(-\frac{u^2}{2v_T^2}\right)I_n\left(-\frac{uv_H}{v_T^2}\right)\times\{\exp[j(n+1)\theta][-j(jn-\nu/\omega_c)-1] + \exp[j(n-1)\theta][j(jn-\nu/\omega_c)-1]\}\,du\,d\theta$$
$$= 2\pi\int_0^{\infty}(-j)\frac{\omega_c}{\nu}u^2\exp\left(-\frac{u^2}{2v_T^2}\right)\left[I_1\left(-\frac{uv_H}{v_T^2}\right)-I_{-1}\left(-\frac{uv_H}{v_T^2}\right)\right] = 0, \quad (17)$$

wherein use was made of

$$I_{n+1}(x) - I_{n-1}(x) = -(2n/x)I_n(x). \quad (18)$$

If $v_H \ll v_T$ then it is clear that the $n=0$ term will dominate in Eq. (15) since for small arguments $|I_0| \gg |I_1|, |I_2|, |I_3|, \ldots$. Furthermore if $|\omega_c| \gg \nu$, inspection of Eq. (15) shows that if $v_H \ll v_T$ the following approximation can be made:

$$f_0 \approx f_{0L}(1 + v_H v_x/v_T^2). \quad (19)$$

Consider now the drifted Maxwellian distribution function

$$f_{0D} \triangleq \frac{N_0}{(2\pi v_T^2)^{3/2}}\exp\left(-\frac{(v_x-v_0)^2}{2v_T^2}\right)\exp\left(-\frac{v_y^2+v_z^2}{2v_T^2}\right), \quad (20)$$

where v_0 is the carrier drift velocity. Since the function f_{0D} is the displaced equilibrium distribution function f_{0L}, it is found by a Taylor expansion that

$$f_{0D} = f_{0L}[v_x \to (v_x - v_0)] = f_{0L}(v_x) - v_0\frac{\partial f_{0L}(v_x)}{\partial v_x} + \frac{v_0^2}{2!}\frac{\partial^2 f_{0L}(v_x)}{\partial v_x^2} + \cdots, \quad (21)$$

which for $v_0 \ll v_T$ yields

$$f_{0D} \simeq f_{0L}(1 + v_0 v_x/v_T^2). \quad (22)$$

Comparison of Eqs. (19) and (22) indicates that when $v_H \ll v_T$ and $|\omega_c| \gg \nu$ the distribution function given in Eq. (15) approximates a drifted Maxwellian with $v_0 = v_H$. For this reason it is commonly assumed that the rf field properties related to the carrier stream are unaffected unless the drift velocity is comparable to the thermal velocity. This assumption will be argued against by deriving the dispersion relation for the quasistatic hybrid mode using the distribution function given by Eq. (15).

III. THE QUASISTATIC HYBRID MODE: GENERAL SOLUTION

Assume that the quasistatic conditions are appropriate so that in the effective dielectric constant

$$\boldsymbol{\epsilon} = \mathbf{I} + (1/j\omega\epsilon)\sum_s \boldsymbol{\sigma}_s, \quad (23)$$

where $\mathbf{I}$ is the unit matrix and $\boldsymbol{\sigma}_s$ is the conductivity tensor of the sth carrier species; any element ϵ_{ij} is much smaller than the wave refractive index $|k^2c^2/\omega^2|$. Thus for the slow waves of interest if the rf field variation is taken as $\exp[j(\omega t - kx)]$ only the component E_{1x} is significant. The linearized Boltzmann equation, with $\mathbf{B}_0 = B_0\hat{z}$ and $\mathbf{E}_0 = E_{0y}\hat{y}$, is then given for each carrier species s by

$$j(\omega - kv_{xs})f_{1s} + \eta_s B_0\left(v_{ys}\frac{\partial f_{1s}}{\partial v_{xs}} - v_{xs}\frac{\partial f_{1s}}{\partial v_{ys}}\right) + \eta_s E_{0y}\frac{\partial f_{1s}}{\partial v_{ys}} + \eta_s E_{1x}\frac{\partial f_{0s}}{\partial v_{xs}} = -\nu_s\left(f_{1s} - \frac{N_{1s}}{N_{0s}}f_{0s}\right), \quad (24)$$

where the collision term which conserves particles properly has been taken on the right-hand side[2] and N_{1s} is the rf number density,

$$N_{1s} = \int\int_{-\infty}^{\infty}\int f_{1s}\,dv_{xs}\,dv_{ys}\,dv_{zs}. \quad (25)$$

The transformation of Eq. (6) can be applied to Eq. (24)

for each carrier species s to obtain

$$\frac{\partial f_{1s}}{\partial\theta}-j(a_s-b_s\cos\theta)f_{1s}=\frac{\eta_sE_{1x}}{\omega_{cs}}\left(\frac{\partial f_{0s}}{\partial v_{xs}}\right)-\frac{\nu_s}{\omega_{cs}}\frac{N_{1s}}{N_{0s}}f_{0s}, \quad (26)$$

where

$$a_s=\frac{\omega-kv_s-j\nu_s}{\omega_{cs}} \quad \text{and} \quad b_s=\frac{ku_s}{\omega_{cs}}. \quad (27)$$

An integrating factor is used to solve Eq. (26) with the following result:

$$f_{1s}=\exp[j(a_s\theta-b_s\sin\theta)]\int_{c_2}^{\theta}\left[\frac{\eta_sE_{1x}}{\omega_{cs}}\left(\frac{\partial f_{0s}}{\partial f_{xs}}\right)-\frac{\nu_s}{\omega_{cs}}\frac{N_{1s}}{N_{0s}}f_{0s}\right]\times\exp[-j(a_s\theta'-b_s\sin\theta')]d\theta', \quad (28)$$

where c_2 is a constant to be determined by the requirement $f_{1s}(\theta)=f_{1s}(\theta+2\pi)$. To evaluate the integrand in Eq. (28) it is necessary to solve explicitly for $\partial f_{0s}/\partial v_{xs}$. This is carried out in the Appendix for the distribution function f_{0s} of Eq. (15).

The Bessel function identities given by

$$\exp(\pm jb_s\sin\theta)=\sum_{m=-\infty}^{\infty}J_m(b_s)\exp(\pm jm\theta) \quad (29)$$

enable the solution of Eq. (28) to be written as follows:

$$f_{1s}=\sum_{l,m=-\infty}^{\infty}\exp(ja_s\theta)\exp(-jl\theta)J_l(b_s)J_m(b_s)\times\int_{c_1}^{\theta}\left[\frac{\eta_sE_{1x}}{\omega_{cs}}\left(\frac{\partial f_{0s}}{\partial v_{xs}}\right)-\frac{\nu_sN_{1s}}{\omega_{cs}N_{0s}}f_{0s}\right]\exp[j(m-a_s)\theta']d\theta'. \quad (30)$$

This integrand is then readily solved using Eqs. (15) and (A2). The result, together with the volume relationship

$$\int\int_{-\infty}^{\infty}\int dv_{xs}dv_{ys}dv_{zs}=\int_0^{\infty}\int_0^{2\pi}\int_{-\infty}^{\infty}u_s\,du_s\,d\theta\,dv_{zs}, \quad (31)$$

permits the rf number density of Eq. (25) to be found as (where now the carrier subscript s is suppressed for clarity)

$$N_1=N/D, \quad (32)$$

where

$$N=-\frac{\eta E_{1x}N_0\exp(-v_H^2/2v_T^2)}{v_T^2}\times\sum_{l,n=-\infty}^{\infty}\sum_{i=1}^{4}\int_0^{\infty}\frac{\exp(-u^2/2v_T^2)J_l(ku/\omega_c)\mathcal{E}_i(l,n)}{\omega-kv_H-j\nu-l\omega_c}u\,du, \quad (33)$$

$$D=1+j\frac{\nu}{v_T^2}\exp\left(-\frac{v_H^2}{v_T^2}\right)\sum_{l,n=-\infty}^{\infty}\int_0^{\infty}\frac{\exp(-u^2/2v_T^2)J_l(ku/\omega_c)I_n(-uv_H/v_T^2)}{\omega-kv_H-j\nu-l\omega_c}\times\left[J_{l-n}\left(\frac{ku}{\omega_c}\right)+\frac{juv_H\{2(jn-\nu/\omega_c)J'_{l-n}(ku/\omega_c)-j[2(l-n)\omega_c/ku]J_{l-n}(ku/\omega_c)\}}{2v_T^2[1+(jn-\nu/\omega_c)^2]}\right]u\,du, \quad (34)$$

and

$$J'_{l-n}\left(\frac{ku}{\omega_c}\right)=\frac{\omega_c}{k}\frac{dJ_{l-n}}{du}\left(\frac{ku}{\omega_c}\right), \quad (35)$$

and where now, with the arguments of I_n always understood as $-uv_H/v_T^2$,

$$\mathcal{E}_1(l,n)=\frac{1}{2j}\left[\frac{dI_n}{du}-\left(\frac{u}{v_T^2}+\frac{n}{u}\right)I_n\right]J_{l-n-1}\left(\frac{ku}{\omega_c}\right),$$

$$\mathcal{E}_2(l,n)=\frac{1}{2j}\left[\frac{dI_n}{du}-\left(\frac{u}{v_T^2}-\frac{n}{u}\right)I_n\right]J_{l-n+1}\left(\frac{ku}{\omega_c}\right),$$

$$\mathcal{E}_3(l,n)=\frac{v_H}{4v_T^2[1+(jn-\nu/\omega_c)^2]}\left\{u\frac{dI_n}{du}\left(j(n-1)-\frac{\nu}{\omega_c}\right)+I_n\left[\frac{n\nu}{\omega_c}-\left(jn-j-\frac{\nu}{\omega_c}\right)\frac{u^2}{v_T^2}\right]\right\}\left[J_{l-n-2}\left(\frac{ku}{\omega_c}\right)+J_{l-n+2}\left(\frac{ku}{\omega_c}\right)\right],$$

$$\mathcal{E}_4(l,n)=\frac{v_H}{v_T^2[1+(jn-\nu/\omega_c)^2]}\left\{-j\frac{u}{2}\frac{dI_n}{du}+I_n\left[j\left(\frac{u^2}{2v_T^2}-1\right)+\frac{n}{2}\left(jn-\frac{\nu}{\omega_c}\right)\right]\right\}J_{l-n}\left(\frac{ku}{\omega_c}\right). \quad (36)$$

IV. CHARACTERISTICS OF THE CARRIER-HEATED MODE IN THE RESONANCE APPROXIMATION

The general solution given by Eqs. (32)–(36) is too complex to analyze directly. Assume then that the carrier species of interest (e.g., electron) satisfies the condition $|\omega_c|\gg\nu$ even when a value for ν appropriate to the carrier-heated state is selected. In that case the resonance approximation is certainly reasonable over suitably restricted regions of (ω,k) space and Eq. (32) becomes

$$N_1\simeq-\frac{\eta E_{1x}N_0\exp(-v_H^2/2v_T^2)}{v_T^2(\omega-kv_H-j\nu'_l-l\omega_c)}\times\sum_{n=-\infty}^{\infty}\sum_{i=1}^{4}\int_0^{\infty}\exp\left(\frac{-u^2}{2v_T^2}\right)J_l\left(\frac{ku}{\omega_c}\right)\mathcal{E}_i(l,n)u\,du, \quad (37)$$

where ν'_l is the effective collision frequency for the lth resonance given by

$$\nu'_l=\nu\left(1-\left[\exp\left(\frac{-v_H^2}{v_T^2}\right)v_T^{-2}\right]\times\sum_{n=\infty}^{\infty}\int_0^{\infty}\exp\left(-\frac{u^2}{2v_T^2}\right)J_l\left(\frac{ku}{\omega_c}\right)I_n\left(-\frac{uv_H}{v_T^2}\right)\left\{J_{l-n}\left(\frac{ku}{\omega_c}\right)+juv_H\left[2\left(\frac{jn}{\nu/\omega_c}\right)J'_{l-m}\left(\frac{ku}{\omega_c}\right)-j\left(\frac{2(l-n)\omega_c}{ku}\right)J_{l-m}\left(\frac{ku}{\omega_c}\right)\right]\times\{2v_T^2[1+(jn-\nu/\omega_c)^2]\}^{-1}\right\}u\,du\right). \quad (38)$$

Particular attention is now given to the $l=0$ mode. The effects of electron carrier heating are studied by examining the perturbation of this mode from the nonheated state for the electron-hole interaction wherein for the electrons $|\omega_c|\gg\nu$ has been assumed.

In the absence of carrier heating the relevant quasistatic dispersion equation can be found either by solving the Boltzmann equation for the drifted-electron Maxwellian distribution function[3] ignoring $\mathbf{E}_0$ or by solving Eqs. (32)–(36) with $(v_H/v_T)\to 0$ and applying Poisson's equation. In either case it can be shown in the resonance approximation that

$$1+\frac{l\omega_c\omega_p^2\exp(-\lambda)I_l(\lambda)}{k^2v_T^2[l\omega_c-(\omega-kv_0-j\nu_l'')]}+[h]=0, \tag{39}$$

where $[h]$ represents the hole term which is not of direct significance, ω_p is the electron plasma frequency given by $\omega_p^2=\eta qN_0/\epsilon$, v_0 is the electron drift velocity, and

$$\lambda \triangleq (kv_T/\omega_c)^2. \tag{40}$$

The electron effective collision frequency for the lth resonance is given by

$$\nu_l''=\nu[1-\exp(-\lambda)I_l(\lambda)]. \tag{41}$$

If Eq. (39) is solved for $l=0$ it may be assumed that there exists a *noninteracting* electron carrier wave defined by

$$\omega_r=kv_0, \tag{42}$$

with damping decrement,

$$\omega_i=\nu_0''=\nu[1-\exp(-\lambda)I_0(\lambda)], \tag{43}$$

so that as $\lambda\to 0$ (i.e., $B_0\to\infty$), $\omega_i\to 0$ and this mode is therefore undamped.

On the other hand, the dispersion equation in the resonance approximation for $l=0$ with the effects of carrier heating included can be obtained from Eq. (37) together with Poisson's equation:

$$\nabla\cdot\mathbf{E}_1=-jkE_{1x}=\sum_s(q_s/\epsilon)N_{1s}, \tag{44}$$

where q_s is the carrier charge, as

$$1+\frac{j\omega_p^2\exp(-v_H^2/2v_T^2)}{kv_T^2(\omega-kv_H-j\nu_0')}\sum_{n=-\infty}^{\infty}\sum_{i=1}^{4}\int_0^\infty\exp\left(-\frac{u^2}{2v_T^2}\right)\times J_0\left(\frac{ku}{\omega_c}\right)\mathcal{E}_i(0,n)u\,du+[h]=0, \tag{45}$$

where again $[h]$ represents the hole term. Inspection of this result indicates that one of the effects of carrier heating is to perturb the $l=0$ electron carrier wave away from the solution given by Eqs. (42) and (43) such that it is now in general an interacting mode. Since the solution of Eq. (43) yields a mode approaching marginal instability as the magnetic field increases, this perturbation need not be large to drive the root into the instability region. Note also that the adopted viewpoint in which the growth rate ω_i must exceed the hole collision frequency[4] (when the dispersion equation is solved with the latter set to zero) is erroneous due to the presence of the static magnetic field.

A further effect of carrier heating is seen by examining the electron drift velocity parallel to the wave vector. For the present case this is obtained from Eq. (15) for the $\hat{x}$ direction as

$$v_0=(1/N_0)\iiint_{-\infty}^{\infty}v_xf_0\,dv_x\,dv_y\,dv_z=v_H+v_\theta,$$

where

$$v_\theta=-\frac{v_H}{2\pi v_T^4}\exp\left(-\frac{v_H^2}{2v_T^2}\right)\sum_{n=-\infty}^{\infty}\frac{1}{1+(jn-\nu/\omega_c)^2}\times\int_0^{2\pi}\int_0^\infty u^3\exp\left(-\frac{u^2}{2v_T^2}\right)I_n\left(-\frac{uv_H}{v_T^2}\right)$$

$$\times\left[\left(jn-\frac{\nu}{\omega_c}\right)\sin\theta\cos\theta-\cos^2\theta\right]\exp(jn\theta)\,du\,d\theta,$$

$$=\frac{v_H}{2v_T^4}\exp\left(-\frac{v_H^2}{2v_T^2}\right)\int_0^\infty\frac{u^3}{1+\nu^2/\omega_c^2}\exp\left(-\frac{u^2}{2v_T^2}\right)\times\left[I_0\left(-\frac{uv_H}{v_T^2}\right)-I_2\left(-\frac{uv_H}{v_T^2}\right)\right]du,$$

$$=\frac{v_H}{1+\nu^2/\omega_c^2}\left\{\exp\left(-\frac{v_H^2}{2v_T^2}\right)\left[M\left(2;1;\frac{v_H^2}{2v_T^2}\right)\right]-\frac{v_H^2}{2v_T^2}\right\},$$

M is the confluent hypergeometric function[5] and $v_H>0$ has been assumed.

Inspection of this result for the range of interest, $v_H\leq v_T$, indicates that $v_0>v_H$. To approximate the effect of the carrier heating the holes are ignored and the coupling term is assumed negligible in Eq. (45) so that

$$\omega-kv_H-j\nu_0'\simeq 0. \tag{46}$$

This indicates that the perturbation from the synchronous unheated state described by Eq. (42) is such that $(\omega_r/k)<v_0$. The above suggests that the $l=0$ carrier-heated mode has a negative electrokinetic energy density and this aspect is now examined.

V. ELECTROKINETIC ENERGY DENSITY OF THE CARRIER-HEATED HYBRID MODE

Since the quasistatic assumption has been made, $\nabla\times\mathbf{B}_1=0$ and it follows that

$$\sum_s J_{1x}^{(s)}+j\omega\epsilon E_{1x}=0, \tag{47}$$

from which it can readily be found that

$$\sum_s W_k^{(s)}+\tfrac{1}{2}\epsilon|E_{1x}|^2=0, \tag{48}$$

where $W_k^{(s)}$ is the electrokinetic energy density of the sth carrier species and is defined by

$$W_k^{(s)}=-(2\omega_i)^{-1}\mathrm{Re}(E_{1x}^*J_{1x}^{(s)});\quad \omega_i\neq 0. \tag{49}$$

Equation (48) indicates that $\Sigma_s W_k^{(s)}$ must be negative for unstable ($\omega_i<0$) interaction to occur. From Eq. (47) and Poisson's equation, Eq. (44), it can be shown that

$$\sum_s J_{1x}^{(s)}=(\omega/k)\sum_s q_sN_{1s}, \tag{50}$$

so that if the carrier species are assumed to be separately conserved in number, then $J_{1x}^{(s)}=(\omega/k)q_sN_{1s}$.

To apply this concept to the present case, since the complexity of Eqs. (32)–(36) precludes direct analysis, it is assumed that $v_H\ll v_T$ so that in the integrand of Eq. (33) the major contribution will come from the $I_0(-uv_H/v_T^2)$ and dI_1/du terms in Eq. (36). In addition, since $l=0$ the integrand of Eq. (33) contains $J_0(ku/\omega_c)$ so that those functions $\mathcal{E}_i(0,n)$ whose n value also introduce a factor $J_0(ku/\omega_c)$ will correlate to give the largest products. This is especially true when $|\omega_c|>kv_T$ (e.g., $|\omega_c|=10kv_T$) since in addition to the correlation property the exponential factor in the integrand limits u to values such that $J_0\gg|J_1|$, $|J_2|$, etc. Proceeding in this manner it is found that, if $|\omega_c|\gg\nu$,

$$\sum_{n=-\infty}^{\infty}\sum_{i=1}^{4}\mathcal{E}_i(0,n)\simeq\frac{jv_H}{v_T^2}\left(\frac{u^2}{2v_T^2}-1\right)J_0\left(\frac{ku}{\omega_c}\right), \tag{51}$$

where $I_0(-uv_H/v_T^2) \simeq 1$ has been used, and from Eq. (38)

$$\nu_0' \simeq \nu_0'' = \nu[1 - \exp(-\lambda)I_0(\lambda)]. \qquad (52)$$

Applying Eq. (51) to Eq. (37) with $l = 0$ we obtain the following form for the charge density:

$$\rho_1 \triangleq qN_1 \simeq -\frac{j\omega_p^2 \epsilon E_{1x} v_H [\exp(-v_H^2/2v_T^2)]}{2v_T^4(\omega - kv_H - j\nu_0')} I(\lambda), \qquad (53)$$

where

$$I(\lambda) \triangleq 2\int_0^\infty \exp\left(-\frac{u^2}{2v_T^2}\right) uJ_0^2\left(\frac{ku}{\omega_c}\right)\left(\frac{u^2}{2v_T^2} - 1\right) du, \qquad (54)$$

which can be expressed as a real-valued function of λ in the form[6]

$$I(\lambda) = v_T^2 e^{-\lambda}[(\tfrac{3}{2}\lambda - 2)I_0(\lambda) + (1 - 2\lambda)I_1(\lambda) + \tfrac{1}{2}\lambda I_2(\lambda)]. \qquad (55)$$

If the wave number k is assumed real, the electrokinetic energy density is then derived using Eqs. (49), (50), and (53) with $\omega = \omega_r + j\omega_i$ as

$$W_k = \frac{\omega_p^2 \epsilon \,|\, E_{1x} \,|^2 v_H [\exp(v_H^2/2v_T^2)][kv_H - (\omega_r/\omega_i)\nu_0']}{4kv_T^4 \,|\, \omega - kv_H - j\nu_0' \,|^2} I(\lambda). \qquad (56)$$

Without loss of generality it can be assumed that ω_r, k, $v_H > 0$ so that $\omega_i < 0$ corresponds to unstable field growth. Inspection of Eq. (56) shows that the sign of W_k is dependent upon the sign of $I(\lambda)$. By comparison with Eq. (54) it is seen that those carriers of the heated Maxwellian distribution which have an "effective" thermal speed perpendicular to the static magnetic field $\{u = [(v_x - v_H)^2 + v_y^2]^{1/2}\}$ less than the carrier velocity $\sqrt{2} \times v_T$ give a negative contribution to the carrier electrokinetic energy density. Similarly, those carriers of the distribution f_0 with $u > \sqrt{2}\, v_T$ give a positive contribution. Inspection of Eq. (55) shows that the value of λ $[\lambda = (kv_T/\omega_c)^2]$ determines which of these two carrier groups dominates and that, approximately, if $\omega_c \gtrsim \frac{1}{2}\sqrt{3} \times kv_T$, the carrier mode possesses a negative electrokinetic energy density. On the other hand when $\omega_c \lesssim \frac{1}{2}\sqrt{3} \times kv_T$ the electrokinetic energy density is positive and there will now be a contribution to the wave damping in addition to the collisional damping due to this thermal effect.

To some extent this result is intuitively reasonable since it is to be expected that the motion of the hot carriers of the distribution will be a regular in-phase charge oscillation [i.e., $\mathrm{Re}(\rho_1/E_{1x}) > 0$ for such carriers], whereas on the other hand the cool carriers ($u < \sqrt{2}\, v_T$) are strongly affected by the carrier heating and execute out-of-phase oscillations with $\mathrm{Re}(\rho_1/E_{1x}) < 0$.

Thus the electron carrier heating provides additional motion of the electrons by means of which they can interact with the holes and supply energy for the field growth. It now only remains to verify that this hybrid-hybrid mode interaction is unstable.

VI. INSTABILITY OF THE CARRIER-HEATED $l = 0$ ELECTRON HYBRID MODE INTERACTING WITH UNHEATED HOLES

If it is assumed that for the holes $\nu_h \gg |\,\omega_{ch}\,|$, inspection of Eqs. (32)–(36) indicates that carrier heating should have little effect on hole motion especially if $l \neq 0$. In order to study the interaction of the carrier-heated $l = 0$ electron mode with hole cyclotron harmonics the unheated form of Eq. (39) is selected for the hole contribution to the dispersion relation whereas the approximate form developed for the electrons in Eq. (53) is used to determine the electron contribution using Poisson's equation, Eq. (44).

The dispersion equation is then found as

$$1 = \frac{\Gamma_e}{\omega - kv_H - j\nu_0'} + \frac{m\omega_{ch}\Theta_h}{\omega - j\nu_h'' - m\omega_{ch}}, \qquad (57)$$

where the hole effective collision frequency ν_h'' is found from Eq. (41) for the m value chosen,

$$\Gamma_e \triangleq \frac{\omega_{pe}^2 v_H [\exp(-v_H^2/2v_{Te}^2)]}{2kv_{Te}^4} I(\lambda_e), \qquad (58)$$

$$\Theta_h \triangleq \frac{\omega_{ph}^2}{k^2 v_{Th}^2} \exp(-\lambda_h) I_m(\lambda_h), \qquad (59)$$

and where

$$\lambda_s \triangleq \left(\frac{kv_{Ts}}{\omega_{cs}}\right)^2; \quad s = e, h \qquad (60)$$

has been introduced to distinguish the carrier species. Note that $\Gamma_e < 0$ if and only if $I(\lambda_e) < 0$ in the reference frame of interest (i.e., ω_r, k, $v_H > 0$). The solution for real k is

$$2\omega = S_h + S_e \pm [(S_h - S_e)^2 + 4\Gamma_e m\omega_{ch}\Theta_h]^{1/2}, \qquad (61)$$

where

$$S_h = m\omega_{ch}(1 + \Theta_h) + j\nu_h'' \qquad (62)$$

and

$$S_e = kv_H + \Gamma_e + j\nu_0'. \qquad (63)$$

To illustrate the instability select a k value which satisfies

$$kv_H + \Gamma_e = m\omega_{ch}(1 + \Theta_h) \qquad (64)$$

and use this in Eq. (61) to find the solution separates as

$$\omega_r = m\omega_{ch}(1 + \Theta_h)$$

and

$$2\omega_i = (\nu_h'' + \nu_0') \pm (\nu_h'' - \nu_0')\left(1 - \frac{4\Gamma_e m\omega_{ch}\Theta_h}{(\nu_h'' - \nu_0')^2}\right)^{1/2}. \qquad (65)$$

Inspection shows that for $\omega_r > 0$ it is necessary that Γ_e and hence $I(\lambda_e) < 0$, or equivalently from Eq. (56), $W_k^{(e)} < 0$. If the lower sign is chosen in Eq. (65) and coupling is assumed small so that the square root is formally approximated by $(1 - \delta)^{1/2} \simeq 1 - \frac{1}{2}\delta$, Eq. (65) leads to the condition for instability ($\omega_i < 0$) that

$$\frac{|\,\Gamma_e\,|\, m\omega_{ch}\Theta_h}{|\,\nu_h'' - \nu_0'\,|} > \nu_0'; \quad \Gamma_e < 0. \qquad (66)$$

This condition should be easily satisfied at a fixed value of E_{0y} if the magnetic field is sufficiently large since it is seen in the limiting case $B_0 \to \infty$ that $\nu_0' \to 0$ [Eq. (52)] concurrent with $\Gamma_e < 0$ from Eqs. (55) and (58).

It is of interest to compare the strength of this interaction with that associated with Eq. (39); namely, the interaction of the fundamental ($l=1$) electron cyclotron mode with the hole harmonics described in Eq. (57), with carrier heating neglected. The relevant dispersion equation is found from Eqs. (39) and (57) as

$$1+\frac{|\omega_{ce}|\Theta_e}{(\omega-kv_H-j\nu_1''+|\omega_{ce}|)}-\frac{m|\omega_{ch}|\Theta_h}{(\omega-j\nu_h''-m|\omega_{ch}|)}=0, \tag{67}$$

where

$$\Theta_e=(\omega_{pe}^2/k^2v_{Te}^2)\exp(-\lambda_e)I_1(\lambda_e) \tag{68}$$

and the hole term is identical with that of Eq. (57). Corresponding to the wave number near the instability of Eq. (64) it is found for the present case that

$$kv_H=|\omega_{ce}|(1+\Theta_e)+m|\omega_{ch}|(1+\Theta_h), \tag{69}$$

and again $\omega_r=m|\omega_{ch}|(1+\Theta_h)$. In addition corresponding to the condition for the instability of Eq. (66) it is found that

$$\frac{\omega_{pe}^2\omega_{ph}^2 m|\omega_{ch}||\omega_{ce}|\exp(-\lambda_e)I_1(\lambda_e)\exp(-\lambda_h)I_m(\lambda_h)}{k^4v_{Te}^2v_{Th}^2|\nu_h''-\nu_1''|}>\nu_1''. \tag{70}$$

For direct comparison Eq. (66) for the $l=0$ carrier-heated mode is written in fully expanded form with $\lambda_e \ll 1$ as

$$\{\omega_{pe}^2\omega_{ph}^2 m|\omega_{ch}|kv_H[\exp(-v_H^2/2v_{Te}^2)]\exp(-\lambda_e)I_0(\lambda_e)$$
$$\times\exp(-\lambda_h)I_m(\lambda_h)\}(k^4v_{Te}^2v_{Th}^2|\nu_h''-\nu_0'|)^{-1}>\nu_0'. \tag{71}$$

For the same system parameters and working frequency ($\omega_r\ll|\omega_{ce}|$) comparison of Eqs. (70) and (71) shows that instability is much more readily achieved (and the growth rate ω_i is much larger) for the carrier-heated $l=0$ mode case than for the unheated $l=1$ mode case. The reasons for this are as follows:

(i) $\nu_1''\gg\nu_0'$, $\lambda_e\ll 1$;

(ii) $I_0(\lambda_e)\gg I_1(\lambda_e)$, $\lambda_e\ll 1$;

(iii) The interaction associated with Eq. (70) occurs at $k\simeq|\omega_{ce}|/v_H$ when $m|\omega_{ch}|\ll|\omega_{ce}|$, whereas for the interaction associated with Eq. (71) $k\simeq\omega_r/v_H$. This fact alone makes the left-hand side of Eq. (71) a factor of $(|\omega_{ce}|/m|\omega_{ch}|)^3$ larger than the left-hand side of Eq. (70)

VII. DISCUSSION AND CONCLUSIONS

The carrier-heated $l=0$ electron hybrid mode is an important new wave in solid-state plasmas. In the noninteracting state it is similar to the helicon wave in that $v_0 \gtrsim \omega_r/k$ and also collisional damping can be decreased to negligible values by the static magnetic field. It possesses the advantage over the helicon wave that although the latter is restricted to the upper frequency limit,

$$\omega_{max}=\omega_p^2v_0^2/|\omega_c|c^2 \quad (|\omega_c|\gg\nu), \tag{72}$$

where

$$c=(\mu_0\epsilon)^{-1/2} \tag{73}$$

and μ_0 is the permeability of free space, the carrier-heated mode with $|\omega_c|\gg\nu$ must only satisfy $\omega_r\ll|\omega_c|$ and hence this mode can exist at much higher frequencies than the helicon. It possesses the disadvantage that the Poynting vector (although nonzero in actuality) is sharply reduced from the magnitude associated with helicon-wave interactions.

Although no numerical work has been completed it is believed that the carrier-heated mode must be responsible for part of the microwave emission which has been reported in InSb. This is because of the strong nature of the carrier-heated electron-hole interaction and the fact that experimental evidence[7] supports the thesis of interactions occurring at the hole cyclotron harmonics.

It is also pointed out that in the present study the field E_{0y} need not be the applied electric field but can represent the Hall field if this is much larger than the applied field. Additional studies, not presented here, show that when ($\mathbf{E}_0 \parallel \mathbf{k} \perp \mathbf{B}_0$) a new $l=0$ mode arises due to carrier heating although this mode is not synchronous in nature.

The present work suggests that it could be of interest to study carrier-heating effects associated with the cyclotron normal-mode geometry ($\mathbf{k}\parallel\mathbf{B}_0\parallel\mathbf{E}_0$) to determine whether or not similar wave phenomena can occur. With minor modifications the present work is of course also applicable to carrier-heated electron-ion interactions in gaseous plasmas.

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APPENDIX

For clarity the carrier species subscript s will be taken as implicitly understood in the following. From Eq. (6) it is true that

$$\frac{\partial f_0}{\partial v_x}=\frac{\partial f_0}{\partial u}\left(\frac{\partial u}{\partial v_x}\right)+\frac{\partial f_0}{\partial\theta}\left(\frac{\partial\theta}{\partial v_x}\right)=\cos\theta\left(\frac{\partial f_0}{\partial u}\right)-\frac{\sin\theta}{u}\left(\frac{\partial f_0}{\partial\theta}\right). \tag{A1}$$

This form is then applied to the carrier-heated distribution function, Eq. (15), to obtain

$$\frac{\partial f_0}{\partial v_x}=\frac{N_0}{(2\pi v_T^2)^{3/2}}\sum_{n=-\infty}^{\infty}\exp\left(-\frac{u^2+v_H^2+v_z^2}{2v_T^2}\right)\exp(jn\theta)$$
$$\times\left\{\left[\cos\theta\left(\frac{dI_n}{du}-\frac{u}{v_T^2}I_n\right)-j\frac{n}{u}\sin\theta I_n\right]\right.$$
$$\times\left(1-\frac{uv_H}{v_T^2}\frac{(jn-\nu/\omega_c)\sin\theta-\cos\theta}{1+(jn-\nu/\omega_c)^2}\right)$$
$$\left.+\frac{v_H}{v_T^2[1+(jn-\nu/\omega_c)^2]}I_n\right\}, \tag{A2}$$

where the argument of I_n is understood as $-uv_H/v_T^2$.

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