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Flow Field of a Flame in a Channel

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A stationary flame is stabilized in a two-dimensional channel in such a way that it closely corresponds to the case of a flame propagating in a channel. The entire flow field of unburned and burned gases is mapped by taking stroboscopic photographs of small particles suspended in the combustible gases. The asymptotic flow field of the burned gases is analyzed in terms of flow conditions at the flame. The possibility of obtaining an analytic flow field on the basis of constant flame speed is examined. It appears that no solution exists unless the channel walls in the region of burned gases are displaced inward. This investigation, together with the previous work on the flow field of a Bunsen flame, leads to some general remarks about the structure and stability of laminar flames. The various predictions are in agreement with experiments. In particular, the convex flames are possible and concave ones impossible within the framework of the theory based on constant flame speed.

INTRODUCTION

IT HAS long been recognized that the flow field of a laminar flame affects its shape and stability. Quite early Michelson[1] calculated the flame shape of a Bunsen flame by assuming that fully developed laminar flow in a pipe remains unchanged as it approaches the flame. His calculations were based entirely on the *kinematics* of the flow and no attempt was made to show that the flow of unburned and burned gases associated with the calculated flame shape is dynamically possible. Until recently the theoretical and experimental work has been limited to flame shape and in some cases to kinematics of the flow at the flame. Although the importance of dynamics of the flow has been recognized, because of the complexity of the problem not much work has been done along this line.

In the present paper the emphasis is on the dynamics rather than the kinematics of the flow. In order to make the problem tractable, it is necessary to assume that (i) the zone of combustion can be replaced by a surface of discontinuity across which the density drops and correspondingly the normal velocity increases and the tangential velocity is continuous; (ii) the flow is inviscid on either side of the flame; (iii) the velocity of the unburned gases normal and relative to the flame is constant, i.e., the flame speed is constant; (iv) the flow on both sides of the flame is incompressible and the density has a constant value of ρ_1 in the unburned and ρ_2 in the burned gases. The inclusion of flame quenching and viscous effects at the walls or the flame holder raises serious problems and it is hoped that useful information can be obtained within this framework.

The simplest problem is the flow associated with a flame propagating with a uniform velocity in a two-dimensional channel. The unsteady problem can be reduced to the steady case by superimposing a uniform velocity in the opposite direction. There is no satisfactory explanation for the fact that the observed flame is always *convex* toward the unburned gases. The curved flame shape is not entirely due to viscous effects at the walls; if this were true a shock wave would also be curved. A straight flame is not realized in a channel because it is unstable. In fact, Landau[2] has shown that a density discontinuity moving with a uniform velocity is unstable if the density across it decreases as in the case of a flame and is stable if the density across it increases as in the case of a shock wave.

The next, in order of increasing difficulty, is the flow field of a Bunsen flame for which the free streamlines of the burned gases as well as the flame shape are unknown boundaries which have to be determined along with the flow field. This case has been studied experimentally and theoretically.[3] Although a complete solution was not possible, enough information was obtained to understand the dynamics of the flow field of a Bunsen flame. Some of the results are generally applicable and are therefore summarized here. The vorticity generated by a two-dimensional flame can be determined from the flow conditions at the flame, thus

$$\rho_1 u_{1_n}(\omega_2 - \omega_1) = \left(\frac{\rho_1 - \rho_2}{\rho_2}\right) u_{1_n} \frac{\partial \rho_1 u_{1_n}}{\partial s} + \frac{\rho_1 - \rho_2}{2} \frac{\partial u_t^2}{\partial s}, \quad (1)$$

where ω is the vorticity, u_{1_n} the flame speed, u_t the

[1] W. Michelson, Ann. Physik. Chem. (Widemann) 37 (1889).

[2] L. Landau, Acta Physicochim. U. R. S. S. 19, 77 (1944).

[3] Uberoi, Kuethe, and Menkes, Phys. Fluids 1, 150 (1958).

tangential velocity at the flame, s the distance along the flame, and the subscripts 1 and 2 denote the conditions in the unburned and burned gases, respectively. The asymptotic velocity at $x = +\infty$ of a streamline in the burned gases is given terms of its initial velocity at $x = -\infty$ in the unburned gases and its condition at the flame front,

$$\tfrac{1}{2}\rho_2 u_{+\infty}^2 = (p_{-\infty} - p_{+\infty}) + \tfrac{1}{2}\rho_1 u_{-\infty}^2 - \tfrac{1}{2}\rho_1 u_{1_n}^2(\rho_1/\rho_2) - \tfrac{1}{2}\rho_1 u_t^2(1 - \rho_2/\rho_1). \quad (2)$$

In Eqs. (1) and (2) we have assumed inviscid flow but the flame speed and the density jump can vary along the flame. By considering the curvatures of the flame, the streamlines going through the flame, and the asymptotic velocities, it was possible to show that *no solution exists for a flame concave toward the unburned gases* (e.g., Bunsen flame tip) *within the framework of the theory based on constant flame speed.*

FLOW FIELD OF A FLAME PROPAGATING IN A CHANNEL

We now discuss some details of the flow field of a flame propagating in a channel when reduced to the steady case by superimposing uniform flow in the direction opposite that of propagation (see Fig. 1). The flow of the unburned gases is uniform at $x = -\infty$ and the flow of the burned gas at $x = +\infty$ is parallel but nonuniform due to the vorticity generated by the flame. At the flame the normal velocity has a constant value u_{1_n} in the unburned and $u_{1_n}\rho_1/\rho_2$ in the burned gases and the flow is bent toward the normal through an angle

$$\theta = \tan^{-1}\left\{\frac{u_t}{u_{1_n}}\left(1 - \frac{\rho_2}{\rho_1}\right)\bigg/\left(1 + \frac{\rho_2 u_t^2}{\rho_1 u_{1_n}^2}\right)\right\}. \quad (3)$$

The central and the wall streamlines must not suffer any deflection going through the flame. Therefore the flame is either normal ($u_t = 0$) or tangent ($u_t = \infty$), to the flow at these points. Equation (2) shows that as long as there is a finite density jump across the flame, neither the normal nor the tangential velocity can become infinite since this would make $(p_{-\infty} - p_{+\infty})$ infinite. Consequently the flame cannot be tangent to the wall. If the flame is normal to the wall then it has concave curvature which is not possible within the framework of the theory based on constant flame speed. The net result is that *no* solution exists for a flame of constant flame speed propagating in a channel. However, under the same assumption as that used above, Ball[4] has obtained numerically a solution for the case at hand. In fact he finds that, within the accuracy of his numerical work, the flame tends to be tangent to the wall and correspondingly the velocity is becoming infinite. He failed to notice that the flame cannot become tangent to the wall. His numerical solution violates the boundary conditions at the wall and he effectively assumes that the flame meets the wall at a small angle and therefore the wall streamline, after going through the flame, becomes detached from the wall.

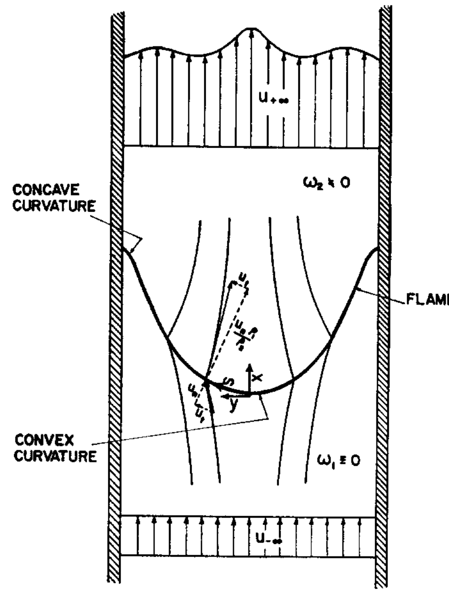

FIG. 1. Flow field of a flame in a channel.

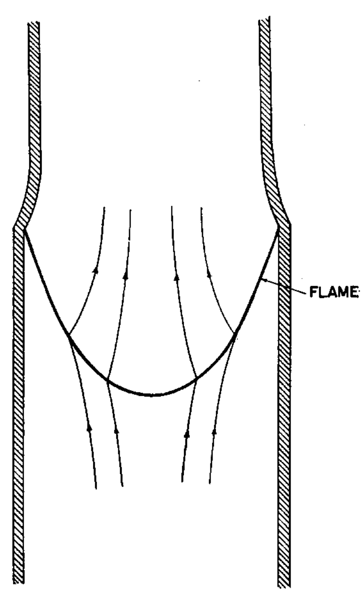

FIG. 2. The channel wall shape effectively assumed by Ball in his numerical calculation of the flow field.

[4] G. Ball, "Two-dimensional flame in a laminar flow channel," Harvard University Combustion Tunnel Lab. Rept. (1951).

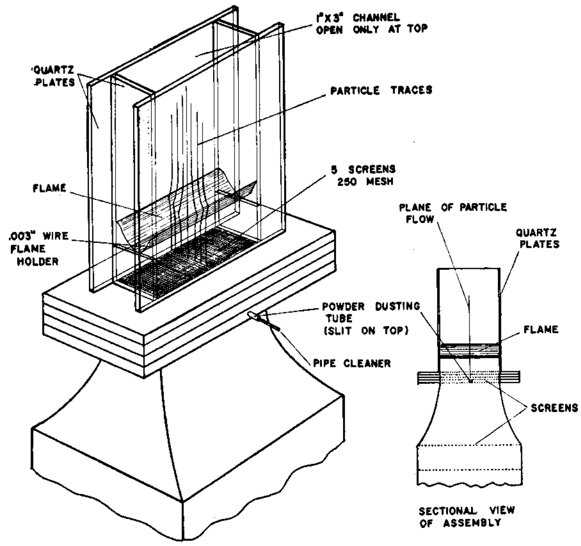

FIG. 3. Sketch of the burner assembly.

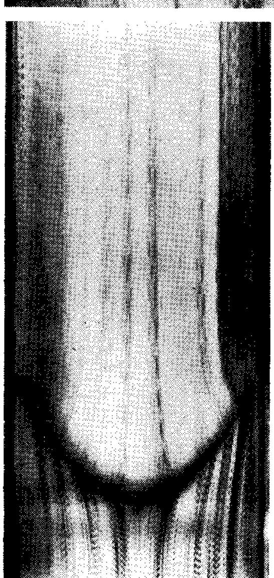

FIG. 4. Photographs of the flow field.

The fact that he got parallel flow far downstream means that he assumed effectively that the walls in the burned gases are displaced inward from their original position (see Fig. 2). The solution is no longer unique and depends on the inward displacement of the wall.

Next, we want to examine in detail the measured flow field to find (i) how close the actual flow comes to the ideal conditions considered above; (ii) the effect of the boundary layer and quenching at the walls; and (iii) a consistent set of assumptions which can be used as a basis for theoretical investigation and the correlation of experimental data.

OBSERVED FLOW FIELD OF A FLAME IN A CHANNEL

A two-dimensional lean acetylene-air flame was stabilized on 0.003-in. diameter chromium-nichrome alloy wire in a 1×3 in. rectangular quartz channel (see Fig. 3). The flow upstream of the flame was made uniform by using a number of very fine screens. Five- to ten-μ titanium oxide particles were introduced in the combustible gases through a tube. The wake of the tube is suppressed by screens placed immediately downstream of it so that all the particles are confined to a narrow sheet. The particle tracks were photographed with an $f2.3$ 7-in. focal length lens using flash bulb illumination and a rotating disk with uniformly spaced openings as the camera shutter. The photograph used to measure the detailed flow field and an enlargement showing in greater details the flow near the flame are shown in Fig. 4. The length of the particle track is proportional to the velocity at this point. The effect of particle inertia is quite small and over-all experimental error is $\pm$ 3%. There is a definite distinction between a propagating flame and that stabilized in a channel. The former has a rounded shape whereas the latter appears more like a vee. In the present case, the flame holder was made so small that it introduced minimum disturbance and the flame has the rounded appearance and corresponds very closely to a propagating flame. A reduction of the flame holder diameter had no effect on the flame shape but the flame flashed back occasionally. Perfect two-dimensional flame with no variation in flame shape along its depth was not obtained immediately on all attempts, but after a few trials and some care perfect flame could be obtained.

The location of the surface of discontinuity which in the analysis replaces the actual zone of combustion is determined in the following way. Instead

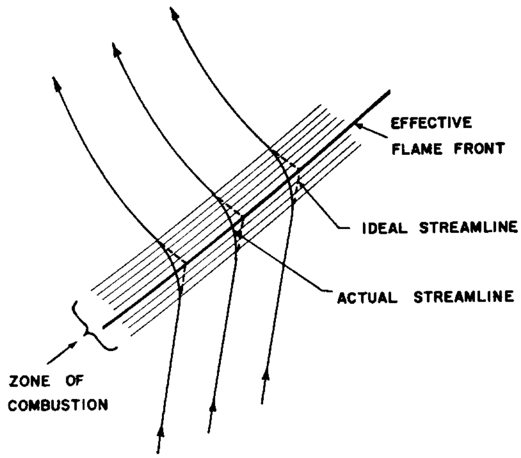

FIG. 5. Procedure for determining the position of the effective flame front.

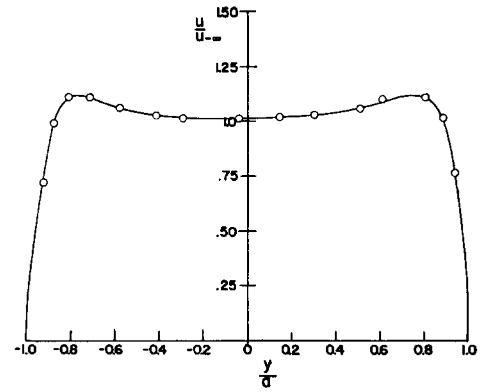

FIG. 7. Velocity profile at $x/a = -0.7$.

of following the actual particle path we extend the particle tracks from either side into the combustion zone as continuation of the paths outside this zone. The point at which the extended tracks from two sides meet is taken as the position of the effective flame front. This procedure is indicated in Fig. 5. The visible flame and the ideal flame front are shown in Fig. 6 and the latter is nearly parabolic and quite symmetrical with respect to the x axis. This indicates that flow field in the plane of particles is symmetrical. This is one of the advantages of introducing dust particles in one plane. The visible flame is determined by the total flame brightness along its depth and angle between the camera axis and the perpendicular to $x - y$ plane. The asymmetry of the visible flame is probably due to misalignment of the camera and slight asymmetry of the flame along its depth which is not present in the plane of the dust particles. This asymmetry was not detectable by eye.

The velocity ahead of the flame at $x/a = 0.7$ is shown in Fig. 7, where $2a$ is the channel width and x is the distance along the channel length measured from the foot of the flame. The average velocity is taken to be the effective uniform $u_{-\infty}$ at $x = -\infty$ and all velocities are given in terms of this velocity. The flame has already had some effect on the flow, the velocity has decreased in the center and increased near the wall, and close to the wall the boundary layer is noticeable. If the last screen is too far from the flame then the viscous boundary layer becomes large; on the other hand, if the screen is placed too close to the flame then we cannot assume that the velocity is uniform in front of the flame. A compromise position of $x/a = -1.5$ was chosen for the last screen. We define a mass stream function ψ such that $d\psi = \rho v dn$ where v is the velocity along the streamline and dn the increment in distance normal to the streamline. The $\psi/\rho_1 a u_{-\infty}$ *versus* y/a at $x/a = -0.7$ was obtained by integrating the velocity profile and is shown in Fig. 8. We can assign a value of the nondimensional stream function to every particle at this station and from the photograph we can determine its velocity and

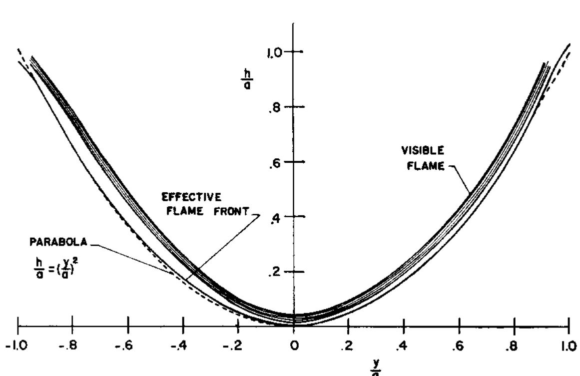

FIG. 6. Shape of the visible flame and effective flame front.

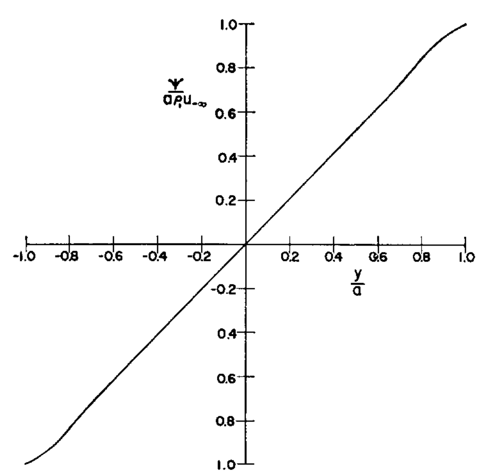

FIG. 8. Stream function at $x/a = -0.7$.

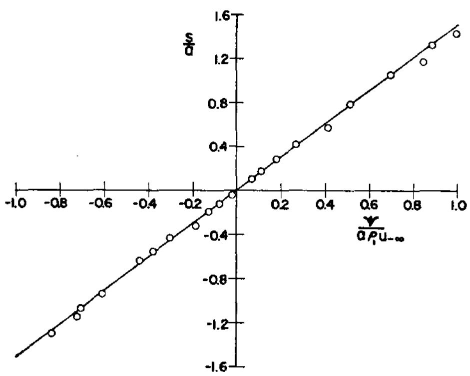

FIG. 9. Stream function at the flame.

position at every point. In other words, the measurements give the velocity *and* the stream function at every point.

The measured $\psi/\rho_1\ au_{-\infty}$ *versus* s/a curve is shown in Fig. 9, since it is a straight line the normal velocity

$$\frac{u_{1_n}}{u_{-\infty}} = \frac{\partial(\psi/\delta_1 au_{-\infty})}{\partial(s/a)}$$

has a constant value of 0.67 everywhere except possibly near the wall. The normal and tangential velocities on both sides of the flame can be determined from the photograph. The results are shown in Fig. 10 and for the flame speed u_{1_n} the results agree with that obtained from the ψ *versus* s curve within a few percent and we will take 0.67 as the flame speed for use in further work. The average ratio of normal velocities $u_{1_n}/u_{1_n} = \rho_1/\rho_2$ is approximately 5.0. The tangential velocity is, of course, continuous across the flame and the difference in the measured tangential velocities on the two sides is due to experimental scatter. We will take their average value as the tangential velocity.

At $x/a = 2.8$ the streamlines have become parallel so that we may consider the velocity profile at this station as the profile at $x = \infty$. The measured

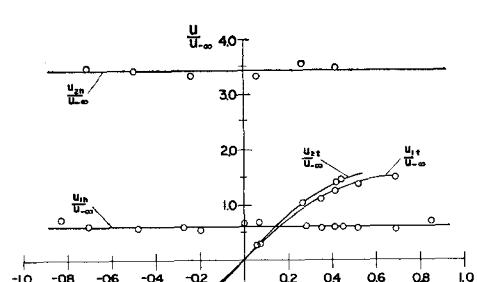

FIG. 10. Normal and tangential velocities at the flame.

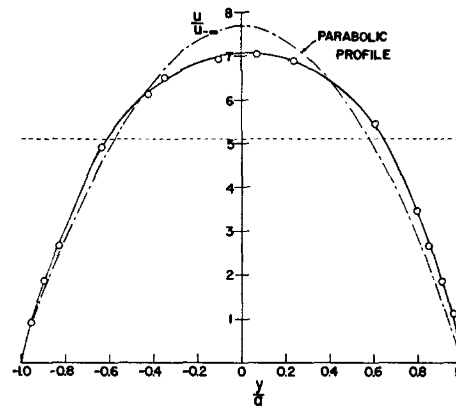

FIG. 11. Velocity profile of the burned gases at $x/a = 2.8$. Dashed line is the parabolic velocity distribution for the same volume flow. Dotted line is the average value of $u_{+\infty}/u_{-\infty}$ which equals ρ_1/ρ_2.

velocity profile is shown in Fig. 11. An average value of 5.1 for ρ_1/ρ_2 was obtained by comparing the volume flow at this station with that $x/a = -0.7$. We can determine the variation of the stream function with y, $d\psi/\rho_1 au_{-\infty} = \rho_2 u_{+\infty}\ dy/\rho_1 au_{-\infty}$, by integrating the $u_{+\infty}/u_{-\infty}$ *versus* y/a curve and assuming that $\rho_1/\rho_2 = 5.1$. The same result can be obtained from the photograph by locating the asymptotic post-flame position of a stream line whose asymptotic pre-flame location is known. The results obtained by these two methods agree as shown in Fig. 12, indicating that the density ratio ρ_1/ρ_2 is the same for all stream lines. The density is, of course, somewhat higher near the walls but this makes negligible contribution to the total mass flow. A more direct procedure would be to compute the density drop for each stream tube by using the relation $(\rho_1 u\Delta y)_{x/a=-0.7} = (\rho_2 u\Delta y)_{x/a=2.8}$ where Δy is the stream tube width. This was done and within the experimental scatter gave $\rho_1/\rho_2 = 5.1$ for every

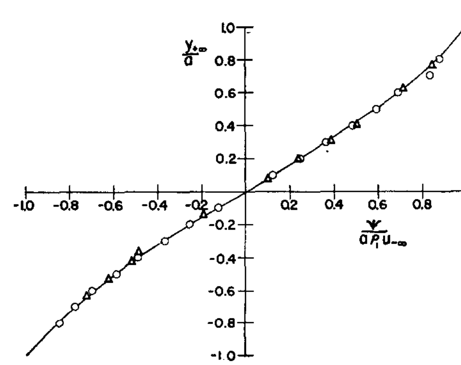

FIG. 12. Stream function at $x/a = 2.8$. Circles correspond to the value obtained by integrating the velocity profile using $\rho_1/\rho_2 = 5.1$ and triangles to the direct measurement of the stream line position.

streamline. This value of ρ_1/ρ_2 will be used for further work.

Using the *y versus* ψ curve of Fig. 12 we can plot $u_{+\infty}/u_{-\infty}$ as a function of ψ and this is shown in Fig. 13. Equation (2) shows that assuming inviscid flow the asymptotic velocity of a streamline can be determined from the normal and tangential velocities at the flame and the pressure drop $(p_{-\infty} - p_{+\infty})$. We have not measured $(p_{-\infty} - p_{+\infty})$ but it can be computed by considering the change in the momentum of either the entire flow or the central streamline. The latter procedure gives a value of 10.76 for $(p_{-\infty} - p_{+\infty})/\frac{1}{2}\rho_1 u_{-\infty}^2$. Using this pressure drop and the measured value of other quantities appearing in Eq. (2), the predicted values of $u_{+\infty}/u_{-\infty}$ are compared with those measured in Fig. 13. The agreement gets worse as we approach the walls where the effect of viscosity becomes quite important.

The flame generated vorticity is conserved along a stream line if we neglect viscosity and assume that ρ_2 is constant. The best place to measure vorticity is at $x/a = 2.8$ where $\omega = -(\partial u/\partial y)$. The vorticity was measured as a function of y and expressed as a function of ψ, using the ψ *versus* y curve of Fig. 12. The vorticity can be computed from Eq. (1) by using the measured tangential velocities at the flame, the ratio ρ_1/ρ_2, and assuming constant flame speed. The vorticity thus computed is compared with that measured in Fig. 14. Near the walls the viscosity has a dominant effect on the flow and the vorticity exceeds the values predicted on the basis of inviscid theory.

DISCUSSION

The measurement of the flow field of a flame in a channel shows that except near the walls the flame speed is constant. In the theoretical analysis we neglected flame quenching and the flame exists right up to the wall where it must be either tangent ($u_t = \infty$) or normal ($u_t = 0$) to the wall. Neither one of these conditions is possible within the framework of theory based on constant flame speed and inviscid flow. Since a flame can be realized experimentally in the theory, we must have neglected some important effects at the walls. These are: (i) quenching of the flame at the wall; (ii) decrease in flame speed at the wall; (iii) density gradient at the wall due to heat conduction; and (iv) viscous effect or the boundary layer at the wall. We could dispose of the question of boundary conditions at the wall simply by assuming that the flame is quenched at the wall, but the photographs of the present flame stabilized in a channel show that the flame exists right up to the wall and the quenching is confined to an extremely narrow region at the wall. Furthermore, from a theoretical point of view, the assumption of quenching means that we relax the boundary conditions at the flame instead of the wall.

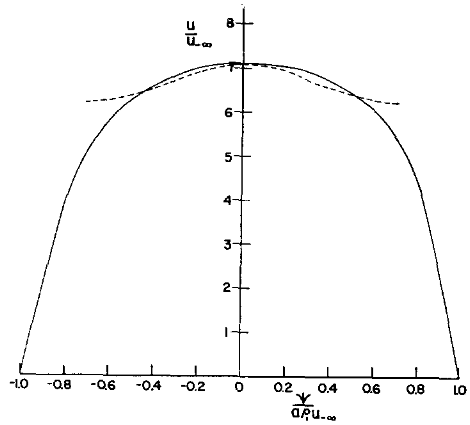

FIG. 13. Velocity profile as a function of ψ at $x/a = 2.8$. Dashed line is computed from Eq. (2).

Equation (2) shows that the flame cannot be tangent to the wall as long as there is a density jump across it. If the flame is normal to the wall then it is concave towards the unburned gases and it has been shown that such a flame cannot exist unless the flame speed *increases* at the wall. Therefore, the second of the aforementioned effects is also unfavorable.

The viscosity and heat conduction are both diffusive in nature but the effect of the former is more important for the following reason. In the unburned gases the effect of the flame is to slow down the central flow and accelerate the outer flow which thins out the boundary layer. In the burned gases the central flow accelerates and the outer flow

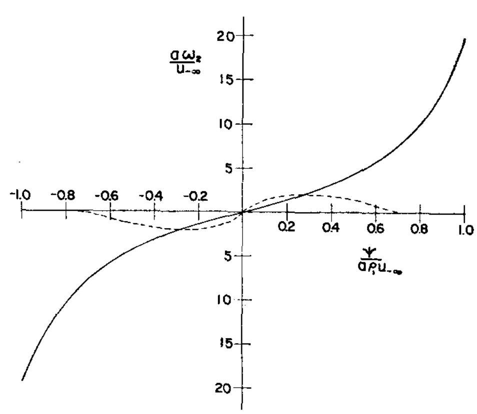

FIG. 14. Vorticity distribution at $x/a = 2.8$. Dashed line is computed from Eq. (1).

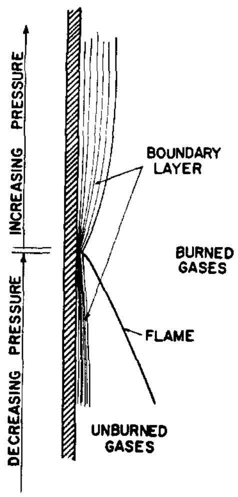

FIG. 15. Boundary layer at the wall showing its thickness in burned gases due to adverse pressure gradient.

slows down which thickens the boundary layer. All this is visible in the photograph of the flow field (see Figs. 4 and 15), so that the third and fourth effects mentioned above thicken the boundary layer at the wall in the region of the burned gases. As far as the main body of the flow is concerned, a thick boundary layer has the same effect as displacement of the solid wall into the fluid. This is exactly what we have to do in order to make Ball's numerical solution dynamically possible (see Fig. 2). Now, the flame need not be either tangent or normal to the wall; it can meet the wall at a grazing angle.

In short, we have to relax the boundary condition at the walls by inward displacement of the walls in the region of burned gases or at the flame by assuming that the flame is quenched before reaching the walls. The above considerations show that the former is more likely the case. It is possible that the problem is overdetermined within the framework of simplified theory and instead of fixing the walls in the region of the burned gases we should only require that the flow should be parallel at infinity and the wall position is determined accordingly. This may not lead to a unique solution and it would be more satisfactory to look for a solution which gives the minimum inward displacement of the wall. It may be possible to determine the displacement by considering some over-all or integral properties of the flow without solving for the detailed flow field. The importance of the boundary layer, relative to the over-all flow, decreases as the Reynolds number or the channel size is increased. Beyond a certain channel size a steady flame with uniform speed of propagation is not possible, which agrees with the experimental facts.*

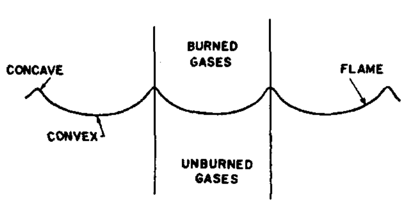

FIG. 16. Profile of cellular flame.

It is known that under some circumstances a flat flame is unstable and may become corrugated. The flow between the two streamlines passing through the neighboring ridges may be taken as a flow through a channel (see Fig. 16). The question arises as to what we can say about this case in view of the foregoing discussion. In the case of a Bunsen flame a concave flame is possible if the flame speed *increases* as we approach the tip of the flame. We have a similar situation here. In fact, we can say that whenever a cellular flame develops the radius of curvature of the *concave* part of the flame will always be comparable with the flame thickness in order that the flame velocity may increase to make the flow dynamically possible. There is no such restriction on the convex part of the flame. These predictions are in agreement with the observation of cellular flames.

ACKNOWLEDGMENTS

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* The reviewer of this paper points out that the concave curvature of the flame occurs when it approaches the boundary layer at the wall where the velocity is rapidly decreasing. This is true from a kinematical point of view. However, one has to show further that the associated flow field is dynamically possible and is consistent with the rest of the flow, which is not very much affected by viscosity. The above discussion is relative to this point.