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# SPACE-CHARGE RECOMBINATION OSCILLATIONS IN DOUBLE-INJECTION STRUCTURES

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The small-signal impedance is derived for a double-injection structure operating in the low-injection square-law regime. Space-charge effects are found to be essential. Spontaneous voltage oscillations are found to occur above a threshold voltage $V_{\mathrm{th}}$. The theory gives quantitative agreement with experimental results for Au-doped Si *p-i-n* diodes. These include the temperature dependence of $V_{\mathrm{th}}$ and the threshold frequency as well as the voltage dependence of the frequency above threshold.

Spontaneous oscillations in forward-biased double-injection structures with deep traps, first observed by Holonyak and Bevacqua,[1] have been observed in several semiconductors doped with a wide variety of impurities.[2-7] A number of attempts, all using the quasineutrality approximation,[8-11] have been made to explain the origin of this instability. In this letter we show that this instability is correctly described only if we include space-charge effects in the oscillating fields.

Our starting point is the steady-state description of long double-injection structures in the low-injection square-law regime.[12,13] Distinguishing the steady-state variables with a subscript zero, the essential results are

$$j_0 = \tfrac{9}{8} e\mu_n\mu_p(p_0\tau_n - n_0\tau_p)(V_0^2/L^3), \tag{1}$$

$$E_0 = (3V_0/2L)(1 - x/L)^{1/2}. \tag{2}$$

The quantity $(p_0\tau_n - n_0\tau_p)$ is, in general, proportional to a Boltzmann factor $e^{-\Delta E/kT}$, where $\Delta E$ is the activation energy of the dominant deep-trap impurity level; thus it is strongly temperature-dependent.

Assuming the time-dependent variables vary in time as $e^{st}$ and are small compared with their dc values, they satisfy the equations

$$sn - \frac{1}{e}\frac{dj_n}{dx} = -\frac{n}{\tau_n}, \tag{3}$$

$$sp + \frac{1}{e}\frac{dj_p}{dx} = -\frac{p}{\tau_p}, \tag{4}$$

$$s(P - N) = \frac{p}{\tau_p} - \frac{n}{\tau_n}, \tag{5}$$

$$\epsilon\frac{dE}{dx} = e(P - N + p - n), \tag{6}$$

where the fluctuating currents are given by

$$j_n = e\mu_n(E_0 n + n_0 E), \quad j_p = e\mu_p(E_0 p + p_0 E). \tag{7}$$

In these equations we have assumed that the carrier lifetimes have the constant values of the low-injection square-law regime and that the thermal-emission rates can be neglected.

These equations have as an immediate integral the total time-dependent current

$$j = j_n(x) + j_p(x) + \epsilon s E(x). \tag{8}$$

Now we use Eqs. (5–8) to express $n$ and $p$ in terms of $E$ and $dE/dx$. Then we insert these expressions into the linear combination of (3) and (4) obtained by multiplying (3) by $\tau_p/\mu_n$, (4) by $\tau_n/\mu_p$, and adding. The resulting equation for $E$ is

$$\frac{-\mu_n\mu_p\tau_n\tau_p}{(1+s\tau_n)(1+s\tau_p)}\frac{s\epsilon}{j_0}E_0\frac{d}{dx}E_0\frac{d}{dx}E$$

$$+\frac{s(\tau_n - \tau_p)\mu_n\mu_p\tau_n\tau_p}{(\mu_n\tau_n + \mu_p\tau_p)(1+s\tau_n)(1+s\tau_p)}E_0\frac{d}{dx}\frac{E}{E_0}$$

$$-\left(\frac{8L^3}{9V_0^2}\frac{\mu_n\tau_n+\mu_p\tau_p+s(\mu_n+\mu_p)\tau_n\tau_p}{(\mu_n\tau_n+\mu_p\tau_p)(1+s\tau_n)(1+s\tau_p)}\right.$$
$$\left.+\frac{\mu_n\tau_n-\mu_p\tau_p+s(\mu_n-\mu_p)\tau_n\tau_p}{(1+s\tau_n)(1+s\tau_p)}\frac{s\epsilon}{j_0}\right)E_0\frac{dE}{dx}$$
$$+\left(1+\frac{s\epsilon}{j_0}E_0\right)\frac{E}{E_0}=\frac{j}{j_0}. \qquad (9)$$

This equation must be solved subject to the standard boundary conditions[14]:

$$E(0)=E(L)=0. \qquad (10)$$

The discussion of the stability of the steady-state solution is now, in principle, straightforward. We solve (9) subject to (10) and form the impedance function

$$Z(s)=\frac{1}{j}\int_0^L dx\,E(x). \qquad (11)$$

We then apply Nyquist's criterion: The spontaneous oscillations of the device are associated with zeros or poles of $Z(s)$ in the right half $s$ plane.

For $E_0(x)$ given by (2) we have found the solution of (9) and the resulting impedance numerically The impedance has poles which move into the right half-plane when $V_0$ exceeds a threshold value. These results will be published in a subsequent paper. We find, however, that similar results are obtained by approximating $E_0$ in (9) by a uniform field, and it is the solution to this approximate equation that we will discuss here. Setting $E_0 = V_0/L$, Eq. (9) becomes

$$A\frac{d^2E}{dx^2}+B\frac{dE}{dx}+CE=\frac{j}{j_0}, \qquad (12)$$

where

$$A=-\mu_n\mu_p\tau_n\tau_p\frac{V_0}{L}\frac{s\tau_d}{(1+s\tau_n)(1+s\tau_p)}, \qquad (13a)$$

$$B=\left(\frac{\mu_n\mu_p\tau_n\tau_p}{\mu_n\tau_n+\mu_p\tau_p}s(\tau_n-\tau_p)\right.$$
$$-\frac{8L^2}{9V_0}\frac{\mu_n\tau_n+\mu_p\tau_p+s(\mu_n+\mu_p)\tau_n\tau_p}{\mu_n\tau_n+\mu_p\tau_p}$$
$$\left.-[\mu_n\tau_n-\mu_p\tau_p+s(\mu_n-\mu_p)\tau_n\tau_p]s\tau_d\right)$$
$$\times[(1+s\tau_n)(1+s\tau_p)]^{-1}, \qquad (13b)$$

$$C=(1+s\tau_d)\frac{L}{V_0}, \qquad (13c)$$

and we have introduced the dielectric relaxation time $\tau_d=\epsilon V_0/j_0L$. The solution to (12) subject to (10), which is elementary, leads to the impedance

$$Z(s)=\frac{L}{Cj_0}\left(1-\frac{D_+-D_-}{LD_+D_-}\frac{(e^{D_+L}-1)(e^{D_-L}-1)}{e^{D_+L}-e^{D_-L}}\right), \qquad (14)$$

where

$$D_\pm=\frac{-B\pm(B^2-4AC)^{1/2}}{2A} \qquad (15)$$

One can readily verify that Eq. (14) has zeros at $s=-1/\tau_n$ and $-1/\tau_p$, corresponding to the decaying modes observed in a transient response experiment.[15] There is also a pole at $s=-1/\tau_d$ corresponding to the simple decaying $RC$ mode. Finally, there are poles whenever

$$(D_+-D_-)L=\pm 2n\pi i, \quad n=1,\ 2,\ \ldots\ , \qquad (16)$$

and these are the poles which lead to the observed instabilities.

Equation (16) becomes much simpler if we make the reasonable assumption that $\tau_n$ is much longer than the other times in the problem ($\tau_n \gg \tau_p$, $\tau_d$, $t_n$, $t_p$, where $t_n=L^2/\mu_p V_0$ are the carrier transit times). In this case we can neglect the linear term in $s$ in (13c) and the quadratic terms in the numerator of (13b) and in the denominators of (13a) and (13b). Squaring (16) then leads to a quadratic equation for the poles:

$$\{(\tfrac{4}{9}n\pi\sigma\Delta)^2+(\tau_n+\tau_p)\sigma\Delta+\tfrac{1}{4}[\tau_n+\tau_p+\bar{z}\sigma$$
$$-(\Delta+\bar{z})(\tau_n-\tau_p)]^2\}s^2+[\tau_n+\tau_p-(\Delta+\bar{z})$$
$$\times(\tau_n-\tau_p-\sigma)]s+1=0, \qquad (17)$$

where

$$\Delta=\frac{9\mu_n\mu_p\tau_n\tau_p V_0}{4L^2(\mu_n\tau_n+\mu_p\tau_p)},$$
$$\sigma=\frac{9(\mu_n\tau_n+\mu_p\tau_p)V_0\tau_d}{4L^2}, \qquad (18)$$
$$\bar{z}=\frac{\mu_n\tau_n-\mu_p\tau_p}{\mu_n\tau_n+\mu_p\tau_p}.$$

The quantity $\Delta$ is a dimensionless voltage, and $\sigma$ has the dimensions of time. For the square law (1) $\sigma$ is independent of $V_0$ and $L$, but it is strongly $T$-dependent owing to the strong $T$ dependence of $j_0$.

For small $\Delta$ the roots of (17) have a negative real part; as $\Delta$ is increased these roots move to the right, crossing into the right half-plane when the linear term in (17) vanishes. This gives for the threshold voltage and frequency

$$\Delta_{\text{th}}=\frac{\tau_n+\tau_p}{\tau_n-\tau_p-\sigma}-\bar{z}, \qquad (19)$$

$$2\pi f_{\text{th}}\equiv -is_{\text{th}}$$
$$=\{(\tfrac{4}{9}n\pi\sigma\Delta_{\text{th}})^2+(\tau_n+\tau_p)\sigma\Delta_{\text{th}}+(\tfrac{1}{2}\sigma\Delta_{\text{th}})^2\}^{-1/2}. \qquad (20)$$

Since $\Delta_{\text{th}}$ is independent of $L$, $V_{\text{th}}$ is proportional to $L^2$ and $f_{\text{th}}$ is independent of $L$. However, both $V_{\text{th}}$ and $f_{\text{th}}$ will have strong $T$ dependences arising from the strong $T$ dependence of $\sigma$. As the tem-

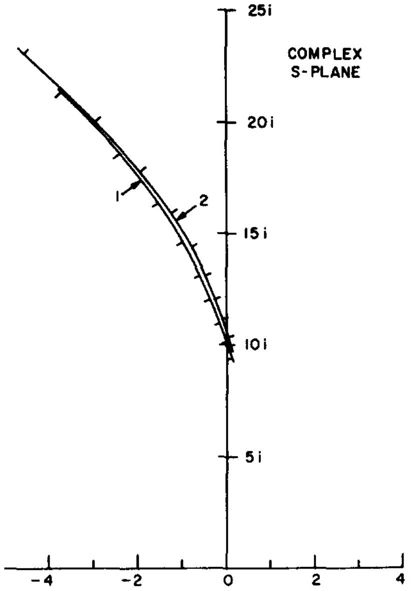

FIG. 1. Path of a pole of $Z(s)$ as the applied voltage is varied. Curve 2 is the path of the $n=1$ pole determined from Eq. (17), while curve 1 is the corresponding result obtained from the numerical solution of Eq. (9). The parameters used are $\tau_n = 36.4$ nsec, $\tau_p = 1.76$ nsec, $L = 100\ \mu$, $\tau_d = 21.3V/V_0$ nsec, $T = 300\,°\mathrm{K}$, and $s$ is in units of $10^6\ \mathrm{sec}^{-1}$. The markers on each curve are at 10-V intervals beginning with 50 V in the upper left corner.

perature is raised $\sigma$ decreases, $f_{\text{th}}$ increases, and $V_{\text{th}}$ approaches a constant. As the temperature is lowered $\sigma$ and $V_{\text{th}}$ increase until $\sigma = \tau_n - \tau_p$. At this point $V_{\text{th}} \to \infty$, and $f_{\text{th}} \to 0$. In practice, however, $V_{\text{th}}$ cannot exceed $V_b$, the breakdown voltage at which the low-injection dc characteristic terminates in a negative resistance.

In order to indicate the validity of our approximations we compare in Fig. 1 the path of a pole obtained from the numerical solution of the exact equation (9) using (2) with the path obtained from (17). The agreement is perhaps better than one would expect, the main difference being a 10% shift in the voltage for corresponding points.

In Fig. 2 we compare the frequency of the oscillation above threshold, identified as the imaginary part of $s$ obtained from (17), with experimental data for 3-$\Omega$ cm $n$-type Si compensated with $\sim 10^{16}\text{-cm}^{-3}$ Au impurities. The theoretical curve was obtained using a measured value of $\tau_n$ and then adjusting $\tau_p$ and $\tau_d$ to fit the experimental values for $V_{\text{th}}$ and $f_{\text{th}}$. This fitting procedure generally gave a value of $\tau_p$ consistent with estimates based on the Au density, the degree of compensation, and the known capture coefficients; it gave a value of $\tau_d$ consistent with the estimated junction area and $i$-region length. For the voltage dependence of $\tau_d$ we used the experimental $j_0$-$V_0$ relation, since the $j_0$-$V_0$ curve was generally steeper than a square law, apparently due to the effects of diffusion. This means that (19) becomes an implicit equation for $V_{\text{th}}$; Eq. (20) is still valid.

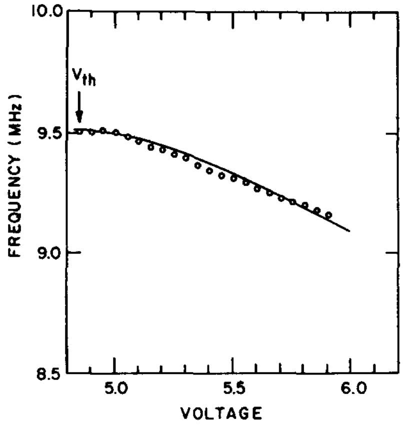

FIG. 2. Oscillation frequency versus the applied voltage for a Au-doped Si $p$-$i$-$n$ diode. The solid line is the theoretical curve given by Eq. (17) with $n=1$, where we have fit the threshold data using the parameters $\tau_n = 80$ nsec, $\tau_p = 2.5$ nsec, $L = 70\ \mu$, $\tau_d = 0.47$ nsec. The device was fabricated on 3-$\Omega$ cm $n$-type Si compensated with $\sim 10^{16}\text{-cm}^{-3}$ Au impurities. $T = 300\,°\mathrm{K}$.

In Fig. 3 the $T$ dependences of $V_{\text{th}}$ and $f_{\text{th}}$ determined from (19) and (20) are compared with our experimental results. These curves were obtained by fitting the data at one temperature (using the above method) and then using $\mu_n \propto T^{-2.5}, \mu_p \propto T^{-2.7}$,[16] and $\tau_d \propto T^{5.2} e^{\Delta E/kT}$, where $\Delta E = 0.35$ eV is the energy of the lower Au level above the valence band.[17]

The electric field associated with a pole is the corresponding solution of the homogeneous equa-

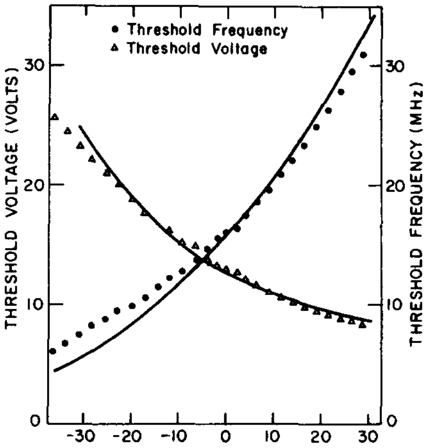

FIG. 3. Temperature dependence of the threshold frequency and voltage for a Au-doped Si $p$-$i$-$n$ diode. The solid lines are theoretical curves determined from (17), where we have fit the experimental data at $T = 5\,°\mathrm{C}$ using $\tau_n = 60$ nsec, $\tau_p = 1$ nsec, $L = 93\ \mu$, and $\tau_d = 0.224$ nsec. The device was fabricated from 3-$\Omega$ cm $n$-type Si doped with $\sim 1.5\times10^{16}\ \mathrm{cm}^{-3}$ Au impurities.

tion (12) ($j=0$) which satisfies (10):

$$E(x,t) \propto Rl\,[(e^{D_+x} - e^{D_-x})e^{st}]. \tag{21}$$

At threshold, using the same approximations used to obtain (17), we find

$$E(x,t) \propto e^{9x/8L} \sin\left(\frac{9}{8\pi\sigma\Delta_{\text{th}} f_{\text{th}}}\frac{x}{L} + 2\pi f_{\text{th}} t\right) \sin n\pi \frac{x}{L}. \tag{22}$$

This has the form of a left running wave modulated by a factor $\sin n\pi(x/L)$ which fits the boundary conditions. Using this solution one can construct the densities of free carriers and charged traps. We find these also appear as modulated left running waves, i.e., waves traveling from the cathode to the anode. The electron and hole densities are ~180° out of phase with the field, with the electrons dominating. The trapped charge density is comparable to the electron charge density but lags it by ~90°. This picture of traveling waves is very similar to that of Konstantinov and Perel,[8] but the instabilities we have found do not follow from their model.

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