frequency. However, we have shown that on a time scale of the order of a mirror transit time a critical level of fluctuations exists, below which no particle losses from the mirror occur. Above this level, some particles will be ejected nonadiabatically on a short time scale, but most particles should be retained for many mirror transits if $\epsilon=\bar{v}_{||}/L_{||}\Omega$ is small. This model has also assumed that the explicit time dependence of the fluctuations is negligible, and while this does not influence J adiabatically, in fact, unless the time dependence is low compared with a mirror transit time, the implied nonconservation of energy will probably produce substantial corrections to the present calculation.

The author gratefully acknowledges several critically clarifying conversations concerning this work and the pre-publication results of M. N. Rosenbluth.

This work was supported in part by the U.S. Atomic Energy Commission under Contract AT(11-1)-2081.

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Comments

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Comments on "Ion Heating by a Modulated Electron Beam"

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(Received 5 May 1971; final manuscript received 6 July 1971)

In a recent paper Haas and Eisner[1] presented experimental results on the heating of plasma ions (H^+, D^+, He^+, and Ar^+) by a modulated electron beam passing through a beam-generated plasma. In their experiment, and in others,[2] it has been shown that energetic ions are produced because of strong beam-driven radial, rf electric fields. They also find that the maximum ion heating effect is observed when the modulating frequency is above a certain critical frequency whose value varies with the square root of the plasma density and is independent of ion mass. These results are explained by the hypothesis that the electron beam excites the lower-hybrid resonance. The lower-hybrid resonance is near the ion-plasma frequency in the low-density limit for plane-wave propagation perpendicular to the magnetic field. When the propagation angle is not exactly 90 deg the lower-hybrid resonance frequency departs from the ion-plasma frequency. An approximate form of the lower-hybrid resonance frequency equation derived for angles near 90 deg is used in Ref. 1 to show that the lower-hybrid resonance frequency is independent of ion mass and proportional to the square root of the density, in agreement with the experiment. The angle of propagation is evaluated as 89 deg from the length/radius (L/R) ratio of the plasma column.

The purpose of this note is to point out that an error in Ref. 1 led to the incorrect conclusion that the lower-hybrid resonance frequency is independent of ion mass for propagation angles near 90 deg. However, some results of dispersion calculations for waves in a bounded plasma are presented which do support the hypothesis of Ref. 1 very well when the boundedness is correctly accounted for. It is found that the resonance frequency in a radially bounded system is strongly dependent on the boundary conditions used at the plasma surface. When the plasma column partially fills the surrounding metal vacuum system, the effect is to make the beam-driven resonance frequency independent of ion mass. In terms of plane waves, the propagation angle is farther from 90 deg than predicted from L/R, and is in the range where the lower-hybrid resonance frequency is independent of ion mass. When the plasma column completely fills the metal tube, as implicitly assumed in Ref. 1, the resonance frequency is strongly mass dependent.

For the frequency of operation in a range such that the inequality $\omega_{ce}^2 \gg \omega^2 \gg \omega_{ci}^2$ is satisfied, the condition for the lower-hybrid resonance in the low density limit

($\omega_{ce}^2 \gg \omega_{pe}^2$) is

$$\omega_{LH}^2 = \omega_{pi}^2 (1 + M/m \cot^2\theta)(1+\cot^2\theta)^{-1}, \quad (1)$$

where ω_{LH} is the lower-hybrid resonant frequency, ω_{pe} and ω_{pi} are the electron and ion plasma frequencies, ω_{ce} and ω_{ci} are the electron and ion cyclotron frequencies, m and M are the electron and ion masses, and θ is the angle of propagation with respect to the steady magnetic field. Contrary to the implication of Eq. (4) in Ref. 1, the resonant frequency is not independent of ion mass at angles of propagation close to 90 deg. For example, the resonant frequency as a function of the angle of propagation is plotted for $\omega_{ce}/\omega_{pe}=5$ in Fig. 1. It clearly shows that at angles close to 90 deg, the lower-hybrid resonant frequency decreases with increasing ion mass. In particular, for $\theta=89$ deg as used in Ref. 1, the resonant frequency for different gases is significantly different. As the angle of propagation moves sufficiently away from 90 deg, the resonant frequency tends to become independent of ion mass. The condition for independence of ion mass is $\cos\theta \gg (m/M)^{1/2}$.

The angle of propagation $\theta = \tan^{-1}(k_\perp/k_{||})$ is found in Ref. 1 by setting $k_{||}=m\pi/L$, $m=1, 2, \cdots$, and using a value for $k_\perp$ determined by the radial boundary condition equation

$$J_n(k_\perp R)=0, \quad n=1, 2, \cdots, \quad (2)$$

where $k_{||}$ and $k_\perp$ are the parallel and perpendicular propagation constants, L is the length of the plasma column and R is the radius. This radial condition is valid for a filled metal plasma tube and its use implies that the rf potential is nonaxisymmetric and zero at the radius $r=R$. However, in the experiments[1,2] the vacuum wall radius is much larger than the plasma radius. Experimentally, it is found[2] that the rf potential and electric field are not zero at the edge of the plasma column and, in fact, the rf field may be measured as a function of radius and axial distance.[3] The values of $k_\perp$ may be different from those given by Eq. (2), and therefore a complete field analysis must be made to accurately predict the resonant frequency.

The formal solution to the plasma waveguide problem has been given by Allis *et al.*[4] in readily applicable form. The results of a quasistatic analysis for the case of a partially filled waveguide are shown in Fig. 2 for H^+, Ne^+, and Ar^+ ions. These curves show the dispersion characteristics of the plasma column $k_{||}(\omega)$ in the frequency range near ω_{pi} for the lowest-order axisymmetric mode. In order to bring out the ion mass dependence, the frequency has been normalized to ω_{pi} for H^+. The angle θ is indicated along each curve. The resonance frequency for the lowest-order axial mode can be found from the intersections of the dispersion curves with the line $k_{||}R=\pi R/L$. For $R=0.5$ cm and $L=40$ cm as in Ref. 1, we have $k_{||}R \times 10^2 \cong 4$ and the predicted resonance frequency is $2.2\,\omega_{pi}(H^+)$ for all three gases. This is higher by a factor of 1.6 compared with that given in Fig. 7 of Ref. 1. However, there is a possibility of experimental error in the measurement of plasma radius and plasma density. The propagation angle θ is in the range where the lower-hybrid resonance

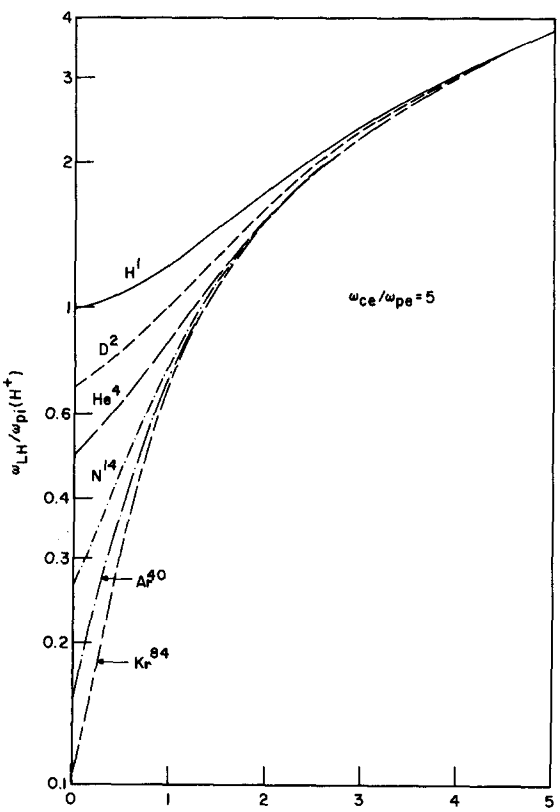

FIG. 1. Lower-hybrid resonant frequency as a function of angle of propagation. Frequency has been normalized with respect to the ion-plasma frequency of H^+.

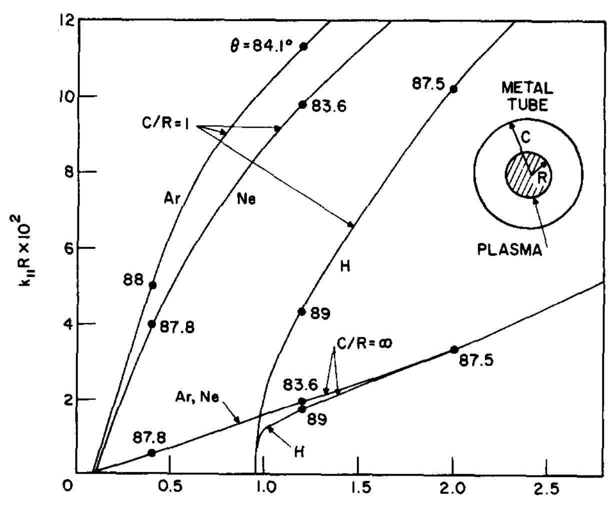

FIG. 2. Normalized axial propagation constant as a function of normalized frequency for a plasma filled waveguide and a partially filled waveguide.

is independent of ion mass on the three curves for the unfilled plasma waveguide ($C/R=\infty$). It can be seen from Fig. 2 that the resonance frequency would still be strongly mass dependent if the filled waveguide curves ($C/R=1$), which correspond to the case used in Ref. 1, were used.

The analysis described above has been studied in detail with the effect of the electron beam included in the dispersion equation, and with axial boundary conditions taken into account by a normal-mode summation.[3] For parameters of experimental interest the electron-beam density is low and it has little effect on the dispersion characteristics in the resonant frequency range. Therefore, plasma dispersion characteristics such as Fig. 2 may be used to predict the resonance frequencies. The analysis predicts higher-order axial modes where $k_{\parallel}R=m\pi R/L$, $m=2, 3, 4, \cdots$. The second and third resonances have been observed experimentally.[3] Details of the analysis will be given in a future publication.

This work was supported by the National Science Foundation under Grant No. GK-15689.

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Reply to the Comments of V. P. Bhatnagar and W. D. Getty

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The "error" referred to in our paper is the term $\Omega i/\Omega e$ in Eq. (4):

$$\omega^2=\Pi e^2\,\frac{[\Omega i/\Omega e+\cot^2\theta]}{1+\cot^2\theta}\,.$$

At an angle of 89 deg, this term results in a frequency change of 27% for molecular hydrogen and 2.2% for argon. At an angle of 88 deg, the change for molecular hydrogen is 9.1% and for argon, less than 1%. This frequency change is well within the scatter of our data points, and considerably less than the factor obtained from the bounded theory.

We do not believe that the use of the bounded theory (i.e., plasma waveguide) as presented by Bhatnagar and Getty is applicable to our experiment, as the vacuum chamber in our apparatus was glass and not metal.

The condition $J_n(K_\perp R)=0$ requires that the amplitude of the wave be zero at the boundary. If the wave is dependent upon the medium for propagation, then the abrupt termination of medium requires that the wave energy be reflected or converted to a form which can propagate in free space. Reshotko has shown that the resonance under consideration is electrostatic (i.e., $\mathbf{K}_\perp\times\mathbf{E}$ is zero), and hence this wave cannot propagate outside the plasma. Thus, these waves should be reflected even in the absence of a metallic boundary. The reflection of an electrostatic wave by a plasma discontinuity should be equivalent to a metallic boundary for an electromagnetic wave. Therefore, we feel justified in using this boundary condition (no metallic boundaries implied) and our results compare quite favorably with this theory.

We have confirmed by measurement, in subsequent experiments, that there are standing waves in a cylindrical plasma column surrounded by a nonconducting boundary.