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VALVE STROKING TO CONTROL TRANSIENT FLOWS IN LIQUID PIPING SYSTEMS

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To my wife and parents

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NOMENCLATURE

Symbol	Meaning	Units
A	Cross-sectional area of pipe	$ft.^2$
A,A'	Denotes point position in the x-t plane	
A	Subscript denoting grid-point location	
a	Celerity of transient pressure pulse	ft./sec.
B	A ratio; B = a/g	sec.
B,B'	Denotes point position in the x-t plane	
B	Subscript denoting grid-point location	
C	A constant term	
C	Subscript denoting grid-point location	
C+,C-	Characteristic curves in the x-t plane	
C_1, C_2, C_3	Collection of known terms	
D	Diameter of pipe	ft.
D	Subscript denoting grid-point location	
E	Subscript denoting grid-point location	
F	Variable function denoting head loss per reach as a function of velocity; $F=\Delta h_f(V)$	ft.
F	Subscript denoting grid-point location	
F( )	Functional relationship of the indicated argument	
f	Darcy-Weisbach friction factor	
f	Subscript denoting final conditions	
f( )	Functional relationship of the indicated argument	
G	Subscript denoting grid-point location	
G( )	Functional relationship of the indicated argument	

NOMENCLATURE (CONT'D)

Symbol	Meaning	Units
$G'(\ )$	Ordinary derivative of function with respect to the indicated argument	
g	Acceleration of gravity	ft./sec.^{2}
H	Piezometric head as a function of distance and time; $H = H(x,t)$	ft.
H	Subscript denoting grid-point location	
$H(\ )$	Head as a function of the indicated argument	ft.
$H'(\ )$	Ordinary derivative of head with respect to the indicated argument	
H_o	Initial steady-state value of head at valve	ft.
H_f	Final steady-state value of head at valve	ft.
H_m	Extreme value of head desired in system	ft.
H'_m	Extreme value of head obtained in system	ft.
H'_r	Head at downstream side of orifice	ft.
HGL	Hydraulic Grade Line	
$h_f(V)$	Total head loss as a function of the velocity; $h_f(V) = fLV^2/D2g$	ft.
I, I'	Integer counter	
i	Subscripted integer counter	
j	Subscript denoting conditions at junction	
K	A constant term	
K_o	Orifice coefficient	ft.$^{1/2}$/sec.
K_v	Valve coefficient	ft.$^{1/2}$/sec.
κ_e	Minor-loss coefficient at pipe entrance	
L	Total length of pipe	ft.
m	Subscript denoting extreme conditions	

NOMENCLATURE (CONT'D)

Symbol	Meaning	Units
N	Number of reaches per pipe	
N	Subscripted integer counter	
n	Subscripted integer counter	
O,O'	Denotes point position in the x-t plane	
o	Subscript denoting initial conditions	
P	Pressure; $P = P(x,t)$	lb./ft.^{2}
P,P'	Denotes point position in the x-t plane	
P	Subscript denoting grid-point location	
Q	Volumetric flow rate; $Q = Q(x,t)$	ft.^{3}/sec.
Q(t)	Volumetric flow rate at a point as a function of time only	ft.^{3}sec.
R	A region of the unique solution of a boundary-data problem	
r	Subscript denoting conditions at reservoir	
SS	Steady State	
t	Independent variable time	sec.
t	Subscript denoting partial differentiation	
t_c	Time of complete valve closure	sec.
t_f	Duration of transient condition in system	sec.
V	Mean cross-sectional velocity as a function of distance and time; $V = V(x,t)$	ft./sec.
V( )	Velocity as a function of the indicated argument	ft./sec.
V'( )	Ordinary derivative of velocity with respect to the indicated argument	
v	Subscript denoting conditions at valve	
x	Independent variable distance	ft.

NOMENCLATURE (CONT'D)

Symbol	Meaning	Units
x	Subscript denoting partial differentiation	
Z	Elevation of pipe centerline above datum	ft.
α	Angle between pipe centerline and the horizontal	
β	Dimensionless time ratio	
γ	Specific weight of fluid	lb./ft.^{3}
ΔH	Change in piezometric head	ft.
ΔH_m	Maximum head rise at valve	ft.
$\Delta h_f(V)$	Head loss per reach of pipe as a function of the velocity	ft.
Δt	Vertical grid height	sec.
$\Delta t'$	Time increment less than a complete Δt	sec.
ΔV	Change in fluid velocity	ft./sec.
Δx	Horizontal grid width	ft.
θ	Curve in the x-t plane	
τ	Dimensionless variable valve relationship	
ϕ	Curve in the x-t plane	
ψ	Curve in the x-t plane	
1,2,3	Subscript denoting pipe or valve number	
I,II,III	Region in the x-t plane	

I. INTRODUCTION

The elimination or effective control of objectionable transients in the operation of fluid systems is becoming an increasingly important consideration in the design of hydroelectric generating stations, water supply systems, cyclic components of otherwise continuous flow processes, and hydraulic systems developed for industrial and commercial applications. With the ever-increasing utilization of the modern high-speed digital computer for rapid analysis and feedback operations, eventual on-line computer control of large and complex hydraulic systems is rapidly becoming a distinct possibility. Historically, transient control has proceeded on the basis of analysis rather than design; within the past decade, however, procedures have been developed which permit a design, or synthesis, approach. These procedures not only permit a more formal and systematic treatment but also result in more elegant and sophisticated control techniques than were previously attainable.

This study attempts to identify underlying principles that govern any design approach, to formalize the existing valve-stroking theory, and to extend this theory to include a greater number of boundary conditions in simple and complex piping systems.

1.1 Literature Review

The control of undesirable transient pressures and velocities in fluid systems has been the subject of intensive study by all investigators of the waterhammer phenomenon. This phenomenon has been analyzed by arithmetic methods since the turn of the century, by graphical methods developed about forty years ago, and starting about 1960, by numerical methods particularly suited for solution on the high-speed digital computer.

With a few notable exceptions, prior to the early 1960's control of objectionable transients was achieved by utilizing the available analysis procedures. In general, changes of the appropriate positions of the valves and other boundary control devices are assumed as functions of time; the system is analyzed for those assumed controller motions; and if the hypothesized motions prove to be inadequate, other, presumably slower [8], boundary motions are assumed. The analysis process is continually repeated until a satisfactory solution is obtained. Even for relatively simple systems, reliance upon such tedious and restrictive procedures ranges from being cumbersome and inconvenient at best to totally inadequate at worst.

Because of the severe limitations imposed by the analysis-oriented procedures, investigators have attempted to develop a synthesis, or design, approach to the determination of proper control-valve operation in transient pipe flow. It would appear that Streeter [26,27,28,30,31] has been the greatest single contributor to the development of suitable synthesis procedures, and he has called this area of study "Valve Stroking". In its most general form, valve stroking may be defined as the synthesis of external boundary conditions to create a desired transient. Hence the term also refers to control by means other than a valve -- for example, the specified variation in elevation of the free surface in a reservoir. Implicit in this definition is that the desired transient is subject to one or more elements of control: i.e., the extreme values of pressure are limited to physically acceptable ones, the period of the transient may be arbitrarily determined, residual transients are eliminated or reduced in magnitude, etc.

The earliest reference in the available literature to the fundamental distinction between the analysis and synthesis procedures appears to be in an article written by Knapp.(14) Considering the design of automatic shutoff valves installed in distribution systems to terminate the flow of water in case of a sudden rupture of the pipeline, he was the first to enunciate this distinction: "It is the usual practice to assume the time and closure law of the valve and then to check the maximum surge caused by the valve closure. It is the purpose of this paper to determine the characteristics and the hydraulic design of self-acting shutoff valves in such a manner that the waterhammer,..., remains within permissible limits."

His solution was to provide a large quick-closing main valve whose control mechanism could be of extremely simplified design. It would be closed in no more than 2L/a seconds following the rupture, where L is the distance from the valve to the upstream constant-head reservoir and _a_ is the speed of the pressure pulse. To control the rise in pressure that would occur when the negatively-directed low pressure wave reflects back to the valve as a positively-directed normal pressure wave accompanied by a significantly higher velocity of the fluid, he proposed the installation of a smaller bypass valve with a more complex control mechanism. Directing his attention to the design of the control motion for this bypass valve and using the graphical method developed by Allievi, he was able to limit the resulting pressure rise to a predetermined maximum value.

The only possible criticism of his procedure is that upon complete closure of the bypass valve an undesirable residual transient still exists in the pipeline. However, it would be more than twenty-five years before it was recognized that this residual transient could be eliminated, and considering the state of the art at that time (1937), Knapp's contribution was both remarkable and significant.

In his classical study first published in France in 1949, Bergeron[2] considered the comparatively more familiar problem of the complete closure of a valve at one end of a pipeline which terminates in a constant-head reservoir at the other end. His solution was similar to Knapp's in that he was able to limit the resulting pressure rise to a predetermined maximum value. Again, after the liquid column in the pipe has been brought to rest, a significant pressure gradient still exists in the fluid which results in the creation of an undesirable residual transient. Nevertheless, he demonstrated that the resulting transient pressures are less severe than would be created if either of two other techniques are used to determine the valve motion for the same time of closure: valve closure such that (1) the fluid velocity at the valve is reduced linearly from its initial steady-state value to zero, and (2) the effective area of the valve is reduced linearly from the initial open position to zero.

With regard to this latter closure, perhaps it should be mentioned that linear valve-closure studies are abundant in the literature (Ref. 2,18,19); by their essential character, however, such studies are analysis oriented and generally only provide useful standards for comparison with the results of comparable synthesis investigations.

Kerr[10,11,12] also studied the problem of valve closure in a simple system (a single pipeline with a valve downstream, a constant-head reservoir upstream). He concluded that the valve closure motion ideally should be such that the fluid velocity at the valve reduces linearly from its initial steady-state value to zero. In one of his studies he compared the maximum pressure rise created by a closure according to his suggested criterion with that created by a closure in which the stem position of the valve is changed linearly in the same time. He reported that for a flow reduction from 5 feet per second to zero in a time of closure equal to 10L/a the maximum pressure rise created in the former case was 60 feet and in the latter case was 210 feet.

Such disparate results are understandable when one recognizes that, for the majority of valves usually installed in water distribution systems, during the first 50% of the stem motion the reduction in flow is negligible and that for even 75% of the stem motion reduction in the flow generally is on the order of only 10%. Kerr suggested that one practical solution would be the design of valves utilizing a two-speed closure: a rapid initial linear stem motion to some intermediate partially closed position followed by a much slower linear stem motion to full closure.

Kerr also studied the effect of valve closure from a partially opened position using the same valve motion and rate of closure for the valve that are determined by the linear velocity-reduction criterion. He noted that under some circumstances the maximum pressure rise resulting from the partial closure is worse than that created during the corresponding full closure condition.

Other investigators who also recognized that valve closure in a simple pipe can be achieved without exceeding a predetermined maximum pressure rise were Streeter[24] and Ruus,[20,21] with the latter's goal being the development of a theory for the optimum closure of hydraulic turbine gates. Their methods are identical to the procedures developed earlier by Knapp and Bergeron: in one round-trip wave-travel time the head at the valve is increased linearly from its initial steady-state value to the prescribed maximum value; that maximum is then maintained until the required fluid deceleration occurs and the valve is closed. As in the earlier studies cited above, upon complete closure of the valve a residual transient still exists in the pipeline.

Záruba(35) referred to a "perfect" relationship for closing a valve and credited it to the work of Krivčenka. Although this reference is unavailable for review, it is evident that Krivčenka's closure method is identical to those of Knapp, Bergeron, Streeter, and Ruus.

Ruus(21) appears to be the first investigator who attempted to modify his procedure to accommodate the effect of fluid friction. He was able to arrive at a solution by utilizing the existing techniques (Ref. 2,18) for incorporating frictional effects into the graphical method of analysis.

The first study which succeeded in eliminating the objectionable residual transient described above was that of Streeter.(26) Again considering a simple frictionless pipe and aided by the visual insight afforded by utilization of the dependent variable graph (Allievi's graphical method), he presented a procedure by means of which the downstream valve motion can be determined which would create a controlled transient between any desired initial and final steady-uniform flow conditions. The essential features of his procedure are as follows:

1. During the first one round-trip wave-travel time

in the pipe, the flow is organized such that the velocity is uniform and a predetermined straight hydraulic grade line is established, sloped positively or negatively depending upon whether the flow is being decreased or increased.

2. During the second phase, the uniform grade line is maintained and the flow is decelerated or accelerated uniformly.
3. During the final one round-trip wave-travel time, the hydraulic grade line and velocity are brought to their final steady-uniform values.

As a consequence of this procedure, the transient condition created is a highly controlled one. The extreme values of pressure can be predetermined and will not be exceeded, all pressures and velocities are linear functions of time and distance, the velocities are not permitted to reverse, and the transient ceases when the valve motion ceases. Included in that work were techniques by which friction corrections could be applied to the preceding solution and also a study which applied the basic procedure to several reaches of pipe of constant diameter but with a varying wall thickness.

In that study Streeter explicitly restricted the time of valve closure to a duration of $4L/a$ or greater. For situations involving complete closure of the valve, the time of closure t_c can be related to the maximum prescribed head rise at the valve ΔH_m by the following relationship:

$$t_c = \frac{2L}{a} + \frac{V_o L}{g \Delta H_m} \qquad \left(t_c \geq \frac{4L}{a}\right),$$

where V_o represents the initial steady-uniform velocity in the pipe and g is the acceleration of gravity.

Considering the earlier procedure which prescribes the maximum head rise at the valve but which also results in the creation of residual pressure fluctuations, the following relationship between valve closure time and maximum head rise can be obtained:

$$t_c = \frac{L}{a} + \frac{V_o L}{g\Delta H_m} \qquad (t_c \geq \frac{2L}{a}).$$

Comparing the two methods for the same time of valve closure, the ratio of the maximum pressure rise created by closure according to the older procedure to the maximum pressure rise sustained during closure according to Streeter's suggested technique is given by

$$\frac{\beta-2}{\beta-1} \qquad (\beta \geq 4),$$

where the common time of closure is $t_c = \beta L/a$. Thus the older procedure has the advantage of creating a lesser maximum rise in pressure, whereas the newer procedure succeeds in eliminating the objectionable residual fluctuations in pressure.

When compared to a closure predicated on the linear velocity-reduction criterion studied by Bergeron and suggested by Kerr, Streeter's recommended closure procedure is clearly superior. Not only does it eliminate the residual transient, but also the ratio of the maximum pressure rise created by closure according to the former procedure to the maximum pressure rise sustained during closure according to Streeter's procedure may be shown to be

$$2 - \frac{4}{\beta} \qquad (\beta \geq 4),$$

which has a value equal to or in excess of 1.0.

Streeter[27] subsequently broadened his procedure when he presented a study demonstrating that in one round-trip wave-travel time any known unsteady condition in a frictionless pipe can be organized into a condition of uniform velocity and predetermined straight hydraulic grade line. Once this condition is established, the procedure is identical to that described above. This concept was applied to the control of the transient condition created following the failure of power to a centrifugal pump. While the problem of transient control in centrifugal pump systems has long intrigued numerous investigators of the waterhammer phenomenon, Streeter's study appears to be the only existing one to suggest a suitable synthesis procedure. Other studies (e.g., a recent investigation by Kinno[13]) are generally restricted to the development of guidelines for valve operation that have been obtained using the existing analysis techniques.

Streeter[28] further extended these early valve-stroking concepts to include the control of decreasing or increasing flow in frictionless series pipe systems.

In the latest and most significant study reported to date, Streeter[30] substantially broadened the scope of the valve-stroking principle by developing a theory which provides for an exact solution when frictional effects are included. This method superseded the earlier technique[26,28] which was only an approximation and which yielded poor results when the effect of fluid friction was significant. The procedure developed was first applied to the problem of changing from some initial to some final steady-uniform flow condition in a simple pipe. The essential features of the suggested procedure are identical to that of the earlier study[26,28] for the frictionless case: in one round-trip wave-travel time the flow is organized such that the velocity is uniform and a straight hydraulic grade line is established; these conditions are maintained during a central phase of the transient; during the final one round-trip wave-travel time the velocity and grade line are brought to their final steady-uniform values. Again the transient condition created is highly controlled: extreme values of pressure are identified and not exceeded, pressures and velocities are nearly linear functions of time and distance, the velocities are not permitted to reverse, and the transient ceases when the valve motion ceases. As in his earlier study, Streeter restricted the time of valve closure to a duration of 4L/a or greater.

The method was then readily extended to complex systems by apportioning the flow changes among the branches as a linear relation of the initial to the final steady-uniform values in each pipe. By combining these two procedures, values of head and velocity may be determined at each external boundary in the system. Thus, motion of the valves or other adjustable boundary conditions can be determined at those control points to create the desired transient. This general procedure was formulated by considering the manner in which the method-of-characteristics solution of the unsteady flow equations proceeds on the independent variable graph (the x-t plane). Essential to this procedure was the realization that if known values of velocity and head exist for a specified period of time along one boundary, the characteristics equations can be employed to determine the history of the transient throughout the x-t plane.

Streeter also presented a suggested valve-stroking procedure for a single pipeline where one boundary condition is a known constant relationship between velocity and head, and he concluded the study with a description of experimental verification of the valve-stroking principle applied to a simple system.

Streeter and Wylie[31] incorporate much of the preceding work in their discussion of the topic; in addition, they provide explicit procedures for the application of the valve-stroking concept to series, branching, parallel, and complex piping systems.

Záruba[34,35] appears to be the only other recent investigator who has attempted to develop suitable synthesis procedures, and his work parallels some of Streeter's studies. He suggested a procedure for determining the proper control-valve closure motion to create a predetermined maximum pressure rise in frictionless series pipe systems, and that procedure is identical to Streeter's.[28] In his study Záruba restricted the time of valve closure to a duration of

$$t_c \geq 4 \sum_{i=1}^{N} \frac{Li}{ai} ,$$

where N represents the number of pipes connected in series. (Although Streeter didn't explicitly restrict the valve closure time in his treatment of the same problem, that restriction is at least implicit in his procedure also.)

Záruba's most recent effort was the development of a closure procedure for a simple pipeline where frictional effects are important; again, the procedure that he suggested is identical to Streeter's.[30,31]

1.2 Scope of Investigation

This study of valve stroking has been divided into two major phases. The first phase includes: (1) an examination of the fundamental equations that form the foundation of all modern computer-oriented transient analysis and synthesis investigations, and (2) the development of additional synthesis procedures to supplement the existing valve-stroking theory for simple and complex piping systems. The second phase concerns a description of the laboratory experimental system and the subsequent laboratory studies which verify some of the new concepts presented in this thesis.

The method-of-characteristics solution of the transient flow equations is examined in Chapter II. A fundamental property of the characteristic equations -- that their application is not restricted to advancing the resulting numerical solutions forward in time only -- is presented, and this property is demonstrated to be of essential importance in the formulation of the valve-stroking theory.

The existing valve-stroking procedures for simple piping systems are evaluated in Chapter III. It is demonstrated that, theoretically at least, the minimum time for control valve motion in such systems is $2L/a$. The consequence of this significant development relative to the control of a system consisting of a single pipe with an upstream constant-head reservoir is presented. A suitable valve-stroking procedure for a pipe in which a constant velocity forms one boundary condition is developed. The chapter concludes with an examination of a third type of simple system, a single pipe in which one boundary condition is a known constant relationship between head and velocity. A valve-stroking procedure is suggested that permits a predetermined extreme value of pressure to be developed and maintained at the valve during a central phase of the transient.

In Chapter IV attention is directed to the development of valve-stroking procedures for elementary complex systems. The existing method for a series system is evaluated; it is demonstrated that this procedure rarely allows the extreme value of pressure encountered in the system to be identified a priori. An alternative technique is suggested that overcomes this disadvantage. Branching systems are next considered and the same disadvantage is found to exist in those systems. Again, an appropriate alternative procedure is suggested. The chapter concludes with the introduction of the concept of passive valve stroking and the application of this concept to the development of a suitable valve-stroking procedure for a representative branching system.

Chapter V concerns a description of the laboratory experimental system, and the discussion of the results of the various laboratory studies are included in Chapter VI. Experimental verification of the applicable valve-stroking theory has been obtained and is presented for a simple system (a closure and an opening, both in 3L/a), a series system (two closure studies), and a passive branching system (two closure studies).

In Chapter VII the principal conclusions derived from this investigation are summarized.

II. BASIC CONCEPTS AND EQUATIONS OF THE TRANSIENT PHENOMENON

The basic partial differential equations describing the unsteady flow of liquids in elastic closed conduits are presented in this chapter, and their solution by the method of characteristics is indicated. Fundamental uniqueness properties of the characteristic solutions are examined relative to the concept of domain of dependence and to selected characteristic initial-value and boundary-data problems in the solution plane.

The basic equations are then reduced to a simplified form and a second-order finite-difference solution of the simplified equations is proposed. Computational schemes suitable for execution on a high-speed digital computer are presented for both transient analysis and synthesis procedures. The manner in which the existing initial conditions influence the transient solution is discussed, and the singular fashion by which the final steady-state solution may be established without residual transient fluctuations is indicated.

2.1 Fundamental Equations

The fundamental partial differential equations governing the unsteady flow of liquids in rigid closed prismatic circular conduits have been developed[31] from considerations of mass conservation and the equation of motion and are, respectively

$$\frac{a^2}{g} V_x + VH_x + H_t + V\sin\alpha = 0 \qquad (1)$$

$$VV_x + V_t + gH_x + \frac{fV|V|}{2D} = 0 \,. \qquad (2)$$

The independent variables t and x represent time and distance along the pipe, respectively, and when used as subscripts denote partial differentiation. The dependent variables V and H are the velocity and the elevation of the hydraulic grade line above some fixed datum. The pressure, P, and H are related by the expression $P = \gamma(H-Z)$, Z being the elevation of the centerline of the pipe above the datum and γ the specific weight of the fluid. The variable f represents the Darcy-Weisbach friction factor, D the pipe diameter, and α the angle between the horizontal and the centerline when measured positively downward.

In these equations the pipe is assumed to be full with the minimum pressure always greater than vapor pressure. The velocity is assumed to be one-dimensional with a uniform distribution across the cross-section, and pressure is considered similarly with a value equal to that existing at the centerline. The liquid and pipe walls are assumed to be perfectly elastic, and the friction is evaluated assuming that the instantaneous value of wall shear stress is equal to the shear stress sustained during the corresponding condition of steady flow. In the friction term the absolute value of V is introduced to maintain the direction of the shear force opposite to that of the velocity.

2.2 The Method of Characteristics

Equations (1) and (2) are two simultaneous quasi-linear partial differential equations of the first order. They are of the hyperbolic type and are amenable to solution by the method of characteristics (Ref. 1,6,16,22). These solutions are

$$\left.\begin{aligned} &\frac{dV}{dt} + \frac{g}{a}\frac{dH}{dt} + \frac{fV|V|}{2D} + \frac{g}{a} V\sin\alpha = 0 \qquad (3) \\ &\frac{dx}{dt} = V + a \qquad (3a) \end{aligned}\right\} C^{+}$$

$$\left.\begin{aligned} &\frac{dV}{dt} - \frac{g}{a}\frac{dH}{dt} + \frac{fV|V|}{2D} - \frac{g}{a}V\sin\alpha = 0 \qquad (4) \\ &\frac{dx}{dt} = V - a \qquad (4a) \end{aligned}\right\} C^-.$$

Thus, the original two simultaneous partial differential equations have been transformed into four ordinary differential equations. The significance of these equations may be explained as follows: along a curve in the x-t plane defined by Equation (3a) -- the C+ characteristic curve -- Equation (3) is valid, and along a curve defined by Equation (4a) -- the C- characteristic curve -- Equation (4) is valid. According to the derivation, every solution of the original system of Equations (1) and (2) satisfies this characteristic system of Equations (3) to (4a). It may be shown that the converse is also true.[6]

Before proceeding further with a discussion of the techniques that can be developed to construct numerical approximations to the solution of these equations, attention is now directed to a summary of some fundamental uniqueness properties of the characteristic solutions. For a thorough discussion of these properties and proofs of the corresponding uniqueness theorems, the reader is referred to the work of Courant and Friedrichs.[6]

2.2.1 Domain of Dependence

With reference to Figure 1, let a curve Θ be given in the x-t plane and along Θ continuous values of $V(x,t)$ and $H(x,t)$ are arbitrarily prescribed. It is assumed that the curve Θ with its prescribed values of V and H has nowhere a characteristic direction. The C+ characteristic which intersects the curve Θ at A and the C- characteristic which intersects Θ at B intersect at point P. Then it may be shown that the values of the solutions V, H at the point P depend only upon the initial values

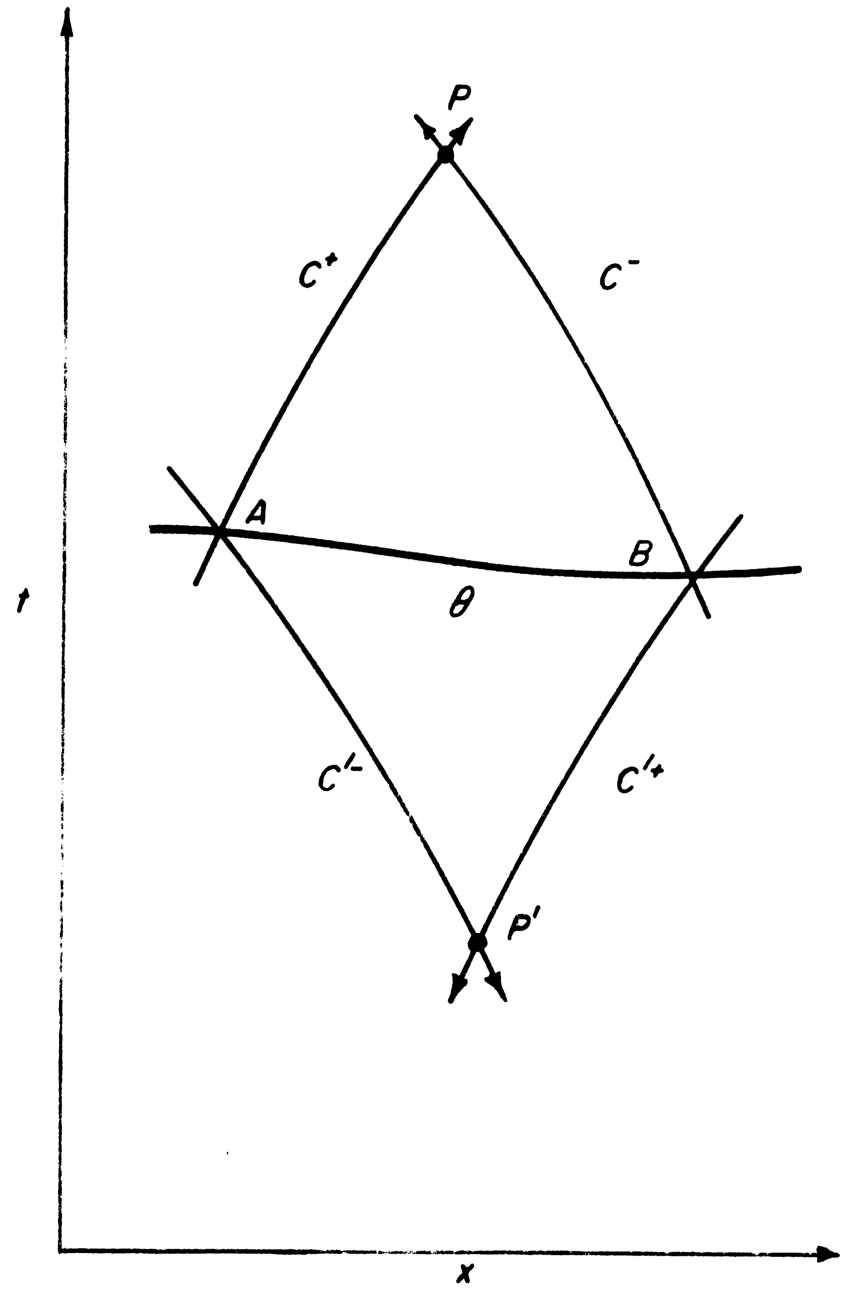


Figure 1. Domains in the x-t plane in which the solution of the initial-value problem can be established.

of V and H on the section of θ intercepted by the two characteristics through P. This interval on the curve θ intercepted by the two characteristics is called the <u>domain of dependence</u> of the point P.

From this notion, the uniqueness of the solution of Equations (1) and (2) in the region ABP bounded by the two characteristics through the point P and the domain of dependence AB cut out by them from the initial curve θ may be demonstrated.

In the preceding discussion no side of the initial curve θ is distinguished. Therefore, the interval AB on the curve θ is also the domain of dependence for the point P', and the solution of Equations (1) and (2) in the region ABP' bounded by the two characteristics C'_+ and C'_- and the interval AB also exists and is likewise unique. Therefore, a unique solution of the initial-value problem exists on both sides of the curve θ.

Intrinsic to the validity of this concept is the recognition that in formulating Equations (3) to (4a) from Equations (1) and (2) no explicit or implicit constraints relative to time exist. Thus, solutions may be advanced forward or backward in time, as the case may be, and the only consideration relative to the existence of unique solutions is the orientation of initial-value curves in the solution plane and the nature of the initial data existing on those curves.

2.2.2 Characteristic Initial-Value Problem

In the preceding section, the initial-value problem of a non-characteristic curve was considered. In this section the characteristic initial-value problem is examined. With reference to Figure 2, suppose θ is a C+ characteristic. Then Equation (3) demonstrates that along θ the values V(x,t) and H(x,t) cannot both be prescribed arbitrarily since Equation (3) is an ordinary differential equation in V and H along θ and

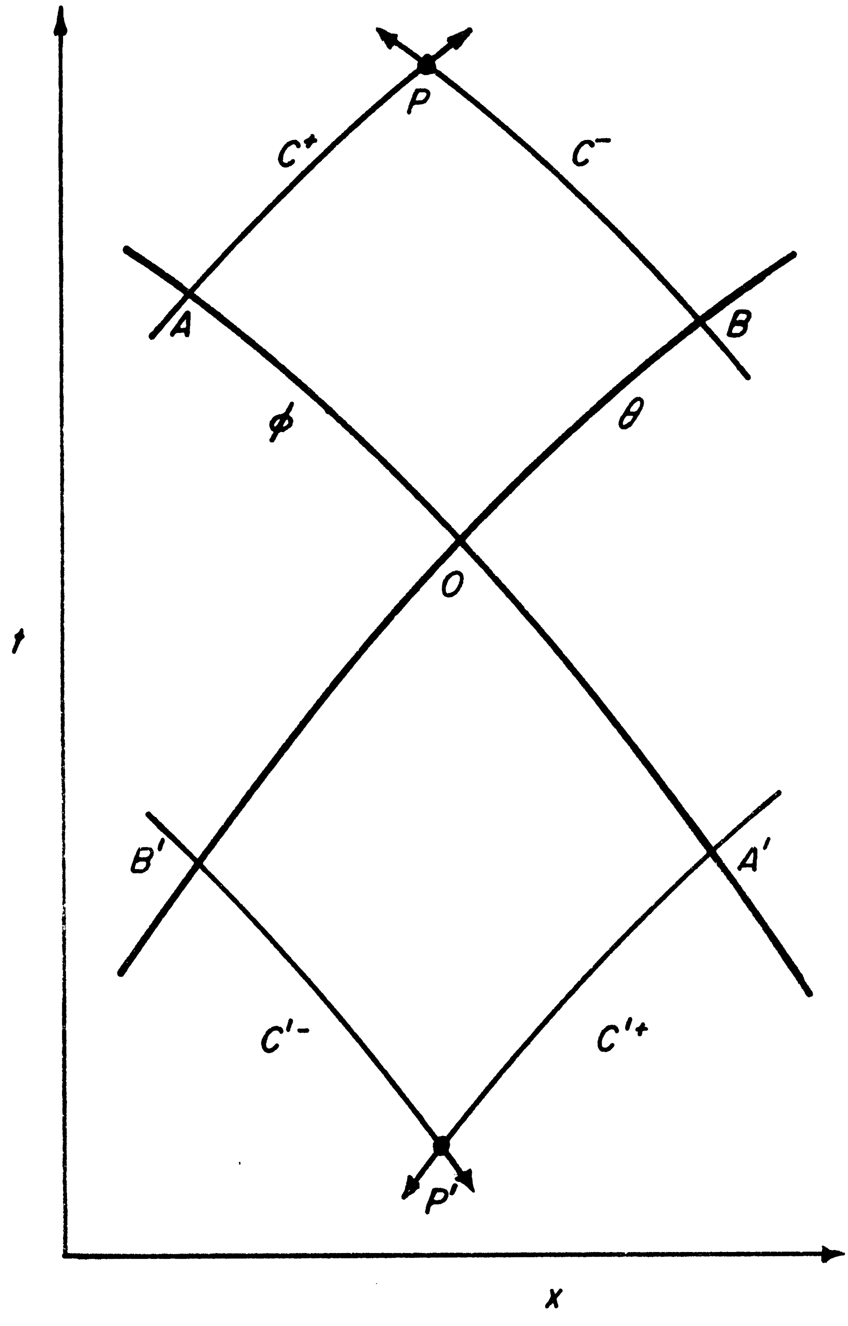


Figure 2. Domains in the x-t plane in which the solution of the characteristic initial-value problem can be established.

establishes a relation between the dependent variables. Consequently, only one function may be prescribed along Θ and, at a single point, the value of the other function.

In many applications the initial-value problem is posed not for a non-characteristic initial curve but for initial data along two intersecting characteristic arcs. This characteristic initial-value problem is formulated for the characteristic differential equations as follows. Let one assume that compatible values of V(x,t) and H(x,t) exist along two characteristic segments, a C+ characteristic Θ and a C- characteristic ∅, as in Figure 2. The C+ characteristic which intersects the curve ∅ at A and the C- characteristic which intersects the curve Θ at B intersect at point P. Then it may be shown that the values of the solutions V,H at the point P depend only upon the intial values along AOB intercepted by the two characteristics through P. Again, this interval AOB is called the domain of dependence of P and the uniqueness of the solution of Equations (1) and (2) in the region AOBP can be demonstrated.

As before, no side of the intersection of Θ and ∅ is distinguished. Therefore, if compatible values of V(x,t) and H(x,t) exist along Θ and ∅, then the interval B'OA' is the domain of dependence of the point P' and the solution of Equations (1) and (2) in the region B'OA'P' also exists and is likewise unique.

2.2.3 Boundary-Data Problems

In this section uniqueness properties relative to problems in which the initial data are prescribed on two or more non-characteristic arcs are examined. Referring to Figure 3(a), Θ and ∅ are two non-characteristic arcs meeting at point O and enclosing an angular region R of the x-t plane. Assigning an arbitrary direction to each of the two families of characteristics (in this example both directions are assigned

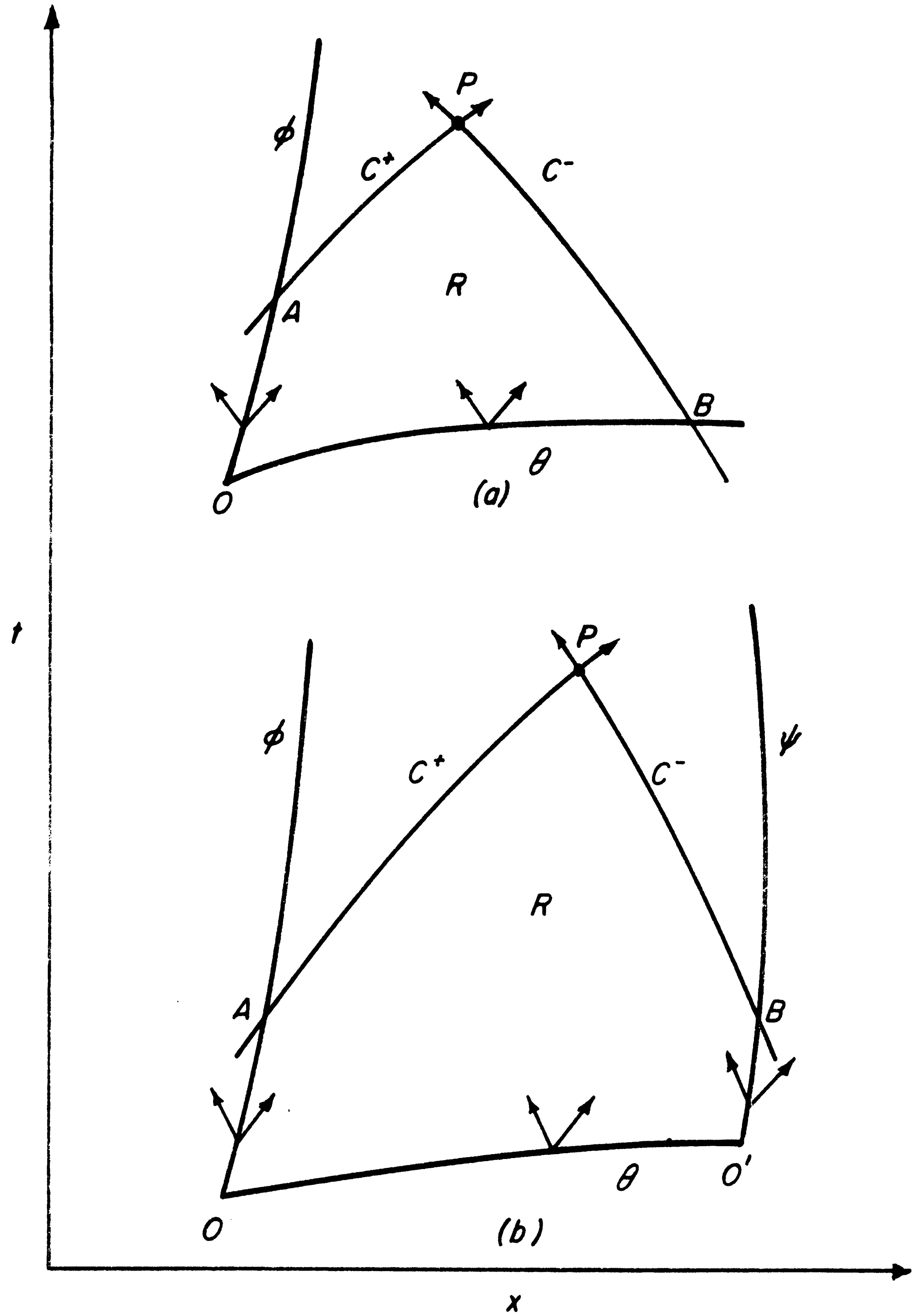


Figure 3. Domain of dependence on (a) a space-like and a time-like arc, and (b) a space-like and two time-like arcs.

as positive, where positive and negative are associated with increasing and decreasing values of time, respectively), it is seen that two characteristics issuing from points on Θ enter region R while only one characteristic issuing from points on ϕ enters R. Then it may be stated that two data on Θ -- both V(x,t) and H(x,t) -- and one datum on ϕ -- either V(x,t) or H(x,t) -- determine the solution in R. If the data are prescribed in this manner, then it may be shown that the solution of Equations (1) and (2) in the region R is unique. Again, the designation "domain of dependence of the point P" for the segment AOB is justified.

Before further characterizing these problems which possess a unique solution, it becomes convenient to introduce the concepts of space-like and time-like directions. A direction is called space-like if both characteristic directions with $dt > 0$ lie on the same side of it; a direction is called time-like if it separates the characteristic directions with $dt > 0$. In this last example, for instance, Θ is space-like and ϕ is time-like.

Essential to the formulation of these boundary-data problems is the following observation: the number of initial data prescribable on an arc depends upon the number of characteristics which, when drawn from a point on the arc in the arbitrarily assigned directions of the characteristic families, enter the region of interest.

The fashion in which these concepts may be extended to three boundaries is demonstrated in Figure 3(b). Arbitrarily assigning positive directions to each of the two families of characteristics, then for a unique solution to exist in the region R two data must be prescribed on the space-like arc Θ and one datum each on the time-like arcs ϕ and ψ. Again the segment AOO'B is designated as the domain of dependence of the point P. It is pertinent to note here that when the arcs ϕ and ψ are parallel to the t-axis and the arc θ is parallel to the x-axis, then the region R assumes the familiar configuration encountered in the conventional transient analysis solutions.

A central concept in the consideration of such boundary-data problems is that the assigned directions of the two families of characteristics are arbitrary. If, as in Figure 4(a), the positive direction is assigned to the C+ characteristic family and the negative direction to the C- characteristic family, then it is observed that two characteristics issuing from points on the time-like arc ϕ enter R while only one characteristic issuing from points on the space-like arc θ enters R. Again, it may be shown that if two data are prescribed on ϕ and one datum on θ the solution of Equations (1) and (2) in the region R exists and is unique. As before, the segment AOB is designated as the domain of dependence of the point P.

In Figure 4(b) a limiting case of the last problem is considered. As before, two data are prescribed on the time-like arc ϕ and one datum (or two compatible data) on the space-like arc θ which now is a characteristic curve. Again, the solution of Equations (1) and (2) exists in R and is unique, and the segment AOB is the domain of dependence of the point P.

A particularly significant consequence of this latter type of boundary-data problem is illustrated in Figure 4(b) and is that along any other time-like arc in R (for example, the arc ψ) no datum may be prescribed. Arc ψ lies within the region of the unique solution of the given boundary-data problem, and hence the solution along ψ is unique and dependent upon the data prescribed on ϕ and θ.

When ϕ and ψ are parallel to the t-axis and θ coincides with a characteristic curve, this last boundary-data problem will be shown to be of fundamental importance in the formulation of the valve-stroking theory.

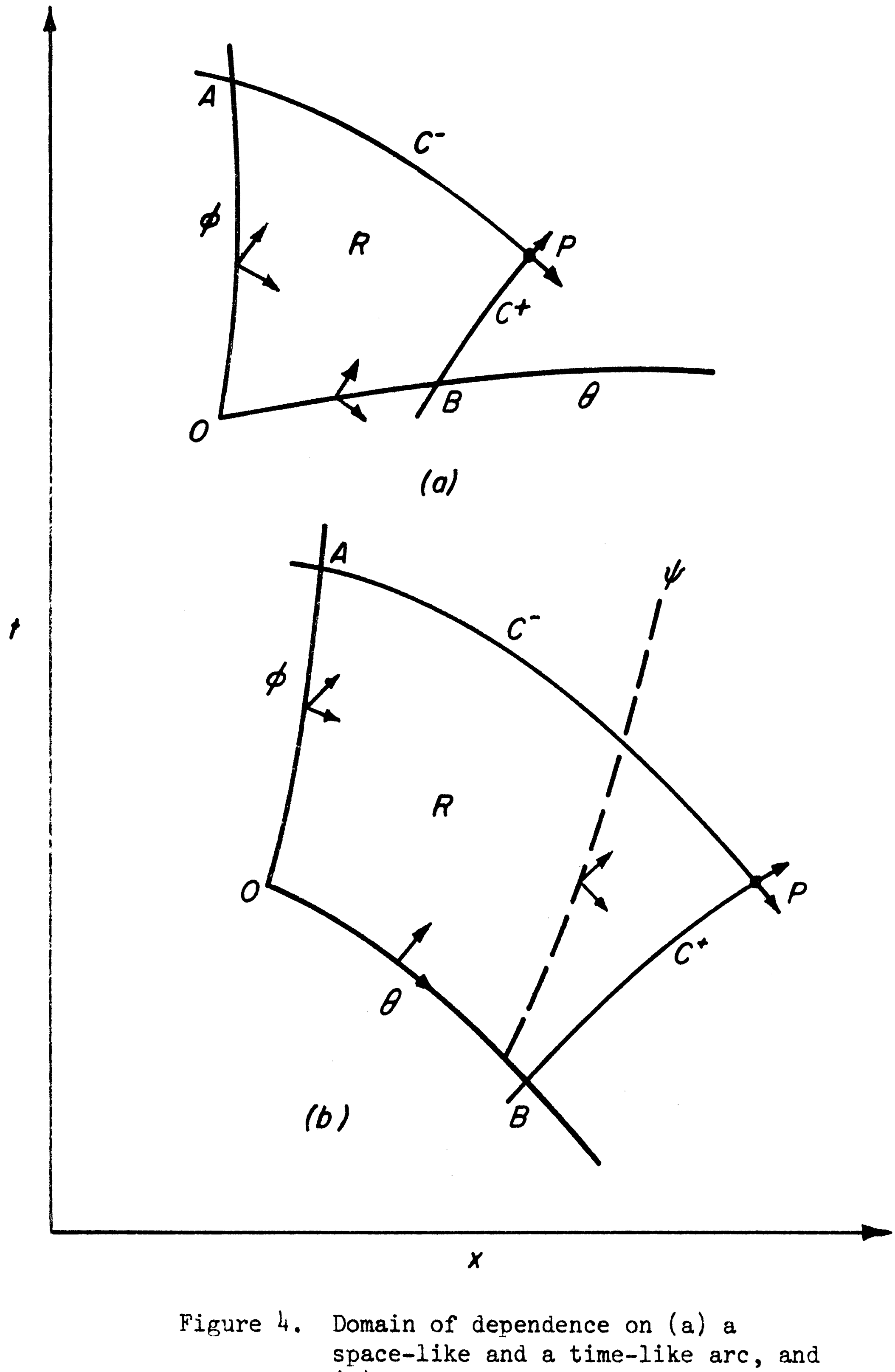


Figure 4. Domain of dependence on (a) a space-like and a time-like arc, and (b) the limiting case of a space-like and a time-like arc.

2.3 Simplified Equations and Finite-Difference Solutions

In Equations (1) and (2) the nonlinear convective terms and the **V sin α** term can be shown[31] to have a negligible effect upon the consequent solutions and are often deleted. (At least one justification is that these terms do not include time derivatives and yet are not commonly incorporated into the corresponding steady-state solutions.) The simplified continuity equation and equation of motion are then obtained and are, respectively

$$\frac{a^2}{g}V_x + H_t = 0 \tag{5}$$

$$V_t + gH_x + \frac{fV|V|}{2D} = 0 . \tag{6}$$

The corresponding characteristic ordinary differential equations obtained from the solution of Equations (5) and (6) by the method of characteristics then become

$$\left.\begin{aligned} &\frac{dV}{dt} + \frac{g}{a}\frac{dH}{dt} + \frac{fV|V|}{2D} = 0 && (7)\\ &\frac{dx}{dt} = +a && (7a) \end{aligned}\right\} C^+$$

$$\left.\begin{aligned} &\frac{dV}{dt} - \frac{g}{a}\frac{dH}{dt} + \frac{fV|V|}{2D} = 0 && (8)\\ &\frac{dx}{dt} = -a && (8a) \end{aligned}\right\} C^- .$$

Again, the significance of these equations parallels that of the earlier set: along a straight line of slope $dx/dt = a$ in the x-t plane Equation (7) is valid, and along a straight line of slope $dx/dt = -a$ Equation (8) is valid. (In this development and throughout the remainder of this investigation, unless otherwise indicated, the wave speed a will be considered to be constant for a given pipe.)

Numerical approximations to the solution of Equations (7) to (8a) may be constructed by utilizing either the linear first-order or the trapezoidal second-order finite-difference approximations.(16) Although the first-order approximations are widely used and have been demonstrated to be generally adequate,(31) for reasons to be explained later some motivation does exist for the employment of the second-order approximations. The formula for this approximation is

$$\int_{x_0}^{x_1} f(x)dx \approx \frac{1}{2}\left[f(x_o) + f(x_1)\right](x_1 - x_0).$$

With reference to Figure 5, it is assumed that all conditions are known at points A and B. The C+ characteristic through A and the C- characteristic through B intersect at point P where conditions are unknown and are to be determined. Equations (7) to (8a) are now multiplied by dt and integrated utilizing the second-order finite-difference approximation. The results are

$$\left.\begin{aligned} &V_P - V_A + \frac{H_P - H_A}{B} + \frac{1}{2}\left(\frac{f_A V_A |V_A|}{2D} + \frac{f_P V_P |V_P|}{2D}\right)(t_P - t_A) = 0 \qquad (9) \\ &x_P - x_A = a(t_P - t_A) \qquad (9a) \end{aligned}\right\} C^+$$

$$\left.\begin{aligned} &V_P - V_B - \frac{H_P - H_B}{B} + \frac{1}{2}\left(\frac{f_B V_B |V_B|}{2D} + \frac{f_P V_P |V_P|}{2D}\right)(t_P - t_B) = 0 \qquad (10) \\ &x_P - x_B = -a(t_P - t_B), \qquad (10a) \end{aligned}\right\} C^-$$

where the subscripts are used to define the location of the known or unknown quantities, and the constant $B = a/g$. Since the friction factor f may ordinarily be determined as a function of velocity, $f = f(V)$, then these four equations are sufficient to determine the values of the four

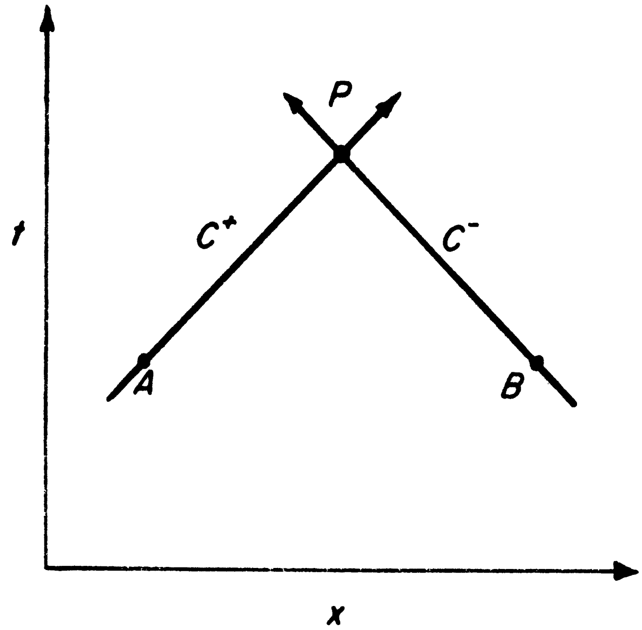


Figure 5. Intersection of the C^+ and C^- characteristics in the x-t plane.

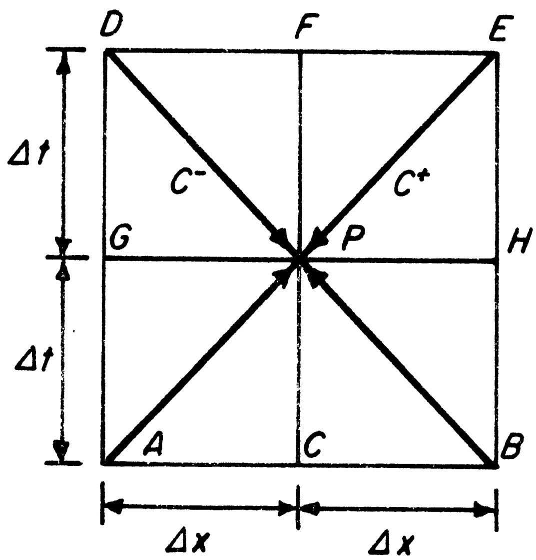


Figure 6. Notation for finite-difference equations.

unknowns V_P, H_P, x_P, and t_P.

There are several ways[16,22,31] of employing the set of Equations (9) to (10a) to obtain an orderly approximate numerical solution to the original set of simplified partial differential equations. One possibility is to use a double-staggered grid of characteristics. Such a grid is particularly simple in this case and, in fact, is identical with a grid of specified-time intervals. Since Equations (9a) and (10a) are independent of V and H, the grid can be immediately determined and is independent of the repeated solutions of Equations (9) and (10). If a pipe of length L is considered to be divided into N equal reaches, then $x_P - x_A = x_B - x_P = \Delta x = L/N$. From Equations (9a) and (10a), $t_P - t_A = t_P - t_B = \Delta t = \Delta x/a = L/(aN)$. Therefore, for a particular pipe and an arbitrarily selected value of N, the grid assumes a rectangular shape with a constant mesh spacing defined by Δx and Δt above.

As a convenience in reducing the friction term in the finite-difference equations and in order to better appreciate its significance, one such term is now examined, for example $\frac{f_A V_A |V_A|}{2D}(t_P - t_A)$. Since $t_P - t_A = L/(aN)$, this term now becomes

$$\frac{f_A V_A |V_A|}{2D} \cdot \frac{L}{aN} = \frac{1}{B} \frac{f_A L}{DN} \frac{V_A |V_A|}{2g} \quad .$$

Thus, this term reduces to $\frac{1}{B}\Delta h_f(V_A)$ where $\Delta h_f(V_A)$ is the corresponding steady-state head loss sustained over a section of pipe of length L/N when the velocity of the fluid flowing is V_A. The variable F may be defined to represent this function which, for a specific pipe, can be determined as a known function of velocity only:

$$F = \Delta h_f(V).$$

As has already been observed and demonstrated with regard to the characteristic ordinary differential equations, no constraints relative to time exist. Therefore, in addition to Equations (9) and (10) a set of companion equations that are the result of finite-difference integrations along the negatively-directed characteristic lines also exist. These equations are virtually identical to the respective equations associated with the positively-directed characteristics, differing only in the signs of the friction terms. With reference to Figure 6 and incorporating the notation for the friction term as proposed above, these four final finite-difference equations are

$$V_P - V_A + \frac{H_P - H_A}{B} + \frac{F_P + F_A}{2B} = 0 \qquad C^{+}\nearrow \tag{11a}$$

$$V_P - V_B - \frac{H_P - H_B}{B} + \frac{F_P + F_B}{2B} = 0 \qquad C^{-}\nwarrow \tag{11b}$$

$$V_P - V_E + \frac{H_P - H_E}{B} - \frac{F_P + F_E}{2B} = 0 \qquad C^{+}\swarrow \tag{11c}$$

$$V_P - V_D - \frac{H_P - H_D}{B} - \frac{F_P + F_D}{2B} = 0. \qquad C^{-}\searrow \tag{11d}$$

These equations and the corresponding grid arrangement for which they are valid will be used throughout the remainder of this investigation to numerically model and evaluate the transient phenomenon.

2.4 Interior-Point and Boundary-Condition Equations

In Section 2.2 uniqueness properties of the characteristic solutions were discussed, and selected characteristic initial-value and boundary-data problems were examined. In this section attention is directed to the fashion by which Equations (11a) to (11d) may be utilized to obtain the desired numerical solutions to such problems. Discussion of the particular computational procedures involved in these solutions will be deferred until the need for a specific procedure is encountered in subsequent phases of this investigation.

2.4.1 Points Interior from a Space-Like Arc

In Figure 7 the double-staggered grid of characteristics is established in the solution plane. Arbitrarily assigning the positive direction to both characteristic families, it is assumed that the required values of $V(x,t)$ and $H(x,t)$ are known or prescribed along the space-like line $t = t_o$ (or at least at the N+1 intersections of the vertical grid lines with the line $t = t_o$). Then Equations (11a) and (11b) may be combined to solve for the desired values of V and H at all interior (non-boundary) grid points situated on the line $t = t_o + \Delta t$, as in Figure 7(a).

Subtracting Equation (11b) from (11a) and rearranging, the resulting expression for the desired value of H_P is

$$H_P = \frac{H_A+H_B}{2} + B\frac{(V_A-V_B)}{2} - \frac{F_A-F_B}{4} \quad . \tag{12a}$$

This value of H_P may then be substituted into Equation (11a); after rearrangement, the desired expression for V_P is

$$V_P = V_A - \frac{H_P-H_A}{B} - \frac{F_P+F_A}{2B} \quad . \tag{12b}$$

Since F_P is a function of V_P, this latter equation must be solved by iteration. By basing the first value of F_P upon the value of V existing at point A (i.e., initially evaluating V_P utilizing the first-order finite-difference approximation), and successively improving this value using the previously calculated value of V_P, the solution is readily convergent. (See Appendix A.)

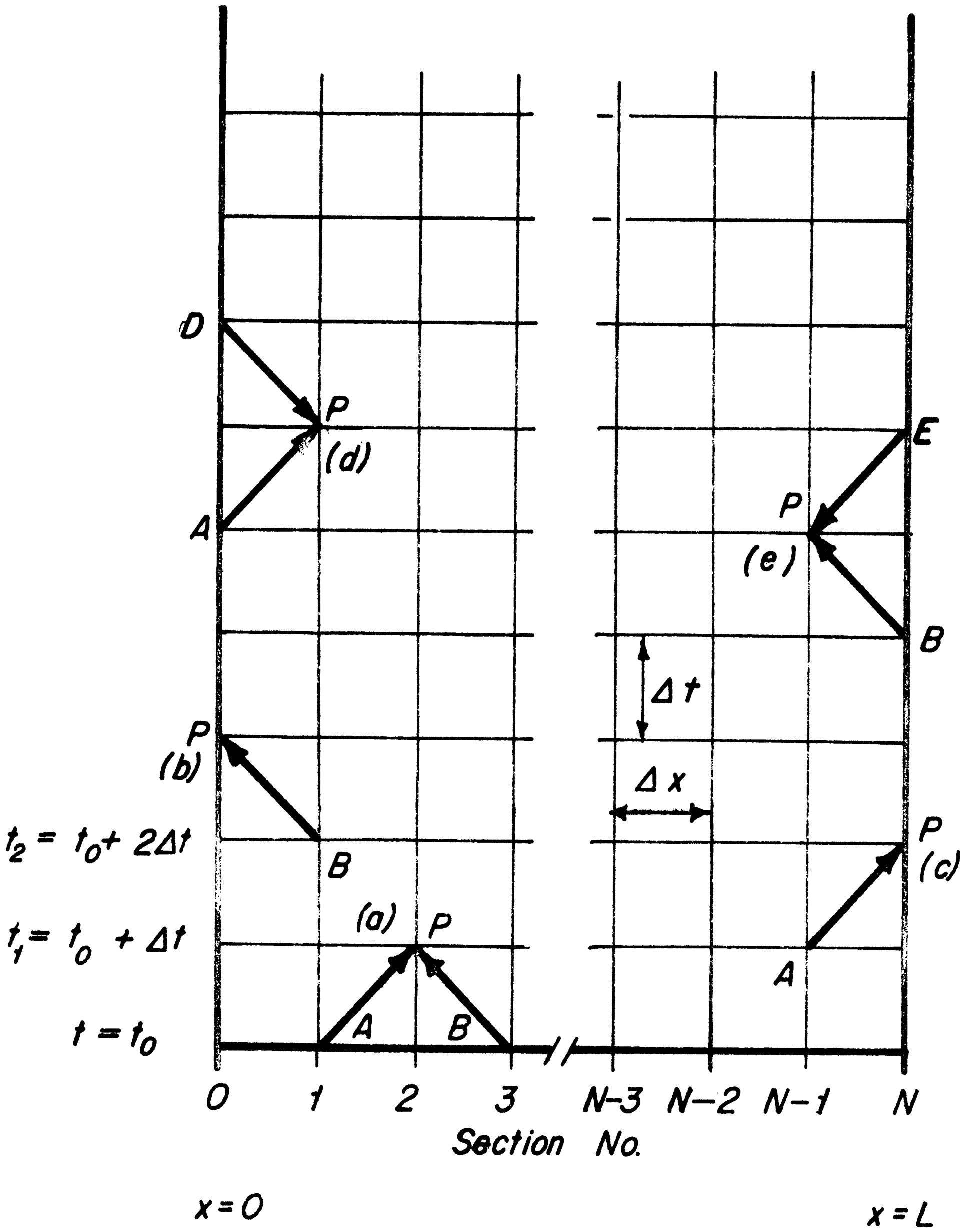


Figure 7. The double-staggered grid of characteristics in the x-t plane. Notation for (a) a space-like interior point, (b) a left-end boundary point, (c) a right-end boundary point, (d) and (e) time-like interior points.

2.4.2 Left-End Boundary Conditions

If the positive direction is arbitrarily assigned to both characteristic families, then it has been observed that one datum along each of the two time-like boundary arcs is essential to the existence of a unique solution in the region of interest. This datum may be either a known value of velocity or head, or it may be some known relationship between these two variables. However the case may be, this auxiliary condition must be determined exterior to the pipe and must always reflect the physical behavior of the particular terminal boundary device under consideration.

With reference to Figure 7(b), for example, let one consider that a constant-head reservoir is located at the upstream (left) boundary of the pipe. The unsteady form of the energy equation[29] may be applied to a control volume of fluid between the reservoir surface and the pipe entrance. By assuming that the time rates of change of mass and internal energy existing within this control volume are negligible, the resulting expression obtained for flow from the reservoir into the pipe is

$$H_r = H_P + \frac{V_P^2}{2g} + k_e \frac{V_P^2}{2g} ,$$

in which H_r is the elevation of the reservoir surface above the arbitrarily selected datum and k_e is the minor-loss coefficient for the entrance. (This coefficient is evaluated assuming that its value is the same as would exist during the corresponding condition of steady flow; this assumption is completely analogous to that concerning the unsteady value of the wall shear stress.)

The above relationship may be further simplified, since quite frequently the kinetic-energy and minor-loss terms are inconsequential and therefore not incorporated into the corresponding steady-state solutions.

Then this simplified boundary condition for the time-like line $x = 0$ is

$$H_P = H_r \tag{13a}$$

and has been experimentally verified in numerous studies.[5,15,30,31] This value may then be substituted into the only available finite-difference equation, Equation (11b); the consequent expression for V_P is

$$V_P = V_B + \frac{H_P - H_B}{B} - \frac{F_P + F_B}{2B} \quad . \tag{13b}$$

This latter equation must be solved by iteration; the same technique that was outlined for the solution of Equation (12b) is equally applicable here.

For flow from the pipe into the reservoir, Equations (13a) and (13b) are still valid; this is true even if the kinetic-energy and minor-loss terms are significant, since for this situation $k_e = -1$.

If the elevation of the reservoir, instead of being constant, varies as a known function of time, the only modification required is to incorporate this variation into Equation (13a): i.e. $H_P = H_r(t)$.

Another example of a typical upstream boundary device would be that of a fixed orifice located just inside the entrance of the pipe from a constant-head reservoir. As with the previous example, a quasi-steady relationship may be developed to model its behavior. Again assuming that the time rates of change of mass and internal energy existing within the adjacent fluid are negligible and that the orifice discharge coefficient is the same as would exist for the corresponding steady-state condition,[5,31] the result is

$$V_P = K_o \sqrt{H_r - H_P}$$

for flow from the reservoir into the pipe or

$$V_P = -K_o \sqrt{H_P - H_r}$$

for flow from the pipe into the reservoir; the parameter K_o, assumed to be constant in this investigation, must incorporate the areas of the pipe and orifice, the discharge coefficient of the orifice, and other constant terms.

Solving these relationships simultaneously with Equation (11b), the resulting expressions for V_P are

$$V_P = \frac{-C_1 + \sqrt{C_1^2 + 4(C_2 - K_o^2 F_P/2)}}{2} \qquad (14a)$$

or

$$V_P = \frac{C_1 - \sqrt{C_1^2 - 4(C_2 - K_o^2 F_P/2)}}{2} \quad , \qquad (14b)$$

in which $C_1 = K_o^2 B$ and $C_2 = K_o^2 (BV_B + H_r - H_B - F_B/2)$: i.e., the C's represent a collection of constant terms that are dependent only upon known values existing at the previous time step. As before, these equations must be solved by iteration. Equation (14a) is valid for flow from the reservoir into the pipe and is used whenever the expression $C_2 - K_o^2 F_P/2$ has a positive value; Equation (14b) is valid for flow from the pipe into the reservoir and is used whenever $C_2 - K_o^2 F_P/2$ is negative. This determination must be made during each successive trial of the iteration procedure.

The ultimate value of V_P may then be substituted into Equation (11b) to obtain the desired value of H_P:

$$H_P = H_B + B(V_P - V_B) + \frac{F_P + F_B}{2} \, . \qquad (14c)$$

2.4.3 Right-End Boundary Conditions

Again assigning the positive direction to both characteristic families and referring to Figure 7(c), consider that a control valve is located at the downstream (right) boundary of the pipe. The valve may be analyzed as an orifice,(15,18,29,30,31) and as with the previous example, a quasi-steady relationship may be developed to model its behavior. Again assuming that the time rates of change of mass and internal energy existing in the adjacent fluid are negligible, that the valve discharge coefficient is the same as would exist for the corresponding steady-state condition, and that the kinetic-energy of the fluid within the pipe is small, the result is

$$V_P = K_V \sqrt{H_P} \quad ,$$

and is valid if the valve discharges into the atmosphere, if the centerline elevation of the valve is selected as the arbitrary datum, and if the value of H_P is always greater than zero (gage), thus eliminating the possibility of air backflow through the valve. The parameter K_V is assumed to be a known function of time (or at least known at every intersection of the horizontal grid lines with the time-like line x = L), and it must incorporate the areas of the pipe and valve, the discharge coefficient of the valve, and other constant terms.

This relationship may be solved simultaneously with the only available finite-difference equation, Equation (11a), with the result that

$$V_P = \frac{-C_1 + \sqrt{C_1^2 + 4(C_2 - K_V^2 F_P/2)}}{2} \quad , \tag{15a}$$

in which $C_1 = K_V^2 B$ and $C_2 = K_V^2(BV_A + H_A - F_A/2)$; as with the previous examples, an iteration solution is required. Substitution of the resulting value of V_P into Equation (11a) yields the desired value of H_P:

$$H_P = H_A - B(V_P - V_A) - \frac{F_P + F_A}{2} \quad . \tag{15b}$$

If, instead of a control valve, a fixed orifice is located at the downstream boundary of a pipe discharging into the atmosphere, the preceding analysis must only be modified to the extent that K_V must be replaced by the fixed-orifice parameter K_o.

Another possible downstream boundary condition would exist if the velocity is some known function of time (for example, at a dead end in which the pressure is always greater than vapor pressure, $V(L,t) = 0$); then

$$V_P = V(L,t) . \tag{16a}$$

From Equation (11a) the desired expression for H_P is

$$H_P = H_A - B(V_P - V_A) - \frac{F_P+F_A}{2} . \tag{16b}$$

In a similar fashion, if the downstream head is a known function of t (an obvious example is provided by the presence of a downstream reservoir, in which case $H(L,t) = H_r$),

$$H_P = H(L,t) ; \tag{17a}$$

and

$$V_P = V_A - \frac{H_P-H_A}{B} - \frac{F_P+F_A}{2B} . \tag{17b}$$

Once again, an iteration procedure is utilized in obtaining the desired solution for V_P.

2.4.4 Points Interior from a Time-Like Arc

In Section 2.2 it was observed that no constraints relative to the characteristic directions exist. With reference to Figure 7(d) for example, if the positive direction is arbitrarily assigned to the C^+ family of characteristics and the negative direction to the C^- family, then it was noted that both $V(x,t)$ and $H(x,t)$ must be known or prescribed along the time-like line $x = 0$ in order for a unique solution to exist in the region interior from the left-end boundary. Assuming that these values are known along that line (or at least at every intersection of the horizontal grid lines with the boundary), then Equations (11a) and (11d) may be combined to solve for the desired values of V and H at all grid points located along the line $x = \Delta x$.

Adding Equations (11a) and (11d) and rearranging, the resulting expression for the desired value of V_P is

$$V_P = \frac{V_A+V_D}{2} + \frac{H_A-H_D}{2B} - \frac{F_A-F_D}{4B} . \qquad (18a)$$

This value of V_P may then be substituted into Equation (11a); after rearrangement, the desired expression for H_P is

$$H_P = H_A - B(V_P - V_A) - \frac{F_P+F_A}{2} . \qquad (18b)$$

Scrutiny of these equations reveals at least one motivation for the utilization of the second-order finite-difference approximations: unlike the corresponding solutions involving space-like points, neither solution requires an iteration procedure. This is in direct contrast to the situation encountered when the first-order approximations are utilized, as was the case in the previous valve-stroking investigations.(30,31)

Similarly, if values of both V(x,t) and H(x,t) are known or prescribed at all grid points along the line $x = L$, then the desired values of V and H at all grid points located on the line $x = L - \Delta x$ may be determined, as in Figure 7(e). Adding Equations (11b) and (11c) and rearranging,

$$V_P = \frac{V_B+V_E}{2} - \frac{H_B-H_E}{2B} - \frac{F_B-F_E}{4B} . \qquad (19a)$$

Again, after substitution of this value of V_P into Equation (11b),

$$H_P = H_B + B(V_P - V_B) + \frac{F_P+F_B}{2} \quad . \tag{19b}$$

These latter two pairs of equations will be of fundamental importance in the formulation of suitable transient synthesis procedures. Since in this system of equations the computation of the desired values of the dependent variables originates at one time-like boundary and procedes across the grid to the other boundary, the necessity for the development of auxiliary boundary-condition equations analogous to those developed in Sections 2.4.2 and 2.4.3 is eliminated.

In the discussion of relevant boundary-data problems presented in Section 2.2.3, the general case of this latter type of problem was considered. There it was observed that, in addition to the two data prescribed along a time-like arc, the knowledge of one datum along a space-like arc is usually essential to the existance of a complete solution. However, as will be demonstrated subsequently in this investigation, the space-like bounding arcs encountered will be characteristic lines along which two compatible data will always exist. Therefore, the necessity to develop supplementary equations essential to the determination of the desired solution is likewise eliminated.

2.5 Complex Systems

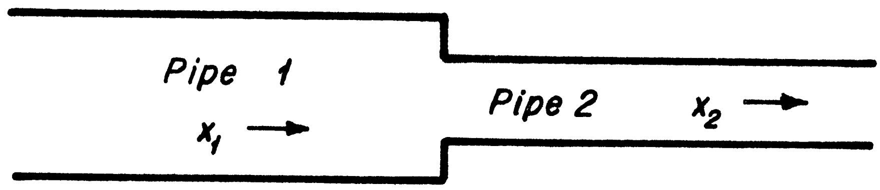
Although the preceding discussion has been restricted to the development of transient-calculation techniques suitable for a single pipe, the transient response of more complex piping systems can be readily analyzed as well.

The transient calculations for each component pipe of a system involve the solution of interior-point equations and boundary-condition equations. As has been demonstrated, at every computational time step the determination of both dependent variables at all interior points is independent of conditions existing at each boundary at that particular instant. Furthermore, if one end of the pipe terminates in an external boundary, i.e., one in which none of the other piping components of the system are connected, then conditions at that boundary may be determined using either the appropriate set of equations developed in Sections 2.4.2 or 2.4.3, or similar equations developed to model the behavior of the specific external boundary device under consideration. What remains, therefore, is merely to consider the analysis of conditions existing at internal boundaries - - interior junction points at which two or more of the system component pipes are connected. In the following sections two typical internal boundary situations are analyzed, and the necessary boundary-condition equations are developed.

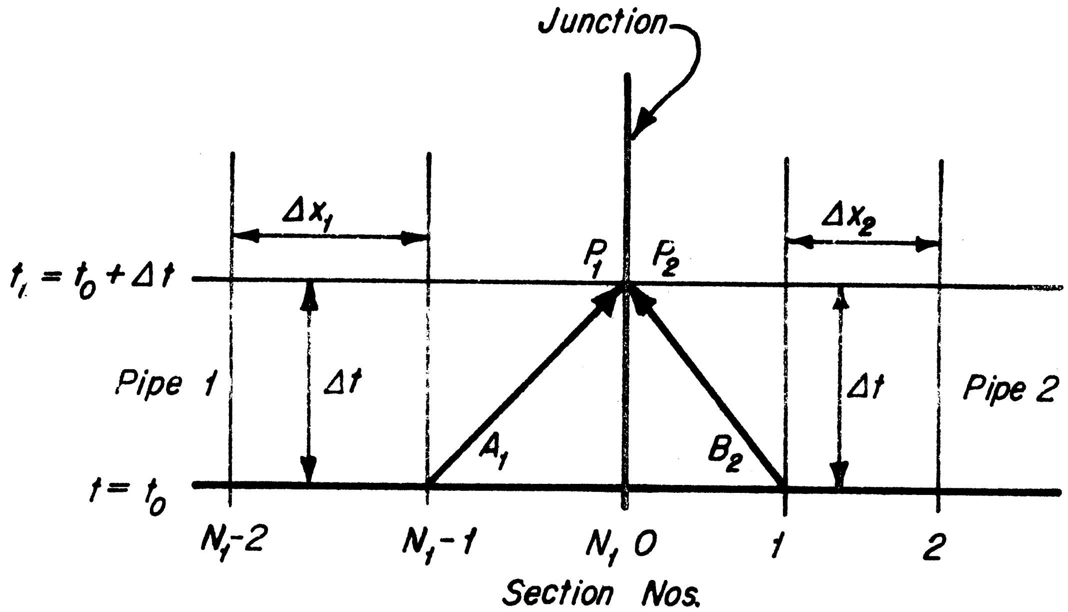
2.5.1 Series-Junction Boundary Condition

With reference to Figure 8, let one first consider the analysis of conditions existing at the junction of two pipes connected in series. (Although in the figure the pipes are depicted as having dissimilar diameters, the following analysis is equally applicable to any combination of discontinuity of diameters, head-loss properties, or wave speeds.)

Assigning the positive direction to both characteristic families and assuming that all conditions are known along the line $t=t_o$, two equations, Equation (11a) for pipe 1 and Equation (11b) for pipe 2, are available to relate the four unknowns V_{P_1}, V_{P_2}, H_{P_1}, and H_{P_2}. The two additional necessary relationships may be developed by application of the unsteady continuity and energy equations to a control volume of fluid adjacent to the junction. Again assuming that the time rates of change of mass and



(a)



(b)

Figure 8. Series-junction boundary condition: (a) definition sketch, (b) solution grids and notation.

internal energy existing within this small volume are negligible and that the kinetic-energy and minor-loss terms are inconsequential, the resulting equations are

$$A_1 V_{P_1} = A_2 V_{P_2}$$

and

$$H_{P_1} = H_{P_2} = H_P \ ,$$

where A_1 and A_2 are the respective areas of pipe 1 and pipe 2.

Combining these four relationships to first eliminate the unknown velocity terms, the result is

$$H_P = \frac{C_3 - C_4 - C_1 F_{P_1}/2 + C_2 F_{P_2}/2}{C_1 + C_2} \ , \qquad (20a)$$

where $C_1 = A_1/B_1$, $C_2 = A_2/B_2$, $C_3 = A_1 V_{A_1} + C_1(H_{A_1} - F_{A_1}/2)$, and $C_4 = A_2 V_{B_2} - C_2(H_{B_2} + F_{B_2}/2)$. This value of H_P may then be substituted into the respective finite-difference equations; the results are

$$V_{P_1} = \frac{C_3 - C_1(H_P + F_{P_1}/2)}{A_1} \qquad (20b)$$

and

$$V_{P_2} = \frac{C_4 + C_2(H_P - F_{P_2}/2)}{A_2} \ . \qquad (20c)$$

Since the F_P's are functions of the respective V_P's, the above set of three equations must be solved by iteration. By initially evaluating F_{P_1} at V_{A_1} and F_{P_2} at V_{B_2} (again, basing the initial evaluations upon the first-order finite-difference approximations) and successively improving these values using the previously calculated values of V_{P_1} and V_{P_2}, the solution of Equations (20a) to (20c) is readily convergent. (See Appendix A.)

One final observation is relevant at this point; this boundary-condition calculation now imposes a restriction upon the heretofore arbitrary grid spacing, since Δt_1 must equal Δt_2. Therefore, it follows that

$$\frac{L_1}{a_1 N_1} = \frac{L_2}{a_2 N_2} .$$

In order for N_1 and N_2 to be suitable integer numbers, minor adjustment of the pipe lengths and/or wave speeds may be required. In reality, however, this presents no real problem, since in most applications wave-speed values generally are not precisely known.[30]

2.5.2 Branching-Junction Boundary Condition

Now consider the analysis of conditions existing at the branching junction of Figure 9. Again assigning the positive direction to both characteristic families and assuming that all conditions are known along the line $t = t_o$, three equations, Equation (11a) for pipe 1 and Equation (11b) for both pipe 2 and pipe 3, are available to relate the six unknowns V_{P_1}, V_{P_2}, V_{P_3}, H_{P_1}, H_{P_2}, and H_{P_3}. The three additional necessary relationships may be obtained from the simplified continuity and energy equations, as before, and are

$$A_1 V_{P_1} = A_2 V_{P_2} + A_3 V_{P_3}$$

and

$$H_{P_1} = H_{P_2} = H_{P_3} = H_P .$$

Combining these six relationships to first eliminate the unknown velocity terms, the result is

$$H_P = \frac{C_4 - C_5 - C_6 - C_1 F_{P_1}/2 + C_2 F_{P_2}/2 + C_3 F_{P_3}/2}{C_1 + C_2 + C_3} , \qquad (21a)$$

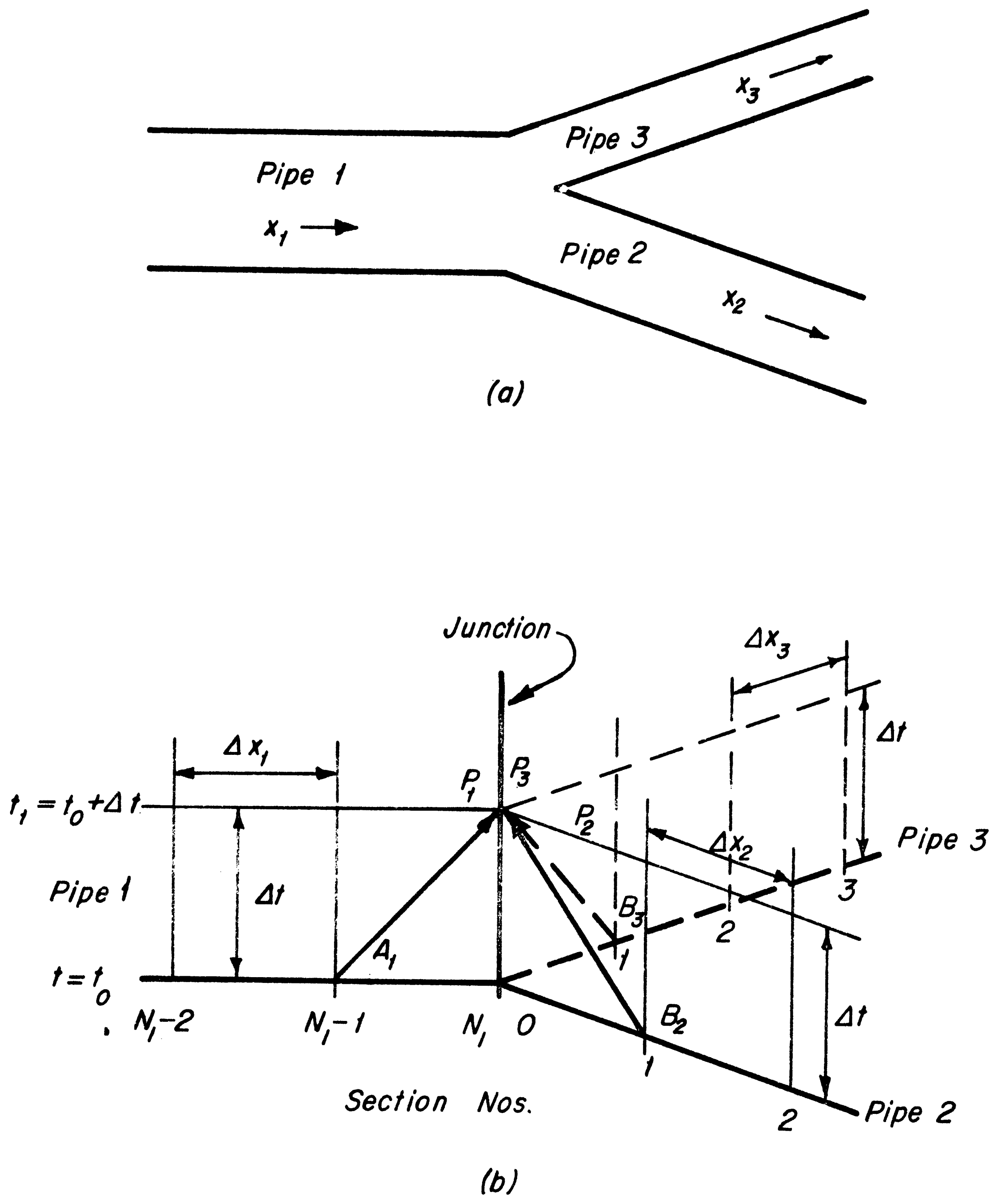


Figure 9. Branching-junction boundary condition: (a) definition sketch, (b) solution grids and notation.

where $C_1 = A_1/B_1$, $C_2 = A_2/B_2$, $C_3 = A_3/B_3$, $C_4 = A_1 V_{A_1} + C_1(H_{A_1} - F_{A_1}/2)$, $C_5 = A_2 V_{B_2} - C_2(H_{B_2} + F_{B_2}/2)$, and $C_6 = A_3 V_{B_3} - C_3(H_{B_3} + F_{B_3}/2)$. Substitution of this value of H_P into the respective finite-difference equations yields:

$$V_{P_1} = \frac{C_4 - C_1(H_P + F_{P_1}/2)}{A_1}, \tag{21b}$$

$$V_{P_2} = \frac{C_5 + C_2(H_P - F_{P_2}/2)}{A_2}, \tag{21c}$$

$$V_{P_3} = \frac{C_6 + C_3(H_P - F_{P_3}/2)}{A_3}. \tag{21d}$$

Again, this set of four equations must be solved by iteration; the same technique that was outlined for the solution of Equations (20a) to (20c) is equally applicable here.

Boundary-condition equations for other internal boundary configurations can be readily developed in a comparable manner.

2.6 Initial and Steady-State Conditions

In concluding this examination of the basic concepts and equations of transient flow, attention inevitably turns toward a consideration of two somewhat related topics: initial conditions and steady-state conditions. The first topic is of interest because of the fashion by which the initial conditions influence and restrict the subsequent transient analysis or synthesis solutions; the second is important because of the dependence of one of the criteria of valve stroking -- elimination of residual transient fluctuations -- upon a thorough understanding of the steady-state phenomenon.

2.6.1 Initial Conditions

With reference to Figure 10, consider that the values of both dependent variables are initially known along the space-like line $t = t_o$ (or at least at the N + 1 intersections of the vertical grid lines with the line $t = t_o$). Then from the discussion in Section 2.2.1, the line $t = t_o$ is the domain of dependence of the region bounded by the positive characteristic line extending upward and to the right from the point $t = t_o$, $x = 0$ and by the negative characteristic line extending upward and to the left from the point $t = t_o$, $x = L$. Therefore, a unique solution exists within this region and is independent of subsequent control-device operation at either boundary.

In addition to the initial existence of both variables along the line $t = t_o$, it is possible that one of the boundary relationships may be invarient. Again referring to Figure 10, suppose that along the left-end boundary one of the two dependent variables (or a relationship between the two) has a constant value. Then from the discussion in Section 2.2.3, a unique solution exists within the triangular region bounded by the line $t = t_o$, the left-end boundary, and the negative characteristic line which extends upward and to the left from the point $t = t_o$, $x = L$ and intersects the line $x = 0$ at $t = t_N$. As before, the solution is unaffected by subsequent control-device operation at the right-end boundary.

Although not intrinsic to the preceding discussion, the consideration of initial steady-state conditions is also of interest. With reference to Figure 10(a), let one assume that steady-state conditions exist along the line $t = t_o$. Then $V_A = V_B = V_C = V_o$, where V_o is the initial steady-uniform velocity in the pipe; $H_A = H_C - F_o$ and $H_B = H_C + F_o$, where F_o is the function

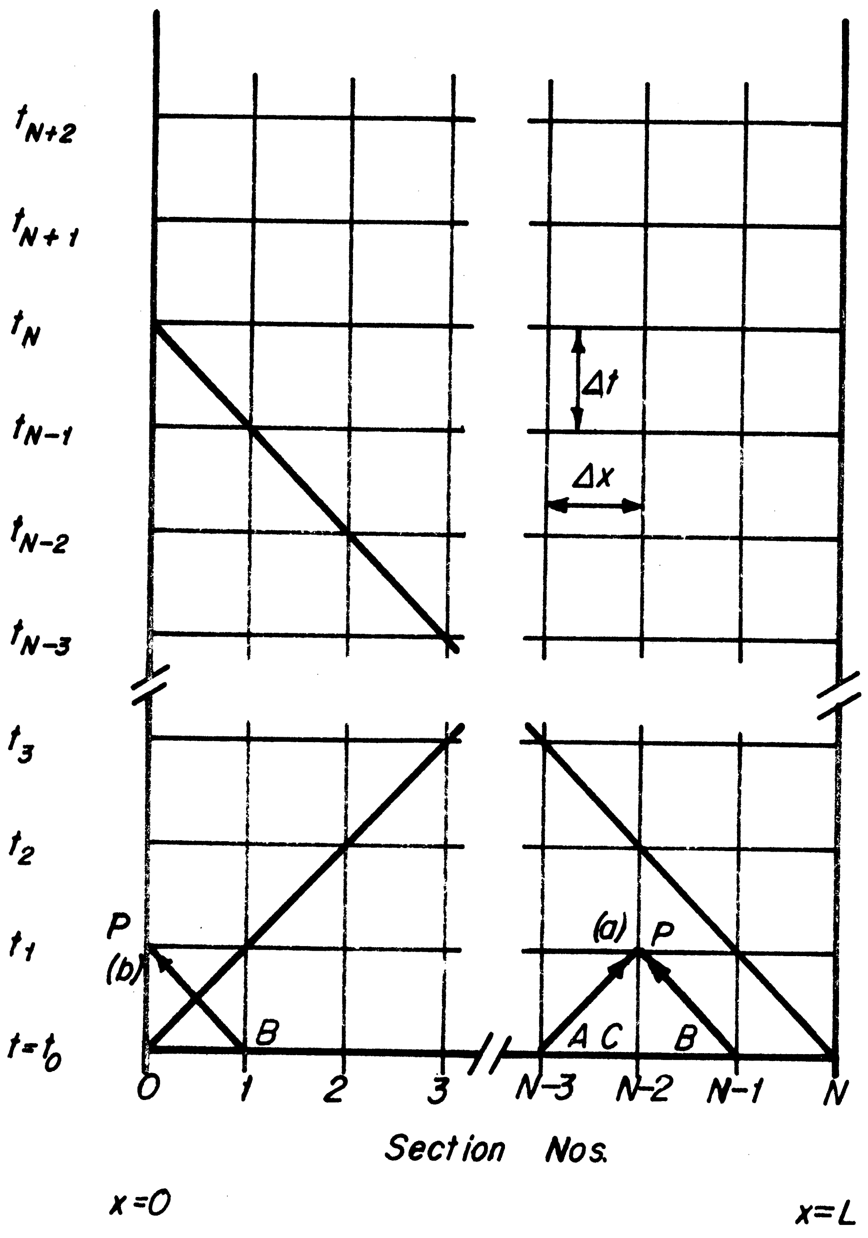


Figure 10. Regions where solution is dependent upon initial conditions. Notation for (a) a steady-state interior point, and (b) a steady-state boundary point.

F evaluated at V_o. (These latter relationships follow directly from the elimination of the time-derivatives from Equations [5] and [6] and from the basic definition of F.)

Incorporation of the above relationships into Equation (12a) and subsequent elimination of canceling terms yields

$$H_P = H_C .$$

After substitution of this result into Equation (12b),

$$V_P + \frac{F_P}{2B} = V_o + \frac{F_o}{2B} .$$

Since the function F is a strictly monotone-increasing function(3) of the velocity V, the only possible conclusion to be derived from this latter equation is that

$$V_P = V_o .$$

Therefore, the point P has been demonstrated to be an initial steady-state point. Since the selection of points A, B, and C was completely arbitrary, then every point within the domain of dependence of the line $t = t_o$ is an initial steady-state point.

If, in addition to the initial existence of steady-state conditions along the line $t = t_o$, one of the boundary relationships is invarient, then the entire region within the domain of dependence of that boundary and the line $t = t_o$ is a region of initial steady-state conditions. For example, assume that a constant-head reservoir is located at the upstream boundary of the pipe. With reference to Figure 10(b), $V_B = V_o$, $H_B = H_r - F_o$, and $H_P = H_r$. After substitution of these identities into Equation (13b),

$$V_P + \frac{F_P}{2B} = V_o + \frac{F_o}{2B} .$$

As before, the only possible conclusion is that

$$V_P = V_o$$

Therefore, from these interior-point and boundary-point evaluations, it is evident that the region of initial steady-state conditions is bounded by the line $t = t_o$, the left-end boundary, and the negative characteristic line which extends upward and to the left from the point $t = t_o$, $x = L$ and intersects the left end boundary at $t = t_N$. Coincidentally, this last example also demonstrates a second interpretation of the characteristic lines: lines along which disturbances in the flow are physically propogated (Ref. 1,6).

2.6.2 Final Steady-State Conditions

As indicated previously, a most significant criteria of valve stroking is the establishment of the final steady-state conditions in a piping system without the existence of residual transient fluctuations. Initially, of course, the transient flow condition originates when either one or both boundary conditions compatible with some initial steady-state condition are altered. Only upon cessation of these boundary modifications (that is, only after these boundary conditions have attained their final steady-state character) will the transient phenomenon subsequently begin to asymptotically (both mathematically and physically) approach the final steady-state solution due to the combined physical processes of reflection of the transient pressure waves and the action of viscous effects.

The singular fashion by which the final steady-state solution may be established without this asymptotic process is indicated in Figure 11. Assuming that one has the freedom to arbitrarily select the characteristic directions (in Chapter III the validity of this assumption is verified), the

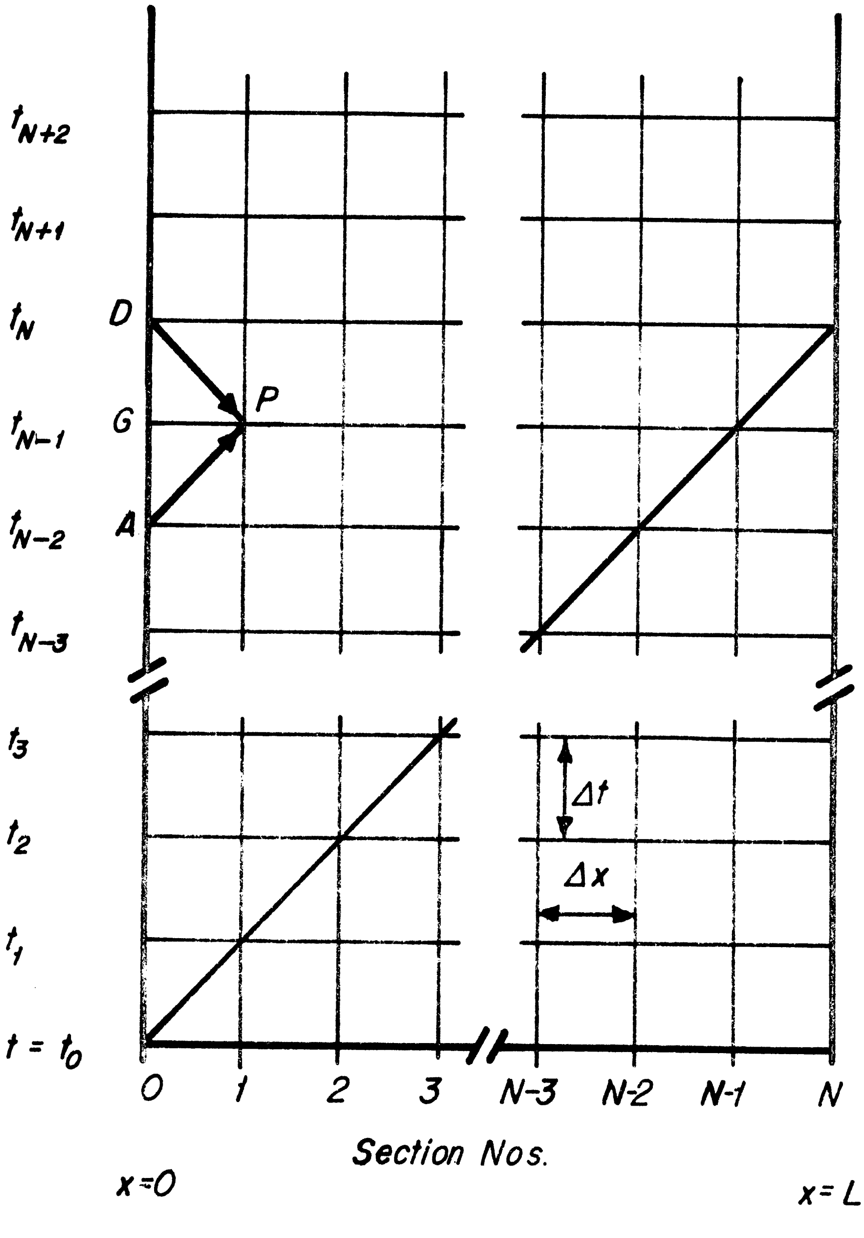


Figure 11. Region of final steady-state solution.

positive direction is assigned to the C^+ family and the negative direction to the C^- family. Recalling that both variables must then be known or prescribed along the left-end boundary, the velocity is arbitrarily prescribed to be V_f, the desired final steady-uniform velocity in the pipe, and the head as H_f, the final steady-state **value** compatible with V_f and either the boundary condition existent at that boundary or other known flow conditions. Then $V_A = V_G = V_D = V_f$ and $H_A = H_G = H_D = H_f$.

Incorporation of these relationships into Equation (18a) and subsequent elimination of canceling terms yields

$$V_P = V_f \ .$$

After substitution of this result into Equation (18b),

$$H_P = H_f - F_f \ ,$$

where F_f is the function F evaluated at V_f. Thus, the point P has been demonstrated to be a final steady-state point. Since the selection of points A, D, and G was completely arbitrary, then final steady-state conditions must exist at every grid point located along the line $x = \Delta x$ on or above the positive characteristic line which extends upward and to the right from the point $t = t_o$, $x = 0$ and intersects the right-end boundary at $t = t_N$.

This latter argument being true for the line $x = \Delta x$, then the same reasoning must hold for the line $x = 2\Delta x$, and so on. Therefore, every grid point within the domain of dependence of that section of the left-end boundary for which $t \geq t_o$ must be a point of final steady-state conditions, ultimately including every point located on the right-end boundary beginning at $t = t_N$.

2.7 Chapter Summary

The transient synthesis procedures developed in subsequent phases of this study will draw abundantly upon the concepts presented in this chapter. As will be demonstrated, an unusual amount of flexibility exists relative to the arbitrary selection of the directions of the two characteristic families. It is possible, for example, to specify both directions as positive during an early stage of the controlled transient and then to arbitrarily assign the negative direction to one of the families during a later stage.

In this investigation a valve-stroking synthesis will refer to the determination of the unknown dependent variables at the two boundaries (and, therefore, the boundary control adjustments required to produce the desired transient condition) either when both variables are specified at one boundary or one variable is specified at each boundary. A characteristics analysis will refer to the determination of the complete history of the transient condition when the unsteady boundary conditions determined by the valve-stroking synthesis are imposed upon the system, and as such it will demonstrate the validity of the valve-stroking computations.

Two broad classifications of valve stroking will be considered in this study. Active valve stroking refers to the synthesis and subsequent operation of a system in which the period of the entire transient may be arbitrarily selected. In a passive valve stroking synthesis and operation, a similar degree of control over the period of the transient is not possible.

The digital computer programs essential to this investigation were executed on the IBM 7090/1410 system at the University of Michigan Computing Center.

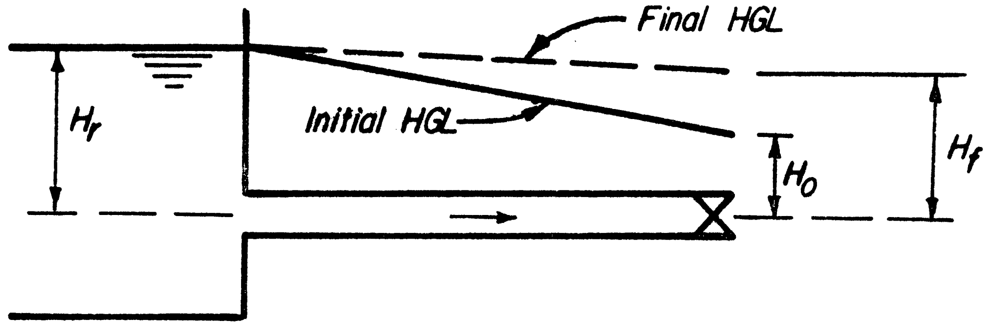
III. VALVE-STROKING CONTROL OF SIMPLE PIPING SYSTEMS

Having presented the fundamental properties and suitable techniques for numerical evaluation of the characteristic solutions in Chapter II, consideration is now given to the application of these concepts and equations to the control of the transient phenomena in simple piping systems.

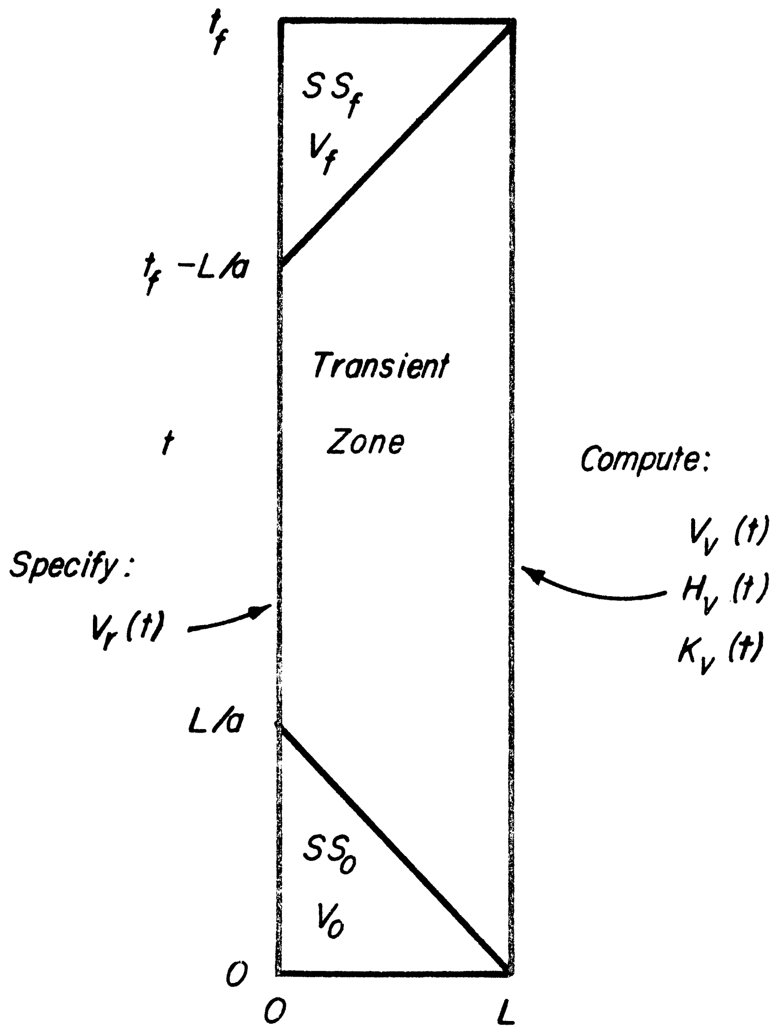
3.1 System with an Upstream Constant-Head Reservoir

Attention is first directed to the most frequently encountered of the simple systems: a single pipe originating at an upstream constant-head reservoir and terminating at a downstream control valve. With reference to Figure 12, consider that it is desired to modify flow conditions in the system from some initial steady-uniform condition (SS_o) to some final steady-uniform condition (SS_f) such that at time $t = t_f$ the final steady-uniform condition is established without the presence of subsequent residual transient fluctuations.

From the discussions presented in Sections 2.2.3, 2.6.1, and 2.6.2, and with reference to Figure 12(b), the following observations are relevant: (1) Because of the presence of the invarient upstream-boundary relationship, $H(0,t) = H_r$, the region of the initial steady-uniform condition is as indicated. (2) If the positive direction is now arbitrarily assigned to the C^+ characteristic family and the negative direction to the C^- characteristic family, then both data must be prescribed along the time-like upstream boundary for a unique solution to exist in the region of interest (recognizing that the requisite compatible data do exist along the space-like characteristic line bounding the initial steady-state region). (3) If the velocity $V_r(t)$ is prescribed



(a)



(b)

Figure 12. Simple system with constant-head reservoir: (a) definition sketch, (b) x-t plane and notation.

along the upstream boundary such that at time $t \geq t_f - L/a$, $V_r(t) = V_f$, then the final steady-uniform condition will be established as desired. (4) Lastly, no datum may be prescribed along the downstream boundary since it lies within the region of the unique solution of the given boundary-data problem. Therefore, the solutions $V_v(t)$, $H_v(t)$, and thus $K_v(t)$ of the valve are solely dependent upon the initial conditions and the data prescribed along the upstream boundary.

Having established the essential element of this problem -- the necessity of specifying the upstream velocity as some function of time -- the next logical step would be to speculate about how best to specify this transient boundary velocity in order to produce the most orderly and controlled transition between the initial and final steady-uniform conditions.

3.1.1 Specification of the Variable Upstream Velocity

Recognizing that along the upstream boundary the hydraulic grade line has a constant value (thus $H_t(0,t) = 0$), one may investigate the consequence of requiring that $H_t(x,t) = 0$ throughout the flow.

With reference to Equation (5), if $H_t = 0$, then $V_x = 0$. Consequently $H = H(x) + C$ and $V = V(t) + K$ are the most general relationships that H and V may satisfy (C and K are constant terms). Then $H_x = H'(x)$ and $V_t = V'(t)$. Subsequent substitution into the simplified equation of motion, Equation (6), yields

$$V'(t) + \frac{fV|V|}{2D} = -gH'(x) \ .$$

Since for a given pipe and fluid $f = f(V)$, the left-hand side of the above equation contains only time-dependent terms while the right-hand side contains only distance-dependent terms. Thus, each side must be equal to a constant and the above expression is now identified as the incompressible surge equation where $H'(x)$ is the slope of the hydraulic grade line. If the extreme permissible value of head desired at the valve is designated as H_m (maximum for closure situations, minimum for situations involving valve openings), this constant slope of the hydraulic grade line becomes $H'(x) = (H_m - H_r)/L$. Thus

$$\frac{dV}{dt} = -g\,\frac{(H_m - H_r)}{L} - \frac{fV|V|}{2D} \tag{22}$$

is suggested as the technique for specifying $V_r(t)$ along the left-end boundary.

One may next investigate the consequences implied by utilization of Equation(22). With reference to Figure 13, assume that the velocity at every grid point along the left-end boundary has been specified according to the second-order finite-difference approximation to Equation (22):

$$V_{i+1} = V_i - \frac{(H_m - H_r)}{NB} - \frac{F_i + F_{i+1}}{2B}\,. \tag{22a}$$

(This approximation follows from the definition of the function F and from $B = a/g$, $L/\Delta x = N$; it is completely analogous to the comparable finite-difference approximations of the characteristic equations and is solved utilizing the same iteration technique outlined for solution of the equations of Chapter II.)

Thus $H_A = H_G = H_D = H_r$ and

$$V_G = V_A - \frac{(H_m - H_r)}{NB} - \frac{F_A + F_G}{2B}$$

$$V_D = V_G - \frac{(H_m - H_r)}{NB} - \frac{F_G + F_D}{2B}\,.$$

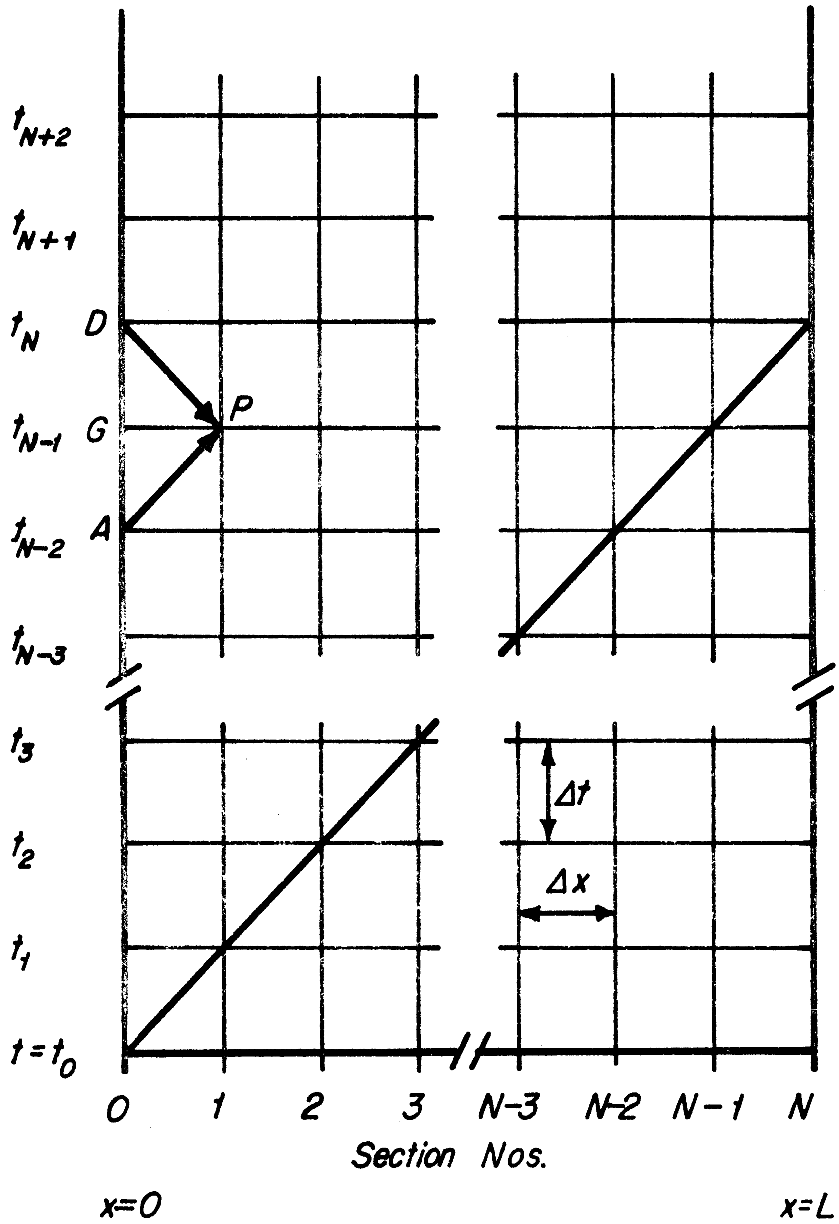


Figure 13. Region of established uniform surge condition.

Incorporation of these relationships into Equation (18a) and subsequent elimination of canceling terms yields

$$V_P = V_G$$

After substitution of this result into Equation (18b),

$$H_P = H_r + \frac{(H_m - H_r)}{N} \quad .$$

Therefore, conditions at the point P have been demonstrated to satisfy the incompressible surge conditions: $V_x = 0$, $H_t = 0$, and H_x = a constant. Since the selection of points A, D, and G was completely arbitrary, then surge conditions must exist at every grid point located along the line $x = \Delta x$ on or above the positive characteristic line which extends upward and to the right from the point $t = t_o$, $x = 0$ and intersects the right-end boundary at $t = t_N$.

This latter argument being true for the line $x = \Delta x$, then the same reasoning must hold for the line $x = 2\Delta x$, and so on. Therefore, <u>every</u> grid point within the domain of dependence of that section of the left-end boundary over which the velocity has been arbitrarily prescribed using Equation (22a) must be a point of incompressible surge conditions. This ultimately includes every point along the right-end boundary lying within the aforementioned domain of dependence, beginning at $t = t_N$, and the head at the boundary must then have the prescribed extreme value H_m.

3.1.2 Study of the Frictionless System

For a system in which viscous effects are so insignificant that they may be safely neglected, the concepts investigated above may be extended and equations describing the entire history of the transient condition throughout the system may be developed. Although such a study would appear to have limited value, ultimately it will be demonstrated that the basic history of the transient condition is only slightly modified by the presence of significant viscous effects; therefore, a thorough analysis of the functionless system provides valuable insight into the nature of the transient condition developed in the system in which viscous effects are important.

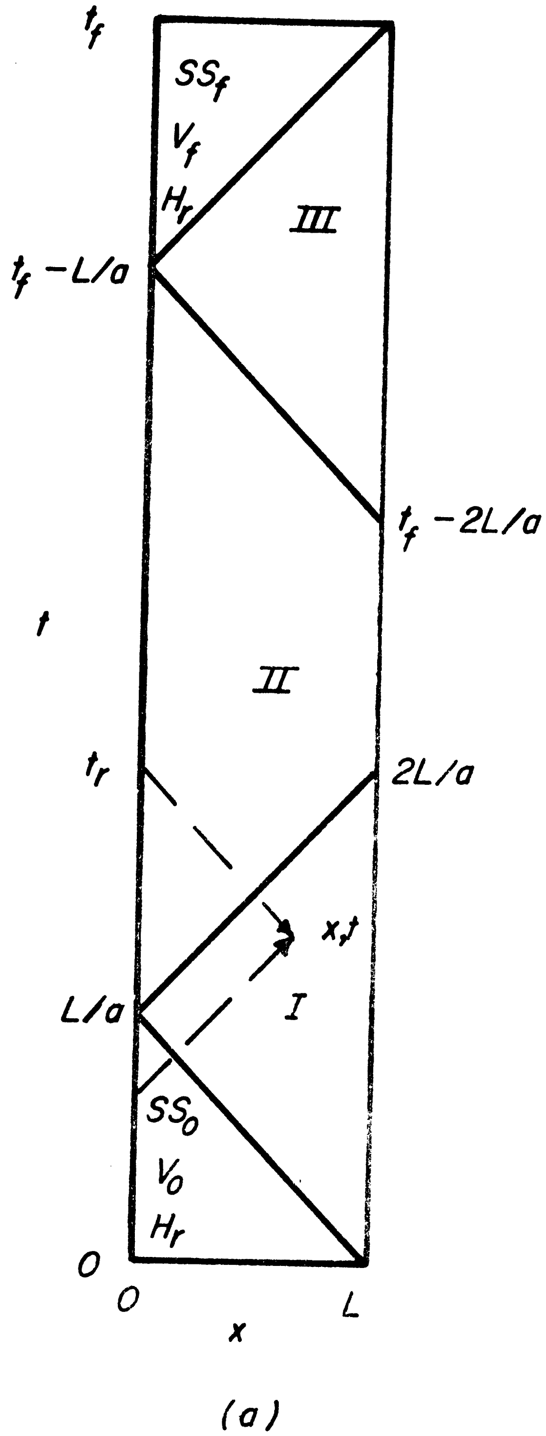
For a given situation (all physical properties and initial and final steady-uniform conditions specified), the time of the complete transient t_f is a function of the arbitrarily selected value of H_m. For the frictionless system, Equation (22) may be integrated. The initial condition is $V = V_o$ when $t = L/a$, the final condition is $V = V_f$ when $t = t_f - L/a$, and the result is

$$t_f = 2L/a + \frac{L}{g}\frac{(V_o - V_f)}{(H_m - H_r)} \quad . \tag{23}$$

In view of the fact that $t_f \geq 2L/a$, the quantity $(H_m - H_r)$ must have the same sign as $(V_o - V_f)$.

The basic pattern of the transient condition imposed upon the system is considerably different for values of $t_f \geq 4L/a$ than it is for values of t_f between $2L/a$ and $4L/a$. Referring to Figure 14(a), one may first consider the pattern for $t_f \geq 4L/a$.

Since conditions in Zone II already have been demonstrated to satisfy the incompressible surge equations, for the frictionless system these equations, after integration and evaluation with the appropriate initial conditions, are



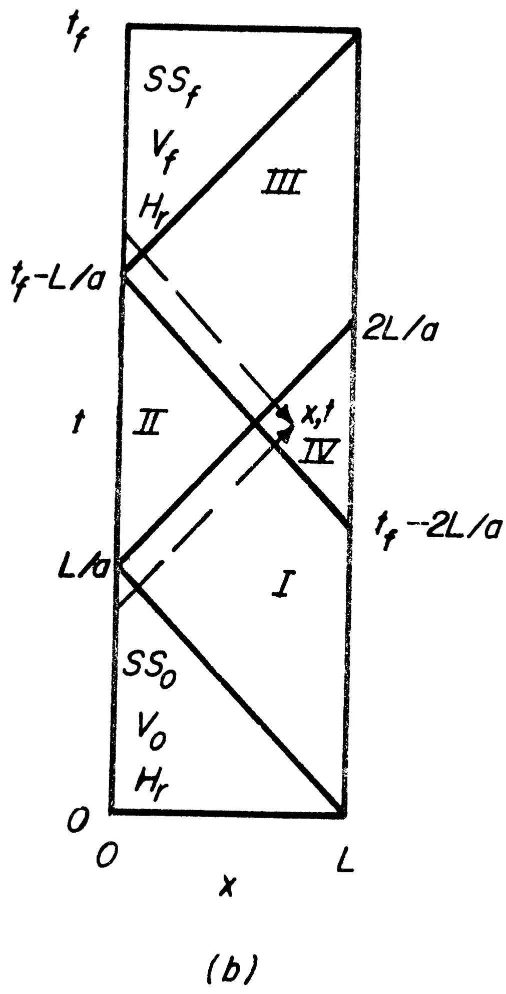


Figure 14. Simple frictionless system with constant-head reservoir. x-t planes for (a) $t_f \geq 4L/a$, and (b) $t_f < 4L/a$.

$$V(t) = V_o - g\,\frac{(H_m - H_r)}{L}\,(t - L/a)\ , \tag{24a}$$

$$H(x) = H_r + \frac{(H_m - H_r)}{L}\,x\ . \tag{24b}$$

Next by solving Equations (18a) and (18b) for a point (x,t) in Zone I simultaneously with Equation (24a) evaluated at time t_r and the condition that $t = t_r - x/a$,

$$V(x,t) = V_o - \frac{g}{2}\,\frac{(H_m - H_r)}{L}\,\left(t + \frac{x-L}{a}\right)\ , \tag{24c}$$

$$H(x,t) = H_r + \frac{a}{2}\,\frac{(H_m - H_r)}{L}\,\left(t + \frac{x-L}{a}\right)\ . \tag{24d}$$

Equations describing the history of the transient condition throughout Zone III may be developed in a similar fashion. These equations are

$$V(x,t) = V_f + \frac{g}{2}\,\frac{(H_m - H_r)}{L}\,\left(t_f - t + \frac{x-L}{a}\right)\ , \tag{24e}$$

$$H(x,t) = H_r + \frac{a}{2}\,\frac{(H_m - H_r)}{L}\,\left(t_f - t + \frac{x-L}{a}\right)\ . \tag{24f}$$

Evaluation of these equations at the right-end boundary enables one to explicitly determine the time-dependent value of the valve coefficient required to produce the desired transition between the initial and final steady-uniform conditions:

$$K_v(t) = \frac{V_o - \frac{g}{2}\frac{(H_m - H_r)}{L}\,t}{\sqrt{H_r + \frac{a}{2}\frac{(H_m - H_r)}{L}\,t}} \qquad 0 \le t \le 2L/a\ , \tag{25a}$$

$$K_v(t) = \frac{V_o - g\,\frac{(H_m - H_r)}{L}\,(t - L/a)}{\sqrt{H_m}} \qquad 2L/a \le t \le t_f - 2L/a\ , \tag{25b}$$

$$K_v(t) = \frac{V_f + \frac{g}{2}\frac{(H_m - H_r)}{L}\,(t_f - t)}{\sqrt{H_r + \frac{a}{2}\frac{(H_m - H_r)}{L}\,(t_f - t)}} \qquad t_f - 2L/a \le t \le t_f\ . \tag{25c}$$

These valve equations are the dimensional equivalents to Streeter's dimensionless valve equations.[26,31]

The limiting case for which the preceding analysis is valid is for $t_f = 4L/a$. This case is of especial interest; the apex of the surge zone (now a triangle) just extends to the right-end boundary at $t = 2L/a$. The value of H_m necessary to produce this situation is

$$H_m = H_r + B \frac{(V_o - V_f)}{2} \quad . \tag{26}$$

One may now consider the transient pattern that develops for values of t_f between 2L/a and 4L/a. As may be readily seen from Figure 14(b), an analysis of the conditions developed in Zones I, II, and III performed in an identical fashion as before would yield the same results. Thus, Equations (24a) to (24f) are still valid. Then solving Equations (18a) and (18b) for a point (x,t) in Zone IV, the results are

$$V = \frac{V_o + V_f}{2} \quad , \tag{24g}$$

$$H = H_r + B \frac{(V_o - V_f)}{2} \quad . \tag{24h}$$

The Zone I and Zone III valve coefficient equations previously developed are still applicable, except that for this situation Equation (25a) is only valid during $0 \leq t \leq t_f - 2L/a$, and Equation (25c) is only valid during $2L/a \leq t \leq t_f$. The constant valve coefficient that obtains during the central phase of the transient condition is then

$$K_v = \frac{\frac{V_o + V_f}{2}}{\sqrt{H_r + B \frac{(V_o - V_f)}{2}}} \qquad t_f - 2L/a \leq t \leq 2L/a \, . \tag{25d}$$

In concluding this examination of the frictionless system, one last observation is relevant. One recognizes that Equations (24h) and (26) are identical. Thus, Equation (24h) represents the maximum (or minimum) value of head that can possibly be developed throughout the system when the proposed valve-stroking control procedures are utilized. Even if an H_m were selected in excess of the value prescribed by Equation (26), thereby resulting in a value of t_f less than 4L/a, the extreme value of head developed throughout the system would still be in accordance with Equation (24h). This immediately implies that if the system can physically accomodate that extreme value, then the minimum possible time of valve-stroking control in a simple system is 2L/a, not the 4L/a restriction of the previous investigations cited in the literature review.

Finally, it is also pertinent to note that most discussions (Ref. 18,19,31) of the waterhammer phenomenon conclude that for "rapid" valve closures in a frictionless system (i.e., $t_f \leq 2L/a$) the maximum value of head rise developed at the valve is given by

$$\Delta H = \frac{a}{g} \Delta V \ .$$

This is the so-called "Joukowsky surge". Yet by using a closure procedure predicated upon the concepts of this section, it is theoretically possible to close a valve in exactly 2L/a seconds, while at the same time restricting the subsequent head rise in the system to only one-half of that value.

3.1.3 Study with Friction Included

For a system in which viscous effects are not negligible, explicit equations describing the entire history of the transient condition can not be developed and one must resort to numerical solutions utilizing the appropriate equations of Chapter II. For the valve-stroking analysis, for example, the left-end boundary velocity at each grid point is specified according to Equation (22a), and Equations (18a) and (18b) are used to advance the resulting solutions across the double-staggered grid of characteristics to the right-end boundary; the value of the valve coefficient of each right-end boundary grid point can then be calculated. For the characteristics analysis used to confirm the valve-stroking analysis, interior-point conditions are evaluated with Equations (12a) and (12b), the left-end boundary conditions with Equations (13a) and (13b), and the right-end boundary conditions with Equations (15a) and (15b). Specific details of these solution procedures are described in Appendix B.

A considerable amount of flexibility exists relative to Equation (22a). Either a value of H_m can be arbitrarily prescribed and the corresponding value of t_f numerically evaluated, or t_f can be prescribed and H_m then evaluated (see Appendix C). In either case the central surge zone will develop within the domain of dependence of the left-hand boundary as discussed in Section 3.1.1.

For values of t_f between 2L/a and 4L/a this zone will be of triangular shape, as in the comparable frictionless system. Designating as H'_m the value of head created at the apex of this zone and developed at time $t_f/2$, then from Equation (24b)

$$H'_m = H_r + \frac{a}{2} \frac{(H_m - H_r)}{L} (t_f - 2L/a) \tag{27}$$

since $x = \frac{a}{2} (t_f - 2L/a)$. In situations involving valve closures (i.e., $V_0 > V_f$), the above expression for H'_m represents the maximum value of head developed in the system. The maximum value of head encountered at the valve occurs at $t = 2L/a$ and is marginally less than the above value. (Recall that in the frictionless case the head developed at the apex of the surge zone and the constant head obtained in Zone IV were identical; in this case, however, wall shear stresses accentuate the fluid deceleration, thus permitting somewhat lower values of head to exist in Zone IV.) In similar situations involving valve openings, the minimum value of head does occur at the valve at t = 2L/a and is marginally less than H'_m .

Yet another possibility should be mentioned relative to the arbitrary selection of t_f. One can always specify t_f and then specify that the upstream velocity vary linearly between V_o and V_f during the time $L/a \leq t \leq t_f - L/a$. Such a strategy results in a very simple and straightforward computational procedure; for values of t_f between 2L/a and 4L/a, valve-stroking control following this procedure may be preferred because of the simplicity of the procedure and because the extreme value of head developed in the system is essentially the same in either case. For values of t_f significantly greater than 4L/a, specification of the upstream velocity according to Equation (22a) is probably preferred because, for the same value of t_f, the extreme value of head developed in the system is less than is obtained utilizing the linear specification.

As previously indicated, the minimum time for valve-stroking control of this simple system is t_f = 2L/a. Although of doubtful practical interest because of the instantaneous valve motions required, it does provide valuable insight into the one restriction inherent in the valve-stroking theory. In closure situations the maximum head (H'_m) develops at a point just inside the left-end boundary at t = L/a. (In this case the entire transient zone is a Zone IV triangle.) Conditions at this point may be evaluated with Equations (24g) and (24h), since the distance over which the friction terms of the finite-difference equations have to be integrated is negligibly small. To create this one-half reduction in velocity at the reservoir at t = L/a, a greater velocity reduction has to occur at the valve because of the line-packing [9,31] phenomenon that exists in any real fluid system. In extremely long pipelines these viscous effects could be sufficiently significant to require that negative velocities develop at the valve during a central period of the transient condition. These negative velocities would be incompatible with the physical operation of a simple control valve and could only be produced by other control devices (i.e., an accumulator) operated in conjunction with the valve.

A computer program which included both the valve-stroking analysis and the confirming characteristics analysis was written to illustrate the effect of system operation according to the principles suggested in the preceding sections. Results of several representative valve-closure studies are illustrated in Figure 15. For convenience, the valve relationships illustrated are the dimensionless tau relationships (Ref. 18,31), obtained by dividing the instantaneous value of the valve coefficient by the initial steady-state value. The pertinent system parameters are: H_r = 100 feet, V_o = 5 feet per second, V_f = 0, L = 4000 feet, a = 3200 feet per second, f = .025, and D = 1 foot; thus L/a = 1.25 seconds, H_o = 61.2 feet, and H_f = 100 feet.

The significant values of head prescribed or developed in the system are summarized in Table I.

These results illustrate remarkably well the fundamental differences in the transient condition developed for various values of t_f; especially well illustrated is the distinction between system control in closure times less than 4L/a and control in times greater than 4L/a.

In summarizing the results of this section, the valve-stroking procedure proposed creates an extremely controlled transient condition

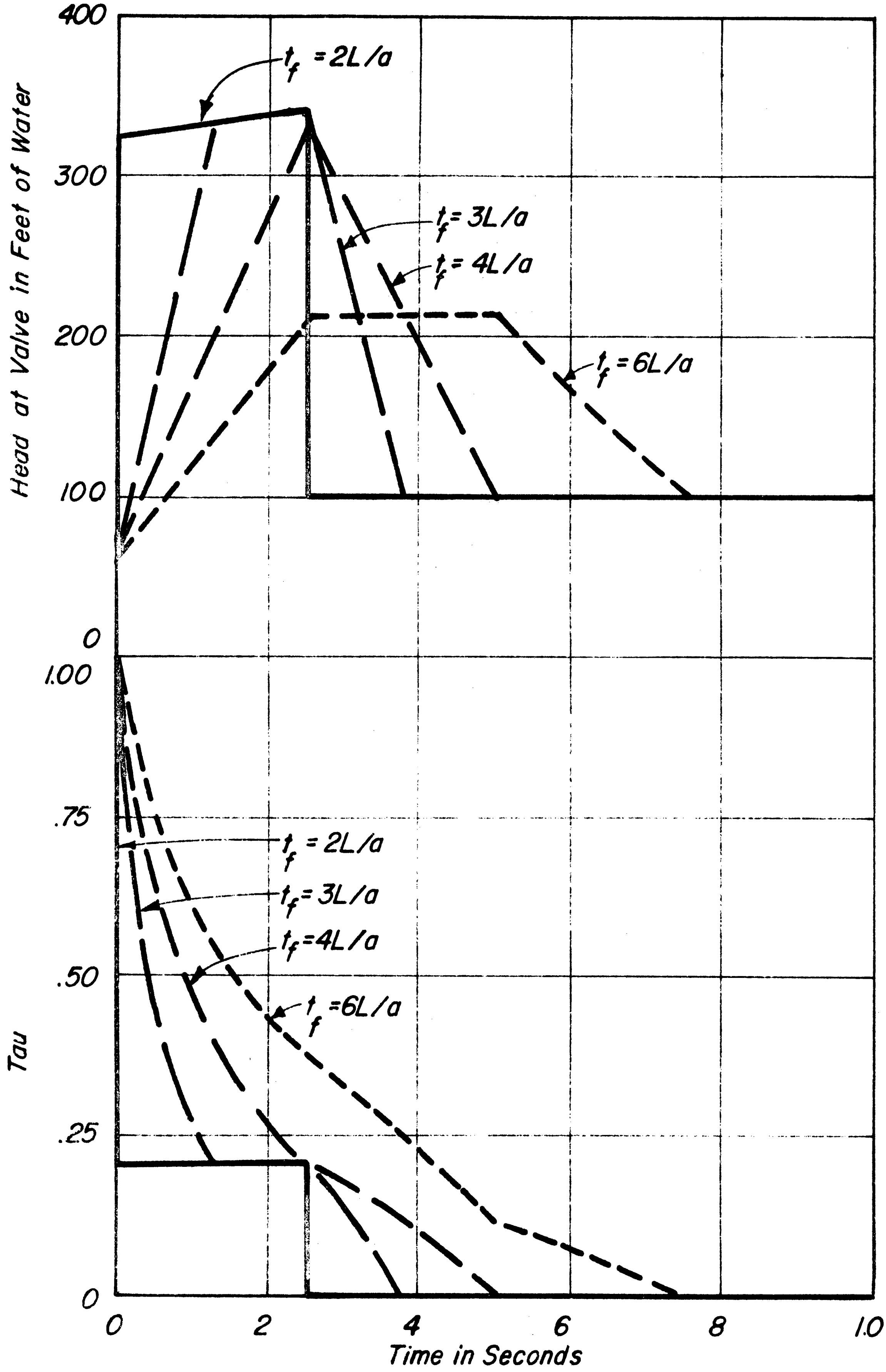


Figure 15. Valve-stroking control of a simple system with constant-head reservoir.

TABLE I

RESUME OF ANALYTICAL COMPUTER RESULTS

t_f seconds	H_m feet	H'_m feet	Max. H at Valve feet
2.50	--	348.4	342.7
3.75	584.2	342.1	339.8
5.00	336.0	336.0	336.0
7.50	212.3	--	212.3

throughout the system: all velocities and pressures are nearly linear functions of time and distance, an arbitrarily selected extreme value of pressure is maintained at the valve during a central phase of the transient condition, and the final steady-uniform condition is established without the development of residual transient fluctuations.

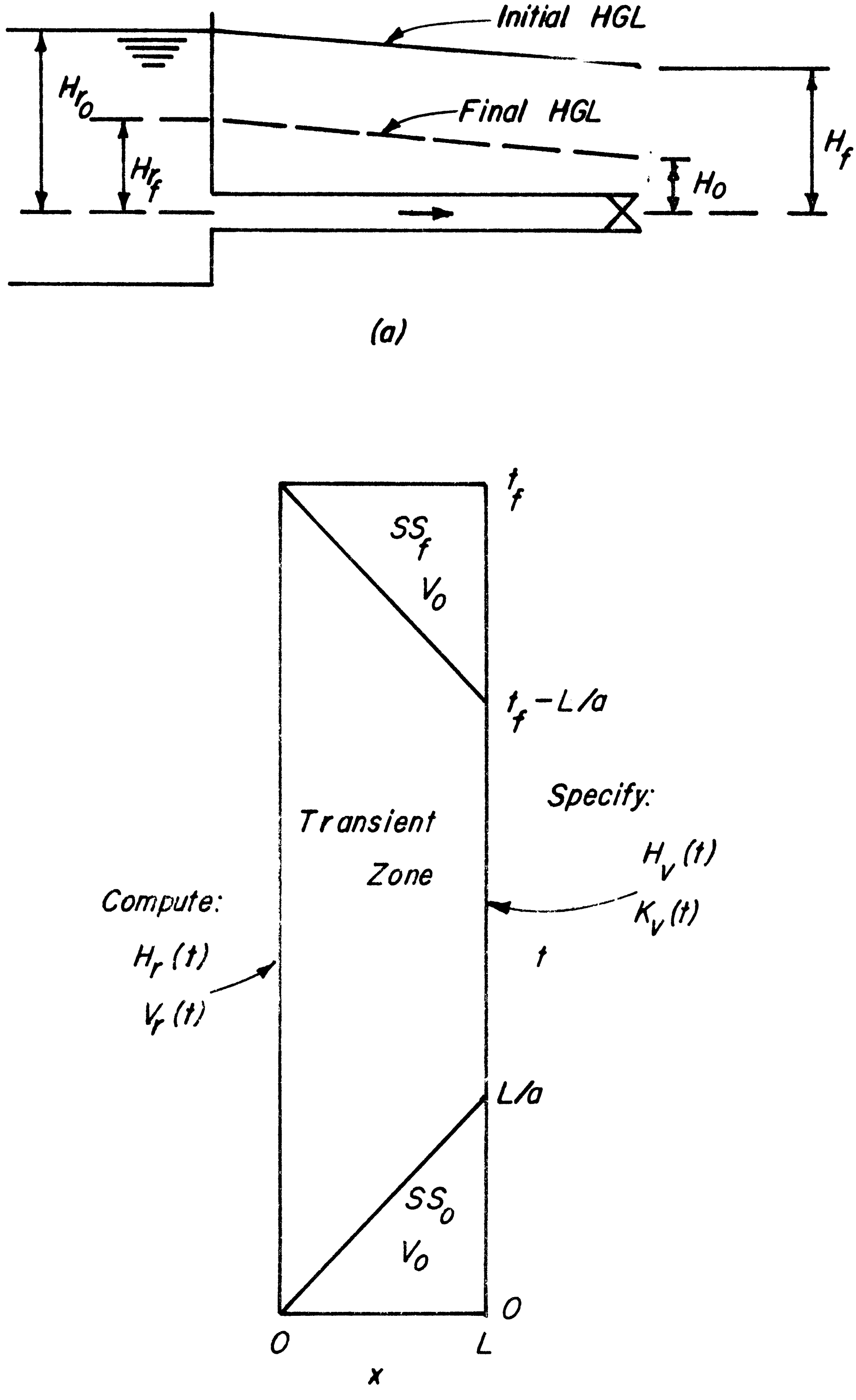
Lastly, it is pertinent to note that although the preceding treatment has been predicated on the assumption that steady-uniform conditions originally exist in the system, the theory developed here is not necessarily restricted to such situations. With reference to the discussion of Section 2.6.1 and Figure 12(b), if the values of both dependent variables are initially known along the line t = 0, then a unique solution does exist within the region labeled "SS_o" (regardless of whether or not it is a steady-state region). Hence, the velocity $V_r(L/a)$ has an established value and Equation (22) or (22a) may then be utilized to prescribe $V_r(t)$ along the left-end boundary. Again the surge condition is established throughout the domain of dependence of the left-end boundary, the extreme value of head is maintained at the valve during the central phase of the transient condition, and the final steady-uniform condition can be established as desired.

3.2 System with a Downstream Constant-Velocity Condition

As another example of a system for which a valve-stroking procedure may be developed, consider the system illustrated in Figure 16. Assume that it is desired to modify the upstream reservoir elevation from an initial elevation H_{r_o} to a final elevation H_{r_f} while maintaining a constant velocity V_o at the downstream valve. As in the first study it will be assumed that steady-uniform conditions originally exist in the system (although, as in the former case, that restriction is not essential to the development of the theory) and that at time $t = t_f$ the final steady-uniform condition is to be established without the presence of subsequent residual transient fluctuations. It will be assumed that the elevation of the reservoir can be precisely controlled to satisfy any arbitrary function of time.

From the discussions referred to in Section 3.1 and with reference to Figure 16(b), the following observations parallel those of Section 3.1: (1) Because of the presence of the invarient downstream-boundary relationship, $V(L,t) = V_o$, the region of the initial steady-uniform condition is as indicated. (2) If the negative direction is now arbitrarily assigned to the C+ characteristic family and the positive direction to the C- characteristic family, then both data must be prescribed along the time-like downstream boundary. (3) If the head $H_v(t)$ is prescribed along the downstream boundary such that at time $t \geq t_f - L/a$, $H_v(t) = H_f$, then the final steady-uniform condition will be established as desired. (4) No datum may be prescribed along the upstream boundary since the solutions $V_r(t)$ and $H_r(t)$ are solely dependent upon the initial conditions and the data prescribed along the downstream boundary.

Having again established the essential element of this problem-- the need to specify the downstream head as some function of time -- one



(b)

Figure 16. Simple system with constant-velocity condition: (a) definition sketch, (b) x-t plane and notation.

again can speculate about how best to do this in order to produce an orderly and controlled transition between the two steady-uniform conditions.

3.2.1 Specification of the Variable Downstream Head

Recognizing that along the downstream boundary the velocity has a constant value (thus $V_t(L,t) = 0$), one may investigate the consequence of requiring that $V_t(x,t) = 0$ throughout the flow.

If $V_t = 0$, then $V = V(x) + C$ is the most general relationship that V can satisfy (C is a constant). Then $V_x = V'(x)$ and from Equation (6),

$$gH'(x) = -\frac{fV|V|}{2D} .$$

It is apparent that $H'(x)$ is also a function of x only, since $f = f(V)$. This equation may then be integrated with respect to x and the result is some function involving only x, $F(x)$. Therefore the most general relationship that H may satisfy is $H = F(x) + G(t) + K$ (K is a constant). Thus $H_t = G'(t)$ can be a function of t only.

The continuity equation, Equation (5), then has the form

$$\frac{a^2}{g} V'(x) = -G'(t) .$$

Again, for a function involving only x to be equal to a function involving only t, the two functions must be equal to a constant. Since $G'(t)$ must thus be constant and represents the rate at which the hydraulic grade line changes elevation,

$$\frac{dH}{dt} = \frac{H_f - H_o}{t_f - 2L/a} \tag{28}$$

is suggested as the technique for specifying $H_v(t)$ along the right-end boundary.

Again, one may now investigate the consequences implied by utilization of Equation (28). With reference to Figure 17,

$$V_B = V_H = V_E = V_o \quad \text{and}$$

$$H_H = H_A + \frac{(H_f-H_o)}{(t_f-2L/a)} \Delta t \ ,$$

$$H_E = H_H + \frac{(H_f-H_o)}{(t_f-2L/a)} \Delta t \ .$$

Incorporation of these relationships into Equation (19a) and subsequent elimination of canceling terms yields

$$V_P = V_o + \frac{1}{B} \frac{(H_f-H_o)}{(t_f-2L/a)} \Delta t \ .$$

After substitution of this result into Equation (19b),

$$H_P = H_H + \frac{F_P+F_o}{2} \ .$$

Therefore, conditions at the point P have been demonstrated to satisfy the restrictions of the preceding analysis: $V_t = 0$, V_x = a constant, and H_t = a constant. Since the selection of points B, E, and H was completely arbitrary, these results permit a conclusion analogous to that of Section 3.1.1: every grid point within the domain of dependence of that section of the right-end boundary over which the head has been arbitrarily prescribed using Equation (28) must be a point at which the specified uniform transient condition is established.

3.2.2 Study of the Frictionless System

As in the first system considered in this chapter, equations describing the entire history of the transient condition throughout a

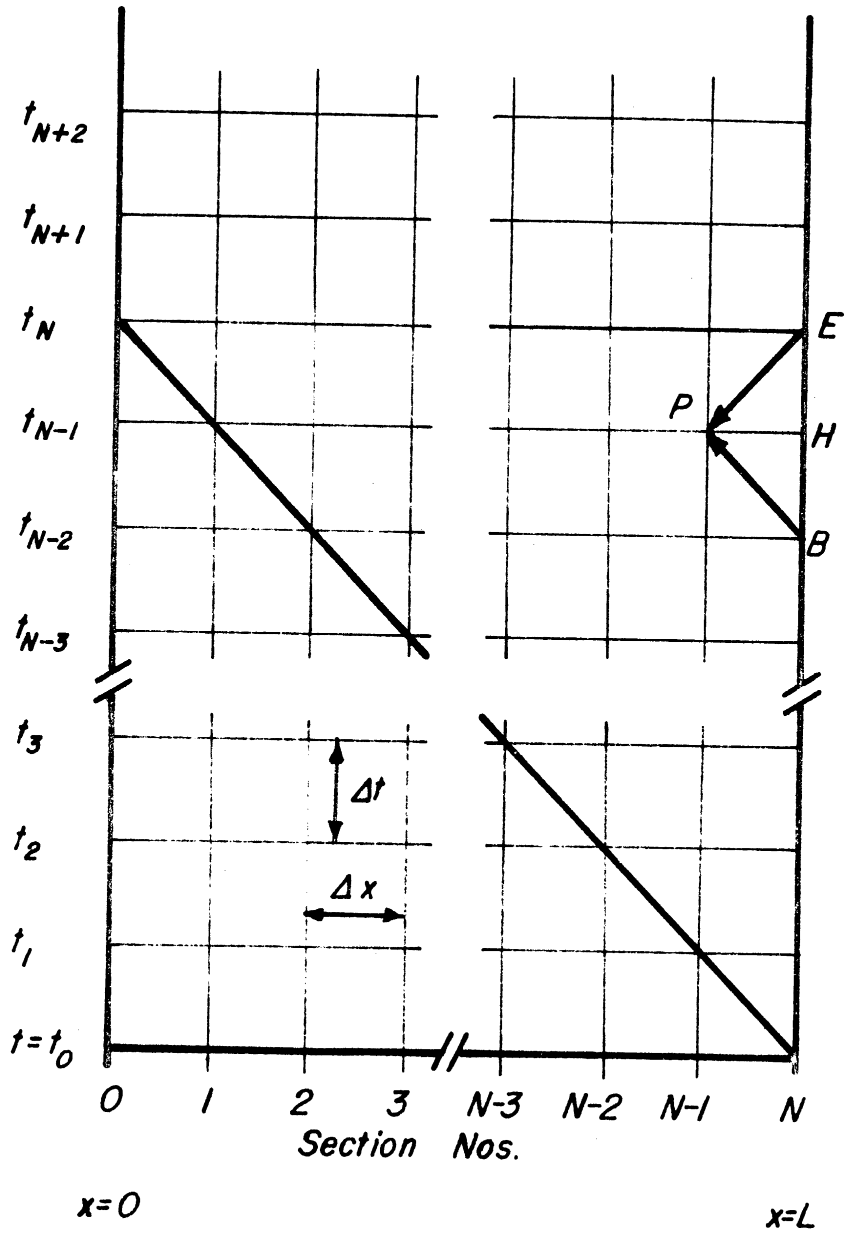


Figure 17. Region of established uniform transient condition.

frictionless system may be developed. Again, they provide insight into the transient condition developed in the system in which viscous effects are important.

As before, the basic pattern of the transient condition imposed upon the system is considerably different for values of $t_f \geq 4L/a$ than it is for values of t_f between $2L/a$ and $4L/a$. Referring to Figure 18(a), one may first consider the pattern for $t_f \geq 4L/a$.

Since conditions in Zone II already have been demonstrated to satisfy the specially restricted forms of Equations (5) and (6), for the frictionless system these equations, after integration and evaluation with the appropriate initial conditions, are

$$V(x) = V_o + \frac{1}{B} \frac{(H_f - H_o)}{(t_f - 2L/a)} \frac{(L-x)}{a} , \quad (29a)$$

$$H(t) = H_o + \frac{(H_f - H_o)}{(t_f - 2L/a)} (t - L/a) . \quad (29b)$$

Next by solving Equations(19a) and (19b) for a point (x,t) in Zone I simultaneously with Equation (29b) evaluated at time t_v and the condition that $t = t_v - (L-x)/a$,

$$V(x,t) = V_o + \frac{1}{2B} \frac{(H_f - H_o)}{(t_f - 2L/a)} (t - \frac{x}{a}) , \quad (29c)$$

$$H(x,t) = H_o + \frac{1}{2} \frac{(H_f - H_o)}{(t_f - 2L/a)} (t - \frac{x}{a}) . \quad (29d)$$

Equations describing the history of the transient condition throughout Zone III may be developed in a similar fashion. These equations are

$$V(x,t) = V_o + \frac{1}{2B} \frac{(H_f - H_o)}{(t_f - 2L/a)} (t_f - t - \frac{x}{a}) , \quad (29e)$$

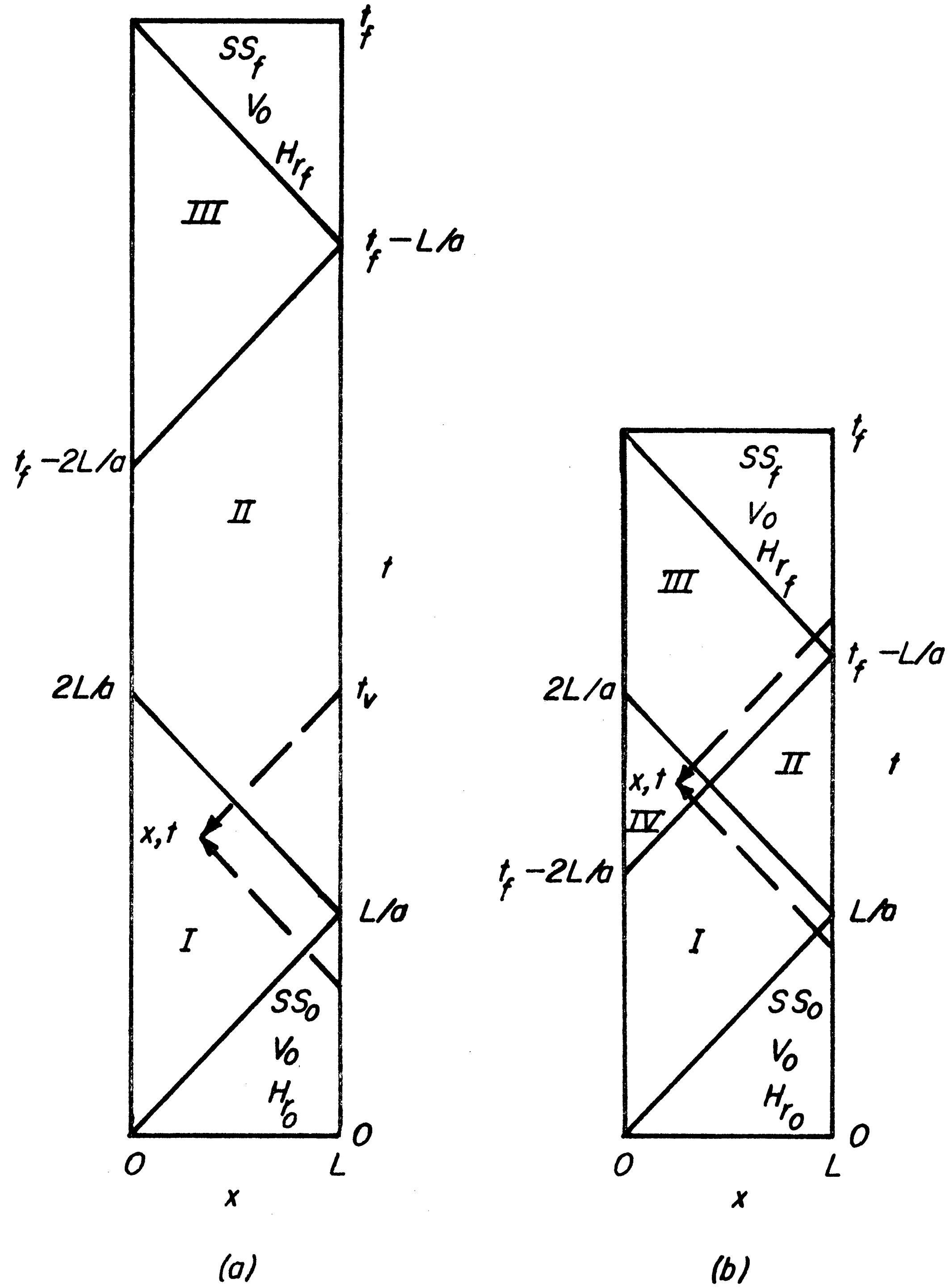


Figure 18. Simple frictionless system with constant-velocity condition. x-t planes for (a) $t_f \geq 4L/a$, and (b) $t_f < 4L/a$.

$$H(x,t) = H_f - \frac{1}{2}\frac{(H_f-H_o)}{(t_f-2L/a)}(t_f - t - \frac{x}{a}) \; . \qquad (29f)$$

Evaluation of these equations at the left-end boundary enables one to determine the time-dependent value of H_r required to produce the desired transition between the initial and final steady-uniform conditions:

$$H_r(t) = H_o + \frac{1}{2}\frac{(H_f-H_o)}{(t_f-2L/a)}\, t \qquad 0 \le t \le 2L/a \; , \qquad (30a)$$

$$H_r(t) = H_o + \frac{(H_f-H_o)}{(t_f-2L/a)}(t - L/a) \qquad 2L/a \le t \le t_f - 2L/a, \qquad (30b)$$

$$H_r(t) = H_f - \frac{1}{2}\frac{(H_f-H_o)}{(t_f-2L/a)}(t_f - t) \qquad t_f - 2L/a \le t \le t_f . \qquad (30c)$$

One may now consider the transient pattern that develops for values of t_f between 2L/a and 4L/a. As may be readily seen from Figure 18(b), an analysis of the conditions developed in Zones I,II, and III performed in an identical fashion as before would yield the same results. Thus, equations (29a) to (29f) are still valid for their respective zones. Solving Equations (19a) and (19b) for a point (x,t) in Zone IV, the results are

$$V = V_o + \frac{(H_f-H_o)}{2B} \; , \qquad (29g)$$

$$H = \frac{H_o+H_f}{2} \; . \qquad (29h)$$

The Zone I and Zone III equations previously developed for $H_r(t)$ are still valid, except that for this situation Equation (30a) is only valid during $0 \le t \le t_f - 2L/a$, and Equation (30c) is only valid during $2L/a \le t \le t_f$. The constant value of H_r that obtains during the central phase of the transient condition is then given by Equation (29h):

$$H = \frac{H_o + H_f}{2} \qquad t_f - 2L/a \leq t \leq 2L/a \; . \tag{29h}$$

Lastly, the time-dependent right-end boundary valve coefficient may be determined by evaluating Equation (29b) at x = L:

$$K_v(t) = \frac{V_o}{\sqrt{H_o + \frac{(H_f - H_o)}{(t_f - 2L/a)}(t - L/a)}} \qquad L/a \leq t \leq t_f - L/a \; . \tag{31}$$

Since the boundary velocity is invarient and the method of specifying the transient elevation of the hydraulic grade line at the valve is independent of viscous effects, Equation (31) is perfectly general and is not restricted only to the frictionless system.

3.2.3 Study with Friction Included

For a system in which viscous effects are not negligible, one must again resort to numerically-oriented solutions. For the valve-stroking analysis, the right-end boundary head at each grid point is specified according to Equation (28), the unsteady valve relationship is computed with Equation (31), and Equations (19a) and (19b) are used to advance the solutions across the grid to the left-end boundary. For the characteristics analysis, interior-point conditions are evaluated with Equations (12a) and (12b), the left-end boundary conditions with Equations (13a) and (13b), and the right-end boundary conditions with Equations (15a) and (15b).

A computer program which included both the valve-stroking analysis and the confirming characteristics analysis was written to illustrate the effect of system operation according to these principles. Results of several representative studies are illustrated in Figure 19. Again, the dimensionless valve relationships are presented. The pertinent

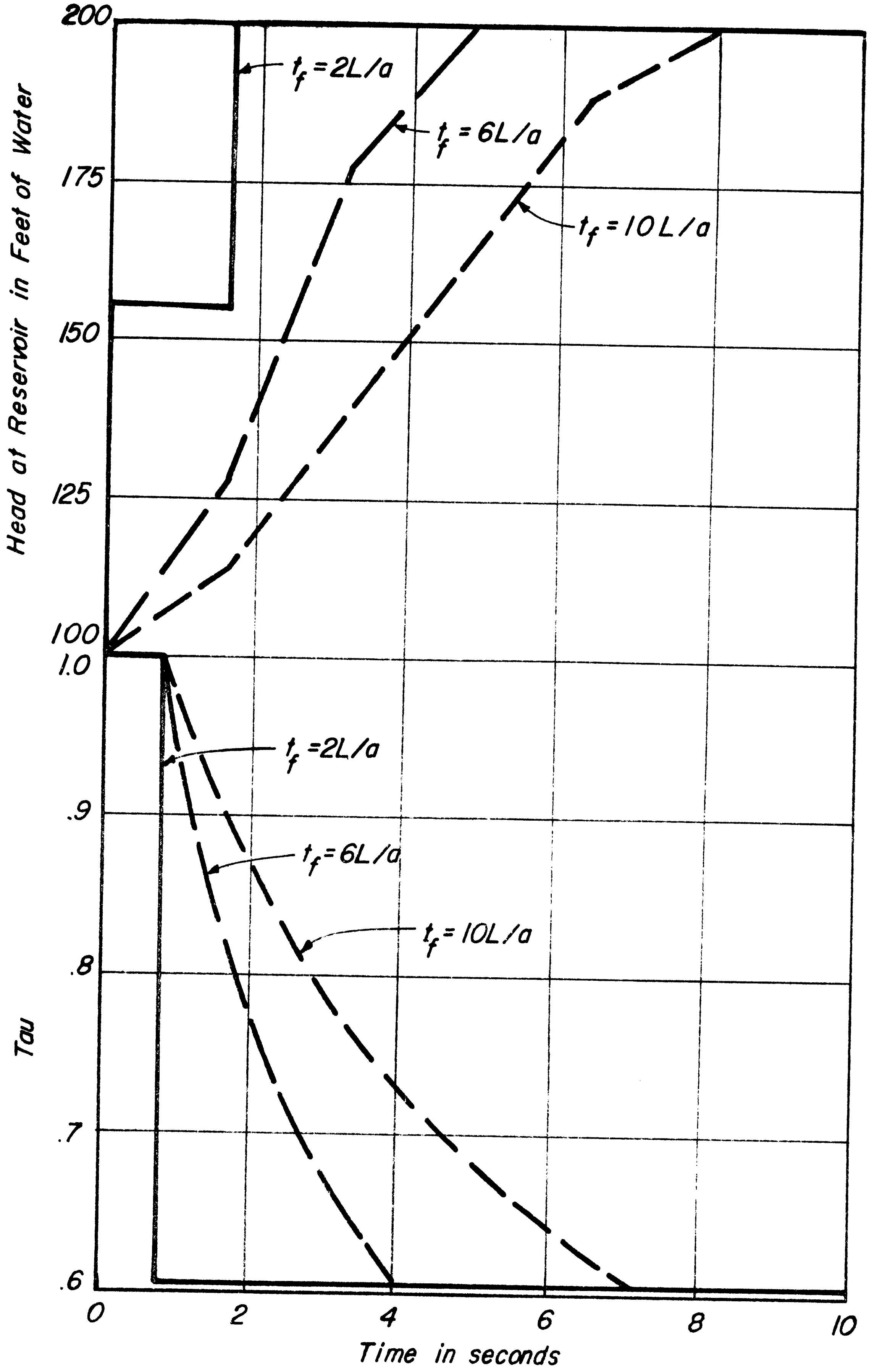


Figure 19. Valve-stroking control of a simple system with constant-velocity condition.

system parameters are: V_o = 3.5 feet per second, H_{r_o} = 100 feet, H_{r_f} = 200 feet, L = 3200 feet, a = 4000 feet per second, f = .035, and D = 0.5 feet; thus L/a = 0.8 seconds, H_o = 57.4 feet, and H_f = 157.4 feet.

The results again confirm the theory; in every case the downstream velocity is maintained at a constant 3.5 feet per second throughout the history of the transient condition and the final steady-uniform condition is established as desired. Of particular interest is the leading case of $t_f = 2L/a$. This is analogous to the one round-trip valve-stroking control of the previous study. Again, the instantaneous control motions required by the theory would appear to exclude most practical applications.

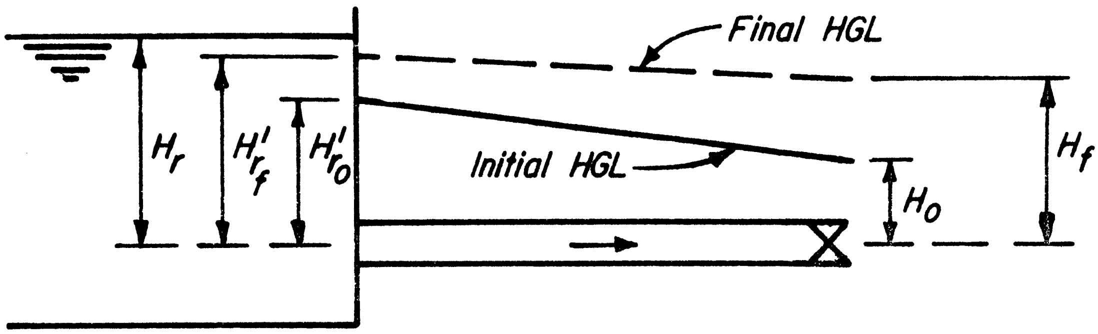
In summary, the proposed valve-stroking procedure creates a highly controlled transient condition throughout the system: all velocities and pressures are nearly linear functions of time and distance, the transient elevations of the hydraulic grade line are everywhere bounded between the initial and final steady-state grade lines, and the final steady-uniform condition is eatablished without the development of residual transient fluctuations.

3.3 System with an Upstream Constant-Relationship Condition

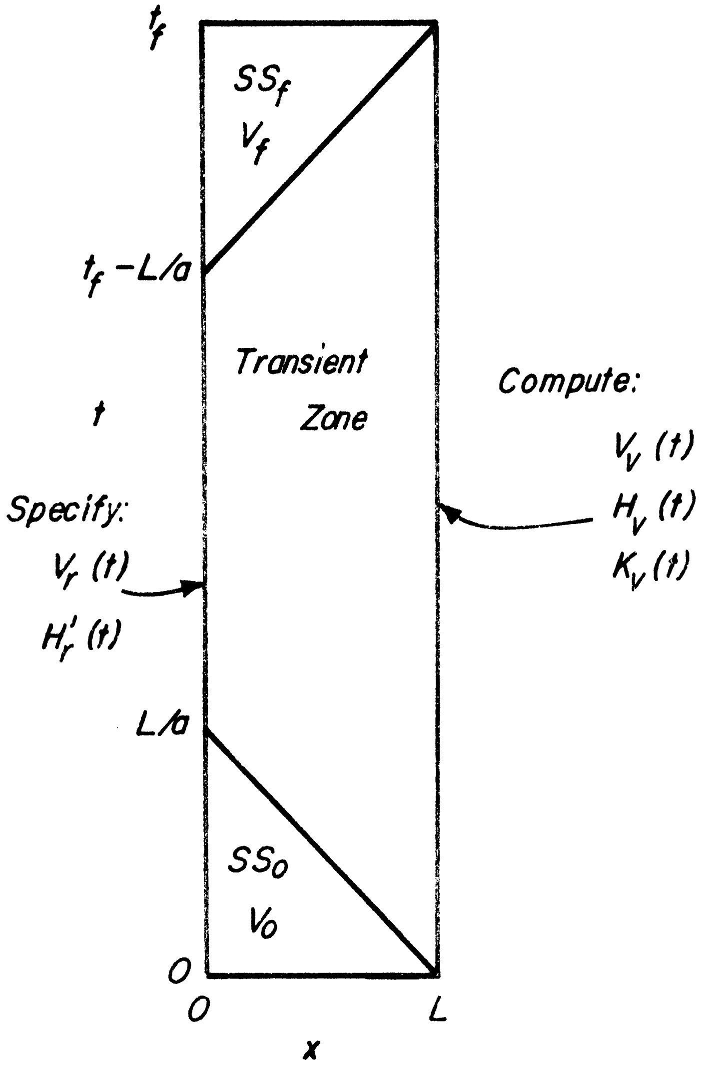
As a third example of a simple system for which a valve-stroking procedure may be developed, consider the system illustrated in Figure 20. Instead of the simple upstream boundary relationship provided by the presence of the constant-head reservoir, the upstream boundary condition is now the constant-relationship condition of the fixed orifice:

$$V_r = K_o \sqrt{H_r - H_r'} \quad .$$

(Before proceeding further, it should be emphasized that any other boundary condition involving a fixed relationship between the dependent variables could be considered, and furthermore the relationship need not



(a)



(b)

Figure 20. Simple system with constant-relationship condition: (a) definition sketch, (b) x-t plane and notation.

be in equation form; a tabular relationship would be entirely acceptable. The H-V relationship of a centrifugal pump provides an obvious example.)

Again consider that it is desired to modify flow conditions in the system from some initial steady-uniform condition (SS_o) to some final steady-uniform condition (SS_f) such that at time $t = t_f$ the final steady-uniform condition is established without the presence of subsequent residual transient fluctuations. (Again, the assumption of an initial steady-uniform condition is adopted for convenience; it is not intrinsic to the theory.)

With reference to Figure 20(b), it is evident that the analysis of the similar boundary-data problem presented in Section 3.1 is equally applicable here; it will not be repeated. Again, the crucial element in the development of a suitable control theory would appear to be that of specifying either of the upstream dependent variables as some arbitrary function of time; the other variable would then be completely specified by the prescribed fixed-orifice relationship.

3.3.1 Specification of the Upstream Variables

An examination of the governing partial differential equations conducted in a similar fashion to those of the previous studies fails to provide any meaningful criteria for upstream variable specification for this system. It is impossible, for example, to establish and maintain a uniform hydraulic grade line throughout the system during a central phase of the transient because of the variable head condition that prevails on the downstream side of the orifice. However, one can arbitrarily specify either of these upstream variables as some prescribed function of time. Following this direction, let us specify that the upstream velocity vary linearly between the initial steady-uniform value (V_f):

$$V_r(t) = V_o + \frac{(V_f - V_o)}{(t_f - 2L/a)}(t - L/a) \qquad L/a \leq t \leq t_f - L/a. \qquad (32)$$

(Recall that in the frictionless constant-head reservoir system the upstream boundary velocity is a linear function of time.)

As with the previous studies, equations describing the entire history of the transient condition throughout a frictionless system may then be developed. While again providing insight into the condition developed in the system in which viscous effects are important, these equations are rather lengthy and will not be presented here. Relative to the extreme value of head developed throughout the system, these equations demonstrate that for $t_f \geq 4L/a$ the extreme value occurs at the valve at $t = t_f - 2L/a$; for values of t_f between $2L/a$ and $4L/a$ a zone analogous to Zone IV of Figure 14(b) is developed throughout which the extreme condition is constantly maintained.

For a system in which viscous effects are not negligible, one must again resort to numerically-oriented solutions. For the valve-stroking analysis the left-end boundary velocity is specified according to Equation (32), the corresponding head is computed from the prescribed fixed-orifice relationship, and Equations (18a) and (18b) are used to advance the solutions across the grid to the right-end boundary; the value of the valve coefficient at each right-end boundary grid point can then be calculated. The confirming characteristics analysis is identical to those previously discussed, except that the left-end boundary conditions are evaluated using Equations (14a) to (14c).

Scrutiny of typical results generated by a program written to illustrate this suggested valve-stroking procedure reveals that whenever $t_f \geq 4L/a$ the extreme value of head encountered throughout the system does develop at the valve at $t = t_f - 2L/a$. For situations involving rapid valve closures in times less than 4L/a, the maximum value of head developed in the system occurs at the apex of Zone IV at time $t_f/2$. As in the earlier study, the maximum value encountered at the valve is marginally less than the system maximum. Pertinent system parameters and results of a typical study are presented in Section 3.3.2.

Scrutiny of these results also reveals a significant deficiency in this procedure: unlike the procedure developed in Section 3.1, specification of the upstream boundary velocity according to Equation (32) does not permit an arbitrarily selected extreme value of pressure to be developed at the valve. If a predetermined extreme value is not to be exceeded, then the appropriate value of t_f and the consequent valve motion required to produce the desired transient condition can only be determined by numerical experimentation. The alternate procedure suggested in the next section overcomes this obvious disadvantage.

3.3.2 Preferred Solution

Recall that in the constant-head reservoir system of Section 3.1 the suggested valve-stroking procedure resulted in the following head-time variation at the valve: (1) a nearly linear transition from H_o to H_m during the first 2L/a seconds; (2) the prescribed value of H_m maintained at the valve during the central phase of the controlled transient condition; (3) a nearly linear transition from H_m to H_f during the last 2L/a seconds. This results suggests the following alternate strategy to the procedure outlined in Section 3.3.1.

From the discussion presented in Sections 2.2.3, 2.6.1, and 2.6.2, and with reference to Figure 21, if the positive direction is arbitrarily assigned to each characteristic family, one datum must be prescribed on each of the time-like boundaries. The fixed-orifice

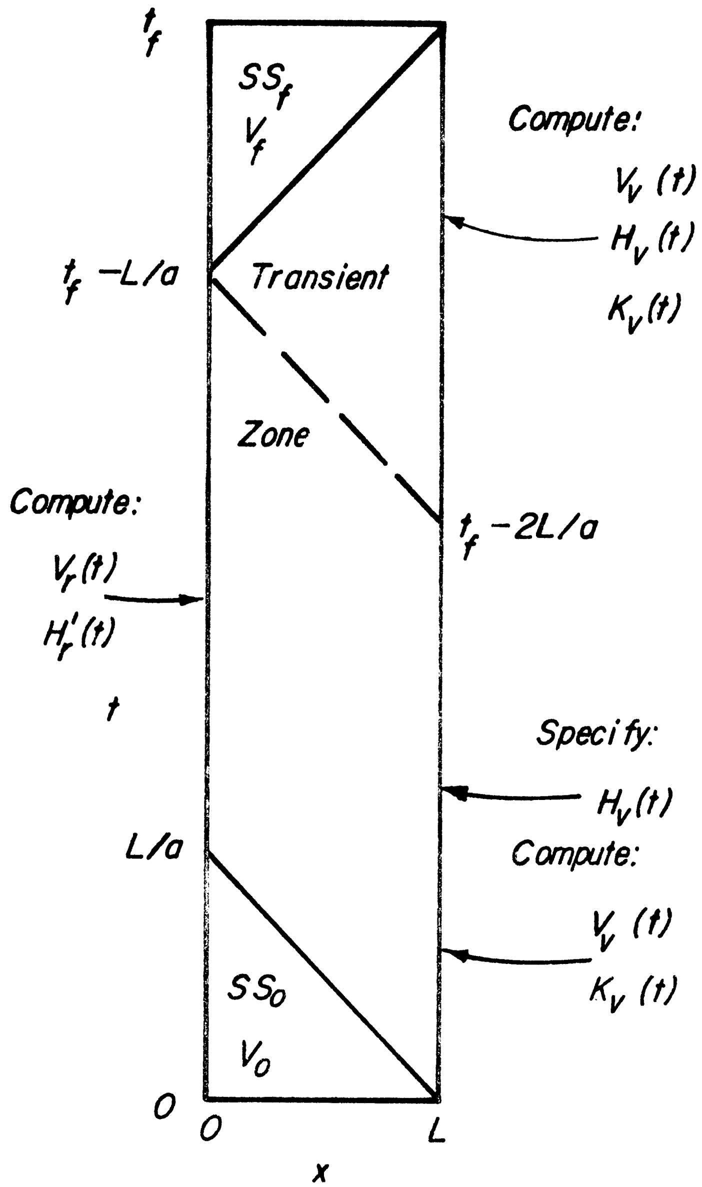


Figure 21. Simple system with constant-relationship condition. x-t plane for preferred solution.

relationship provides the upstream datum. One may arbitrarily specify that the downstream head change linearly during the first 2L/a seconds to an arbitrarily selected extreme valve H_m and that it be maintained at that value during the central phase of the transient condition:

$$H_v(t) = H_o + \frac{(H_m - H_o)}{2L/a} t \qquad 0 \le t \le 2L/a \ , \tag{33a}$$

$$H_v(t) = H_m \qquad 2L/a \le t \le t_f - 2L/a \ . \tag{33b}$$

A unique solution then exists in the region enclosed by the time-like boundaries, the space-like line t = 0, and the negative characteristic line extending from the right-end boundary at $t = t_f - 2L/a$ and intersecting the left-end boundary at $t = t_f - L/a$.

This constant value of H_m is prescribed along the right-end boundary until such time that the corresponding value of velocity obtained L/a seconds later at the left-end boundary equals the final steady-uniform velocity (V_f). At that point one may now arbitrarily assign the positive direction to the C+ characteristic family and the negative direction to the C- characteristic family and specify that along the upstream boundary $V_r(t) = V_f$. Thus the final steady-uniform condition will be established as desired, and a unique solution will exist in the traingular region bounded by the negative characteristic line (the dashed line illustrated in Figure 21) and by the positive characteristic line bounding the final steady-state region.

The necessary numerical solutions again utilize the appropriate equations of Chapter II. The downstream head is prescribed according to Equations (33a) or (33b); the downstream velocity is determined from Equation (17b). Interior point conditions are then evaluated diagonally along a C- characteristic utilizing Equations (12a) and (12b), and the left-end boundary conditions are evaluated using Equations (14a) to (14c).

This solution procedure is terminated when the left-end velocity equals V_f. The velocity along the boundary is then specified to be V_f and Equations (18a) and (18b) are used to advance the solutions in the upper triangular region of the transient zone across the grid to the valve boundary. Specific details of the solution procedure are described in Appendix B. The confirming characteristics analysis is identical to that described in the preceding section.

A computer program which included both the valve-stroking analysis and the confirming characteristics analysis was written to illustrate the effect of system operation according to these concepts. Results of two typical studies are illustrated in Figure 22; results of several studies are summarized in Table II and compared with the results of the computer studies described in Section 3.3.1. The pertinent system parameters are: H_r = 165 feet, V_o = 4 feet per second, V_f = 0, L = 3220 feet, a = 3220 feet per second, f = .025, K_o = .8, and D = 0.5 feet; thus L/a = 1.00 second, H'_{r_o} = 140 feet, H'_{r_f} = 165 feet, H_o = 100 feet, and H_f = 165 feet.

TABLE II

COMPARISON OF ANALYTICAL COMPUTER RESULTS

Upstream Velocity Specification			Downstream Head Specification		
t_f seconds	Max. H at Valve feet	Max. H in System feet	t_f seconds	Max. H Specified feet	Max. H Obtained feet
3.00	344.2	346.2	3.96	344.2	340.2
4.00	340.2	340.2	3.99	340.2	340.0
6.00	258.7	258.7	5.58	258.7	258.7
8.00	228.8	228.8	6.97	228.8	228.8
10.00	213.4	213.4	8.07	213.4	213.4

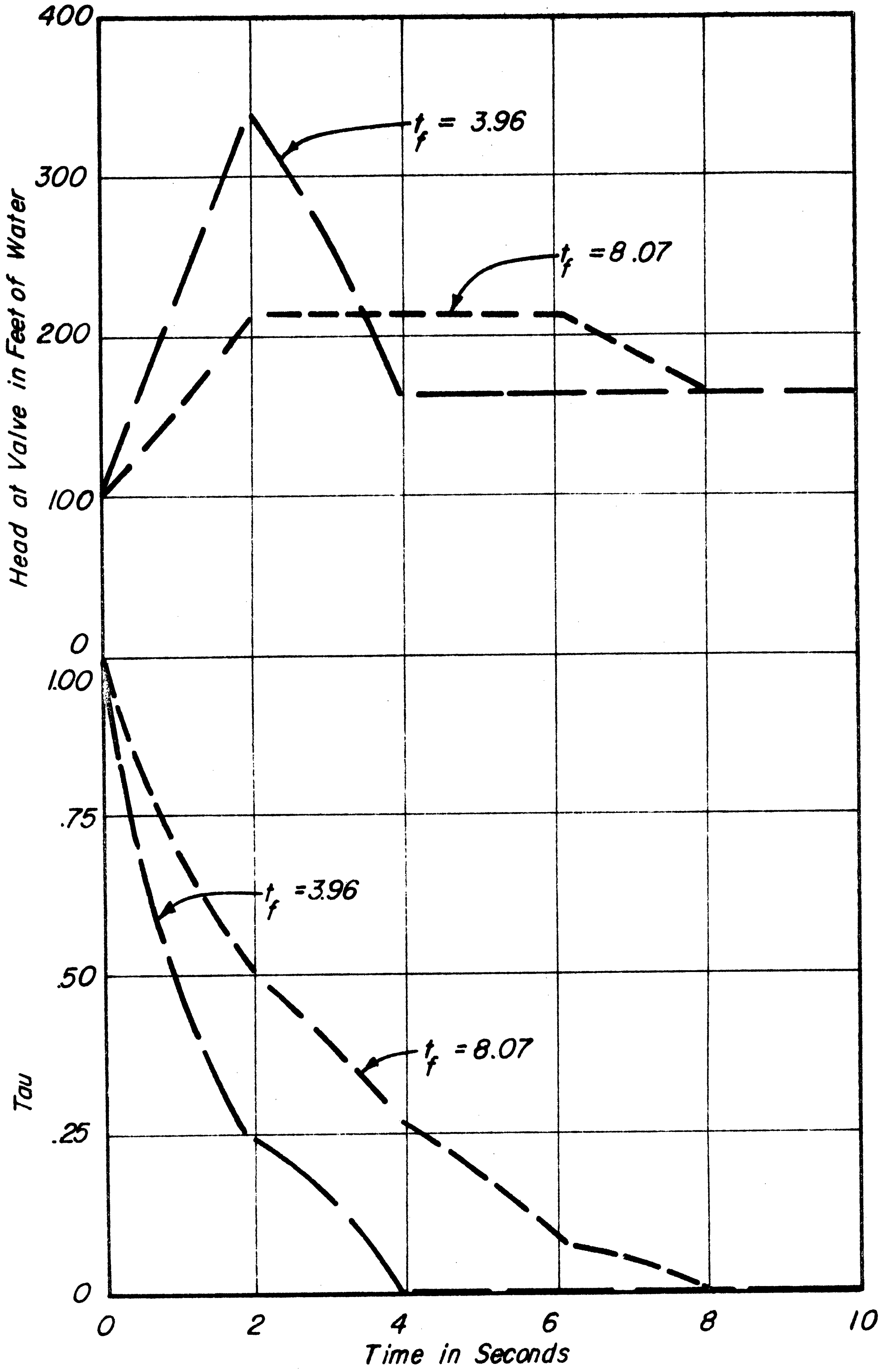


Figure 22. Valve-stroking control of a simple system with constant-relationship condition.

These results illustrate in a most remarkable way the differences in the transient conditions obtained utilizing the two procedures. This is particularly true for the slower valve closures since the head developed at the valve by the procedure outlined in Section 3.3.1 continually rises during the central phase to a maximum value 2L/a seconds before the cessation of the valve motion, whereas the maximum value developed by the preferred procedure proposed in this section is maintained at the valve during the entire central phase of the transient condition. Thus it is not unexpected that, for the same maximum value obtained, the time of valve operation required by the preferred procedure can be considerably less than that required by the former one.

The distinction between system control in closure times less than 4L/a and control in times greater than 4L/a is again illustrated rather well. Note that in the preferred solution technique, a direct analogy exists with the control procedures developed for the constant-head reservoir system. In either case an extreme value of downstream head is selected that may not be attained if, for the specific system conditions that obtain, the consequent value of t_f is less than 4L/a. Since the maximum value of head developed in the system is essentially the same for control times between 2L/a and 4L/a, valve-stroking control following the upstream-velocity specification procedure is probably preferred (because of the simplicity of the computational procedure) for systems that can physically accomodate such rapid closures.

In summary, the proposed valve-stroking procedure creates a highly controlled transient condition throughout the system: unlike the previous studies [30,31] of constant-relationship boundary-condition systems, a predetermined extreme value of pressure can be developed and maintained at the valve during a central phase of the transient condition; again, the final steady-uniform condition is established without the presence of subsequent residual transient fluctuations.

IV. VALVE-STROKING CONTROL OF COMPLEX PIPING SYSTEMS

The valve-stroking concepts and procedures essential to the deterministic control of the transient phenomenon in the simple piping systems of the last chapter may be readily extended to develop suitable control techniques for the elementary complex systems as well.

4.1 Series System

Attention is first directed to the most frequently encountered of the elementary complex systems: a system of two pipes connected in series with the upstream pipe originating at a constant-head reservoir and the downstream pipe terminating at a control valve. With reference to Figure 23, again consider that it is desired to modify flow conditions in the system from some initial steady-state condition to some final steady-state condition such that at time $t = t_f$ the final steady-state condition is established without the presence of subsequent residual transient fluctuations. (Throughout this chapter, as before, the assumption of an initial steady-state condition is adopted for convenience only.)

Recall now that in the development of the control techniques for the simple systems of Chapter III, the essence of the problem -- the really crucial element -- was the necessity to specify one of the boundary dependent variables as an arbitrary function of time. It is instructive to note that a generalized criterion exists relative to either transient analysis or synthesis investigations of all complex systems: if there are m unknown boundary dependent variables and n available equations, then m-n of the variables must be specified to obtain the desired solution. The only restriction is that such specification may not violate one of the governing equations: i.e., specification of all of the

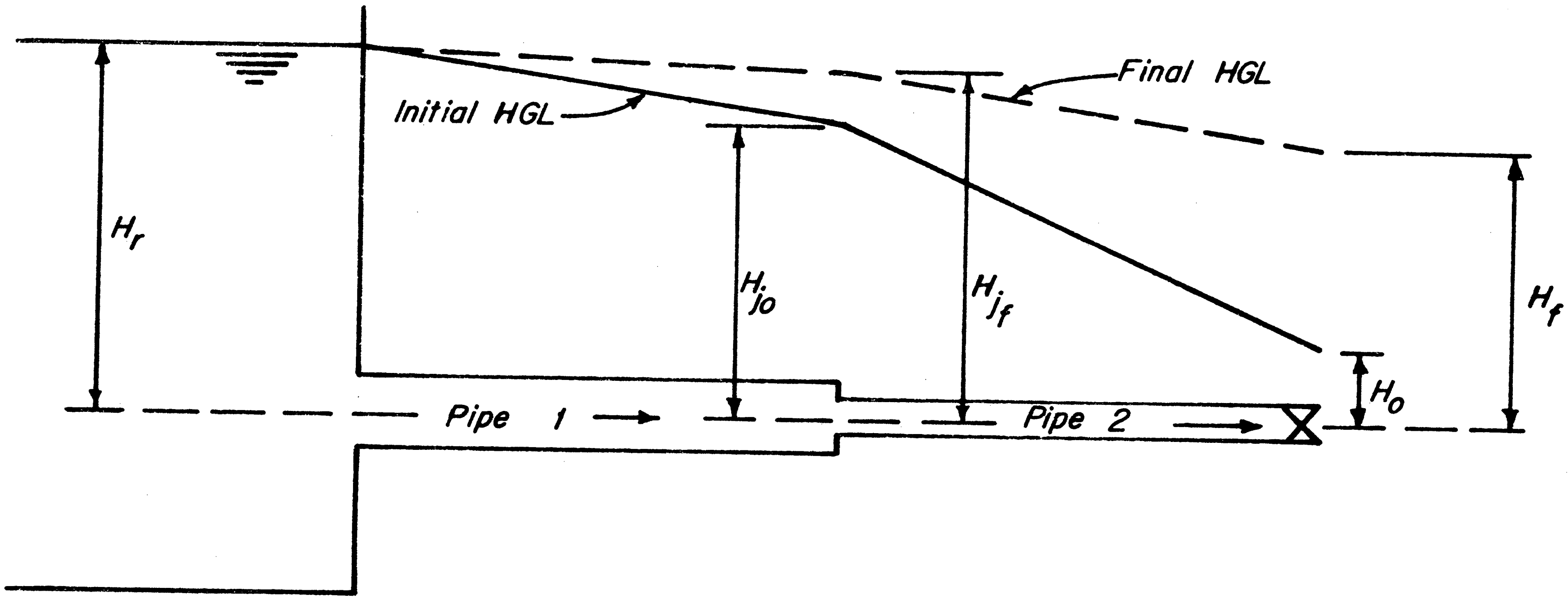


Figure 23. Definition sketch of series system with upstream constant-head reservoir.

velocities at an interior junction would violate the junction continuity equation. In the series system of Figure 23, for example, there are a total of six unknown variables (the velocity at the reservoir, two velocities and the head at the junction, and the velocity and head at the valve) and only five available equations (two characteristic equations for each pipe and the series-junction continuity equation). Therefore, specification of one of the unknown variables completely specifies the transient condition imposed upon the system.

4.1.1 Evaluation of the Existing Technique

From an extension of the discussions presented in Sections 2.2.3, 2.6.1, and 2.6.2, and with reference to Figure 24, the following observations are relevant: (1) Because of the presence of the invarient reservoir-boundary relationship, $H_1(0,t) = H_r$, the regions of the initial steady-uniform conditions are as indicated. (2) If the positive direction is now arbitrarily assigned to the C+ characteristic family and the negative direction to the C- characteristic family, then both data must be prescribed along the time-like reservoir boundary for a unique solution to exist in the region of interest of pipe 1; since both data are thereby established at the upstream boundary of pipe 2 (the existence of the common boundary head and the series-junction continuity equation), a unique solution likewise exists in pipe 2. (3) If the velocity $V_r(t)$ is prescribed along the reservoir boundary such that at time $t \geq t_f - L_1/a_1 - L_2/a_2$, $V_r(t) = V_{1_f}$, then the desired final steady-state condition will be established throughout the system as illustrated. (4) Again, no datum may be prescribed along the downstream boundary since it lies within the region of the unique solution of the given boundary-data problem. The

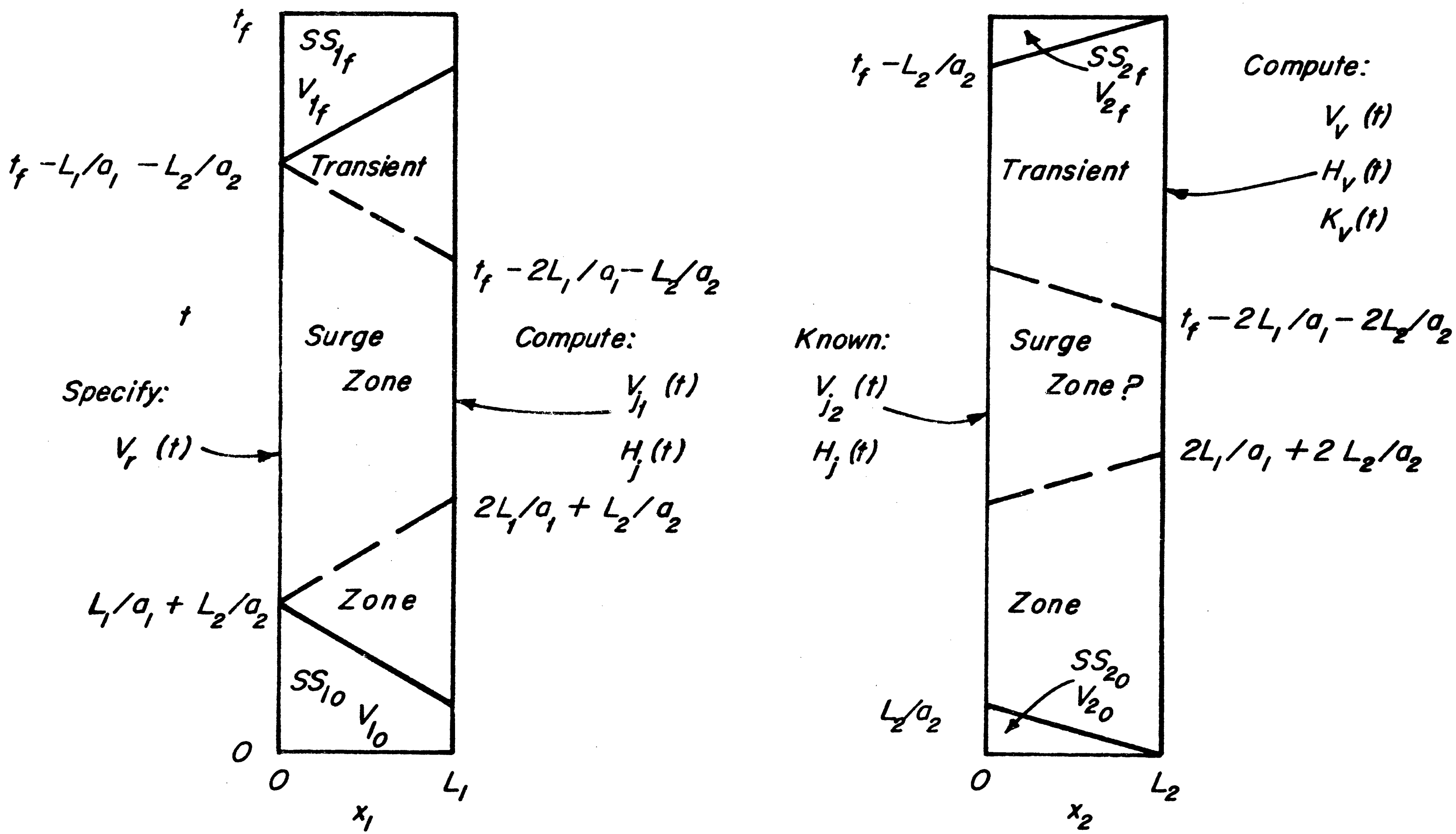


Figure 24. x-t plane and notation for series system with upstream constant-head reservoir.

solutions $V_v(t)$, $H_v(t)$, and $K_v(t)$ are solely dependent upon the initial conditions and the data prescribed along the upstream reservoir boundary.

Streeter and Wylie[31] suggest that the reservoir-boundary velocity be specified according to Equation (22), except that the extreme permissible value of head desired of the value (H_m) is replaced by the extreme permissible value of head desired at the junction (H_{m_j}). Since the discussion presented in Section 3.1.1 is equally applicable here, the existence in pipe 1 of a central zone of incompressible surge conditions is assured.

While the strategy is reasonable and certainly acceptable, experience indicates that during most transient conditions the extreme value of pressure sustained in the system obtains at the valve, not at the junction, and as before it would be desirable to identify and limit that value a priori. Recognizing that at the junction the head developed during the central phase of the transient condition is the constant value H_{m_j} -- thus conditions during that time are identical to those provided by the presence of a constant-head reservoir -- one may speculate about whether a central surge zone is propagated into pipe 2. If this be the case, then the incompressible surge equations for each pipe can be related to permit the selection of H_{m_j} in terms of H_m and the dilemma is favorably resolved.

If a surge zone is propagated into pipe 2, then conditions in each pipe must satisfy Equation (22). Each equation may be multiplied by the respective area of the pipe and equated, since $Q_1 = Q_2$:

$$\frac{dQ}{dt} = -g\,\frac{A_1}{L_1}\left(H_{m_j} - H_r + h_{f_1}(V_1)\right) = -g\,\frac{A_2}{L_2}\left(H_m - H_{m_j} + h_{f_2}(V_2)\right) .$$

The friction term takes the indicated form since

$$\frac{L}{g}\,\frac{fV|V|}{2D} = h_f(V) .$$

After rearrangement,

$$\frac{A_1}{L_1} h_{f_1}(V_1) - \frac{A_2}{L_2} h_{f_2}(V_2) = \frac{A_2}{L_2}(H_m - H_{m_j}) - \frac{A_1}{L_1}(H_{m_j} - H_r) \quad . (34)$$

Since the right-hand side of the above equation must be a constant, then

$$\frac{A_1}{L_1} h_{f_1}\left(\frac{Q}{A_1}\right) - \frac{A_2}{L_2} h_{f_2}\left(\frac{Q}{A_2}\right) = \text{constant}.$$

But this constant must be zero, since $h_{f_1}(0) = h_{f_2}(0) = 0$. Therefore

$$\frac{A_1}{L_1} h_{f_1}\left(\frac{Q}{A_1}\right) = \frac{A_2}{L_2} h_{f_2}\left(\frac{Q}{A_2}\right) \qquad (35a)$$

is the necessary relationship between the viscous resistance properties of each pipe essential to the establishment of a zone of incompressible surge conditions in pipe 2.

If the Darcy-Weisbach friction factors can be considered to have a constant value for each pipe, then the above criterion becomes

$$\frac{f_1}{D_1^3} = \frac{f_2}{D_2^3} \quad . \qquad (35b)$$

One immediately recognizes that either of the above criteria rarely would be satisfied. If they are, however, then the desired relationship between H_{m_j} and H_m may be obtained by rearrangement of the right-hand side of Equation (34):

$$H_{m_j} = \frac{\frac{A_1}{L_1} H_r + \frac{A_2}{L_2} H_m}{\frac{A_1}{L_1} + \frac{A_2}{L_2}} \quad . \qquad (36)$$

One <u>does</u> recognize that a frictionless system satisfies the criterion of Equation (35b) and thereby does permit extreme system pressures to be limited to arbitrarily established values. Relative to the frictionless system, equations describing the entire history of the transient condition throughout the system may be developed analogous to those of the simple system of Section 3.1.2. These equations are so lengthy (because of the several parameters involved) and numerous (because of the compound possibilities of values of t_f and relative values of L/a in each pipe) that they have diminished utility for providing insight into the controlled transient condition developed in the viscous system; they will not be presented here.

For a system in which viscous effects are not negligible, one must again depend upon numerically-oriented solutions. For the valve-stroking analysis the reservoir-boundary velocity is specified according to Equation (22) (modified only to the extent that H_{m_j} replaces H_m), and Equations (18a) and (18b) are used to advance the solutions across the grid of pipe 1 to the junction boundary. The common junction-boundary head and continuity equation establish the junction boundary solutions for pipe 2, and Equations (18a) and (18b) again permit the calculation scheme to advance to the valve boundary; the value of the valve coefficient at each grid point can then be determined. The confirming characteristics analysis is virtually identical to that of the simple systems, except that Equations (20a), (20b), and (20c) must now be utilized to evaluated all conditions at the junction.

A computer program was again written to illustrate the effect of system operation according to the principals suggested in this section. Pertinent system parameters and results of a typical study are presented in Section 4.1.2. In yet another study the friction factors were selected according to Equation (35b); the validity of that criterion was confirmed and a central surge zone did develop in pipe 2 as illustrated in Figure 24.

4.1.2 Preferred Solution

Having demonstrated the inherent deficiency of the existing technique for control of the series system, attention is now directed toward the following alternate strategy: recalling the preferred solution technique developed in Section 3.3.2, and with reference to Figure 25, the positive direction is arbitrarily assigned to each characteristic family and the downstream head is specified to change linearly from H_o to H_m during the first $2L_1/a_1+2L_2/a_2$ seconds and maintained at that value during the central phase of the transient condition:

$$H_v(t) = H_o + \frac{(H_m-H_o)t}{2L_1/a_1 + 2L_2/a_2} \qquad 0 \leq t \leq 2L_1/a_1 + 2L_2/a_2 \,, \quad (37a)$$

$$H_o(t) = H_m \qquad 2L_1/a_1 + 2L_2/a_2 \leq t \leq t_f - 2L_1/a_1 - 2L_2/a_2 \,. \quad (37b)$$

A unique solution again exists in the lower regions of the transient zones enclosed by the time-like boundaries, the space-like line $t = 0$, and the negative (dashed) characteristic lines extending from the valve to the reservoir.

Again, the constant value of H_m is prescribed along the valve boundary until such time that the corresponding value of velocity obtained $L_1/a_1+L_2/a_2$ seconds later at the reservoir boundary equals the final steady-uniform velocity (V_{1_f}). At that point one may again assign the positive direction to the C+ characteristic family and the negative direction to the C- family, specify that along the upstream boundary $V_r(t) = V_{1_f}$, established the final steady-state condition as desired, and obtain unique solutions in the upper triangular region of the transient zone of pipe 1 and the upper trapezoidal region of pipe 2.

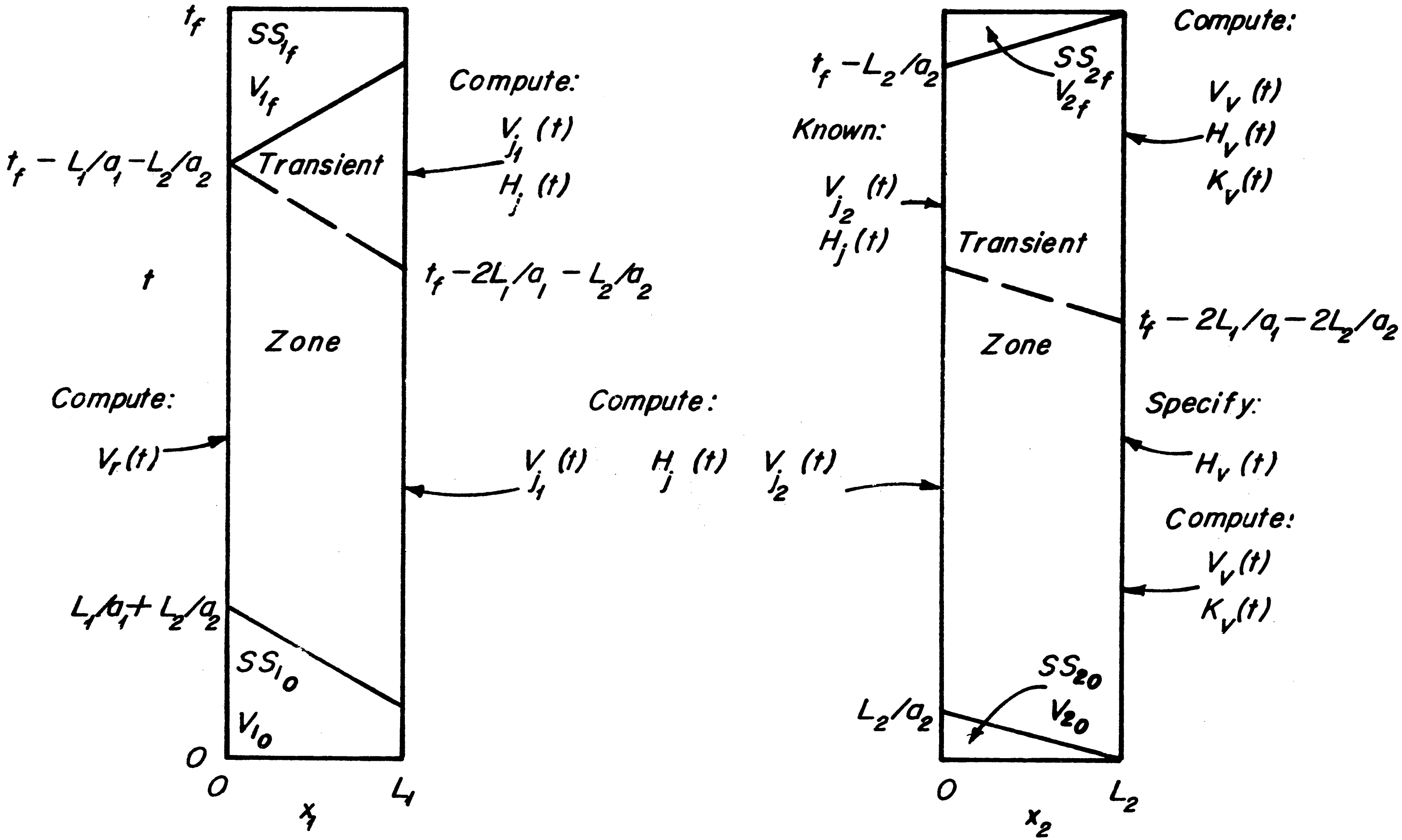


Figure 25. x-t plane and notation for series system with upstream constant-head reservoir --preferred solution.

The necessary numerical solutions are patterned after those of Section 3.3.2. The head at the downstream valve is prescribed according to Equations (37a) or (37b); the downstream velocity is determined from Equation (17b). Interior point conditions are then evaluated diagonally along a C- characteristic as before, and the series-junction boundary conditions are evaluated using the appropriate equations. Again the computations proceed diagonally along the C- characteristic of the upstream pipe, and the reservoir-boundary conditions are evaluated using Equations (13a) and (13b). This solution procedure is terminated when the reservoir-boundary velocity equals V_{1_f}. The velocity along the boundary is then specified to be V_{1_f}, and the remainder of the solution is obtained in the upper regions of the transient zones in a fashion identical to that of the preceding series-system valve-stroking analysis. Specific details of the solution procedures are described in Appendix B. The confirming characteristics analysis is identical to that described in the preceding section.

A computer program which included both the valve-stroking analysis and the confirming characteristics analysis was written to illustrate the effect of system operation according to these concepts. Results of two typical studies are illustrated in Figure 26; results of several studies are summarized in Table III and compared with the results of the computer studies described in Section 4.1.1. The pertinent system parameters are: H_r = 125 feet, V_{o_1} = 2.56 feet per second, V_{o_2} = 4.00 feet per second, $V_{f_1} = V_{f_2} = 0$, L_1 = 3500 feet, L_2 = 4800 feet, A_1 = 3500 feet per second, a_2 = 4000 feet per second, f_1 = .022, f_2 = .020, D_1 = 1.25 feet, and D_2 = 1.00 feet: thus $L_1/a_1 + L_2/a_2$ = 2.2 seconds, H_{j_o} = 118.7 feet, H_o = 94.9 feet, and $H_{j_f} = H_f$ = 125 feet.

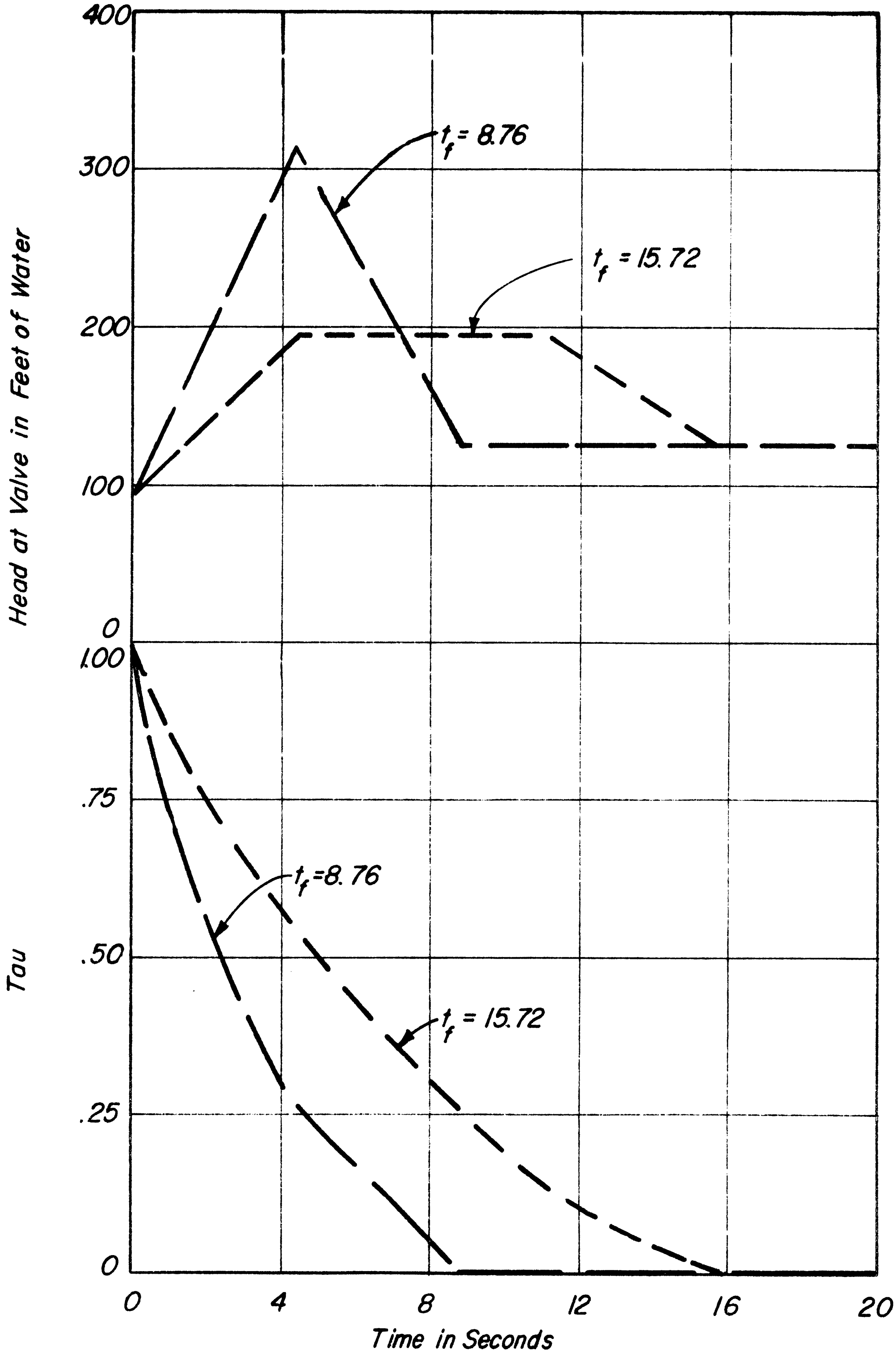


Figure 26. Valve-stroking control of a series system with upstream constant-head reservoir.

TABLE III

COMPARISON OF ANALYTICAL COMPUTER RESULTS -- SERIES SYSTEM

Junction Head Specification			Downstream Head Specification		
t_f seconds	Max. H at Junction feet	Max. H at Valve feet	t_f seconds	Max. H Specified feet	Max. H Obtained feet
8.80	189.2	315.2	8.76	315.2	313.3
10.00	172.7	273.2	9.99	273.2	273.2
12.00	159.6	232.9	11.96	232.9	232.9
14.00	152.0	209.4	13.78	209.4	209.4
16.00	147.0	193.9	15.72	193.9	193.9

These results again confirm the theory and illustrate the differences in the transient conditions obtained utilizing the two procedures. The somewhat more rapid closure times permitted by the valve-head specification technique (for the same ultimate maximum value of head developed in the system) are obtained for the same reasons as they were in the system of Section 3.3.

Especially noteworthy is the obvious extension of the rapid control concepts of the simple systems to the series system as well. As illustrated by the first case presented in Table III, an extreme value of head was selected that was not attained since the consequent value of t_f was less than $4L_1/a_1 + 4L_2/a_2$.

In summary, the proposed valve-stroking procedure creates a controlled transient condition throughout the system and permits the final steady-state condition to be established without residual transient fluctuations. Unlike the previous procedure, the solution technique proposed in this section permits a predetermined extreme value of pressure to be developed and maintained at the valve during a central phase of the transient condition. Furthermore, the procedure does not depend upon the presence of a constant-head upstream reservoir; it is perfectly general and could be applied to any series system with a known constant-relationship upstream boundary condition. Lastly, it may be applied to any number of pipes in series. In such cases the minimum value of t_f is $\Sigma\, 2L/a$, and all other values of valve operation time discussed in this section must be modified accordingly.

4.2 Branching System

Another example of an elementary complex system for which a valve-stroking procedure may be developed is provided by the branching system illustrated in Figure 27. Again consider that a controlled transient condition is to be developed between known initial and final steady-state conditions in the system such that residual transient fluctuations are eliminated.

An analysis of the system indicates that a total of nine unknown boundary dependent variables exist, whereas only seven equations are available. Thus, specification of two of the unknown variables will be required to completely specify the transient condition imposed upon the system.

4.2.1 Evaluation of the Existing Technique

With reference to Figure 28, the existing solution technique (Ref. 30,31) for this system is first examined.

Because of the invarient reservoir-boundary relationship, $H_1(0,t) = H_r$, and the subsequent procedure for specification of velocities at the junction, the regions of the initial steady-uniform conditions are as illustrated. Arbitrarily assigning the positive direction to the C+

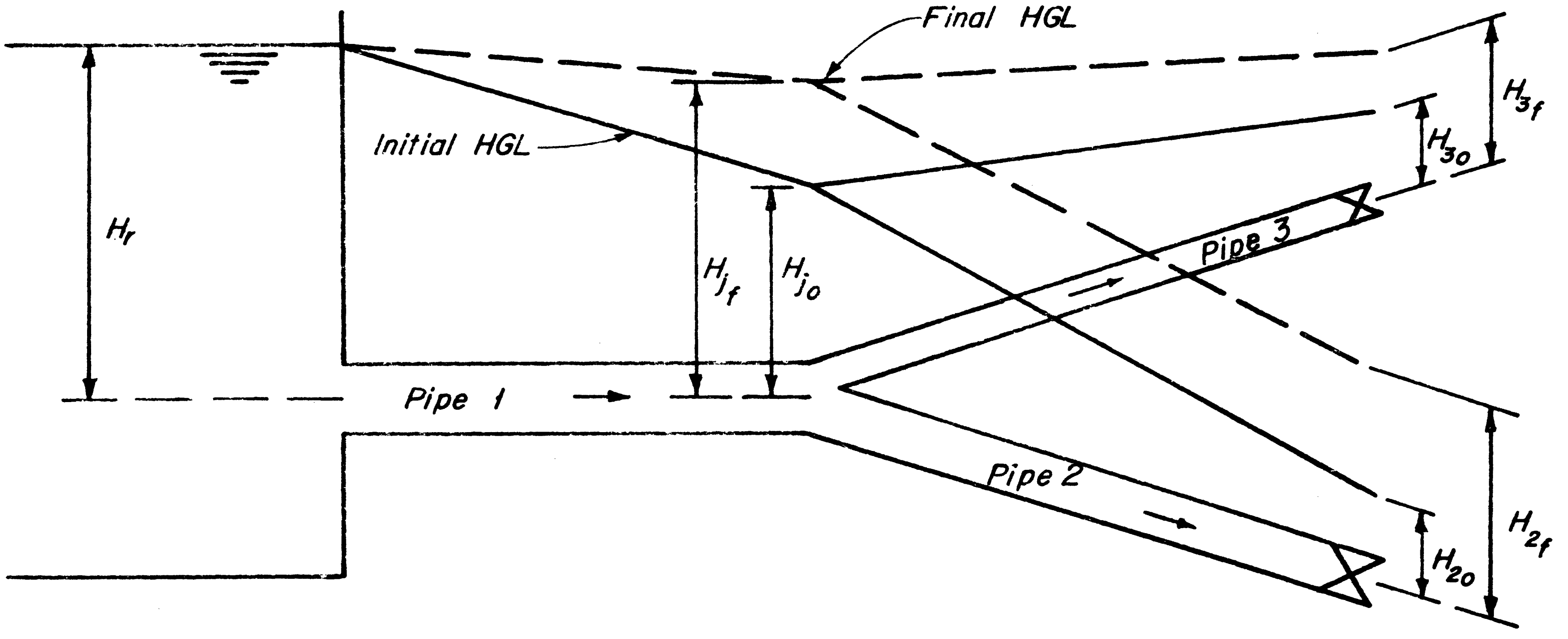


Figure 27. Definition sketch of branching system with upstream constant-head reservoir.

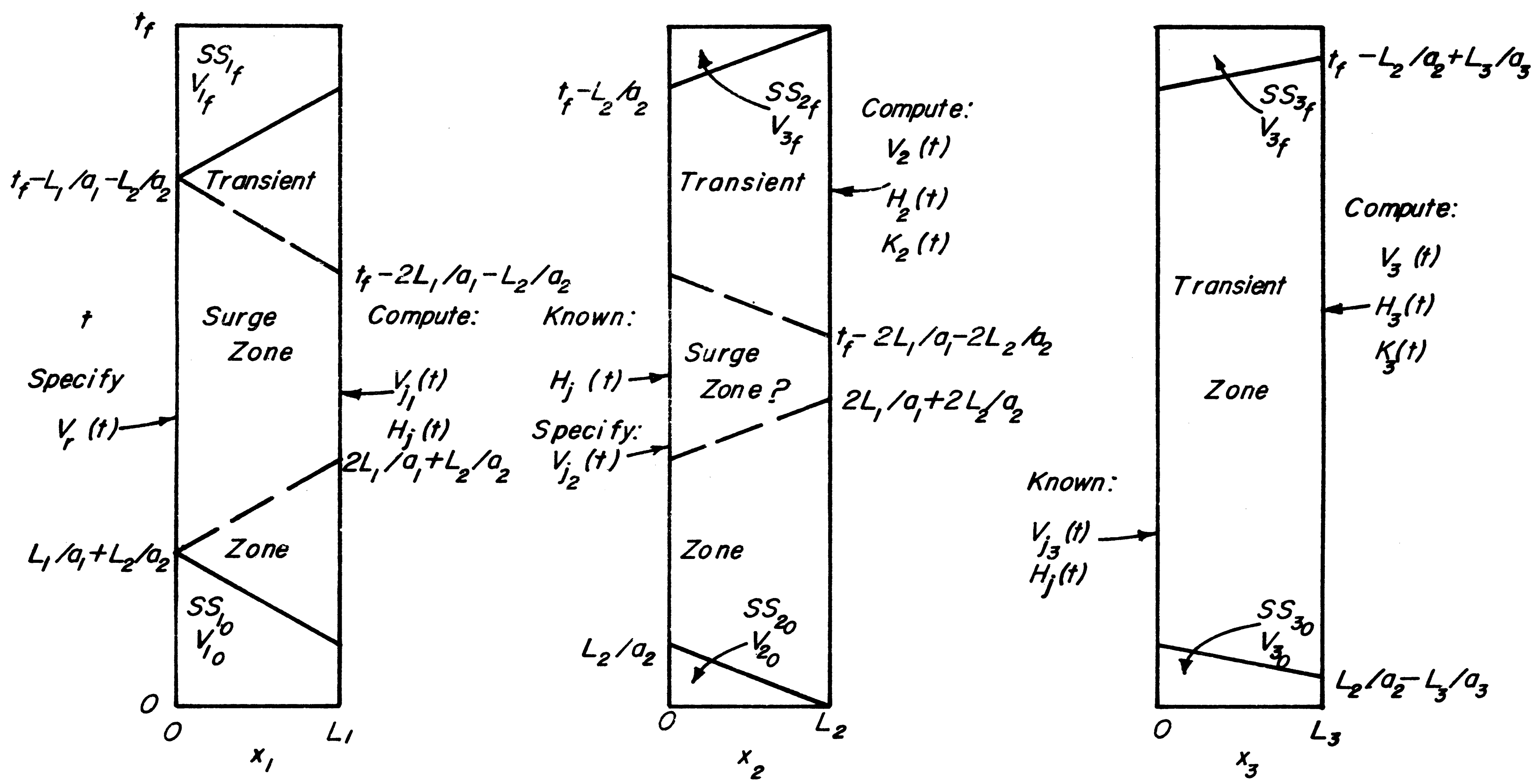


Figure 28. x-t plane and notation for branching system with upstream constant-head reservoir.

characteristic family and the negative direction to the C- family, the reservoir-boundary velocity is specified according to Equation (22) exactly as it was for the series system: an extreme permissible value of head desired at the junction is specified. Thus a unique solution exists in pipe 1, and by specifying that at time $t \geq t_f - L_1/a_1 - L_2/a_2$, $V_r(t) = V_{1_f}$, the final steady-uniform condition in the pipe is established as desired.

Directing attention now to the branching junction, three of the four essential boundary data are now established; the fourth must therefore be specified (the second of the two permissible boundary-data specifications). The flow distribution at the junction is determined by arbitrarily specifying that the change in flow in pipe 2 is to be proportional to the change in flow in pipe 1:

$$\frac{Q_{j_2}(t) - Q_{2_0}}{Q_{2_f} - Q_{2_0}} = \frac{Q_{j_1}(t) - Q_{1_0}}{Q_{1_f} - Q_{1_0}} \quad . \qquad (38)$$

(Application of the continuity equation at the junction demonstrates that the change in flow in pipe 3 is similarly proportional to the change in flow in pipe 1.) Since both data are thereby established at the upstream boundaries of each pipe, unique solutions exist, and the solutions of the dependent variables and unsteady valve coefficients may be determined at each downstream valve boundary. Again the final steady-uniform conditions will be established as desired, since at $t \geq t_f - L_2/a_2$ the head and velocity at the upstream boundary of each pipe is the final steady-uniform value.

As with the existing technique for the series system, the same objection to this strategy may exist because the extreme values of pressure sustained in the system usually develop at the valves, not the junction; again it would be desirable to identify and limit those values a priori. Since conditions in pipe 1 of this system are identical to those encountered in the series system, one may speculate about the possible propagation of surge zones into the downstream pipes as before.

If a surge zone is propagated into pipe 2, for example, conditions in each pipe must again satisfy Equation (22). Equation (38) may be reduced to the form

$$Q_2(t) = CQ_1(t) + K \ ,$$

where

$$C = \frac{Q_{2_f} - Q_{2_0}}{Q_{1_f} - Q_{1_0}} \ , \qquad K = Q_{2_0} - CQ_{1_0} .$$

Then from Equation (22),

$$C \frac{A_1}{L_1} (H_{m_j} - H_r + h_{f_1}(V_1)) = \frac{A_2}{L_2} (H_{m_2} - H_{m_j} + h_{f_2}(V_2))$$

since $dQ_2/dt = C \cdot dQ_1/dt$; H_{m_2} is the extreme value of head desired at valve 2. After rearrangement,

$$C \frac{A_1}{L_1} h_{f_1}(V_1) - \frac{A_2}{L_2} h_{f_2}(V_2) = \frac{A_2}{L_2} (H_{m_2} - H_{m_j}) - C \frac{A_1}{L_1} (H_{m_j} - H_r) \ . \tag{39}$$

Since the right-hand side of the above equation must be a constant, then

$$C \frac{A_1}{L_1} h_{f_1}\left(\frac{Q_1}{A_1}\right) - \frac{A_2}{L_2} h_{f_2}\left(\frac{CQ_1+K}{A_2}\right) = \text{constant.}$$

This constant may be evaluated, since the first term above is zero when $Q_1 = 0$. Therefore

$$C \frac{A_1}{L_1} h_{f_1}\left(\frac{Q_1}{A_1}\right) = \frac{A_2}{L_2}\left(h_{f_2}\left(\frac{CQ_1+K}{A_2}\right) - h_{f_2}\left(\frac{K}{A_2}\right)\right) \tag{40a}$$

is the necessary relationship between the viscous resistance properties of each pipe essential to the establishment of a zone of incompressible surge conditions in pipe 2.

In terms of the Darcy-Weisbach friction factors, the above criterion becomes

$$\frac{f_1}{D_1^{\,3}} = \frac{f_2}{D_2^{\,3}} \cdot \left(1 + \frac{2K}{Q_1}\right) . \tag{40b}$$

One again recognizes that the above criteria would probably never be satisfied by flow conditions prevailing in a real system. A system in which viscous effects are so insignificant that they may be safely neglected would satisfy the criteria, of course, and the desired relationship between H_{m_j} and H_{m_2} can be obtained by rearrangement of the right-hand side of Equation (39):

$$H_{m_j} = \frac{C \dfrac{A_1}{L_1} H_r + \dfrac{A_2}{L_2} H_{m_2}}{C \dfrac{A_1}{L_1} + \dfrac{A_2}{L_2}} . \tag{41}$$

Since flow conditions in pipe 3 are similar to those in pipe 2, a surge zone would also be propagated into pipe 3; a similar relationship involving H_{m_j} and H_{m_3} likewise would exist. The lesser of the two computed values of H_{m_j} could then be used to specify the reservoir-boundary velocity.

4.2.2 Alternate Solution

Let one assume that system parameters are such that it would be desirable to limit the extreme value of head developed at one of the valves to a predetermined value. Again with reference to Figure 28, assume that the extreme value permitted at valve 2 is to be H_{m_2}. Then if the velocity $V_{j_2}(t)$ at the junction is specified by means of the incompressible surge equation over the interval $L_2/a_2 \leq t \leq t_f - L_2/a_2$, and not by Equation (38), then the discussion presented in Section 3.1.1 assures the propagation of a central zone of incompressible surge conditions into pipe 2 as illustrated (assuming, of course, that the surge zone is of sufficient duration to propagate to the valve boundary). Thus

$$\frac{dV_2}{dt} = \frac{-g}{L_2}\left(H_{m_2} - H_{m_j}\right) - \frac{f_2 V_2 |V_2|}{2D_2} \qquad (42)$$

is suggested as an alternate procedure for specifying the flow into pipe 2 at the junction.

Each of the surge equations can then be integrated to determine the duration each is utilized to specify the appropriate boundary velocity, and the results may be equated since the time of specification at the reservoir is $2L_1/a_1$ seconds less than the time of specification at the junction:

$$\frac{2L_1}{a_1} - \frac{L_1}{g} \int_{V_{1_0}}^{V_{1_f}} \frac{dV_1}{(H_{m_j} - H_r + h_{f_1}(V_1))} = \frac{-L_2}{g} \int_{V_{2_0}}^{V_{2_f}} \frac{dV_2}{(H_{m_2} - H_{m_j} + h_{f_2}(V_2))} \qquad (43)$$

Since every parameter of Equation (43) is known except H_{m_j}, H_{m_j} may be determined. (The solution is a numerical one -- see Appendix C.)

A computer program was written to illustrate the effect of system operation according to either of the suggested techniques. For the valve stroking analysis the reservoir-boundary velocity is specified according to Equation (22) (again by replacing H_m with H_{m_j}), and Equations (18a) and (18b) are used to advance the solutions across the grid of pipe 1 to the junction boundary. Either Equation (38) or (42) is utilized to specify the junction-boundary velocity in pipe 2; the junction-boundary velocity in pipe 3 is then determinable from the continuity equation. Equations (18a) and (18b) again advance the solution schemes to each valve boundary, permitting the value of both valve coefficients to be determined at the respective boundary grid points. The confirming characteristics analysis is identical to that of the series system, with the exception that Equations (21a), (21b), (21c), and (21d) must now be utilized to evaluate all conditions at the junction.

Results of a study comparing the two techniques are illustrated in Figures 29 and 30. The pertinent system parameters are: $H_r = 100$ feet, $V_{o_1} = 3.52$ feet per second, $V_{o_2} = 5.00$ feet per second, $V_{o_3} = V_{f_3} = 2.00$ feet per second, $V_{f_1} = .32$ feet per second, $V_{f_2} = 0$, $L_1 = 3600$ feet, $L_2 = 3200$ feet, $L_3 = 1800$ feet, $a_1 = 3600$ feet per second, $a_2 = 4000$ feet per second, $a_3 = 3000$ feet per second, $f_1 = .018$, $f_2 = .020$, $f_3 = .025$, $D_1 = 1.25$ feet, $D_2 = 1.00$ feet and $D_3 = 0.50$ feet; thus $L_1/a_1 + L_2/a_2 = 1.8$ seconds, $H_{j_o} = 90.0$ feet, $H_{2_o} = 65.2$ feet, $H_{3_o} = 84.4$ feet, $H_{j_f} = H_{2_f} = 99.9$ feet, and $H_{3_f} = 94.3$ feet.

The study was arranged to compare the results of the two techniques for the same value of H_{m_j} (and thus the same value of t_f). This value was determined by establishing $H_{m_2} = 175.0$ feet; H_{m_j} was then determined to be 136.7 feet and t_f was subsequently found to be 12.52 seconds.

These results again confirm the theory and illustrate the differences in the transient conditions obtained utilizing the two procedures. Control according to the existing technique results in a maximum head developed at valve 2 of 187.6 feet; control according to the alternate technique suggested here permits the maximum value to be limited to the preselected 175.0 feet. One should note, however, the considerable difference in conditions imposed upon the valve boundary

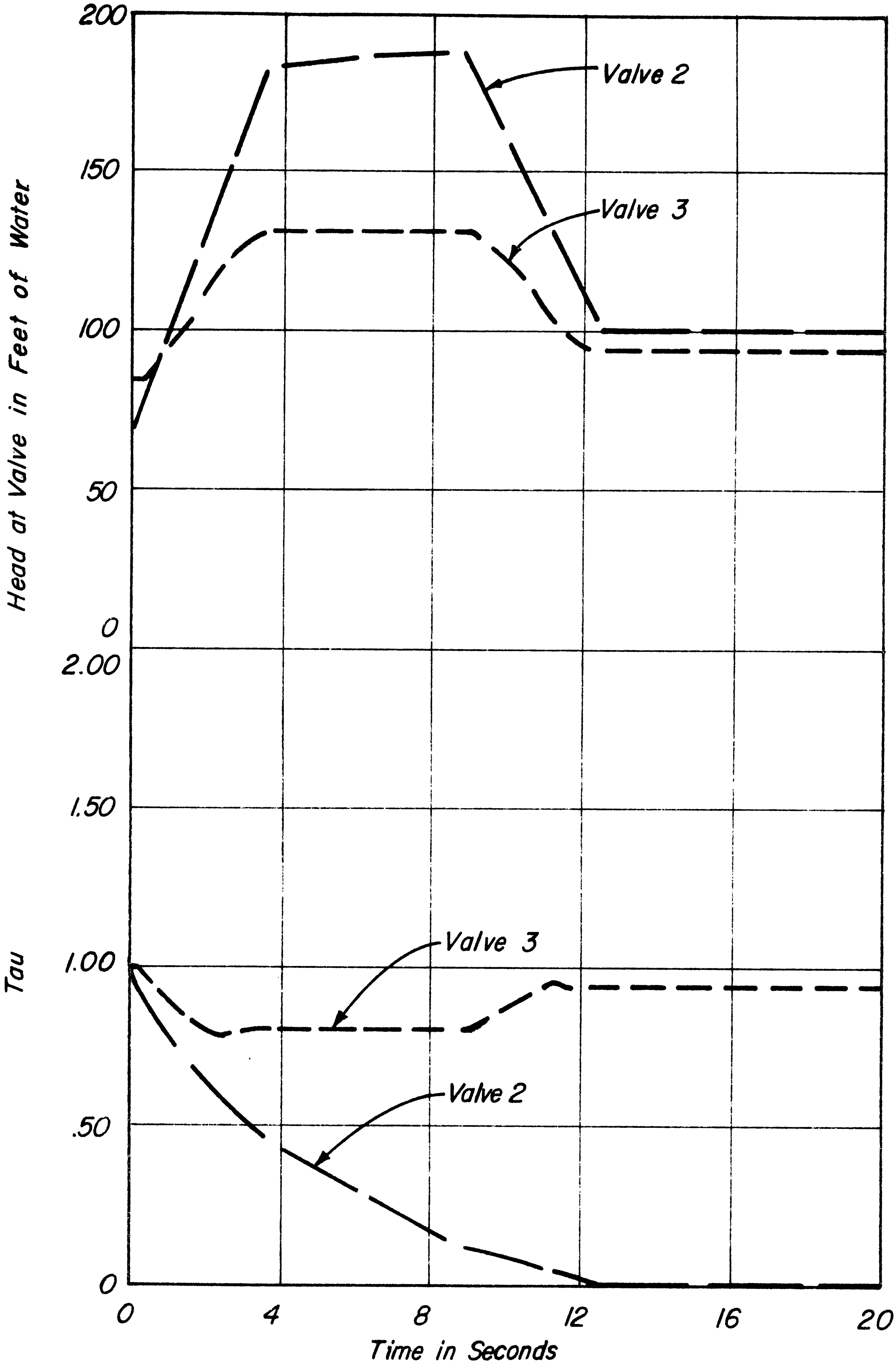


Figure 29. Valve-stroking control of a branching system with upstream constant-head reservoir -- existing procedure.

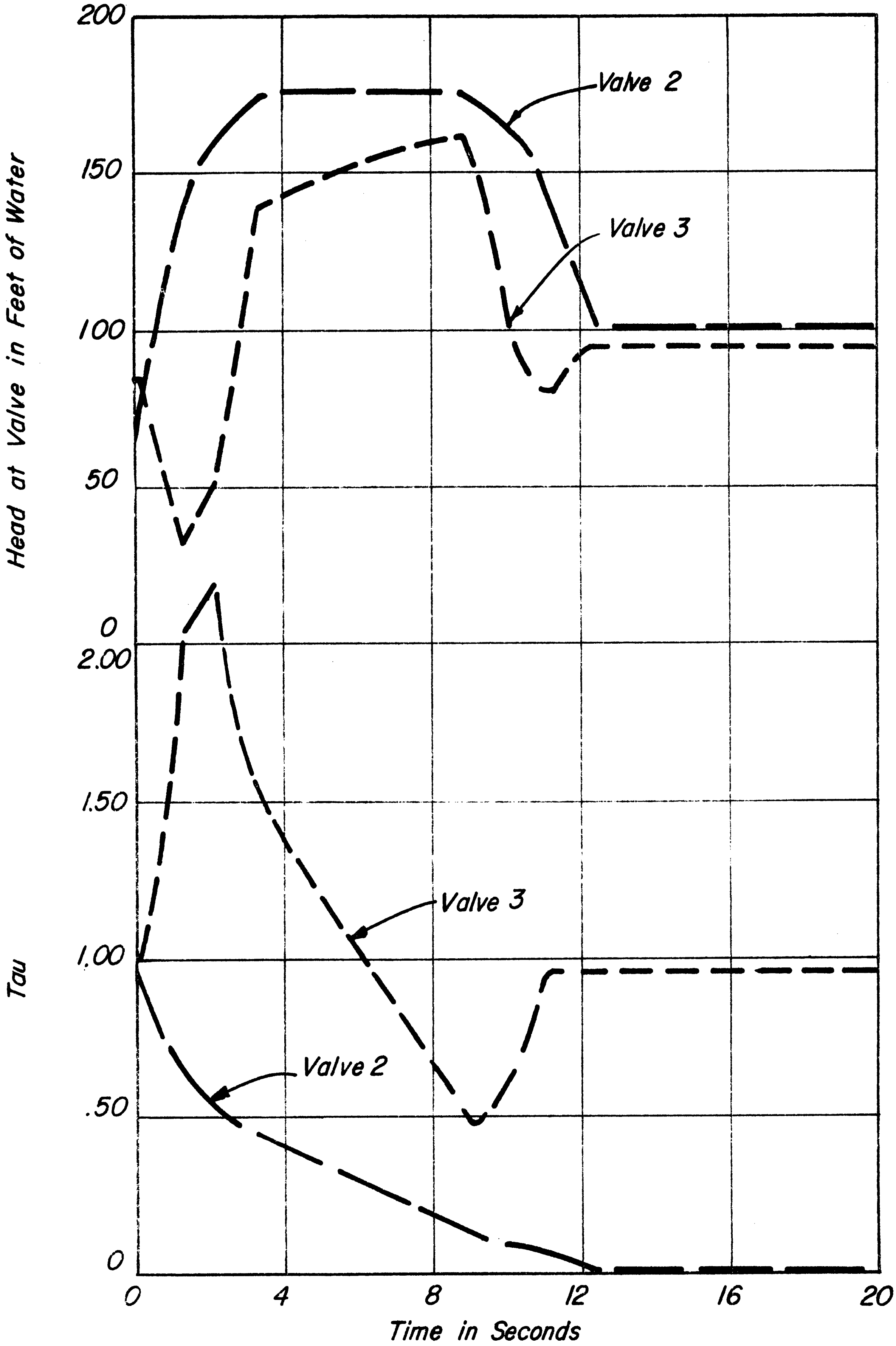


Figure 30. Valve-stroking control of a branching system with upstream constant-head reservoir -- alternate procedure.

of pipe 3. (The constant head and velocity developed there by the existing technique during a central phase of the transient condition is only a circumstance of the data. Since $V_{3_o} = V_{3_f} = 2.00$ feet per second, specification of the junction flows according to Equation (38) results in $V_{j_3}(t)$ being 2.00 feet per second throughout the entire transient condition. Since the head at the junction is the prescribed constant H_{m_j} during the central phase, these two constant variables combine to propagate a zone of steady-uniform conditions into pipe 3.)

In summary, either valve-stroking procedure creates a controlled transient condition throughout the system and again permits the final steady-state condition to be established without the presence of residual transient fluctuations. Unlike the existing technique, the alternate technique proposed in this section does permit a predetermined extreme value of pressure to be developed and maintained at one valve during a central phase of the transient condition. The technique does, however, impose a less orderly condition of operation upon valve 3 and thereby demonstrates what would appear to be a fundamental axiom of transient control in complex systems that must always be appreciated: the more sophisticated and elegant the desired control techniques are, the more sophisticated the control devices will have to be.

4.3 Passive Systems

As an instructive example of the concept of passive valve stroking, consider the following: assume that it is desired to develop a controlled transient condition between known initial and final steady-state conditions in the branching system of Figure 27 subject to the additional operational constraint that a constant outflow be maintained through valve 3 during the entire history of the imposed transient condition. (Recall that in the example reported in the last section both the initial and final steady-uniform velocities through valve 3 were 2.0 feet per second; however, that value was not maintained throughout the entire transient.) This imposed constraint now reduces the number of unknown boundary dependent variables to only eight, while the seven original equations are still available. Here, specification of only one of the unknown variables will be required to completely specify the controlled transient operation of the system.

With reference to Figure 31, the initial steady-uniform regions of each pipe are as indicated due to the presence of the invarient reservoir-boundary relationship, $H_1(0,t) = H_r$, and the invarient valve-boundary relationship, $V_3(L_3,t) = V_{3_o}$. Arbitrarily assigning the positive direction to the C+ characteristic family and the negative direction to the C- family, the reservoir-boundary velocity could then be specified according to Equation (22) following the existing procedure for complex systems: an extreme permissible value of head desired at the junction is specified. Again, a unique solution exists in pipe 1, and denoting as t_r the time at which $V_r(t)$ becomes thereafter equal to V_{1_f}, the final steady-uniform zone in pipe 1 is as illustrated in the figure.

Attention is now directed to pipe 3. One boundary datum has now been established at each boundary -- the junction-boundary head, $H_j(t)$, available from the solution in pipe 1 and the steady valve-boundary velocity, $V_3(t) = V_{3_o}$. Thus, a unique solution exists in pipe 3. After $t = t_r + L_1/a_1$, the junction-boundary head becomes the final steady-state value (H_{j_f}). Note, however, that unlike the situations considered in the previous examples or pipe 1 of this example, in which

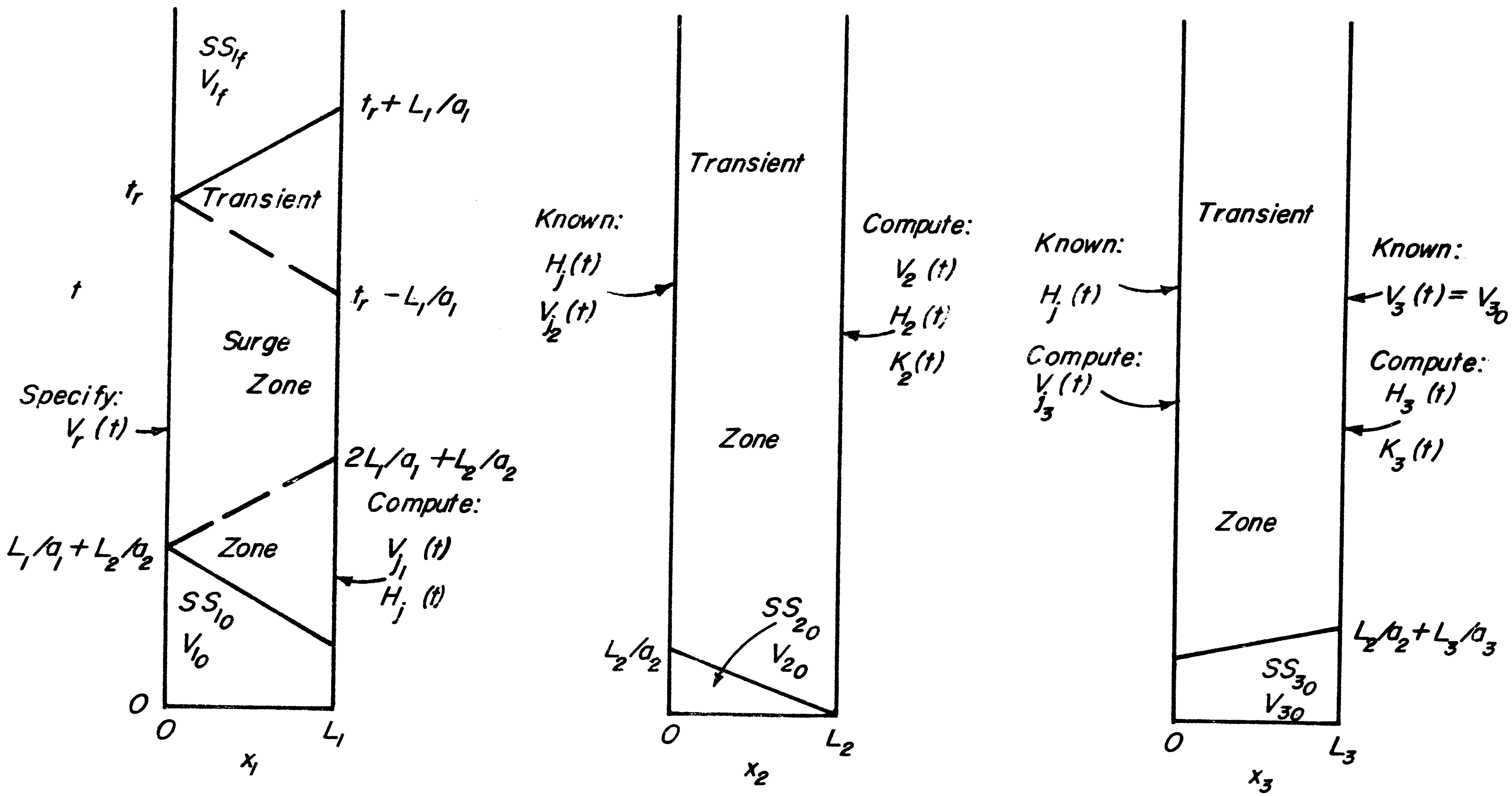


Figure 31. x-t plane and notation for branching system with upstream constant-head reservoir --passive system.

a final steady-state zone is forced by the creation of two steady-state boundary data at one boundary, no final steady-state zone is immediately created in pipe 2. As discussed in Section 2.6.2, only after $t = t_r + L_1/a_1$ will the transient condition subsequently begin to asymptotically approach the final steady-unifrom conditions in pipe 3. Unlike the previous situations considered, the duration of the transient condition imposed upon pipe 3 (and thereby pipe 2) can not be arbitrarily specified and can only be determined by the appropriate numerical evaluations advanced upward on the double-staggered grid of characteristics (see Appendix B). The calculations may be halted thereafter, and the valve coefficient-time relationship may then be determined from the solutions $V_3(t)$ and $H_3(t)$ computed at the right-end boundary.

With reference to pipe 2, both data are thereby established at its upstream boundary (the existance of the common boundary head and the branching-junction continuity equation), and thus a unique solution exists in the pipe; the valve coefficient-time relationship may then be computed from the known solutions $V_2(t)$ and $H_2(t)$.

Therefore, the entire transient condition for this system has been prescribed and valve motions determined that would create the desired condition. Of the system, pipe 1 and its transient condition would be considered as <u>active</u> elements; the period of the transient zone may be arbitrarily selected and the nature of the transient itself can be predetermined. Pipe 3, its prescribed transient condition, and the valve would be considered as <u>passive</u> elements; the period and nature of the controlled transient are not readily predictable. In addition, the valve, instead of creating a transient condition, merely reacts to that condition in order to maintain the required constant outflow (hence the term "passive"). Pipe 2, its transient condition, and valve would have to be considered as passive elements also; the period of the transient is dependent upon the transient condition in pipe 3, its nature is not readily predictable, and the valve, even though it creates the initial disturbance, must necessarily respond to the passive character of the transient created in pipe 3.

It is pertinent to note that although the passive condition of this example was a consequence of an arbitrarily imposed operational constraint, any constant-condition boundary constraint at the downstream end of pipe 3 would have created the same condition; the presence of a constant-head reservoir, a fixed orifice, a centrifugal pump, or a dead end are all commonplace examples.

Also note that the passive condition of the example is independent of the specification of the one free unknown boundary variable. It would have been possible, for example, to select pipe 3 as the active pipe by specifying the transient head at the valve according to Equation (28) of Section 3.2.1. That strategy would alter the sequence of calculations, of course, but would not change the essential, passive character of the system.

An example of a system that is inherently passive is provided by the last of the elementary complex systems, the parallel system of Figure 32. Analysis of this system indicates that a total of eleven unknown boundary dependent variables exist, whereas only ten equations are available (two characteristic equations for each pipe plus the two branching-junction continuity equations). The existing technique[31] for valve-stroking control of the system is to specify the reservoir-boundary velocity according to Equation (22) by again selecting the

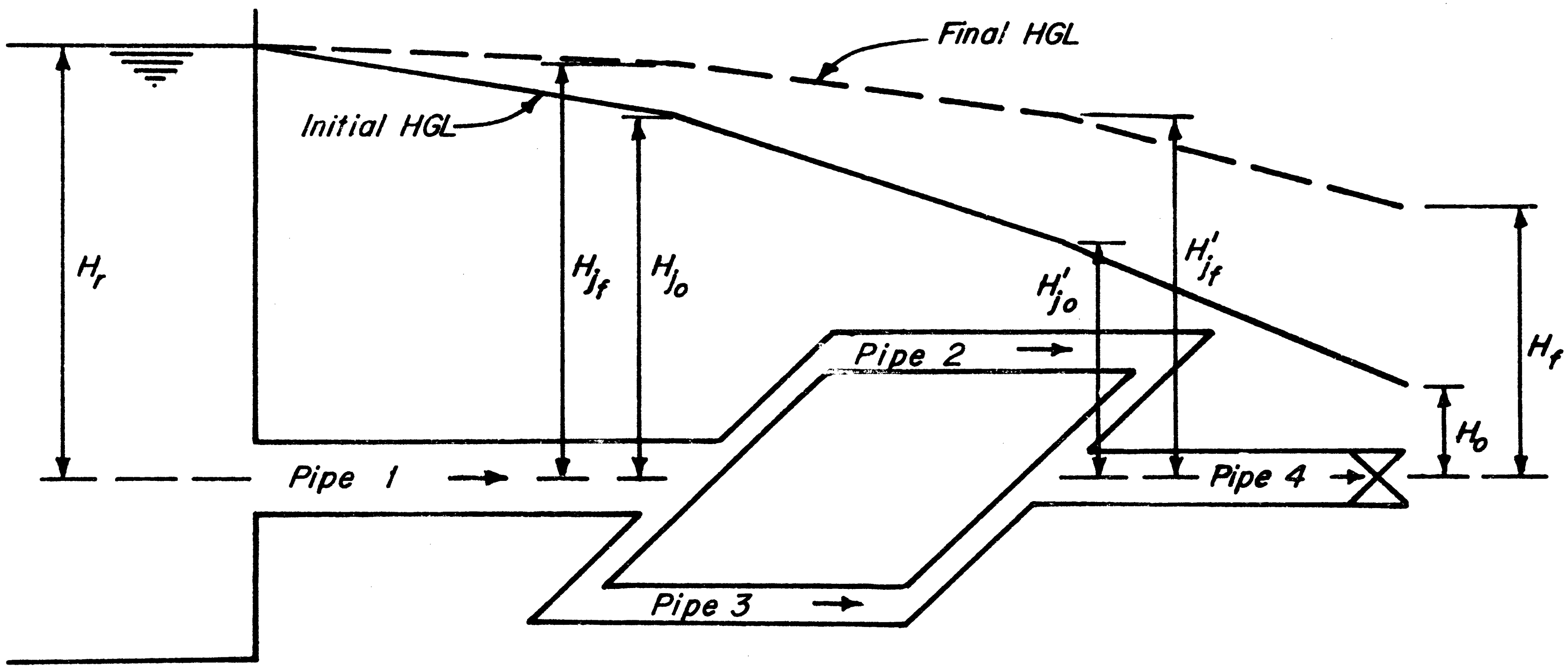


Figure 32. Definition sketch of parallel system with upstream constant-head reservoir.

extreme value of head to be developed at the upstream junction. The transient condition in pipe 1 is then completely specified and the pipe is an active element of the system. With reference to pipes 2 and 3, the number of unknown boundary variables and available equations now both equal five; therefore unique solutions exist in these pipes, and as with the preceding example, the duration of the transient condition imposed upon the remainder of the system -- the passive elements -- can only be determined by subsequent numerical evaluation.

Relative to the more extensive complex systems, the following generalized observation can be made: all multiple series systems are active systems; multiple branching systems may be active if a constant-relationship boundary condition exists at only one of the external boundaries of the system, and all other multiple branching systems must necessarily be passive; all multiple loop systems must be passive systems.

Experimental confirmation of the concept of passive valve-stroking control of a branching system is provided in Chapter VI, and a discussion of proposed alternate valve-stroking techniques for the elementary passive systems is included in the recommendations for further study discussed in Chapter VII.

V. LABORATORY EXPERIMENTAL SYSTEM

In order to obtain experimental confirmation of the previously published valve-stroking theory,[30,31] an experimental system had been constructed in the G. G. Brown Fluids Engineering Laboratory of the University of Michigan. This system was used to obtain additional verification of some of the valve-stroking concepts presented in this investigation.

5.1 General Description of System

The laboratory system (see Plate I) was comprised of three principal elements: the reservoir and piping systems, the control valve and servo systems, and the various instrumentation devices essential to system calibration and transient operation.

A schematic of the flow system is illustrated in Figure 33. One of the building open-channel floor drains was utilized as the sump; its dimensions were approximately 14 inches wide, 18 inches deep, and 36 feet long. Water was supplied to the reservoir from the sump by means of a Worthington Type 1 TM 0 turbine pump. The bypass valve on the downstream side of the pump permitted the water level in the reservoir to be maintained at an essentially constant level over the intended range of system operating conditions.

The high-pressure reservoir was constructed from a 2-foot diameter, 5-foot high welded galvanized steel tank. This reservoir was about half filled with water, and compressed air occupied the space above the water surface. The air pressure in the tank was regulated utilizing a Fisher Type 98LD low pressure differential relief valve; the pressure on the upper side of the valve diaphragm was regulated using a Bellofram

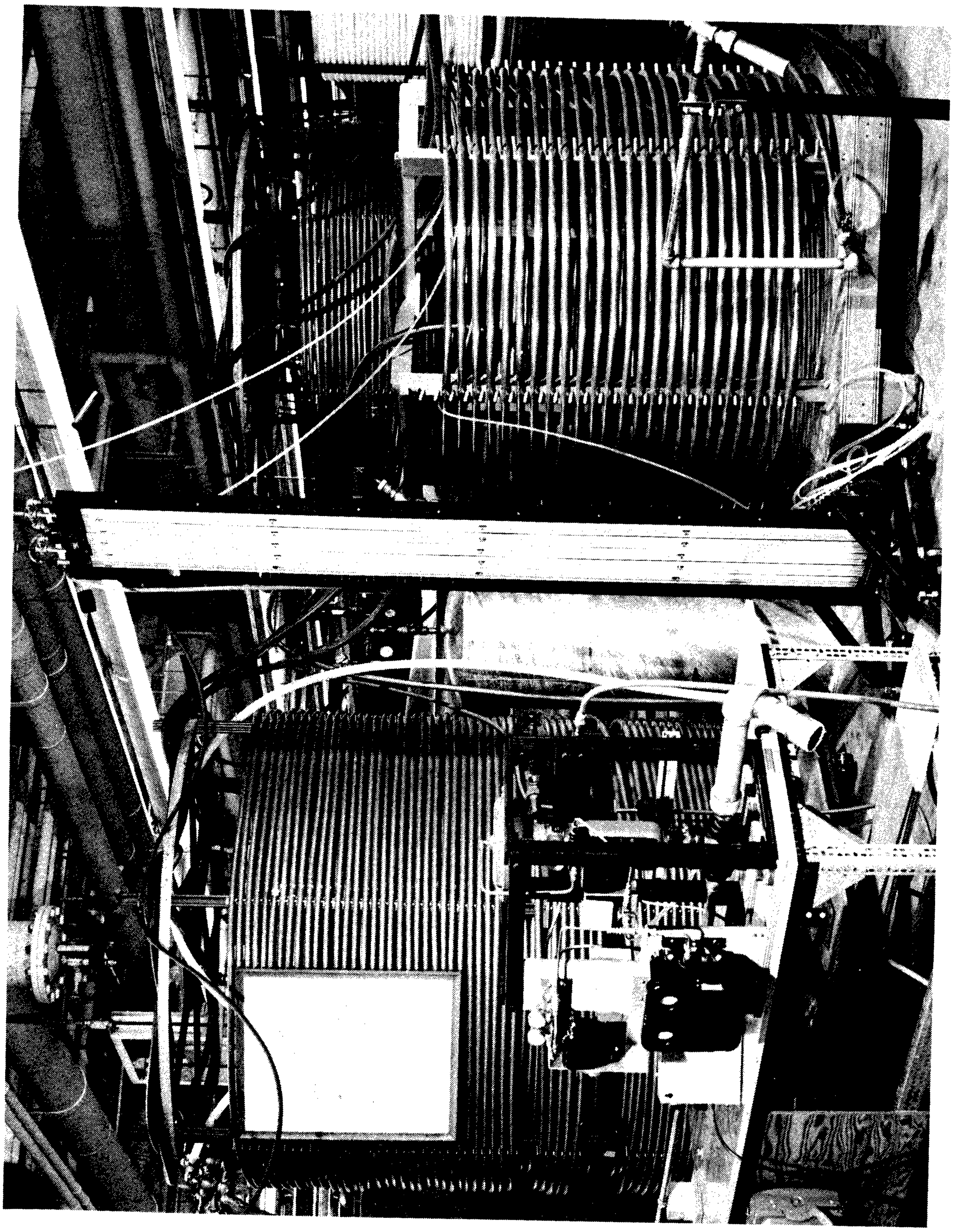


Plate I. Experimental system

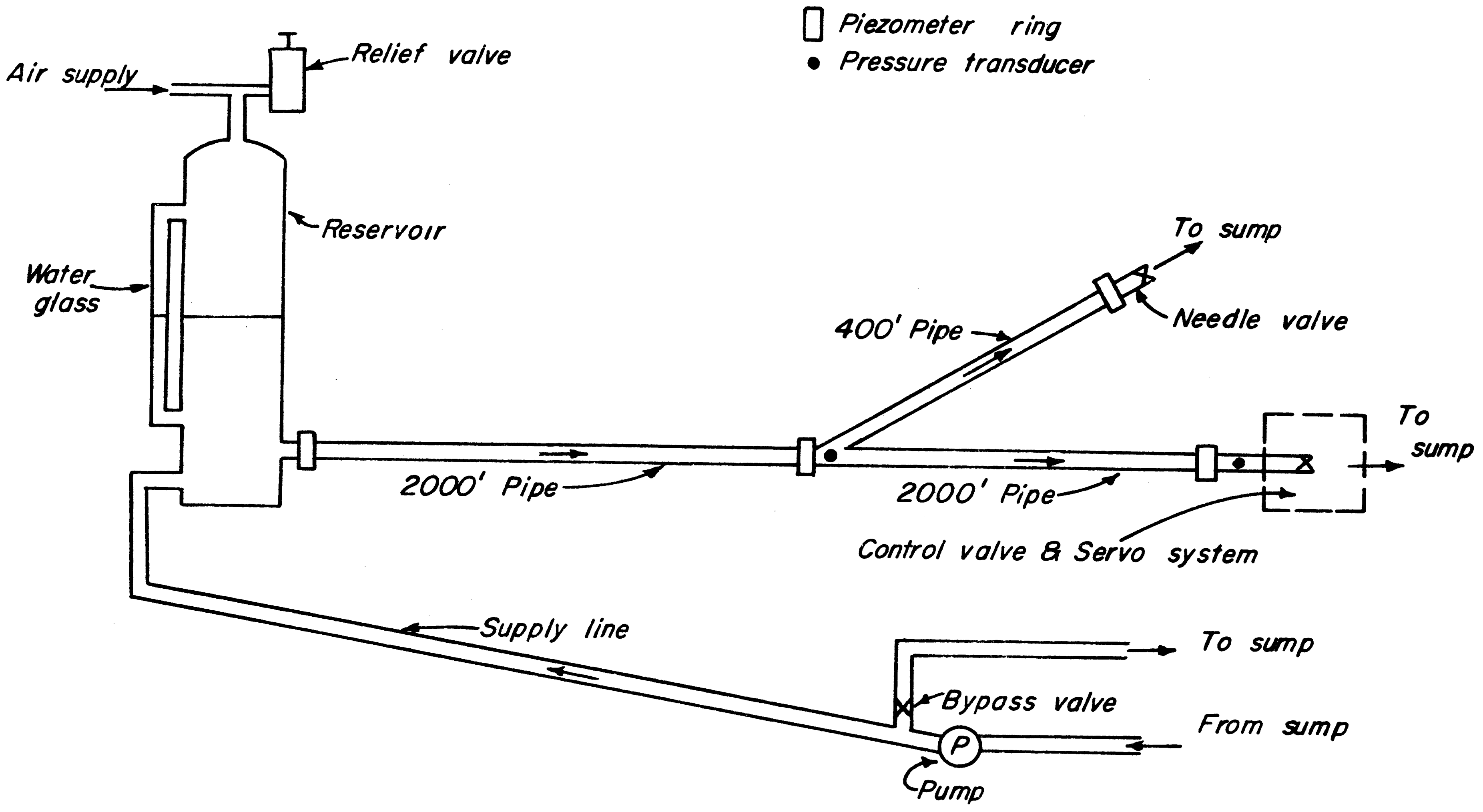


Figure 33. Schematic of laboratory system.

Type 10A pressure regulator (see Plate II).

Of the piping system, the upstream pipe consisted of 2000 feet (nominal) of .95-inch ID copper tubing; the wall thickness was 1/16 of an inch. An identical pipe formed one of the downstream branching pipes, and the other downstream pipe consisted of 400 feet (nominal) of 1.00 inch ID copper tubing. The wall thickness of this pipe was 7/64 of an inch. The 2000-foot pipes were each constructed from 100-foot segments carefully joined together by soldered sleeve connections and loosely coiled in a double helix (a 1000-foot inner coil, followed by a 1000 foot outer coil) on a 6-foot diameter frame constructed of "Unistrut" steel members. The 400-foot pipe was similarly constructed from 100-foot segments and coiled in a helix on a 5-foot diameter wooden frame. Earlier studies[15,25] had demonstrated that this configuration would produce no significant distortion of the transient pressure waves.

At the downstream end of the 400-foot pipe, a 3/4-inch Crane needle valve was installed at the same elevation as the control valve (the arbitrary datum) to provide the function of a fixed orifice.

The system was very versatile: by closing a valve in the 400-foot pipe at the branching junction, a simple system could be obtained. (The actual length of this system, including all connector pipes, was determined to be 4030 feet. That value was used in the simple-system analyses reported in Chapter VI. The nominal lengths were used, however, in the series and branching system analyses, because of the heretofore mentioned Δx-Δt computational constraint.) Finally, by connecting the 400-foot pipe between the reservoir and the two 2000-foot pipes, a series system was obtained.

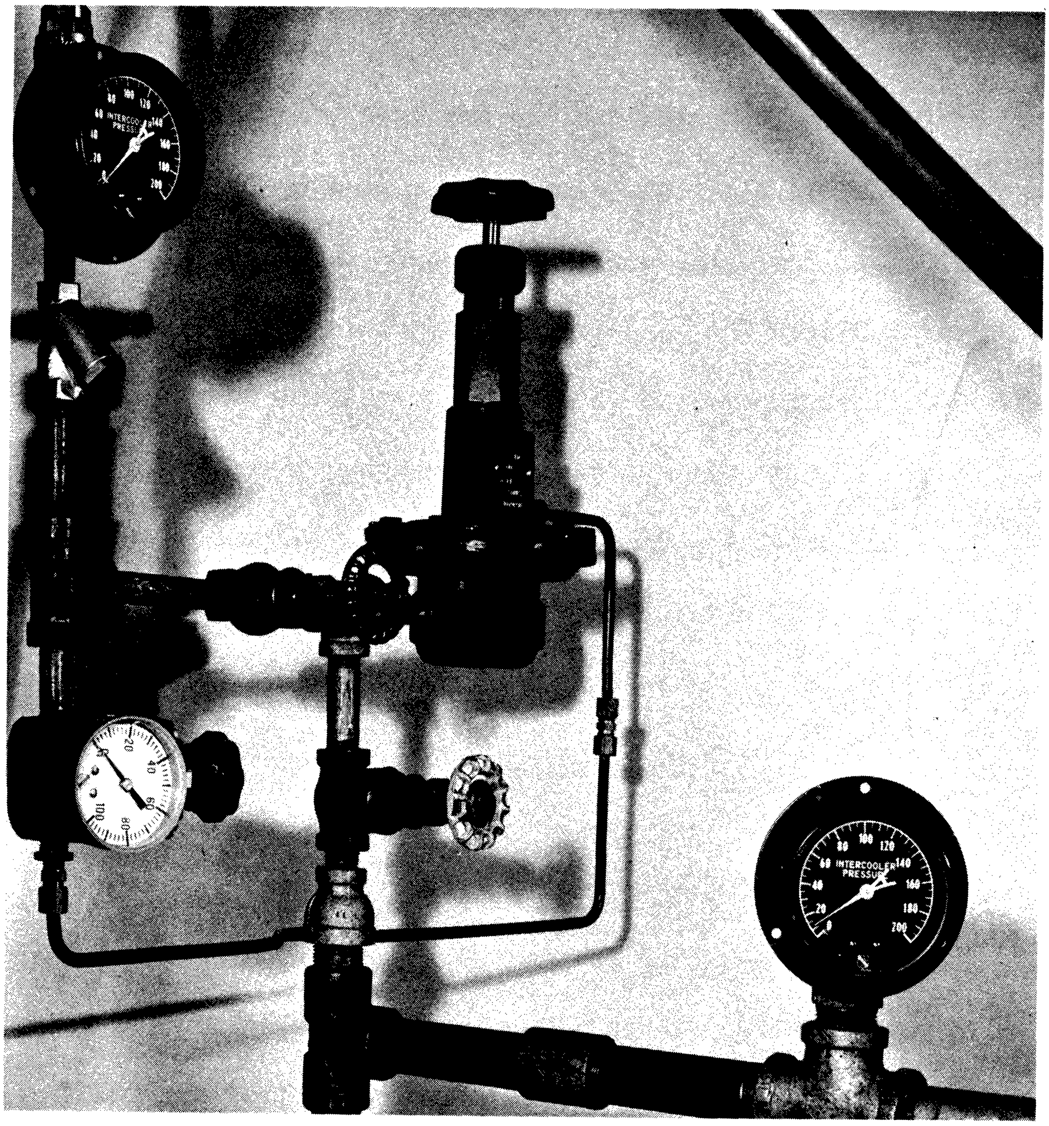


Plate II. Pressure regulator and relief valve.

A schematic of the control valve and low-pressure pneumatic servo system is illustrated in Figure 34. A Minarik Model TF-63SJ6 speed controller was used to regulate the speed of a 1/50-horsepower Bodine motor. This motor in turn rotated the 5.5-inch diameter cam platform on which was mounted a plastic card (see Plate IV). The variable vertical displacement of this card was converted by a Fisher Type 3561 motion transmitter into an equivalent air-pressure signal. Based upon this signal, a Fisher Series 3560 valve positioner would then appropriately position the Fisher Series 6S7A diaphragm control valve (1/4-inch micro-flute type). (See Plates IV and V.)

Four sets of 2-inch diameter piezometer rings were attached to the piping system. The first ring was attached to the downstream 2000-foot pipe about 15 inches upstream from the control valve (see Plate III); the second was located at the branching junction; the third was placed just downstream from the reservoir; and the fourth was positioned on the 400-foot pipe about three feet upstream from the needle valve. 100-inch reservoir-type differential manometers were then connected to these rings and used in the various system and instrumentation calibrations. The manometer fluid was mercury; by connecting the other leg of each manometer to the building constant-head tank (at a measured elevation of 42.2 feet above the datum), somewhat higher pressures were capable of being recorded than would have been possible otherwise.

Two Dynisco APT25-3C strain-gage type pressure transducers were used to monitor the transient pressures at the control valve and junction. The valve transducer was located about 12 inches upstream from the control valve and the junction transducer on an auxiliary 30-inch pipe suspended from the junction. (The length of this pipe was so negligible compared to

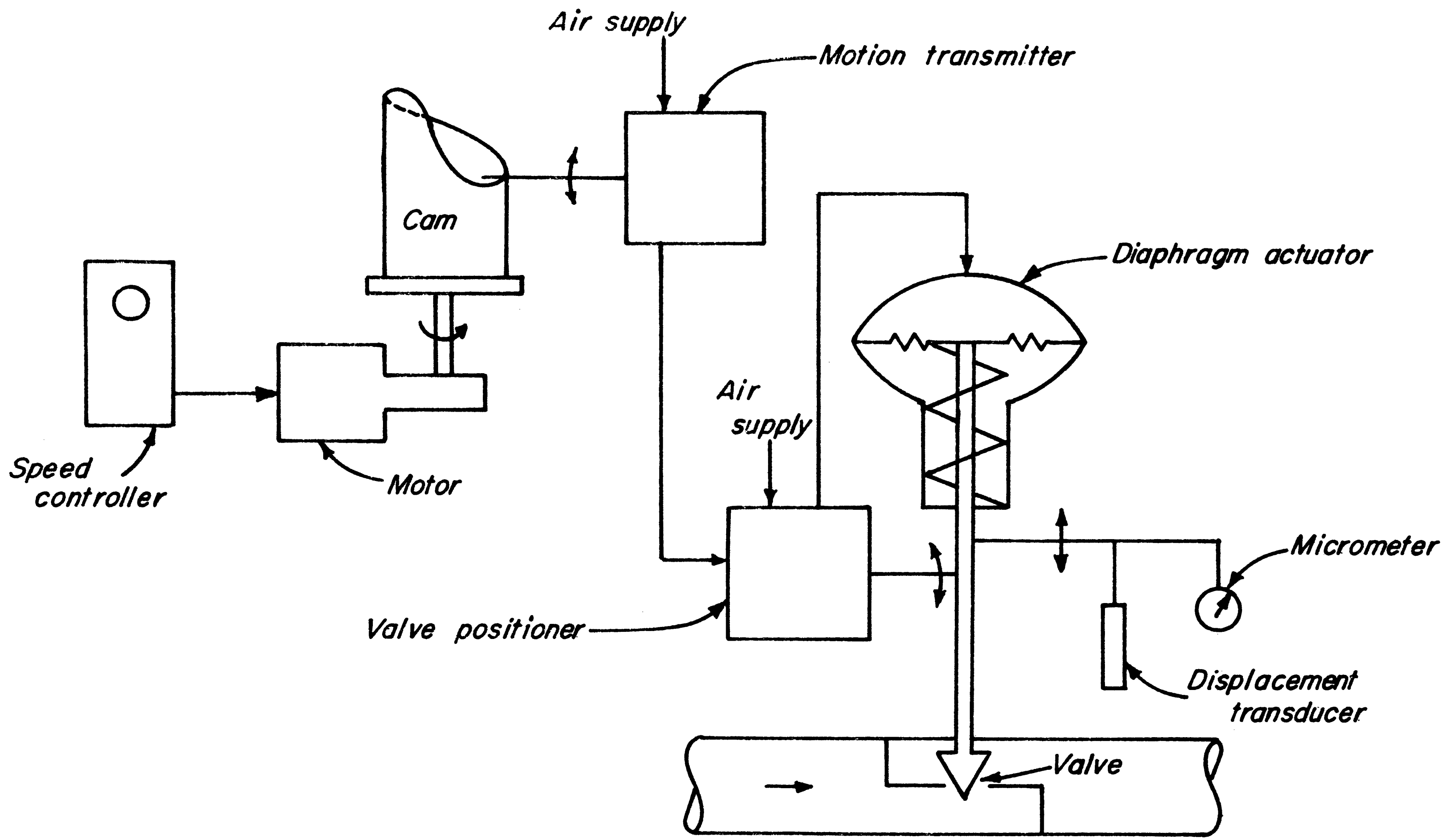


Figure 34. Schematic of control valve and servo system.

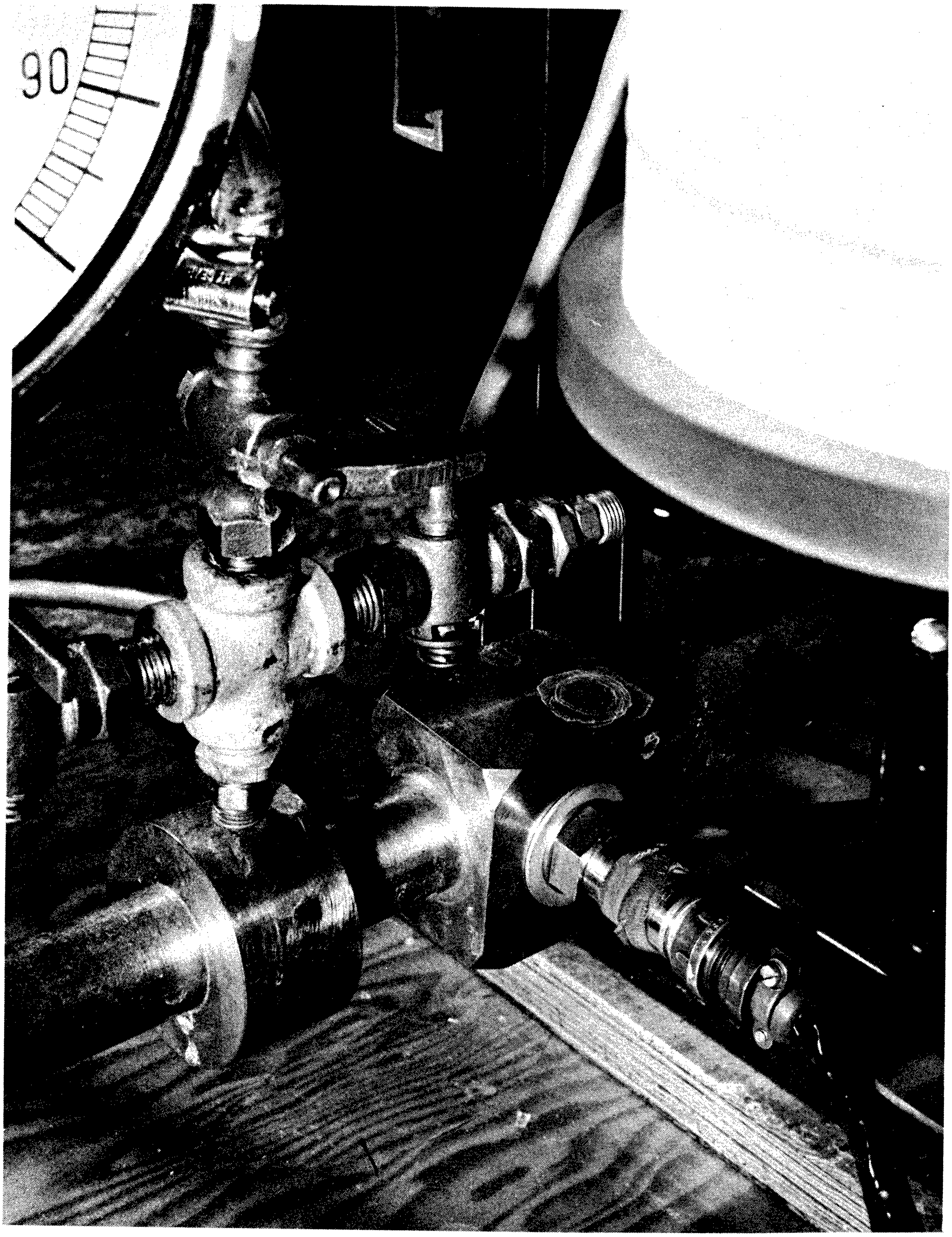


Plate III. Pressure transducer, transducer mounting block, and piezometer ring.

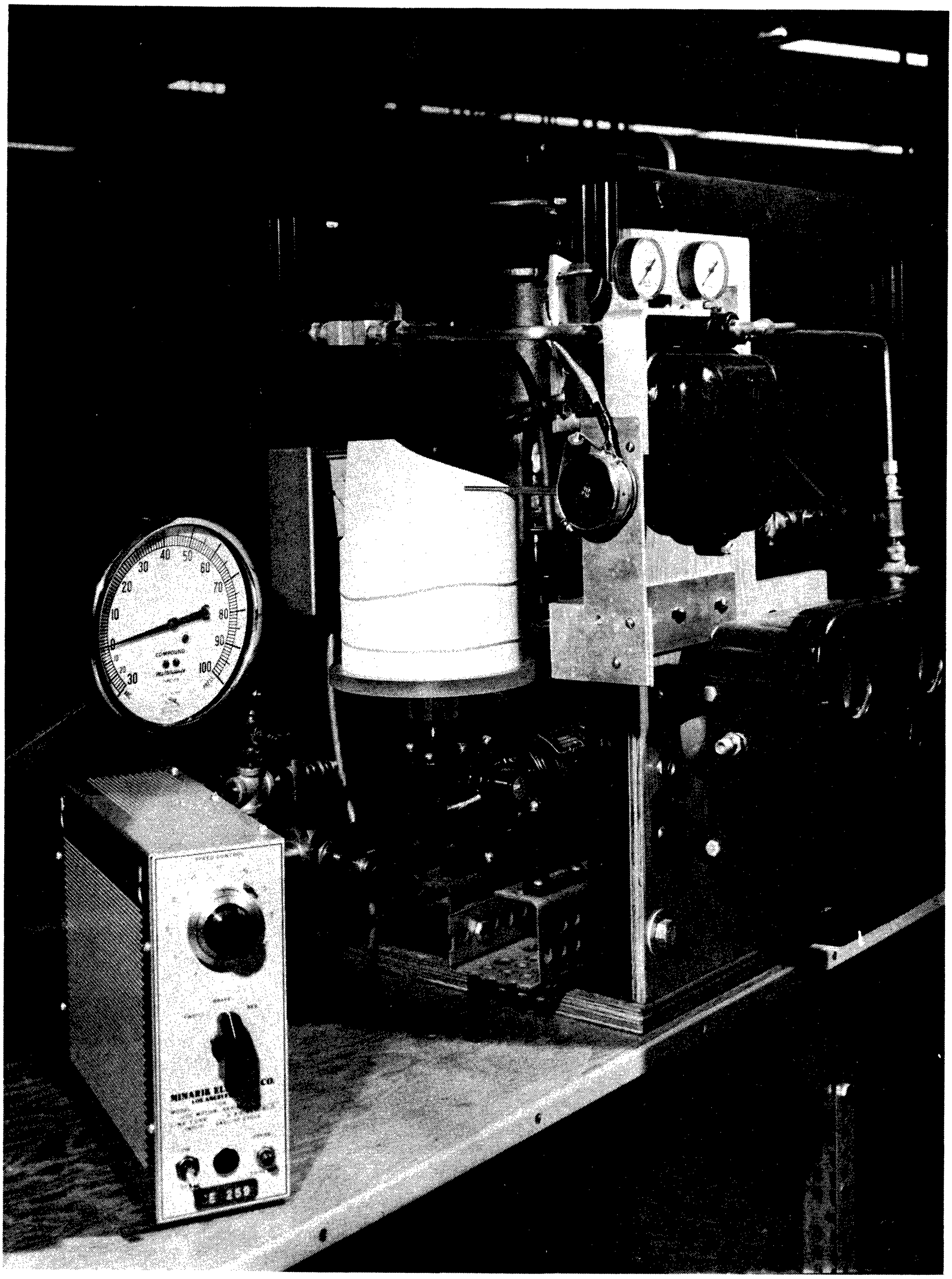


Plate IV. Speed controller, motor, plastic cam, motion transmitter, and valve positioner.

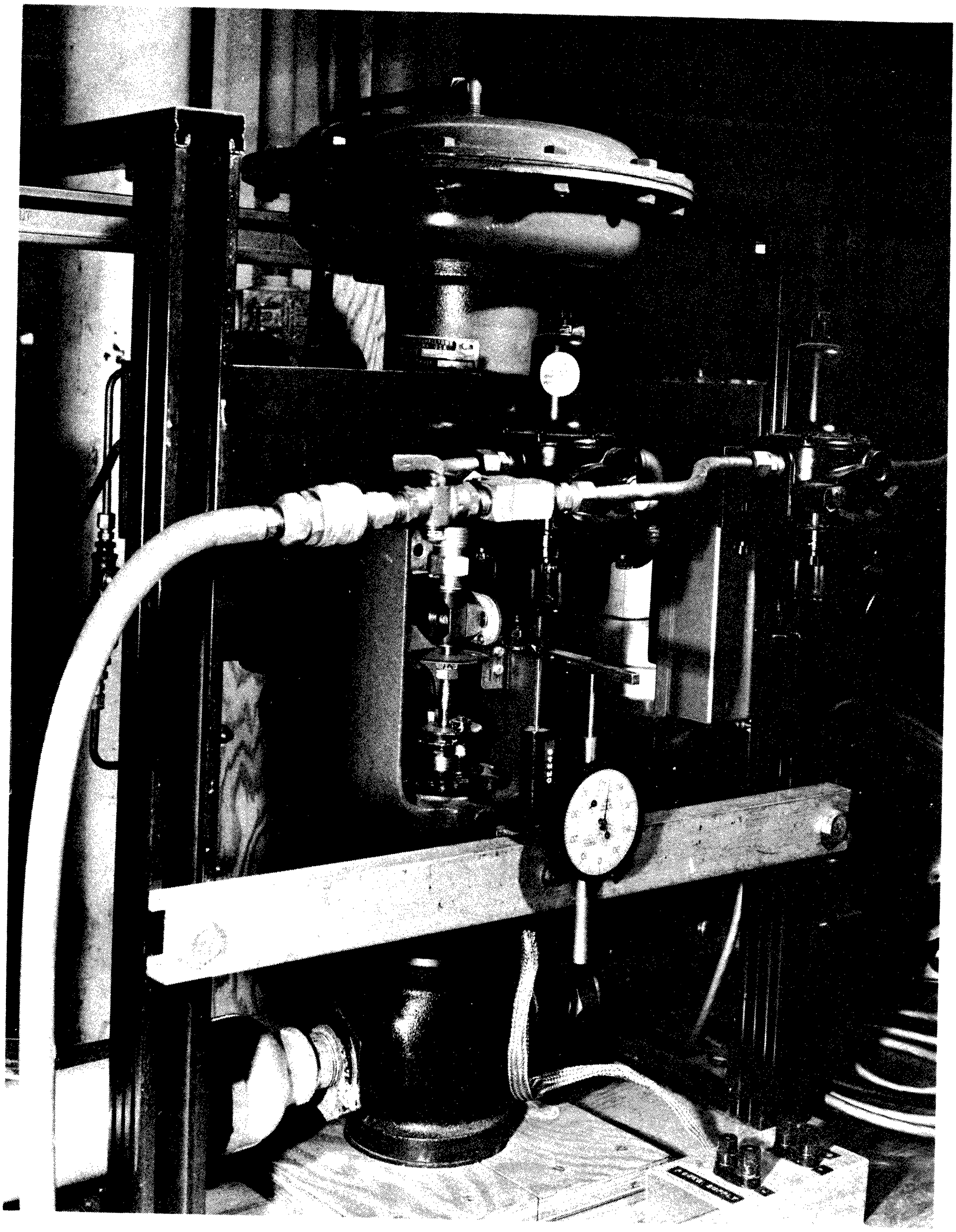


Plate V. Control valve, diaphragm actuator, micrometer, and displacement transducer.

the rest of the system that no subsequent buffering of the imposed transient condition resulted.) Each transducer was mounted nearly flush with the inside wall of the pipe (see Plate III). Sanborn Model 311 transducer amplifier-indicator units were used to supply the input voltage needed to activate the Wheatstone bridge circuit of each transducer and to amplify the response signal. The output of the amplifier was then recorded on photosensitive paper utilizing a Sanborn Model 4500 direct-writing optical oscillograph.

Finally, a micrometer and Sanborn Model 24DCDT-050 displacement transducer (LVDT) were mounted to measure and record, respectively, the vertical stem motion of the control valve (see Plate V). An H-Lab Model 6204A power supply was used to supply the input voltage to the transducer, and its signal was also recorded utilizing the Sanborn optical oscillograph.

5.2 System Calibration

Calibration data essential to establish the frictional-resistance characteristics of the piping system, the discharge coefficients of the control valve and orifice, and the relationship between plastic card height, control-valve stem position, and valve coefficient were obtained for the full range of anticipated system operation. The differential manometers were used to measure the heads, weigh-tank observations were used to determine the flow velocities, and a calibrated metal cam template and the micrometer were used to establish the essential valve and servo relationships. The flow-resistance relationships for the 2000-foot pipes and the 400-foot pipe are illustrated in Figures 35 and 36, respectively; the card height-stem position-valve coefficient relationships are illustrated in Figures 37 and 38. The orifice calibration data for the two cases presented in Chapter VI are summarized in Table IV. Since no apparent trend

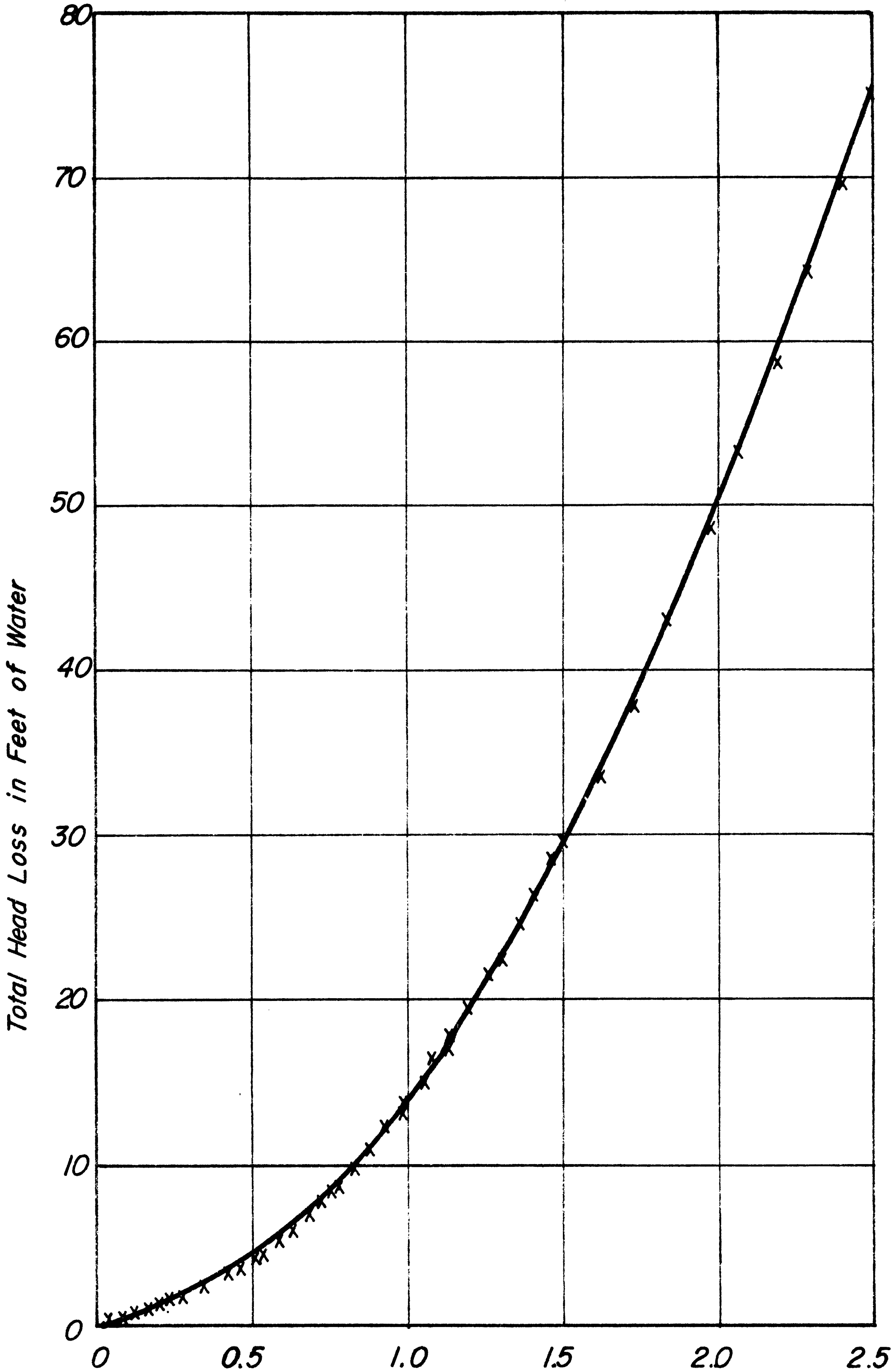


Figure 35. Flow-resistance relationship for the 2000-ft. copper pipe.

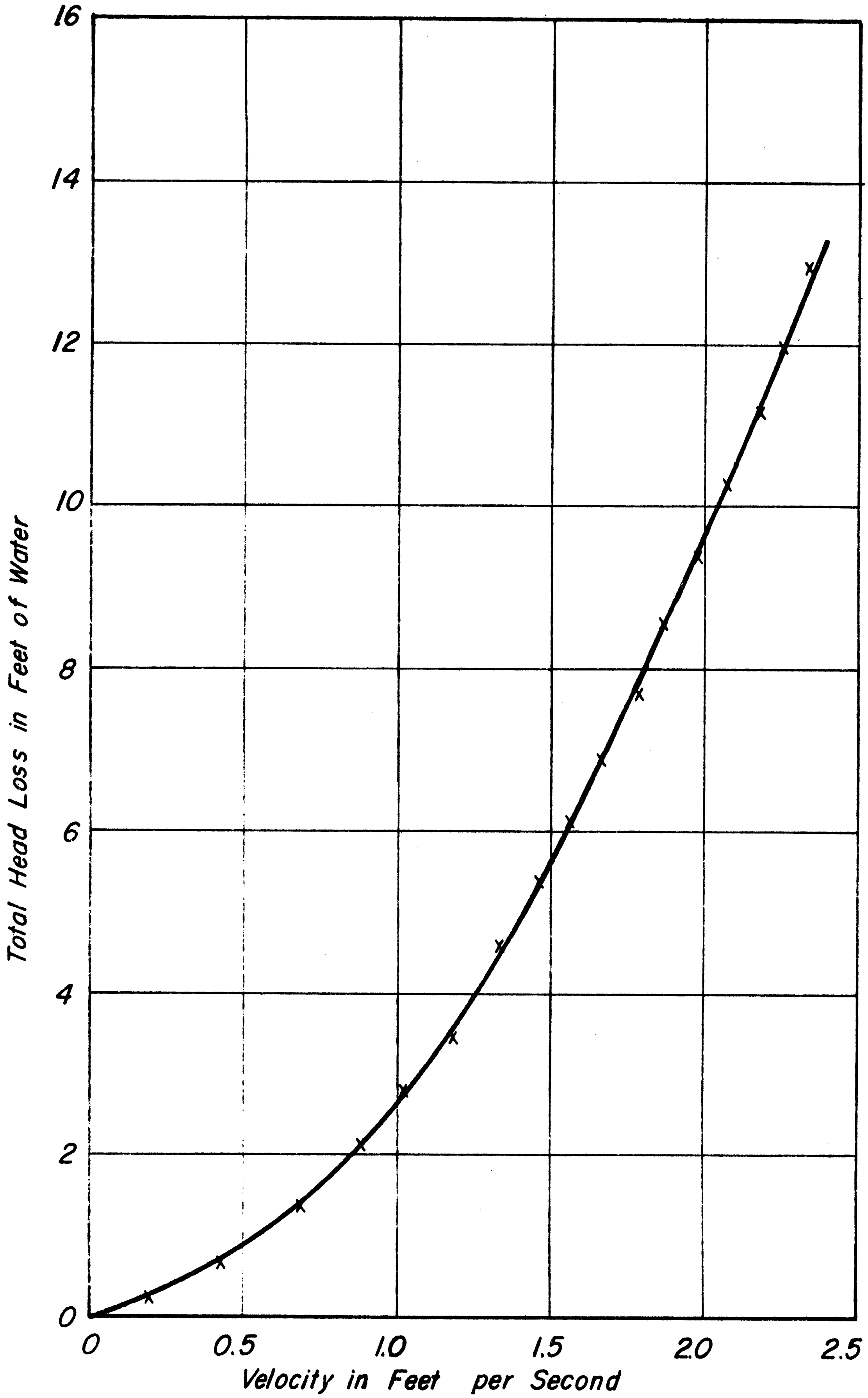


Figure 36. Flow-resistance relationship for the 400-ft. copper pipe.

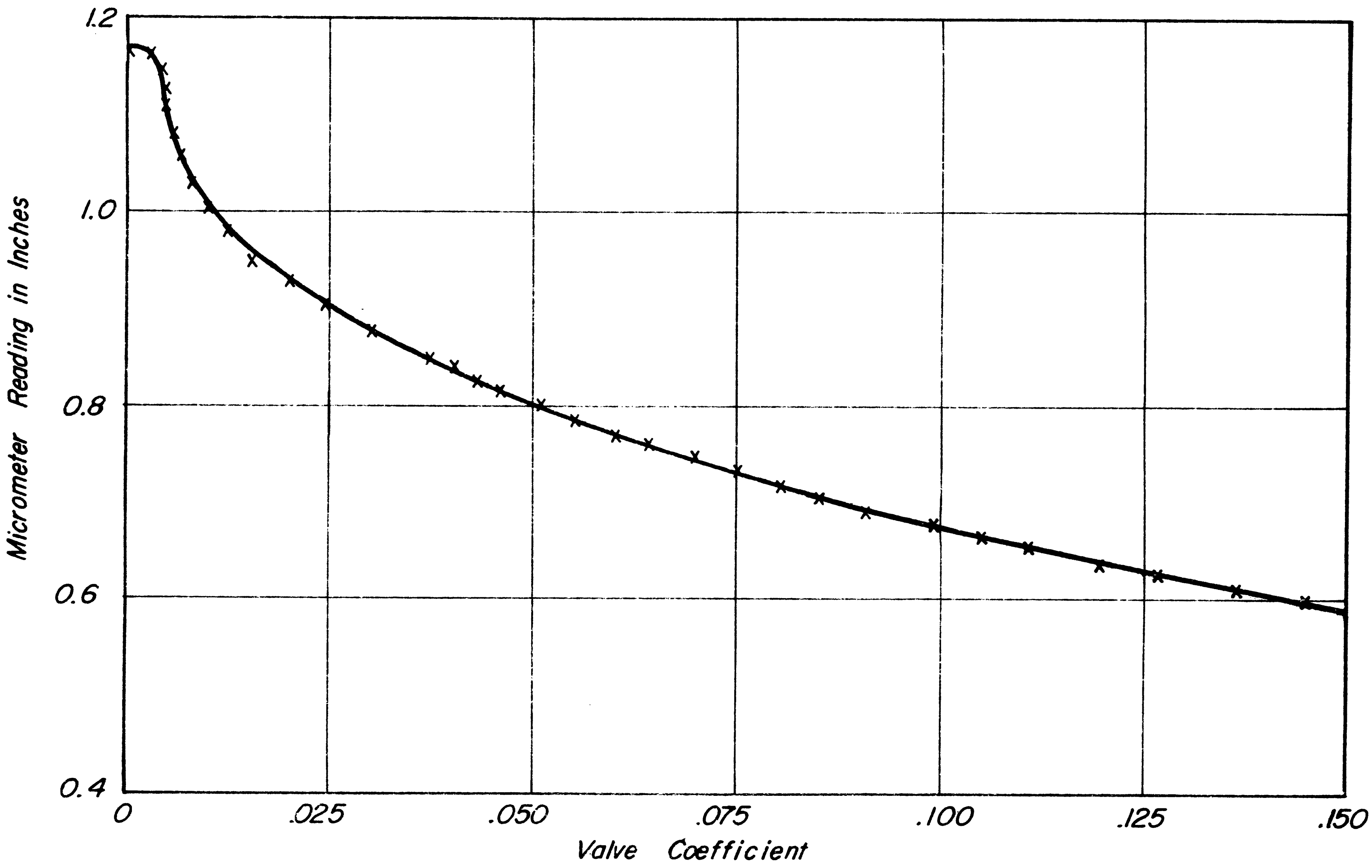


Figure 37. Relationship between valve stem position and valve coefficient.

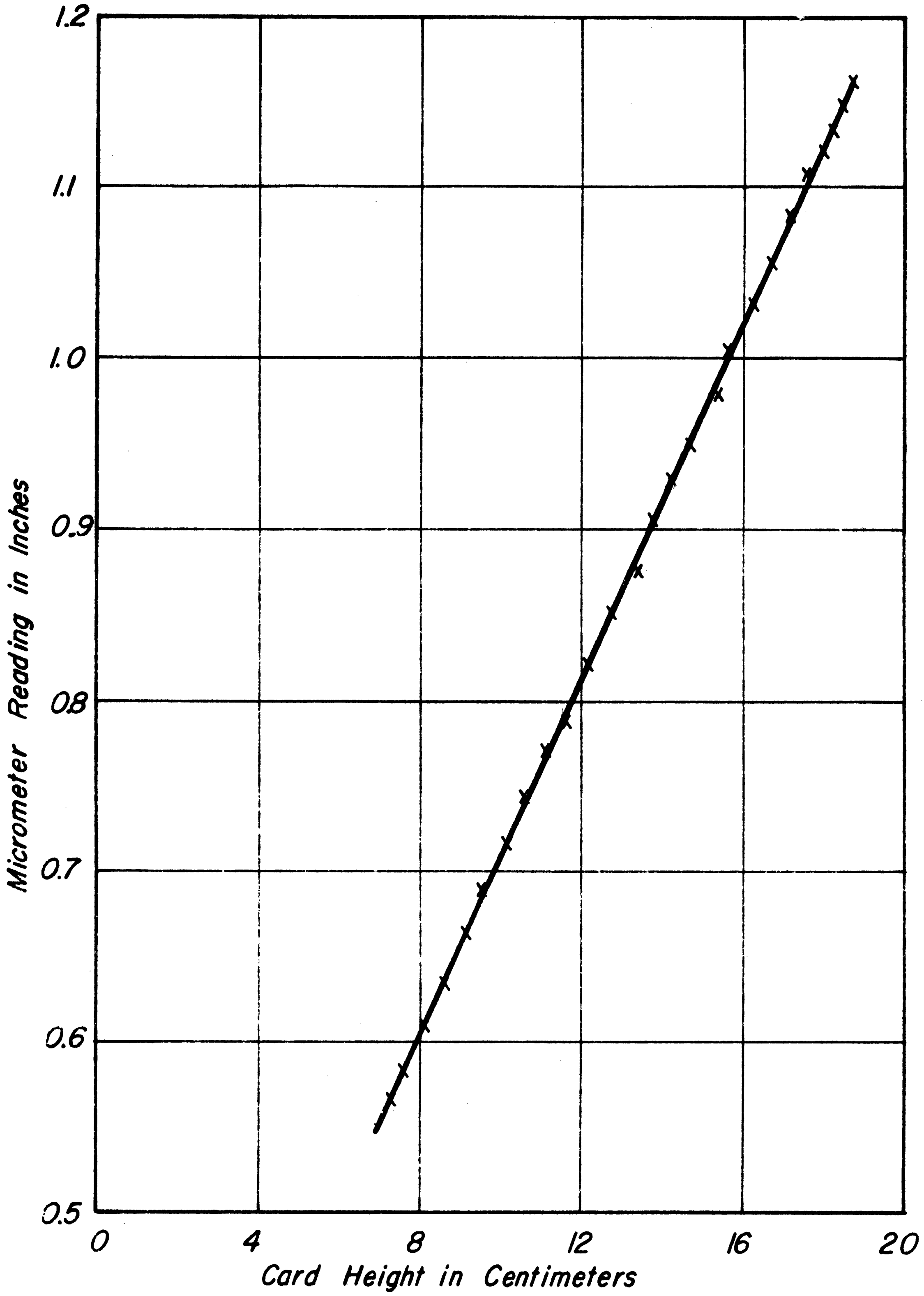


Figure 38. Relationship between valve stem position and card height.

exists in either set of data, the average values were used in the respective theoretical studies of the next chapter.

Earlier studies had established the wavespeed in the 400-foot pipe as being between 4250 and 4350 feet per second; values in this range

TABLE IV

RESUME OF ORIFICE CALIBRATION DATA

Case I			Case II		
Velocity ft./sec.	Head ft.	K_o ft.$^{1/2}$/sec.	Velocity ft./sec.	Head ft.	K_o ft.$^{1/2}$/sec.
1.834	82.14	.2024	.348	136.8	.0298
1.822	80.99	.2024	.338	136.4	.0290
1.812	80.83	.2015	.343	136.6	.0294
1.792	80.52	.1997	.341	136.4	.0292
1.786	79.68	.2001	.337	136.1	.0289
1.775	77.84	.2011	.341	135.0	.0294
1.750	75.38	.2016	.337	133.2	.0292
1.713	72.34	.2014	.330	130.8	.0289
1.686	69.09	.2029	.331	127.0	.0294
1.618	75.17	.2004	.326	122.6	.0294
1.588	61.13	.2031	.315	116.7	.0292
1.562	58.98	.2033	.314	113.4	.0295
1.510	56.73	.2004	.305	110.4	.0290
1.484	54.58	.2009	.302	106.3	.0293
1.457	52.44	.2012	.299	102.9	.0295
1.448	51.34	.2020	.297	100.5	.0296
	Ave. =	.2015		Ave. =	.0293

were used accordingly. Previous evidence had indicated that the wavespeed in the 2000-foot pipes was somewhat pressure-dependent, probably due to the presence of small amounts of free gas in the water and to the fact that the relatively thin-walled pipe was slightly out-of-round at lower pressures. By timing the advance of a small disturbance from the valve transducer to the junction transducer, at various static pressure levels, this relationship was established (as illustrated in Figure 39), and was used as a guide in the selection of wavespeed values for the theoretical studies reported in Chapter VI.

5.3 Experimental Procedures

The calibration data of the last section was entered into the pertinent computer programs of Chapter VI in tabular form; essential information was then computed from this tabular data as necessary utilizing the technique of parabolic interpolation.[29,31] Program output included computer (Calcomp) plots of the theoretical head versus time relationships at the valve and junction, the control-valve stem position versus time relationship, and a drawing of the plastic card profile. This profile was traced on a sheet of plastic and cut with a scissors to the required shape. A short lead-in section was provided to allow the turntable to accelerate to its uniform rotational speed before the control action was to be initiated; a similar run-out section was also provided.

After an elaborate and systematic procedure to bleed the entire system and manometers of entrapped air was completed, the signals from the two pressure transducers were then calibrated utilizing the respective manometers, and conveniently scaled using the gain and zero

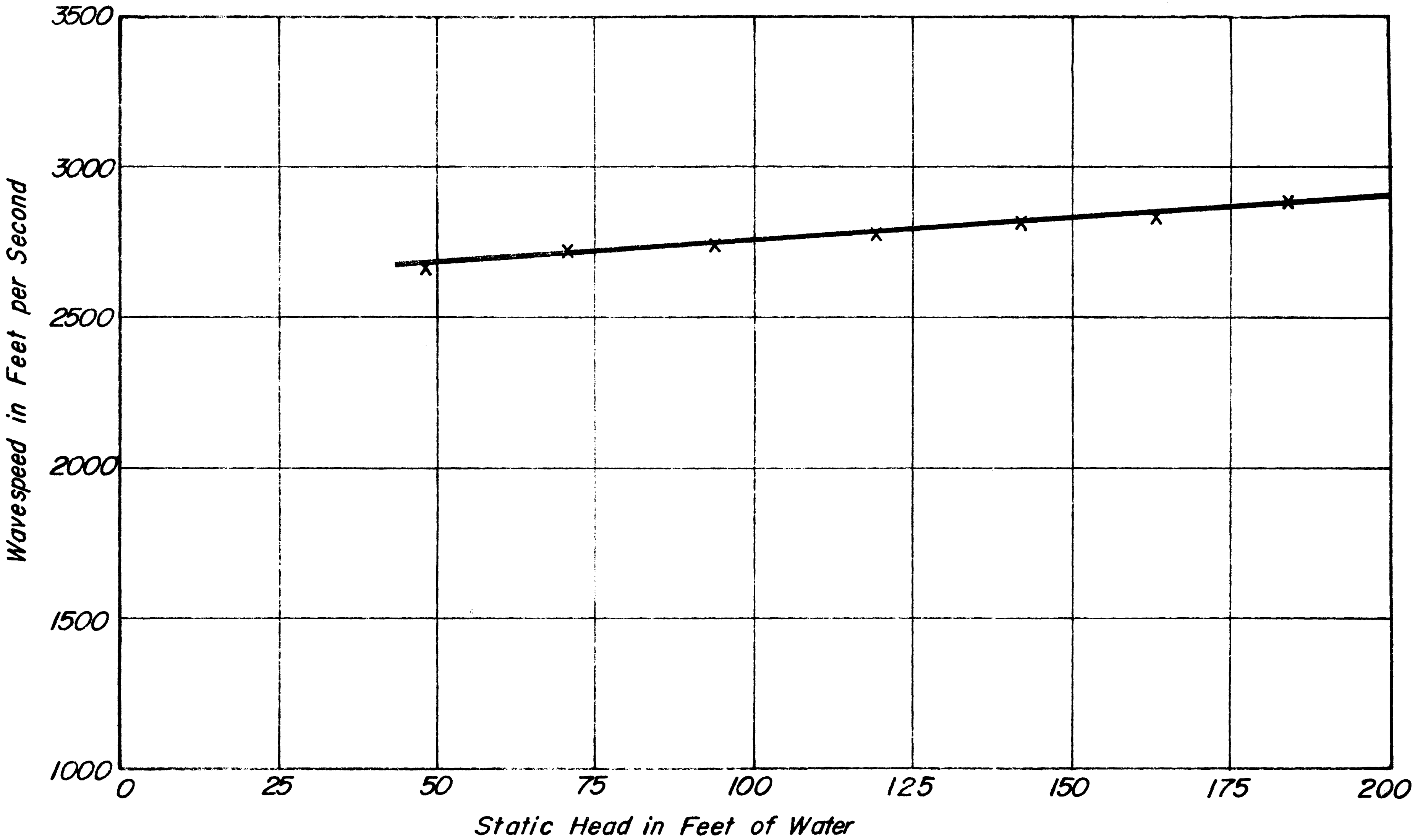


Figure 39. Relationship between wavespeed and head for the 2000-ft. copper pipe.

adjustments of the recorder; the signal from the displacement transducer was calibrated using the micrometer and similarly scaled. The plastic card was then mounted on the turntable, and the speed controller was adjusted to establish the proper time of valve operation. After all instrumentation had been calibrated and checked, the setting of the pressure relief valve was modified, if necessary, to establish the desired value of reservoir head. (In every study of this investigation, a value of 140 feet - corresponding to 93.3 inches manometer reading -- was used.)

The speed controller would then be activated, causing the valve to initiate the transient condition. After the cessation of the transient, the recorded data would then be inspected. Of particular interest was the control-valve stem position versus time relationship. Since the card height versus stem position relationship was not altogether stable, and was also unable to account for the dynamic lag in the servo equipment, subsequent modification of the plastic card profile usually was necessary. The above procedure would then be repeated until the cam profile had been modified such that the desired stem position versus time relationship was truly obtained. The head versus time relationships thus obtained are the ones reported in the several studies of Chapter VI. After the desired valve closure relationship was obtained, several more runs were usually made, and in every case the results were highly repeatable.

VI. COMPARISON AND DISCUSSION OF THEORETICAL AND EXPERIMENTAL RESULTS

This study of valve stroking culminates in a comparison between the theoretical results derived from the appropriate mathematical models and the experimental laboratory results obtained in the system described in Chapter V. Experimental verification of the pertinent valve-stroking theory has been obtained and is presented for a simple system, a series system, and a passive branching system.

6.1 Simple System

Since the recognition that the minimum time for the valve-stroking control of a simple system is 2L/a, not the previously reported 4L/a, is one of the new concepts presented in this investigation, experimental verification was deemed to be a desirable addition to the previously reported confirming laboratory studies.

Because of the presence of the upstream pressure transducer at the approximate midpoint of the 4030-foot pipe (the junction of the basic system), it was decided to investigate a closure situation (Case I) and an opening (Case II), with t_f for each being exactly 3L/a. (Recall from the discussion of Section 3.1.3 that in the closure case the maximum value of head created in the system should then be developed at the midpoint.)

The essential elements of the computer analysis are identical to those discussed in Section 3.1.3. Initial and final steady-uniform velocities were arbitrarily selected for each study, and the value of H_m necessary for t_f to be equal to 3L/a was determined by utilizing the procedure described in Appendix C. Pertinent system parameters for each case are summarized in Table V.

TABLE V

RESUME OF PERTINENT DATA -- SIMPLE SYSTEM

	Case I	Case II
t_f (sec.)	4.44	4.50
a (ft./sec.)	2725	2685
V_o "	1.10	0.10
V_f "	0.10	1.00
H_r (ft.)	140.00	140.00
H_{j_o} "	123.45	139.51
H_{j_f} "	139.51	126.30
H_o "	106.90	139.02
H_f "	139.02	112.60
H_m "	212.68	53.00
H'_m "	176.34	96.50
Extreme H in System (ft.)	176.34	86.60

A comparison between the theoretically predicted and experimentally obtained results for Case I are presented in Figure 40. Again for convenience, the dimensionless tau-time relationship of the valve is illustrated. Note that during the central L/a seconds the value was required to open slightly, a requirement readily attainable on the laboratory control system. Considering the assumptions implicit in the development of the basic theory (especially the assumption of a one-dimensional velocity uniformly distributed across the cross-section, and the assumptions relative to the instantaneous values of wall shear stress and valve coefficient), the probability of slight experimental error in the various system calibrations, and that somewhat inexact values of wavespeed were utilized in the theoretical analysis, the

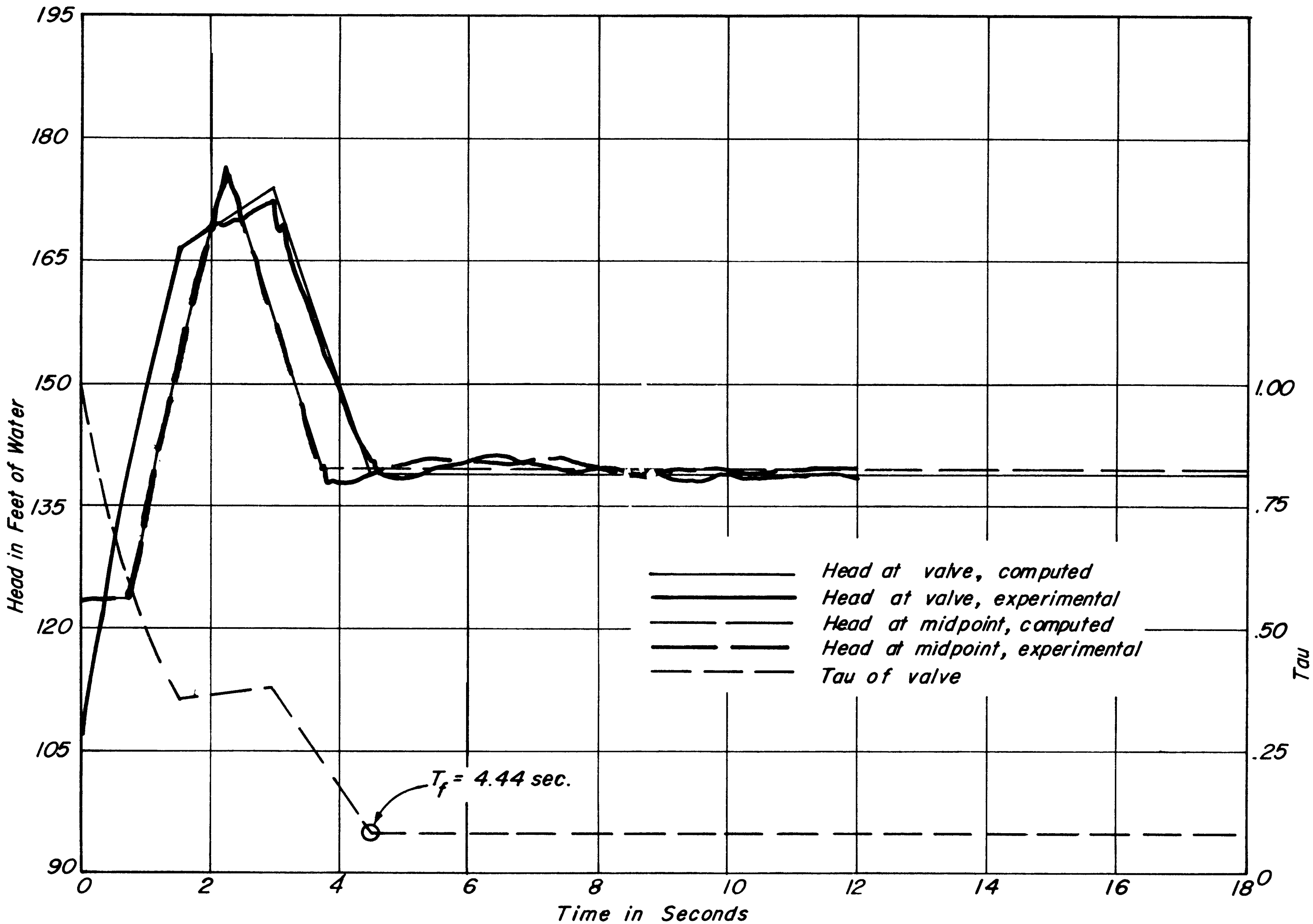


Figure 40. System variables versus time. Simple system. Case I -- closure in 3L/a.

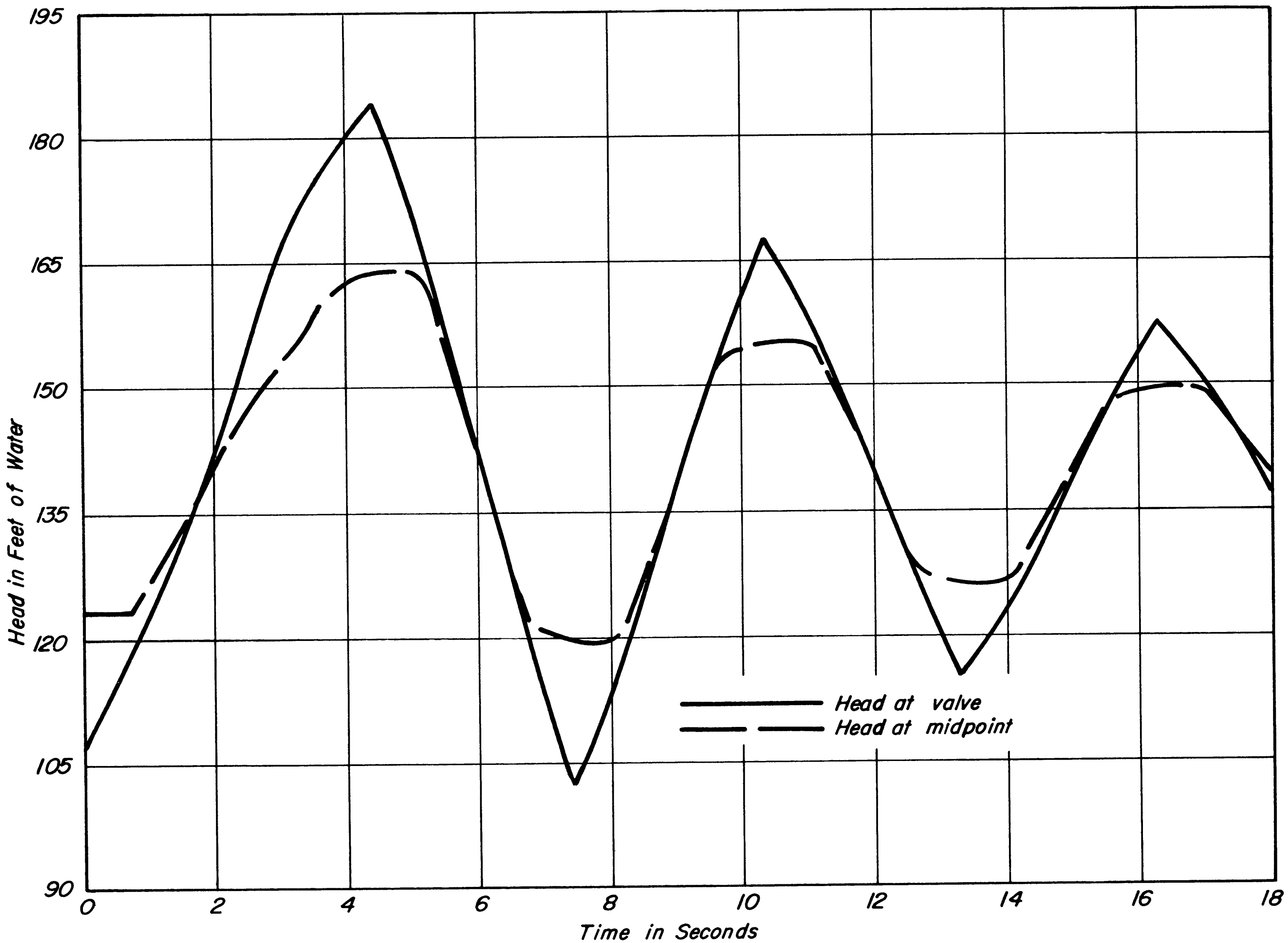


Figure 41. System variables versus time. Simple system. Computed results of a linear tau-time closure to compare with Case I.

results are considered to be excellent. Because of the response characteristics of the transducers and the opportunity for slight error in the transducer calibration and scaling operations, the experimental data is estimated to be accurate within 0.50 feet. Computer results of a comparable linear valve closure in the same time are illustrated in Figure 41; Figures 40 and 41 provide a most remarkable comparison of the results of the two closure strategies.

Comparison between the theoretical and experimental results for Case II are presented in Figure 42. Again, good agreement is obtained for much of the duration of the transient condition; only at the end do the experimentally obtained pressures at the valve and mid-point overshoot the final steady-state values. Since the same computer program was utilized in the theoretical analyses for both Case I and Case II, and the experimental studies were conducted concurrently, the discrepancy between the predicted and observed results can not be attributed to program or system calibration errors.

There is, however, considerable evidence to suggest that the observed discrepancy is due to the effect of gas liberation during the time that system pressures are substantially below the initial static level. The net effect of such liberation is a lower value of wavespeed;[31] a lesser average wavespeed persisting during the latter phase of the transient would produce the observed conditions. Contractor[5] observed and reported the same phenomenon accompanying a sudden reduction in pressure conditions in an earlier study.

Because of the limitations on time available for the laboratory investigations, the probable difficulty in readily developing an improved mathematical model of the phenomenon, and the fact that such an effort would have been decidedly tangential to the stated objectives of this

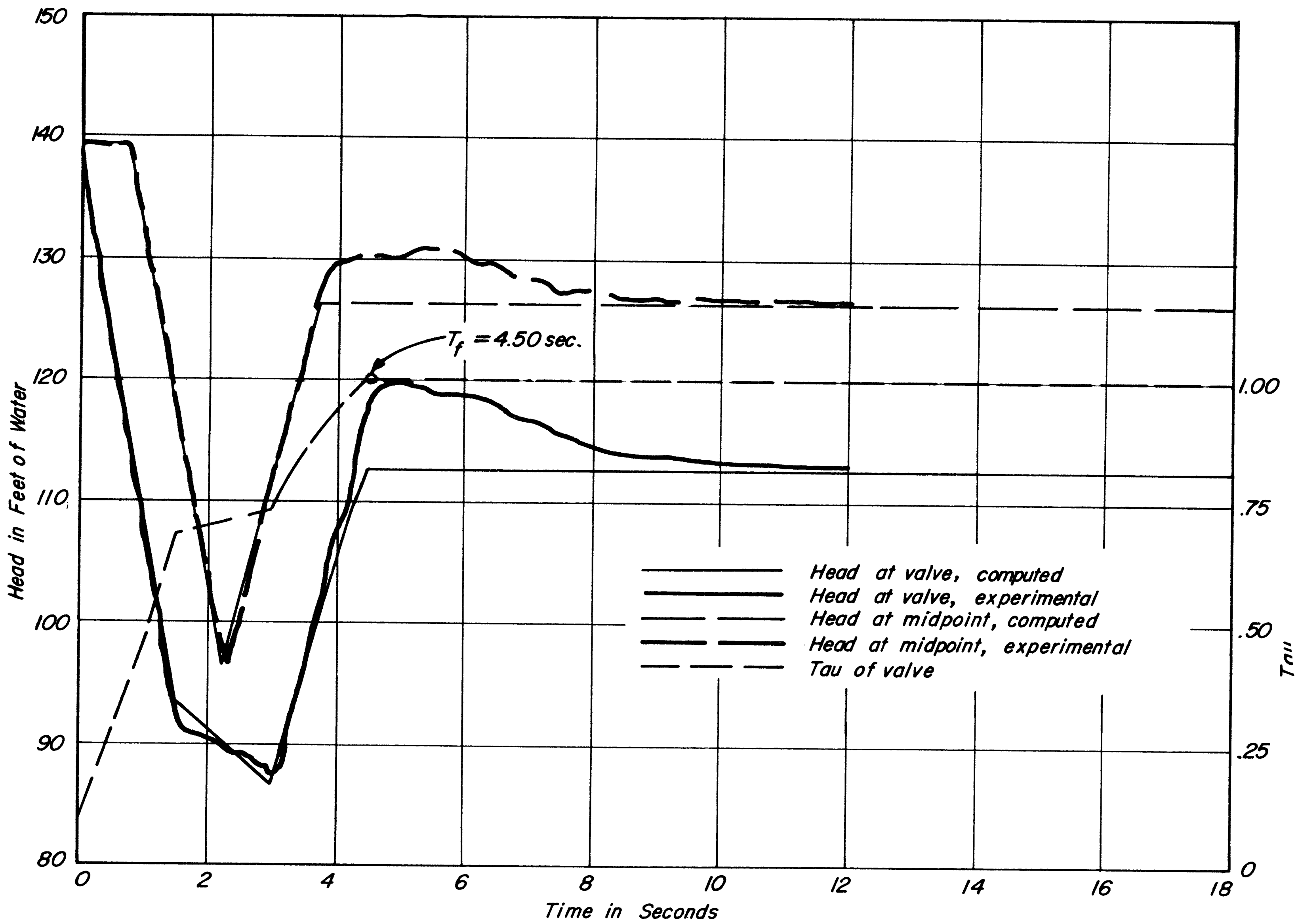


Figure 42. System variables versus time. Simple system. Case II -- opening in 3L/a.

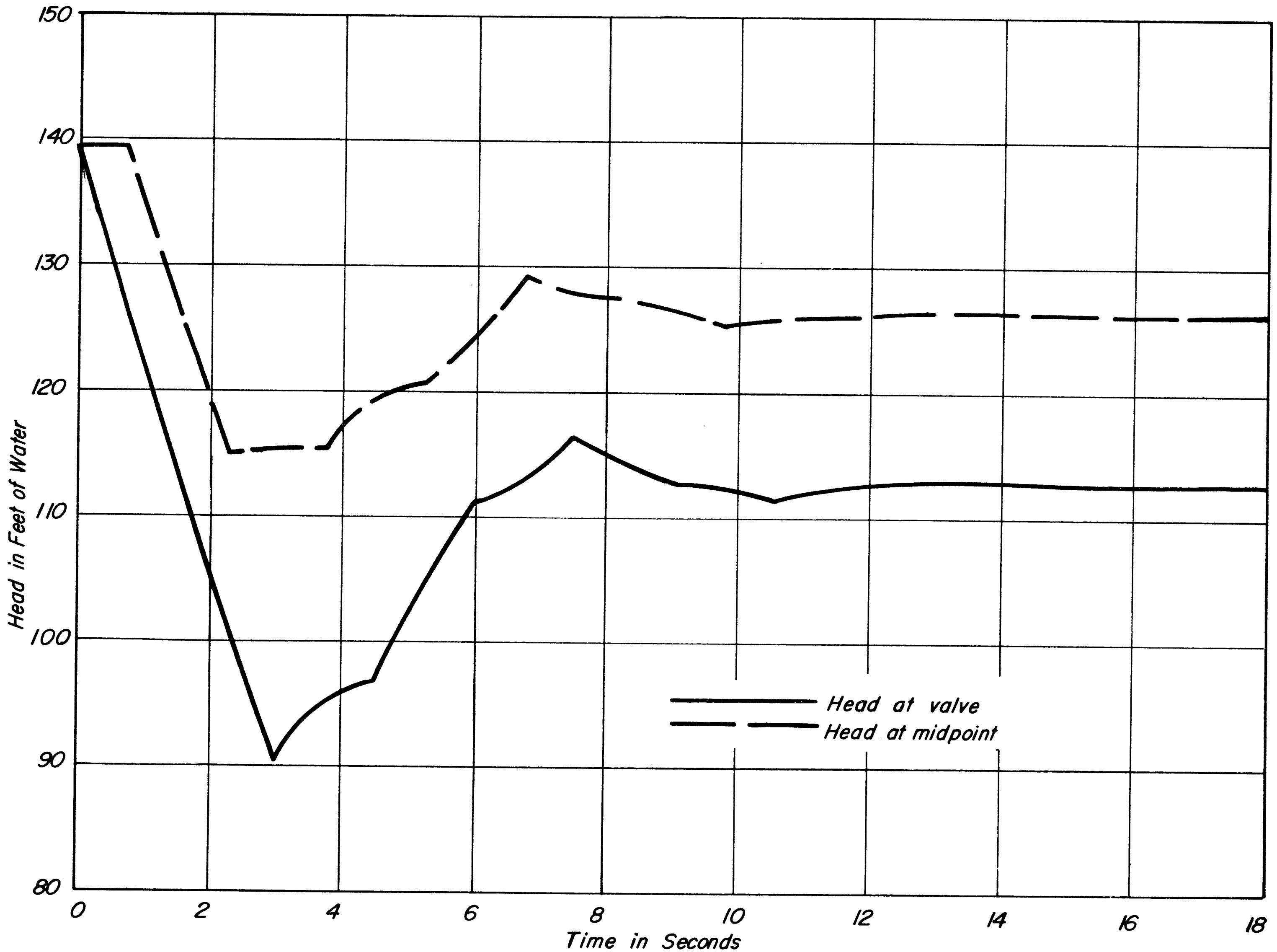


Figure 43. System variables versus time. Simple system. Computed results of a linear tau-time opening to compare with Case II.

investigation this aspect was not pursued further. One should recognize, however, that the minimum system pressures were held within established limits and that, in the case of valve openings, residual transient fluctuations are usually of negligible magnitude and duration.

6.2 Series System*

Experimental verification of the existing technique for valve-stroking control of a series system has also been obtained. The essential elements of the computer analysis are identical to those discussed in Section 4.1.1. Initial and final steady-uniform velocities in the upstream (400-foot) pipe were arbitrarily selected for each study, and the prescribed values of H_{m_j} were similarly established; velocities in the downstream (4000-foot) pipe and all steady-state values of piezometric head were then obtained from the appropriate steady-state equations. The ratios of length to wavespeed for each pipe were such that N_1 was selected to be 1 and N_2 to be 16. Other pertinent system parameters for each case are summarized in Table VI.

Comparisons between the theoretically predicted and experimentally obtained results for these two cases are presented in Figures 44 and 45, respectively. Again, the results are considered to be excellent. The somewhat better agreement obtained in Case II motivated the use of the Case II wavespeed values in the theoretical analyses reported in the next section.

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* The theoretical and experimental studies of this section are the work of Mr. Joel L. Caves, and the author is indebted to him for these results.

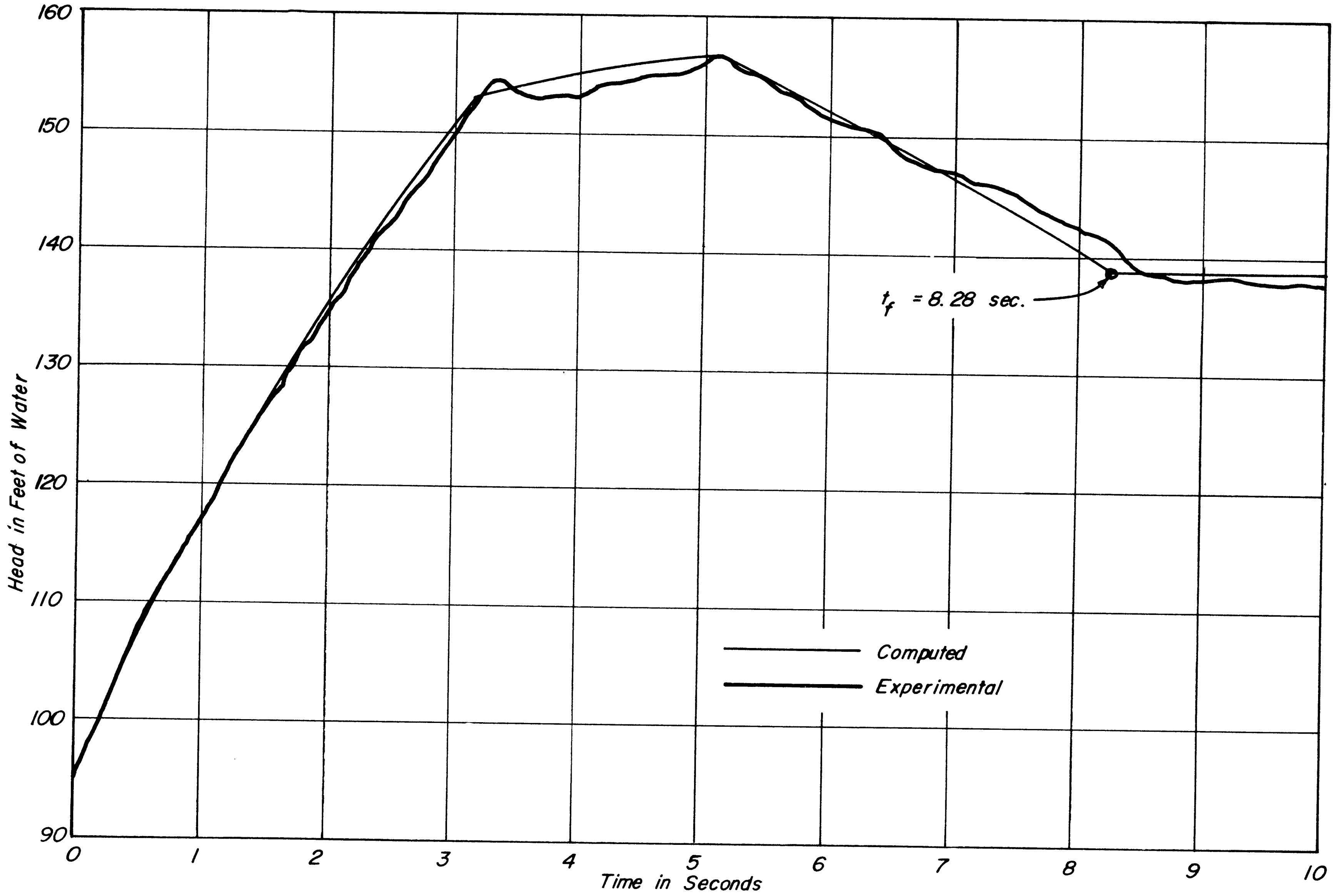

Figure 44. Head at valve versus time. Series system -- Case I.

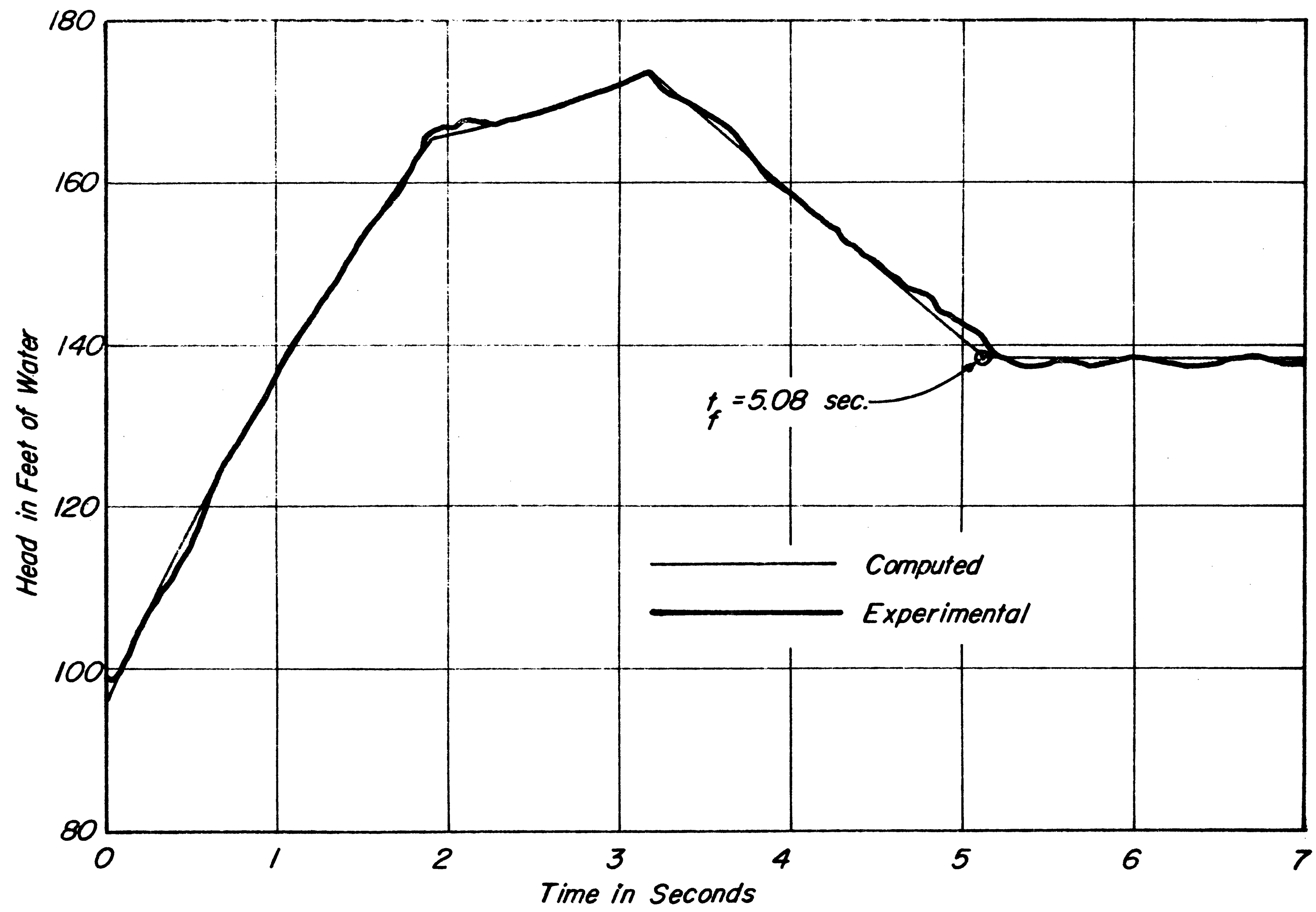

Figure 45. Head at valve versus time. Series system -- Case II.

TABLE VI

RESUME OF PERTINENT DATA -- SERIES SYSTEM

		Case I	Case II
t_f	(sec.)	8.28	5.08
a_1	(ft./sec.)	4344	4262
a_2	"	2709	2664
V_{o_1}	"	1.10	1.10
V_{f_1}	"	0.10	0.10
V_{o_2}	"	1.22	1.22
V_{f_2}	"	.11	.11
H_r	(ft.)	140.0	140.0
H_o	"	95.3	95.3
H_f	"	138.7	138.7
H_{m_j}	"	141.5	145.5

6.3 Branching System

Since the introduction of the passive valve-stroking concept is one of the original contributions of this investigation, experimental verification was also desired.

The essential elements of the computer analysis are identical to those discussed in Section 4.3, except that the fixed-orifice boundary-condition equations (Equations (15a) and (15b), modified) must be utilized at the right-end boundary of pipe 3 (the 400-foot pipe). Initial and final steady-uniform velocities in the upstream (2000-foot) pipe were arbitrarily selected for each study, and the prescribed values of H_{m_j} were similarly established. Both velocities and all steady-state values of piezometric head were then obtained for pipe 3 by solving the appropriate steady-state equations utilizing the method of interval halving.

(For references and a brief discussion of this method, see Appendix C). All steady-state conditions in pipe 2 were then directly determinable. N_1 and N_2 were selected to be 8, N_3 to be 1. Pertinent system parameters for each case are summarized in Table VII.

Comparisons between the theoretically predicted and experimentally obtained results for these two cases are presented in Figures 46 and 48, respectively. Computer results of the comparable linear valve closures are illustrated in Figures 47 and 49, respectively, and again provide interesting comparisons of the results of the two closure strategies. In each case, the slight discrepancy between the theoretical and experimental initial conditions results from the frictional-resistance calibrations. Since the time lapse between system calibration and the subsequent experimental studies was frequently several days, changes in ambient laboratory temperature within that period of time undoubtedly contributed to changes in the resistance properties of the system.

Nevertheless, the results are considered to constitute excellent verification of the passive valve-stroking concept. Considering the significant viscous effects encountered in this system, particularly in the high-throughflow situation of Case I (eighty-odd feet of loss sustained in pipe 1 alone), these results parenthetically demonstrate the intrinsic accuracy of the characteristic equations in the analysis of the transient phenomenon in viscous liquid piping systems.

TABLE VII

RESUME OF PERTINENT DATA -- BRANCHING SYSTEM

		Case I	Case II
t_f	(sec.)	5.72	7.13
a_1	(ft./sec.)	2664	2664
a_2	"	2664	2664
a_3	"	4262	4262
V_{o_1}	"	2.60	1.70
V_{f_1}	"	2.10	.50
V_{o_2}	"	.96	1.37
V_{f_2}	"	.13	.12
V_{o_3}	"	1.48	.30
V_{f_3}	"	1.78	.34
H_r	(ft.)	140.00	140.00
H_{j_o}	"	59.05	103.80
H_{j_f}	"	85.60	135.98
H_{2_o}	"	46.31	78.80
H_{2_f}	"	84.95	135.35
H_{3_o}	"	53.64	105.35
H_{3f}	"	77.93	135.46
H_{m_j}	"	92.00	148.00

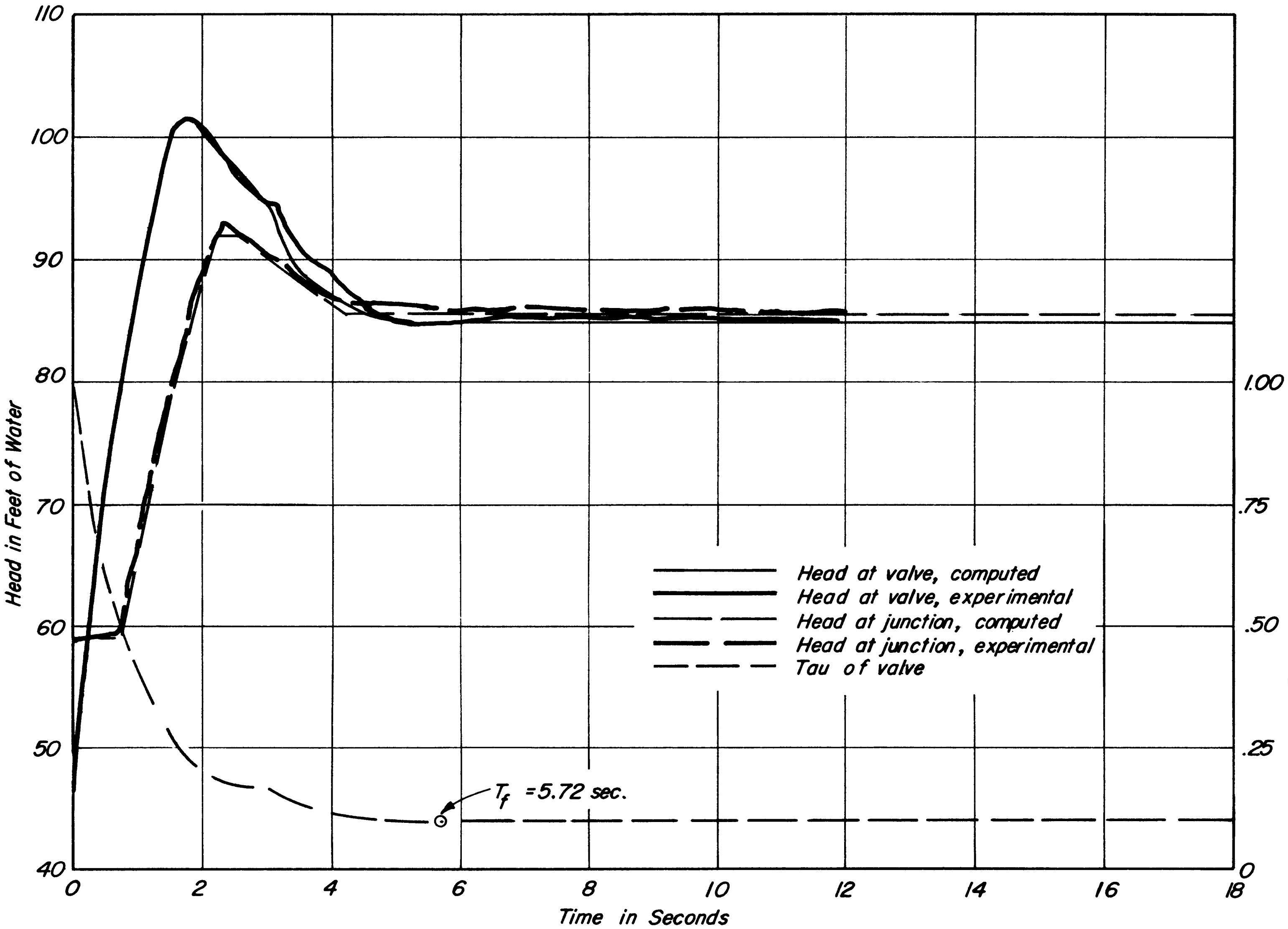

Figure 46. System variables versus time. Branching system -- Case I.

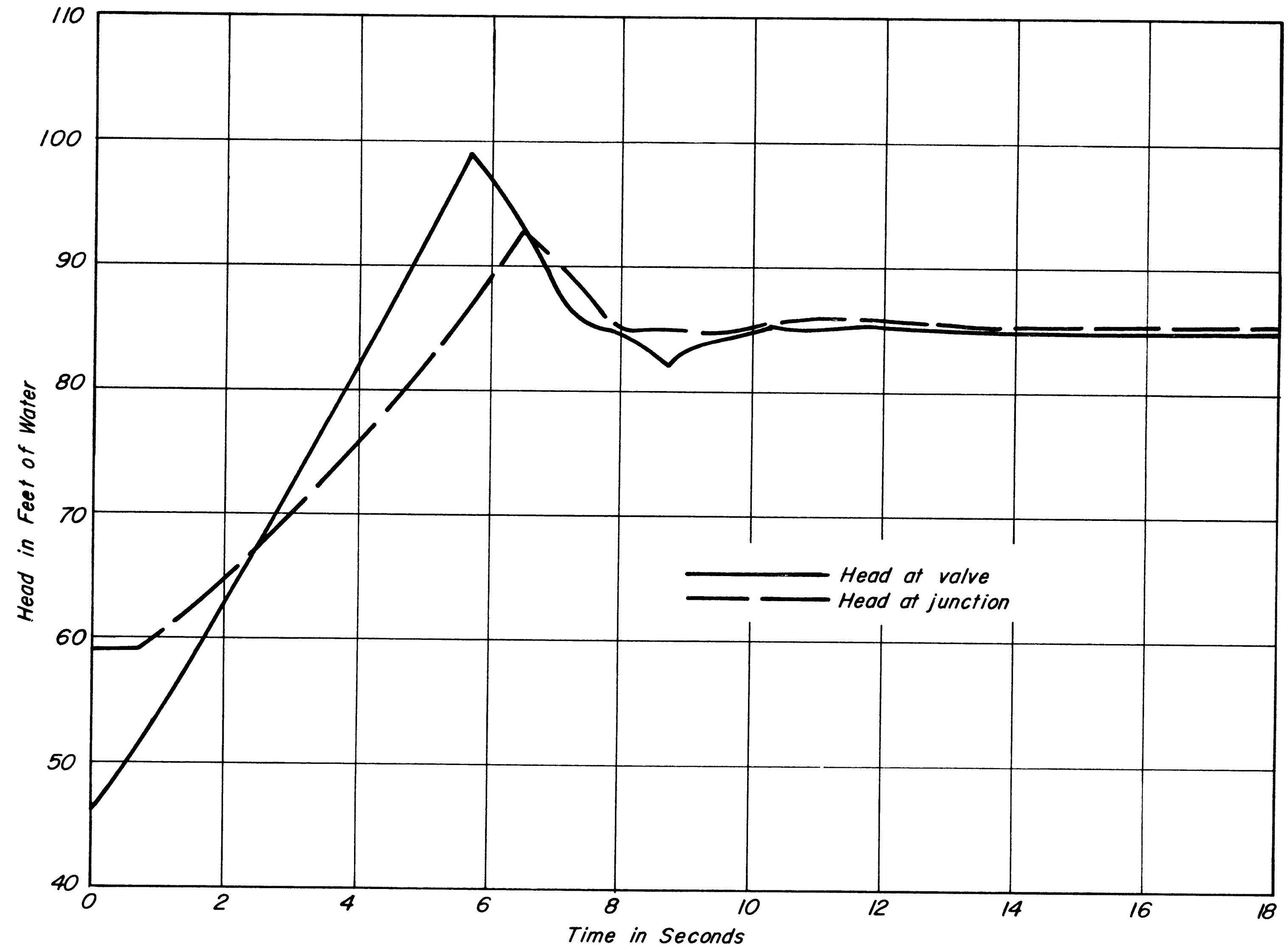

Figure 47. System variables versus time. Branching system. Computed results of a linear tau-time closure to compare with Case I.

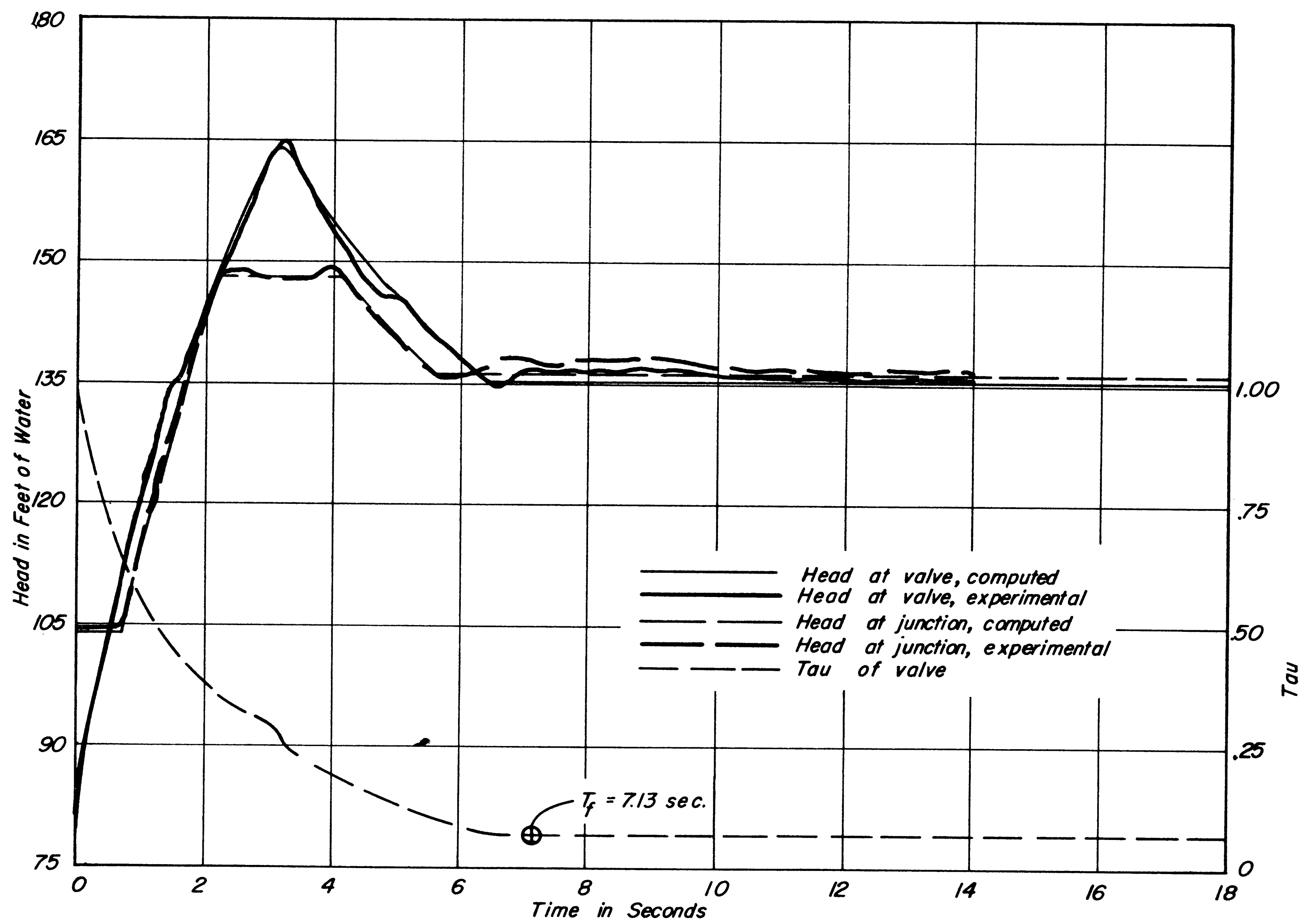

Figure 48. System variables versus time. Branching system -- Case II.

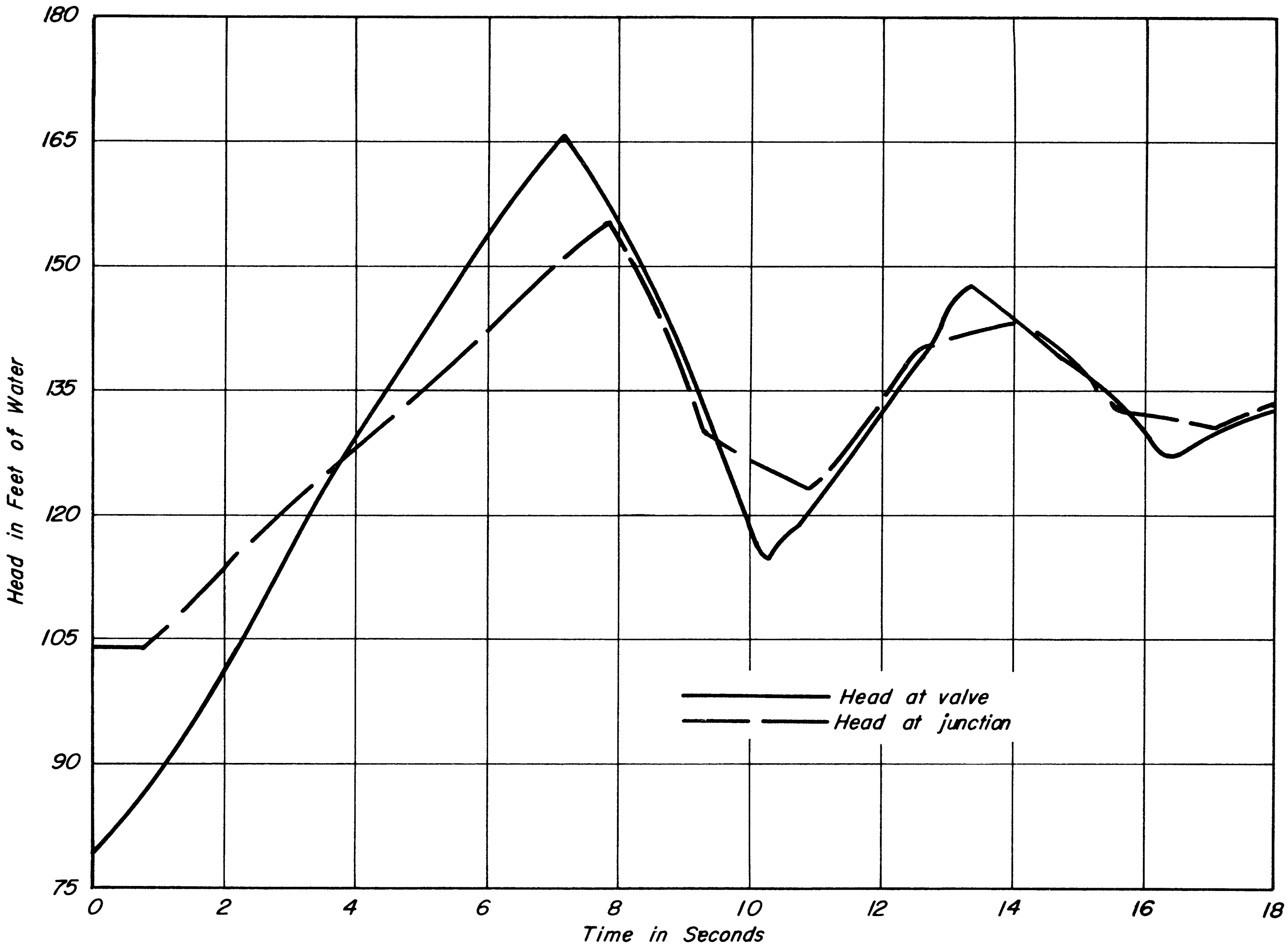

Figure 49. System variables versus time. Branching system. Computed results of a linear tau-time closure to compare with Case II.

VII. SUMMARY AND CONCLUSIONS

The primary objective of this study was to interpret and evaluate the existing valve-stroking theory in terms of the basic principles governing the transient phenomenon in liquid piping systems and to modify and extend the theory to include a greater number of operational constraints in the desired control of the simple and elementary complex piping systems.

The fundamental property of the characteristic equations relative to the arbitrary selection of the characteristic directions was shown to be of essential importance in the formulation of the pertinent theory. Three unique classes of problems--one datum at each boundary, two data at one boundary, and the mixed boundary-data problem--were each demonstrated to be instrumental in the development of procedures appropriate to the desired control action and system configuration.

Valve-stroking control of the three types of simple systems was considered in detail; the recognition that the duration of the transient condition imposed upon such systems may be arbitrarily selected has been presented, and the minimum time for control valve motion in such systems was demonstrated to be 2L/a, not the 4L/a previously reported. The consequence of this development relative to the control of a system consisting of a single pipe with an upstream constant-head reservoir has been discussed. A valve-stroking procedure for a pipe in which a constant velocity forms one boundary condition has been developed that permits complete control of the transient pressures developed in the system. The third type of simple system, a single pipe in which one boundary condition is a known relationship between head and velocity, has been examined and a valve-stroking procedure has been developed that permits a predetermined extreme value of pressure to be developed and maintained at the valve during a central phase of the transient condition.

The deficiency of the existing control procedure for the active complex systems has been demonstrated, and alternate techniques have been proposed that do allow the extreme system pressures to be established a priori. Finally, the concept of a passive system has been introduced and its implications in the control of the more extensive complex systems has been discussed.

Specific conclusions which can be drawn from this study, and which hopefully contribute to the development of systematic control procedures in transient pipe flow, are summarized.

1. A considerable amount of flexibility exists relative to the determination of valve-stroking procedures for the simple systems. Either the duration of the transient or the extreme system pressure may be arbitrarily selected. For systems which can physically accomodate the rapid closures ($t_f \leq 4L/a$), control following the duration-specification procedures is preferred because of the relative simplicity of the computational procedures; for systems in which slower closures must be utilized, the extreme-head procedure is preferred because of the direct nature of the desired solution.
2. Direct procedures can be established to limit extreme system transient pressures to acceptable values in the active complex systems.
3. Valve-stroking control of the passive complex systems is less direct; the duration and nature of the transient condition imposed upon the system can only be determined by numerical evaluation.

4. Because of the persistent changes in velocity that are developed in a system controlled by valve-stroking principles, the second-order finite-difference approximations to the characteristic ordinary differential equations are preferred, as they more accurately model the effects of viscous resistance to the flow. This is, to the author's knowledge, the first study in which the second-order approximations have been so extensively utilized. Because of the unusual circumstance that prevails relative to these solutions--iteration solutions (which are shown to be readily convergent) being required whenever the characteristic directions are both selected as positive, direct solutions otherwise--the program execution time of the second-order system is not significantly increased over the first-order system. (For the programs written during the course of this study, the maximum increase in program execution time was less than 25 percent.)
5. The experimental verification of some of the more advanced valve-stroking concepts presented in this study, together with the previously reported laboratory studies, demonstrates the efficacy of the theory and the intrinsic accuracy of the characteristic equations in the evaluation of the transient phenomenon in viscous liquid piping systems.

Although no direct consideration of the philosophies or procedures involved in the eventual on-line feed-forward computer control of large and complex distribution systems was attempted in this study, it should be recognized that the ultimate development and application of on-line control techniques will be dependent upon the procedures discussed herein and suitable extension of these procedures yet to be investigated. In this regard, the extreme downstream-head specification technique developed in this study appears to be particularly significant; further investigation into the application of this technique to the control of the following elementary complex systems is recommended:

1. The active branching system discussed in Section 4.2. The head at each downstream valve could be prescribed to change linearly from the initial value to the extreme value during the time the initial disturbance travels to the reservoir and is reflected back to the valve. The head at each valve would then remain at its respective preselected value during the central phase of the transient condition. When the velocity at the reservoir becomes the final steady-state value, that final value would then be specified along the reservoir boundary. The flow distribution at the junction in the upper region of the transient zone would then have to be specified; it appears likely that a technique similar to the existing procedure could be utilized. The remainder of the imposed transient condition would then be evaluated as before.
2. The passive branching system discussed in Section 4.3. The head at the valve would be prescribed and the transient condition in the lower region of the transient zone evaluated in a fashion similar to that for the active case. After the reservoir velocity becomes the final steady-state value, the analysis of system conditions in the upper region are identical to that discussed in Section 4.3.
3. The parallel system. Again the head at the valve would be prescribed and conditions in the lower region of the

transient zone could be evaluated. After the reservoir velocity attains its final steady-state value, the final value is again specified at the reservoir and conditions in pipe 1 (the active element) completely determined. Conditions may then be determined in pipes 2 and 3. The solution advances in a spiral fashion around the two pipes if the value of L/a in each pipe is not identical; otherwise the equations must be solved simultaneously. Conditions in pipe 4 may then be determined.

As with the preferred solution developed for the series system, one should recognize that the above procedures would be perfectly general and could be applied to any system with a known constant-relationship upstream boundary condition.

A recent development that has immediate valve-stroking implications is Streeter's study[32] in which he was able to avoid the restriction on the grid spacing of complex systems heretofore imposed by the Δx-Δt constraint. His solution is to arbitrarily select a value of Δt which then establishes the grid spacing for each system element, $\Delta x_i = a_i \Delta t$. The last, short reach of each pipe (and the total length of short pipes) are then modeled using either the lumped equations or the implicit equations, which are then incorporated into the appropriate boundary-condition equation of the respective pipe. It appears likely that similar techniques could be developed for the valve-stroking analyses of complex systems; such an investigation is also recommended for further study.

Lastly, two other investigations deserve especial mention since each could be relevant to future extensions of the valve-stroking theory for liquid piping systems. Wylie[33] has developed techniques for the control of the transient phenomenon in open-channel systems; the computational schemes he developed could be applicable to the analysis and control of piping systems in which the wavespeed is time-dependent. Compelling evidence exists that this is the case in systems constructed of such plastic pipes as Polyvinyl Chloride (PVC). With the ever-expanding utilization of these newer materials in large distribution systems, the analysis and control of undesirable transients in their operation will become an essential design consideration. Stoner[23] has studied the application of valve-stroking theory to the control of gas-distribution systems; he has developed computational procedures for complex multiple loop systems which could be applicable to the liquid systems. In either case, however, one must recognize that the physical aspects of those two systems are decidedly dissimilar to those of the liquid system. The relatively minor inertial considerations of the former differ considerably from the pre-eminent inertial considerations of the latter, and the effective control of inertial effects must necessarily be the essential priority in the development of suitable control strategies for the liquid systems.

APPENDIX A

CONVERGENCE OF ITERATION SOLUTIONS

The interior-point and external boundary-condition equations developed in Chapter II which can not be solved directly may be readily solved by an iterative procedure based upon the method of successive approximations.[17]

As an example, consider the solution for the velocity at a space-like interior point:

$$V_P = V_A - \frac{H_P - H_A}{B} - \frac{F_P + F_A}{2B} . \qquad (12b)$$

This equation is of the form V = G(V) and its desired solution is unique since G(V) is a strictly monotone-decreasing function of V. (Recall that the function F is a strictly monotone-increasing function of V, and that it is the only unknown term on the right-hand side of the equation.)

The solution procedure is depicted geometrically in Figure 1-A, with the arrows indicating the pattern of the iterations. The desired solution V_P lies at the intersection of the 45° line with the curve described by G(V). A trial value of V_P is selected, indicated by V_{P_1}, and the next value is computed by utilizing this first estimate:

$$V_{P_2} = G(V_{P_1}) .$$

The next approximation is then

$$V_{P_3} = G(V_{P_2}) ,$$

and so on, until the nth approximation (or the nth iterate, as it is often called) is

$$V_{P_n} = G(V_{P_{n-1}}) .$$

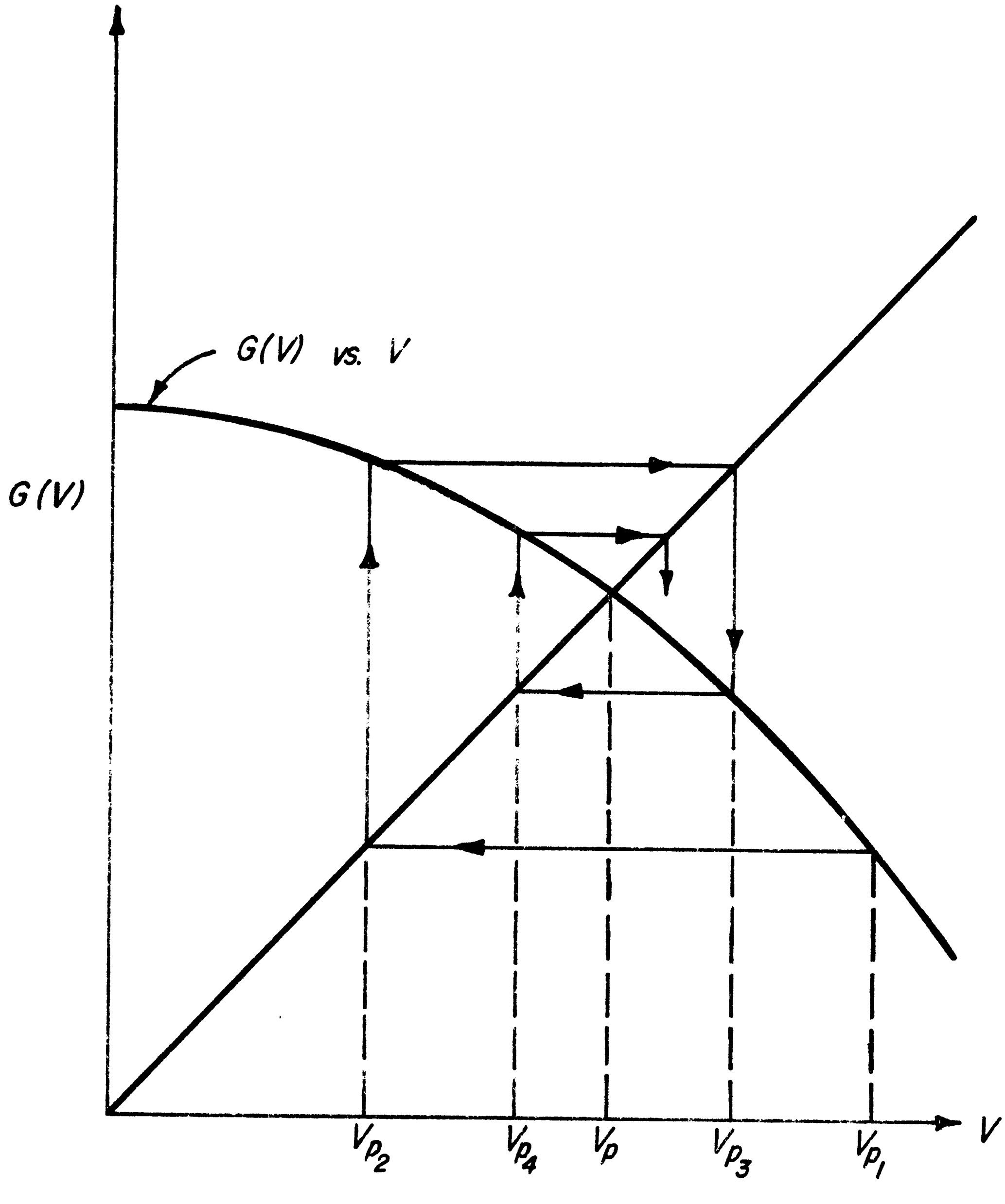

Figure 1-A. Graphical representation of the method of successive approximations.

Delaying for a moment the question of rapidity of convergence, attention is now directed to the more basic question of convergence itself: i.e., do the V_{P_n} converge to the desired solution V_P as n increases? As would appear to be evident from Figure 1-A and can be proven analytically,[17] the sufficient condition on G(V) for this desired convergence is

$$0 < |G'(V)| < 1 . \qquad \text{(A-1)}$$

One can investigate Equation (12b) relative to this criterion, since

$$|G'(V)| = \frac{F'(V)}{2B} = \frac{1}{2B}\frac{d}{dV}\left(\frac{fLV^2}{DN2g}\right)$$

$$= \frac{1}{2B}\left(\frac{2fLV}{DNLg} + \frac{LV^2 f'(V)}{DN2g}\right)$$

$$= \frac{F}{2B}\left(\frac{2}{V} + \frac{f'(V)}{f}\right) .$$

Since the derivative of f with respect to V is generally a negative number (the only exception being for flow in the critical zone, and then the value of the function F is not large), this term can be safely neglected in an assessment of the anticipated maximum value of $|G'(V)|$. Thus

$$|G'(V)| \approx \frac{F}{BV} . \qquad \text{(A-2)}$$

Now consider the following evaluation: for the 2000-ft pipes of the laboratory experimental system, the maximum head loss that can be reasonably extrapolated from the calibration data is 92.5 ft at a velocity of 2.8 ft/sec. For a = 2664 ft/sec, N=8, and g = 32.2 ft/sec^2,

$$|G'(V)| \approx \frac{32.2 \times 92.5}{2664 \times 8 \times 2.8} \approx .05 ,$$

which is safely within the criterion established by Equation (A-1).

The only external boundary-condition equations which can not be solved directly and in which the friction term does not appear in the same form as that of Equation (12b) are those involving an orifice or a valve. In these, the solution of a quadratic in V is encountered. For example, consider the solution for the velocity of flow through a valve located at the downstream boundary of the pipe:

$$V_P = \frac{C_1 + \sqrt{C_1^2 + 4(C_2 - K_v^2 F_P/2)}}{2} \quad . \tag{15a}$$

Again, convergence can be demonstrated a priori since

$$|G'(V)| = \frac{K_v^2 F'(V)}{2\sqrt{C_1^2 + 4(C_2 - K_v^2 F_P/2)}} \quad .$$

Therefore

$$|G'(V)| \leq \frac{K_v^2 F'(V)}{2C_1}$$

since, in order to ensure positive velocities through the valve, $C_2 - K_v^2 F_P/2$ can not be a negative number. Finally, since $C_1 = K_v^2 B$,

$$|G'(V)| \leq \frac{F'(V)}{2B} \quad ,$$

which has already been demonstrated to adequately satisfy the convergence criterion.

In the event that G'(V) might not satisfy the convergence criterion established by Equation (A-1), several alternatives are available: (1) reduction of the time increment Δt, thus creating an equivalent reduction in Δx and F, and, therefore, G'(V); (2) treating f as a constant, thus permitting the direct solution of a quadratic in V; or if neither of the above two alternatives are acceptable, (3) the simultaneous solution of two nonlinear equations in V and f by some suitable numerical procedure.[7,17]

With regard to rapidity of convergence, that the method of successive approximations does not converge as rapidly as other possible solution techniques is readily acknowledged. Generally speaking, the modified method of successive approximations, Newton's method, the Newton-Raphson method, or the method of regula falsi will oftentimes converge more rapidly than the former. Nevertheless, the method of successive approximations is preferred because of its relative simplicity and ease in programming, particularly when the original estimate is reasonably accurate in any case. (Recall that the original evaluations are always based upon the equivalent first-order finite-difference approximations.) In the various computer programs essential to this investigation, a maximum of only eight iterates were necessary to produce values of head and velocity accurate to two and three decimal places, respectively.

Unlike the interior-point and external boundary-condition equations, convergence of the internal boundary-condition equations can not be demonstrated a priori.

For example, consider the solution for the conditions existing at the junction of two pipes connected in series:

$$H_P = \frac{C_3 - C_4 - C_1 F_{P_1}/2 + C_2 F_{P_2}/2}{C_1 + C_2} , \qquad (20a)$$

$$V_{P_1} = \frac{C_3 - C_1 (H_P + F_{P_1}/2)}{A_1} , \qquad (20b)$$

$$V_{P_2} = \frac{C_4 + C_2 (H_P - F_{P_2}/2)}{A_2} . \qquad (20c)$$

Equations (20a) to (20c) are three simultaneous nonlinear algebraic equations in three unknowns. While several solution techniques[7,17] have been shown to be convergent for systems of linear equations, one generally must resort to numerical experimentation in the case of the nonlinear equations.

In this investigation, the solution procedure adopted is based upon the Gauss-Seidel (single step) iteration within successive approximations. Again, relative simplicity and ease in programming motivated its selection, and the procedure is as follows: (1) F_{P_1} and F_{P_2} are initially evaluated at V_{A_1} and V_{B_2}, respectively; (2) H_P is computed from Equation (20a); (3) V_{P_1} is computed from Equation (20b) using the initial value of F_{P_1} and the previously computed value of H_P; V_{P_2} is computed from Equation (20c) in a similar fashion; (4) F_{P_1} and F_{P_2} are then re-evaluated at the previously computed values of V_{P_1} and V_{P_2}, respectively; (5) the process is then continually repeated until the desired convergence obtains. As with the interior-point and external boundary-condition equations, a maximum of only eight iterations were necessary to produce values of head and velocity accurate to two and three decimal places, respectively.

APPENDIX B

SOLUTION PROCEDURES

Orderly and systematic procedures essential to the valve-stroking and characteristics analyses may be developed and catagorized according to the particular boundary-data problem and system considered.

With reference to Figure 1-B, assume that initial conditions are known along each grid point on the space-like boundary and one datum on each grid point of the time-like boundaries. Then conditions at point 1 may be evaluated by utilizing the appropriate left-end boundary-condition equation, interior points 2, 3, ... , N-1, N by utilizing Equations (12a) and (12b), and point N+1 by utilizing the appropriate right-end boundary-condition equation. The cycle is then repeated as indicated; if a total of I such cycles exist, then the total number of grid points at which conditions are evaluated is I(N+1). This procedure is intrinsic to all characteristics analyses, of course, and occasionally is employed in the valve-stroking analysis of a passive system when complete boundary data are available prior to the start of the computational scheme (for example, as they are for pipe 3 of the laboratory branching system). Note that even though the initial conditions may be steady-uniform, the simple and direct nature of the procedure eliminates the need to establish the steady-state zone prior to the beginning of the computations. It will develop during the computational procedure as discussed in Section 2.6.1.

In the valve-stroking analyses of parallel and multiple loop systems, necessary boundary data is frequently generated sequentially, one boundary at a time, as the computation scheme advances around the loop from one pipe, to the next, etc. Again, if one datum is eventually

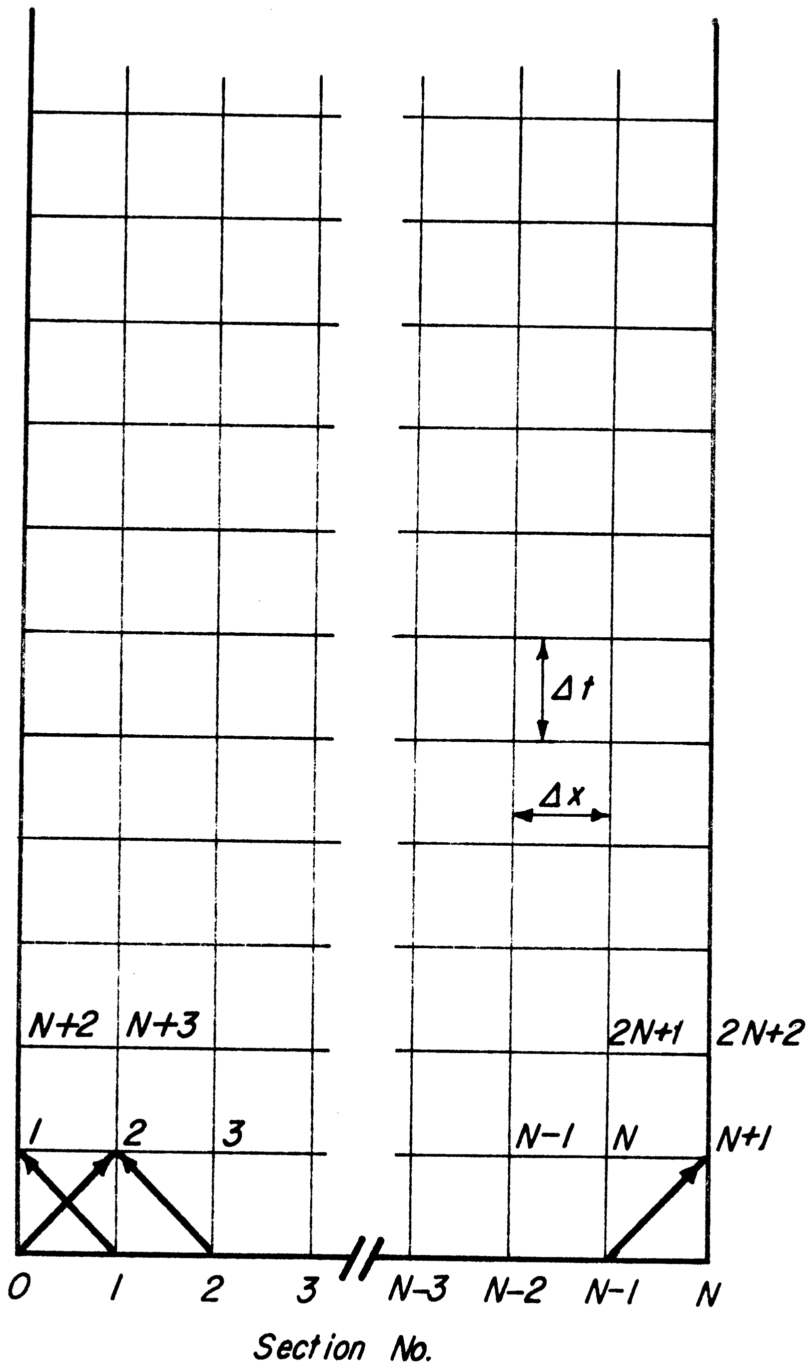

Figure 1-B. Solution procedure for one datum on each boundary.

established at the next grid point on each boundary, the computational procedure for such a pipe is illustrated in Figure 2-B. The same sequence and equations are used as before, but the computations advance diagonally upward along the appropriate characteristic lines. Here it becomes expedient to establish the initial zone a priori. If the pipe is subdivided into N equal reaches, the number of initial calculations is given by the sum of the arithmetic progression

$$N+1 + N + N-1 + \ldots + 3 + 2 + 1.$$

The sum of this progression is[4] $(N+1)(N+2)/2$. Again the total number of computational points in the transient zone is $I(N+1)$ unless the pipe is contiguous to a system element of the mixed-boundary data type (see below); in such cases the total number of points is $(I+1)(N+1)$.

The fundamental procedure essential to the valve-stroking analyses of most of the elementary active systems of Chapters III and IV is illustrated in Figure 3-B. The necessary data are prescribed along the appropriate boundary, and again it becomes expedient to establish the initial and final steady-state zones a priori. Conditions at the time-like interior points are then evaluated using the appropriate equations -- Equations (18a) and (18b), or Equations (19a) and (19b) -- along each successive vertical grid line as indicated, ultimately extending the computations to the other boundary. If I represents the number of grid points along the boundary at which the transient boundary data are specified, then the total number of interior points at which conditions must be evaluated is given by the sum of the progression

$$I+2 + I+4 + \ldots + I+2(N-1) + I+2N.$$

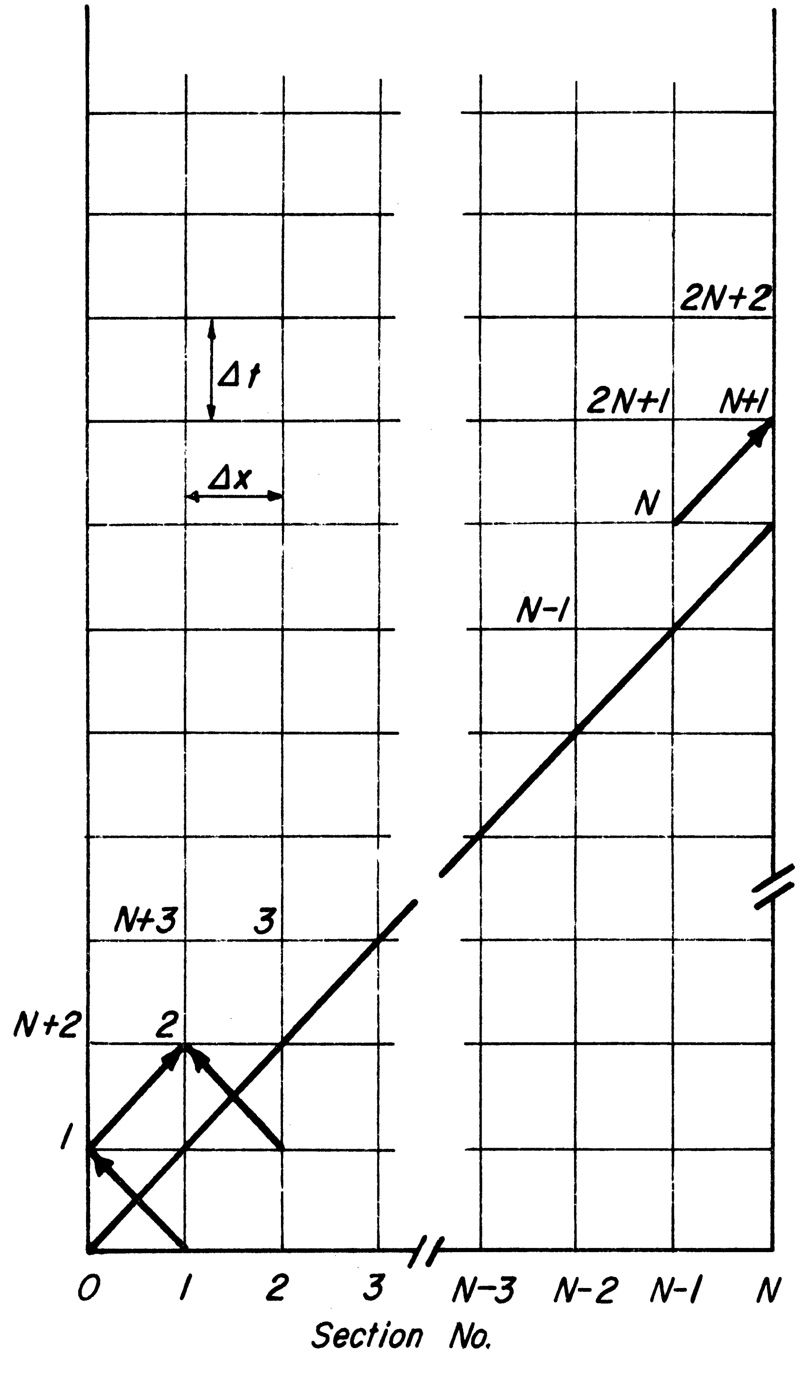

Figure 2-B. Solution procedure for one datum on each boundary -- alternate style.

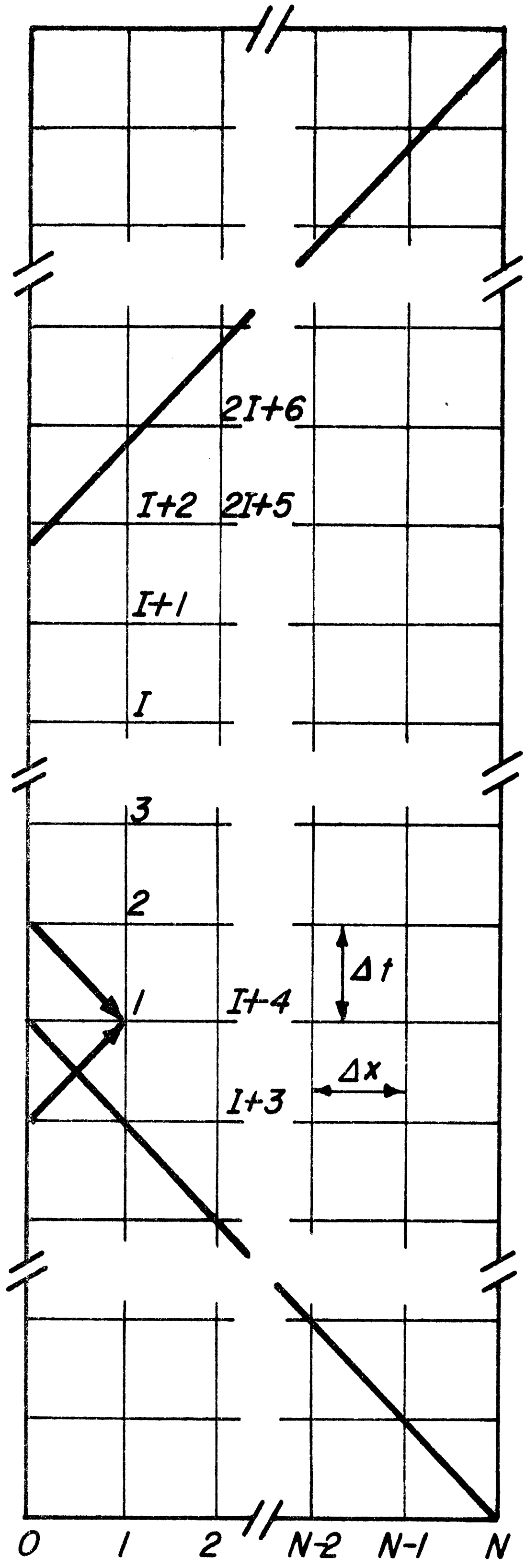

Figure 3-B. Solution procedure for both data on one boundary -- active case.

The sum of this progression is N(I+N+1).

If both data are prescribed along one of the boundaries, as in Figure 4-B, but the pipe is a passive element of a system (for example, pipe 2 of the laboratory branching system), then no final steady-state zone may be arbitrarily established. The solutions along the boundary are terminated after a given period of time, and the transient zone is established on and below the dashed line of the figure as indicated. Again, if I represents the number of grid points along the boundary at which the transient boundary data are specified, then the transient zone consists of a total of IxN time-like interior points. Note that in this case computations may either be advanced along successive vertical grid lines as indicated, or they may be advanced diagonally downward along characteristic lines. (For example, the first diagonal would consist of the points 1, I+1, etc.) In the valve stroking analyses of parallel and multiple loop systems, this latter technique must be used if the necessary boundary data are generated sequentially, as is frequently the case.

The solution procedure essential to a valve-stroking analysis involving the downstream-head specification technique of Sections 3.3.2 and 4.1.2 is illustrated in Figure 5-B. Again the initial zone is first established. Conditions at point 1 may be evaluated by utilizing the appropriate right-end boundary-condition equation, interior points 2, 3, ..., N-1, N again by utilizing Equations (12a) and (12b), and point N+1 by utilizing the appropriate left-end boundary-condition equation. After a period of time this scheme is terminated, and both boundary data are then specified along the left-end boundary; the remainder of the solution procedure is thereafter identical to that of Figure 3-B. (The dashed characteristic line delineates the boundary between the two zones.) If

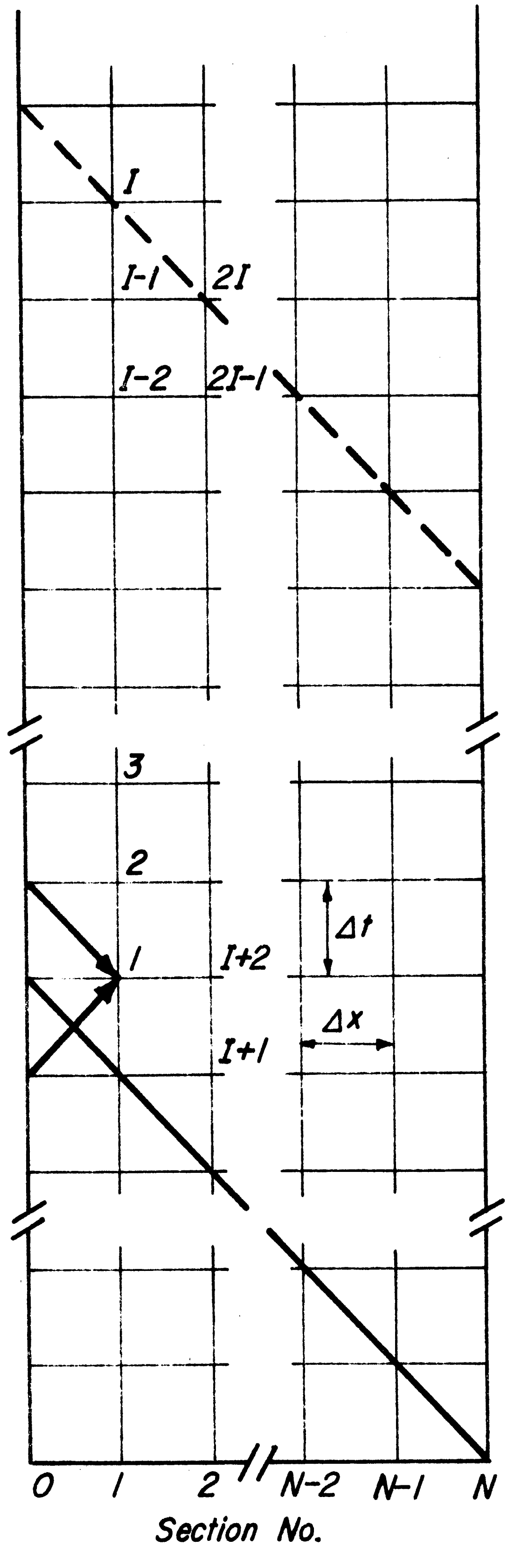

Figure 4-B. Solution procedure for both data on one boundary -- passive case.

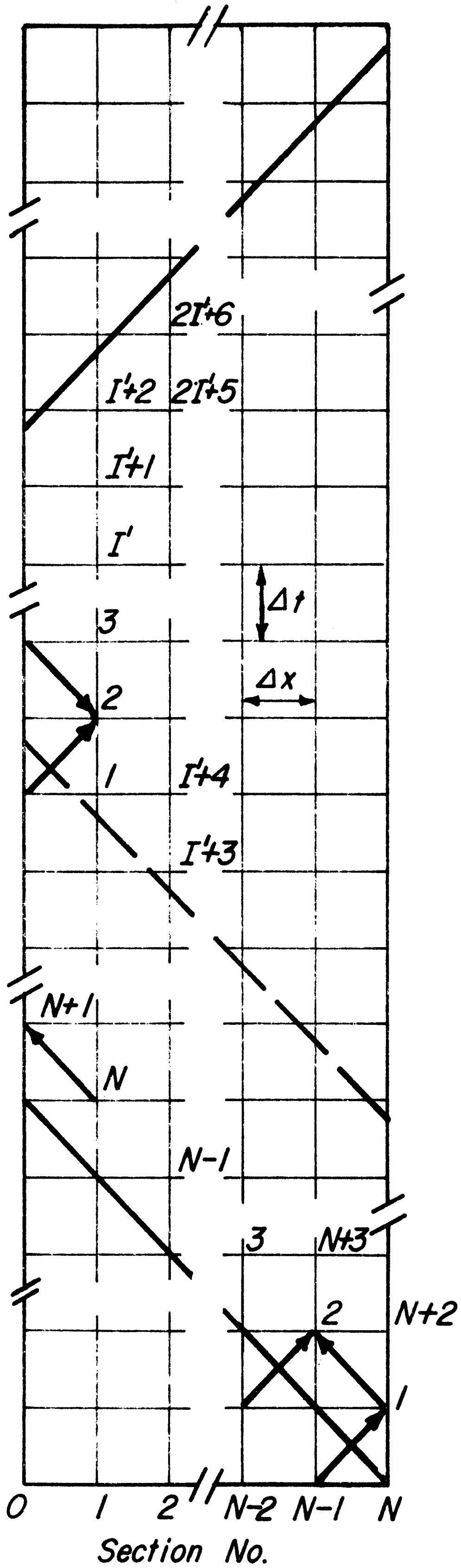

Figure 5-B. Solution procedure for mixed boundary-data problem -- active case.

I represents the number of left-end boundary grid points of the transient zone below the dashed line, and I' the number of boundary grid points above the line, then the transient zone consists of a total of I'xN + (N+1)(I+N+1) points at which conditions must be evaluated. One should recognize that conditions must be evaluated twice on the first diagonal characteristic above the dashed line; the dashed line would rarely coincide with the grid points immediately above it, and its position can only be established after the last cycle of computations along the diagonal has advanced to the furthest upstream boundary.

The extension of this latter solution procedure to a passive element of a system is illustrated in Figure 6-B; it would be intrinsic to the studies proposed in Chapter VII. The procedure in the lower region of the transient zone is identical to that of Figure 5-B and in the upper region to that of Figure 4-B. As with the previous discussion, computations in the upper zone may advance either along successive vertical grid lines or downward along diagonal characteristic lines. The transient zone contains a total of I'xN + (I+1)(N+1) points at which conditions must be evaluated.

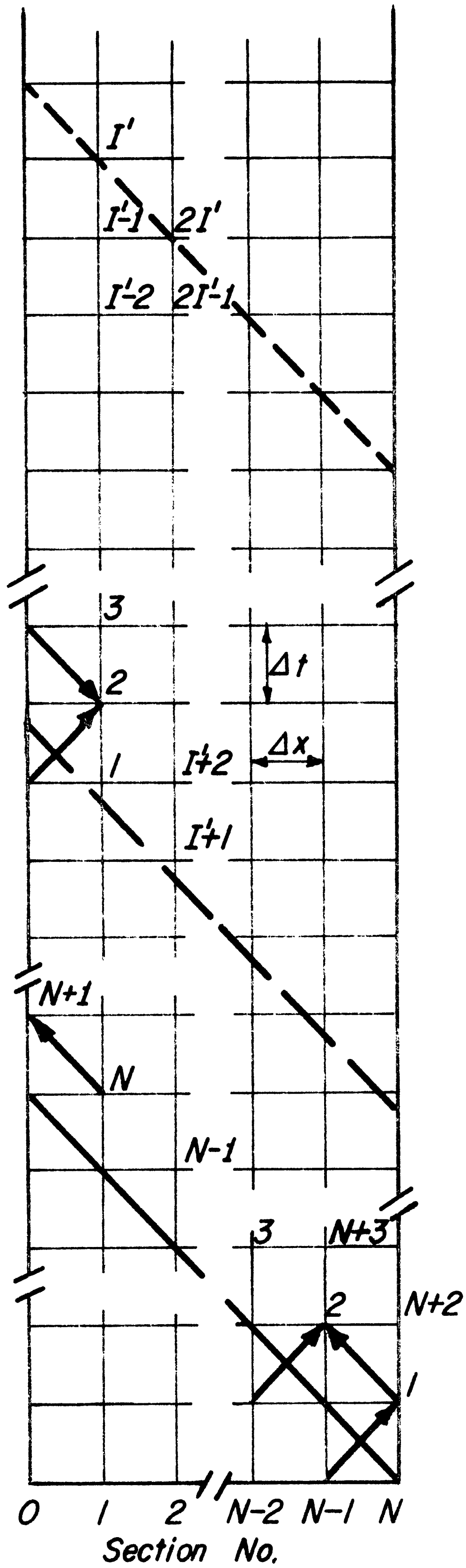

Figure 6-B. Solution procedure for mixed boundary-data problem -- passive case.

APPENDIX C

TRANSIENT DURATION CALCULATIONS

When the transient condition imposed upon the system is specified by selection of the extreme permissible value of head desired at the valve, the duration of the transient (t_f) of the simple system of Section 3.1 may be determined by integration of Equation (22). Thus

$$t_f = 2L/a - \int_{V_o}^{V_f} \frac{dV}{\frac{g}{L}(H_m - H_r) + \frac{fV|V|}{2D}} . \qquad \text{(C-1)}$$

Since $t_f \geq 2L/a$, close examination of Equation (22) reveals the following restriction on H_m: for $V_o > V_f$, $H_m > H_f$; for $V_o < V_f$, $H_m < H_f$.

The above integral may be evaluated numerically in the following manner: With reference to Figure 1-C (G(V) has the indicated shape since the friction term is a strictly-monotone increasing function of V), recall the second-order (trapezoidal) finite-difference approximation to Equation (22):

$$V_{i+1} = V_i - \frac{(H_m - H_r)}{NB} - \frac{F_i + F_{i+1}}{2B} . \qquad \text{(22a)}$$

Beginning at V_o, Equation (22a) is solved (by iteration) to establish V_1 such that the approximate area under the curve of G(V) is equal to Δt, the constant time increment of the solution grid. V_2, V_3, ..., V_{i-1}, V_i, V_{i+1} are established in the same way. (This procedure, of course, thereby establishes the upstream boundary velocity at each point of the solution grid.) The scheme is terminated when the last velocity calculated is less than V_f for closure situations, or exceeds V_f for situations involving valve openings. The time $\Delta t'$ then may be approximated by

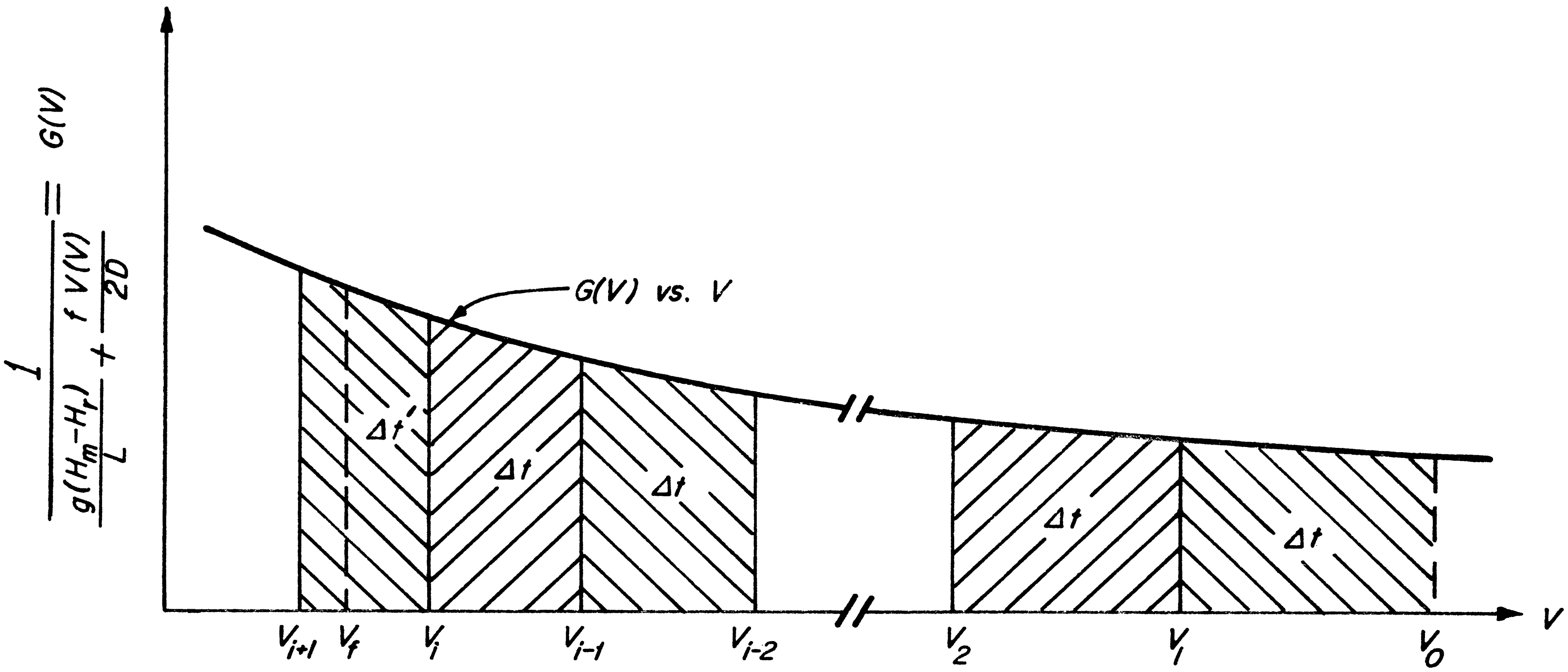

Figure 1-C. Graphical representation of the numerical evaluation of the time-duration integral.

$$\Delta t' = \frac{V_i - V_f}{V_i - V_{i+1}} \Delta t \quad . \tag{C-2}$$

If I represents the number of grid points along the boundary at which the transient velocity data are thereby specified, then the duration of the transient condition is

$$t_f = 2L/a + I\Delta t + \Delta t' \quad . \tag{C-3}$$

Even though the upstream velocity at each grid point is established using a different technique, Equation (C-3) is equally applicable to the downstream-head specification procedure of Section 3.3.2. Of course, the equation must be appropriately modified to include the wave-travel times of the downstream pipes in the series and branching systems of Chapter IV.

The manner by which t_f may be arbitrarily selected and the appropriate value of H_m subsequently determined is now considered. From Figure 1-C it is evident that a change in the value of H_m would merely raise or lower the curve of G(V) relative to its position illustrated. Therefore, the area under the curve between V_o and V_f -- and hence the value of t_f -- must be a strictly-monotone function of H_m. Thus, the value of H_m that would yield the prescribed value of t_f can be readily determined by the method of interval halving[17] applied to the solution procedure outlined for the determination of t_f. This method (also called the bisection method[29,31] or Bolzano's method[7]) permits an exceedingly simple and direct computer-oriented solution when the desired root may be isolated in a known interval, as is the case here. (Recall the restriction on H_m cited above. The other bound on the solution interval can be established without difficulty.)

A direct extension of this procedure is utilized to determine the unknown head to be prescribed at the junction (H_{m_j}) in Equation (43) of the branching system analysis. For closure situations, for example, the left-hand side of the equation is a strictly-monotone decreasing function of H_{m_j} while the right-hand side is a strictly-monotone increasing function. For any specific flow situation encountered, the limits on the solution interval again can be readily established.

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