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THE TRANSMISSION OF ELECTROMAGNETIC RADIATION THROUGH WIRE GRATINGS

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INTRODUCTION

An investigation of the transmission of electromagnetic waves through diffraction gratings whose spacing (D) is smaller than the wavelength (λ) was undertaken as a preliminary to their use in infrared spectroscopy, for example as reflecting surfaces for Fabry-Perot interferometers, polarizers, selective filters, etc. The most important characteristic of such transmission gratings is the Hertz effect. Hertz,[1] using 60-cm waves and a grating of 1-mm diameter wires with a separation of 3 cm, found that complete reflection occurred if the electric field was parallel to the wires and that complete transmission occurred if the electric field was perpendicular to the wires. A qualitative explanation for the phenomenon can be given in terms of currents induced in the wires by the incident field. These currents will be large along the axis of the wire when the electric field is parallel to that axis, but practically nonexistent across the wire for the perpendicular orientation. The radiated or scattered field will be proportional to the induced currents, but will be out of phase with the incident field. Since the final field is formed by the superposition of the incident and scattered fields, the dependence of the transmission on the direction of the electric intensity follows.

Lamb[2] did some of the first theoretical work on this diffraction problem, discussing gratings both of wires and strips. From a consideration of analogous situations existing in hydrodynamics, he constructed the following solutions for the transmission through strip gratings when $\lambda \gg D$, and where A is half the width of a metal strip:

$$T_{\perp} = \frac{1}{1 + x^2} \qquad x = \frac{2D}{\lambda} \ln \sec \frac{\pi A}{D}$$

$$T_{\parallel} = \frac{y^2}{1 + y^2} \qquad y = \frac{2D}{\lambda} \ln \sec \left(\frac{1}{2} - \frac{A}{D} \right)$$

He also obtained similar expressions for wire gratings.

where n is the spectral order and i_1 and i_2 are the angles of incidence and of emergence respectively, it is clear that for $D \leq \lambda/2$, only the zero-order spectrum can be formed. For small values of i_1 and i_2 this will still be true even if D approaches λ in value. This, then, will be the region in which transmission gratings will have high efficiencies, since no energy will be lost into the higher orders. Grating materials of high electrical conductivity will have a negligible resistive loss at microwave frequencies, and consequently the energy will either be transmitted in an undeviated direction or reflected ($T + R = 1$). Since any optical system accepts rays in a finite solid angle subtended by the source, it is of some importance to know the dependence of the transmission on the angular orientation of the grating with respect to the beam. However the grating may be rotated in two different manners. Referring to Fig. 1, a rotation through the angle α reduces the effective grating space, but a rotation through β leaves it unchanged.

Hence the program was to measure the transmitted radiation as a function of the grating space, the angles α and β, and the various widths of wires and strips.

APPARATUS

The source was a 723 AB klystron coupled to a reflecting paraboloid 16 inches in width—the remnants of a war-surplus airborne radar set. The klystron was modulated at 2000 cps and the wavelength of the radiation was determined by the distance between the maxima of a Fabry-Perot interferometer to be 3.20 cm. An attenuator was placed in the wave guide between the klystron and the antenna to reduce standing-wave effects. The radiation was completely plane-polarized, which permitted the parallel and perpendicular orientation of the electric field with repsect to the grating wires.

The detecting system consisted of a paraboloidal mirror focusing the beam onto a crystal, an untuned audio amplifier, and an indicating microammeter. Its overall response was found to be linear by rotating the crystal to known angles relative to the direction of polarization of the beam and checking its response against the expected $\cos^2$ variation. The detector was some 100 feet from the source and the diffraction grating was placed midway between the two. This large distance served both to reduce standing-wave effects and to give plane waves. The gratings were 18 inches square, could be mounted with the elements either horizontal or vertical (i.e. parallel or perpendicular to the electric field), and could be turned about either a horizontal or a vertical axis. A 14-inch square aperture was placed close behind the grating so that only radiation passing through the grating could reach the detector.

The accuracy of the transmission values is thought to be within 1% or 2%. The effect of standing waves, as judged by moving either the source or the grating back and forth, was always within this limit. A check on the overall accuracy of the system was made through the use of 1/8-inch thick glass plates. These particular plates had an optical thickness of very nearly a quarter wavelength since, as the sheets were stacked together one after the other, alternate maxima and minima of nearly constant values were obtained. The index of refraction determined in this manner was then used to calculate the transmitted intensity for one sheet, giving a value about 1-1/2% higher than that measured (part of this difference can be attributed to a small amount of absorption in the glass). An attempt was made to gain an independent check from the transmission through a metal film of known resistance. This has been shown by Hadley and Dennison[10] to be

$$T = \frac{4}{(2 + 377/R)^2},$$

where R is the resistance of the metal film per square. The results differed by some 5% from the calculated value, but this discrepancy was traced to the nonuniformity of the metal film. It was found that uniform films of a sizeable area were somewhat difficult to make and, in view of the satisfactory agreement using glass plates, the matter was dropped. However, it is worth noting that this method is the simplest and most accurate method of obtaining a known transmission provided that a uniform film can be obtained.

The wire gratings consisted of brass or aluminum rods or, for the larger sizes, tubes (since the small skin depth for these frequencies would indicate no appreciable difference between a solid and a hollow cylinder). The strips were of 0.001-inch aluminum foil, cut on a metal shears, with the smaller widths backed by paper.

RESULTS

In Fig. 2, the per cent transmission is plotted vs λ/D for wire gratings having several ratios of D/A, where A is the wire radius. The perpendicular component is at the top of the figure and the parallel is at the bottom. The radiation is incident normally upon the grating in each case. A number of points were calculated from Wessel's work and they agreed within 1% of these measured values so long as $D/A \geq 8$. However for $D/A \leq 4$ there were large differences, which can be expected as a result of the approximation Wessel used. The transmission for the perpendicular component behaves in a most surprising manner for small values of D/A when the wavelength is somewhat greater than D. Presumably this is a resonance phenomenon dependent on the geometry of the opening between grating elements.

Figure 3 is a plot of the transmission calculated from the equations developed by Lamb for cylinders. It is evident that there is no quantitative agreement with these experimental results.

The measured transmission for strip gratings is plotted in Fig. 4 (again at normal incidence). To make an easy comparison with the wire gratings, the half width, A, of the strips is used to correspond to the wire radius in the previous case. It can be concluded from such a comparison that the strips have more scattering effect on the perpendicular component and less on the parallel component than do the comparable size wires. It is noteworthy that no sharp transmission band exists near $\lambda = D$ for the perpendicular component on strip gratings, such as there was for wire gratings. Further work is planned to examine the dependence of this band on the shape of the grating elements.

In Fig. 5, Lamb's equations for strip gratings having several values of D/A are plotted along with the corresponding measured values. There is a fair, although not precise, fit for $\lambda > 2D$.

To give an indication of the effect of the thickness of the strips, several gratings were made from 1/16-inch aluminum (this thickness is about 0.05λ as compared with a thickness of about 0.0008λ for the aluminum foil). In Fig. 6, it can be seen that this results in a decreased transmission for both polarizations. With strips of much greater thickness, the work of Lengyel[11] on metal plate media would be applicable for the parallel component.

The dependence of the transmission on the angular orientation of the grating is shown for several representative cases in Figs. 7-10. The angular resolution of the optical system considered as a telescope is

$$\theta_{min} = \frac{\lambda}{W} = 5^\circ ,$$

where W is the width of the aperture. However, rays leaving the grating at an angle greater than 1° will not fall upon the telescope mirror. Any abrupt change in transmission with angle will therefore be slightly smoothed out. In addition, at angles greater than 20°, the grating holder began to intercept part of the beam (approximately 6% at 30°), but the results have not been corrected for this.

It can be concluded that for large values of λ/D, the angular variations α and β lead to only minor changes in the transmission for either wires or strips. When λ approaches D in value, there are marked effects, and as would be expected the angle α leads to greater changes than does β.

Most of these observations lie outside the range of validity of Honerjager's calculations for the angular variation. One of the conspicuous features of his results is that there should be 100% transmission whenever a spectral order can be found at grazing emergence to the grating. In particular, for $\lambda/D = 0.9$, there should be two such peaks as shown at the top of Fig. 11, one at 6° and the other at 53°. The observed curves do indeed show maxima at these points, but they are far from 100%. The obstruction of radiation by the grating support would account for the reduced transmission in the 50° region, but not at small angles. Moreover, in the 5° region the areas under the two curves should be the same. The explanation for the discrepancy probably lies in the finite size of the grating used. Honerjager's value of 100% results from an infinite value for the infinite series of terms arising from contributions from each wire. Given a finite number of wires, this infinity disappears and the transmission would be less than 100%. The same effect must also take place when $\lambda/D = 1$. Observations at this point are not very exact, since the curves have steep slopes and we could not be certain that λ/D was precisely 1. Other than at these points of 100% transmission for the parallel component, the finite size of the gratings appears to have no appreciable effect.

At the bottom of Fig. 11, comparison between observed and calculated points under less severe conditions shows quite satisfactory agreement.

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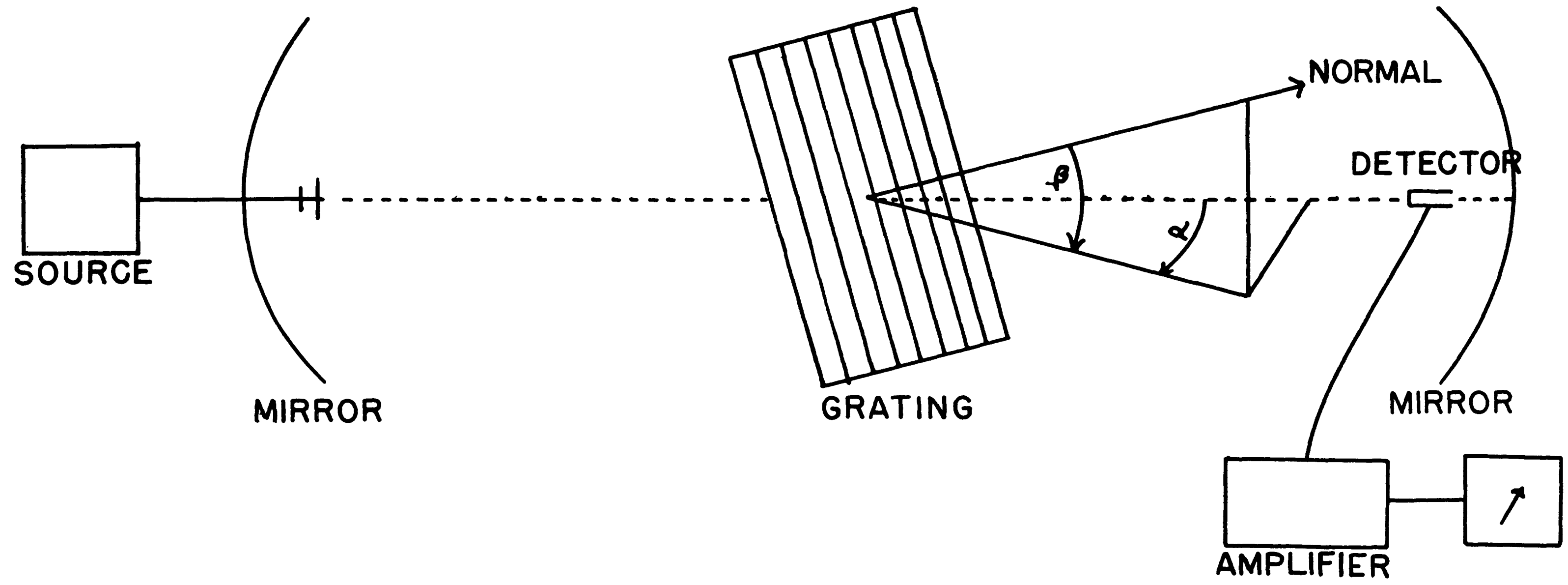
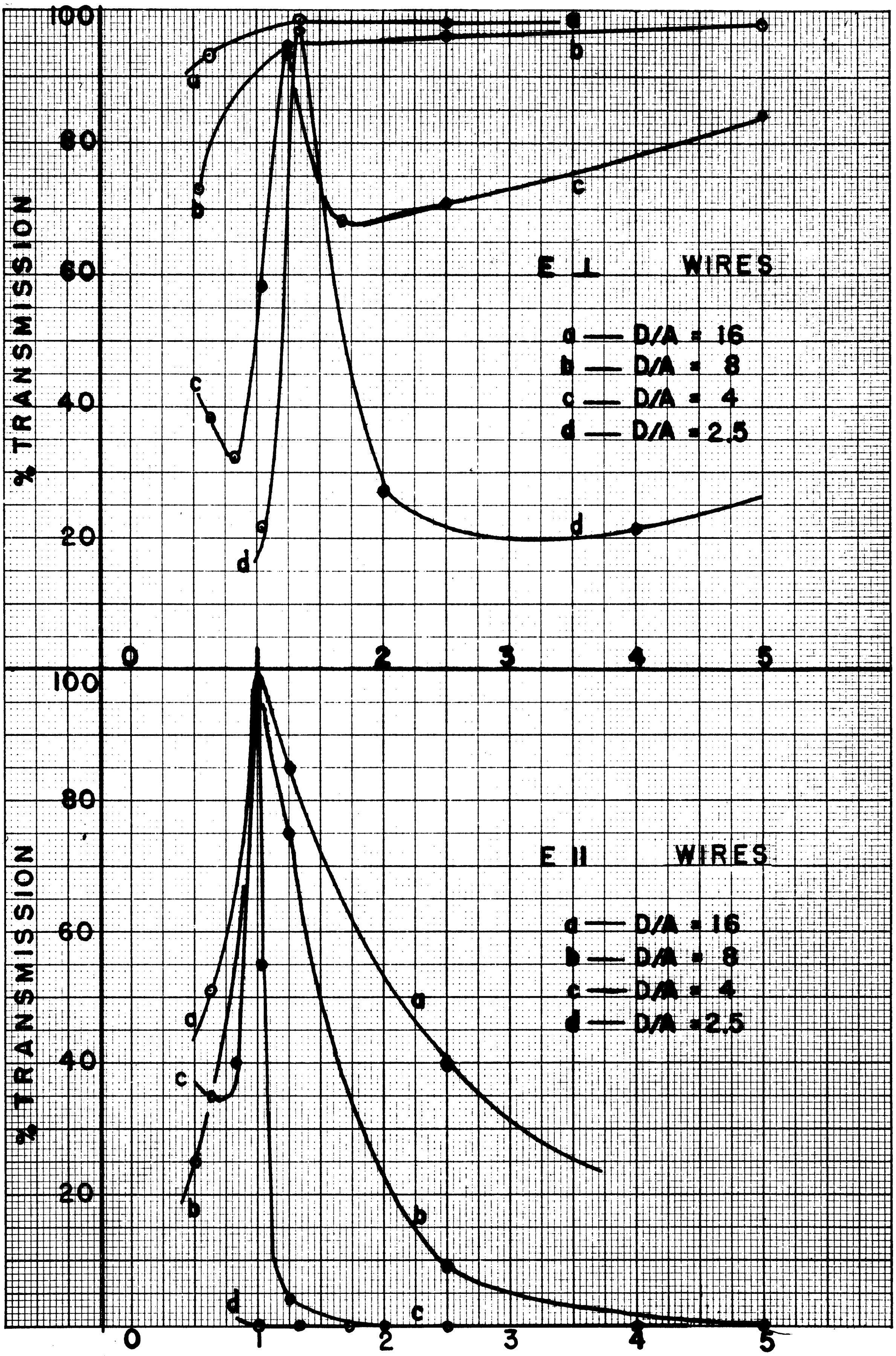

NORMAL
DETECTOR
SOURCE
MIRROR
GRATING
MIRROR
AMPLIFIER
β
α

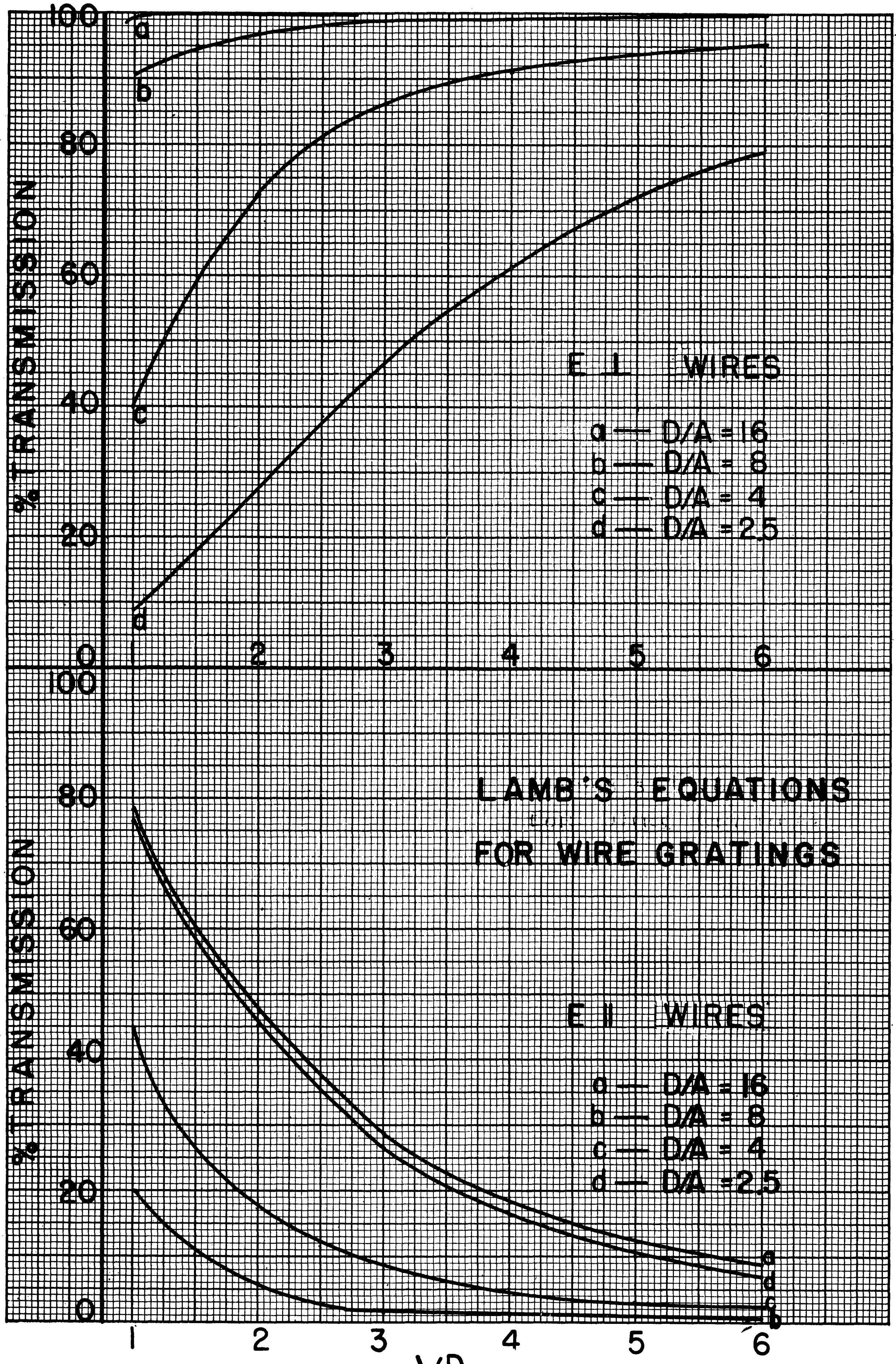
FIGURE I

FIGURE 2



% TRANSMISSION
E ⊥ WIRES
a — D/A = 16
b — D/A = 8
c — D/A = 4
d — D/A = 2.5
E ∥ WIRES
a — D/A = 16
b — D/A = 8
c — D/A = 4
d — D/A = 2.5
λ/D

FIGURE 3



% TRANSMISSION
E ⊥ WIRES
a — D/A = 16
b — D/A = 8
c — D/A = 4
d — D/A = 2.5
LAMB'S EQUATIONS
FOR WIRE GRATINGS
% TRANSMISSION
E ∥ WIRES
a — D/A = 16
b — D/A = 8
c — D/A = 4
d — D/A = 2.5
λ/D

FIGURE 4

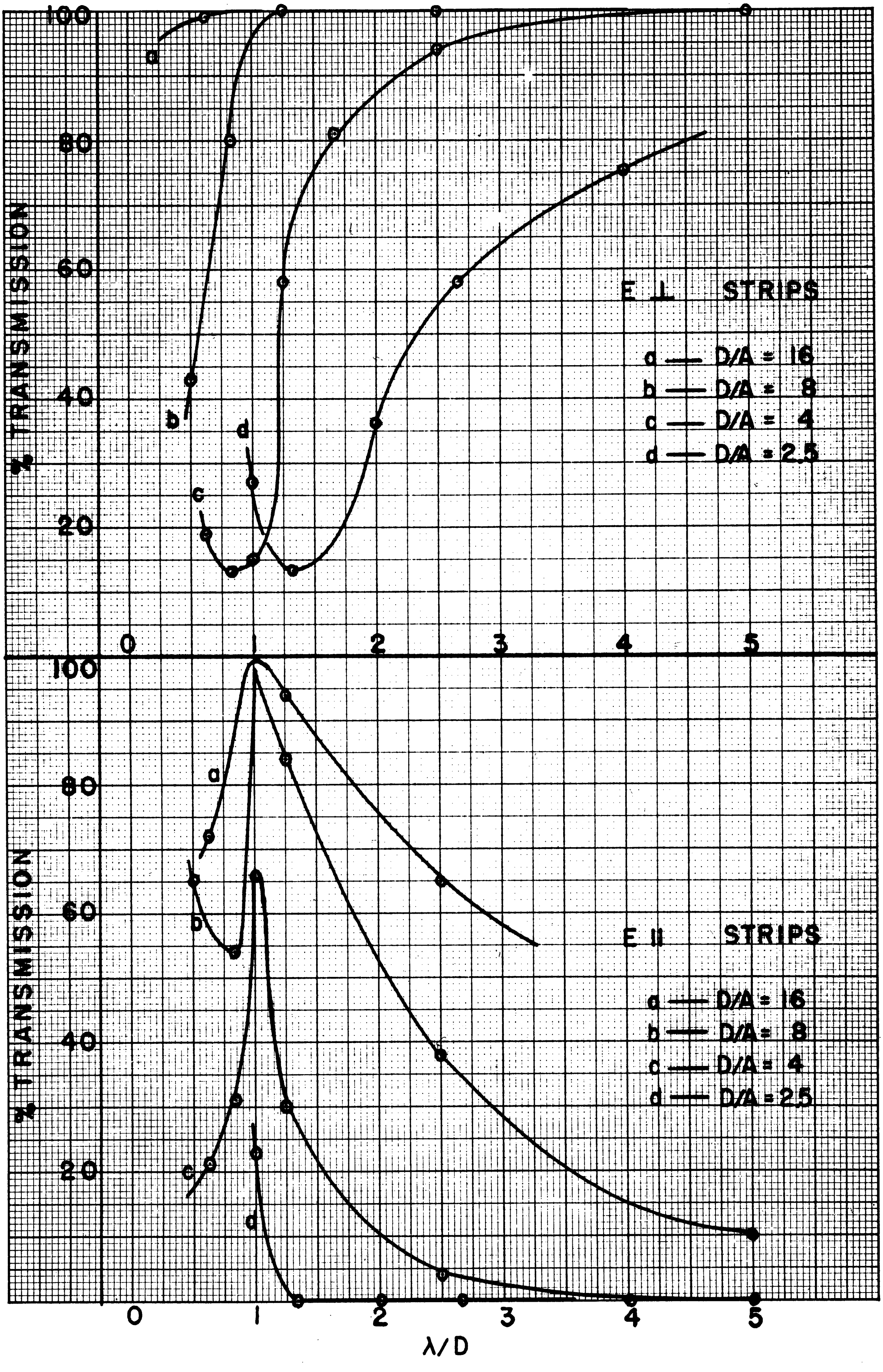
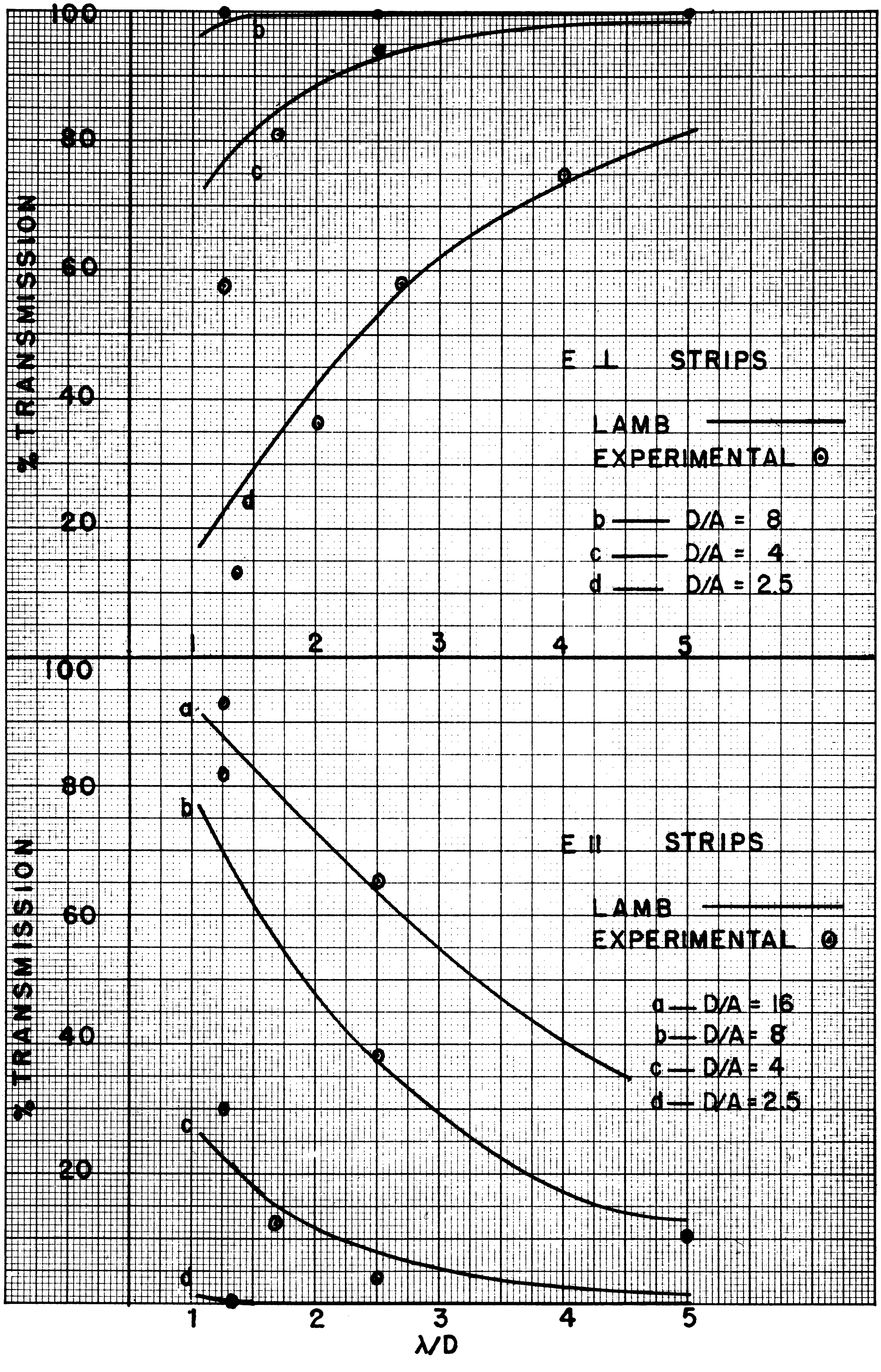

100
80
60
40
20
% TRANSMISSION
a
b
c
d
E ⊥ STRIPS
a — D/A = 16
b — D/A = 8
c — D/A = 4
d — D/A = 2.5
0
1
2
3
4
5
100
80
60
40
20
% TRANSMISSION
a
b
c
d
E ll STRIPS
a — D/A = 16
b — D/A = 8
c — D/A = 4
d — D/A = 2.5
0
1
2
3
4
5
λ/D

FIGURE 5



100
80
60
40
20
% TRANSMISSION
b
c
d
E ⊥ STRIPS
LAMB
EXPERIMENTAL ⊙
b — D/A = 8
c — D/A = 4
d — D/A = 2.5
1
2
3
4
5
100
80
60
40
20
% TRANSMISSION
a
b
c
d
E ∥ STRIPS
LAMB
EXPERIMENTAL ⊙
a — D/A = 16
b — D/A = 8
c — D/A = 4
d — D/A = 2.5
1
2
3
4
5
λ/D

FIGURE 6

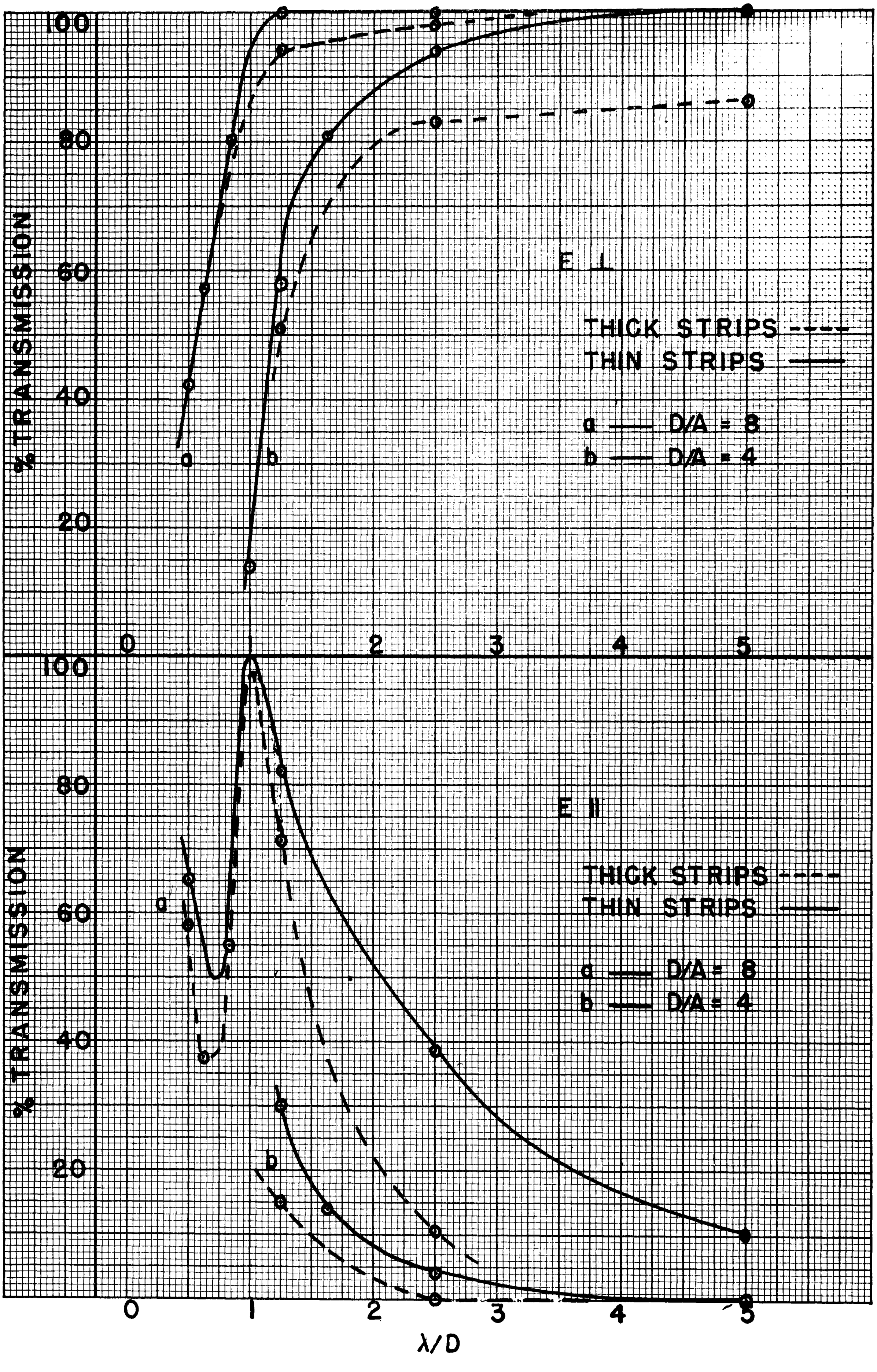
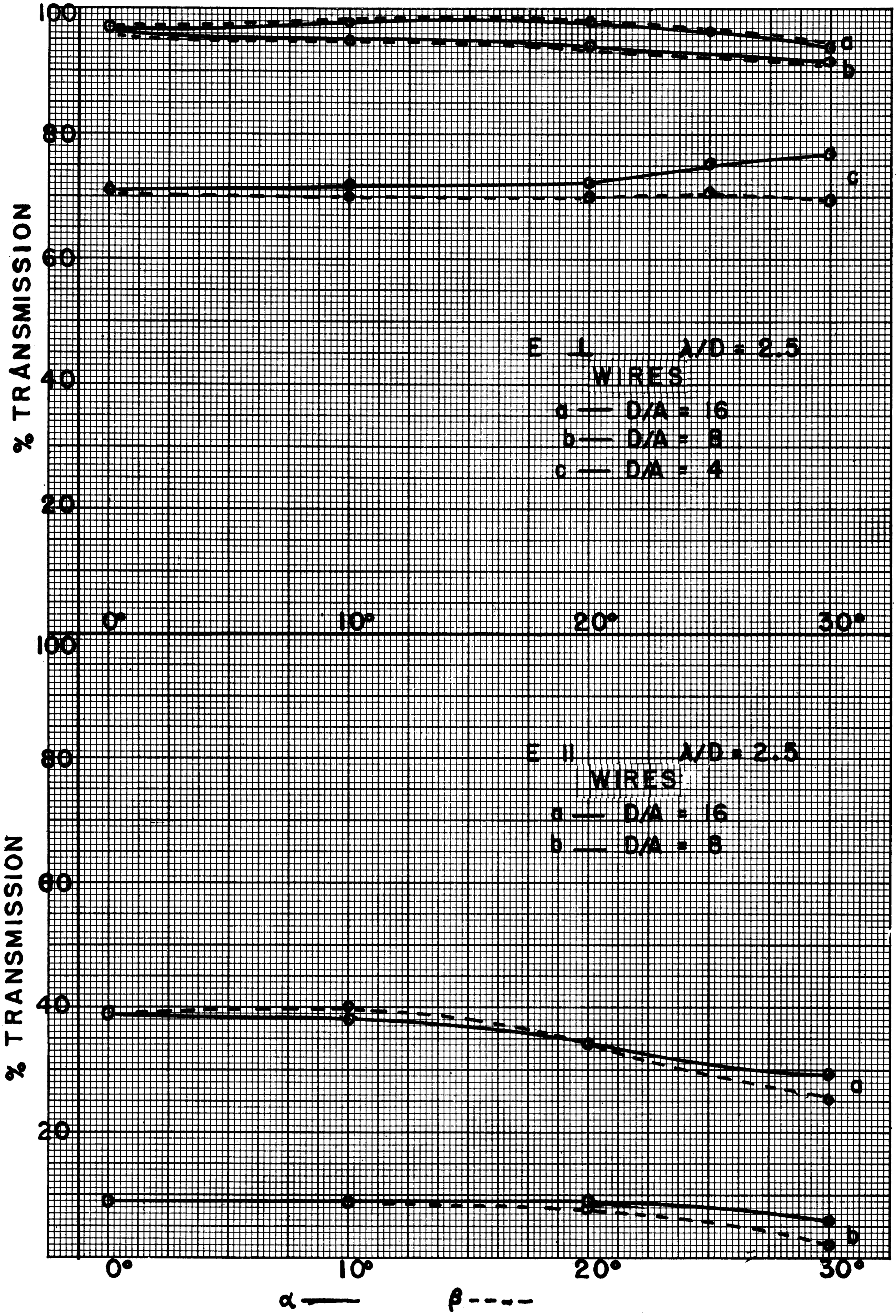

100
80
60
40
20
% TRANSMISSION
a
b
0
2
3
4
5
E ⊥
THICK STRIPS ----
THIN STRIPS ——
a —— D/A = 8
b —— D/A = 4
100
80
60
40
20
% TRANSMISSION
a
b
E ∥
THICK STRIPS ----
THIN STRIPS ——
a —— D/A = 8
b —— D/A = 4
0
1
2
3
4
5
λ/D

FIGURE 7



100
80
60
40
20
% TRANSMISSION
E ⊥ λ/D = 2.5
WIRES
a — D/A = 16
b — D/A = 8
c — D/A = 4
a
b
c
0°
10°
20°
30°
100
80
60
40
20
% TRANSMISSION
E ∥ λ/D = 2.5
WIRES
a — D/A = 16
b — D/A = 8
a
b
0°
10°
20°
30°
α ——— β - - - -

FIGURE 8

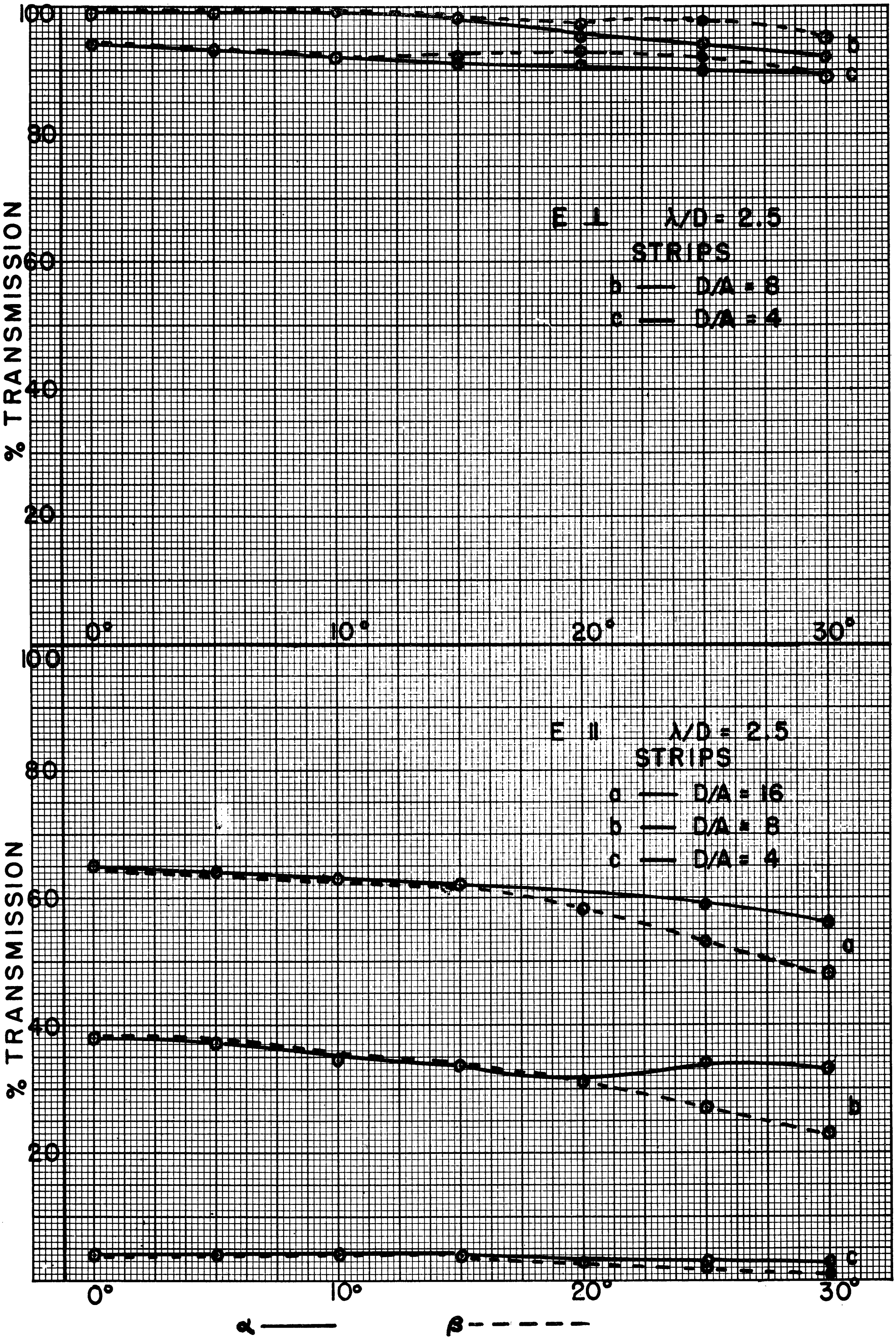
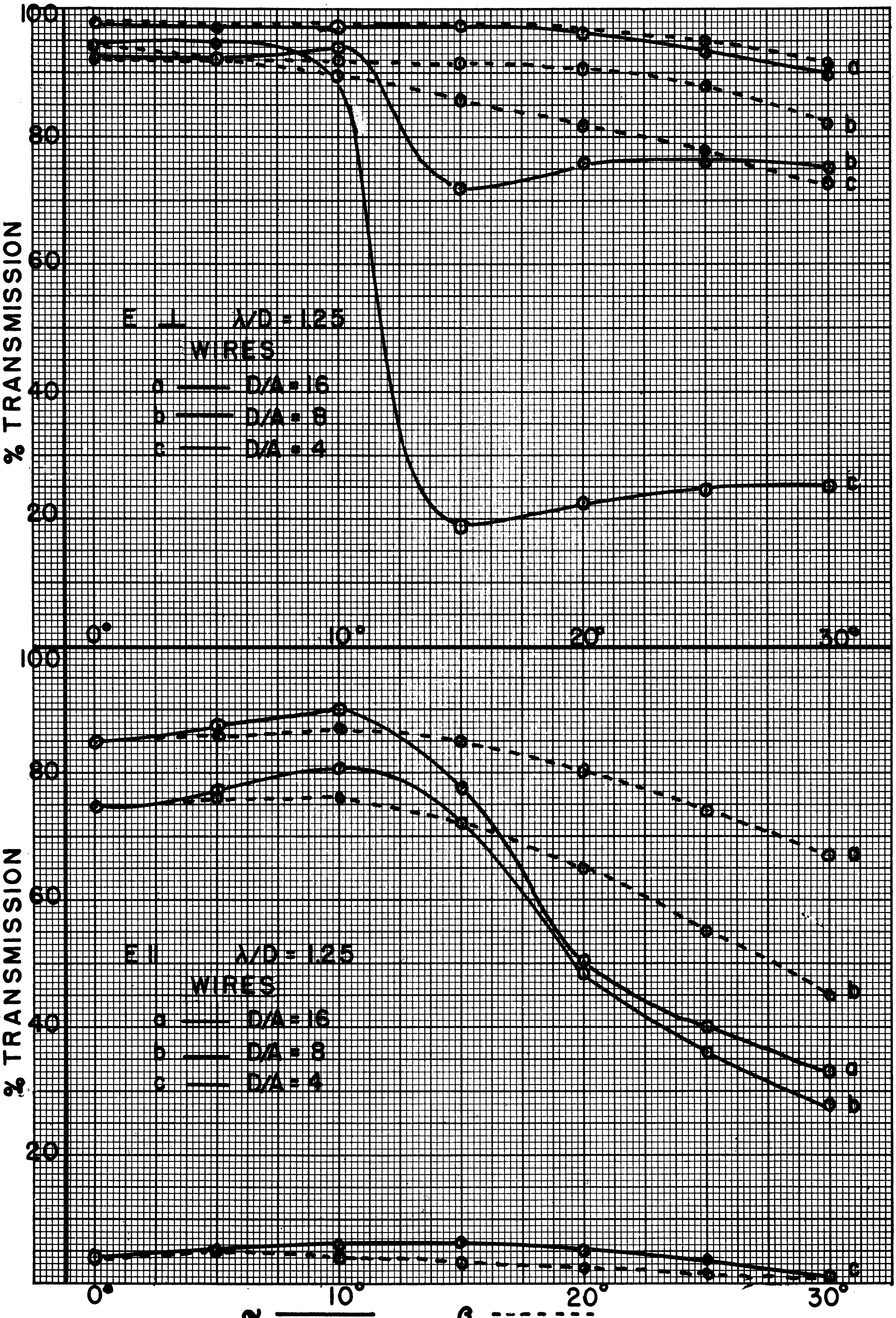
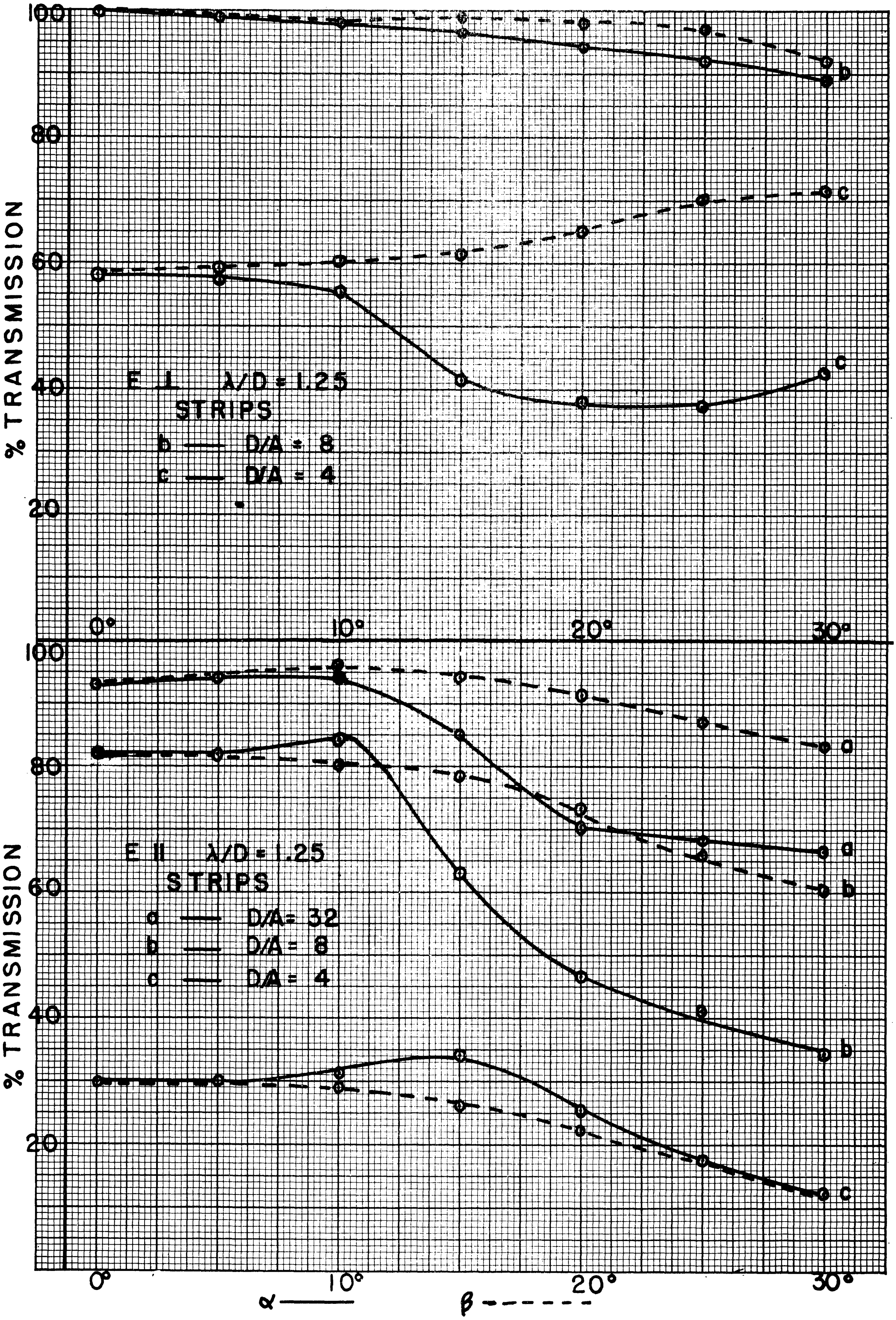

100
80
60
40
20
% TRANSMISSION
E ⊥ λ/D = 2.5
STRIPS
b — D/A = 8
c — D/A = 4
b
c
0°
10°
20°
30°
100
80
60
40
20
% TRANSMISSION
E ∥ λ/D = 2.5
STRIPS
a — D/A = 16
b — D/A = 8
c — D/A = 4
a
b
c
0°
10°
20°
30°
α ——— β - - - - -

FIGURE 9



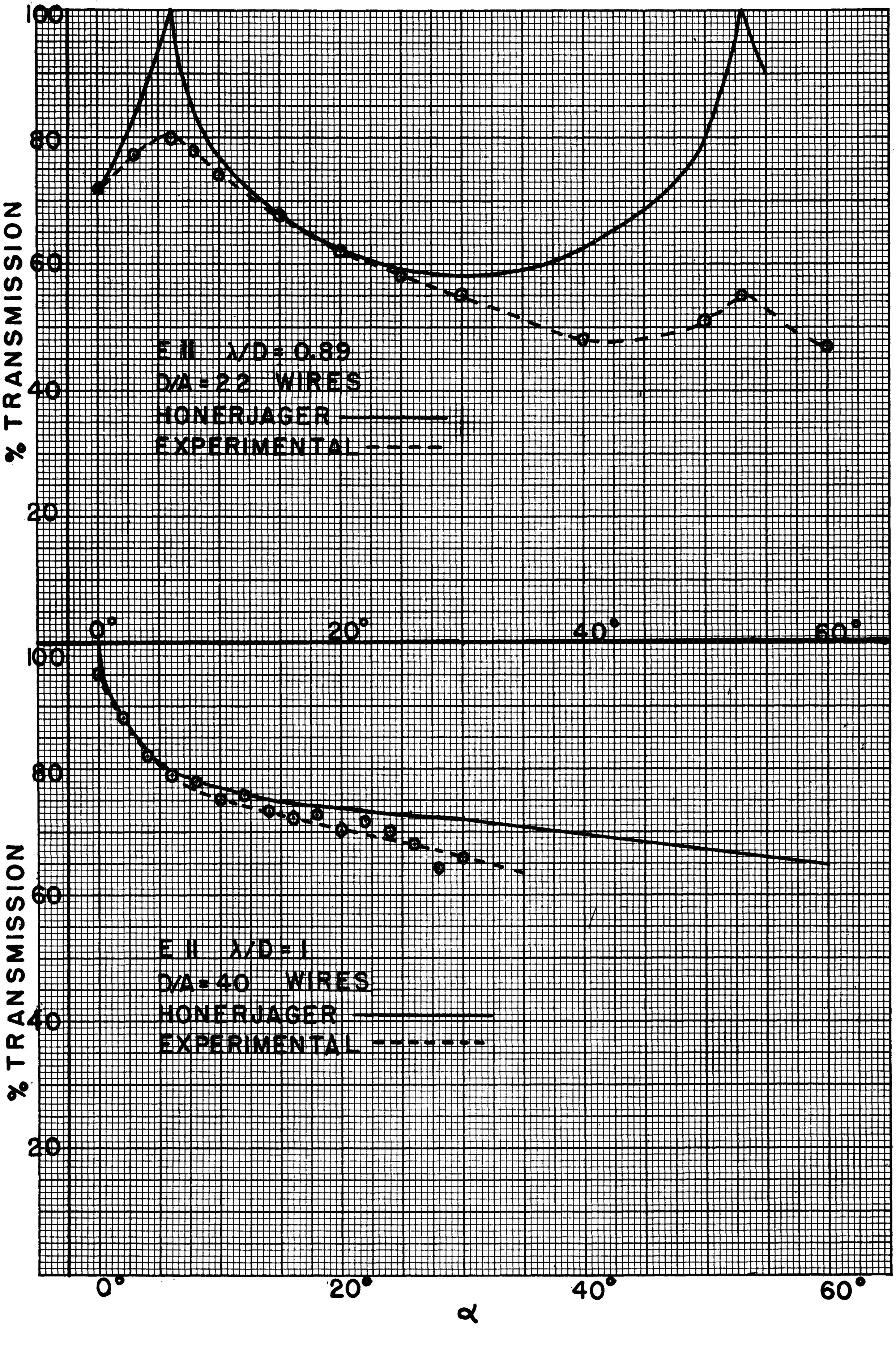
100
80
60
40
20
% TRANSMISSION
E ⊥ λ/D = 1.25
WIRES
a D/A = 16
b D/A = 8
c D/A = 4
a
b
b
c
c
0°
10°
20°
30°
100
80
60
40
20
% TRANSMISSION
E ‖ λ/D = 1.25
WIRES
a D/A = 16
b D/A = 8
c D/A = 4
a
b
a
b
c
0°
10°
20°
30°
α
β

FIGURE 10



100
80
60
40
20
% TRANSMISSION
E ⊥ λ/D = 1.25
STRIPS
b —— D/A = 8
c —— D/A = 4
b
c
c
0°
10°
20°
30°
100
80
60
40
20
% TRANSMISSION
E ll λ/D = 1.25
STRIPS
a —— D/A = 32
b —— D/A = 8
c —— D/A = 4
a
a
b
b
c
0°
10°
20°
30°
α ——
β - - - - -

FIGURE 11


100
80
60
40
20
% TRANSMISSION
E ll λ/D = 0.89
D/A = 22 WIRES
HONERJAGER
EXPERIMENTAL
0°
20°
40°
60°
100
80
60
40
20
% TRANSMISSION
E ll λ/D = 1
D/A = 40 WIRES
HONERJAGER
EXPERIMENTAL
0°
20°
40°
60°
α