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DYNAMICALLY LOADED FOUNDATIONS

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DYNAMICALLY LOADED FOUNDATIONS

by F. E. Richart, Jr.*

for

Symposium on Bearing Capacity and

DYNAMICALLY LOADED FOUNDATIONS

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INTRODUCTION

A dynamically loaded foundation is one for which the effect of rate of loading introduces significant differences between the actual displacements and failure conditions and those which might be computed by conventional static procedures. Often a velocity or acceleration limit must be established for the motion of a satisfactory foundation. The definition of failure is also related to the intended use of the foundation and may vary from an intolerable elastic motion of only several thousandths of an inch to footing displacements under blast loadings on the order of one or two feet.

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The quantities which enter into the analysis, in addition to the considerations needed for static bearing capacity solutions, involve the mass of the footing, the unit weight of the soil, and the strain-rate response of the soil during its deformation. The importance of each quantity must be evaluated for the specific condition of applied loads and definitions of failure.

On Figure 1 are illustrated several rigid shallow footings which are subjected to vertical loads. It is assumed that the footings maintain continuous contact with the soil, that no seepage forces exist, and that the soil will not fail due to liquefaction or development of localized dynamic pore pressures. Within the limits of these assumptions the sketches shown on Figure 1 illustrate a variety of vertical loads which may be applied to a footing. Figure 1(a) shows the usual static dead load applied to a footing while Figure 1(b) illustrates the static dead load plus a slowly changing live load which may vary during the life of a structure. Figure 1(c) illustrates a footing sustaining a static dead load plus a steady-state oscillating vertical load. Figure 1(d) indicates a static plus an impulsive load which often is of low enough magnitude that the footing does not suffer important settlements. Figure 1(e) illustrates a static load plus a blast load capable of forcing the footing into the soil an appreciable distance. For the latter condition it is a question then of how far the footing penetrates during the time the load is applied. Finally, Figure 1(f) illustrates the problem of a projectile striking against the surface of the soil and penetrating until its velocity is brought to zero. The latter case is at the extreme end of the compaction problem in which a compactor hammers the soil surface repeatedly. The discussion which follows will be confined to conditions described by Figures 1(c), (d), and (e).

Figure 2 describes the forces which exist on a footing at any particular instant during a dynamic loading. The external vertical load, Q_z, is shown acting downward. It is resisted by the force developed by accelerating the foundation mass plus the force acting at the base of the footing which represents the dynamic reaction (P) of the supporting soil in response to motion of the footing. The soil reaction may be both time-dependent and displacement-dependent, that is, the resistance may be a function of the rate of loading and the amount of dynamic motion of the footing base. The most significant problem associated with the evaluation of the dynamic response of footings is to determine the soil reaction force, P, and relate this to soil strength properties through an appropriate theory. Analytical solutions are readily available for determining the footing response under steady-state or impulsive loadings after the quantity, P, has been evaluated.

EFFECTS OF STRAIN-RATE ON SOIL PROPERTIES

Any procedure devised to evaluate the strain-rate effect on the stress-strain characteristics of a particular soil must attempt to duplicate as closely as possible the boundary or confining condition which exists in the field. However, up until the past few years it has been customary to evaluate the dynamic stress-strain behavior of soils by means of a dynamic triaxial test. Under these conditions, lateral expansion of the soil sample was permitted and the tangent to the stress-strain curve gradually decreased in slope until it became horizontal at the maximum load. However, an effective confining pressure of undetermined magnitude was introduced by the lateral inertia of the sample. More recently, both static and dynamic stress-strain curves have been obtained for different materials using test equipment which did not permit lateral expansion of the sample (Whitman, Roberts, and Mao, 1960, Davisson, 1963, and Zaccor and Wallace, 1963).

Dynamic triaxial tests of dry sand were carried out by Casagrande and Shannon (1948) using rapid loads applied by a falling beam-type apparatus. They found an increase in shearing strength of 10 to 15 per cent and an increase in modulus of deformation of about 30 per cent above the values found for static loading conditions. Seed and Lundgren (1954) investigated the strength and deformation characteristics of saturated specimens of a fine and a coarse sand under dynamic loadings. The rapid loading was applied by a falling weight impact testing machine. They found an increase in strength of 15 to 20 per cent and about 30 per cent increase in modulus of deformation under the dynamic loading conditions. They noted that dilatancy effects and lack of drainage contributed to the strength under dynamic loads. Taylor and Whitman (1954) reported the results of dynamic triaxial tests of saturated sands in which one of their major objectives was to measure the time change of pore pressure. Whitman and his co-workers have subsequently developed dynamic pore-pressure measuring devices which they feel are adequate for rapid triaxial tests in which the loads may be applied in the order of 10 seconds for clays and 0.01 seconds for sands. Nash and Dixon (1961) devised a somewhat less sensitive dynamic pore-pressure measuring device and presented curves of load versus time, displacement versus time, and pore-pressure versus time for a dynamic triaxial test on a saturated sand. Their test results clearly indicated intermittent adjustments in the soil structure by a temporary drop in axial load accompanied by a corresponding increase in pore pressure. In this test the time required for the soil to adjust its structure was smaller than the time rate of external load application. Essentially their device was a constant rate of strain loading machine which could permit and record these transient unloadings.

Casagrande and Shannon (1948) also studied intensively the behavior of clays and soft rock under dynamic loading conditions. Their studies indicated that the water content of the clay has an important influence on the relation between time of loading and compressive strength. The per cent increase of the fast transient strength over the static compressive strength was greatest for specimens of the highest water content and least for specimens of the lowest water content. The fast transient compressive strength, taken at a time of loading of 0.02 seconds, ranged between 1.4 and 2.6 times greater than the 10 minute static compressive strength for all tests on clays. For the same dynamic loading conditions the modulus of deformation (secant modulus determined at one half the ultimate strength) for all types of clays tested was found to be approximately twice that for the 10 minute static loading. Taylor and Whitman (1954) evaluated this strain-rate effect on ten different cohesive materials. Generally an increase in the strain rate from zero to 1000 per cent per second increased the ultimate strength by a factor of 1.5 to about 3, which is of the same order of magnitude as that found by Casagrande and Shannon. Recent tests by Jackson and Hadala (1964) indicated that the strain-rate factor for the compacted clay used in their tests was of the order of 1.5 to 2.0. This strain-rate factor was again the amount of increase in modulus of deformation and ultimate strength above that obtained for static conditions. Much work still needs to be done to evaluate the strain-rate sensitivity of different soils to single loading tests and to repeated loadings in both the unconfined and confined conditions. Direct shear or ring shear devices have been adopted for both impulsive loadings (Saxe, Graves, and Schimming, 1964) and slowly repeated loadings (Converse 1961), for example. The effects of repeated loads on compaction of sands and on the modifications of soil strength and structure of clay will contribute useful information for evaluating the soil response under dynamically loaded footings.

DYNAMIC MODULUS OF SOILS

In addition to the strain-rate effects which might be evaluated for soils which will be subjected to large strains, it is often necessary to evaluate the stiffness of soil under conditions of small deformations. When the deformations are of a magnitude that the soil structure is not significantly changed by repeated deformations, the dynamic behavior of a foundation may be estimated by use of an elastic theory. The soil property required for use in such a theory is a modulus of elasticity or a modulus of deformation for relatively small displacements. This quantity may be evaluated by laboratory tests or by in-situ field measurements.

Laboratory test data for both sand and clay have been obtained by Wilson and Dietrich (1960) while information for clean cohesionless sands has been prepared by Shannon, Yamane, and Dietrich (1960), and by Hardin and Richart (1963). The work by Wilson and Sibley (1962) points out the probable modifications of the modulus of deformation as a function of the amplitude of motion involved. A field method for evaluating the shear modulus of soil in-situ has been developed by personnel of the U. S. Army Engineer Waterways Experiment Station and is summarized in the reports by Fry (1963) and Ballard (1964). This method has been used with reasonable success for both cohesionless and cohesive materials.

THEORY OF RESPONSE OF FOOTINGS

The most popular theory to be used for estimating the behavior of a rigid footing subjected to a dynamic load is the single degree of freedom system with viscous damping. The terms in this theory are illustrated by the equation of equilibrium below:

$$m_o \frac{d^2z}{dt^2} + c\frac{dz}{dt} + kz = Q$$

in which m_o = the mass of the footing

C = damping factor

k = spring constant

z = vertical displacement

Q = external force

The early work by DEGEBO (1933) attempted to evaluate damping quantities from tests of model footings subjected to vertical vibrations. This was relatively unsuccessful. Later developments by the DEGEBO group, and others, eliminated consideration of the damping term but attempted to make theory fit test data by modifying the mass of soil which moves "in phase" with the footing. This approach has also not been successful. A solution based upon the theory of elasticity, in which the soil is considered as a semi-infinite elastic homogeneous isotropic body and the footing is represented by pressures distributed over a circular area of a surface, was first proposed by Reissner (1936) and has been improved by Sung (1953), Quinlan (1953) and others within the past decade. Richart (1962) has demonstrated the use of this theory in the study of vibration of foundations.

It has been demonstrated by Hsieh (1962), Hall and Richart (1963) and Lysmer (1964) that information obtained from the elastic half-space theory may be interpreted as the spring constant and viscous damping factor needed for successful application of the single degree of freedom viscous damped theory. Lysmer (1964) also illustrated how the elastic half-space theory may be utilized to evaluate the response of a rigid footing resting on the elastic body when subjected to transient or impulsive loads.

With the assistance of the elastic half-space theory to provide spring constants and damping factors for use in the single degree of freedom system, it now becomes possible to introduce these quantities into the Phase-plane graphical procedure for determining footing response for a damped system. The Phase-plane procedure is described in Jacobsen and Ayre (1958).

FIELD AND LABORATORY TESTS FOR FOOTINGS UNDERGOING SMALL MOTIONS

In this section, results will be considered for footing responses under conditions where no appreciable permanent settlement is developed. In addition to the elimination of settlement, there are many instances in which vibrations of the order of $\pm$ 1/1000" cannot be tolerated by personnel or by structural components. The limits of allowable vibrations were summarized by Richart (1962).

A comprehensive series of field tests was carried out by the personnel of the U. S. Army Engineer Waterways Experiment Station during 1960 to 1963 and the results are summarized in a technical report (Fry, 1963b). Five sizes of footing ranging from approximately 5 feet to 16 feet in diameter were subjected to vertical oscillation by means of a mechanical oscillator. The first series of tests was conducted at Vicksburg with the footings resting upon a uniform deposit of loess, classified as a silty clay, and the second series was run at Eglin Field, Florida, on a deposit of uniform fine sand. This work represents the most significant collection of information regarding the behavior of footings loaded by vibratory loads obtained up to the present time.

Figure 3 includes a summary of the information obtained from the amplitude of oscillation of the WES footings when subjected to vertical exciting forces. The black dots on Figure 3 indicate the test results obtained from the WES site while the open circles correspond to results from the Eglin Field location. The ordinates represent the ratio of amplitude of vertical oscillation computed by the Reissner-Sung theory (based on the elastic semi-infinite body) to the measured amplitude of oscillation from test results. The abscissa of the diagram represents the measured maximum acceleration of the footing compared to the acceleration of gravity. From Figure 3 it is seen that the agreement between measured and computed values is relatively good considering the wide range of footing sizes used and the appreciable difference in soil characteristics. At the present state of the art in evaluating vibrations of foundations, an amplitude estimate within a factor of 2 is generally considered quite satisfactory.

A limited number of tests on a footing 1 foot in diameter resting on the surface of dense Ottawa sand were conducted at the University of Michigan using an impulsive load. The amplitude of motion was restricted to such values that the footing did not suffer measurable permanent settlement after the loading test was completed. Figure 4 shows the measured acceleration of the footing as well as the values of acceleration computed from the elastic half-space theory by Lysmer's theoretical solution. In this case the agreement between measured and computed values is good. Figure 5(b) shows the agreement between the computed value of displacement versus time and that obtained by the graphical phase-plane procedure. Since both of these are theoretical solutions the purpose is mainly to illustrate that the approximate loading diagram represented by the rectangular blocks in Figure 5(a) gives a response which is very close to that obtained using the loading diagram shown by the continuous lines. It should be noted that the value of shear modulus for the sand, used in the theoretical solutions, was determined from the graphs given by Hardin and Richart (1963) after computing the void ratio of the sand and an average confining pressure below the periphery of the footing.

SUMMARY OF DYNAMIC BEARING CAPACITY

Footings on sand

One of the original aims of the DEGEBO group in Germany in the early 1930's was to evaluate the effect of vibrations on the settlement of footings resting on sand. Partly because of the insensitivity of amplitude-measuring equipment, these footings were subjected to vertical accelerations which in many cases produced a jumping and hammering action, and settlement did occur. A decade later, Tschebotarioff and McAlpin (1946) made small-scale plunger tests on clay and sand and found the sands were extremely sensitive to vibration. They found that with a vibratory force of only $\pm$ 2% of the static load, the settlements were much greater than for repeated static loads. This result was undoubtedly discouraging to engineers who must design foundations for machines. However, a dynamic force of only 2% of the static load may produce a much higher contact force at the base of a vibrating footing, depending upon how close the operating frequency is to resonance and how much damping is involved. Fortunately a large footing has an effective geometrical or dispersion damping. Further tests for the purpose of estimating the effects of vibration and impact on compaction of sand were conducted by the group at CalTech (Converse, 1952) primarily to determine the efficiency and depth of compaction produced by vibrating footings. There is still need for further research on the magnitude of vibrations and impact forces which may be successfully resisted by both dry and saturated cohesionless materials without developing significant compaction.

One step beyond the compacting effect is the behavior and settlement of a footing when it is loaded by a blast-type loading. Under conditions of rapidly applied pulse-type loadings the footing will be accelerated rapidly and forced into the soil, sometimes to appreciable distances. Thus the problem of dynamic bearing capacity includes additional effects of the type and characteristic of the impulsive load, the mass and shape of the footing even if it is assumed to be rigid, and the unit weight, strength, and deformation characteristics of the underlying soil. In order to evaluate the soil resistance characteristics, many model and a few prototype tests have been conducted. Model tests on sand have been conducted by Taylor and Whitman (1954), Selig and McKee (1961), Shenkman and McKee (1961), Fisher (1962), White (1964), Cunny and Sloan (1961), and Vesic, et al (1965), for example. One of the most significant results from these tests is an indication that, even for dense sand, if the rate of loading is high enough the failure mode is by punching of the footing into the soil rather than by the general shear failure which is exhibited under static loading conditions. Heller (1964), in evaluating the failure modes for impact loaded footings on dense sand, has estimated that if the footing acceleration exceeds about 13 g, the failure mode will be by punching. He interprets the lateral inertia developed by sand particles in being shoved aside from beneath the footing as an effect comparable to a significant depth of embedment. Vesic (1963) has demonstrated that even for dense sand the failure of deep footings corresponds to a punching type of failure rather than local shear or a general shear type of failure. For conditions in which a shallow footing on sand is accelerated to less than 13 g it is possible for general shear type of failures to occur, with bearing capacities still appreciably higher than those predicted by static methods. For relatively slow rates of loading of footings resting on a sand surface, a minimum value for bearing capacity was observed (Vesic, Banks, and Woodard, 1965). The slight increase in bearing capacity under very slow rates of loading is probably caused by the fact that the sand particles have time to adjust their position when a new load increment is added. If this time is not allowed, there will be an apparent drop of shearing strength below that obtained for long-time loadings and, of course, when the rate of loading is increased significantly, inertia effects contribute lateral restraint and the bearing capacity then is increased.

Footings on clay

Relatively few organizations have run model tests of the dynamic bearing capacity of footings on clay. The most interesting of these tests have been run by personnel of the U. S. Army Engineer Waterways Experiment Station and results are presented in reports by Sloan (1962), Jackson and Hadala (1964), and Carroll (1963). The report by Carroll includes an evaluation of the test information obtained by Jackson and Hadala and a survey of the theoretical procedures which are available for interpretation of these test results. The tests by Jackson and Hadala were made on beds of compacted buckshot clay prepared in movable carts, which were moved at the time of the tests into a position below the WES dynamic loading machine. The dynamic loading machine is described in detail in the report by Sloan (1962). Load was applied to square footings from 5 1/2 to 16 inches on a side with different programmed loading patterns. A careful attempt was made in designing these tests to provide model parameters on the basis of dimensional analysis, whereby the test results should be applicable to prototype conditions. This included careful control of the shear strength of the compacted clay and the use of different clay strengths in tests of different footing sizes.

Typical test information is reproduced in Figure 6 which corresponded to Test 9-1 as reported by Jackson and Hadala. An impulsive load was applied at the top of a loading column to force the square bearing plate into the soil. The failure mode was by punching of the footing into the clay, in contrast to the general shear failure obtained under static loading. Figure 6(a) illustrates the computed column load, Figure 6(b) shows the measured column acceleration in terms of the acceleration of gravity, Figure 6(c) represents the load cell reaction measured by a load cell located between the loading column and the square footing, and finally, Figure 6(d) shows the average displacement of the rectangular plate. Similar results have been obtained for more than 85 tests of square footings on the buckshot clay. From the results of these tests, Jackson and Hadala concluded that the principles of similitude can be employed quite advantageously in the conduct and analysis of small-scale footing tests. This leads to a certain amount of confidence in prediction of dynamic displacements of prototype footings subjected to comparable blast loadings. For these footings they found a unique relation between the maximum strength parameter which is $P_{max}/\tau_f B^2$

and the maximum displacement parameter, z/B .

(P is the load applied to the soil surface, B is the width of the square plate, and τ_f is the static shearing resistance of the clay, obtained from the unconfined test.)
This relation ties in the ratio of average vertical pressure to the shear strength of the soil, and the displacement as a proportion of the plate width. From the static plate bearing tests and laboratory unconfined tests they obtained a reasonable estimate of the relation between the dynamic strength parameter and displacement parameter $\left(P_{max}/\tau_f B^2 \text{ vs. } z_{max}/B \right)$
This relation can be obtained approximately by first multiplying the plate bearing loads by a factor of 1.5, then plotting the results in dimensionless form. This means that if we can determine a static load-displacement diagram and multiply this by a suitable strain-rate factor, we can obtain an approximate dynamic load-strain diagram. The load-displacement curve for tests on footings in Cart 9 are shown in Figure 7.

The report by Carroll (1963) summarizes much of the work done on dynamic bearing capacity of clays and, in particular, presents an analysis of the test results obtained by Jackson and Hadala. He reviewed the several methods for evaluating dynamic bearing capacities which are now available and discusses the applicability of each. In his comparisons of the static and dynamic stress-strain properties of buckshot clay he found that the two-parameter rectangular hyperbola was found to give a simple, fairly accurate, and useful mathematical formulation of the unconfined unconsolidated stress-strain relation. He also found that the time-dependence of the stress-strain properties of the buckshot clay could be represented by a single static and a single dynamic curve at comparable water contents and confining pressures. He illustrated how the stress-strain information could be utilized to develop a load-settlement relation by Skempton's (1951) procedure. The predicted curves agreed quite well with the mean experimental curve.

In his conclusions, Carroll emphasized that the spring-mass-dashpot dynamic system may be coerced to give valid predictions if the time-dependent parameters of the system can be defined. He also noted that if a relation

$$\frac{v_d t_o}{B} \leq 1$$

where v_d = dilatational wave velocity in the clay,

t_o = the rise time to peak load,

and B = the width of the square plate, then the elementary wave theory with constrained dynamic stress-strain curve should reliably predict the footing displacements. He also found that if the relation

$$\frac{v_r t_o}{B} \geq 2.5$$

(in this case v_r is the rod wave velocity), inertia stresses exerted only minor influence on the displacement. In his analysis he noted that the theory for the footing on a semi-infinite elastic body should be helpful in clarifying the nature of the problem.

It should be noted that if the rise time, t_o is interpreted as one quarter of the period of oscillation for a footing on the elastic half space, the parameters noted above may be interpreted in terms of the dimensionless frequency parameter, a_o, in the Reissner-Sung theory as

$$\frac{v_d t_o}{B} = \frac{\sqrt{\frac{2(1-\mu)}{1-2\mu}} \cdot v_s}{2r_o \cdot 4f_o} = \frac{\pi}{4}\sqrt{\frac{2(1-\mu)}{1-2\mu}} \cdot \frac{1}{a_o} \leq 1$$

If we introduce μ = 3/8, as estimated by Carroll, the above expression indicates that if $a_o \geq 1.8$ the elementary wave theory is acceptable. For the other criterion,

$$\frac{v_r t_o}{B} = \frac{\sqrt{2(1+\mu)} \cdot v_s}{2r_o \cdot 4f_o} = \frac{1.3}{a_o} \geq 2.5$$

or if $a_o \leq 0.5$, the inertia effects are unimportant. By comparing these rather arbitrary limits with the curves on Fig. 7 of Richart (1962), it is seen that the first of these criteria is reasonable. Above $a_o \approx 1.8$ no significant peaks occur on the amplitude magnification curves. Below $a_o \approx 0.5$ the conditions required for a resonance condition to develop demand an intensity of footing loading (i.e. a high b-value) which would ordinarily not be used. Thus his criteria are reasonable, but since a theory is available, the writer would prefer to use it directly.

The model tests conducted on footings resting on clay should be continued, particularly with different depths of embedment. The results should be compared with large scale tests whenever possible. It is interesting to note that the dynamic load-displacement results from these tests can be duplicated rather well by multiplying the value of load at each point by a "strain-rate factor" of 1.5 to 2.0. The strain rate factor is comparable to that determined from static and dynamic triaxial tests of clays.

SUGGESTED METHOD FOR ESTIMATING DYNAMIC RESPONSE OF FOOTINGS

The limited test information available to date provides a basis for a tentative design procedure. Any such procedure will naturally be modified and improved as more experience and information is accumulated.

The most significant part of the procedure is an evaluation of a dimensionless load-settlement diagram which should be established on the basis of the most complete information available. It is apparent that from modelling tests, dynamic load-settlement relations can be established from tests on different sizes of dynamically loaded footings. It also appears that the load-settlement curve thus obtained may be quite similar to a curve obtained from the static load-settlement relations, for which the loads at each value of settlement are multiplied by a strain-rate factor of the order of 1.5 to 2.0. For a preliminary design it would be anticipated that computations for the footing behavior would be made on the basis of a lower and upper limit for the load displacement curve, perhaps using the 1.5 and 2.0 strain-rate factors, or if it is necessary to be quite conservative, to use a 1.0 strain-rate factor and determine the displacements resulting from the impulsive loads.

In order to use an analytical procedure which includes the effects of the curved load settlement diagram, it is convenient to approximate the curve with a series of straight lines. An example of this is shown on Figure 8(a) for which the curved soil load-footing displacement curve for Cart 9 from the Jackson and Hadala tests are approximated by two straight lines, the first defines a spring constant (load/displacement) of 47,600 lbs. per inch and the second defines a spring constant of 4600 lbs. per inch. These are used to approximate the load-deflection curve which was chosen as 1.8 times the value obtained from the static tests shown in Figure 7. If the range of deformation had continued, it might have been necessary to use three or more straight-line segments.

With the straight-line approximations to the load displacement curve, as in Figure 8(2), we can now utilize the phase-plane procedure to evaluate the dynamic response of the footings. In the computations for displacement, from an initial value of zero up to a settlement indicated by 0.11"on Figure 8(a) the spring reaction, k_1 enters into the problem. At the time the displacement equals the value of 0.11", we can evaluate the velocity and displacement of the footing at that particular instant and stop the problem. Then we start the problem again with the initial velocity and displacement just computed, but now introduce k_2, the spring constant for Regime II into the solution. This means we now have an initial velocity and displacement, determined by the coordinates of point 1' and the subsequent vibration is governed by the spring constant k_2 and the damping factor.

The critical part of the phase-plane solution (after the empirical load vs. deflection diagram has been established) involves an estimate of the equivalent viscous damping factor to be introduced into the solution. Lysmer (1964) has shown that a very good approximation can be made to the theoretical solution if we neglect the frequency-dependent terms in the equivalent damping factor indicated by the elastic half-space theory. Using the notation introduced by Hall and Richart (1963) this leads to an expression for the damping ratio (i.e. ratio of existing damping to the critical damping) as

$$\frac{R_v}{R_{vc}} \approx \frac{0.85}{\sqrt{1-\mu}\sqrt{b}}$$

where $b = W_o/\gamma r_o^3$, the mass ratio. The damping ratio is designated by the symbol ν in Jacobsen and Ayre. The expression for the damping ratio indicated above is based on the "rigid base" pressure distribution. For large settlements, it is better to use the value based on a uniform pressure distribution. Thus when we know the dead weight of the footing, the unit weight of the soil, and the radius of a circular footing, or, where a square footing is involved, we may interpret this as a circular footing of equal bearing area, we can establish a damping ratio. For the 8 inch square footing used in Test 9-1 the damping ratio amounts to 0.22.

This damping ratio is then used in the Phase-plane solution as indicated in Figure 8(b). For further elaboration of the Phase-plane solution for problems involving viscous damping, the reader is referred to Jacobsen and Ayre (1958), page 203.

The load-time diagram indicated in Figure 6 for Test 9-1 from Jackson and Hadala has been reproduced as Figure 9(a). Also shown on Figure 9(a) are equivalent rectangular pulses of load acting over increments of time. Of course, this load-time diagram is required as a forcing function for the Phase-plane solution. Figure 9(b) indicates the displacement-time diagram obtained by the Phase-plane solution as compared to that obtained from test measurements. The oscillations in the Phase-plane solution are introduced by the assumed instantaneous changes in computed load which introduce a bouncing effect of the footings The use of smaller blocks of loading time will minimize these oscillations.

CONCLUSIONS

Tentative conclusions from this brief review of the problem of dynamically loaded foundations are enumerated below.

1. For small amplitude vertical steady-state vibrations of shallow foundations, the use of the theory based on the elastic half-space is satisfactory for design purposes.

The psuedo-elastic response of the soil may be established by laboratory or field tests.

2. For small amplitude vertical dynamic response of shallow footings subjected to impulsive loads, an adequate evaluation may be based on the Phase-plane solution after the psuedo-elastic soil properties have been established.
3. Dynamic loading tests of shallow footings on sand and on clay have indicated that failure occurs by "punching" rather than by a general shear failure, as often occurs under static loads. The limited number of tests available up to the present time indicate that a "strain rate factor" may be applied to the static load-settlement curve for a given footing to establish a dynamic load-settlement curve. When presented on a dimensionless plot, this information can be used for design of footings of different sizes.
4. Because the maximum load a dynamically loaded footing can support depends on the length of time the load is applied, compared to the time required for the footing to respond, the term "dynamic bearing capacity" has little meaning and should not be used. We are concerned with the time-dependent motions of the footing in response to a given loading pulse. An estimate of this motion can be obtained by use of the graphical Phase-plane procedure based on a series of straight-line approximations to the curved load-settlement diagram and a damping ratio determined from the elastic half-space theory.

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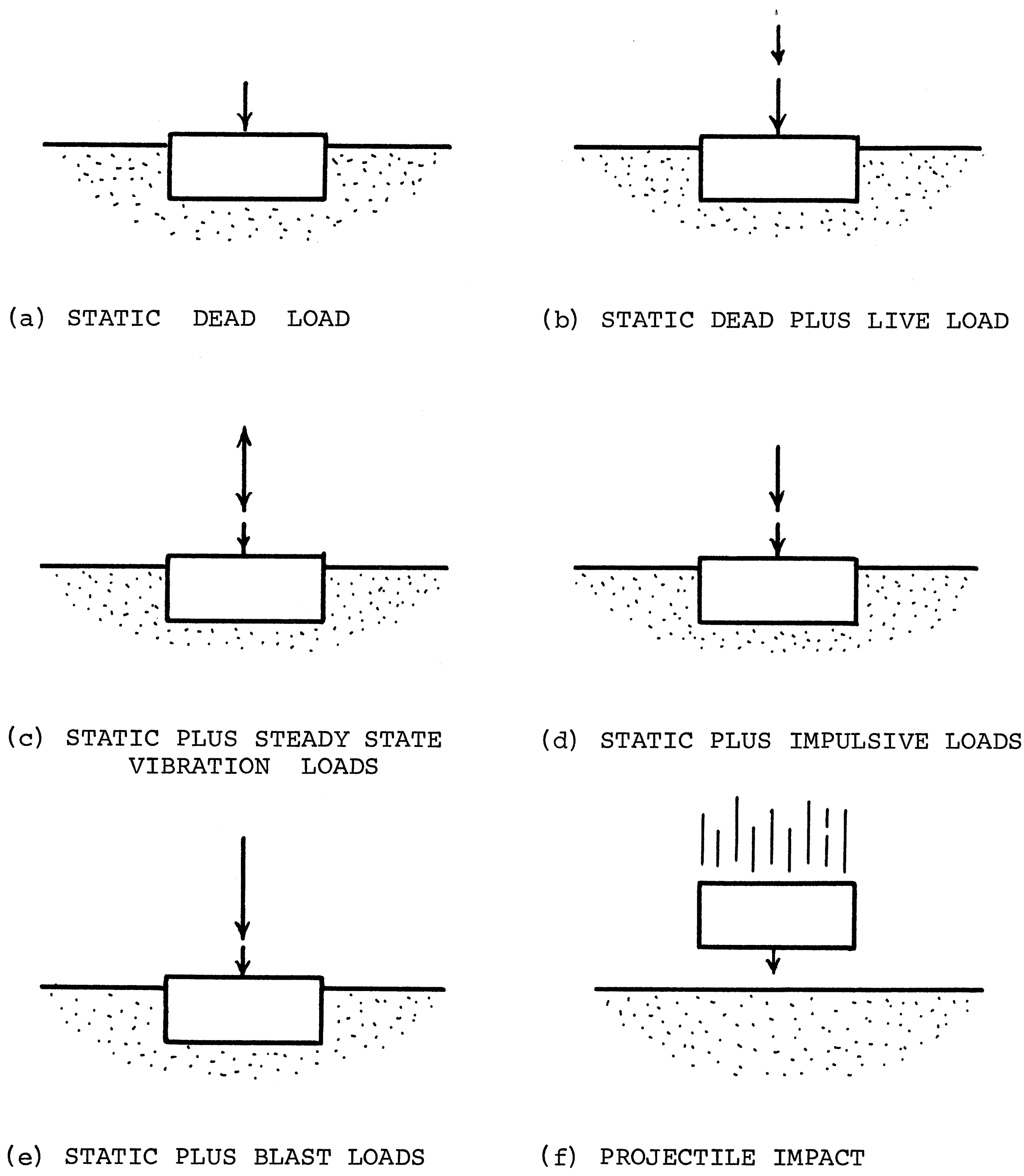
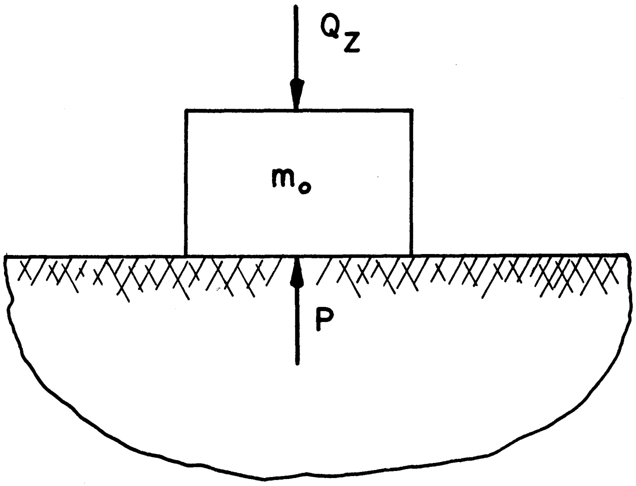


FIGURE 1. VERTICAL LOADS FOR SHALLOW FOOTINGS.



$$M_o \frac{d^2z}{dt^2} = Q_z - P$$

$$\text{USING } P = R_V \frac{dz}{dt} + K_V Z$$

$$M_o \frac{d^2z}{dt^2} + R_V \frac{dz}{dt} + K_V Z = Q_z$$

FIGURE 2. FORCES ACTING ON A DYNAMICALLY LOADED FOOTING

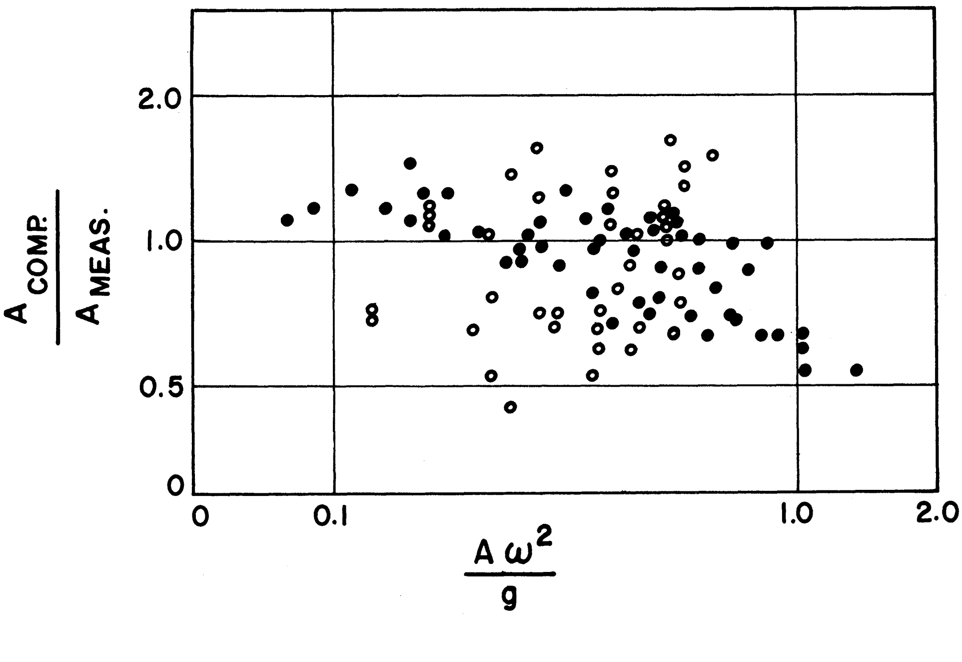


FIGURE 3. COMPARISON OF COMPUTED AND MEASURED VERTICAL AMPLITUDES OF MOTION FROM 81 TESTS OF MODEL FOOTINGS.

(BASED ON DATA FROM WES . 1963)

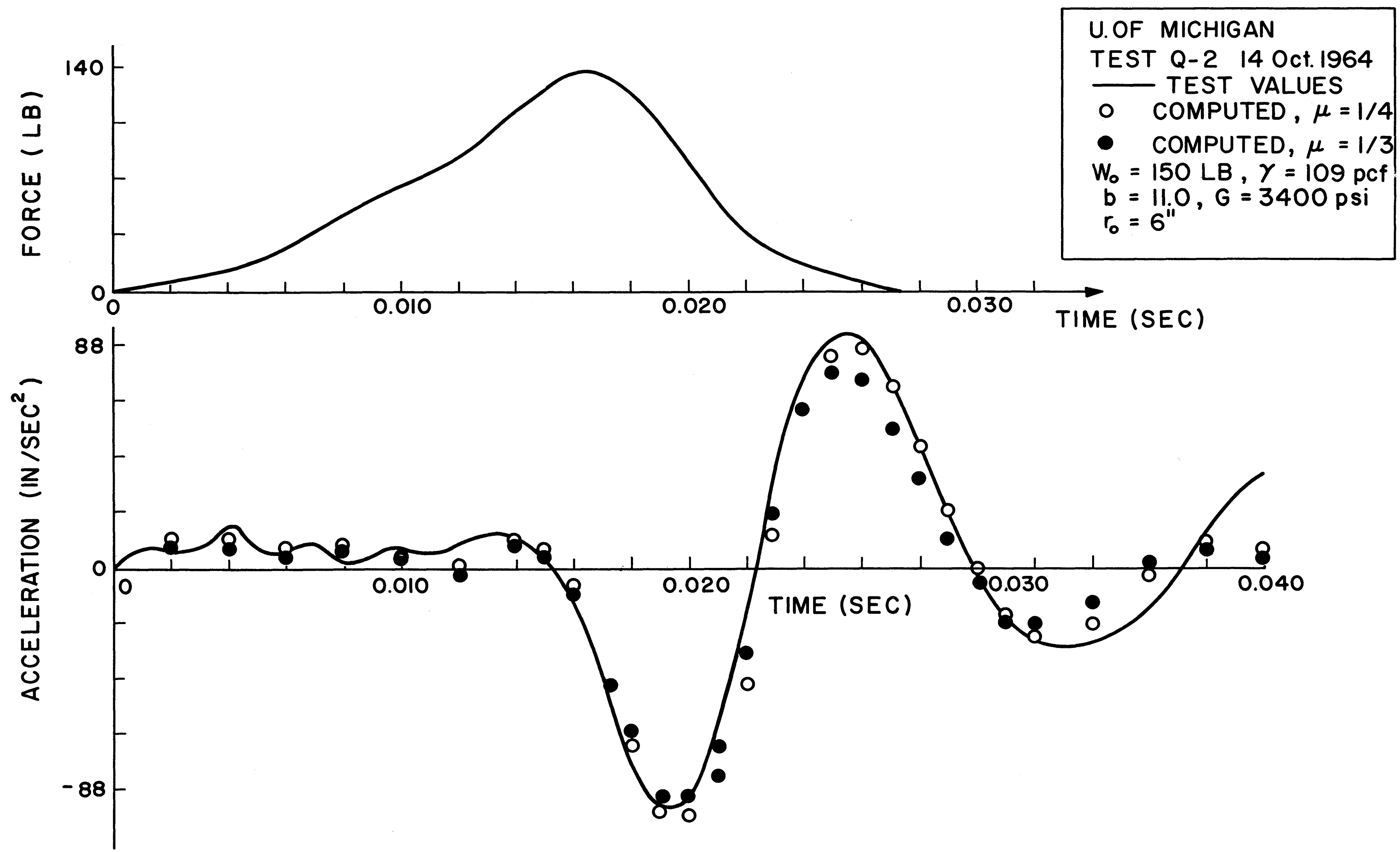


FIGURE 4. COMPARISON BETWEEN COMPUTED AND MEASURED ACCELERATIONS OF 1' DIAMETER IMPULSIVELY LOADED FOOTING ON SAND.

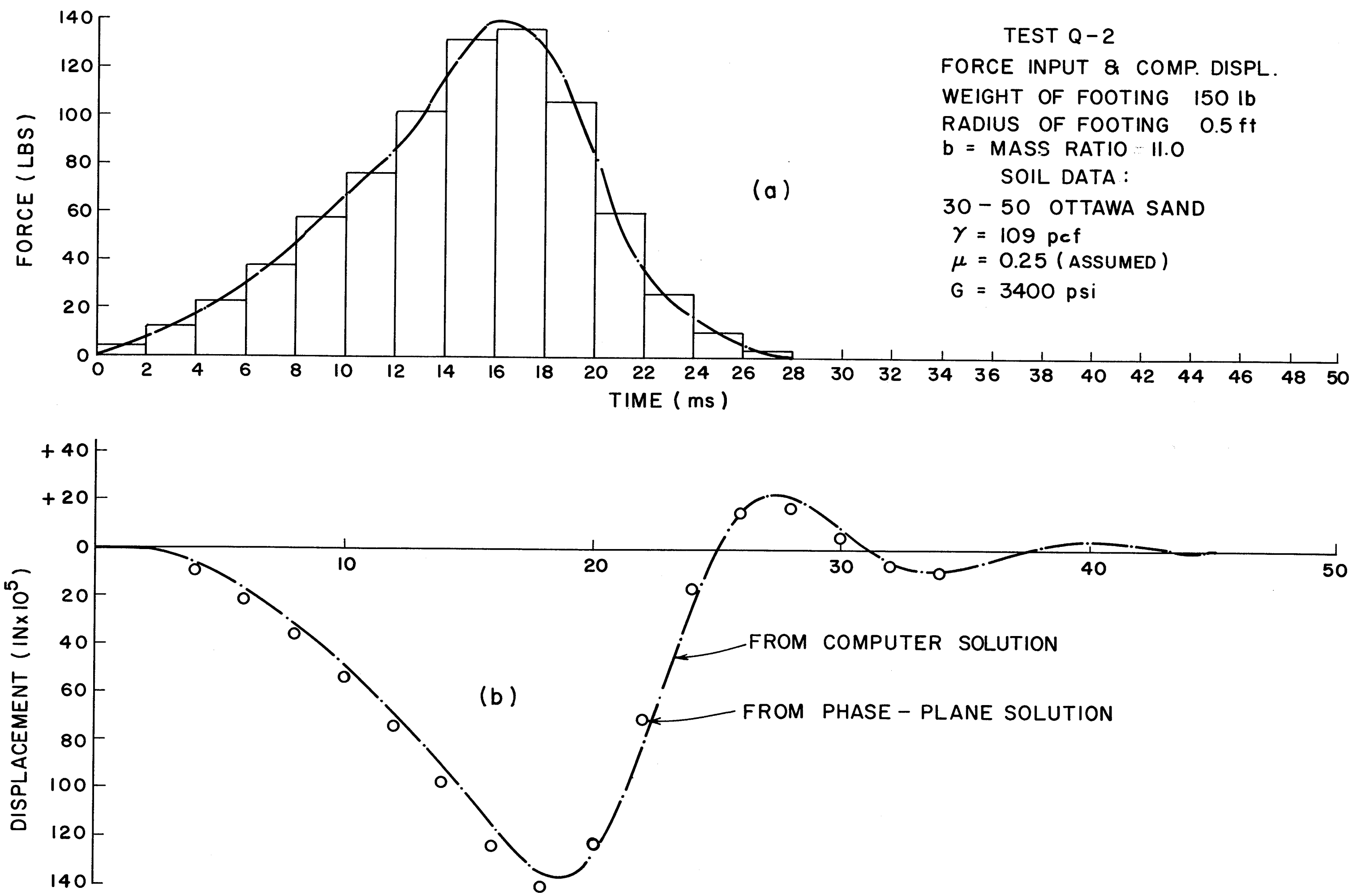


FIGURE 5. (a) FORCE-TIME, AND (b) DISPLACEMENT-TIME CURVES FROM IMPULSIVE LOADING TESTS OF 1' DIAMETER FOOTING ON SAND. (UNIVERSITY OF MICHIGAN, 1964)

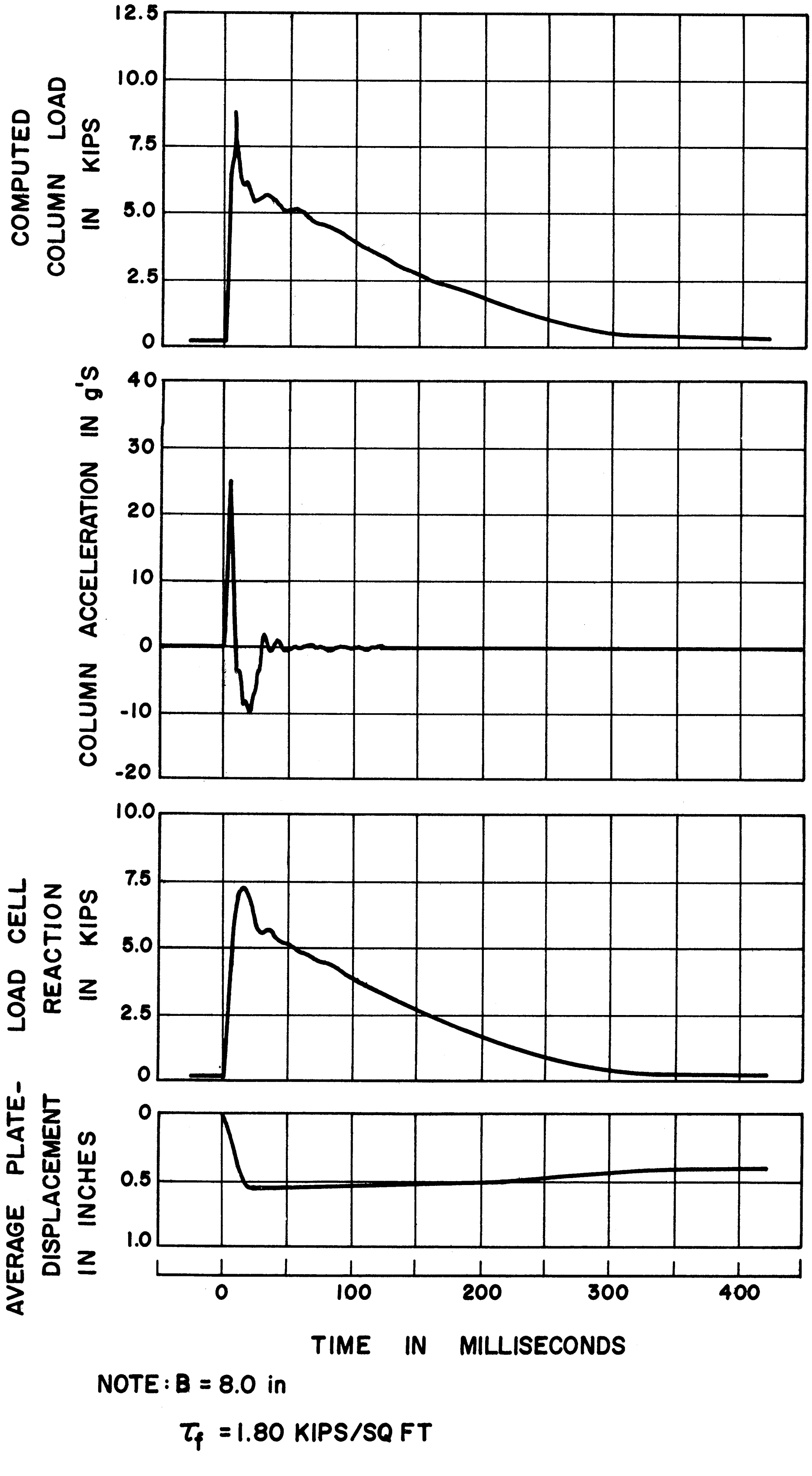


FIGURE 6. DYNAMIC TEST 9 - 1 FOR 8" SQUARE FOOTING ON BUCKSHOT CLAY. (FROM JACKSON AND HADALA, 1964)

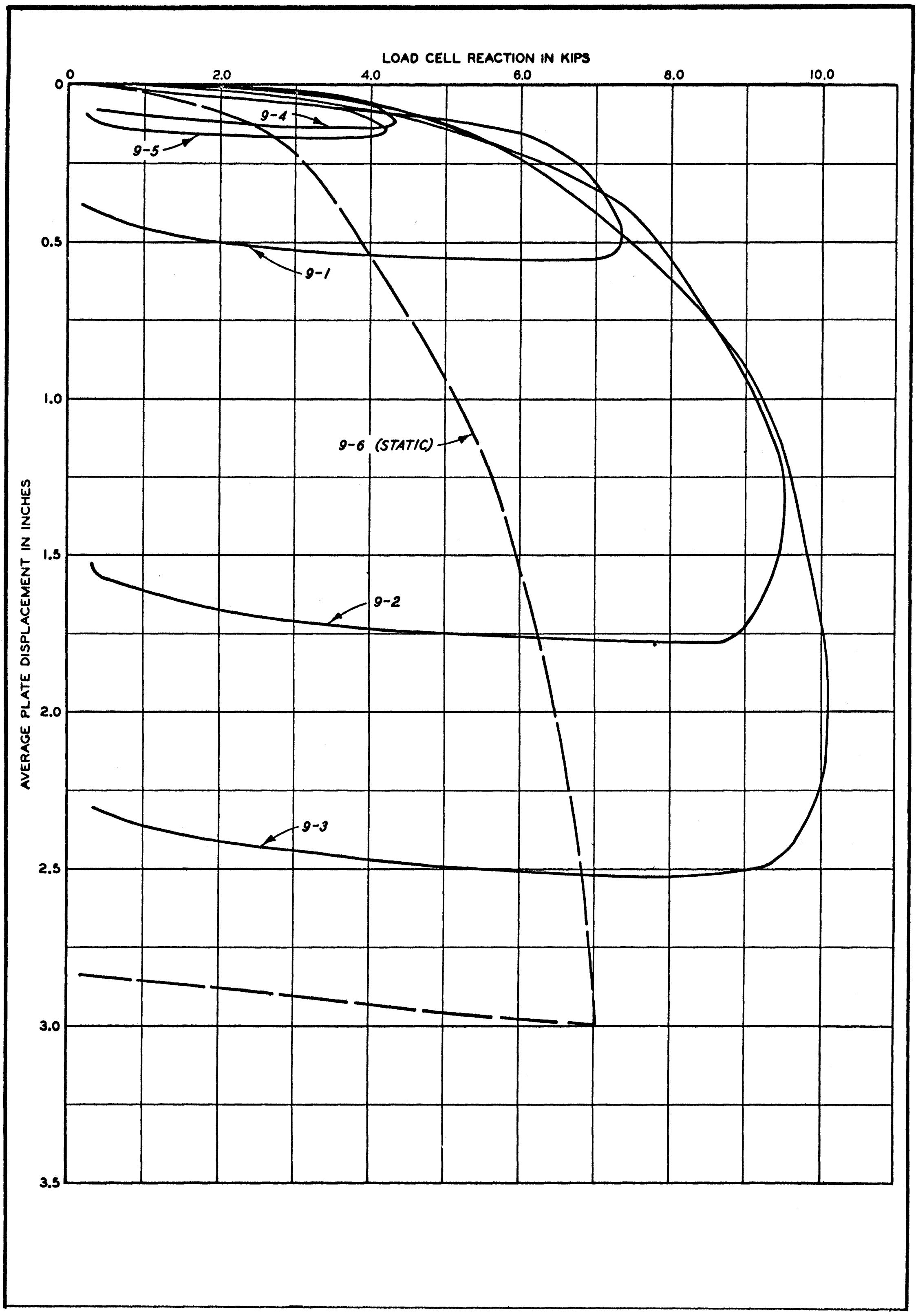


FIGURE 7. STATIC AND DYNAMIC LOAD-DISPLACEMENT CURVES FOR TESTS IN CART 9. (FROM JACKSON AND HADALA, 1964)

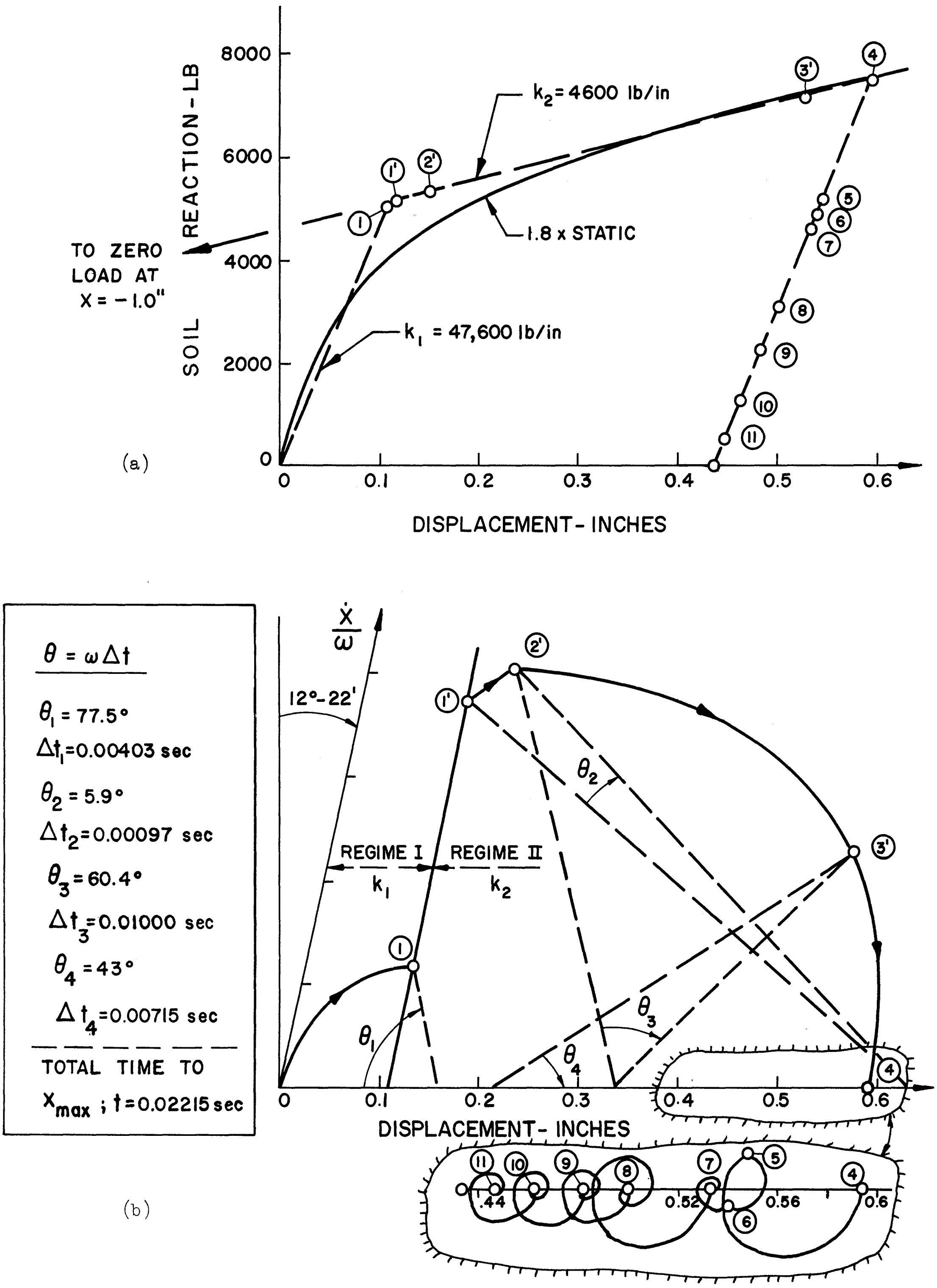


FIGURE 8. (a) STRAIGHT-LINE APPROXIMATIONS TO DYNAMIC LOAD-SETTLEMENT CURVE, (b) PHASE-PLANE SOLUTION FOR TEST 9-1.

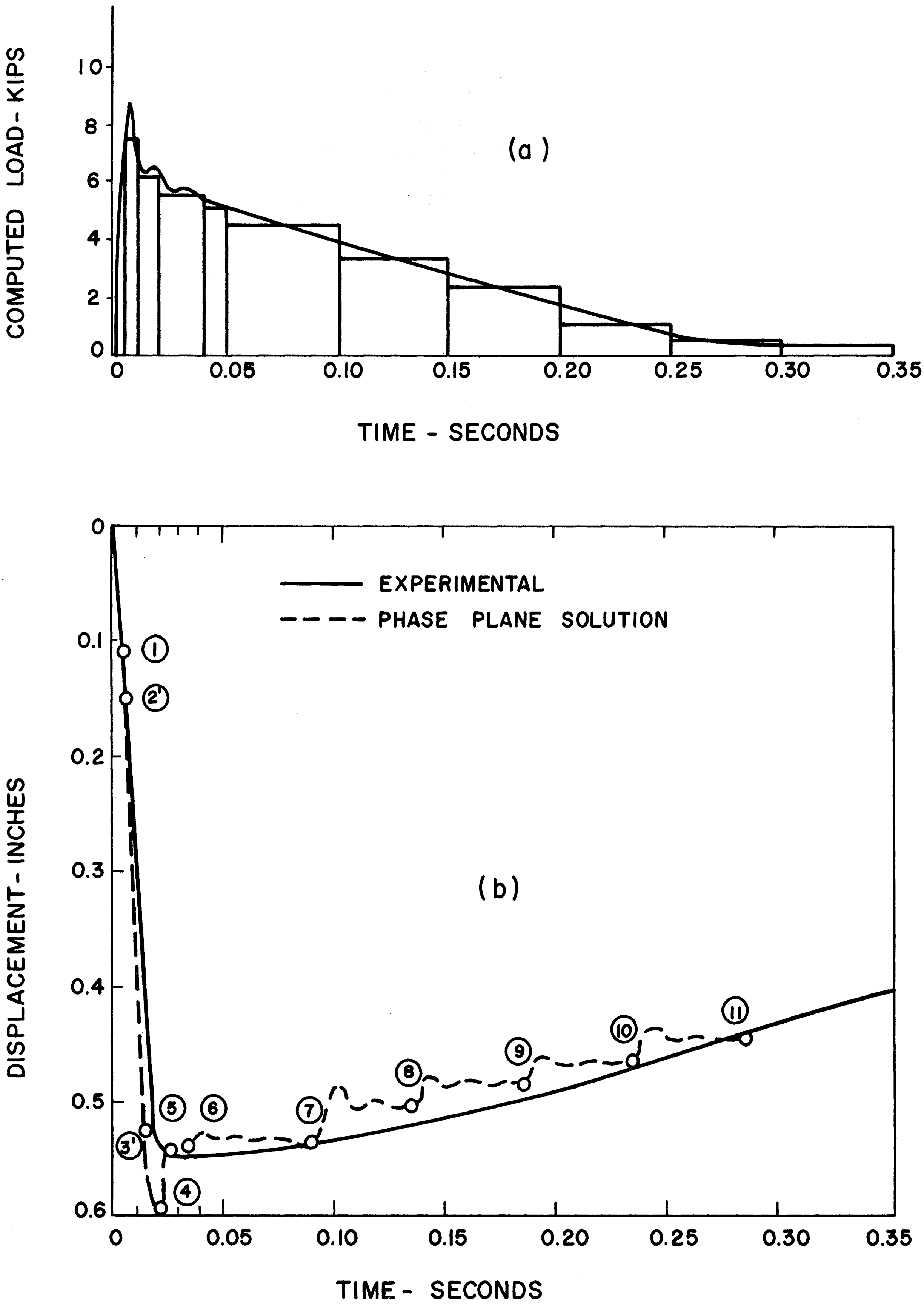


FIGURE 9. (a) LOAD-TIME, AND (b) DISPLACEMENT-TIME CURVES FOR TEST 9-1. (FROM JACKSON AND HADALA, 1964)

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