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SOME USEFUL TECHNIQUES FOR OVERCOMING THE FREQUENCY LIMITATIONS OF CONVENTIONAL DISTRIBUTED AMPLIFIERS

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Department of Electrical Engineering
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By: P. H. Rogers

Approved by: J. A. Boyd

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SOME USEFUL TECHNIQUES FOR OVERCOMING THE FREQUENCY LIMITATIONS OF CONVENTIONAL DISTRIBUTED AMPLIFIERS[1]

1. INTRODUCTION

Many factors affect the operation of distributed amplifiers at high frequencies. The tubes themselves impose the limitations on the high frequency capability of the circuit. Some of these limitations are briefly summarized below.

The distributed amplifier is considered as two artificial transmission lines which are joined by active elements. The tube interelectrode capacities provide the shunt capacities of the artificial transmission lines. However, at VHF frequencies the tubes no longer appear as pure capacitances in the transmission line structures. The cathode lead inductance, electron transmit time, and Miller effect result in a significant input grid conductance which increases approximately as the square of frequency. Hence, the artificial grid line can only be approximately realized in the VHF region.

Screen lead inductance can be increased to decrease the input conductance; however, practical considerations relegate this subterfuge to a second order role.

As the operating frequency increases the inductances of the grid and plate leads approach series resonance with the input and output capacities

1. The author is indebted to Mr. L. A. Beattie and Mr. B. F. Barton for reviewing the manuscript and for their comments.

respectively. Ginzton[1] has shown that negative mutual inductances can be incorporated advantageously in the grid and plate lines to cancel or even overcompensate for these grid and plate lead inductances. This cancellation becomes increasingly difficult as the frequency of operation is extended since the series inductances of the artificial lines must be decreased as the bandwidth is increased. The minimum physical spacing between tubes determines a minimum practical series inductance for the lines.

The prescription of the series inductance together with a practical coefficient of coupling limits the negative mutual inductance which can be reflected into the grid and plate leads. This paper shows that an optimum compromise in the interest of bandwidth exists between the ideas of a small series inductance and a substantial mutual inductance. An additional complication in the conventional circuit is that the minimum physical spacing between tubes determines a minimum practical series inductance for the lines.

In the following section, two techniques are discussed which may be employed to partially circumvent frequency limitations of conventional distributed amplifiers.

2. ANALYSIS OF LIMITATIONS IMPOSED BY TUBE GRID AND PLATE LEAD INDUCTANCES

The conventional distributed amplifier equivalent circuit is shown in Figure 1. This circuit utilizes negetive mutual coils to obtain an m-derived line for which m may be greater than 1. The advantages of this technique are pointed out in Reference 1. Higher cut-off frequencies can be obtained with

[1] E.L. Ginzton, W.R. Hewlett, J.H. Jasberg, J.D. Noe: "DISTRIBUTED AMPLIFICATION," Proc. IRE. 36, pp. 956-69, 1948.

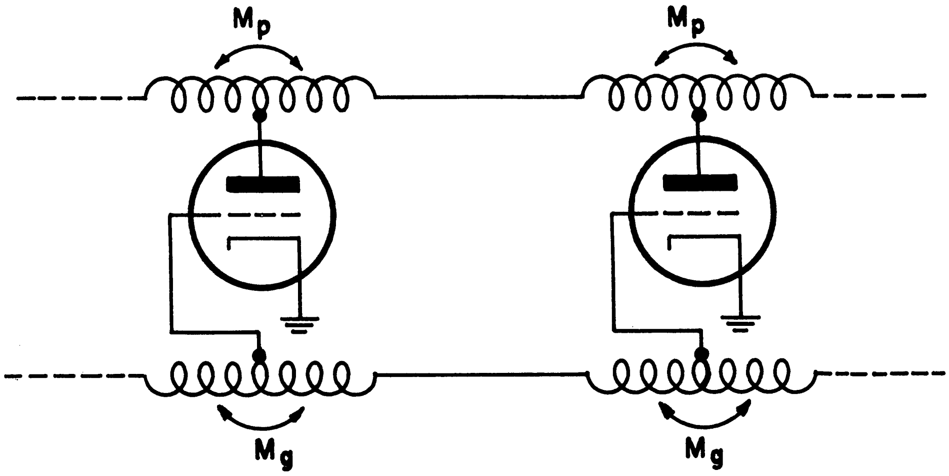

FIG. I CONVENTIONAL DISTRIBUTED AMPLIFIER EQUIVALENT CIRCUIT

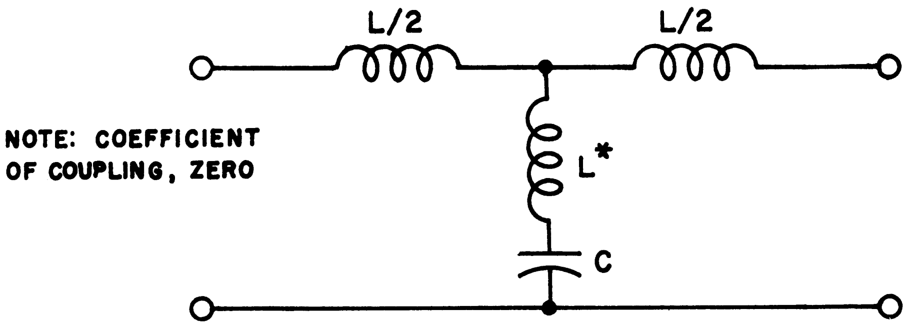

FIG. 2 SECTION OF DISTRIBUTED AMPLIFIER GRID LINE

this circuit than with a constant-k line with the same nominal impedance.

As the design cut-off frequency is raised higher the series inductance of the constant-k line must be decreased. With a limited realizable coefficient of coupling, the negative mutual inductance which can be reflected into the grid or plate leads must also decrease. It is to be noted that if the minimum inductance to span the spacing between tubes is used, the desired mutual inductance is not physically obtainable.

Let us consider a grid circuit in which the mutual inductance is zero. This circuit is shown in Figure 2. Here L^* is the lead inductance, C is the input capacitance of the tube, and L is the inductance between tubes. The characteristic impedance of this line is given by the square root of the short circuit impedance times the open circuit impedance.

$$\begin{aligned} Z_o &= \sqrt{Z_{oc} \quad Z_{sc}} \\ &= \sqrt{\left(\frac{j\omega L}{2}\right)\left(j\omega L^* - \frac{j}{\omega C}\right) + \frac{j\omega L}{2}(j)\left(\frac{\omega L}{2} + \omega L^* - \frac{1}{\omega C}\right)} \qquad (1) \\ &= \sqrt{\frac{L}{C}\left[1 - \omega^2\left(L^*C + \frac{LC}{4}\right)\right]} \end{aligned}$$

If ω_o is defined by

$$\omega_o^2 = \frac{1}{L^*C + \frac{LC}{4}} , \qquad (2)$$

then,

$$Z_o = \sqrt{\frac{L}{C}}\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2} . \qquad (3)$$

In addition, ω_r is defined as the series resonant frequency of the lead inductance and the input capacitance and ω_k is the upper cut-off frequency of the filter section without lead inductance. That is:

$$\omega_r^2 = \frac{1}{L^* C} \tag{4}$$

$$\omega_k^2 = \frac{4}{LC} \; . \tag{5}$$

Equation 2 may be written as

$$\omega_o^2 = \frac{1}{\dfrac{1}{\omega_r^2} + \dfrac{1}{\omega_k^2}} \tag{6}$$

or

$$\omega_o = \frac{\omega_r}{\sqrt{1 + \left(\dfrac{\omega_r}{\omega_k}\right)^2}} \; . \tag{7}$$

Equation 7 is plotted in Figure 3 where it can be seen that the highest frequency obtainable with this circuit is the series resonant frequency ω_r. It will also be noted that an increase in upper cut-off frequency above about 0.7 ω_r is obtained only with a disproportionately large increase in ω_k. In terms of circuit elements this means a very rapidly decreasing series inductance in the line and thus a rapidly decreasing characteristic impedance, as ω_o approaches ω_r. The nominal impdeance level for a constant-k line is given by

$$R_{ok} = \sqrt{\frac{L_k}{C_k}} \; .$$

Using the above equation, the nominal impedance level for the structure shown in Figure 2 is given by

$$R_o = R_{ok}\sqrt{1 - \left(\frac{\omega_o}{\omega_r}\right)^2} \tag{8}$$

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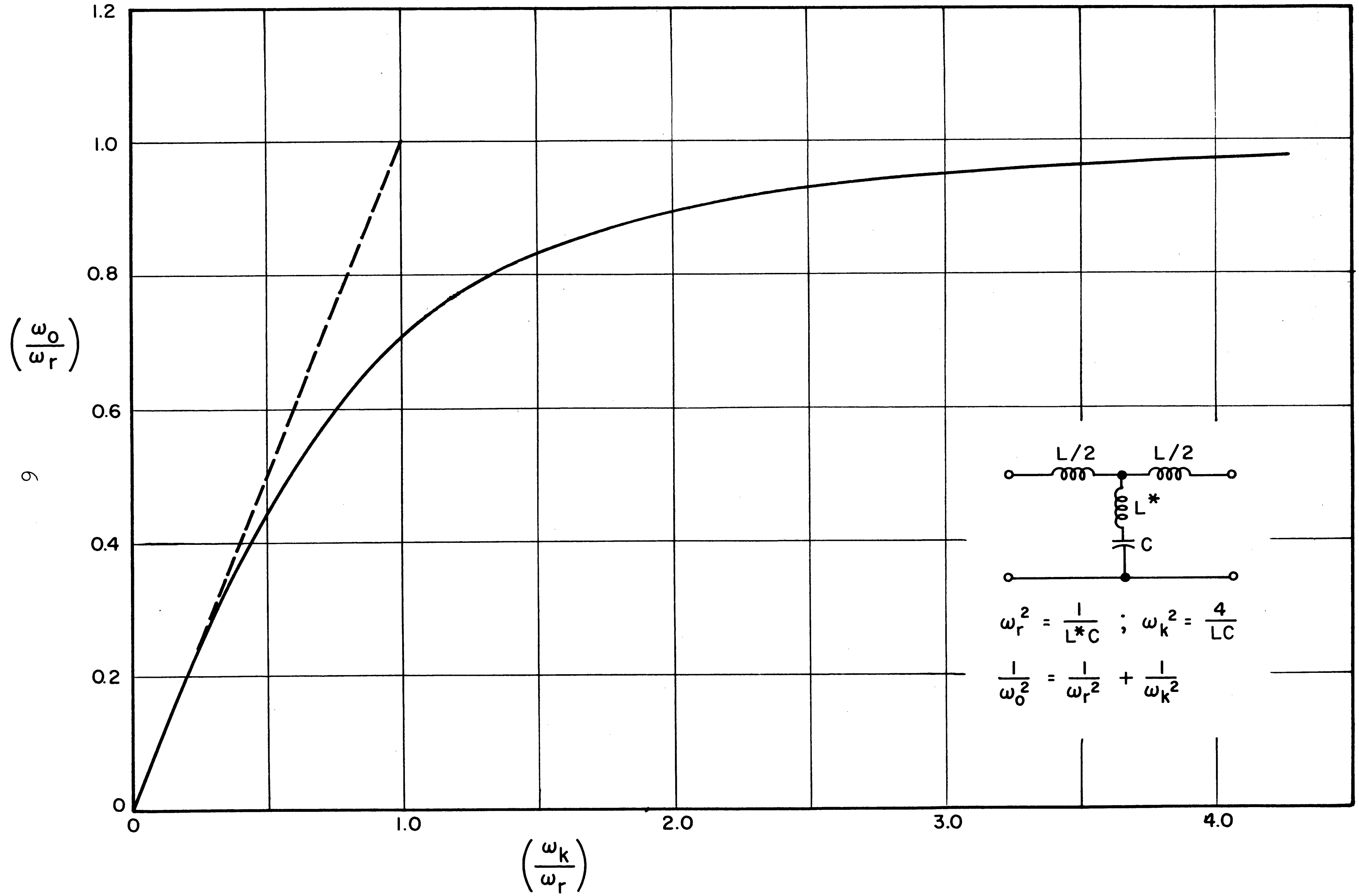

FIG. 3 VARIATION OF DISTRIBUTED AMPLIFIER CUT-OFF FREQUENCY AS A FUNCTION

As an example consider a 4X150A tube where f_r is approximately 430 mc. Equation 8 is plotted in Figure 4, assuming this value of f_r and a value of 20$\mu\mu$f for C. The constant-k impedance level that would be applicable for a 4X150A distributed amplifier, if no lead inductance were present, is drawn for comparison purposes, i.e. $\omega_r = \infty$. In practical 4X150A distributed amplifiers with cut-off frequencies up to about 300 mc the grid lead inductance can at least be cancelled with negative mutual coils so that the impedance level can be at least that of the constant-k curve. However, for upper cut-off frequencies between 300 mc and the series resonant frequency of 427 mc, the impedance level would be between the two curves. It would be exactly on the lower curve when the series inductance required equals the minimum practical inductance necessary to make connections between tubes, since the coefficient of coupling is then essentially zero.

Figure 5 (derivation in Appendix) shows the frequency response for several values of ω_k/ω_r. (These curves assume matched conditions on the plate line and the grid line.) The rising characteristic is normally flattened due to the presence of grid conductance which increases with the square of frequency in high frequency distributed amplifiers[1]. The fact that the curves rise more rapidly with increasing ω_k/ω_r means that the number of tubes to be used to flatten the response curve increases above that employed with a constant-k line[2]. The increased number of tubes helps in some measure to compensate for the low line impedance indicated by the above discussion, since the power output varies as the square of the number of tubes.

2.1 The Use of Dummy Sections to Increase the Cut-off Frequency of Distributed Amplifiers.

The use of dummy sections such as are shown in Figure 6 allows some of

1 Ginzton, et al, op. cit., page 2.
2 P.H. Rogers, "Large Signal Analysis of Distributed Amplifiers," University of Michigan, Electronic Defense Group Technical Report No. 52, July 1955.

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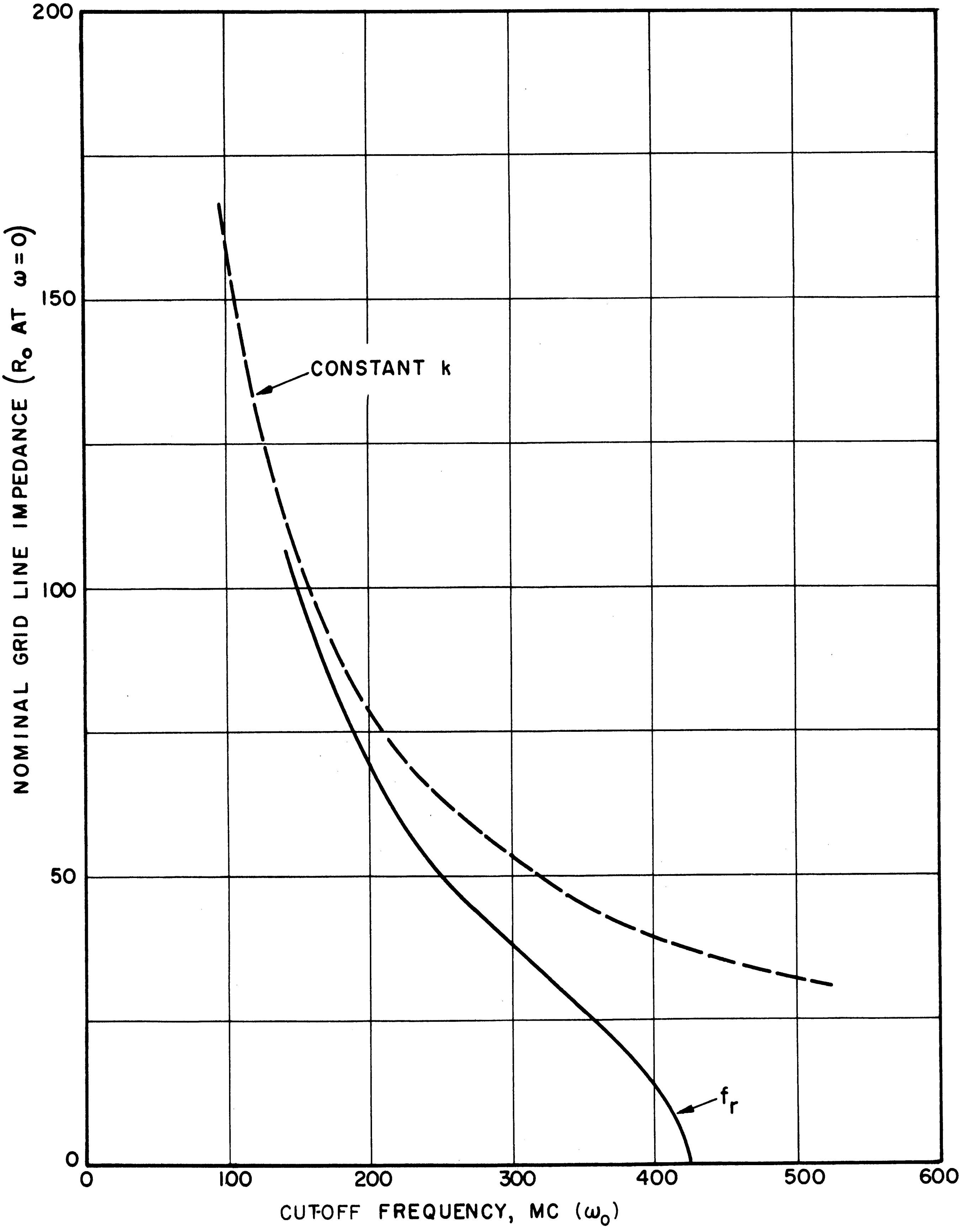

FIG. 4 GRID LINE IMPEDANCE LEVEL OF A 4X150A DISTRIBUTED AMPLIFIER WITHOUT MUTUAL INDUCTANCE

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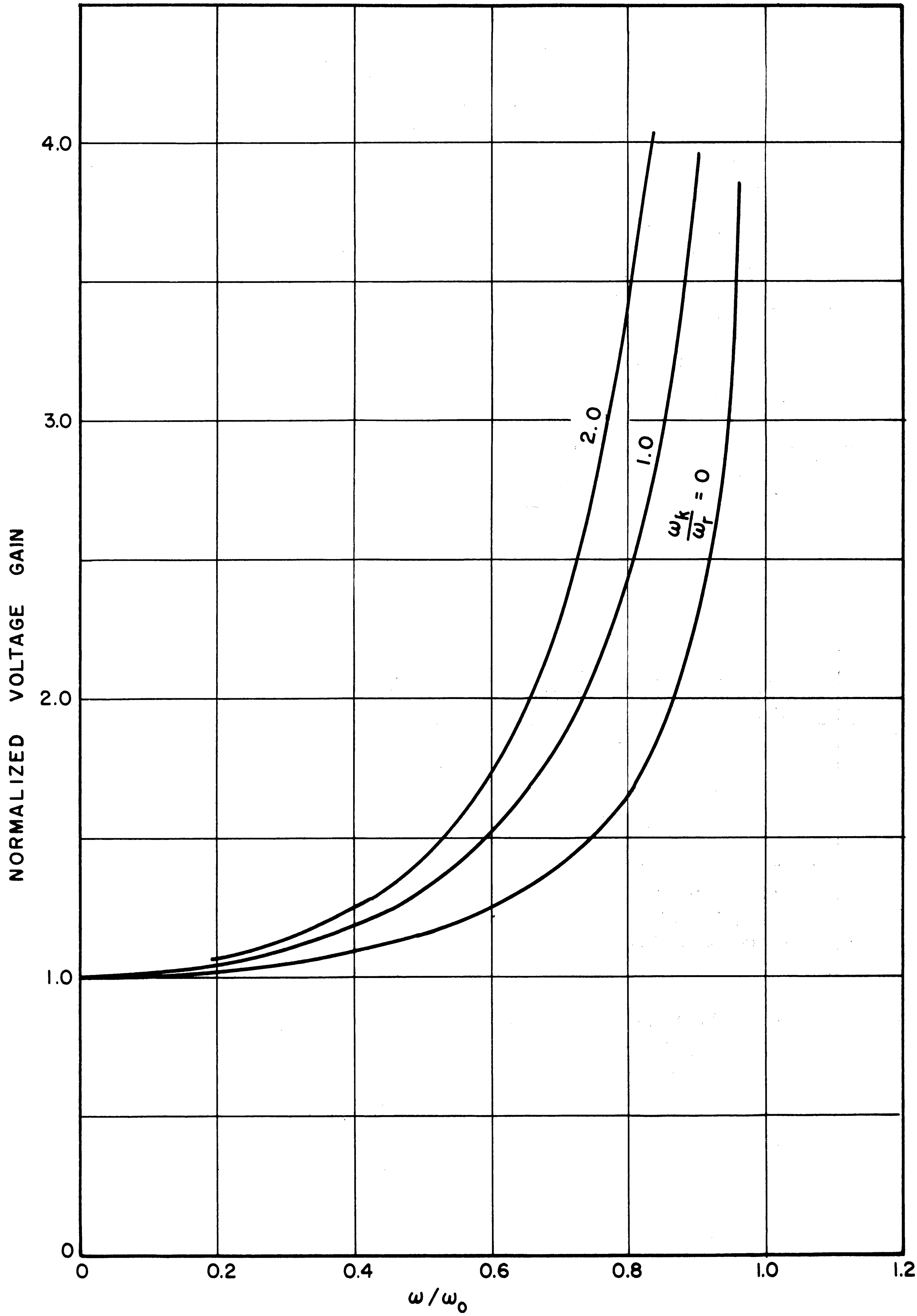

FIG. 5 NORMALIZED FREQUENCY RESPONSE (VOLTAGE) FOR VARIOUS VALUES OF $\left(\frac{\omega_k}{\omega_r}\right)$

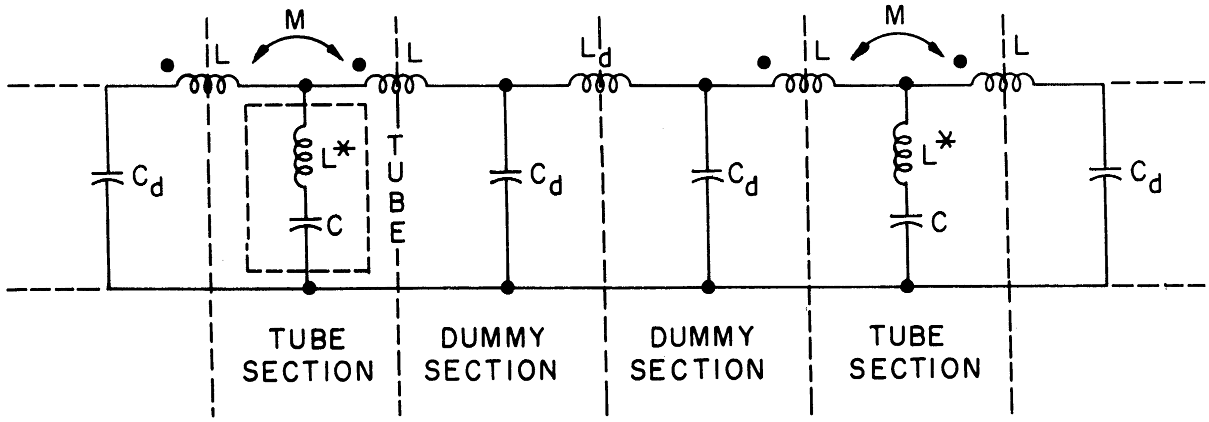

FIG. 6 ARTIFICIAL LINE FOR DISTRIBUTED AMPLIFIERS UTILIZING TWO DUMMY SECTIONS BETWEEN TUBES.

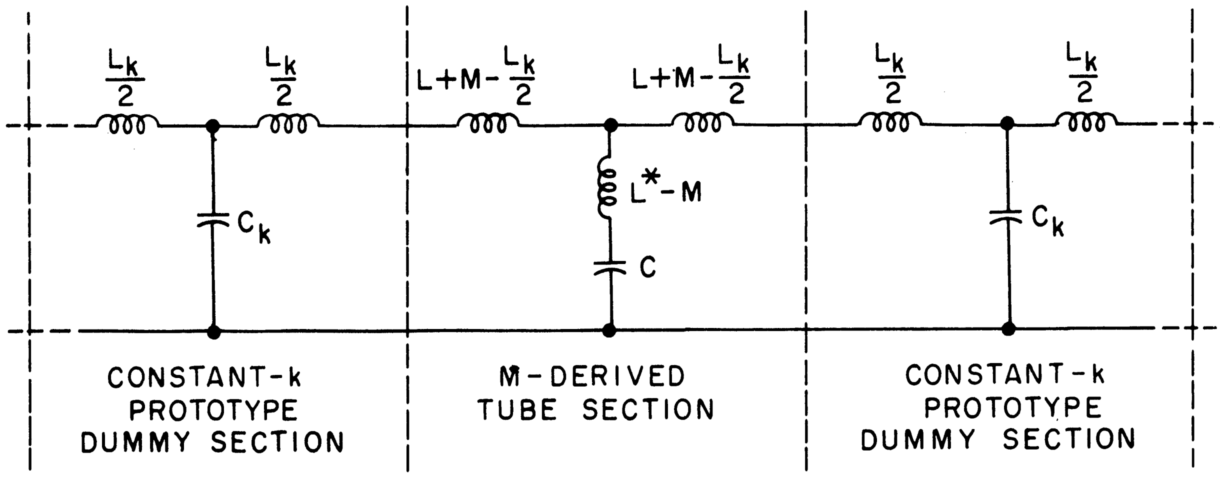

FIG. 7 ARTIFICIAL LINE WITH INDIVIDUAL SECTION ELEMENTS DEPICTED

the frequency limitations of the conventional distributed amplifier to be surpassed. Their use also removes the necessity of using minimum tube spacing. Many advantages accrue from using a wider tube spacing. For example consider a 4X150A distributed amplifier. Standard Eimac sockets can be used instead of loctal sockets, and the necessity for shielding between plates is removed. The latter has the great advantage of reducing the output capacity and hence increasing the impedance level of the plate line and therefore the gain.

The dummy tee sections can be inserted between the m-derived tube sections with little effect except phase shift since the mid-series impedance of an m-derived filter section is of the same form as that of a constant-k filter. Hence the image impedances are approximately matched if the tube loading is not too severe. Figure 6 can be redrawn for analysis purposes as shown in Figure 7. If the elements shown in the tube sections in Figure 7 are equated to the elements of an m-derived filter, of which the constant-k dummy sections are prototypes, the following equations result:

$$L + M - \frac{L_k}{2} = \frac{mL_k}{2} \tag{9}$$

$$C = mC_k \tag{10}$$

$$L^* - M = \frac{1 - m^2 L_k}{4m} \tag{11}$$

The coefficient of coupling of the negative mutual coils, k, is given by the relation

$$k = \frac{M}{L} \; . \tag{12}$$

If Eqs 9 and 11 are solved simultaneously for L_k, then Eq 13 results:

$$L_k = L^* \quad \frac{4m(k + 1)}{m^2(k-1) + 2mk + k + 1} \tag{13}$$

Substituting Eq 10 and Eq 13 into the equation for the cutoff frequency, one has

$$\omega_o^2 = \frac{4}{L_k C_k} = \frac{1}{\dfrac{L^* m(k+1)}{m^2(k-1) + 2mk + k + 1} \dfrac{C}{m}} , \quad (14)$$

or

$$\left(\frac{\omega_o}{\omega_r}\right)^2 = \frac{m^2(k-1) + 2mK + k + 1}{k + 1} . \quad (15)$$

where ω_r is the series resonant angular frequency of the grid lead inductance and the input capacitance. If this expression is maximized with respect to m, the optimum value of m in so far as upper cutoff frequency is concerned is

$$m_{opt} = \frac{k}{1-k} . \quad (16)$$

If this value of m is substituted into Eq 15, then

$$\left(\frac{\omega_o}{\omega_r}\right)^2 \Bigg|_{m = m_{opt}} = \frac{1}{1-k^2} , \quad (17)$$

or

$$\frac{f_o}{f_r} = \frac{1}{\sqrt{1-k^2}} \quad (18)$$

Equation 18 as well as the optimum value of m given by Eq 16 are plotted in Figure 8. For comparison purposes the upper cut-off frequency for $m = 1$ is also plotted. It can be seen that the value of m is not very critical for variations in the coefficient of coupling in the range $k < 0.7$. It is interesting to note that below a coefficient of coupling of 0.5 the highest cut-off frequency is obtained with $m < 1$. From Figure 8 it can be seen that it is possible to operate distributed amplifiers above the series resonant frequency of the tube elements if the circuit shown in Figure 6 is used. Practical considerations dictate the maximum coefficient of coupling that can be realized with the small coils required and hence the upper cut-off frequency limit is spelled out in Figure 8 for a given coefficient of coupling.

As an example of the usefulness of this circuit consider a 4X150A distributed amplifier. The maximum frequency obtained with conventional circuitry and this tube is about 300 mc. This upper frequency limit is surpassed in the circuit of Figure 7 for three reasons: (1) the frequency degrading aspects of the lead length necessary to span the tube spacing is removed (this inductance becomes the useful inductance of the dummy section), (2) the inductance contribution per section of the negative mutual coils is halved ($2L + 2M$ in the conventional circuit is replaced by $L + M$ in the modified circuit), (3) all of the inductance L can be utilized in obtaining the desired mutual inductance M thus allowing smaller L's for the realization of a given M. There is actually an interplay of the above factors, but the overall result is that the spanning inductance no longer places a lower bound on the inductance per section and the inductance of the negative mutual section is reduced. If the circuit shown in Figure 6 is used, it is theoretically possible to realize a cutoff frequency of 500 mc with a coefficient of coupling of about 0.55.

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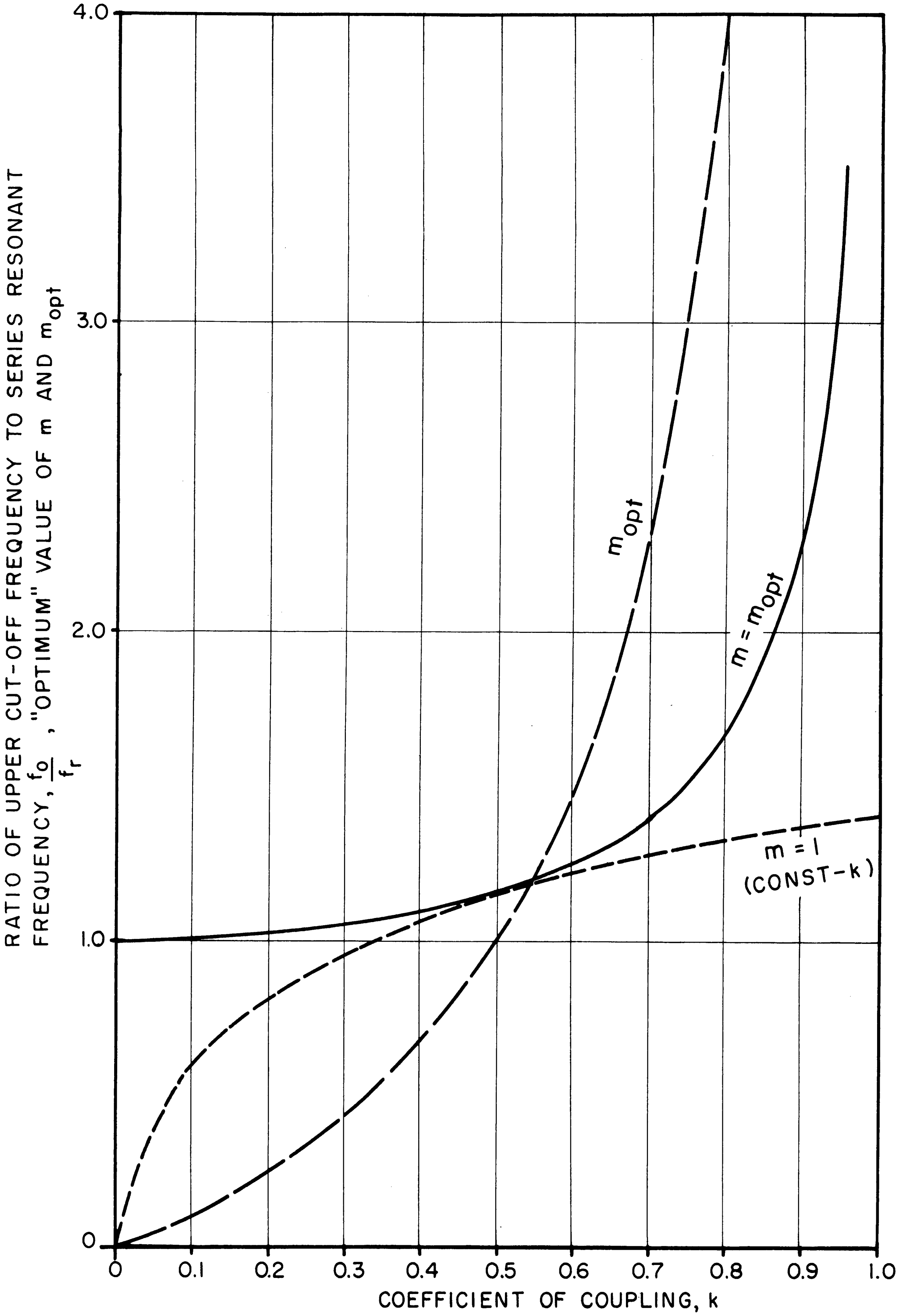

FIG. 8 VARIATION OF MAXIMUM UPPER CUT-OFF FREQUENCY OF CIRCUIT SHOWN IN FIG. 6 AND "OPTIMUM" VALUE OF m AS A FUNCTION OF COEFFICIENT OF COUPLING OF NEGATIVE MUTUAL COILS.

3. DISTRIBUTED AMPLIFIERS USING TUBES WITH TWO GRID LEADS AND TWO PLATE LEADS

An increase in the upper cut-off frequency can be achieved in a practical distributed amplifier if tubes with two grid leads and two plate leads are used. The major difficulty with the circuit shown in Figure 6 is the high coefficient of coupling that is required to obtain a cut-off frequency substantially above the series resonant frequency. In small air core coils it is virtually impossible to obtain coefficients of coupling greater than 0.6 to 0.7. Hence in practice distributed amplifiers that are designed in a fashion similar to Figure 6 have an upper cut-off limit of about 1.4 times the series resonant frequency of either the grid of the plate circuit depending upon which is lower. However, the same basic idea as developed in Section 2 can be used to obtain higher cut-off frequencies with tubes having two grid leads and two plate leads. Figure 9 shows the circuit to be treated in this section. Here the dotted coils represent the inductance of each grid lead; the input capacity of the tube is not represented, but is assumed to be between grid and ground. These values are repeated in the constant-k sections between tubes. The number of these constant-k sections that are used is dependent upon the number required to span the distance between tubes.

The cut-off angular frequency for the circuit shown in Figure 9 is given by

$$\omega_0 = \frac{2}{\sqrt{LC}} \tag{19}$$

where L is the inductance of one grid lead plus any socket and wiring inductance necessary to get to the capacitor C. The capacitance C is a fixed capacitor whose value equals the input capacitance of the tube. Equation 19 indicates that the highest frequency that can be realized with this circuit is twice the series resonant frequency of the tube plus socket lead inductance and the input capacity of the tube.

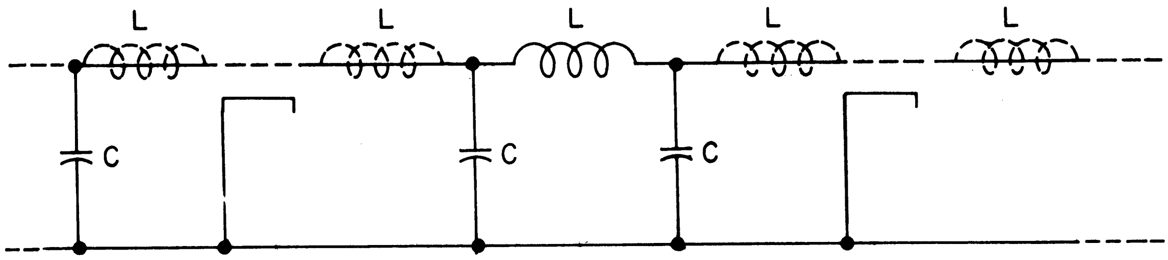

FIG.9 GRID LINE OF DISTRIBUTED AMPLIFIER WITH TUBES HAVING TWO GRID LEADS.

If a 4X150A were modified as shown in Figure 10, the equivalent circuit for the tube with its lead inductances would be as shown in Figure 11. The values of inductance given are calculated from the geometry using equations given in Terman's "Radio Engineers Handbook". The maximum cut-off frequency that could be obtained is about 1000 mc. This high frequency may not be usable in a distributed amplifier since it may not be possible to obtain useful gain at this bandwidth due to the input conductance at high frequencies. However, this is the limit on bandwidth of the grid line with the circuit shown. The use of this modified tube appears promising above about 300 mc.

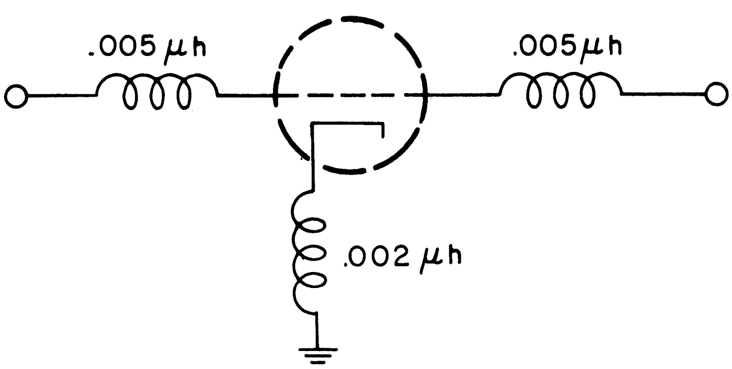

FIG. 11 EQUIVALENT CIRCUIT OF MODIFIED 4X150A

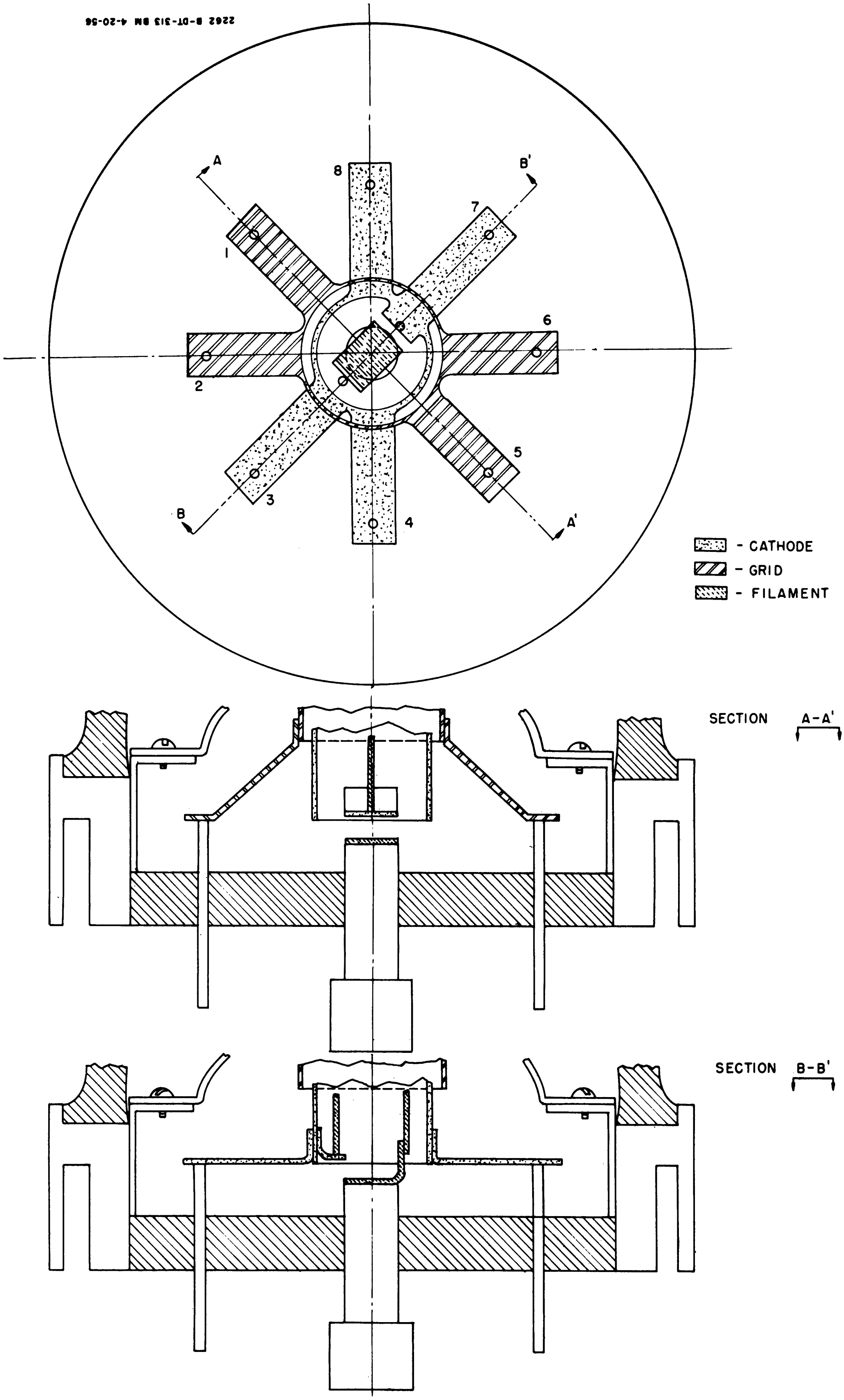

FIG. 10 PROPOSED MODIFICATION OF 4X-150-A BASE CONNECTIONS.

4. CONCLUSIONS

It should be possible to substantially increase the cut-off frequency of a standard distributed amplifier by using dummy filter sections in both the grid and plate lines. This increase is bought at the expense of circuit complexity, but this cost is not considered excessive. The question of useful gain at these higher frequencies must also be considered. For any given tube and its associated lead inductances in the amplifier, the proper number of tubes for a flat response can be determined either experimentally or in the case of constant-k lines, by Ginzton's equation. The flat gain can then be determined knowing the transconductance of the tubes. For any given application there is a minimum gain that is useful and this figure must be exceeded by the amplifier under consideration or else the cut-off frequency must be reduced so that useful gain is obtained.

If distributed amplifiers are constructed with tubes having two grid leads and two plate leads, some additional advantage can be obtained in cut-off frequency over that attainable with standard tubes. There is always the question of whether the amplifier will have useful gain at these higher frequencies and this must be checked for each tube considered. A measure of the increase in cut-off frequency to be obtained is indicated by the example of a 4X150A distributed amplifier. A cut-off frequency of about 500 mc should be obtainable if the circuit shown in Figure 6 is used. This amplifier employs conventional tubes. Finally, if tubes were constructed as shown in Figure 10, an upper cut-off frequency of about 1000 mc is theoretically possible. There is grave doubt whether a 4X150A amplifier could be constructed to operate satisfactorily to this limit, but the two grid leads effectively remove the lead inductances as the limiting factor in high frequency distributed amplifiers.

APPENDIX A

DERIVATION OF GAIN EQUATION FOR A DISTRIBUTED AMPLIFIER WITH NO TUBE LEAD COMPENSATION

Since the phase shift a signal experiences as it traverses any one of the multiple paths between the input and the output of a distributed is the same, the output voltage is the arithmetic sum of the output voltages as the tubes are turned on one at a time. Also the frequency responce of the final amplifier is exactly the same as that of any one stage. Hence the gain for a single tube is calculated here.

A.I GRID NETWORK

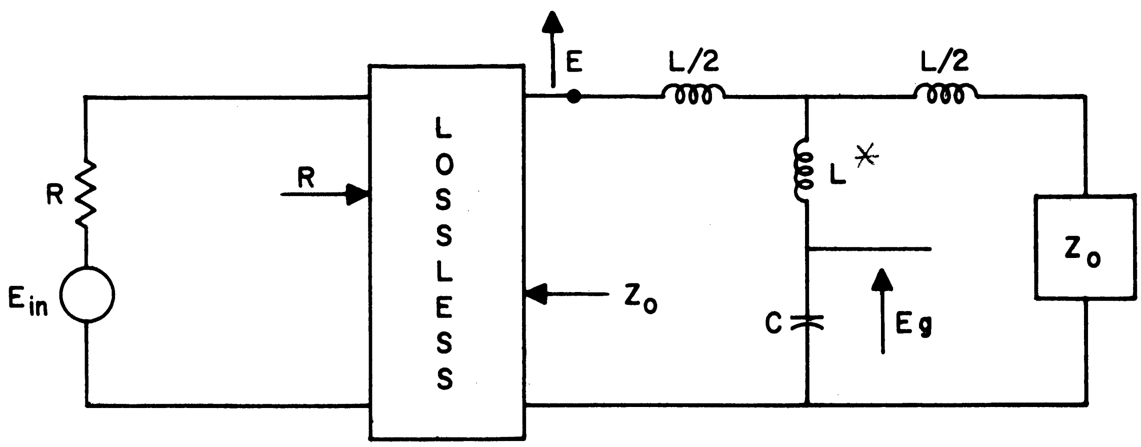

FIG. A-1

From Figure A-1 the driving power

$$P_{dr} = \frac{E_{in}^{2}}{4R} = \frac{E^{2}}{Z_o}$$

or

$$E = \frac{E_{in}}{2}\sqrt{\frac{Z_o}{R}}$$

This network can be considered as equivalent to the one in Figure A-2a and b

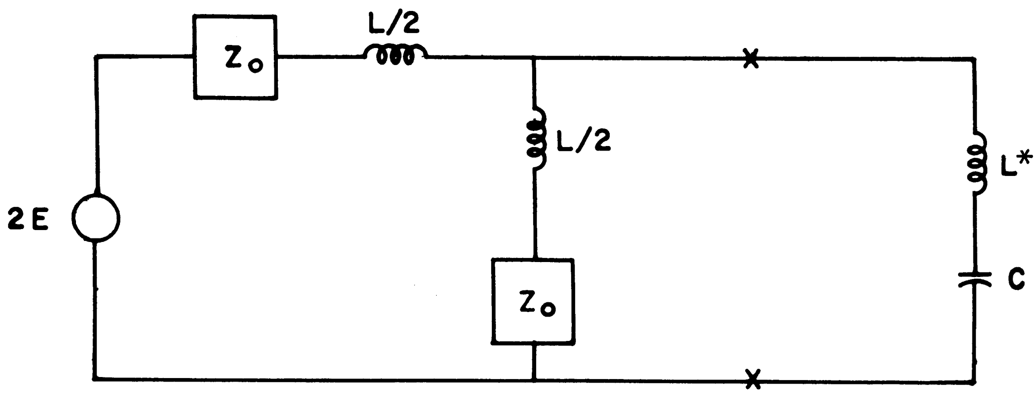

FIG. A-2a

or

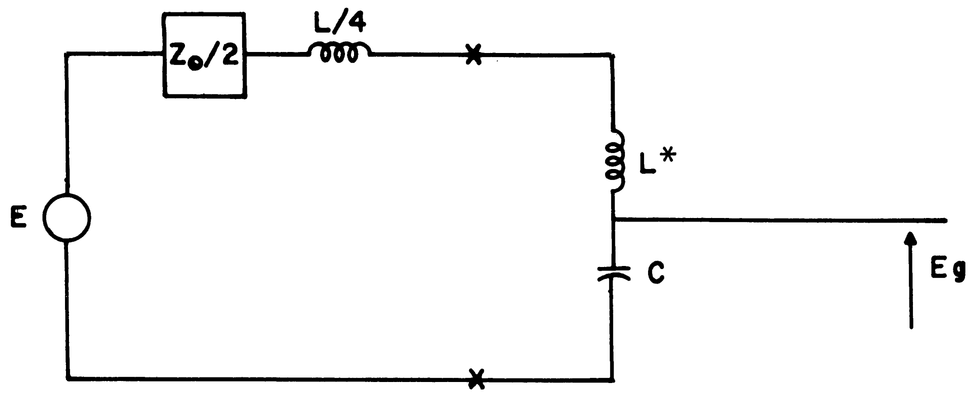

FIG. A-2b

From Figure A-2b it is obvious that the grid voltage is

$$E_g = E \quad \frac{\frac{1}{j\omega C}}{\frac{Z_o}{2} + j\omega\frac{L}{4} + j\omega L^* + \frac{1}{j\omega C}}$$

or

$$E_g = E \quad \frac{1}{1 - \omega^2\frac{LC}{4} - \omega^2 L^* C + j\omega C\,\frac{Z_o}{2}}$$

$$= \frac{E}{1 - \left(\frac{\omega}{\omega_k}\right)^2 - \left(\frac{\omega}{\omega_r}\right)^2 + j\omega C\,\frac{Z_o}{2}} = \frac{E}{1 - \left(\frac{\omega}{\omega_o}\right)^2 + j\frac{\omega C}{2}\sqrt{\frac{L}{C}}\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2}}$$

$$= \frac{E}{\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2}\left[\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2} + j\left(\frac{\omega}{\omega_k}\right)\right]}$$

$$\left|E_g\right| = \frac{|E|}{\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2}\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2 + \left(\frac{\omega}{\omega_k}\right)^2}}$$

$$= \frac{\left|E_{in}\right|}{2} \quad \frac{\sqrt[4]{\frac{L}{C}}\;\sqrt[4]{1 - \left(\frac{\omega}{\omega_o}\right)^2}}{\sqrt{R}\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2}\sqrt{1 - \left(\frac{\omega}{\omega_r}\right)^2}}$$

A.II PLATE NETWORK

The plate network can be represented by Figure A-3.

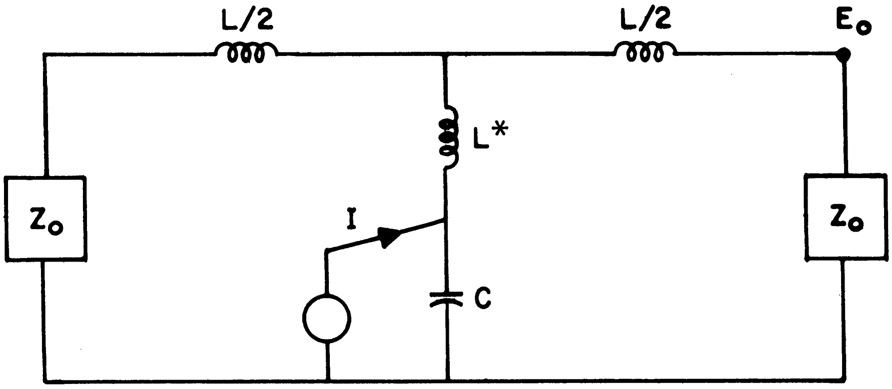

FIG. A-3a

or

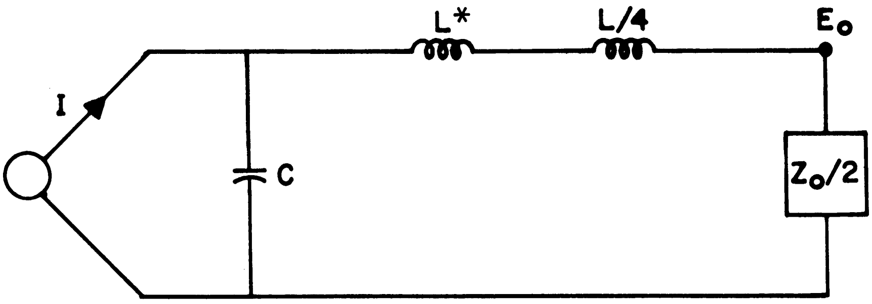

FIG. A-3b

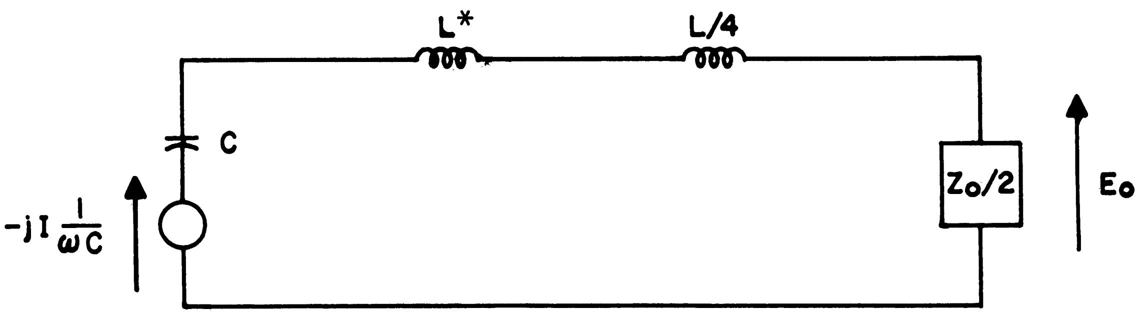

FIG. A-3c

$$E_o = \frac{I}{j\omega C} \; \frac{\frac{Z_o}{2}}{\frac{Z_o}{2} + j\omega L^* + j\omega\frac{L}{4} + \frac{1}{j\omega C}}$$

$$= I \; \frac{\frac{Z_o}{2}}{1 - \omega^2 L^* C - \omega^2\frac{LC}{4} + j\omega C\frac{Z_o}{2}}$$

This denominator is exactly the same as encountered in grid line equations and can be written as

$$|E_o| = |I| \; \frac{\frac{Z_o}{2}}{\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2} \; \sqrt{1 - \left(\frac{\omega}{\omega_r}\right)^2}}$$

The output voltage

$$|E_{out}| = |E_o|\sqrt{\frac{R}{Z_o}}$$

Therefore,

$$|E_{out}| = \frac{|I|}{2} \; \frac{\sqrt{R} \cdot \sqrt{Z_o}}{\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2} \cdot \sqrt{1 - \left(\frac{\omega}{\omega_r}\right)^2}}$$

$$= \frac{|I|}{2} \; \frac{\sqrt{R} \cdot \sqrt[4]{\frac{L}{C}}}{\sqrt[4]{1 - \left(\frac{\omega}{\omega_o}\right)^2} \; \sqrt{1 - \left(\frac{\omega}{\omega_r}\right)^2}}$$

Now

$$|I| = G_m |E_g|$$

$$|E_{out}| = \frac{G_m \quad |E_{in}|}{2 \times 2} \quad \frac{\sqrt[4]{\frac{L}{C}}}{\sqrt{R}} \quad \frac{1}{\sqrt[4]{1 - \left(\frac{\omega}{\omega_o}\right)^2} \quad \sqrt{1 - \left(\frac{\omega}{\omega_r}\right)^2}} \quad x$$

$$\frac{\sqrt{R} \quad \sqrt[4]{\frac{L}{C}}}{\sqrt[4]{1 - \left(\frac{\omega}{\omega_o}\right)^2} \quad \sqrt{1 - \left(\frac{\omega}{\omega_r}\right)^2}}$$

$$\left|\frac{E_{out}}{E_{in}}\right| = \frac{G_m}{4} \quad \frac{\sqrt{\frac{L}{C}}}{\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2} \quad \sqrt{1 - \left(\frac{\omega}{\omega_r}\right)^2}}$$

$$\left|\frac{E_{out}}{E_{in}}\right| = \frac{K}{\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2 \left[1 - \left(\frac{\omega}{\omega_o}\right)^2 \quad \left(\frac{1}{1 + \left(\frac{\omega_r}{\omega_k}\right)^2}\right)\right]}}$$

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