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Effects of off great-circle propagation on the phase of long-period surface waves

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Summary. Surface wave phase corrections for departures from great-circle propagation are computed using two-point ray-tracing through the aspherical earth model M84C of Woodhouse & Dziewonski (1984). For Rayleigh and Love waves with periods in the range 100–250 s, we determine whether these corrections provide significant variance reductions in source determinations compared with corrections calculated assuming great-circle propagation through the heterogeneous structure. For most source–receiver geometries, the off great-circle travel-time effects are small (< 10 s) for second and third orbits (e.g. R2 and R3), and their application in source determinations does not significantly reduce the data variance. This suggests that for the low-order heterogeneous models currently available the geometrical optics approximation is valid for long-period low orbit surface waves. Off great-circle phase anomalies increase quasi-linearly with increasing orbit number, indicating that the geometrical optics approximation degrades for higher orbits, which emphasizes the importance of developing higher order approximations for free-oscillation studies.

Key words: lateral heterogeneity, surface waves, two-point ray tracing

Introduction

The effects of lateral heterogeneity of the Earth on the amplitude and phase of long-period surface waves as well as on the eigenfrequencies of free oscillations are now well recognized (e.g. Dahlen 1974; Woodhouse & Dahlen 1978; Jordan 1978; Woodhouse & Girnius 1982; Lay & Kanamori 1985; Woodhouse & Wong 1986). The application of these data to the retrieval of aspherical Earth structure or source parameters has generally required the use of the geometrical optics approximation. In terms of travelling-wave analysis, this requires that observed phase anomalies be dependent only on the average structure along the great-circle path connecting the source and receiver, and be independent of the location of the source and receiver on the great-circle path as well as independent of the structure off the great-

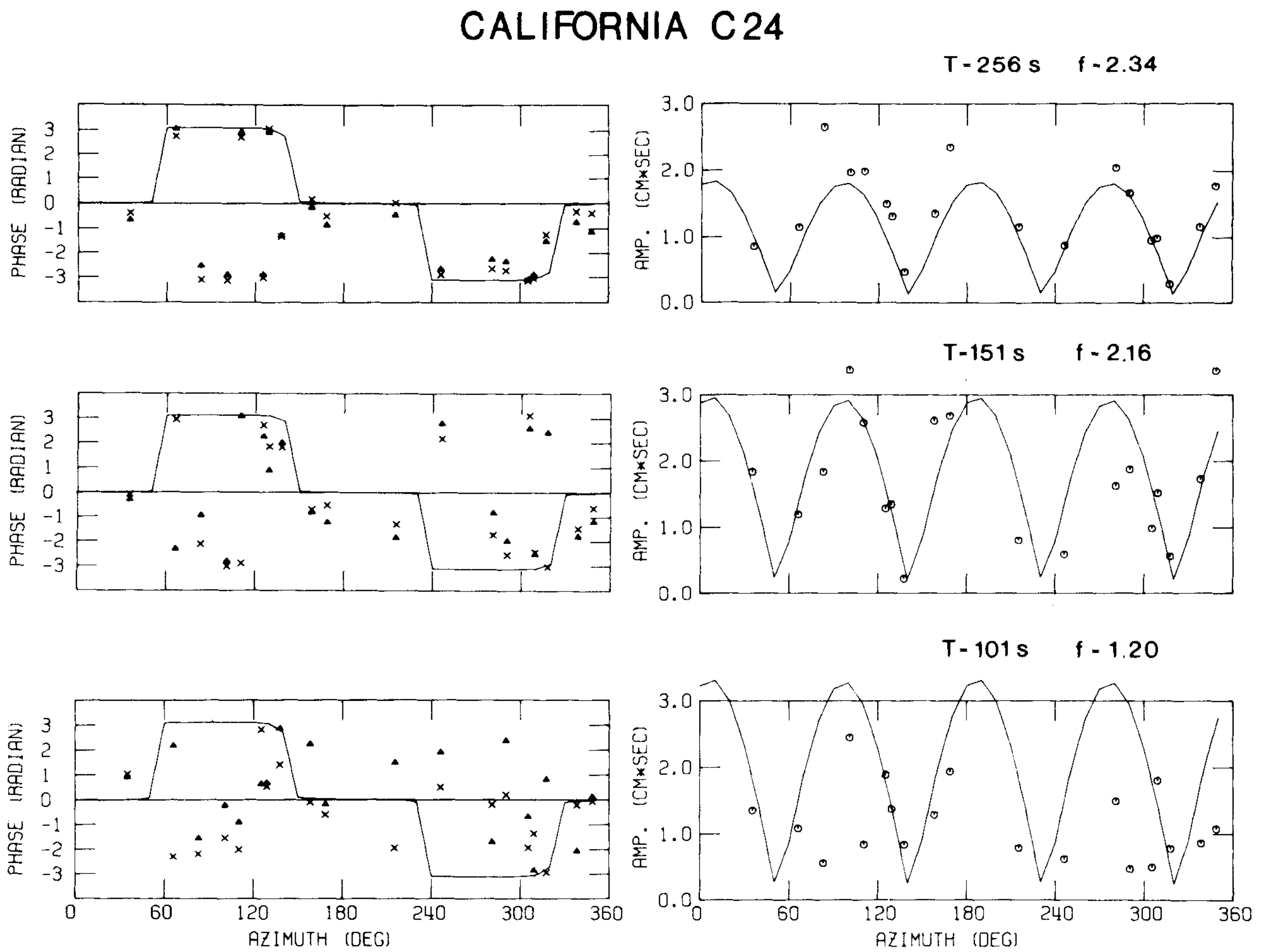

Figure 1. Rayleigh wave phase and amplitude source spectra from the 1980 November 8 Eureka, California, earthquake. The phase is corrected for propagation through either spherically symmetric earth model PREM (triangles) or through aspherical earth model M84C (X's). Solid lines are theoretical curves for the source solution determined using 256 s period data obtained by Tanimoto & Kanamori (1986). The ratio of data variance in the homogeneous case to the heterogeneous case (*f*) indicates significant improvement using earth model M84C at 256 and 151 s, but not at 101 s.

circle path. Several observations of departures from this approximation have been made for both free oscillations (Jobert & Roult 1976; Silver & Jordan 1981; Woodhouse & Girnius 1982) and long-period surface waves (Masters, Priestly & Gilbert 1984; Lay & Kanamori 1985; Schwartz & Lay 1985; Roult, Romanowicz & Jobert 1986; Woodhouse & Wong 1986). The availability of global models of mantle heterogeneity has allowed exact surface-wave ray-tracing calculations to be performed, and substantial deflections from great-circle paths have been reported. A technique incorporating these deflections to better constrain lateral variations in the Earth is presently being developed by Woodhouse & Wong (1986). Our purpose in this note is to investigate the effects of the off great-circle propagation on the determination of source mechanisms.

The accurate determination of surface wave initial phase is important in the inversion of long-period source spectra for the seismic moment tensor. Until recently, spherically symmetric earth models have been used to correct the observed phase for propagation effects. Nakanishi & Kanamori (1982) demonstrated that the use of regionalized velocity models can improve surface wave moment tensor inversions at long periods. Their propagation corrections were computed assuming that surface-wave phase velocities are expressed as path integrals of phase velocities encountered along great-circle paths, which is the same assumption used in obtaining the regionalized models. We shall refer to this technique as the 'great-circle correction'. Tanimoto & Kanamori (1986) have also obtained significant variance reductions in their surface wave moment tensor inversions by correcting the phase spectra using recent global models of upper mantle structure. They show that while such corrections are successful in improving earthquake source determinations at periods greater than 200 s, they are less successful at periods between 100 and 200 s.

An example of the great-circle correction procedure for the 1980 Eureka, California, earthquake is shown in Fig. 1. The three panels show the phase and amplitude of the Rayleigh wave source spectra at periods of 256, 151 and 101 s, respectively. The phase data are shown with great-circle corrections for propagation through either spherically symmetric earth model PREM (Dziewonski & Anderson 1981) (triangles in Fig. 1) or through aspherical Earth model M84C (Woodhouse & Dziewonski 1984) (x's in Fig. 1). The great-circle phase corrections calculated for propagation through earth model M84C can achieve values as large as 0.5 rad at 256 s and 2 rad at 101 s. The theoretical curves were computed using the solution obtained for a period of 256 s by Tanimoto & Kanamori (1986). The relative fit of the data to the predicted curves is measured by taking the ratio of the variance obtained using the spherically symmetric earth model to the variance found using the laterally heterogeneous earth model. Ratios significantly greater than one imply that the heterogeneous earth model improves the source determination. For this example, the ratio (f) increases with period. The variance reductions using model M84C are statistically significant for periods of 256 and 151 s, but not for 101 s.

Tanimoto & Kanamori (1986) suggested that the failure of laterally heterogeneous earth

Table 1. 1980 earthquakes analysed.

Event	Date (m d)	Time (h m s)	Latitude (deg)	Longitude (deg)	Depth (km)	τ (s)	Region
K4	02 23	05 51 03.2	43.530N	146.753E	44	20	Kur. Is.
C8	06 09	03 28 18.9	32.220N	114.985W	05	15	S. Cal.
V16	07 29	03 11 56.3	13.101S	166.338E	48	19	Van. Is.
P18	09 26	15 20 37.1	03.225S	142.237E	33	14	Papua
C24	11 08	10 27 34.0	41.117N	124.253W	19	30	N. Cal.
A25	11 11	10 36 58.2	51.422S	28.796E	10	30	Ind. O.

models to reduce scatter in source phase-spectra at periods less than 200 s may be due to substantial deviations of surface wave ray-paths from great circle paths. Shorter period waves are most sensitive to shallow mantle structure, which has the strongest lateral velocity variations, thus the path deflections are expected to increase. In this note we test this hypothesis by computing phase corrections due to off great-circle propagation using two-point ray-tracing through the laterally heterogeneous earth model M84C. For both Rayleigh and Love waves we determine if these corrections provide significant variance reductions in source determinations compared with great-circle corrections.

Data and method

Our dataset consists of source spectra computed from long-period Rayleigh and Love wave seismograms from six earthquakes recorded at stations of the IDA and GDSN networks. The earthquakes were chosen from a set of 28 large, shallow events in 1980 which were analysed by Kanamori & Given (1982), Nakanishi & Kanamori (1982, 1984), and Tanimoto & Kanamori (1986). Table 1 lists the relevant parameters for the earthquakes analysed here (the labelling of the earthquakes established in the earlier papers is retained). Fig. 2 shows the spatial distribution of both the earthquakes and stations used. The geographic distribution of events ensures that our results are globally representative. The focal mechanisms shown in Fig. 2 and the source process times (*T*) in Table 1 were taken from Tanimoto & Kanamori (1986) or Nakanishi & Kanamori (1982).

Spectra were determined for second and third orbits (R2, R3, G2, G3) at periods of 201, 151, and 101 s and the phase spectra were corrected for propagation using both great-circle corrections and corrections obtained by exact ray-tracing through the aspherical earth model

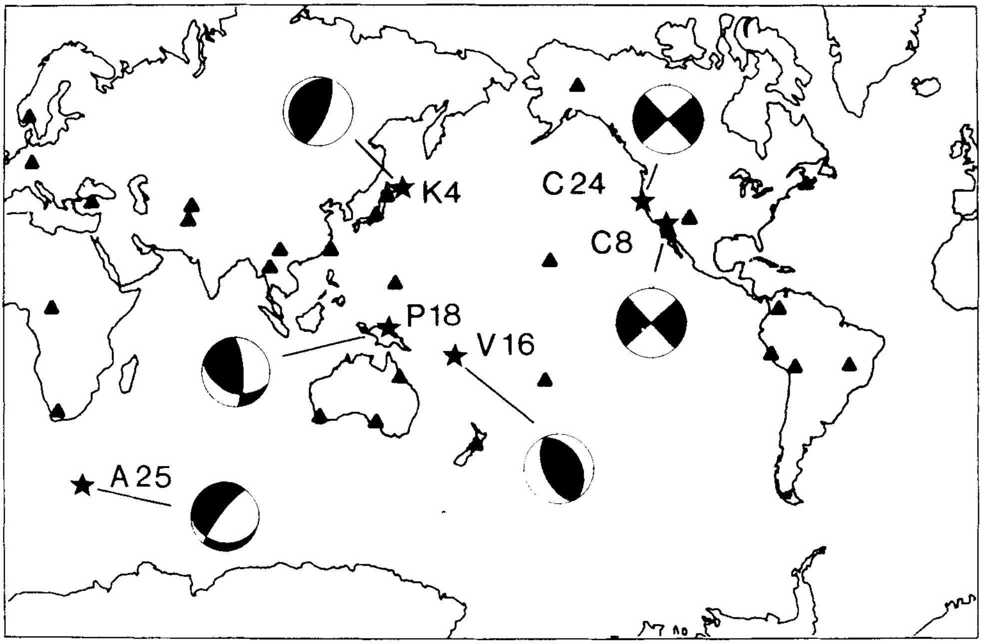

Figure 2. Map of earthquake (stars) and IDA and GDSN station (triangles) locations used in this study. Focal mechanisms shown were taken from Nakanishi & Kanamori (1982) and Tanimoto & Kanamori (1986).

M84C. We concentrated only on the phase effects of propagation through laterally heterogeneous structure because amplitude corrections based on geometrical ray theory are of questionable validity for these long-period waves. The resulting source spectra were then compared with predicted spectra based on the stable 256 s period moment tensor solutions (Fig. 2) to determine if the data variance was significantly reduced by accounting for the off great-circle propagation. We chose to compare the computed source spectra with prescribed models rather than inverting each period for a new mechanism because we believe that the 256 s solutions provide the best estimate of the true faulting geometries. The relatively small scatter in the amplitude and phase of the source spectra at 256 s, as well as the reduced size of the propagation corrections due to the suppressed variations in observed phase velocity allow for more stable source inversions at 256 s than at the shorter periods. The large scatter in the source spectra at shorter periods introduces instabilities into the source inversions, and we were often unable to successfully invert the 101 s data for any reasonable mechanism.

A shooting algorithm was employed to locate the ray-path travelling from source to receiver through heterogeneous earth model M84C. Julian's (1970) ray-tracing equations were numerically integrated using a predictor–corrector method. The use of analytical velocity derivatives in the ray-tracing equations, which are easily computed from the spherical harmonic representation of the velocity field, allowed ray-paths to be very accurately determined. Great-circle azimuths from source to receiver were used as initial azimuths and perturbed using a variation of Newton's method until a ray came within 5 km of the receiver. Convergence was typically achieved after three iterations for R2 and four iterations for R3. To ensure that the ray-paths found were the first arrivals and that substantial multipathing was not occurring, we performed several experiments of shooting a large number of rays at small azimuthal increments bracketing the great-circle azimuth to determine if more than one ray arrived at a given station. The computed ray-path variation was found to be smoother than previously determined (Lay & Kanamori 1985; Schwartz & Lay 1985) and little multipathing was observed.

Results

Rayleigh wave phase anomalies, defined as the difference in travel-time accumulated along the actual ray from that along the artificial great-circle ray-path, computed at 151 s for all six earthquakes are shown in Fig. 3. The phase anomalies are small for R2 (circles) and only slightly larger for R3 (triangles). The phase anomalies do not show any systematic azimuthal patterns and are two-sided functions. For R1 we would expect the phase anomalies to all be negative since Fermat's principle requires that the first orbit be a minimum travel-time path. For higher orbits, Fermat's principle requires only that the travel times be stationary; they are not absolute maxima or minima. From our dataset it appears that both R2 and R3 orbits tend more often to be slower than the great-circle paths.

Phase anomalies computed for Rayleigh waves at periods greater than 150 s are smaller than those shown in Fig. 3 due to the decrease in mantle heterogeneity with depth. The small travel-time corrections due to off great-circle propagation for periods greater than 150 s suggests that their application in source determinations will have little effect. Rayleigh wave phase anomalies computed at 101 s, and Love wave phase anomalies at 151 and 101 s are somewhat larger and we will investigate their effect on source determinations.

Fig. 4 shows R2, R3, G2, and G3 ray-paths from the Vanuatu Islands (V16) source region to the GDSN stations. The ray-paths were computed by ray-tracing through the 101 s Rayleigh and Love wave M84C structures. Ray-path deflections are evident as the lack of

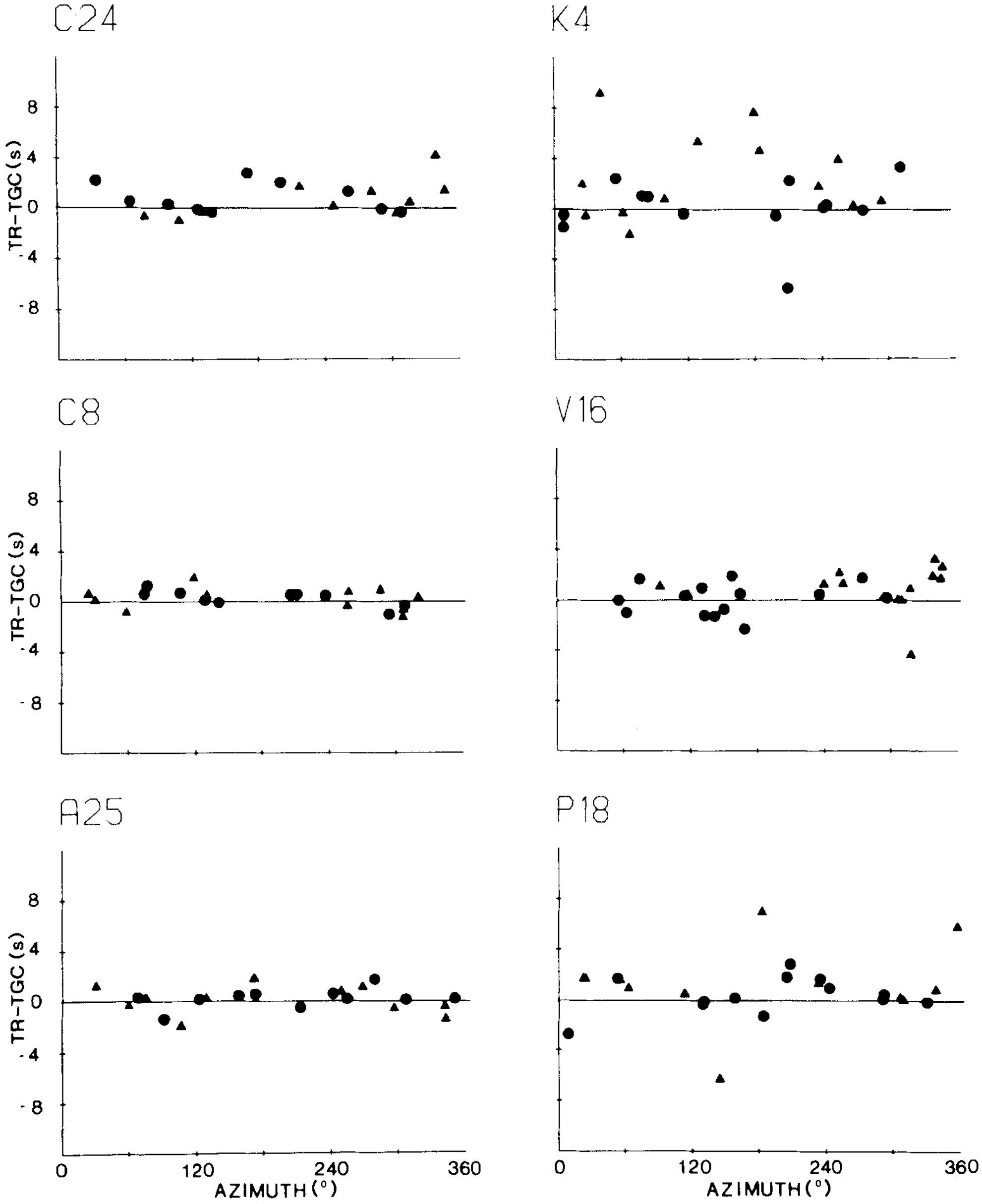

Figure 3. The difference between Rayleigh wave travel times accumulated along actual ray-paths (TR) and artificial great-circle paths (TGC) computed at 151 s for all six earthquakes. Circles indicate R2 paths and triangles R3 paths.

complete focusing of the ray-bundles at the source and the antipode, and by imperfect overlap in the successive orbit ray-paths arriving at the same station. These effects are enhanced for Love waves relative to Rayleigh waves due to an increased variation in the corresponding phase velocities. Very little multipathing is observed for the R2 and R3 orbits, while multipathing is more common but still rare for the G2 and G3 orbits. Also shown in Fig. 4 are phase anomalies computed at 151 s (circles) and 101 s (triangles) for the Vanuatu Islands

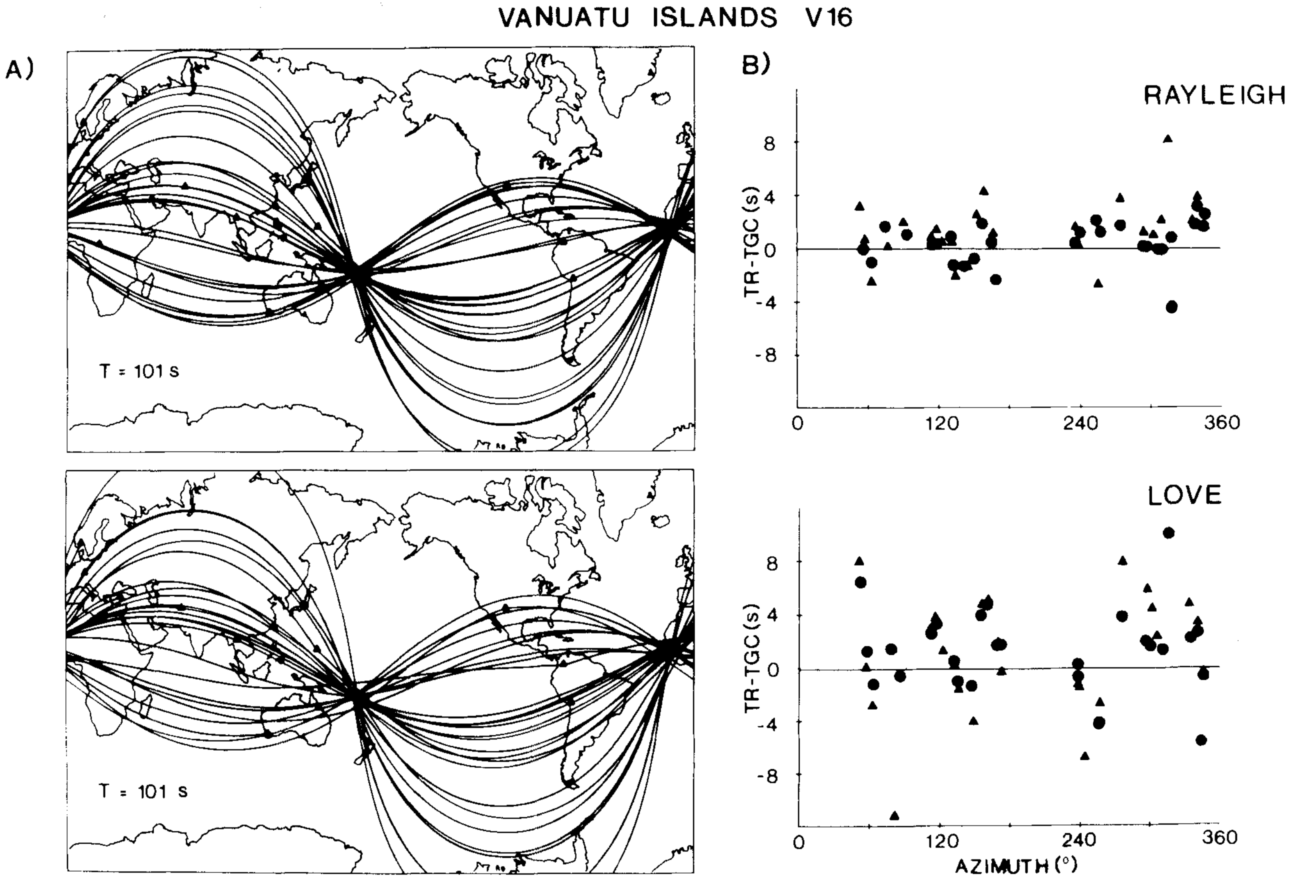

Figure 4. (a) Rayleigh wave (top) and Love wave (bottom) ray-paths traced from the Vanuatu Islands earthquake to GDSN stations through the 101 s earth model M84C. The maps are centred on the source area and the lack of complete focusing at the source as well as the antipode illustrates the path deflections from great-circle paths. (b) Differences in travel time along actual ray-paths (TR) and artificial great-circle paths (TGC) for 151 s (circle) and 101 s (triangle) Rayleigh and Love waves for the Vanuatu event. The anomalies are larger at shorter periods and for Love-waves compared with Rayleigh waves of the same period.

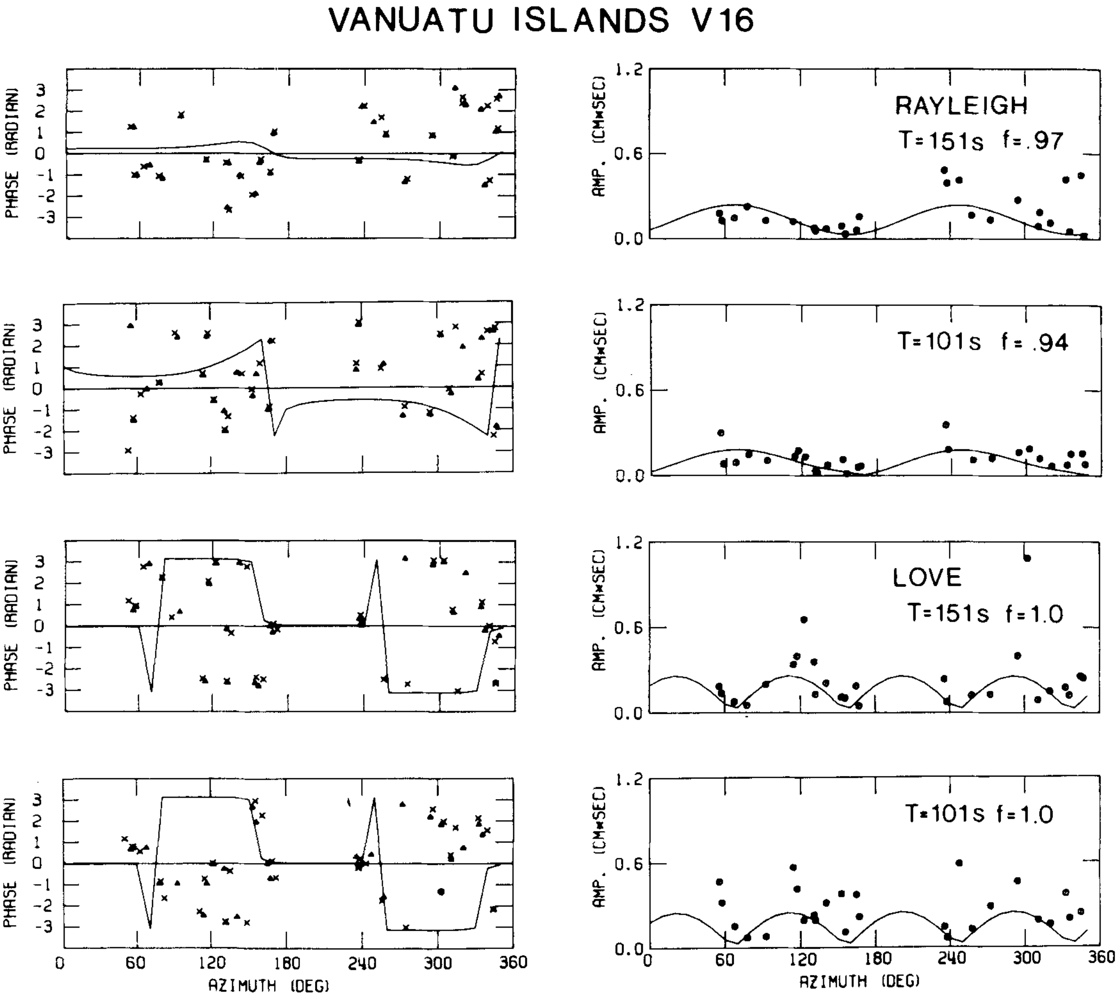

Figure 5. Phase and amplitude of Rayleigh and Love wave source spectra from the Vanuatu Islands event corrected for great circle (triangle) and actual raypath (X's) propagation through earth model M84C. The solid lines are theoretical curves for the source solution found using 256 s period data by Nakanishi & Kanamori (1982). The *f* ratio of about unity in all cases indicates that the off great-circle corrections are unable to decrease the data variance.

earthquake. As expected from examination of the ray-paths, the phase anomalies are larger for Love waves compared with Rayleigh waves, and they increase with decreasing period.

Application of the off great-circle phase corrections to the Rayleigh and Love wave source spectra from the Vanuatu Islands event has little effect on the data variance at both 151 and 101 s (Fig. 5). The data variance is slightly increased for Rayleigh waves and slightly decreased for Love waves. The off great-circle propagation corrections are small for Rayleigh waves but can be quite large for Love waves, especially at 101 s. Although the largest corrections are nearly of the same magnitude as those made going from a spherically symmetric earth model to an aspherical earth model, they are not systematically in the proper direction necessary to reduce data variance.

Fig. 6 shows an example of off great-circle propagation corrections applied to Rayleigh wave source spectra of the Kurile Islands earthquake (K4). Again these corrections have no

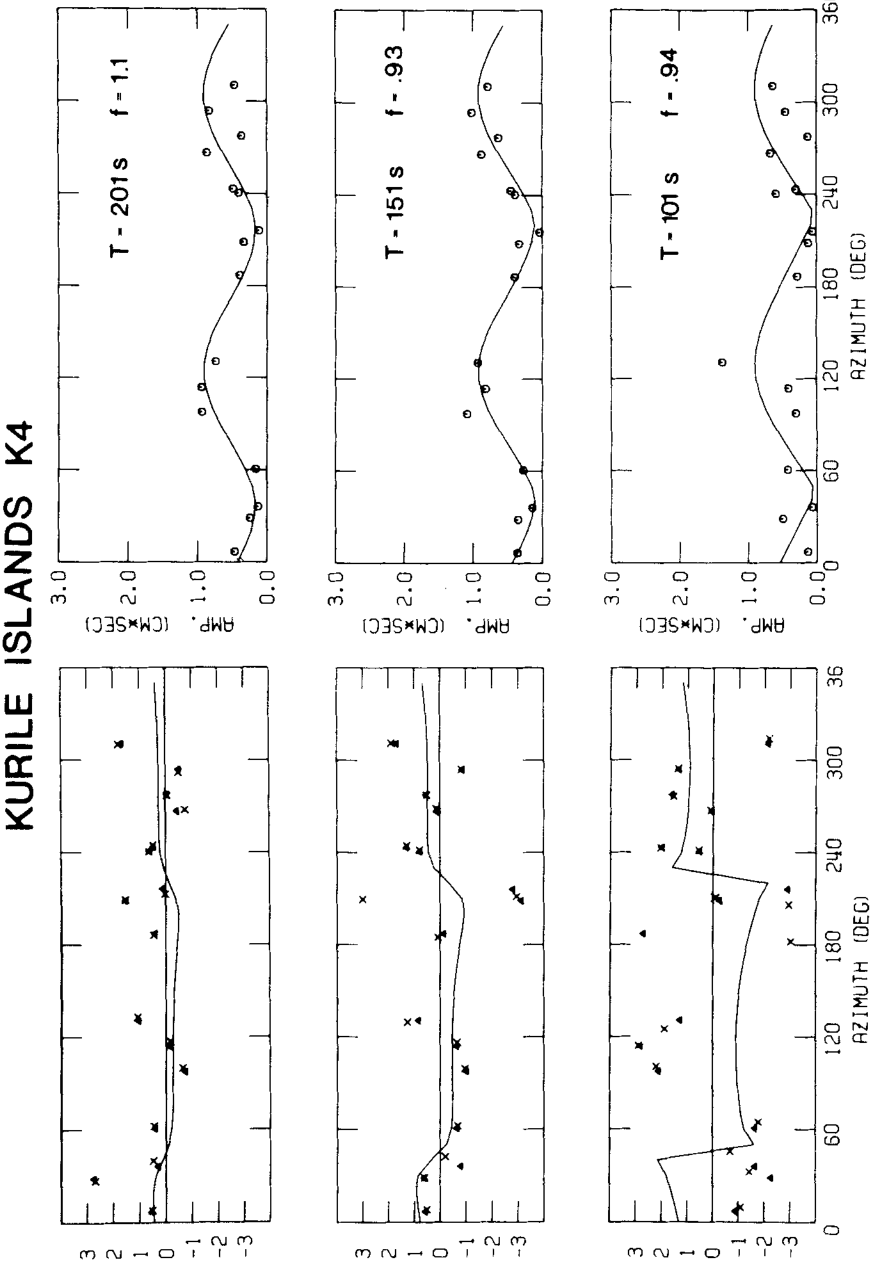

Figure 6. Same as Fig. 5 for Rayleigh waves from the Kurile Islands event (K4).

significant effect on the data variance at 201, 151 and 101 s. For all other events studied, the application of off great-circle propagation corrections was unable to significantly reduce the data variance at all periods tested.

Discussion

The failure of off great-circle corrections to improve source determinations at periods less than 200 s suggests that the aspherical upper mantle velocity model used to determine these corrections lacks resolution at shorter periods. It also confirms that the geometrical optics approximation is valid for low-orbit numbers. Therefore the inclusion of actual ray-path

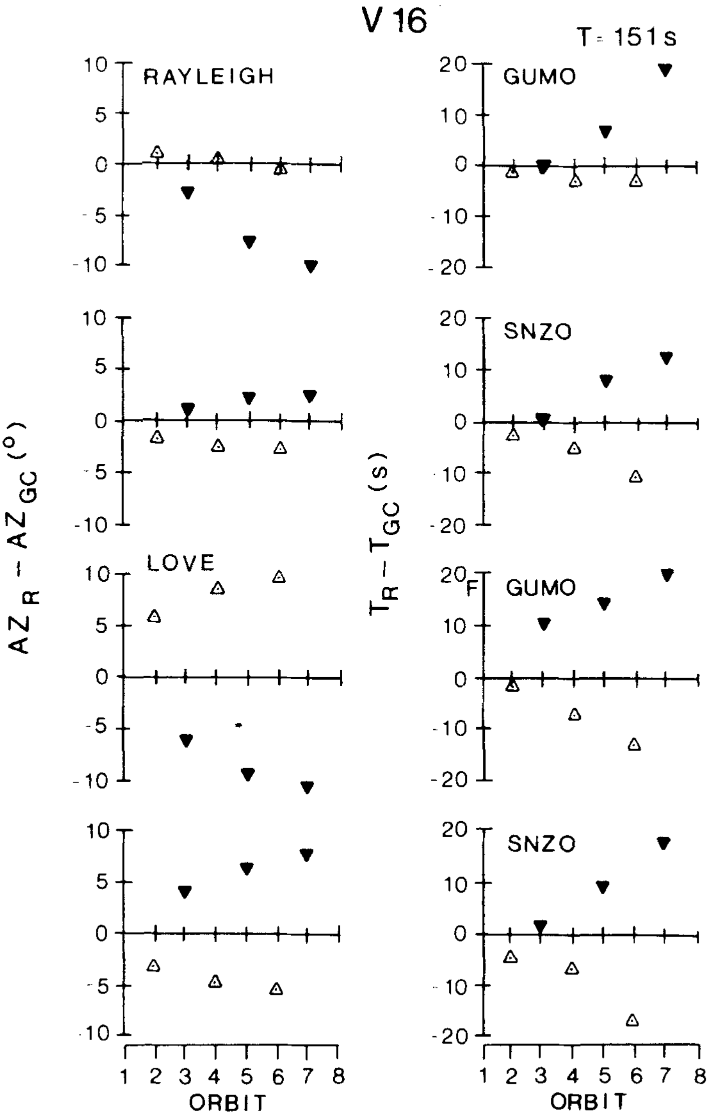

Figure 7. Rayleigh and Love wave azimuth and travel-time anomalies from great circle values as a function of orbit number for two GDSN stations recording event V16. Anomalies accumulate linearly with orbit number and even (open triangles) and odd (solid triangles) orbits show distinctive trends.

calculations when inverting low-orbit surface waves for low-order models of mantle heterogeneity is probably unnecessary.

Woodhouse & Wong (1986) show theoretically that in the presence of slight heterogeneity, to first order, phase anomalies referenced to a spherically symmetric earth structure and azimuthal deviations from great circle azimuths should increase linearly with orbit number. Their expressions indicate that the azimuthal deviations will have opposite slopes for odd and even orbits but the phase anomalies will have the same slope for both orbits. They show examples for Love and Rayleigh waves at periods between 150 and 300 s that agree with the geometrical optics approximation, as well as examples that violate it. We have computed azimuthal deviations from great-circle azimuths and phase anomalies accumulated along actual ray-paths referenced to great circle paths for 151 s higher orbit Rayleigh and Love waves from the Vanuatu Islands event (Fig. 7). We also observe trends not predicted by the geometrical optics approximation. While most azimuthal deviations show the predicted behaviour, some are not truly linear and others show the same sign for both odd and even orbits. Our phase anomalies, which measure deviations from the geometrical optics approximation, also increase quasi-linearly with increasing orbit number (Fig. 7) and for most cases these deviations are of different sign for odd and even orbits. Although these deviations are small for low orbits, they become substantial at higher orbit numbers.

The geometrical optics approximation appears to be valid for low-orbit surface waves. Its breakdown for higher orbits, which are important in free oscillation studies, emphasizes the importance of developing higher order approximations.

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