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START-OSCILLATION CONDITIONS IN MODULATED AND UNMODULATED O-TYPE OSCILLATORS

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ABSTRACT

The starting conditions for the O-type backward-wave oscillator are computed for large values of C, QC and d using both digital and analog methods. A general method of solving complex polynomials called the "downhill" method is applied both to the secular equation and then to the r-f voltage equation to obtain starting conditions. The analog computer is used to solve simultaneously, by trial and error method, the linear circuit and ballistic differential equations. The analog method is applied to the modulated BWO in order to determine the effects of modulations on the starting conditions.

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INTRODUCTION

The O-type backward-wave oscillator has found wide application as a tunable signal source where relatively low power levels are acceptable. The chief limitation in building high-power tunable oscillators is the lack of a broadband circuit which has a high characteristic impedance for the backward-wave mode. Most O-type oscillators are built using the helix operated on the minus-one space harmonic and for normal beam current values have characteristic C values near 0.02 and below. Larger values of C have been achieved at S-band by Putz and Luebke[1] in their work on a 100-watt oscillator using a folded-line r-f structure. Even with values of C between 0.05 and 0.08 the efficiency of their device was around 8-10 percent. In recent years the interdigital line has been used in O-type backward-wave oscillators; its higher backward-wave impedance suggests that oscillators with higher C values will be possible.

Many analyses[2,3,4,5] of the O-type oscillator have been made using various linear theories to compute the starting conditions, generally assuming small C values. A linear type of analysis can be used because the oscillator is operating linearly just at start oscillation.

One of the purposes of this paper is to present a summary of start-oscillation conditions for the O-type backward oscillator for large values of C, QC and d computed using a digital method. It is shown how they may also be calculated on an analog computer. One of the advantages of the analog computer method is that the oscillator can be studied under modulation conditions. This study of the modulated BWO is the second objective of this paper, wherein a study of the effect of modulation on the oscillator starting conditions is presented.

BALLISTIC AND CIRCUIT EQUATIONS

The linear theory to be used in calculating the start-oscillation conditions for large C has been described by Rowe[6] previously and was developed following a procedure similar to Johnson's small-C derivation[5]. The interaction with the backward-wave mode of the circuit is accounted for by introducing an impedance parameter into the equations which is the negative of the forward-wave impedance parameter used in traveling-wave amplifier studies. It is also convenient to define a passive-mode parameter Q which is the negative of that used in the forward-wave equations, in addition to the change in C noted above.

The linearized equations for the oscillator are easily integrated to give the propagation constants. The wave amplitudes are determined from a knowledge of the input boundary conditions at the entrance to the interaction region. An oscillation condition exists for a given C, QC and d when the proper combination of b and CN_s is found such that the amplitude of the r-f wave on the structure vanishes for $z > 0$. N_s is the structure length in stream wavelengths. Heffner[2] and Johnson[4] used essentially this method in their work except that Heffner used the beam current as the driving source for the circuit equation whereas Johnson more correctly used the space derivative of the current. In Bernier's[7] analysis of the forward-wave amplifier he also used the current as the driving source in the transmission-line equation.

The linearized ballistic equations for the BWO are identical to those for the small-signal traveling-wave amplifier. They may be written as follows:

$$\eta E_T + j\omega v + u_o \frac{dv}{dz} = 0 \quad , \qquad (1)$$

$$\rho_o \frac{dv}{dz} + u_o \frac{d\rho}{dz} + j\omega\rho = 0 \quad , \qquad (2)$$

where

ρ = magnitude of electron charge-to-mass ratio,

E_T = total z-directed electric field intensity acting on the beam (sum of the circuit and space-charge fields, $E_c + E_{sc}$),

v = amplitude velocity perturbation,

ρ = amplitude of charge-density perturbation,

u_o = average beam velocity,

ρ_o = average beam charge density,

and all perturbed quantities are assumed to have a time variation of exp $j\omega t$. It has been assumed that $v \ll u_o$ and that $\rho \ll \rho_o$.

The circuit equation is the forced transmission line equation. Note that the righthand side of this equation has the opposite sign to that of the corresponding TWA equation.

$$\frac{d^2V_c}{dz^2} + \beta_o^2 V_c + j \frac{R}{Z_o} \beta_o V_c = - \omega(Z_o\beta_o + jR)\,\rho \quad , \qquad (3)$$

where

V_c = r-f circuit potential (potential from which the slow-wave field E_c is derivable),

β_o = radian wave number of circuit,

Z_o = characteristic impedance of circuit, and

R = series loss of circuit (ohms/meter).

The above formulation of the interaction equations is convenient for both the digital and analog methods to be used in solving the equations for the start-oscillation conditions.

The form of the space-charge field expression used is that given by Pierce.

$$E_{sc} = \frac{-j\Gamma^2 i}{\omega C_1} , \qquad (4)$$

where C_1 is the capacitance defined by Pierce.

In order to facilitate digital solution of the above equations it is convenient to assume that all r-f quantities vary exponentially with distance as exp $-\Gamma z$ in addition to their exponential time variation. Since it is expected that the actual propagation constant of the system will be only slightly different from the system cold propagation constant, Pierce's[8] method will be used and the following terms defined

$$-\Gamma_n \overset{\triangle}{=} - j\beta_e + \beta_e C\delta_n , \qquad (5)$$

$$-\Gamma_1 \overset{\triangle}{=} - j\beta_e - j\beta_e Cb + \beta_e Cd , \qquad (6)$$

where $\delta_n = x_n + jy_n$, the perturbation propagation constant and the remainder of the symbols have the usual meanings (see Pierce[8] or Rowe[6]). The Γ_1 of Eq. 6 is the cold-circuit propagation constant and the Γ_n are the actual wave propagation constants with the stream present.

The interaction equations may now be combined as algebraic equations in δ_n using the definitions given in Eqs. 5 and 6. The resultant is a fourth-degree polynomial with complex coefficients and of course complex roots. This is given below for the BWO.

$$\begin{aligned}\delta^4\left(\frac{C}{2} - 2C^3QC\right) &- j\delta^3(1 - 8C^2QC)\\ &+ \delta^2\left[(1 + C^3 - 4C^2QC)\nu + C(C + 10QC) + \nu^2\left(\frac{C}{2} - 2C^3QC\right)\right]\\ &+ j\delta\left[\nu(-2C^2 + 8CQC) + \nu^2(4C^2QC) - 2(C + 2QC)\right]\\ &\qquad - 1 + \nu(4QC - C) + \nu^2(2CQC) = 0 \quad , \qquad (7)\end{aligned}$$

where

$$\nu = b + jd.$$

The input boundary conditions are applied to the stream convection current density and stream velocity and to the r-f wave on the circuit at $z = 0$ to obtain a wave-amplitude matrix. The solution of this matrix for the excited wave amplitudes on the circuit is given in reference 6 and need not be repeated here. Analogously to the TWA, the r-f voltage as a function of distance in the BWO may be written as

$$\frac{V_t(\theta)}{V_c(0)} = e^{-j\frac{\theta}{C}} \sum_{i=1}^{3} \left(\frac{V_{ci}}{V_c(0)}\right) e^{\delta_i \theta} \quad , \qquad (8)$$

where $\theta \triangleq 2\pi CN_s$ = the radian length of the tube. The fourth root has been eliminated since this wave does not interact with the electron stream.

DIGITAL CALCULATION OF STARTING CONDITIONS

In order to determine a starting condition for the BWO it is necessary to search the $b - CN_s$ plane for points at which the right-hand side of Eq. 8 becomes zero for particular values of C, QC and d. A general method known as the "downhill" method has been worked out to find both the complex roots of Eq. 7 and the start-oscillation condition from Eq. 8. The quartic polynomial given in Eq. 7 may be separated into real and imaginary parts and written in the following form:

$$P(\delta) = \sum_{i=0}^{4} (A_i + jB_i)\, \delta^i \quad . \tag{9}$$

The following definitions are made to simplify the coefficients of Eq. 7.

$$R = C\left(\frac{1}{2} - 2C^2QC\right) \qquad U = b - 2QC(b^2 - d^2)$$

$$S = 10QC + \frac{1}{2}\left(b^2 - d^2\right) \qquad V = 4QC$$

$$T = 1 - 4bQC \qquad W = 1 - 8C^2QC \quad . \tag{10}$$

In terms of the above definitions the components of Eq. 9 become

$$\begin{aligned} A_0 &= -(T+CU) & B_0 &= d(V-CT) \\ A_1 &= -2CB_0 & B_1 &= -V + 2CA_0 \\ A_2 &= (b+CS) - C^2A_0 & B_2 &= d(1+Cb) - C^2B_0 \\ A_3 &= 0 & B_3 &= -W \\ A_4 &= R & B_4 &= 0 \quad . \end{aligned} \tag{11}$$

The downhill method for obtaining roots of a polynomial $P(\delta)$ consists of searching the surface over the complex plane generated by

$$S = |R[P(\delta)]| + |I[P(\delta)]| \quad . \tag{12}$$

When S becomes 0, the value of δ is then a root of the polynomial. The nature of Eq. 12 is such that if a point δ_1 produces S_1, there is a point δ_2 where $S_2 < S_1$ unless S_1 is zero. Thus for a continuous path from δ_1 to δ_2 there is a point δ_3 on this path such that $S_2 < S_3 < S_1$. Hence it is clear that during the search for roots, one always proceeds downhill to the roots. Mechanization of this method requires a systematic scheme for searching the surface S.

A wheel of points is selected with a center point and the polynomial is evaluated at each of these points and the point for which S is a minimum becomes the center point for the next wheel. This process is repeated until the value of S reaches some minimum limit, usually of the order of 10^{-6}. The value of δ giving this minimum is considered to be the root.

After the roots of the polynomial are determined for a given set of C, QC and d then the same technique is applied to the r-f voltage equation and the solution of

$$\sum_{i\;1}^{3} \left(\frac{V_{ci}}{V_c(0)} \right) e^{\delta_i \theta} = 0 \qquad (13)$$

is sought. The initial center point of the wheel and the initial spacing of the endpoints are arbitrarily selected. Of course, there is more than one combination of b and CN_s which will satisfy Eq. 13, as was shown in reference 6. The minimum b-CN_s combination is the desired one and two typical plots of these are shown in Figs. 1 and 2*. The parameter $(\beta-\beta_e)L$ is proportional to the product of CN_s and b. The results indicate that the value of CN_s at start oscillation becomes relatively independent of QC as C is increased beyond 0.1. The effect of both loss and space charge is to increase the required starting length at constant C. As expected the required CN_s decreases with increasing C while the required b increases.

ANALOG CALCULATION OF STARTING CONDITIONS

An alternate method for determining the start-oscillation conditions is to program the differential equations on an analog computer

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* Starting conditions for a wide range of parameters are tabulated in Appendix A. Plots of the starting conditions are given in Appendix B.

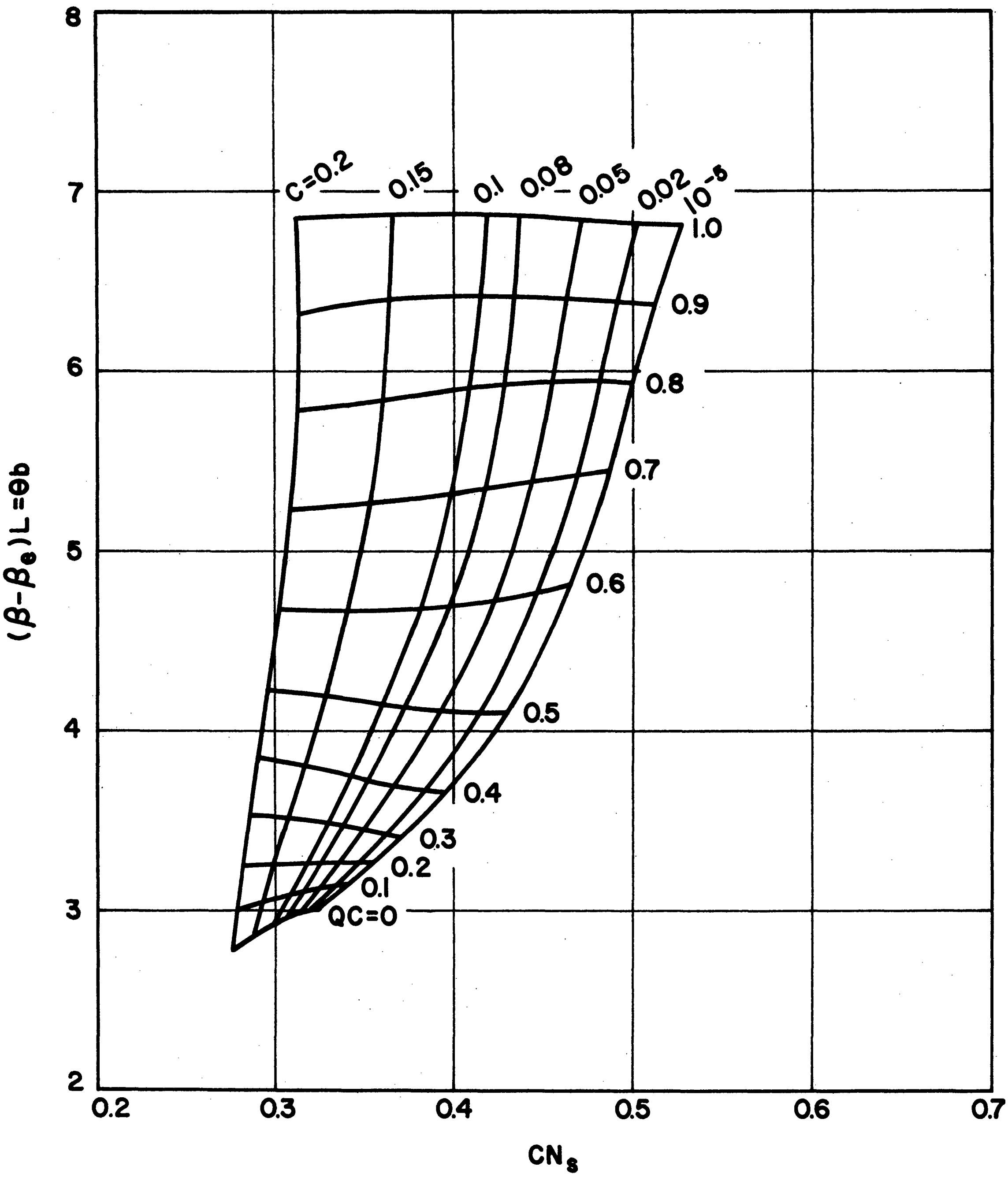

FIG. I O-TYPE BWO STARTING CONDITIONS. (d=0.1)

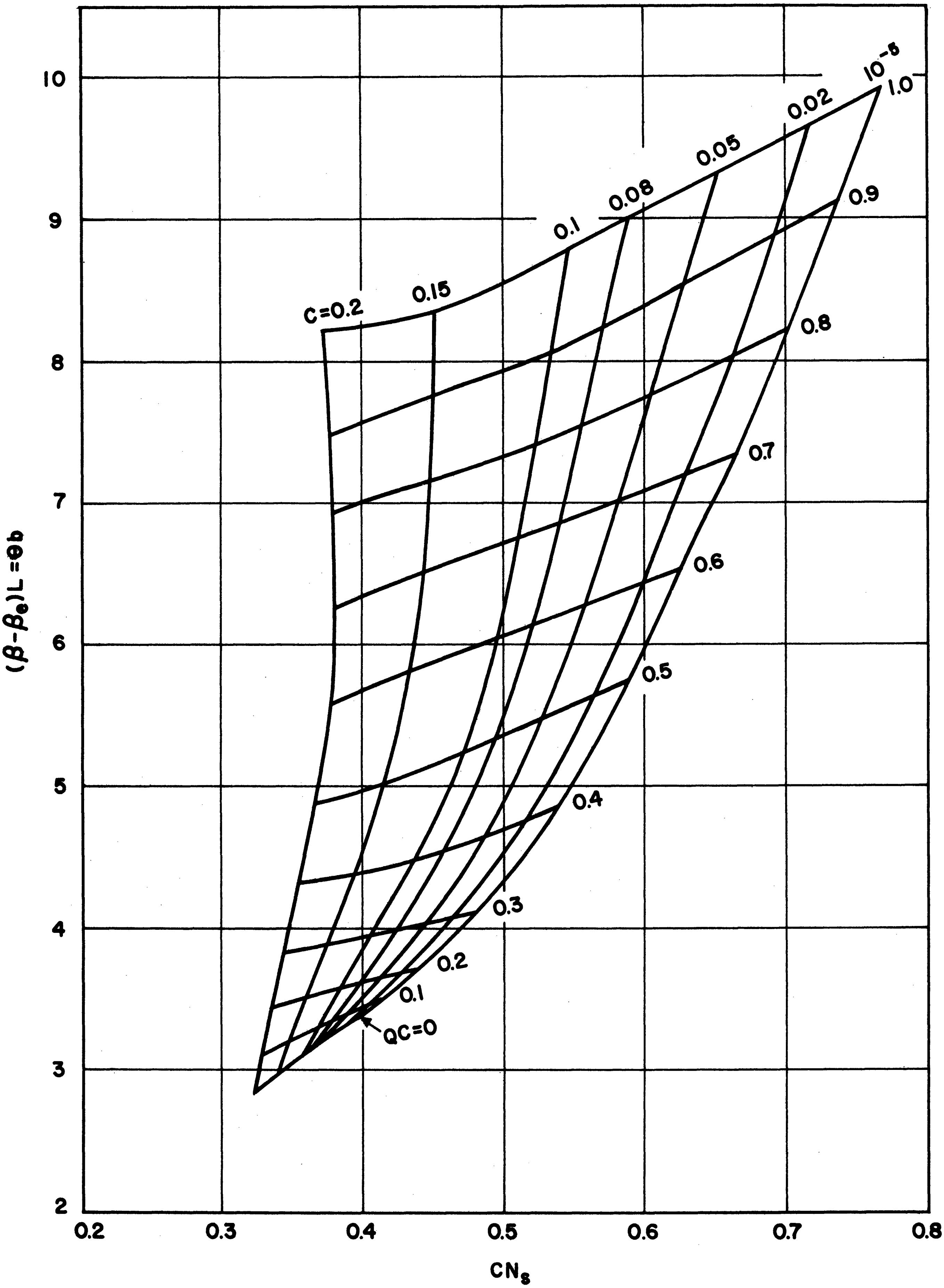

FIG. 2 O-TYPE BWO STARTING CONDITIONS. (d=0.5)

and obtain the solutions by a trial and error procedure. The advantage of using an analog computer is that one can easily vary parameters merely by changing a potentiometer setting. Grow[9] has previously determined the start-oscillation conditions for the unmodulated BWO using an analog computer. He found the r-f circuit voltage by using a fourth-order linear differential equation obtained by combining the circuit and ballistic equations and eliminating the beam charge density, electron velocity and current from the resulting expressions.

The method used in this paper is to program the circuit and ballistic equations as simultaneous linear differential equations. The advantage of this program is that one can study the variation of current, charge density and velocity in addition to the circuit voltage. This leads to a better appreciation of the entire phenomenon of start-oscillation. The program is also applied to the determination of the start-oscillation conditions of a modulated BWO.

The interaction equations were given in Eqs. 1 through 3. An alternate form for the space-charge field expression is used in the analog computation. It can be shown that for a beam of finite radius the space-charge field is given by[10]

$$E_{sc} = j \frac{u_o^2}{\eta I_o} \left(\frac{v_c}{u_o}\right) \left(\frac{\omega_p R}{\omega}\right)^2 \rho \ , \qquad (14)$$

where

ω_p = the plasma radian frequency,

v_c = phase velocity of the circuit wave,

R = plasma frequency reduction factor, and

I_o = average beam current.

Equation 14 is more amenable to analog computation than is Eq. 4. These different forms for the space-charge field result in different starting conditions for the cases of large C and large QC, as will be noted later. This was pointed out previously by Grow[9].

It is found convenient to introduce several normalizations before programming. A normalized distance variable is introduced according to the following definition.

$$y = \beta_e C z \quad , \tag{15}$$

where β_e is the stream phase constant. The normalized interaction equations then become

$$\frac{1}{2}\frac{dV_c}{dy} + j\,\frac{4CQC}{(1-2C\sqrt{QC})^2}\,\frac{\rho}{1+Cb} = \frac{dv}{dy} + j\,\frac{v}{C} \quad , \tag{16}$$

$$\frac{dv}{dy} + \frac{d\rho}{dy} + j\,\frac{\rho}{C} = 0 \quad , \tag{17}$$

and

$$\frac{d^2V_c}{dy^2} + \left(\frac{1+Cb}{C}\right)^2 V_c + 2j\,\frac{d}{C}\,(1+Cb)\,V_c = \left[4C(1+Cb) + j8C^2d\right]\rho \; . \tag{18}$$

Equations 16 through 18 are solved on an analog computer after separating into real and imaginary parts. This is accomplished by assuming the following forms for the dependent variables.

$$V_c = V_R + jV_I \quad ,$$
$$v = v_R + jv_I \quad ,$$
$$\rho = \rho_R + j\rho_I \quad . \tag{19}$$

The magnitude of any of the above variables may be found by using a standard analog computer resolver for converting cartesian coordinates into cylindrical coordinates. The boundary conditions must be specified before Eqs. 16 through 19 may be integrated. These are the usual conditions that the beam enters the interaction region carrying no r-f information other than noise, the amplitude of the r-f circuit wave at this position is arbitrary*, and the r-f output is matched. Noise on the beam is neglected in determining the circuit wave amplitudes. The boundary conditions are

$$v(0) = \rho(0) = 0 \quad ,$$

and

$$\frac{V_c(0)}{I_c(0)} = -Z_o \quad , \tag{20}$$

which lead to

$$\left(\frac{dV_c}{dy}\right)_o = -V_c(0)\left[j\,\frac{1+Cb}{C} - 2d\right] \quad . \tag{21}$$

The problem can be scaled by using the following approximate maximum values, assuming a unit voltage

$$|\rho| \approx \frac{0.4}{C_o^2} \quad , \tag{22}$$

$$|v| \approx \frac{0.625}{C_o} \quad , \tag{23}$$

and a time scale of

$$P = C_o \frac{d}{dy} \quad . \tag{24}$$

The procedure to find an oscillating point for a given C, QC and d is to vary b and plot the circuit voltage V_c as a function of y for each value of b. A typical set of plots is shown in Fig. 3. The b that forces the r-f circuit voltage to go to zero represents a start-oscillation condition. The starting length is determined from the value of y at which the line voltage vanishes.

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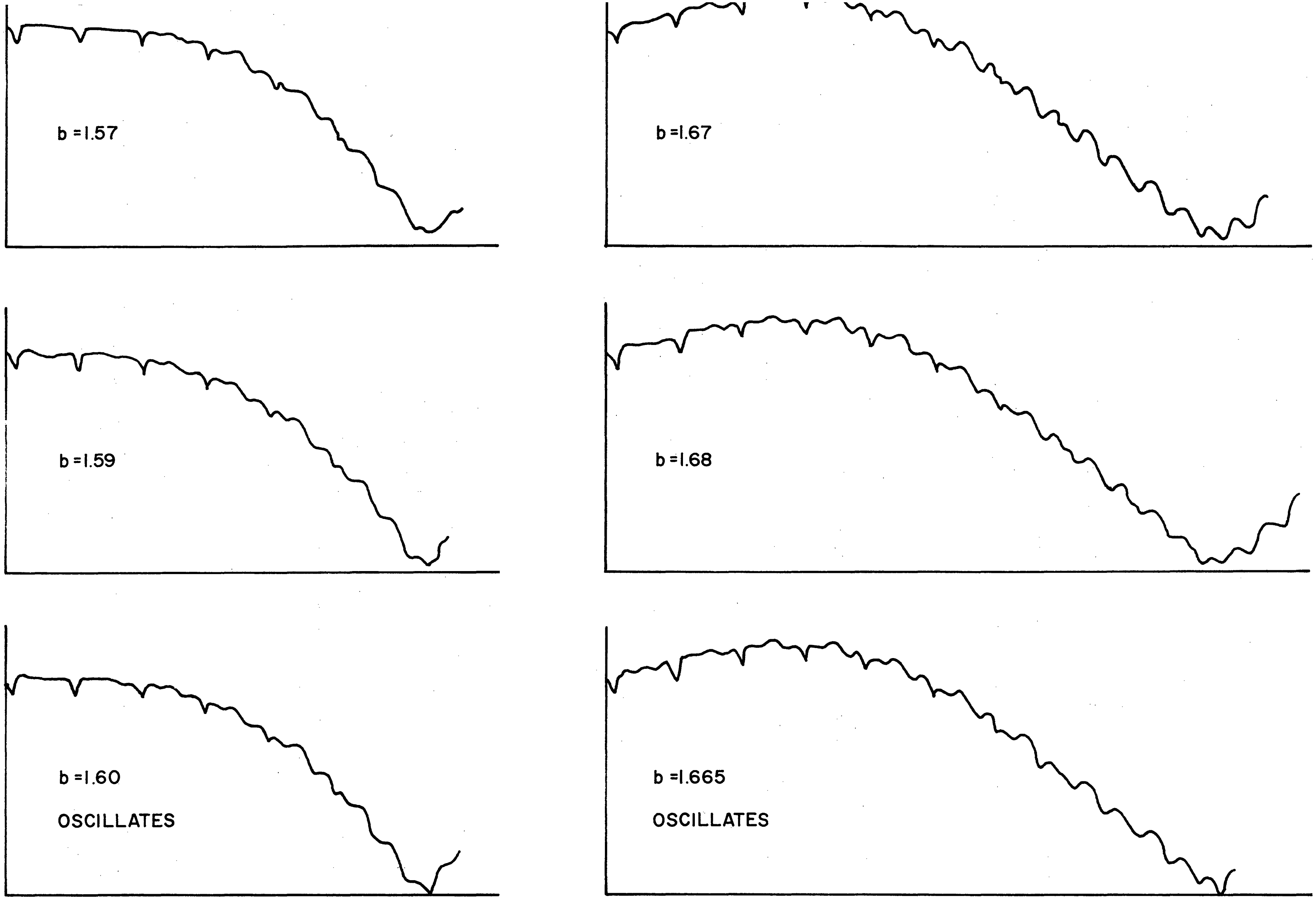
* This is true owing to the linearity of the problem.



a. C=0.05, QC=0.10, d=0

b. C=0.05, QC=0.50, d=0.25

FIG. 3 TRIAL AND ERROR RUNS FOR DETERMINING START-OSCILLATION CONDITIONS SHOWING R-F VOLTAGE VS. LENGTH.

A set of oscillating points as determined from the analog computer is shown below in Table I. A comparison is made with digital computer calculations in which the aforementioned implicit relation is solved. Note that good agreement is obtained between the two methods for all cases except for simultaneously large C and QC. This discrepancy at large QC for large C values is a result of the different space-charge expressions used. These results do disagree with those of Grow[9]. Since the results of Table I were found using two independent methods it is the authors' opinion that there may be some errors in Grow's work.

TABLE I

Comparison of Start-Oscillation Conditions Determined by Analog and Digital Methods

C	QC	d	b at Start-Oscillation		CN_s for Oscillation	
			Analog	Digital	Analog	Digital
0.05	0	0	1.57	1.559	0.3000	0.3013
0.05	0.10	0	1.61	1.575	0.3096	0.3099
0.05	0.20	0	1.62	1.593	0.3167	0.3202
0.05	0.50	0	1.71	1.690	0.3748	0.3691
0.05	0	0.25	1.48	1.467	0.3334	0.3314
0.05	0.10	0.25	1.51	1.484	0.3406	0.3433
0.05	0.20	0.25	1.52	1.503	0.3581	0.3579
0.05	0.50	0.25	1.67	1.652	0.4464	0.4351
0.10	0	0	1.62	1.594	0.2898	0.2891
0.10	0.5	0	1.83	1.872	0.3614	0.3399
0.20	0	0	1.67	1.664	0.2675	0.2662
0.20	0.5	0	2.17	2.329	0.3153	0.2826

MODULATION OF A BWO

A detailed modulation study of the BWO would involve determining the change in frequency and power level of the operating oscillator as its average beam potential and current are varied by the modulating signal.

The characteristics of the operating* or unmodulated BWO can be determined only from a nonlinear analysis of the device, since it is the nonlinearities that ultimately determine the operating levels and the frequency. Nonlinear analyses of the unmodulated BWO have been carried out by Sedin[11] and by Rowe[6] using the Lagrangian or particle approach. Sedin modified Nordsieck's[12] TWA equations by changing the sign of the circuit impedance, while Rowe developed more general interaction equations and also changed the sign of the impedance. In each of these papers the authors experienced considerable difficulty and expense in determining operating points. The major difficulty was that the boundary conditions at the beam entrance have to be found by a trial and error procedure. This, just as in the small-signal case, involves integrating the BWO equations for a series of trial boundary conditions until a zero line voltage is found to exist.

In a low-frequency beam-modulation study of the BWO, it is necessary to include, in addition to the variable average beam parameters, a variable frequency as a function of the average beam conditions. Allowing the frequency to vary means that dispersion and impedance variation effects must be accounted for; this implies that an r-f structure must be specified. After selecting an r-f backward-wave structure, a set of quasi-stationary equations can be derived to describe the modulation characteristics. The solution of these equations would involve a difficult trial and error procedure which not only entails guessing the boundary conditions, but would further require guessing the new frequency.

In view of the difficulties that would be encountered, it was decided not to attempt to solve the modulation problem using the Lagrangian

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* Operating refers to operation at current levels above the start-oscillation point.

approach. Due to the severe bunching effects shown theoretically by Sedin and experimentally by Gewartowski[13], it was decided that a hydrodynamical model using nonlinear equations would not be valid. The study presented below will discuss only the effect of modulation on the start-oscillation conditions.

THE EFFECT OF MODULATION ON THE START-OSCILLATION CONDITIONS

The equations developed above for the start-oscillation conditions of a BWO can easily be adapted to study the effect of low-frequency beam modulations on the start-oscillation conditions. A low-frequency beam modulation can be introduced by postulating that the beam average potential and current vary as

$$V_o = V_{o1}\left(1 + \frac{\Delta V}{V_{o1}}\right) \quad , \qquad (25)$$

$$I_o = I_{o1}\left(1 + \frac{\Delta I}{I_{o1}}\right) \quad , \qquad (26)$$

where V_{o1}, I_{o1} = the unmodulated quantities, and

ΔV, ΔI = the variations caused by the modulation.

It is convenient to introduce some additional modulation parameters. Therefore, define:

$$\xi_1 = \left(1 + \frac{\Delta V}{V_{o1}}\right)^{1/2} , \qquad (27a)$$

$$\xi_2 = \left(\frac{1 + \frac{\Delta I}{I_{o1}}}{M^3}\right)^{1/2} , \qquad (27b)$$

$$M^3 = \frac{1 + \frac{\Delta V}{V_{o1}}}{1 + \frac{\Delta I}{I_{o1}}} \quad , \qquad (27c)$$

$$\zeta_1 = \frac{\beta_o(M)}{\beta_o(M=1)} \quad , \tag{27d}$$

$$\zeta_2 = \frac{Z_o(M=1)}{Z_o(M)} \quad , \tag{27e}$$

$$\zeta_3 = \frac{\omega(M)}{\omega_o} \quad , \tag{27f}$$

and

$$F(M) = \frac{R^2(M)}{R^2(M=1)} \, \frac{1}{\zeta_1} \quad . \tag{27g}$$

M = 1 indicates no modulation.

When the interaction equations are revised to allow for the variations of Eqs. 25 and 26, the frequency is included as a function of the modulation, and the terms introduced in Eqs. 27 are used the following set of normalized equations results.

$$\xi_2 \frac{dv}{dy} + \xi_1 \frac{d\rho}{dy} + j \frac{\rho}{C_o} = 0 \quad , \tag{28}$$

$$\frac{1}{2} \frac{dV_c}{dy} + \frac{4C_o QC_o}{(1-2C_o\sqrt{QC_o})^2} \, \frac{\rho}{1+C_ob_o} \, \frac{F(M)}{\zeta_3} = \xi_1 \frac{dv}{dy} + j \frac{v}{C_o} \quad , \tag{29}$$

and

$$\frac{d^2V_c}{dy^2} + \left(\frac{1+C_ob_o}{C_o}\right)^2 \left(\frac{\zeta_1}{\zeta_3}\right)^2 V_c + 2j \frac{d_o}{\omega} (1+C_ob_o) \frac{\zeta_1\zeta_2}{\zeta_3} V_c$$

$$= \left[4C_o(1+C_ob_o) \frac{\zeta_1}{\zeta_2\zeta_3} + j \frac{8C_o^2 d_o}{\zeta_3} \right] \rho \quad , \tag{30}$$

where C_o, QC_o, b_o and d_o refer to the unmodulated parameters.

The boundary conditions now become

$$v(0) = \rho(0) = 0$$

$$\left(\frac{dV_c}{dy}\right)_{y=0} = -V(0)\left[j\,\zeta_2\zeta_3\,\frac{1+C_o b_o}{C_o} - 2d_o\,\zeta_2\zeta_3\right] . \qquad (3.1)$$

As mentioned above, in order to solve this set of equations it is necessary to specify a structure. The r-f circuit selected is the bifilar helix. The important properties of the bifilar helix as a backward-wave structure are given in the literature[14,15] and are shown in Fig. 4. The normalized frequency parameters ζ_1, ζ_2, ζ_3 may be calculated using Fig. 4. ζ_1 and ζ_2 as a function of ζ_3 are shown in Fig. 5. F as a function of ζ_3 is shown in Fig. 6. The reduction factor for a hollow beam[16] was used and the unmodulated ka for all plots was taken as 0.30.

The program to solve Eqs. 28 through 30 is only slightly different from the one for the unmodulated BWO. The procedure for finding the effect of modulation on the start-oscillation conditions (i.e., the new frequency and the new starting length) is to determine first the unmodulated operating point using the method given in the previous section. The frequency as a function of voltage modulation can then be estimated from the ω-β diagram for the bifilar helix. Such an estimate is shown in Fig. 7. The problem is run on the computer first using the ζ_3 determined from Fig. 7 for a specific $\Delta V/V_{01}$; ζ_3 is then varied according to the data for Figs. 5 and 6 as well as the other parameters until an oscillating point is found. This procedure is illustrated in Fig. 8.

The effect of a beam-voltage modulation on the required normalized start-oscillation length and normalized starting frequency of a typical BWO is shown in Figs. 9 through 11. It can be seen that the initial values are quite accurate at low modulation amplitudes, but are in error as

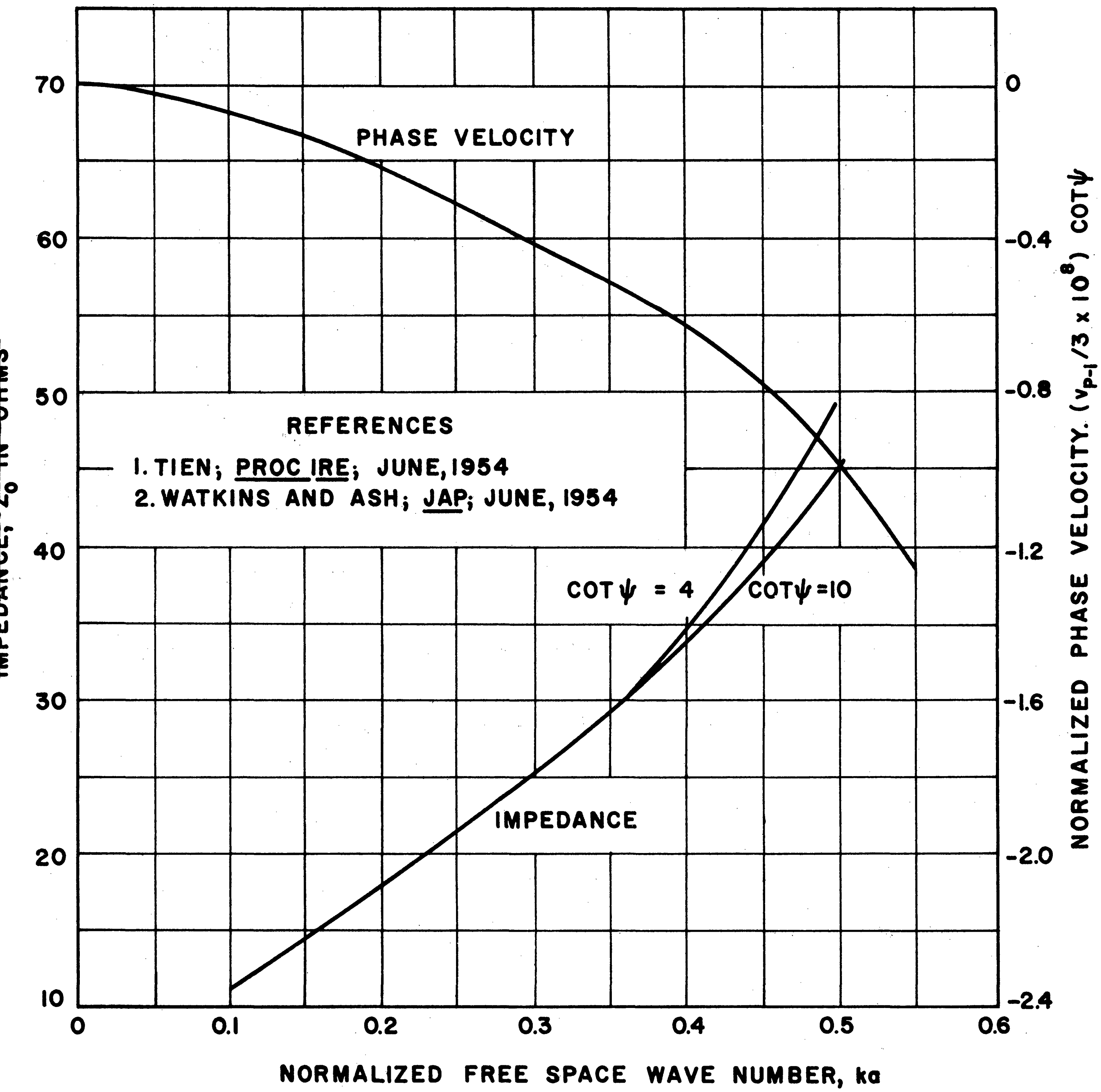

FIG. 4 IMPEDANCE[1] AND PHASE VELOCITY[2] CHARACTERISTICS OF (-1) MODE OF BIFILAR HELIX.

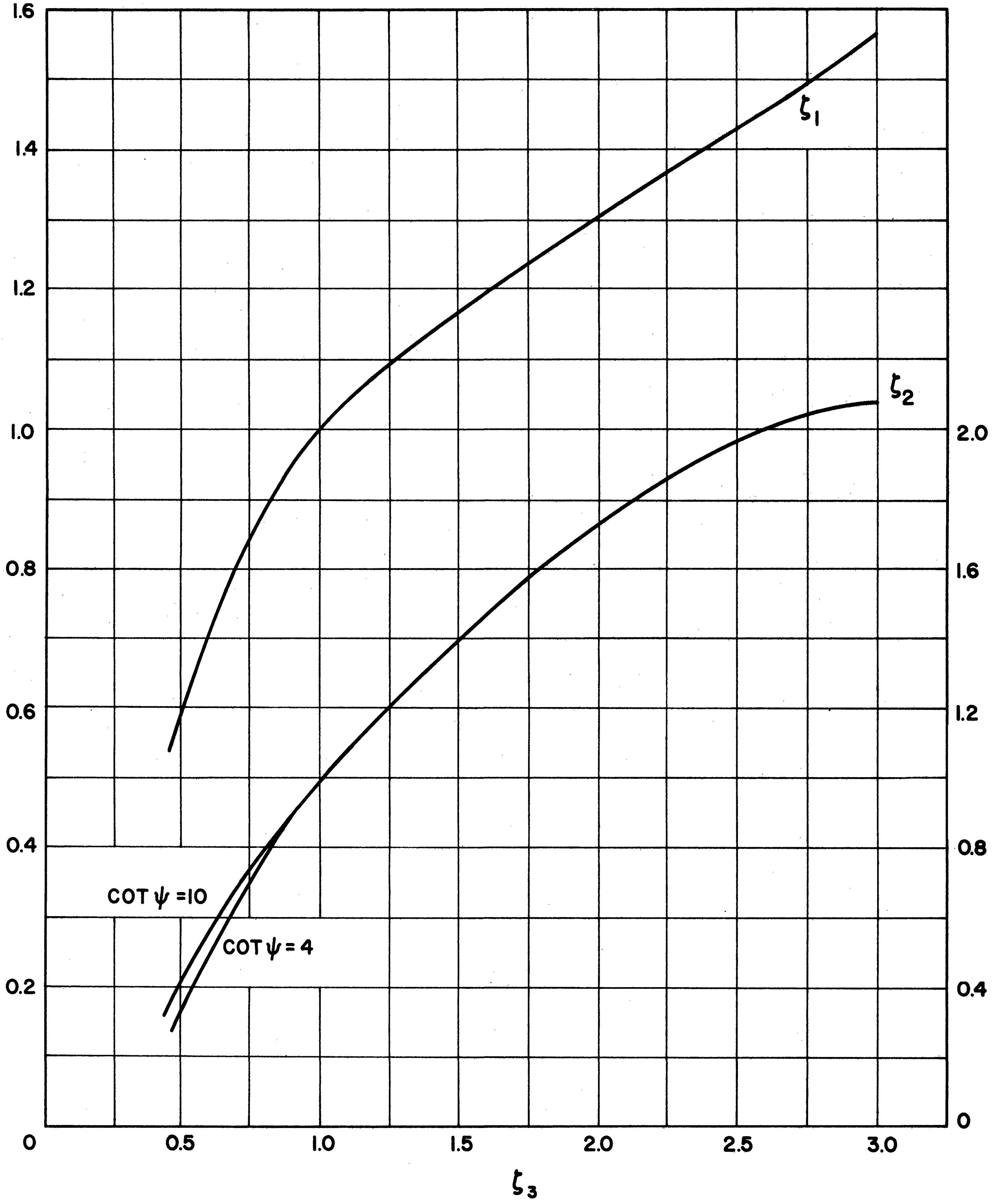

FIG. 5 BWO MODULATION PARAMETERS ζ_1 AND ζ_2 AS FUNCTIONS OF FREQUENCY, ζ_3. (BIFILAR HELIX; AT $\Delta V=0$, $ka=0.30$)

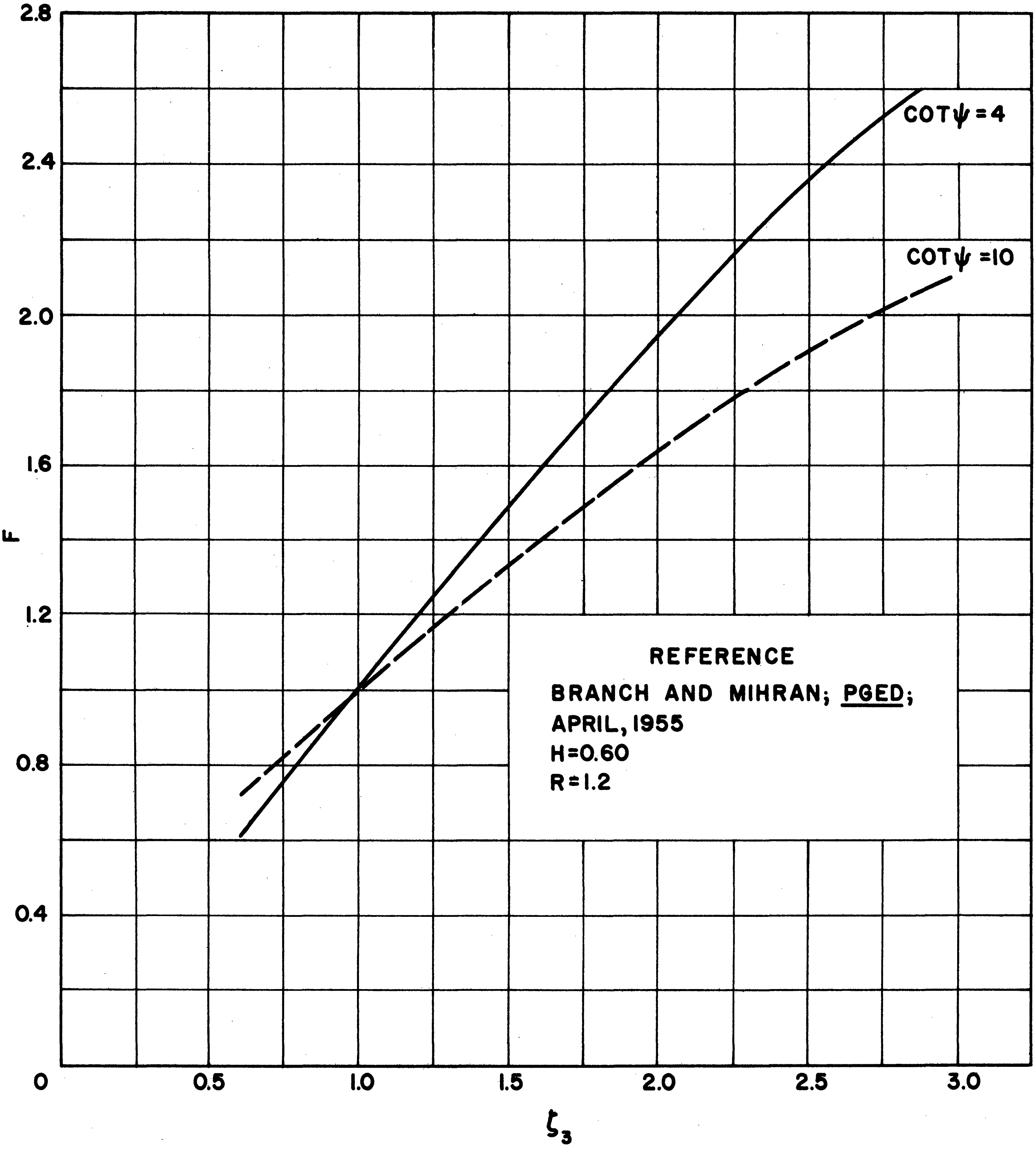

FIG. 6 BWO MODULATION PARAMETER F AS FUNCTION OF FREQUENCY, ζ_3. (BIFILAR HELIX; AT $\Delta V=0$, $ka=0.30$; ANNULAR BEAM, $H=0.60$, $R=1.2$)

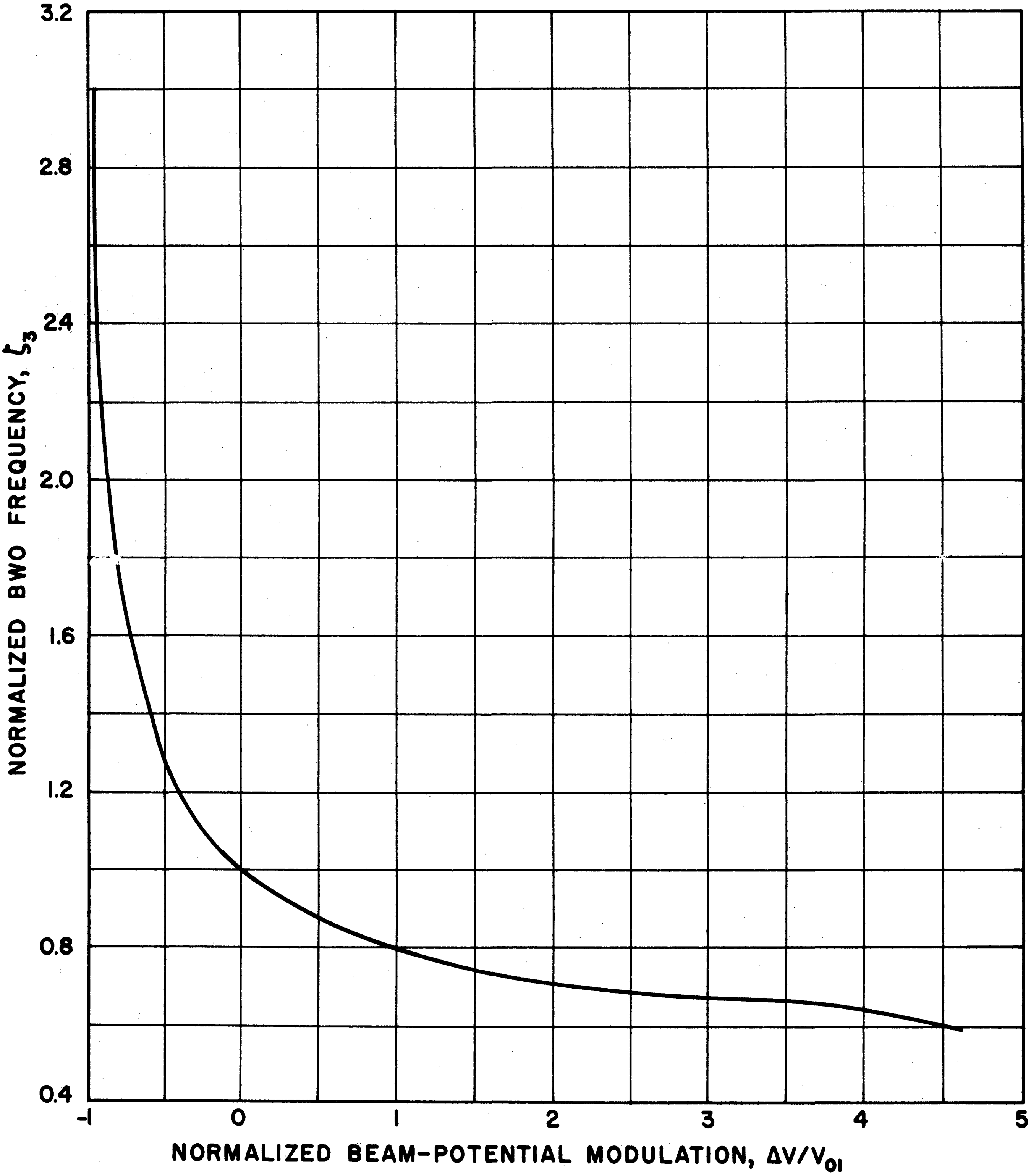

FIG. 7 FIRST ESTIMATE OF NORMALIZED FREQUENCY, ζ_3, OBTAINED FROM ω-β DIAGRAM. (BIFILAR HELIX; AT $\Delta V=0$, $ka=0.30$)

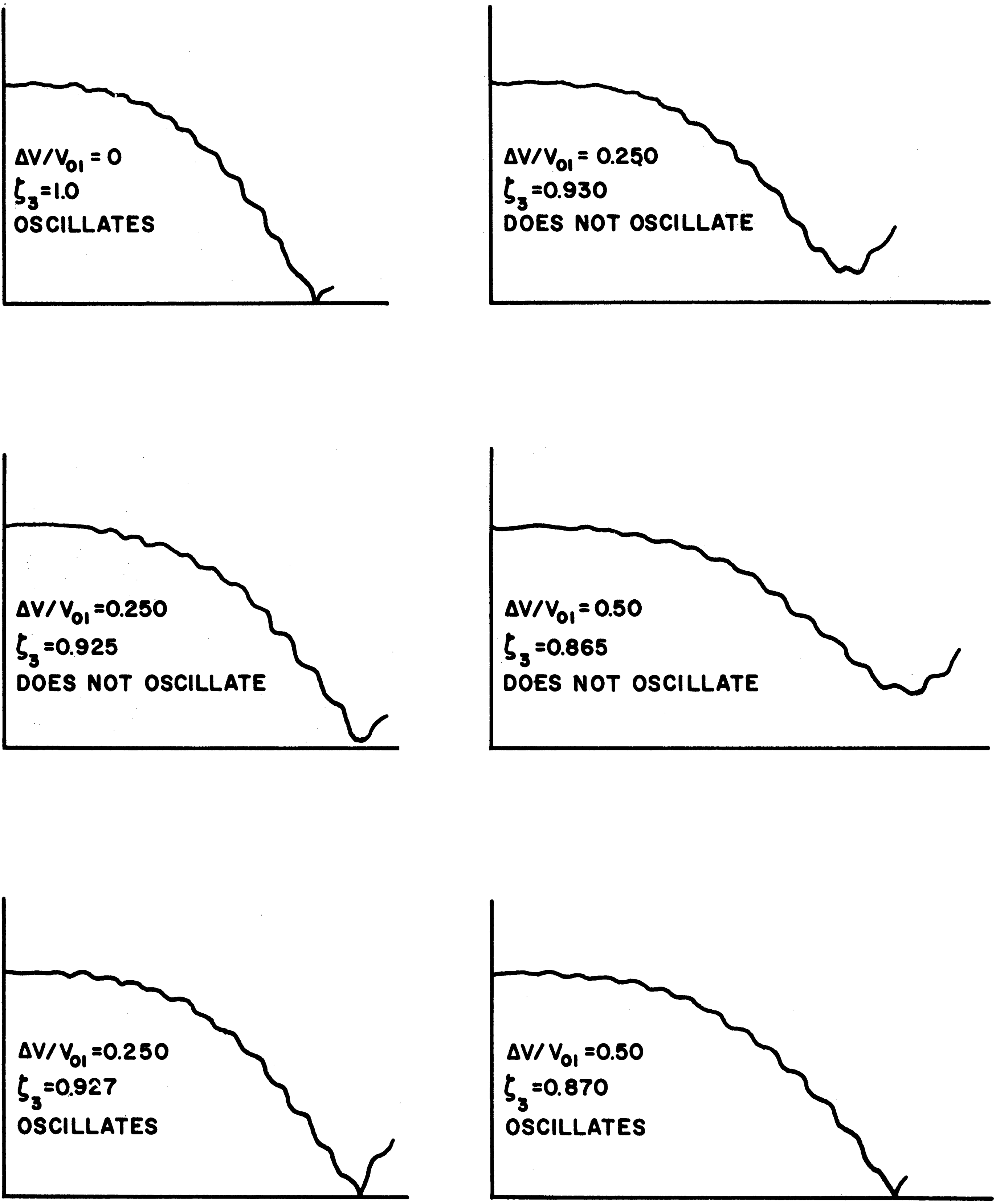

FIG. 8 TRIAL AND ERROR PROCEDURE FOR FINDING START-OSCILLATION CONDITIONS WITH BEAM-POTENTIAL MODULATION.

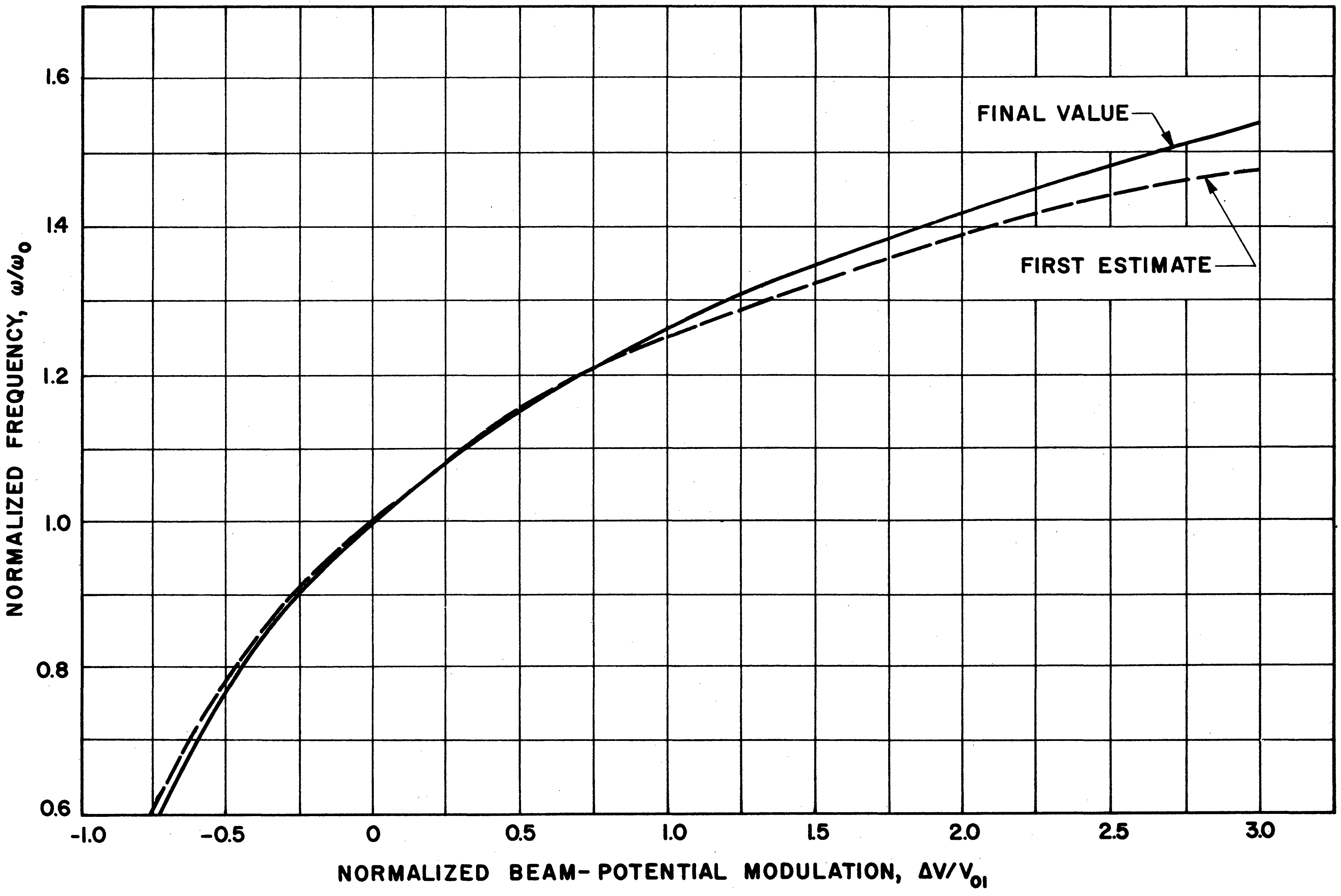

FIG. 9 VARIATION OF START-OSCILLATION FREQUENCY AS FUNCTION OF BEAM-POTENTIAL MODULATION. (C_0=0.05, QC_0=0, d_0=0, b_0=1.56; BIFILAR HELIX; ANNULAR BEAM, H=0.60, R=1.2; AT ΔV=0, ka=0.30)

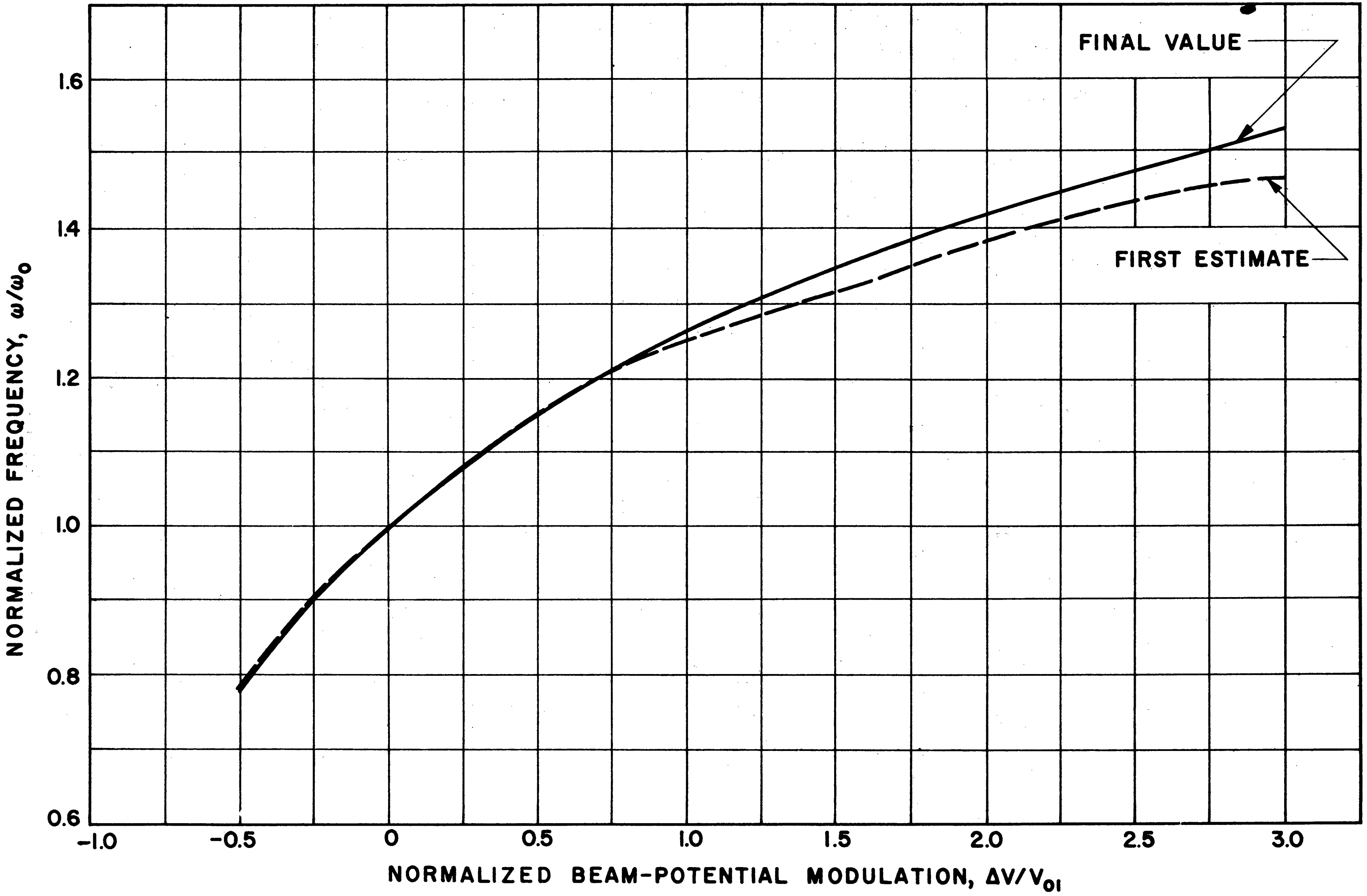

FIG. 10 VARIATION OF START-OSCILLATION FREQUENCY AS FUNCTION OF BEAM-POTENTIAL MODULATION. (C_0=0.05, QC_0=0.20, d_0=0, b_0=1.59; BIFILAR HELIX; ANNULAR BEAM, H=0.60, R=1.2; AT ΔV=0, ka=0.30)

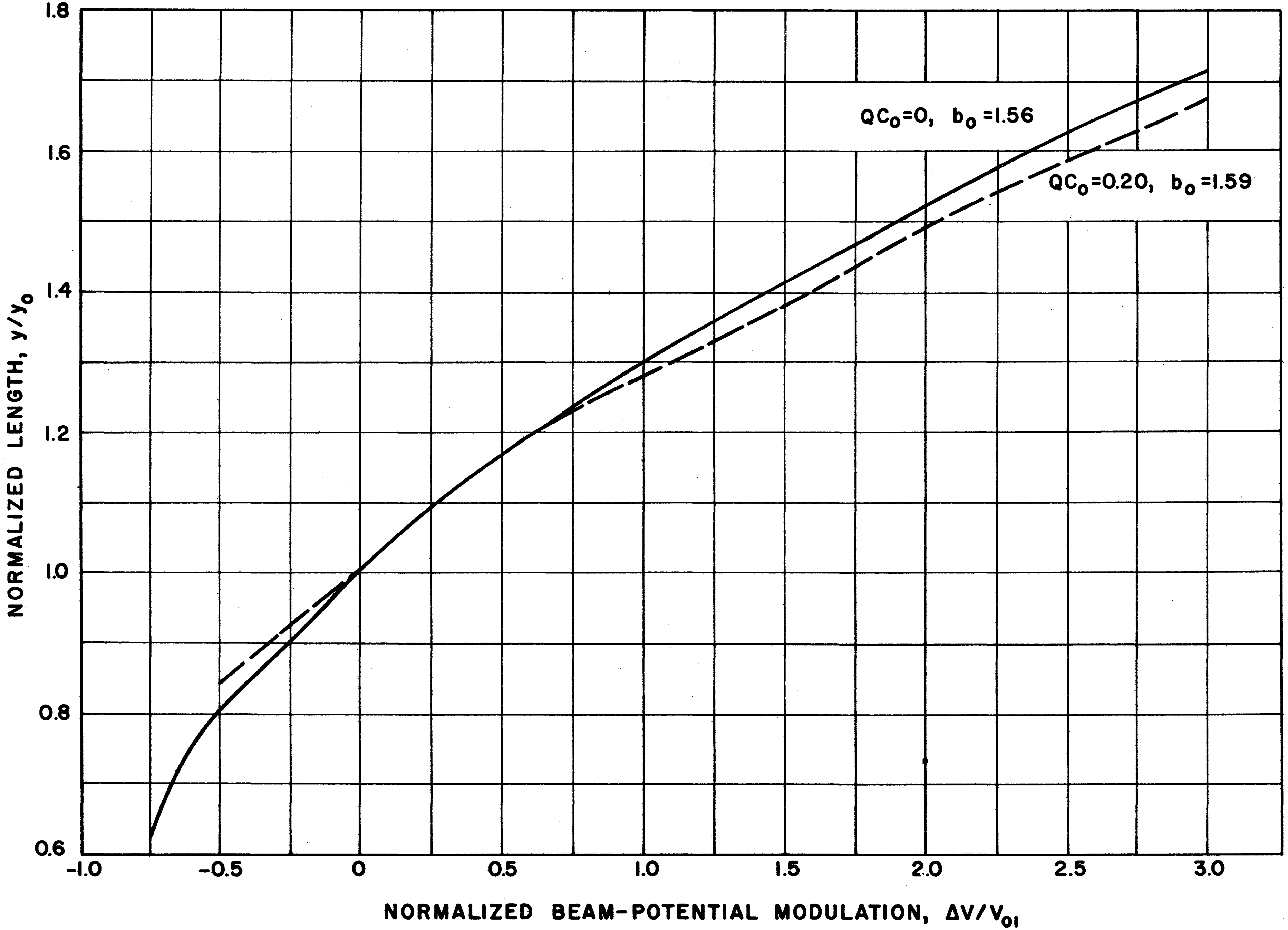

FIG. 11 VARIATION OF START-OSCILLATION LENGTH AS FUNCTION OF BEAM-POTENTIAL MODULATION. ($C_0=0.05$, $d_0=0$, BIFILAR HELIX; ANNULAR BEAM, $H=0.60$, $R=1.2$; AT $\Delta V=0$, $ka=0.30$)

$\Delta V/V_{01}$ increases. This particular method of calculating the start-oscillation conditions is recommended when the variation of starting frequency and required lengths is desired once an unmodulated operating point has been chosen. This method has the advantage of using only one set of parameter values.

As the current or the tube length is increased beyond the start-oscillation conditions nonlinearities rapidly set in to limit the power output for a specific d-c beam input. As mentioned above, the Eulerian model must be abandoned in a nonlinear description of the BWO because of the severe bunching that takes place. The Lagrangian equations for the modulated BWO can be derived in a manner similar to the derivation of the nonlinear TWA equations.

CONCLUSIONS

The start-oscillation conditions for the O-type backward-wave oscillator have been computed for large values of C, QC, and d, first using a digital technique and then an analog technique. The digital program involves the downhill method of finding the complex roots of the secular equation and this method is also applied to find points in the b - CN_s plane which correspond to the first-order oscillation conditions, i.e., values of $z > 0$ where the r-f voltage is zero. The analog computer method involves the solution of the circuit and ballistic equations as simultaneous linear differential equations. Specific results of the digital calculation are given and a comparison between digital and analog results indicates excellent agreement.

The analog computer technique is also applied to a study of the effect of beam-potential modulation on the starting conditions of the backward-wave oscillator. The unmodulated starting conditions are used as initial estimates in a trial and error procedure. These results are summarized in several graphs.

APPENDIX A: O-TYPE BWO STARTING CONDITIONS

Parameter	Range
C	10^{-5} to 0.2
QC	0 to 1.0
d	0 to 1.0

O-Type BWO Starting Conditions
C = 0.00001, d = 0

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.522	0.3141	3.003	0.72523+j0.15048	-0.72523+j0.15048	-j1.8228
0.1	1.512	0.3242	3.080	0.43181+j0.16643	-0.43181+j0.16643	-j1.8451
0.2	1.504	0.3363	3.178	-j0.19310	j0.56219	-j1.8733
0.3	1.500	0.3514	3.311	-j0.47439	j0.88342	-j1.9091
0.4	1.505	0.3712	3.510	-j0.65280	j1.1029	-j1.9549
0.5	1.533	0.3990	3.843	-j0.79850	j1.2825	-j2.0165
0.6	1.625	0.4347	4.438	-j0.95390	j1.4400	-j2.1112
0.7	1.769	0.4591	5.104	-j1.1212	j1.5816	-j2.2298
0.8	1.892	0.4713	5.602	-j1.2659	j1.7094	-j2.3355
0.9	1.989	0.4807	6.008	-j1.3898	j1.8270	-j2.4264
1.0	2.072	0.4913	6.396	-j1.5002	j1.9366	-j2.5083
				C = 0.00001, d = 0.05		
0	1.507	0.3199	3.029	0.73346+j0.14570	-0.72088+j0.15874	0.037417-j1.8113
0.1	1.497	0.3307	3.111	0.44361+j0.15398	-0.42889+j0.18293	0.035282-j1.8341
0.2	1.489	0.3437	3.216	0.030664-j0.18531	-0.013527+j0.55900	0.032863-j1.8632
0.3	1.486	0.3601	3.362	0.025370-j0.46798	-0.0056185+j0.88203	0.030248-j1.9002
0.4	1.493	0.3819	3.581	0.025954-j0.64731	-0.0036092+j1.1022	0.027655-j1.9478
0.5	1.528	0.4127	3.963	0.026975-j0.79683	-0.0025947+j1.2823	0.025619-j2.0138
0.6	1.635	0.4497	4.619	0.026661-j0.95961	-0.0019044+j1.4404	0.025243-j2.1154
0.7	1.776	0.4721	5.268	0.025586-j1.1252	-0.0014414+j1.5818	0.025855-j2.2326
0.8	1.891	0.4844	5.756	0.025111-j1.2665	-0.0011532+j1.7095	0.026042-j2.3346
0.9	1.984	0.4952	6.174	0.025138-j1.3880	-0.00095943+j1.8269	0.025821-j2.4233
1.0	2.066	0.5079	6.593	0.025399-j1.4979	-0.00081837+j1.9365	0.025420-j2.5046

C = 0.00001, d = 0.1

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.492	0.3261	3.056	0.74186+j0.14062	-0.71636+j0.16676	0.074500-j1.7991
0.1	1.482	0.3375	3.143	0.45578+j0.14131	-0.42594+j0.19899	0.070159-j1.8224
0.2	1.475	0.3515	3.257	0.062369-j0.17974	-0.027623+j0.55727	0.065254-j1.8522
0.3	1.472	0.3693	3.416	0.051406-j0.46282	-0.011391+j0.88096	0.059984-j1.8904
0.4	1.482	0.3934	3.663	0.052432-j0.64327	-0.0072831+j1.1016	0.054851-j1.9400
0.5	1.526	0.4277	4.100	0.054060-j0.79733	-0.0051928+j1.2823	0.051133-j2.0109
0.6	1.644	0.4650	4.804	0.052859-j0.96648	-0.0037787+j1.4409	0.050920-j2.1185
0.7	1.780	0.4859	5.434	0.050954-j1.1292	-0.0028724+j1.5820	0.051919-j2.2330
0.8	1.889	0.4987	5.920	0.050315-j1.2671	-0.0023076+j1.7095	0.051992-j2.3317
0.9	1.979	0.5113	6.358	0.050545-j1.3872	-0.0019235+j1.8268	0.051379-j2.4187
1.0	2.061	0.5267	6.821	0.051046-j1.4973	-0.0016400+j1.9365	0.050594-j2.5003
				C = 0.00001, d = 0.25		
0	1.446	0.3465	3.148	0.76803+j0.12349	-0.70189+j0.18942	0.18385-j1.7587
0.1	1.436	0.3606	3.255	0.49442+j0.10189	-0.41690+j0.24456	0.17248-j1.7829
0.2	1.430	0.3784	3.400	0.16031-j0.17736	-0.070013+j0.56163	0.15970-j1.8143
0.3	1.433	0.4019	3.617	0.13316-j0.45478	-0.029318+j0.87986	0.14616-j1.8551
0.4	1.453	0.4350	3.971	0.13425-j0.64097	-0.018474+j1.1014	0.13423-j1.9133
0.5	1.532	0.4798	4.618	0.13431-j0.81323	-0.012799+j1.2837	0.12849-j2.0022
0.6	1.665	0.5141	5.379	0.12946-j0.99041	-0.0092379+j1.4424	0.12978-j2.1173
0.7	1.782	0.5340	5.978	0.12706-j1.1431	-0.0071327+j1.5826	0.13007-j2.2214
0.8	1.877	0.5522	6.512	0.12724-j1.2742	-0.0057835+j1.7095	0.12854-j2.3125
0.9	1.965	0.5738	7.086	0.12806-j1.3939	-0.0048251+j1.8269	0.12676-j2.3984
1.0	2.060	0.5973	7.730	0.12776-j1.5106	-0.0040877+j1.9367	0.12633-j2.4860

C = 0.00001, d = 0.5

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.367	0.3894	3.345	0.81512+j0.087307	-0.67547+j0.22263	0.36035-j1.6769
0.1	1.358	0.4106	3.504	0.56534+j0.029922	-0.40120+j0.31182	0.33586-j1.7001
0.2	1.356	0.4388	3.738	0.31765-j0.21500	-0.12627+j0.59220	0.30862-j1.7331
0.3	1.370	0.4794	4.126	0.27646-j0.47457	-0.058272+j0.88644	0.28181-j1.7819
0.4	1.433	0.5376	4.840	0.27119-j0.67827	-0.035949+j1.1065	0.26476-j1.8612
0.5	1.554	0.5898	5.759	0.26182-j0.87242	-0.024347+j1.2885	0.26253-j1.9702
0.6	1.666	0.6262	6.554	0.25787-j1.0387	-0.018022+j1.4448	0.26016-j2.0720
0.7	1.765	0.6642	7.368	0.25773-j1.1838	-0.014132+j1.5838	0.25639-j2.1655
0.8	1.869	0.7037	8.265	0.25608-j1.3206	-0.011425+j1.7106	0.25534-j2.2594
0.9	1.971	0.7362	9.118	0.25412-j1.4486	-0.0094772+j1.8280	0.25535-j2.3510
1.0	2.065	0.7663	9.941	0.25396-j1.5659	-0.0080499+j1.9375	0.25409-j2.4364
				C = 0.00001, d = 1.0		
0	1.199	0.5441	4.098	0.92356-j0.030165	-0.61833+j0.27341	0.69477-j1.4421
0.1	1.197	0.6117	4.602	0.72855-j0.16281	-0.36666+j0.41664	0.63811-j1.4513
0.2	1.225	0.7260	5.588	0.59551-j0.41129	-0.17608+j0.67090	0.58057-j1.4846
0.3	1.311	0.8948	7.370	0.54934-j0.67465	-0.096236+j0.92102	0.54689-j1.5573
0.4	1.423	1.096	9.803	0.53126-j0.90590	-0.061767+j1.1267	0.53051-j1.6438
0.5	1.539	1.341	12.97	0.52206-j1.1102	-0.043839+j1.3006	0.52178-j1.7295
0.6	1.652	1.659	17.22	0.51664-j1.2955	-0.033192+j1.4530	0.51655-j1.8101
0.7	1.762	2.114	23.39	0.51315-j1.4681	-0.026270+j1.5899	0.51312-j1.8834
0.8						
0.9						
1.0						

C = 0.02, d = 0

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.537	0.3089	2.983	0.72403+j0.15658	-0.72403+j0.15658	-j1.8500
0.1	1.537	0.3185	3.075	0.43101+j0.16444	-0.43101+j0.16444	-j1.8818
0.2	1.539	0.3299	3.190	-j0.20372	j0.55230	-j1.9195
0.3	1.545	0.3440	3.338	-j0.49352	j0.86516	-j1.9644
0.4	1.559	0.3621	3.546	-j0.68074	j1.0766	-j2.0185
0.5	1.593	0.3869	3.873	-j0.83400	j1.2479	-j2.0872
0.6	1.681	0.4188	4.425	-j0.99038	j1.3969	-j2.1841
0.7	1.828	0.4436	5.094	-j1.1616	j1.5304	-j2.3086
0.8	1.963	0.4559	5.624	-j1.3162	j1.6505	-j2.4258
0.9	2.075	0.4642	6.051	-j1.4507	j1.7604	-j2.5286
1.0	2.169	0.4726	6.441	-j1.5708	j1.8624	-j2.6212
				C = 0.02, d = 0.05		
0	1.520	0.3146	3.005	0.73227+j0.15223	-0.71981+j0.16469	0.037540-j1.8374
0.1	1.521	0.3248	3.103	0.44288+j0.15268	-0.42836+j0.18055	0.035475-j1.8699
0.2	1.523	0.3370	3.225	0.029756-j0.19538	-0.012907+j0.54891	0.033151-j1.9085
0.3	1.530	0.3522	3.385	0.024653-j0.48664	-0.0053058+j0.86370	0.030653-j1.9545
0.4	1.545	0.3721	3.613	0.025214-j0.67465	-0.0033843+j1.0757	0.028170-j2.0105
0.5	1.586	0.3995	3.981	0.026282-j0.83079	-0.0024268+j1.2476	0.026145-j2.0829
0.6	1.687	0.4332	4.592	0.026299-j0.99397	-0.0017906+j1.3972	0.025492-j2.1865
0.7	1.834	0.4562	5.258	0.025332-j1.1655	-0.0013509+j1.5306	0.026019-j2.3114
0.8	1.964	0.4682	5.776	0.024753-j1.3170	-0.0010722+j1.6505	0.026319-j2.4254
0.9	2.070	0.4773	6.208	0.024662-j1.4491	-0.00088578+j1.7603	0.026224-j2.5256
1.0	2.162	0.4842	6.620	0.024853-j1.5681	-0.00075170+j1.8623	0.025899-j2.6171

C = 0.02, d = 0.1

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.504	0.3205	3.029	0.74068+j0.14760	-0.71541+j0.17256	0.074729-j1.8243
0.1	1.504	0.3314	3.132	0.45512+j0.14071	-0.42565+j0.19624	0.070525-j1.8573
0.2	1.507	0.3445	3.262	0.060611-j0.18917	-0.026420+j0.54691	0.065808-j1.8967
0.3	1.514	0.3611	3.435	0.050005-j0.48094	-0.010771+j0.86255	0.060766-j1.9438
0.4	1.532	0.3829	3.687	0.050999-j0.66993	-0.0068385+j1.0751	0.055839-j2.0018
0.5	1.580	0.4132	4.104	0.052810-j0.82964	-0.0048691+j1.2475	0.052059-j2.0784
0.6	1.694	0.4480	4.768	0.052272-j0.99933	-0.0035599+j1.3975	0.051288-j2.1884
0.7	1.839	0.4694	5.424	0.050428-j1.1698	-0.0026911+j1.5308	0.052263-j2.3123
0.8	1.962	0.4814	5.935	0.049561-j1.3181	-0.0021445+j1.6505	0.052584-j2.4229
0.9	2.065	0.4917	6.379	0.049581-j1.4482	-0.0017759+j1.7603	0.052195-j2.5210
1.0	2.156	0.5036	6.822	0.050029-j1.5666	-0.0015078+j1.8622	0.051479-j2.6118
				C = 0.02, d = 0.25		
0	1.455	0.3403	3.110	0.76696+j0.13189	-0.70125+j0.19483	0.18429-j1.7814
0.1	1.455	0.3536	3.233	0.49391+j0.10345	-0.41715+j0.24083	0.17325-j1.8154
0.2	1.459	0.3702	3.393	0.15661-j0.18425	-0.067549+j0.55001	0.16094-j1.8566
0.3	1.469	0.3917	3.617	0.12976-j0.47160	-0.027804+j0.86118	0.14805-j1.9071
0.4	1.498	0.4215	3.968	0.13116-j0.66470	-0.017436+j1.0745	0.13628-j1.9722
0.5	1.576	0.4616	4.571	0.13237-j0.83965	-0.012103+j1.2483	0.12973-j2.0649
0.6	1.713	0.4954	5.332	0.12833-j1.0215	-0.0087180+j1.3988	0.13039-j2.1865
0.7	1.843	0.5144	5.956	0.12549-j1.1842	-0.0066739+j1.5313	0.13119-j2.3022
0.8	1.951	0.5296	6.490	0.12519-j1.3251	-0.0053730+j1.6506	0.13019-j2.4044
0.9	2.047	0.5466	7.030	0.12604-j1.4523	-0.0044649+j1.7602	0.12842-j2.4991
1.0	2.144	0.5664	7.631	0.12648-j1.5740	-0.0037788+j1.8622	0.12730-j2.5928

C = 0.02, d = 0.5

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.370	0.3816	3.286	0.81439+j0.098306	-0.67518+j0.22742	0.36079-j1.6963
0.1	1.372	0.4013	3.459	0.56487+j0.035183	-0.40178+j0.30673	0.33691-j1.7299
0.2	1.379	0.4273	3.703	0.31279-j0.21703	-0.12335+j0.57854	0.31057-j1.7727
0.3	1.401	0.4639	4.085	0.27133-j0.48584	-0.055849+j0.86651	0.28452-j1.8302
0.4	1.466	0.5158	4.752	0.26774-j0.69401	-0.034341+j1.0785	0.26660-j1.9148
0.5	1.592	0.5655	5.655	0.25975-j0.89332	-0.023160+j1.2526	0.26341-j2.0309
0.6	1.716	0.5984	6.451	0.25499-j1.0690	-0.016985+j1.4012	0.26200-j2.1442
0.7	1.824	0.6294	7.214	0.25482-j1.2217	-0.013244+j1.5324	0.25843-j2.2470
0.8	1.932	0.6636	8.056	0.25443-j1.3638	-0.010683+j1.6514	0.25626-j2.3480
0.9	2.043	0.6936	8.903	0.25258-j1.4996	-0.0088194+j1.7611	0.25624-j2.4489
1.0	2.148	0.7184	9.694	0.25166-j1.6259	-0.0074479+j1.8630	0.25579-j2.5451
				C = 0.02, d = 1.0		
0	1.195	0.5276	3.961	0.92430-j0.012223	-0.61826+j0.27682	0.69396-j1.4595
0.1	1.203	0.5883	4.446	0.72837-j0.14820	-0.36685+j0.40959	0.63848-j1.4804
0.2	1.237	0.6878	5.346	0.59200-j0.40091	-0.17374+j0.65546	0.58174-j1.5239
0.3	1.328	0.8332	6.954	0.54633-j0.66922	-0.093619+j0.89899	0.54729-j1.6063
0.4	1.448	0.9968	9.067	0.62883-j0.90510	-0.059455+j1.0975	0.53063-j1.7043
0.5	1.572	1.182	11.68	0.52002-j1.1133	-0.041847+j1.2639	0.52183-j1.8029
0.6	1.694	1.399	14.89	0.51498-j1.3014	-0.031465+j1.4086	0.51648-j1.8975
0.7	1.812	1.665	18.96	0.51181-j1.4748	-0.024753+j1.5379	0.51294-j1.9869
0.8						
0.9						
1.0						

C = 0.05, d = 0

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.559	0.3013	2.951	0.72187+j0.16588	-0.72187+j0.16588	-j1.8911
0.1	1.575	0.3100	3.067	0.42957+j0.16169	-0.42957+j0.16169	-j1.9387
0.2	1.593	0.3202	3.204	-j0.22017	j0.53864	-j1.9919
0.3	1.614	0.3327	3.374	-j0.52310	j0.83954	-j2.0518
0.4	1.644	0.3484	3.598	-j0.72438	j1.0397	-j2.1204
0.5	1.690	0.3690	3.921	-j0.89034	j1.1999	-j2.2016
0.6	1.778	0.3954	4.418	-j1.0522	j1.3374	-j2.3056
0.7	1.924	0.4191	5.067	-j1.2283	j1.4598	-j2.4380
0.8	2.078	0.4318	5.638	-j1.3978	j1.5696	-j2.5731
0.9	2.212	0.4390	6.100	-j1.5492	j1.6694	-j2.6957
1.0	2.328	0.4449	6.506	-j1.6862	j1.7614	-j2.8076
				C = 0.05, d = 0.05		
0	1.541	0.3068	2.969	0.73011+j0.16214	-0.71784+j0.17378	0.037728-j1.8771
0.1	1.556	0.3160	3.090	0.44156+j0.15090	-0.42732+j0.17728	0.035761-j1.9253
0.2	1.575	0.3269	3.235	0.028439-j0.21112	-0.012012+j0.53504	0.033574-j1.9794
0.3	1.597	0.3404	3.416	0.023628-j0.51560	-0.0048670+j0.83802	0.031241-j2.0406
0.4	1.628	0.3575	3.658	0.024151-j0.71749	-0.0030706+j1.0388	0.028921-j2.1109
0.5	1.680	0.3802	4.013	0.025244-j0.88548	-0.0021896+j1.1994	0.026948-j2.1953
0.6	1.778	0.4085	4.564	0.025637-j1.0528	-0.0016238+j1.3375	0.025988-j2.3051
0.7	1.930	0.4312	5.229	0.024957-j1.2317	-0.0012242+j1.4599	0.026269-j2.4405
0.8	2.079	0.4431	5.788	0.024299-j1.3989	-0.00096143+j1.5696	0.026664-j2.5732
0.9	2.208	0.4504	6.249	0.024050-j1.5480	-0.00078579+j1.6694	0.026737-j2.6932
1.0	2.321	0.4572	6.666	0.024107-j1.6834	-0.00066090+j1.7613	0.026555-j2.8033

C = 0.05, d = 0.1

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.522	0.3125	2.990	0.73853+j0.15815	-0.71361+j0.18146	0.075081-j1.8626
0.1	1.538	0.3223	3.115	0.45389+j0.13992	-0.42495+j0.19249	0.071072-j1.9114
0.2	1.557	0.3340	3.267	0.058051-j0.20396	-0.024674+j0.53264	0.066626-j1.9663
0.3	1.580	0.3486	3.460	0.047991-j0.50919	-0.0098981+j0.83677	0.061910-j2.0286
0.4	1.613	0.3673	3.723	0.048932-j0.71183	-0.0062159+j1.0381	0.057287-j2.1008
0.5	1.670	0.3923	4.117	0.050887-j0.88234	-0.0044067+j1.1992	0.053523-j2.1887
0.6	1.780	0.4222	4.722	0.051183-j1.0554	-0.0032400+j1.3376	0.052060-j2.3045
0.7	1.934	0.4437	5.393	0.049695-j1.2358	-0.0024391+j1.4601	0.052747-j2.4414
0.8	2.079	0.4550	5.993	0.048602-j1.4004	-0.0019220+j1.5697	0.053323-j2.5713
0.9	2.203	0.4629	6.407	0.048314-j1.5474	-0.0015749+j1.6693	0.053264-j2.6889
1.0	2.313	0.4708	6.843	0.048551-j1.6815	-0.0013262+j1.7613	0.052778-j2.7977

C = 0.05, d = 0.25

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.467	0.3313	3.055	0.76490+j0.14444	-0.69989+j0.20319	0.18499-j1.8158
0.1	1.484	0.3433	3.200	0.49282+j0.10568	-0.41723+j0.23572	0.17442-j1.8659
0.2	1.503	0.3580	3.382	0.15114-j0.19534	-0.063905+j0.53393	0.16277-j1.9229
0.3	1.530	0.3768	3.621	0.12507-j0.49708	-0.025710+j0.83486	0.15065-j1.9885
0.4	1.572	0.4018	3.968	0.12655-j0.70282	-0.015956+j1.0370	0.13941-j2.0671
0.5	1.652	0.4351	4.516	0.12903-j0.88467	-0.011079+j1.1993	0.13206-j2.1684
0.6	1.791	0.4667	5.253	0.12659-j1.0727	-0.0079818+j1.3386	0.13140-j2.2993
0.7	1.940	0.4849	5.911	0.12344-j1.2504	-0.0060425+j1.4607	0.13261-j2.4332
0.8	2.069	0.4968	6.460	0.12242-j1.4077	-0.0048095+j1.5698	0.13239-j2.5547
0.9	2.184	0.5085	6.976	0.12289-j1.5500	-0.0039633+j1.6692	0.13109-j2.6667
1.0	2.292	0.5220	7.516	0.12378-j1.6833	-0.0033403+j1.7612	0.12957-j2.7743

C = 0.05, d = 0.5

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.375	0.3704	3.201	0.81275+j0.11460	-0.67431+j0.23498	0.36157-j1.7257
0.1	1.393	0.3880	3.395	0.56386+j0.042681	-0.40246+j0.29978	0.33861-j1.7759
0.2	1.416	0.4107	3.653	0.30541-j0.22102	-0.11894+j0.55943	0.31355-j1.8349
0.3	1.452	0.4417	4.029	0.26360-j0.50451	-0.052310+j0.83868	0.28872-j1.9071
0.4	1.523	0.4847	4.637	0.26190-j0.72138	-0.031950+j1.0396	0.27007-j2.0022
0.5	1.652	0.5295	5.498	0.25662-j0.92784	-0.021474+j1.2025	0.26487-j2.1289
0.6	1.796	0.5586	6.303	0.25111-j1.1181	-0.015544+j1.3407	0.26445-j2.2604
0.7	1.922	0.5816	7.024	0.25010-j1.2853	-0.011990+j1.4617	0.26191-j2.3811
0.8	2.041	0.6062	7.772	0.25080-j1.4382	-0.0096223+j1.5703	0.25884-j2.4960
0.9	2.162	0.6313	8.575	0.25026-j1.5849	-0.0079072+j1.6697	0.25766-j2.6107
1.0	2.284	0.6520	9.355	0.24889-j1.7257	-0.0066249+j1.7617	0.25775-j2.7244
				C = 0.05, d = 1.0		
0	1.188	0.5050	3.769	0.92465+j0.014015	-0.61779+j0.28253	0.69317-j1.4852
0.1	1.212	0.5564	4.235	0.72775-j0.12735	-0.36720+j0.39995	0.63948-j1.5249
0.2	1.258	0.6369	5.033	0.58605-j0.38713	-0.17029+j0.63359	0.58427-j1.5851
0.3	1.357	0.7534	6.423	0.54151-j0.66352	-0.089875+j0.86798	0.54840-j1.6822
0.4	1.489	0.8747	8.185	0.52431-j0.90826	-0.056144+j1.0566	0.53186-j1.7989
0.5	1.627	1.002	10.24	0.51629-j1.1245	-0.039036+j1.2128	0.52278-j1.9169
0.6	1.763	1.135	12.57	0.51196-j1.3199	-0.029053+j1.3472	0.51712-j2.0321
0.7	1.895	1.276	15.19	0.50919-j1.4999	-0.022648+j1.4663	0.51349-j2.1437
0.8	2.022	1.431	18.18	0.50734-j1.6680	-0.018259+j1.5739	0.51095-j2.2513
0.9	2.146	1.604	21.63	0.50610-j1.8269	-0.015100+j1.6725	0.50903-j2.3552
1.0	2.265	1.802	25.65	0.50517-j1.9783	-0.012739+j1.7639	0.50760-j2.4554

C = 0.08, d = 0

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.580	0.2939	2.919	0.71929+j0.17533	-0.71929+j0.17533	-j1.9328
0.1	1.613	0.3015	3.057	0.42787+j0.15921	-0.42787+j0.15921	-j1.9976
0.2	1.649	0.3104	3.216	-j0.23731	j0.52635	-j2.0681
0.3	1.688	0.3212	3.406	-j0.55399	j0.81593	-j2.1452
0.4	1.735	0.3345	3.646	-j0.77038	j1.0057	-j2.2304
0.5	1.796	0.3515	3.967	-j0.95094	j1.1559	-j2.3272
0.6	1.890	0.3728	4.428	-j1.1228	j1.2835	-j2.4428
0.7	2.036	0.3938	5.037	-j1.3050	j1.3960	-j2.5848
0.8	2.205	0.4066	5.633	-j1.4886	j1.4967	-j2.7378
0.9	2.362	0.4130	6.130	-j1.6587	j1.5878	-j2.8830
1.0	2.503	0.4170	6.559	-j1.8153	j1.6712	-j3.0180
C = 0.08, d = 0.05						
0	1.560	0.2992	2.933	0.72751+j0.17218	-0.71542+j0.18306	0.037920-j1.9172
0.1	1.593	0.3072	3.076	0.43992+j0.14930	-0.42596+j0.17434	0.036047-j1.9829
0.2	1.629	0.3168	3.242	0.027182-j0.22761	-0.011162+j0.52257	0.033986-j2.0543
0.3	1.669	0.3283	3.443	0.022662-j0.54593	-0.0044626+j0.81436	0.031807-j2.1325
0.4	1.717	0.3428	3.698	0.023147-j0.76278	-0.0027840+j1.0048	0.029643-j2.2195
0.5	1.782	0.3614	4.048	0.024228-j0.94480	-0.0019710+j1.1554	0.027749-j2.3190
0.6	1.885	0.3844	4.553	0.024863-j1.1207	-0.0014636+j1.2833	0.026606-j2.4395
0.7	2.039	0.4052	5.191	0.024535-j1.3070	-0.0011059+j1.3961	0.026577-j2.5859
0.8	2.207	0.4170	5.781	0.023907-j1.4899	-0.00086179+j1.4967	0.026961-j2.7381
0.9	2.360	0.4231	6.273	0.023548-j1.6579	-0.00069711+j1.5877	0.027155-j2.8809
1.0	2.497	0.4276	6.708	0.023475-j1.8127	-0.00058071+j1.6712	0.027111-j3.0139

C = 0.08, d = 0.1

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.540	0.3046	2.948	0.73591+j0.16877	-0.71134+j0.19056	0.075446-j1.9013
0.1	1.573	0.3132	3.096	0.45231+j0.13923	-0.42392+j0.18911	0.071621-j1.9676
0.2	1.609	0.3234	3.270	0.055581-j0.21964	-0.022996+j0.51986	0.067427-j2.0399
0.3	1.650	0.3359	3.482	0.046086-j0.53888	-0.0090900+j0.81303	0.063016-j2.1193
0.4	1.700	0.3517	3.756	0.046968-j0.75631	-0.0056445+j1.0040	0.058688-j2.2080
0.5	1.770	0.3721	4.138	0.048964-j0.94013	-0.0039765+j1.1550	0.055025-j2.3105
0.6	1.882	0.3967	4.690	0.049855-j1.1206	-0.0029312+j1.2833	0.053088-j2.4363
0.7	2.042	0.4169	5.349	0.048935-j1.3100	-0.0022060+j1.3962	0.053283-j2.5860
0.8	2.207	0.4278	5.932	0.047792-j1.4916	-0.0017223+j1.4968	0.053942-j2.7367
0.9	2.355	0.4340	6.423	0.047260-j1.6576	-0.0013965+j1.5877	0.054148-j2.8771
1.0	2.489	0.4390	6.867	0.047255-j1.8108	-0.0011651+j1.6711	0.053922-j3.0083
				C = 0.08, d = 0.25		
0	1.480	0.3227	3.001	0.76230+j0.15695	-0.69802+j0.21183	0.18575-j1.8507
0.1	1.514	0.3332	3.168	0.49135+j0.10777	-0.41695+j0.23115	0.17562-j1.9184
0.2	1.551	0.3459	3.370	0.14572-j0.20756	-0.060292+j0.51949	0.16460-j1.9929
0.3	1.594	0.3619	3.625	0.12057-j0.52424	-0.023745+j0.81068	0.15321-j2.0758
0.4	1.651	0.3827	3.971	0.12206-j0.74404	-0.014573+j1.0026	0.14255-j2.1703
0.5	1.740	0.4097	4.478	0.12530-j0.93617	-0.010091+j1.1546	0.13482-j2.2841
0.6	1.881	0.4375	5.171	0.12456-j1.1312	-0.0072838+j1.2838	0.13275-j2.4255
0.7	2.047	0.4549	5.850	0.12166-j1.3235	-0.0054693+j1.3967	0.13384-j2.5780
0.8	2.200	0.4647	6.423	0.12007-j1.4992	-0.0043044+j1.4969	0.13426-j2.7220
0.9	2.337	0.4725	6.937	0.11997-j1.6599	-0.0035123+j1.5876	0.13358-j2.8561
1.0	2.464	0.4807	7.443	0.12068-j1.8103	-0.0029391+j1.6710	0.13229-j2.9838

C = 0.08, d = 0.5

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.380	0.3598	3.120	0.81045+j0.13064	-0.67292+j0.24291	0.36253-j1.7555
0.1	1.415	0.3751	3.334	0.56245+j0.049706	-0.40284+j0.29359	0.34045-j1.8239
0.2	1.455	0.3946	3.606	0.29788-j0.22628	-0.11446+j0.54191	0.31664-j1.9007
0.3	1.507	0.4204	3.980	0.25589-j0.52540	-0.048892+j0.81315	0.29306-j1.9898
0.4	1.587	0.4554	4.542	0.25542-j0.75333	-0.029603+j1.0041	0.27425-j2.0986
0.5	1.721	0.4941	5.344	0.25309-j0.96742	-0.019865+j1.1568	0.26684-j2.2368
0.6	1.882	0.5204	6.156	0.24769-j1.1724	-0.014225+j1.2857	0.26660-j2.3877
0.7	2.031	0.5378	6.864	0.24558-j1.3569	-0.010841+j1.3976	0.26532-j2.5301
0.8	2.168	0.5540	7.547	0.24611-j1.5253	-0.0086296+j1.4973	0.26257-j2.6645
0.9	2.302	0.5716	8.266	0.24690-j1.6849	-0.0070605+j1.5879	0.26022-j2.7960
1.0	2.438	0.5884	9.013	0.24648-j1.8402	-0.0058847+j1.6712	0.25946-j2.9279

C = 0.08, d = 1.0

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.181	0.4846	3.594	0.92410+j0.039488	-0.61685+j0.28887	0.69288-j1.5106
0.1	1.221	0.5276	4.047	0.72671-j0.10770	-0.36758+j0.39135	0.64099-j1.5705
0.2	1.281	0.5924	4.769	0.57944-j0.37544	-0.16690+j0.61318	0.58759-j1.6492
0.3	1.389	0.6850	5.976	0.53608-j0.66090	-0.086273+j0.83926	0.55031-j1.7621
0.4	1.536	0.7772	7.501	0.51894-j0.91644	-0.052985+j1.0190	0.53416-j1.8991
0.5	1.688	0.8659	9.183	0.51225-j1.1425	-0.036414+j1.1660	0.52429-j2.0374
0.6	1.840	0.9505	10.99	0.50799-j1.3488	-0.026806+j1.2913	0.51893-j2.1748
0.7	1.989	1.033	12.91	0.50581-j1.5393	-0.020707+j1.4014	0.51502-j2.3090
0.8	2.134	1.113	14.93	0.50421-j1.7183	-0.016553+j1.5002	0.51246-j2.4403
0.9	2.275	1.193	16.85	0.50321-j1.8879	-0.013584+j1.5902	0.51049-j2.5686
1.0	2.413	1.272	19.28	0.50251-j2.0498	-0.011377+j1.6730	0.50898-j2.6944

C = 0.1, d = 0

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.594	0.2891	2.897	0.71733+j0.18171	-0.71733+j0.18171	-j1.8171
0.1	1.640	0.2959	3.048	0.42657+j0.15769	-0.42657+j0.15769	-j2.0382
0.2	1.687	0.3039	3.222	-j0.24919	j0.51888	-j2.1213
0.3	1.739	0.3134	3.425	-j0.57541	j0.80121	-j2.2110
0.4	1.799	0.3252	3.675	-j0.80254	j0.98457	-j2.3089
0.5	1.872	0.3399	3.998	-j0.99395	j1.1286	-j2.4178
0.6	1.973	0.3581	4.441	-j1.1747	j1.2502	-j2.5438
0.7	2.121	0.3768	5.021	-j1.3629	j1.3568	-j2.6942
0.8	2.299	0.3891	5.621	-j1.5560	j1.4520	-j2.8597
0.9	2.473	0.3951	6.139	-j1.7394	j1.5379	-j3.0214
1.0	2.632	0.3982	6.586	-j1.9106	j1.6163	-j3.1743
				C = 0.1, d = 0.05		
0	1.573	0.2942	2.908	0.72552+j0.17891	-0.71356+j0.18932	0.038051-j1.9443
0.1	1.618	0.3014	3.065	0.43865+j0.14832	-0.42488+j0.17253	0.036237-j2.0225
0.2	1.666	0.3100	3.245	0.026375-j0.23910	-0.010618+j0.51501	0.034255-j2.1065
0.3	1.719	0.3202	3.459	0.022049-j0.56703	-0.0042104+j0.79963	0.032173-j2.1975
0.4	1.780	0.3330	3.724	0.022510-j0.79456	-0.0026066+j0.98365	0.030108-j2.2970
0.5	1.856	0.3491	4.072	0.023570-j0.98713	-0.0018353+j1.1281	0.028277-j2.4085
0.6	1.965	0.3688	4.553	0.024308-j1.1711	-0.0013614+j1.2500	0.027065-j2.5384
0.7	2.122	0.3876	5.166	0.024204-j1.3636	-0.0010304+j1.3568	0.02684-j2.6940
0.8	2.300	0.3990	5.767	0.023657-j1.5570	-0.00080009+j1.4520	0.027154-j2.8598
0.9	2.470	0.4045	6.279	0.023259-j1.7388	-0.00064306+j1.5379	0.027395-j3.0196
1.0	2.626	0.4078	6.728	0.023114-j1.9083	-0.00053218+j1.6163	0.027429-j3.1705

C = 0.1, d = 0.1

QC	$b_{\text{osc.}}$ at	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.552	0.2996	2.921	0.73390+j0.17588	-0.70957+j0.19672	0.075695-j1.9274
0.1	1.597	0.3072	3.083	0.45107+j0.13881	-0.42304+j0.18704	0.071989-j2.0063
0.2	1.646	0.3164	3.271	0.053984-j0.23064	-0.021915+j0.51213	0.067954-j2.0912
0.3	1.699	0.3275	3.495	0.044875-j0.55956	-0.0085847+j0.79826	0.063733-j2.1834
0.4	1.761	0.3413	3.724	0.045718-j0.78758	-0.0052898+j0.98284	0.059595-j2.2846
0.5	1.842	0.3589	4.154	0.047700-j0.98163	-0.0037075+j1.1277	0.056030-j2.3988
0.6	1.959	0.3801	4.677	0.048866-j1.1694	-0.0027326+j1.2499	0.053890-j2.5339
0.7	2.123	0.3987	5.318	0.048360-j1.3655	-0.0020580+j1.3569	0.053721-j2.6930
0.8	2.301	0.4092	5.916	0.047297-j1.5586	-0.0015991+j1.4521	0.054325-j2.8584
0.9	2.467	0.4145	6.425	0.046654-j1.7386	-0.0012879+j1.5378	0.054657-j3.0161
1.0	2.619	0.4180	6.879	0.046501-j1.9066	-0.0010675+j1.6162	0.054589-j3.1652

C = 0.1, d = 0.25

QC	$b_{\text{osc.}}$ at	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.488	0.3171	2.965	0.76027+j0.16524	-0.69650+j0.21771	0.18628-j1.8742
0.1	1.534	0.3265	3.147	0.49017+j0.10909	-0.41656+j0.22838	0.17645-j1.9547
0.2	1.583	0.3379	3.362	0.14213-j0.21640	-0.057895+j0.51073	0.16582-j2.0419
0.3	1.639	0.3521	3.626	0.11766-j0.54338	-0.022501+j0.79566	0.15490-j2.1375
0.4	1.708	0.3702	3.939	0.11913-j0.77340	-0.013720+j0.98127	0.14463-j2.2443
0.5	1.805	0.3934	4.461	0.12269-j0.97416	-0.0094575+j1.1270	0.13683-j2.3686
0.6	1.949	0.4181	5.119	0.12297-j1.1752	-0.0068331+j1.2501	0.13392-j2.5187
0.7	2.125	0.4346	5.803	0.12051-j1.3772	-0.0051120+j1.3573	0.13466-j2.6841
0.8	2.294	0.4434	6.392	0.11870-j1.5662	-0.0039947+j1.4522	0.13536-j2.8447
0.9	2.449	0.4491	6.901	0.11822-j1.7411	-0.0032370+j1.5378	0.13507-j2.9961
1.0	2.594	0.4546	7.408	0.11867-j1.9055	-0.0026927+j1.6161	0.13408-j3.1408

C = 0.1, d = 0.5

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.383	0.3530	3.068	0.80856+j0.14120	-0.67170+j0.24838	0.36326-j1.7755
0.1	1.430	0.3668	3.295	0.56130+j0.054143	-0.40293+j0.28984	0.34175-j1.8569
0.2	1.482	0.3841	3.577	0.29277-j0.23056	-0.11140+j0.53109	0.31875-j1.9468
0.3	1.546	0.4068	3.951	0.25078-j0.54061	-0.046679+j0.79731	0.29601-j2.0485
0.4	1.635	0.4369	4.489	0.25086-j0.77717	-0.028072+j0.98205	0.27732-j2.1683
0.5	1.773	0.4710	5.248	0.25034-j0.99738	-0.018817+j1.1286	0.26859-j2.3156
0.6	1.945	0.4955	6.056	0.24554-j1.2120	-0.013401+j1.2517	0.26797-j2.4798
0.7	2.111	0.5100	6.764	0.24283-j1.4093	-0.010130+j1.3581	0.26741-j2.6387
0.8	2.263	0.5220	7.421	0.24285-j1.5906	-0.0080084+j1.4525	0.26528-j2.7893
0.9	2.409	0.5344	8.091	0.24389-j1.7618	-0.0065234+j1.5379	0.26275-j2.9356
1.0	2.557	0.5473	8.791	0.24436-j1.9279	-0.0054208+j1.6162	0.26117-j3.0814
				C = 0.1, d = 1.0		
0	1.175	0.4720	3.485	0.92325+j0.056076	-0.61595+j0.29339	0.69294-j1.5273
0.1	1.227	0.5099	3.933	0.72578-j0.095218	-0.36781+j0.38615	0.64227-j1.6015
0.2	1.298	0.5656	4.613	0.57466-j0.36878	-0.16463+j0.60033	0.59021-j1.6938
0.3	1.412	0.6444	5.717	0.53196-j0.66103	-0.083899+j0.82132	0.55217-j1.8182
0.4	1.570	0.7221	7.124	0.51518-j0.92448	-0.050960+j0.99559	0.53601-j1.9695
0.5	1.733	0.7907	8.612	0.50893-j1.1587	-0.034731+j1.1370	0.52603-j2.1228
0.6	1.898	0.8552	10.20	0.50508-j1.3735	-0.025387+j1.2568	0.52053-j2.2760
0.7	2.059	0.9125	11.81	0.50300-j1.5733	-0.019483+j1.3615	0.51672-j2.4266
0.8	2.217	0.9665	13.47	0.50170-j1.7618	-0.015486+j1.4550	0.51401-j2.5749
0.9	2.372	1.016	15.14	0.50076-j1.9414	-0.012638+j1.5399	0.51210-j2.7209
1.0	2.524	1.061	16.83	0.50024-j2.1137	-0.010531+j1.6177	0.51052-j2.8647

$C = 0.15,\ d = 0$

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.630	0.2774	2.840	0.71156+j0.19781	-0.71156+j0.19781	-j2.0321
0.1	1.708	0.2820	3.025	0.42272+j0.15426	-0.42272+j0.15426	-j2.1443
0.2	1.790	0.2873	3.231	-j0.28058	j0.50251	-j2.2632
0.3	1.877	0.2938	3.466	-j0.63245	j0.76768	-j2.3898
0.4	1.973	0.3016	3.739	-j0.88936	j0.93637	-j2.5255
0.5	2.082	0.3112	4.071	-j1.1123	j1.0674	-j2.6725
0.6	2.214	0.3226	4.487	-j1.3227	j1.1752	-j2.8346
0.7	2.380	0.3347	5.005	-j1.5355	j1.2690	-j3.0175
0.8	2.582	0.3440	5.582	-j1.7563	j1.3522	-j3.2202
0.9	2.799	0.3486	6.129	-j1.9778	j1.4268	-j3.4308
1.0	3.011	0.3497	6.617	-j2.1934	j1.4945	-j3.6394

$C = 0.15,\ d = 0.05$

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.605	0.2823	2.847	0.71766+j0.19584	-0.70801+j0.20517	0.038389-j2.0131
0.1	1.683	0.2871	3.037	0.43486+j0.14613	-0.42153+j0.16850	0.036711-j2.1261
0.2	1.766	0.2929	3.249	0.024465-j0.26970	-0.0093393+j0.49852	0.034911-j2.2461
0.3	1.854	0.2998	3.493	0.020623-j0.62334	-0.0036347+j0.76610	0.033048-j2.3740
0.4	1.951	0.3083	3.780	0.021030-j0.88054	-0.0022059+j0.93544	0.031213-j2.5112
0.5	2.063	0.3187	4.131	0.022013-j1.1041	-0.0015286+j1.0662	0.029553-j2.6604
0.6	2.199	0.3311	4.574	0.022867-j1.3165	-0.0011247+j1.1749	0.028294-j2.8259
0.7	2.373	0.3436	5.123	0.023189-j1.5326	-0.00085120+j1.2689	0.027699-j3.0130
0.8	2.580	0.3525	5.714	0.022972-j1.7556	-0.00065738+j1.3522	0.027722-j3.2184
0.9	2.796	0.3564	6.261	0.022614-j1.9769	-0.00052105+j1.4268	0.027943-j3.4287
1.0	3.006	0.3572	6.748	0.022369-j2.1914	-0.00042405+j1.4945	0.028091-j3.6360

C = 0.15, d = 0.1

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.581	0.2873	2.854	0.72796+j0.19367	-0.70424+j0.21236	0.076347-j1.9938
0.1	1.659	0.2925	3.049	0.44732+j0.13788	-0.42016+j0.13244	0.072915-j2.1076
0.2	1.742	0.2987	3.269	0.050175-j0.26017	-0.019345+j0.49531	0.069245-j2.2286
0.3	1.831	0.3062	3.523	0.042035-j0.61501	-0.0074249+j0.76467	0.065464-j2.3576
0.4	1.929	0.3154	3.824	0.042782-j0.87258	-0.0044842+j0.93461	0.061776-j2.4965
0.5	2.044	0.3268	4.196	0.044667-j1.0970	-0.0030954+j1.0657	0.058502-j2.6480
0.6	2.186	0.3401	4.670	0.046192-j1.3117	-0.0022667+j1.1746	0.056149-j2.8171
0.7	2.367	0.3530	5.249	0.046577-j1.5311	-0.0017070+j1.2689	0.055204-j3.0084
0.8	2.577	0.3613	5.851	0.046170-j1.7558	-0.0013157+j1.3521	0.055371-j3.2158
0.9	2.792	0.3646	6.397	0.045343-j1.9766	-0.0010434+j1.4268	0.055772-j3.4254
1.0	3.000	0.3652	6.883	0.044944-j2.1899	-0.00085010+j1.4944	0.055977-j3.6312
				C = 0.15, d = 0.25		
0	1.509	0.3038	2.879	0.75420+j0.18580	-0.69172+j0.23276	0.18771-j1.9339
0.1	1.587	0.3102	3.094	0.48648+j0.11212	-0.41486+j0.22223	0.17857-j2.0497
0.2	1.671	0.3180	3.340	0.13329-j0.24105	-0.051976+j0.49171	0.16888-j2.1732
0.3	1.763	0.3276	3.628	0.11074-j0.59535	-0.019601+j0.76161	0.15905-j2.3059
0.4	1.866	0.3396	3.983	0.11211-j0.85422	-0.011694+j0.93274	0.14977-j2.4498
0.5	1.992	0.3544	4.436	0.11597-j1.0826	-0.0079726+j1.0646	0.14219-j2.6093
0.6	2.154	0.3707	5.017	0.11808-j1.3062	-0.0057488+j1.1743	0.13785-j2.7906
0.7	2.355	0.3834	5.672	0.11733-j1.5349	-0.0042774+j1.2689	0.13713-j2.9928
0.8	2.569	0.3896	6.288	0.11564-j1.7611	-0.0032918+j1.3522	0.13783-j3.2019
0.9	2.777	0.3917	6.834	0.11454-j1.9788	-0.0026195+j1.4267	0.13826-j3.4077
1.0	2.976	0.3920	7.332	0.11425-j2.1885	-0.0021416+j1.4943	0.13807-j3.6089

C = 0.15, d = 0.5

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.390	0.3372	2.944	0.80263+j0.16708	-0.66764+j0.26255	0.36539-j1.8262
0.1	1.470	0.3467	3.202	0.55763+j0.064417	-0.40253+j0.28153	0.34528-j1.9437
0.2	1.556	0.3587	3.508	0.27969-j0.24404	-0.10352+j0.50690	0.32421-j2.0705
0.3	1.654	0.3739	3.887	0.23819-j0.58356	-0.041364+j0.76147	0.30355-j2.2091
0.4	1.773	0.3934	4.383	0.23897-j0.84626	-0.024385+j0.93225	0.28579-j2.3639
0.5	1.929	0.4160	5.042	0.24185-j1.0876	-0.016250+j1.0649	0.27477-j2.5419
0.6	2.127	0.4344	5.804	0.23994-j1.3276	-0.011466+j1.1750	0.27190-j2.7427
0.7	2.337	0.4439	6.518	0.23678-j1.5608	-0.0085058+j1.2695	0.27209-j2.9498
0.8	2.540	0.4483	7.155	0.23534-j1.7821	-0.0065940+j1.3525	0.27162-j3.1527
0.9	2.735	0.4511	7.752	0.23552-j1.9940	-0.0052835+j1.4268	0.27012-j3.3512
1.0	2.927	0.4536	8.342	0.23649-j2.1999	-0.0043355+j1.4942	0.26820-j3.5479
				C = 0.15, d = 1.0		
0	1.160	0.4438	3.235	0.91950+j0.096270	-0.61271+j0.30563	0.69397-j1.5685
0.1	1.245	0.4700	3.678	0.72267-j0.066010	-0.36822+j0.37485	0.64631-j1.6818
0.2	1.347	0.5067	4.287	0.56166-j0.35595	-0.15884+j0.57071	0.59795-j1.8125
0.3	1.482	0.5565	5.184	0.51926-j0.66883	-0.077856+j0.78037	0.55936-j1.9709
0.4	1.667	0.6070	6.350	0.50565-j0.95349	-0.046132+j0.94219	0.54124-j2.1602
0.5	1.867	0.6426	7.539	0.49839-j1.2148	-0.030669+j1.0712	0.53303-j2.3594
0.6	2.066	0.6706	8.707	0.49633-j1.4562	-0.022026+j1.1789	0.52644-j2.5576
0.7	2.267	0.6940	9.884	0.49484-j1.6859	-0.016615+j1.2719	0.52251-j2.7570
0.8	2.468	0.7105	11.02	0.49333-j1.9074	-0.012987+j1.3541	0.52039-j2.9570
0.9	2.667	0.7218	12.10	0.49269-j2.1219	-0.010438+j1.4279	0.51847-j3.1565
1.0	2.866	0.7301	13.14	0.49252-j2.3315	-0.0085716+j1.4951	0.51677-j3.3560

C = 0.2, d = 0

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.664	0.2662	2.783	0.70456+j0.21401	-0.70456+j0.21401	-j2.1051
0.1	1.779	0.2682	2.998	0.41795+j0.15123	-0.41795+j0.15123	-j2.2576
0.2	1.901	0.2707	3.233	-j0.31487	j0.48919	-j2.4197
0.3	2.032	0.2738	3.495	-j0.69561	j0.73835	-j2.5924
0.4	2.173	0.2778	3.792	-j0.98770	j0.89404	-j2.7772
0.5	2.329	0.2826	4.136	-j1.2499	j1.0126	-j2.9761
0.6	2.507	0.2882	4.541	-j1.5019	j1.1101	-j3.1923
0.7	2.716	0.2941	5.018	-j1.7560	j1.1934	-j3.4297
0.8	2.959	0.2987	5.553	-j2.0194	j1.2667	-j3.6902
0.9	3.230	0.3004	6.096	-j2.2920	j1.3320	-j3.9690
1.0	3.514	0.2992	6.607	-j2.5689	j1.3910	-j4.2579

C = 0.2, d = 0.05

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.637	0.2708	2.785	0.71252+j0.21277	-0.70118+j0.22117	0.038743-j2.0836
0.1	1.752	0.2730	3.005	0.43011+j0.14417	-0.41721+j0.16498	0.037187-j2.2371
0.2	1.874	0.2757	3.200	0.022702-j0.30345	-0.0081660+j0.48522	0.035547-j2.4002
0.3	2.005	0.2792	3.518	0.019335-j0.68593	-0.0031263+j0.73680	0.033875-j2.5742
0.4	2.148	0.2835	3.826	0.019700-j0.97823	-0.0018572+j0.89314	0.032243-j2.7605
0.5	2.306	0.2888	4.184	0.020591-j1.2408	-0.0012625+j1.0121	0.030754-j2.9614
0.6	2.488	0.2949	4.610	0.021450-j1.4939	-0.00091528+j1.1097	0.029548-j3.1803
0.7	2.701	0.3011	5.111	0.021984-j1.7501	-0.00068667+j1.1933	0.028784-j3.4212
0.8	2.951	0.3055	5.664	0.022095-j2.0160	-0.00052641+j1.2666	0.028513-j3.6849
0.9	3.224	0.3067	6.214	0.021933-j2.2897	-0.00041185+j1.3320	0.028559-j3.9651
1.0	3.508	0.3051	6.714	0.021722-j2.5665	-0.00032925+j1.3909	0.028686-j4.2538

C = 0.2, d = 0.1

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.609	0.2756	2.787	0.72072+j0.21138	-0.69759+j0.22817	0.077040-j2.0619
0.1	1.725	0.2780	3.013	0.44259+j0.13702	-0.41627+j0.17846	0.073853-j2.2162
0.2	1.848	0.2810	3.262	0.046617-j0.29305	-0.016958+j0.48183	0.070509-j2.3804
0.3	1.979	0.2848	3.542	0.039448-j0.67689	-0.0063948+j0.73537	0.067115-j2.5556
0.4	2.123	0.2895	3.862	0.040118-j0.96946	-0.0037798+j0.89232	0.063829-j2.7435
0.5	2.283	0.2953	4.237	0.041849-j1.2325	-0.0025601+j1.0116	0.060877-j2.9465
0.6	2.469	0.3020	4.685	0.043455-j1.4870	-0.0018490+j1.1094	0.058560-j3.1682
0.7	2.688	0.3084	5.210	0.044353-j1.7455	-0.0013818+j1.1931	0.057192-j3.4126
0.8	2.943	0.3126	5.780	0.044417-j2.0136	-0.0010562+j1.2665	0.056800-j3.6793
0.9	3.218	0.3133	6.335	0.044036-j2.2881	-0.00082553+j1.3319	0.056949-j3.9605
1.0	3.501	0.3112	6.846	0.043636-j2.5646	-0.00066005+j1.3909	0.057182-j4.2488

C = 0.2, d = 0.25

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.528	0.2912	2.797	0.74669+j0.20601	-0.68556+j0.24809	0.18930-j1.9953
0.1	1.644	0.2944	3.042	0.48174+j0.11474	-0.41211+j0.21700	0.18079-j2.1519
0.2	1.768	0.2984	3.316	0.12466-j0.26977	-0.046189+j0.47649	0.17195-j2.3189
0.3	1.902	0.3035	3.627	0.10428-j0.65428	-0.016974+j0.73203	0.16312-j2.4978
0.4	2.050	0.3098	3.993	0.10552-j0.94774	-0.0099028+j0.89031	0.15481-j2.6905
0.5	2.219	0.3175	4.372	0.10931-j1.2132	-0.0066345+j1.0103	0.14775-j2.9002
0.6	2.419	0.3258	4.952	0.11228-j1.4731	-0.0047353+j1.1088	0.14287-j3.1315
0.7	2.657	0.3326	5.552	0.11315-j1.7395	-0.0034984+j1.1929	0.14076-j3.3868
0.8	2.923	0.3354	6.159	0.11242-j2.0127	-0.0026573+j1.2664	0.14065-j3.6601
0.9	3.199	0.3344	6.722	0.11136-j2.2876	-0.0020753+j1.3319	0.14111-j3.9419
1.0	3.477	0.3311	7.235	0.11066-j2.5622	-0.0016617+j1.3908	0.14139-j4.2278

C = 0.2, d = 0.5

QC	b at osc.	CN_s	$(\beta-\beta_e)L$	δ_1	δ_2	δ_3
0	1.396	0.3225	2.829	0.79503+j0.19219	-0.66213+j0.27724	0.36796-j1.8780
0.1	1.513	0.3277	3.116	0.55286+j0.073567	-0.40119+j0.27477	0.34919-j2.0371
0.2	1.640	0.3343	3.444	0.26610-j0.26211	-0.095186+j0.48663	0.32994-j2.2080
0.3	1.779	0.3428	3.832	0.22589-j0.63449	-0.036340+j0.73048	0.31130-j2.3929
0.4	1.939	0.3533	4.304	0.22683-j0.93016	-0.020927+j0.88895	0.29494-j2.5952
0.5	2.130	0.3654	4.889	0.23141-j1.2035	-0.013768+j1.0097	0.28320-j2.8197
0.6	2.361	0.3757	5.573	0.23266-j1.4766	-0.0096318+j1.1087	0.27780-j3.0696
0.7	2.621	0.3805	6.266	0.23090-j1.7526	-0.0070328+j1.1930	0.27696-j3.3377
0.8	2.890	0.3801	6.901	0.22891-j2.0272	-0.0053381+j1.2666	0.27724-j3.6128
0.9	3.159	0.3768	7.479	0.22792-j2.2986	-0.0041843+j1.3319	0.27707-j3.8907
1.0	3.428	0.3724	8.021	0.22787-j2.5688	-0.0033625+j1.3907	0.27628-j4.1716
				C = 0.2, d = 1.0		
0	1.143	0.4194	3.011	0.91355+j0.13477	-0.60799+j0.31890	0.69621-j1.6088
0.1	1.265	0.4351	3.459	0.71837-j0.039426	-0.36820+j0.36565	0.65160-j1.7666
0.2	1.404	0.4565	4.027	0.54726-j0.34860	-0.15266+j0.54435	0.60717-j1.9432
0.3	1.572	0.4843	4.784	0.50328-j0.68836	-0.071500+j0.74457	0.56997-j2.1456
0.4	1.785	0.5132	5.756	0.49432-j0.99814	-0.041404+j0.89546	0.54881-j2.3798
0.5	2.032	0.5309	6.777	0.48746-j1.2935	-0.026832+j1.0139	0.54108-j2.6357
0.6	2.285	0.5377	7.719	0.48388-j1.5753	-0.018800+j1.1115	0.53660-j2.8975
0.7	2.539	0.5394	8.605	0.48311-j1.8476	-0.013903+j1.1946	0.53245-j3.1624
0.8	2.797	0.5384	9.462	0.48307-j2.1158	-0.010669+j1.2675	0.52924-j3.4322
0.9	3.062	0.5347	10.29	0.48257-j2.3837	-0.0084058+j1.3324	0.52745-j3.7087
1.0	3.333	0.5280	11.06	0.48172-j2.6532	-0.0067674+j1.3911	0.52664-j3.9922

APPENDIX B: GRAPHS OF THE O-TYPE BWO STARTING CONDITIONS

Parameter	Range
C	10^{-5} to 0.2
QC	0 to 1.0
d	0 to 1.0

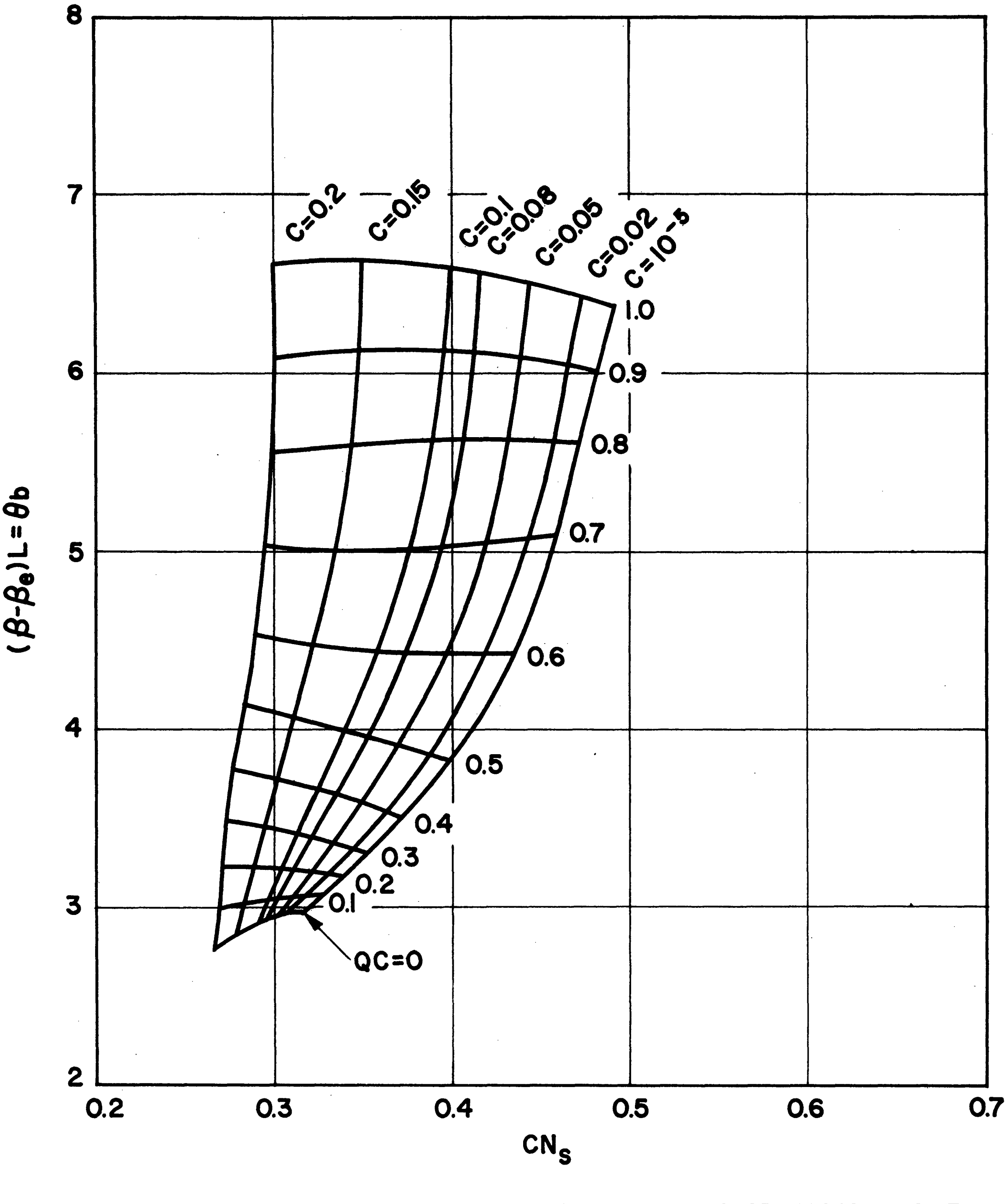

FIG. B.1 O-TYPE BWO STARTING CONDITIONS. d=0

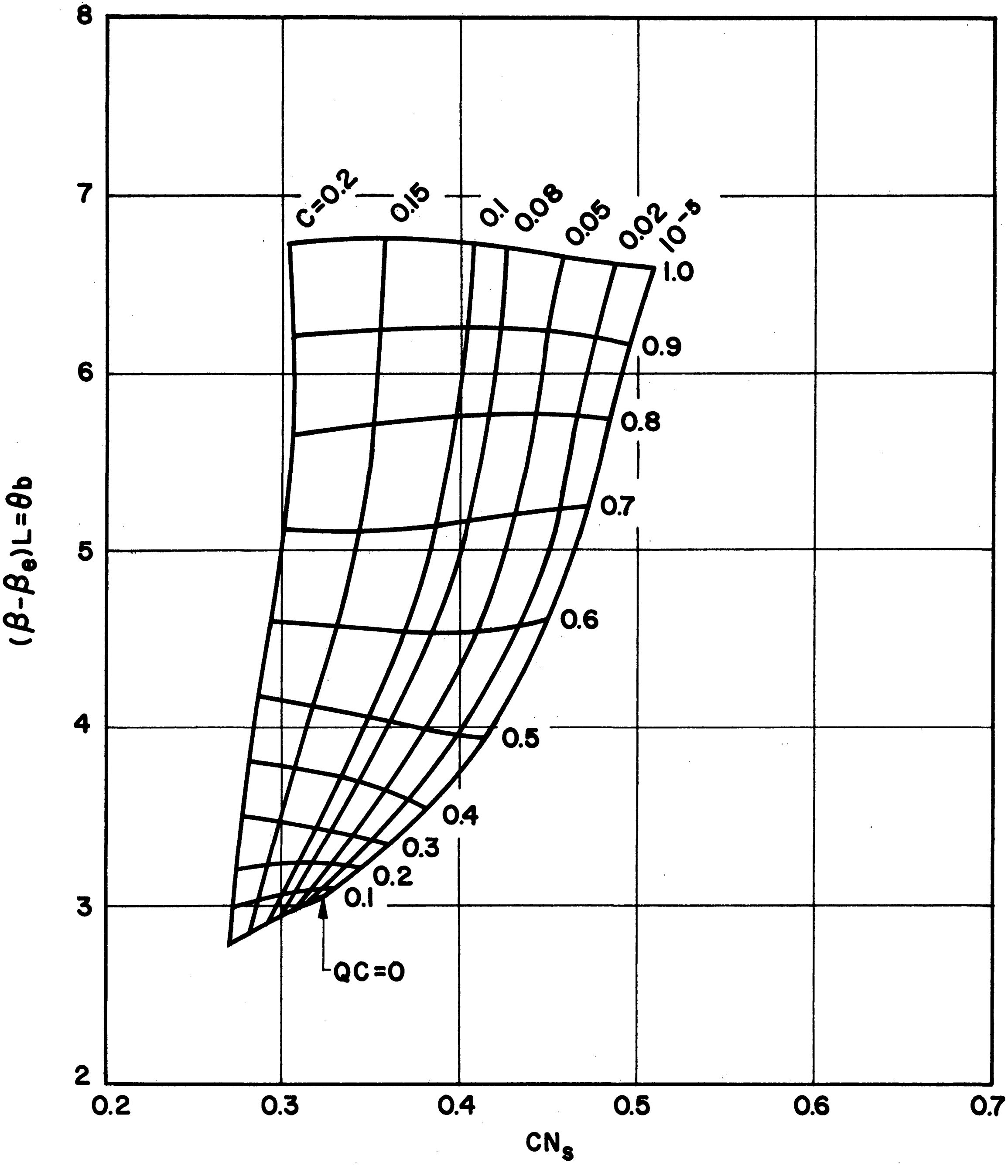

FIG. B.2 O-TYPE BWO STARTING CONDITIONS. d=0.05

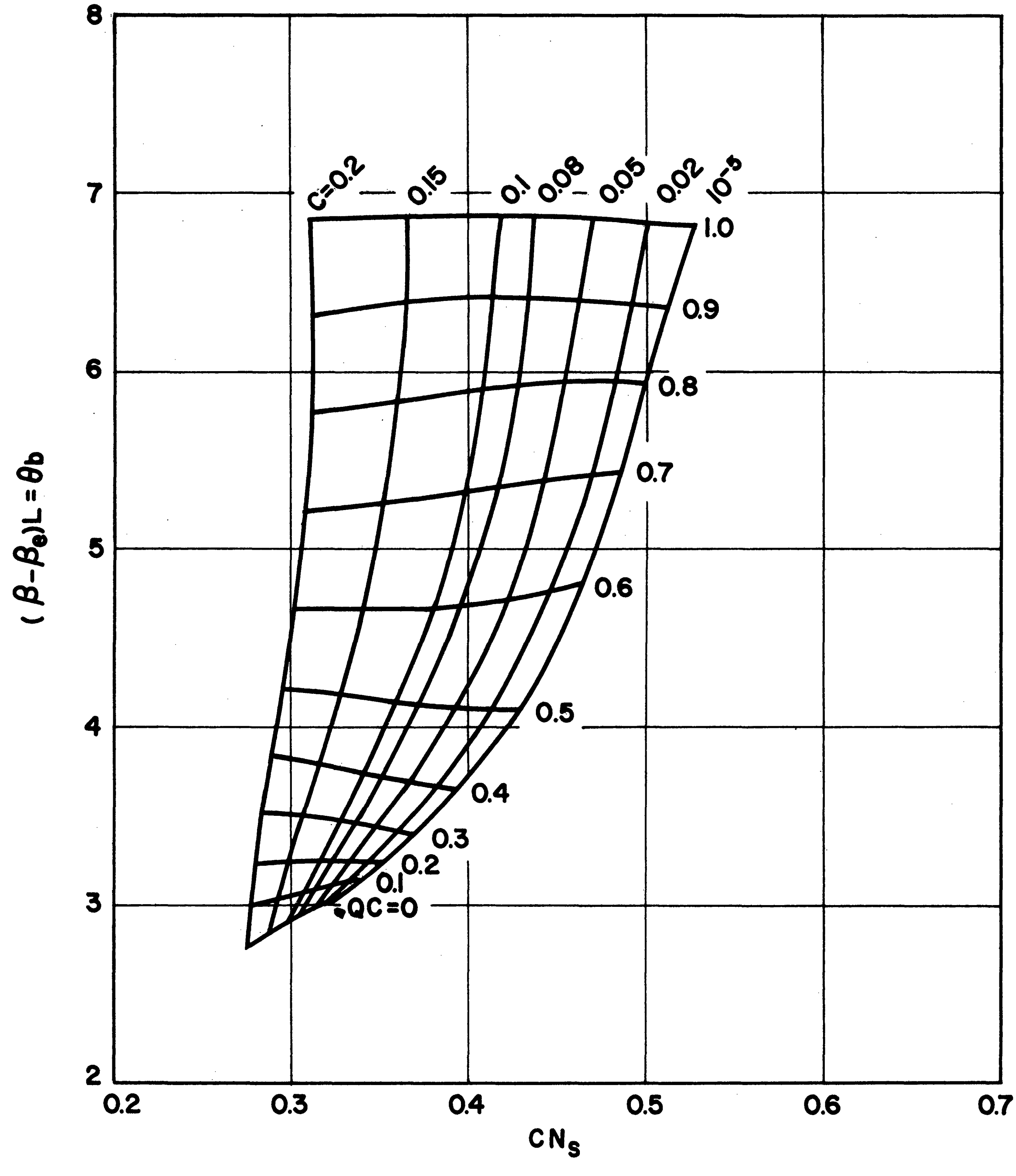

FIG. B.3 O-TYPE BWO STARTING CONDITIONS. d=0.1

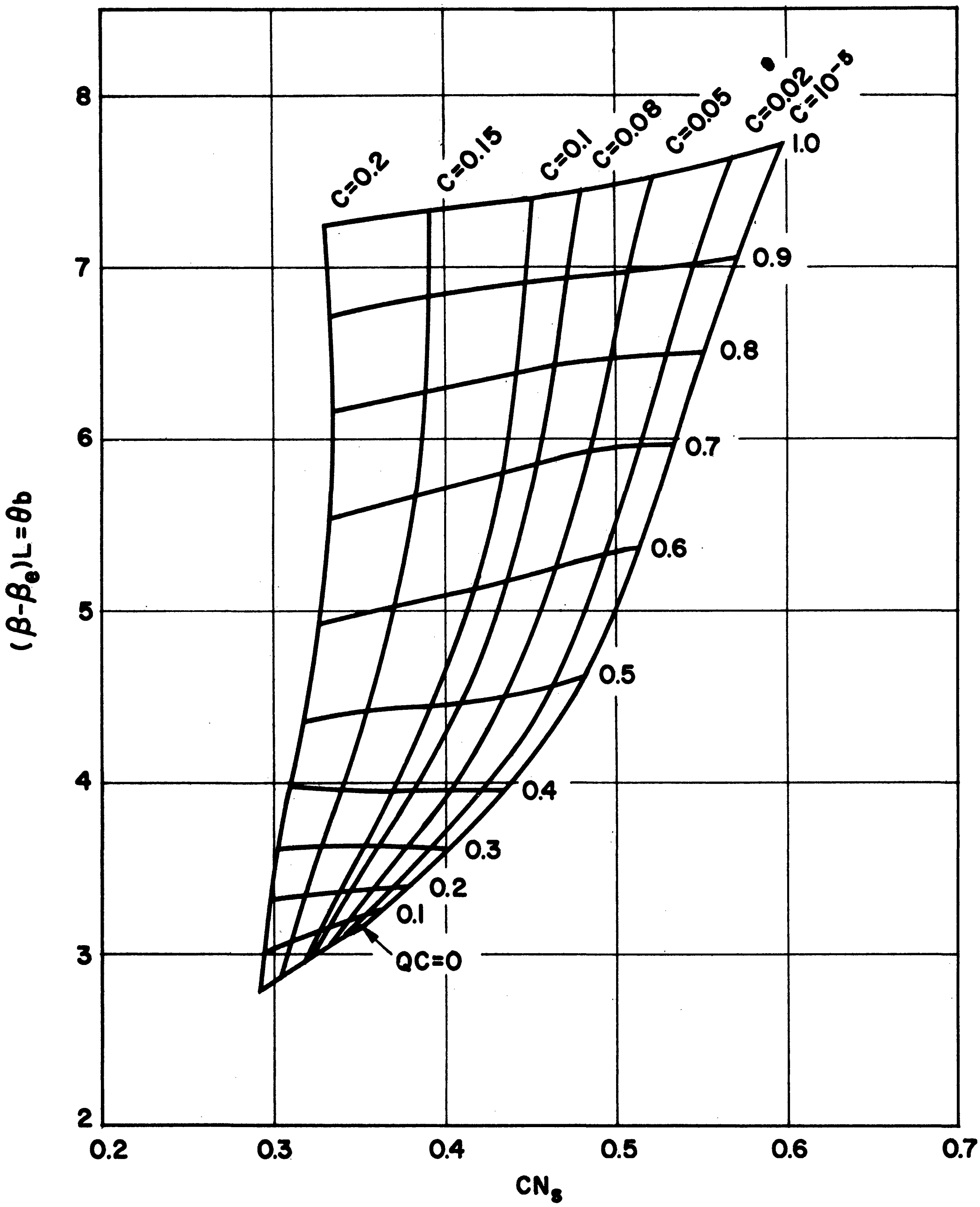

FIG. B.4 O-TYPE BWO STARTING CONDITIONS. d=0.25

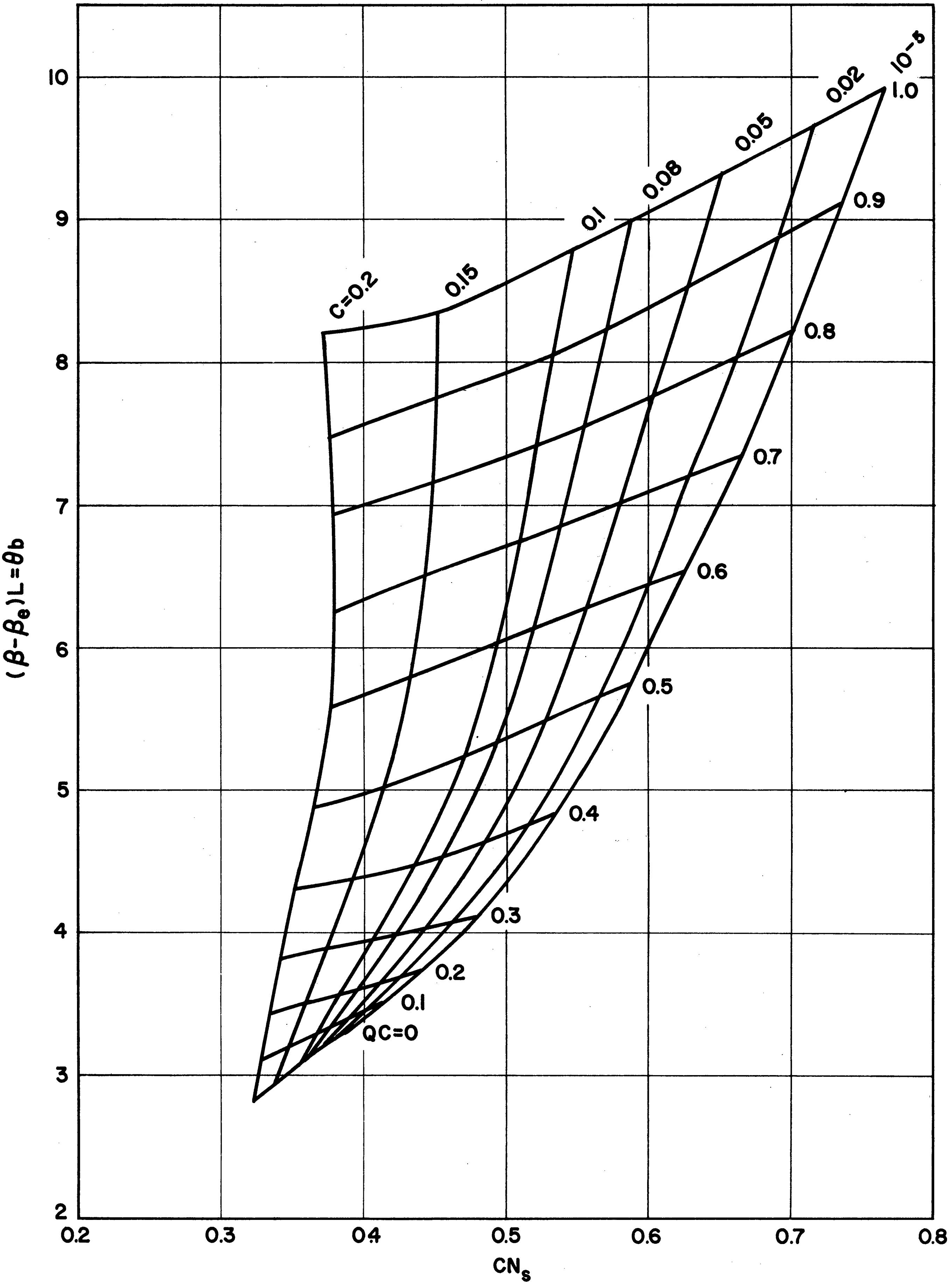

FIG. B.5 O-TYPE BWO STARTING CONDITIONS. d=0.5

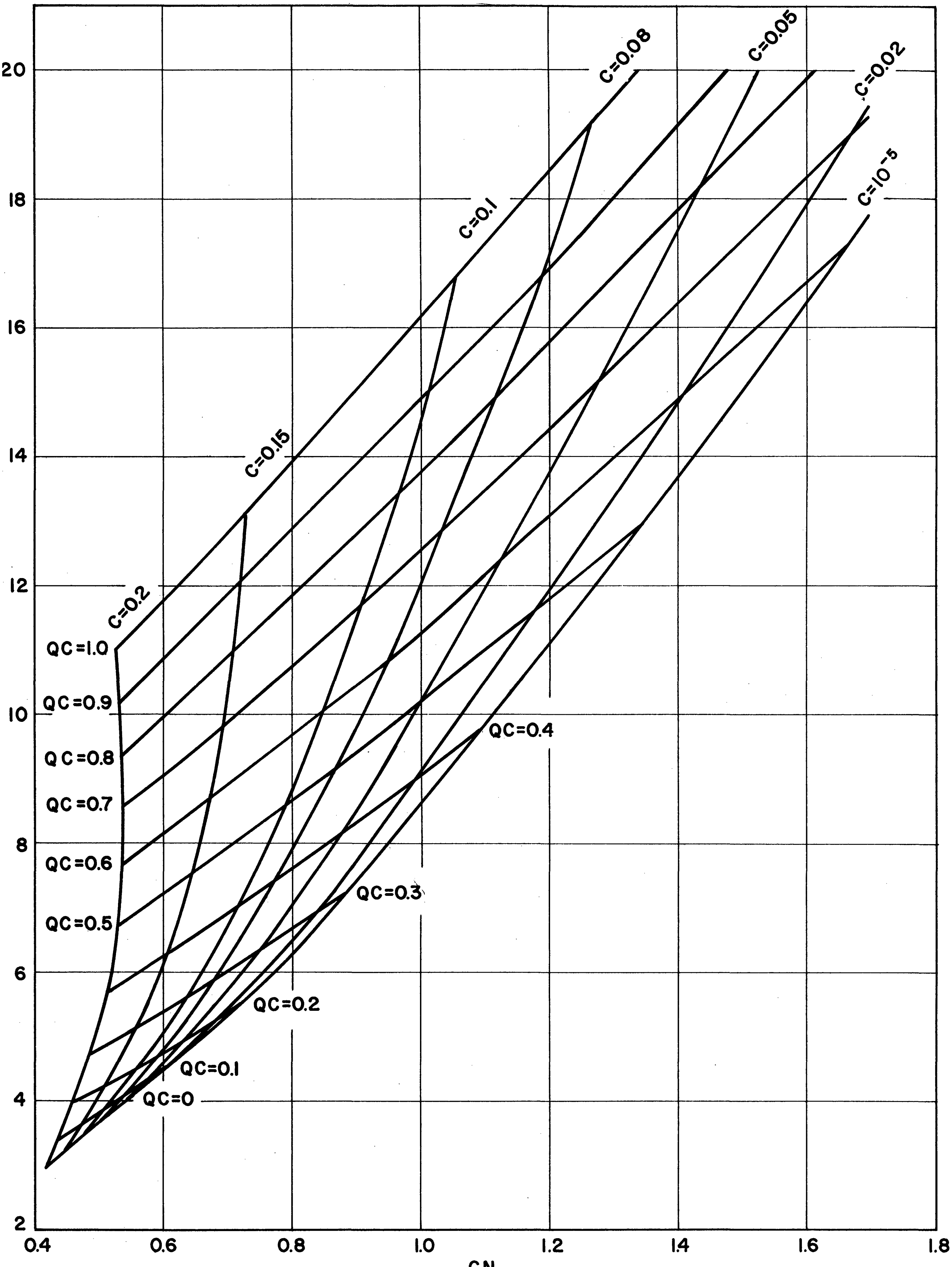

FIG. B.6 O-TYPE BWO STARTING CONDITIONS. d=1.0

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3. Analog Calculation of Starting Conditions
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5. Effect of Modulation on the Start-Oscillation Conditions

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