THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

MODES OF VIBRATION OF TALL CONCRETE CHIMNEYS WITH STEEL LINING

W. S. Rumman

May, 1963

IP-619

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

SUMMARY

The natural frequencies and the mode shapes for the first four fundamental modes of vibration are calculated for tall reinforced concrete chimneys with steel liners. The method of solution utilizes the well known Stodola numerical procedure and is based on the Bernoulli-Euler flexural theory. Numerical solutions are obtained for chimneys with steel liners simply supported at top and bottom, for chimneys with steel liners fixed at the bottom only and for chimneys with no liners. Some graphs are presented to illustrate the effect of certain major parameters on the solution. All the numerical results were obtained by the use of the IBM 7090 computer at The University of Michigan.

INTRODUCTION

The need for air pollution control in the last 10 or 15 years has led to the construction of tall reinforced concrete chimneys and at the same time to an increase in the velocity of the flue gases (100 - 120 feet/sec. is not unusual). In the past the majority of these chimneys were constructed as shown in Figure 1 (a) using corbel-supported brick lining and fiber glass or fused silica insulation between the lining and the concrete shell. In the last 3 or 4 years, many chimneys of this type have shown signs of acid attack on the concrete shell which is the result of the hot gases working their way through the brick lining and their condensation on the relatively cold concrete. The condensate is acidic due to the high sulfuric content of the fuel. The problem of acid attack is an acute one and lots of research is being conducted now to correct it.

A very promising solution to the acid problem is the use of independent steel liners made of acid resistant Corten steel as shown in Figure 1 (b). A few chimneys have been built in the last few years using the steel liners but not enough time has elapsed to evaluate their effectiveness.

The dynamic behavior of chimneys with steel liners is undoubtedly different from that of conventional chimneys where corbel-supported brick lining is used. The purpose of this paper therefore, is (1) to explore this problem and to outline a method of finding the frequencies and the mode shape, (2) to study the variations in the frequencies and the mode shapes with respect to certain parameters of the problem and (3) to compare the results of the steel lined chimneys with the unlined chimneys.

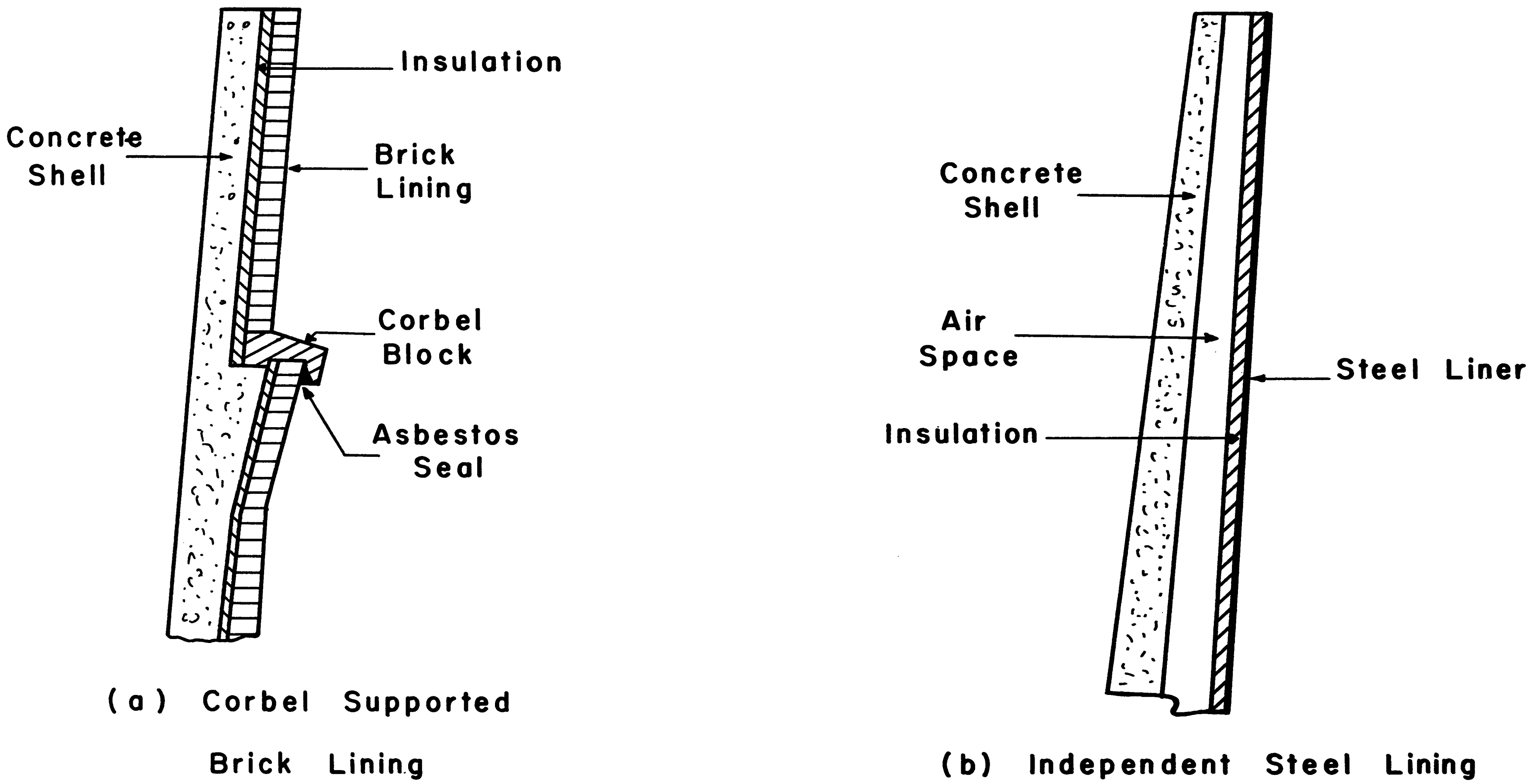

Figure 1. Concrete Chimneys with Two Types of Lining.

Notation: Letter symbols adopted for use in this paper are defined when they first appear and are arranged in the appendix.

METHOD OF SOLUTION

A concrete chimney with a steel liner is no different from a cantilever beam except for the additional end conditions imposed by the liner. Ignoring shear deformations and rotary inertia effects, the basic differential equation for the free vibration of a beam with zero damping is given by:

$$\frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 Z}{\partial x^2}\right) + m \frac{\partial^2 Z}{\partial t^2} = 0 \qquad (1)$$

where $Z(x, t)$ are the displacements in the beam, $m(x)$ is the mass intensity per unit length, x is the distance along the beam and t is the time.

For any solution of Equation (1) of the form

$$Z = Y(x)\, T(t) \qquad (2)$$

We obtain, by substituting (2) in Equation (1)

$$\frac{\partial^2}{\partial x^2} (EI \overset{''}{Y} T) + m Y \ddot{T} = 0 \qquad (3)$$

where $\overset{''}{Y} = \frac{d^2Y}{dX^2}$ and $\ddot{T} = \frac{d^2T}{dt^2}$

Dividing both sides of Equation (3) by mYT we get:

$$\frac{1}{mY} \frac{d^2}{dx^2} (EI \overset{''}{Y}) + \frac{\ddot{T}}{T} = 0 \qquad (4)$$

or

$$\frac{1}{mY} \frac{d^2}{dx^2} (EI \overset{''}{Y}) = - \frac{\ddot{T}}{T} = \omega^2 \text{ (constant)} \qquad (5)$$

$\therefore \ \ddot{T} + \omega^2 T = 0$

and $T = A \cos \omega t + B \sin \omega t$

$$\therefore \frac{1}{mY} \frac{d^2}{dx^2} (EI \ddot{Y}) = \omega^2$$

or

$$\frac{d^2}{dx^2} (EI \ddot{Y}) = m \omega^2 Y \qquad (6)$$

Equation (6) is solved numerically using the Stodola process which is outlined briefly.

Consider an assumed deflected shape of the beam

$$Y^{[o]} = a_1 Y_1 + a_2 Y_2 + \ldots + a_i Y_i + \ldots \qquad (7)$$

where Y_i is the exact shape of the i^{th} mode and a_i is constant.

The inertia loads for any mode $a_i Y_i$ will be equal to $m\, a_i\, Y_i\, \omega_i^2$.

If ω_i is taken equal to unity then two integrations of the inertia load $m\, a_i\, Y_i$ will give the bending moments (M) and two further integrations of the M/EI will give the derived displacements $\dfrac{Y_i}{\omega_i^2}$. Thus from an assumed $Y^{[o]}$ one obtains by the successive four integrations $Y^{[1]}$ equals to:

$$Y^{[1]} = \frac{a_1 Y_1}{\omega_1^2} + \frac{a_2 Y_2}{\omega_2^2} + \frac{a_3 Y_3}{\omega_3^2} + \ldots + \frac{a_i Y_i}{\omega_i^2} + \ldots$$

Similarly assuming $Y^{[1]}$ as the deflection curve one obtains

$$Y^{[2]} = \frac{a_1 Y_1}{\omega_1^4} + \frac{a_2 Y_2}{\omega_2^4} + \ldots + \frac{a_i Y_i}{\omega_i^4} + \ldots$$

If this process is continued, then:

$$Y^{[n-1]} = \frac{1}{\omega_1^{2(n-1)}}\left[a_1 Y_1 + \left(\frac{\omega_1}{\omega_2}\right)^{2(n-1)} a_2Y_2 + \ldots\ldots \right.$$
$$\left. + \left(\frac{\omega_1}{\omega_i}\right)^{2(n-1)} a_iY_i + \ldots \right] \quad (8)$$

and

$$Y^{[n]} = \frac{1}{\omega_1^{2n}}\left[a_1 Y_1 + \left(\frac{\omega_1}{\omega_2}\right)^{2n} a_2Y_2 + \ldots\ldots \right.$$
$$\left. + \left(\frac{\omega_1}{\omega_i}\right)^{2n} a_iY_i + \ldots \right] \quad (8a)$$

Since $\omega_1 < \omega_2 < \omega_3 \ldots\ldots$, the terms containing $Y_2, Y_3 \ldots \; Y_i \ldots$ in Equations (8) and 8 (a) will decrease rapidly with the increase in n and their contribution to the shape can be neglected. Thus the process will converge to the first mode and can be continued until the computed shape $Y^{[n]}$ has the same configuration as the assumed shape $Y^{[n-1]}$ to any specified degree of accuracy. The first mode frequency is then obtained by dividing the maximum value of $Y^{[n-1]}$ by the corresponding value of $Y^{[n]}$. Thus with all but the first term in Equations (8) and 8 (a) neglected we get:

$$\frac{Y^{[n-1]}(\bar{x})}{Y^{[n]}(\bar{x})} = \frac{a_1 Y_1 (\dot{x})}{\omega_1^{2(n-1)}} \Big/ \frac{a_1 Y_1 (\bar{x})}{\omega_1^{2n}} \quad (9)$$

where $\bar{x}$ is the value of x for which $Y^{[n-1]}$ is maximum.

From Equation (9) we get:

$$\omega_1^2 = \frac{Y^{[n-1]}(\bar{x})}{Y^{[n]}(\bar{x})} \quad (9a)$$

For the higher modes the assumed shape should be "purified" by removing the lower mode shapes through the use of the orthogonality relationship:

$$\int_0^L m\, Y_i\, Y_j\, dx = 0 \qquad i \neq j \tag{10}$$

For example, the second mode can be obtained by removing the first mode shape from the assumed shape. Thus for an assumed shape Y, the purified shape Yp can be written as:

$$Y_p = Y - A\, Y_1$$

where A is a constant.

$(Y - A\, Y_1)$ is a shape that does not include the first mode, therefore

$$\int_0^L m\, Y_1\, (Y - AY_1)\, d\, x = 0$$

from which
$$A = \frac{\int m\, Y\, Y_1\, d\, x}{\int m\, {Y_1}^2\, d\, x}$$

In general the purified shape for the i^{th} mode is:

$$Y_p = Y - \frac{\int m\, Y\, Y_1\, d\, x}{\int m\, {Y_1}^2\, d\, x} Y_1 - \frac{\int m\, Y\, Y_2\, d\, x}{\int m\, {Y_2}^2\, d\, x} Y_2 \,\cdots\cdots - \frac{\int m\, Y\, Y_{(i-1)}\, d\, x}{\int m\, Y^2_{(i-1)} d\, x} Y_{(i-1)} \tag{11}$$

The Stodola process will therefore converge to the i^{th} mode if the assumed shape for every cycle is purified of all the lower modes according to Equation (11). It is apparent that this method will require that the modes be determined successively starting with the first mode.

THE COMPUTER SOLUTION

In the following discussion the subscripts "c" and "s" will be used to designate the concrete shell and the steel liner respectively. The concrete shell is divided into N_{C1} equal elements in the region above the bottom of the liner and into N_{C2} equal elements for the remainder of the length L_C as shown in Figures 2 and 3. The steel shell is divided into N_S equal elements.

The procedure of finding the first mode is as follows:

1. Assume any deflected shape for the structure. The specific shape assumed is a zero deflection at the bottom of the concrete shell and unity at all the other stations of the concrete shell and the steel liner.
2. Compute the values of the intensity of the dynamic load which is $m\,\omega^2 Y$ with ω taken equal to 1. Find the end reactions on the steel liner due to this load as shown in Figures 2 (b) and 3 (b) and apply equal and opposite forces on the concrete shell.
3. Find the values of the bending moments (M) at the different stations assuming a second degree variation in the loading. Compute the values of $\frac{M}{EI}$.
4. Assuming a second degree variation in the values of $\frac{M}{EI}$ and using Newmark's Numerical procedure[2], calculate the displacements at all stations in the concrete shell starting with zero displacement and zero slope at the bottom. In case of the structure with the cantilever

---

[2] "Numerical Procedure for Computing Deflections, Moments, and Buckling Loads" by N. M. Newmark, Trans. Am. Soc. C. E., Vol. 108, 1943.

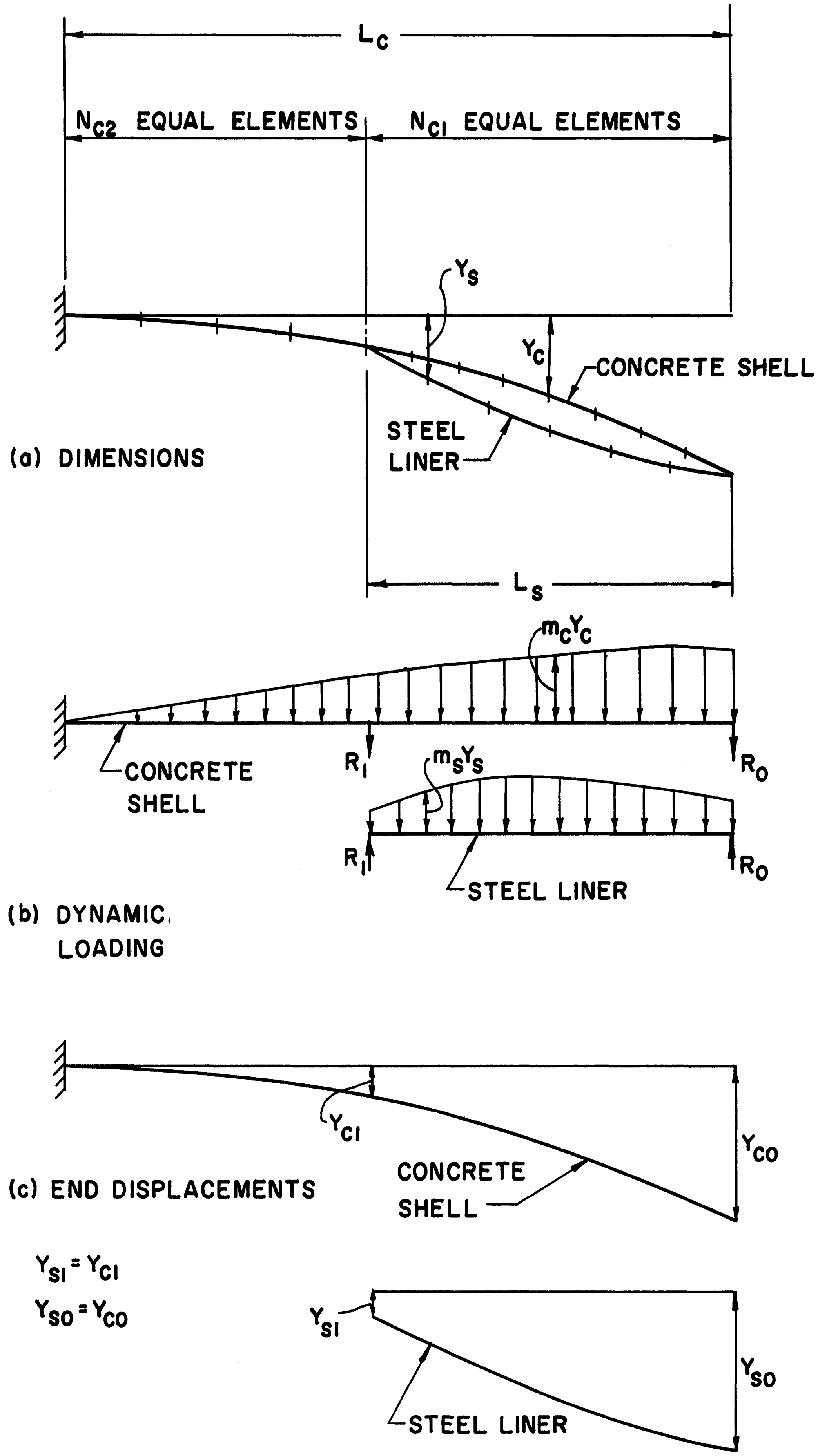

Figure 2. Chimney with Simply Supported Liner.

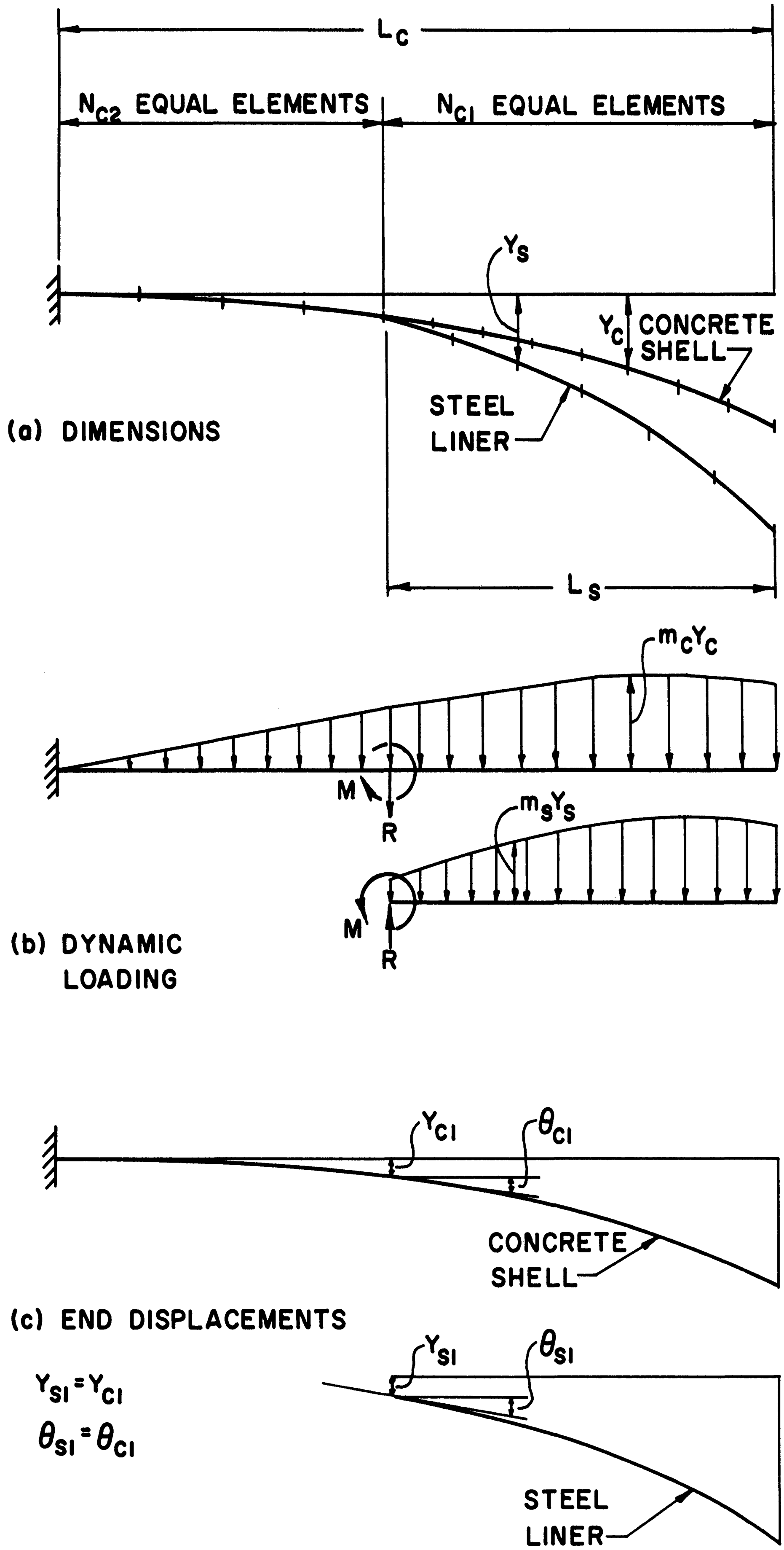

Figure 3. Chimney with Cantilever Liner.

liner (Figure 3) compute the slope of the elastic curve of the concrete shell at the location of the bottom of the liner.

5. Using end displacements for the steel liner compatible with the displacements of the concrete shell as shown in Figures 2 (c) and 3 (c), compute the displacements at all the stations of the liner.

6. Normalize the computed shape by making the maximum displacement equal to +1. This maximum could occur in either the concrete shell or the liner.

7. Compare the normalized shape of Step (6) with the assumed shape of Step 1. The comparison is made on the basis of the difference between the absolute values of the two shapes at all the stations of both the concrete shell and the liner.

   (I) If this difference does not exceed .000001 at any location, then the computed shape is taken equal to the assumed shape and the square of the frequency is obtained by finding the ratio between the maximum value of the assumed shape and the corresponding value of the computed shape; that is

$$\omega^2 = \frac{\text{Max. value of assumed } Y}{\text{Corresponding value of computed } Y} \tag{12}$$

   Note that the maximum value of assumed Y is always equal to +1.

   (II) If the difference exceeds .000001 then the process is repeated by assuming the normalized shape of Step (6) as the deflected shape and repeating the process starting with Step (2).

The process is thus repeated until convergence is obtained.

A higher mode frequency is obtained as follows:

(a) Assume any shape. In this case the same shape as in Step (1) is assumed.

(b) Do Steps 2, 3, 4 and 5 as for the first mode.

(c) Purify the computed shape from the lower modes according to Equation (11).

(d) Normalize (c) in the same way as was done in the first mode.

(e) Compare the shape of (d) to that of the assumed shape. The comparison test is identical to that of the first mode.

(I) If the comparison test is not satisfied then repeat the process starting with (b) and using as the assumed shape that obtained in Step (d).

(II) If the comparison test is satisfied the process is stopped and the square of the frequency is obtained by finding the ratio between the maximum value of the assumed shape and the corresponding value of the computed-purified shape of Step (c).

For the higher modes:

$$\omega^2 = \frac{\text{Max. value of assumed } Y}{\text{Corresponding Value of Computed-Purified } Y} \qquad (13)$$

EXAMPLES

Two examples are given in this paper. The first deals with the linearly tapered chimneys and the second is a specific example of an actual chimney.

Example 1: Linearly Tapered Chimneys

In this example the general problem of the linearly tapered chimney is considered in which the outside diameter of the concrete shell as well as its thickness varies linearly along the height. The steel liner is also linearly tapered in both diameter and thickness. It should be mentioned that this example approximates most of the actual chimneys.

The notation for this example is listed below:

E_c, E_s = Modulus of elasticity of concrete shell and steel liner, respectively.

ρ_c, ρ_s = Mass density of concrete shell and steel liner, respectively.

The rest of the notation is illustrated in Figure 4 where the first subscript "c" or "s" refers to the concrete shell or the steel liner, respectively and the second subscript "o" or "n" refers to the top or bottom, respectively.

The moment of inertia at any section of the concrete shell or the steel liner is approximated by:

$$I = \frac{\pi\, d^3 t}{8}$$

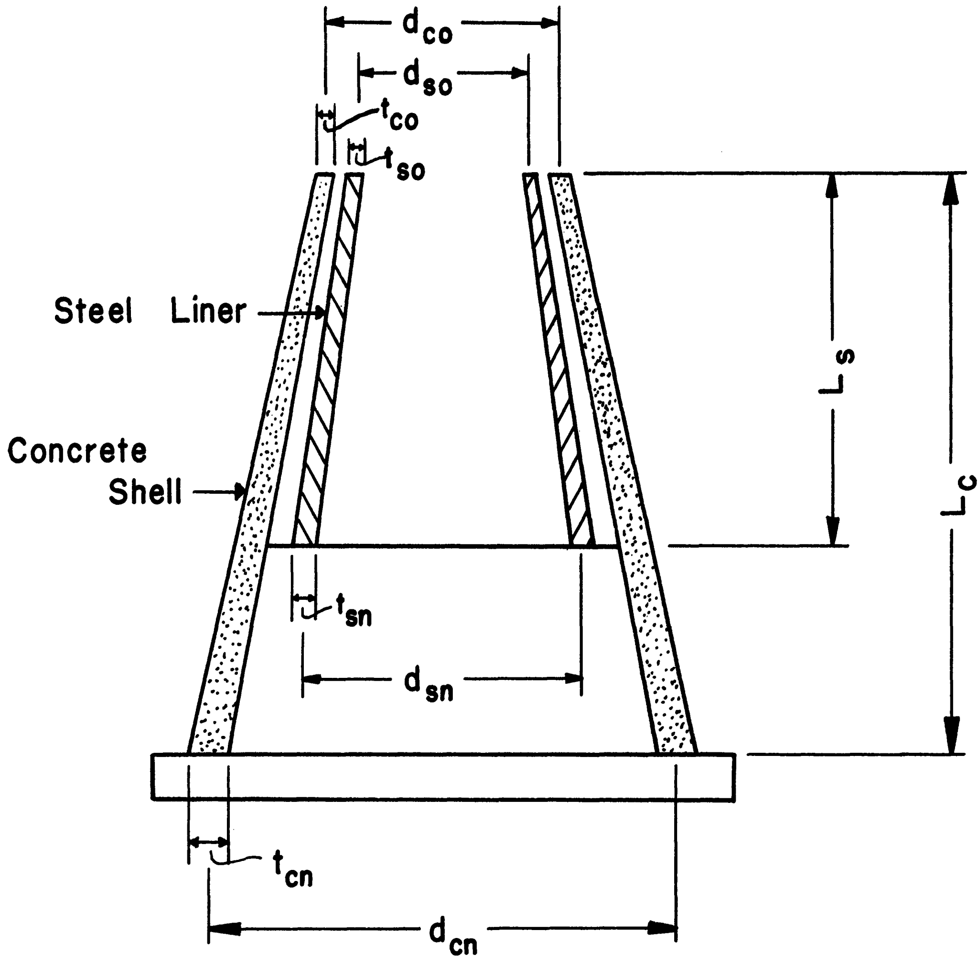

Figure 4. Linearly Tapered Chimney.

where I = moment of inertia

d = the mean diameter

t = the thickness

The above approximation is a very good one for all practical problems of this type where the d/t ratio is in the order of 20 or more.

The structure as shown in Figure 4 can be described in terms of the following nine dimensionless parameters:

$$\frac{L_s}{L_c}, \quad \frac{E_s}{E_c}, \quad \frac{\rho_s}{\rho_c}, \quad \frac{d_{co}}{d_{cn}}, \quad \frac{t_{co}}{t_{cn}}$$

$$\frac{d_{so}}{d_{co}}, \quad \frac{t_{so}}{t_{co}}, \quad \frac{d_{so}}{d_{sn}}, \quad \text{and} \quad \frac{t_{so}}{t_{sn}}$$

The modes of vibration can be computed as a function of any parameter if the other parameters are kept constant.

Solutions are obtained for three cases:

Case I - Chimneys with simply supported liners

Case II - Chimneys with cantilever liners

Case III - Chimneys with no liners

In Figures 5, 6 and 7, the frequencies for the first four modes of vibration are plotted for Case I as a function of $\frac{d_{co}}{d_{cn}}$ for different values of $\frac{t_{co}}{t_{cn}}$ and of $\frac{L_s}{L_c}$. Note that the curves for $\frac{L_s}{L_c} = 0$ represent the solution for Case III. The other parameters of the problem are kept constant and have been given the values shown in the figures. These values were chosen to represent as closely as possible the practical dimensions of steel lined chimneys.

Figures 8 and 9 give the frequencies for Case II (the chimneys with the cantilever liner).

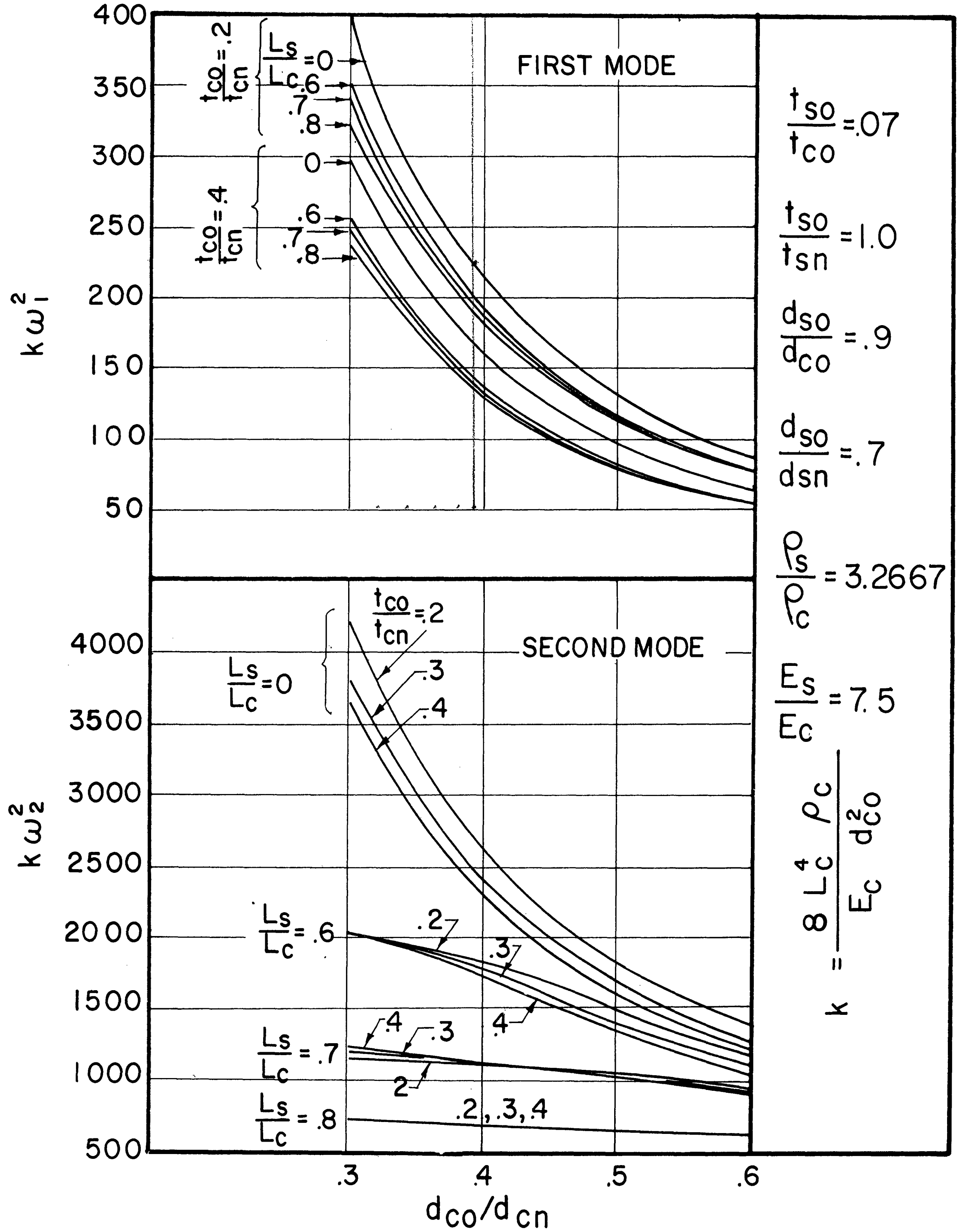

Figure 5. First and Second Mode Frequencies vs. d_{co}/d_{cn} Ratio
Linearly Tapered Chimneys - Simply Supported Liner.

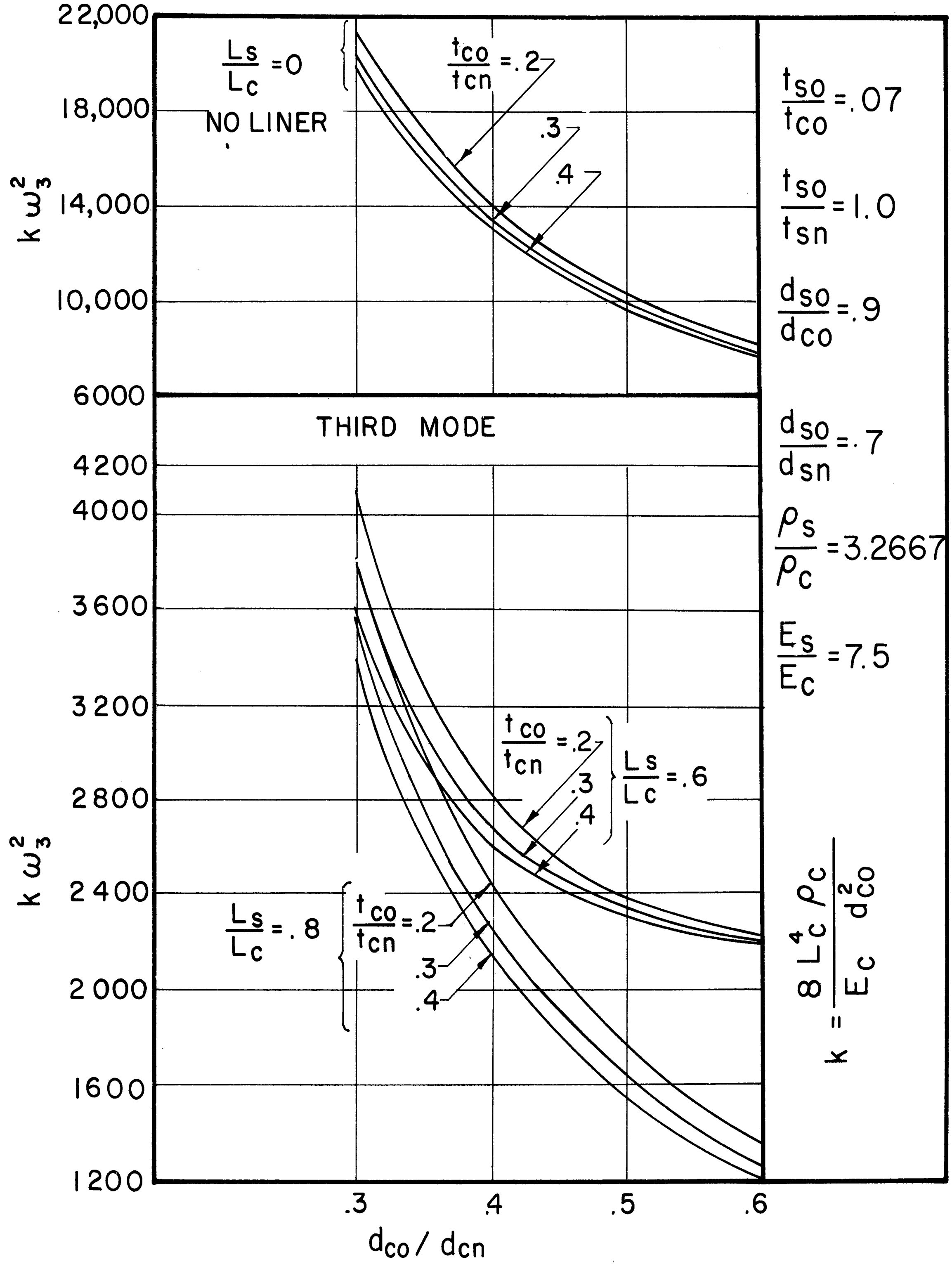

Figure 6. Third Mode Frequencies vs. d_{co}/d_{cn} Ratio Linearly Tapered Chimneys - Simply Supported Linear.

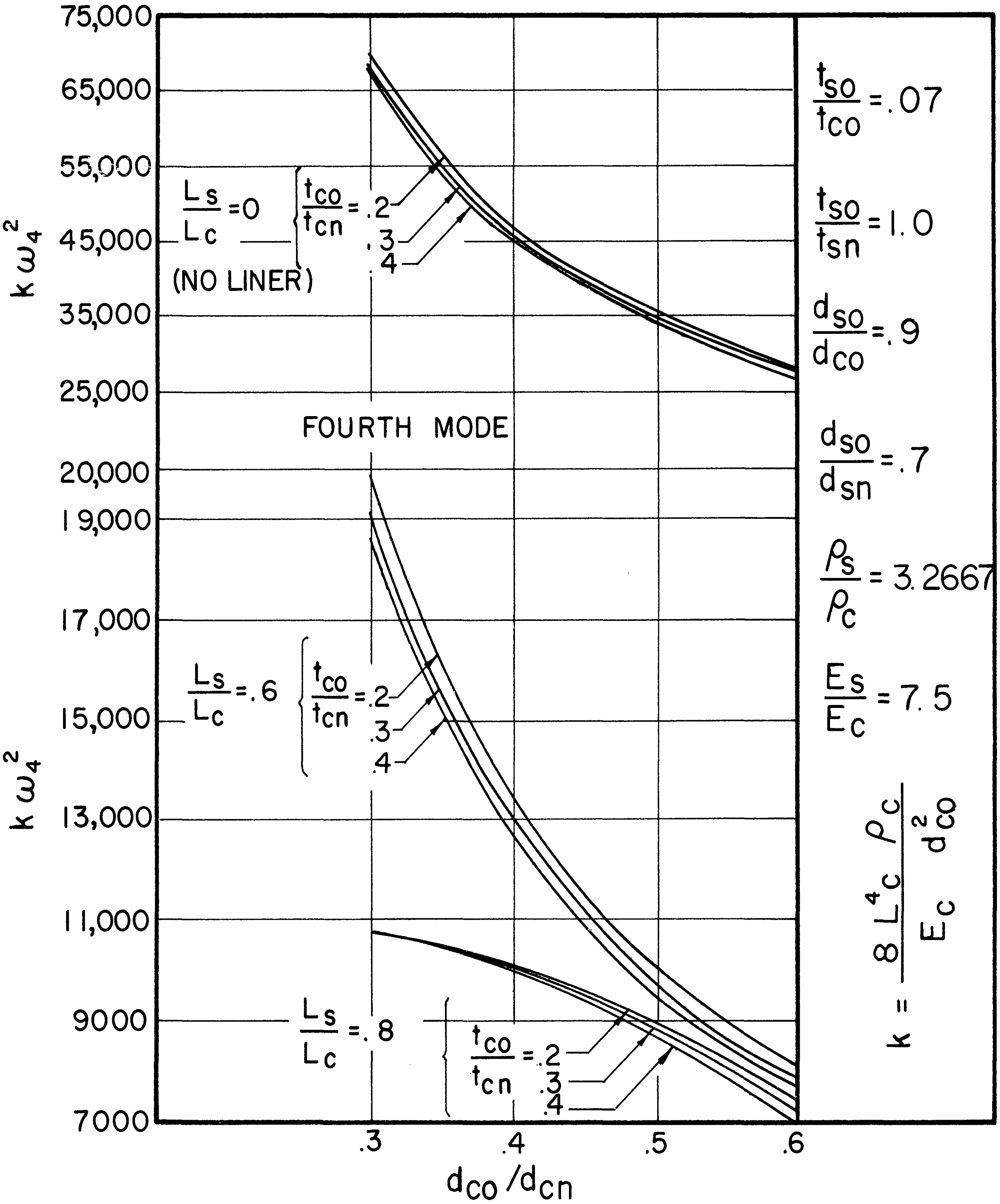

Figure 7. Fourth Mode Frequencies vs. d_{co}/d_{cn} Ratio Linearly Tapered Chimneys - Simply Supported Liner.

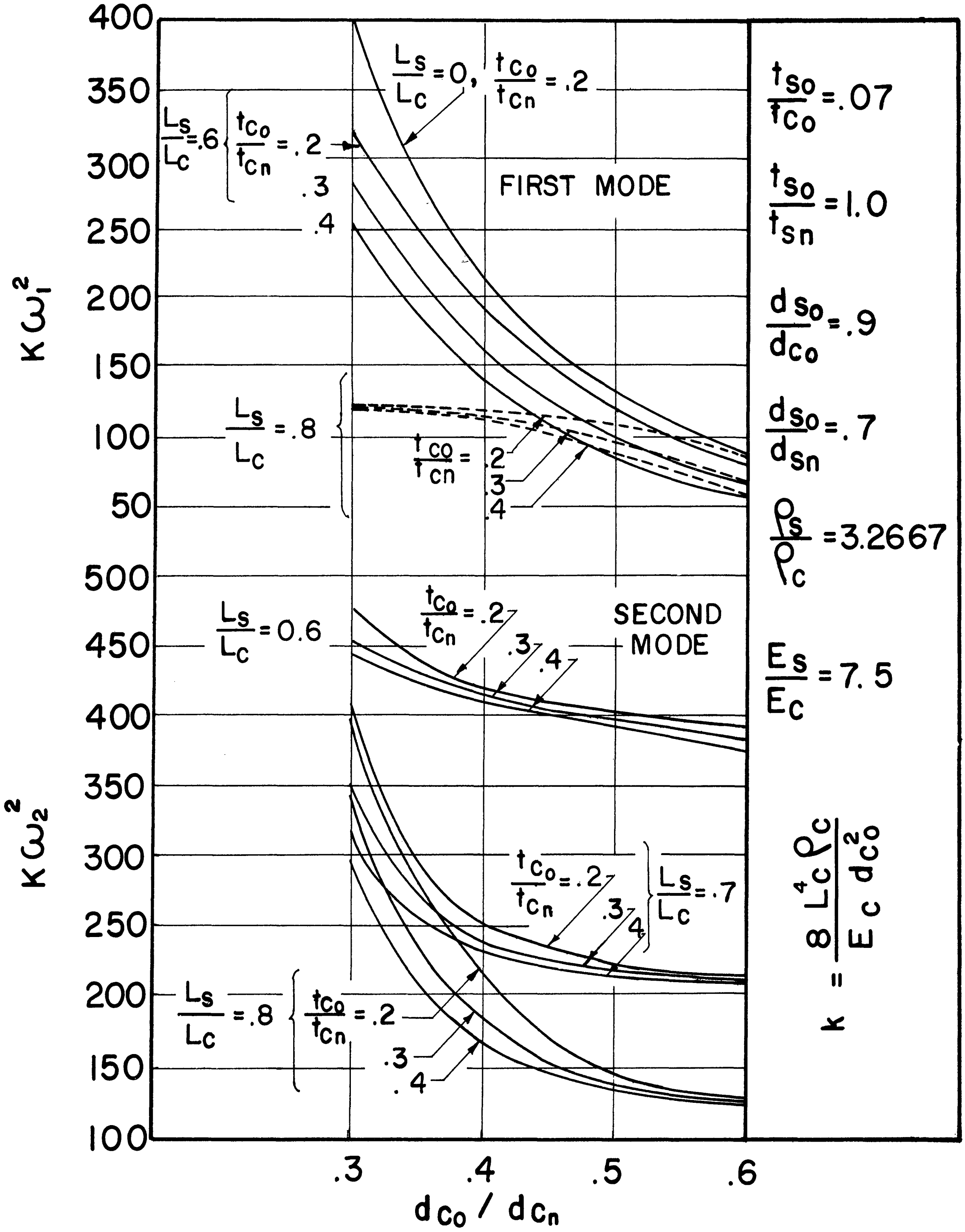

Figure 8. First and Second Mode Frequencies vs. d_{co}/d_{cn} Ratio Linearly Tapered Chimneys - Cantilever Liner.

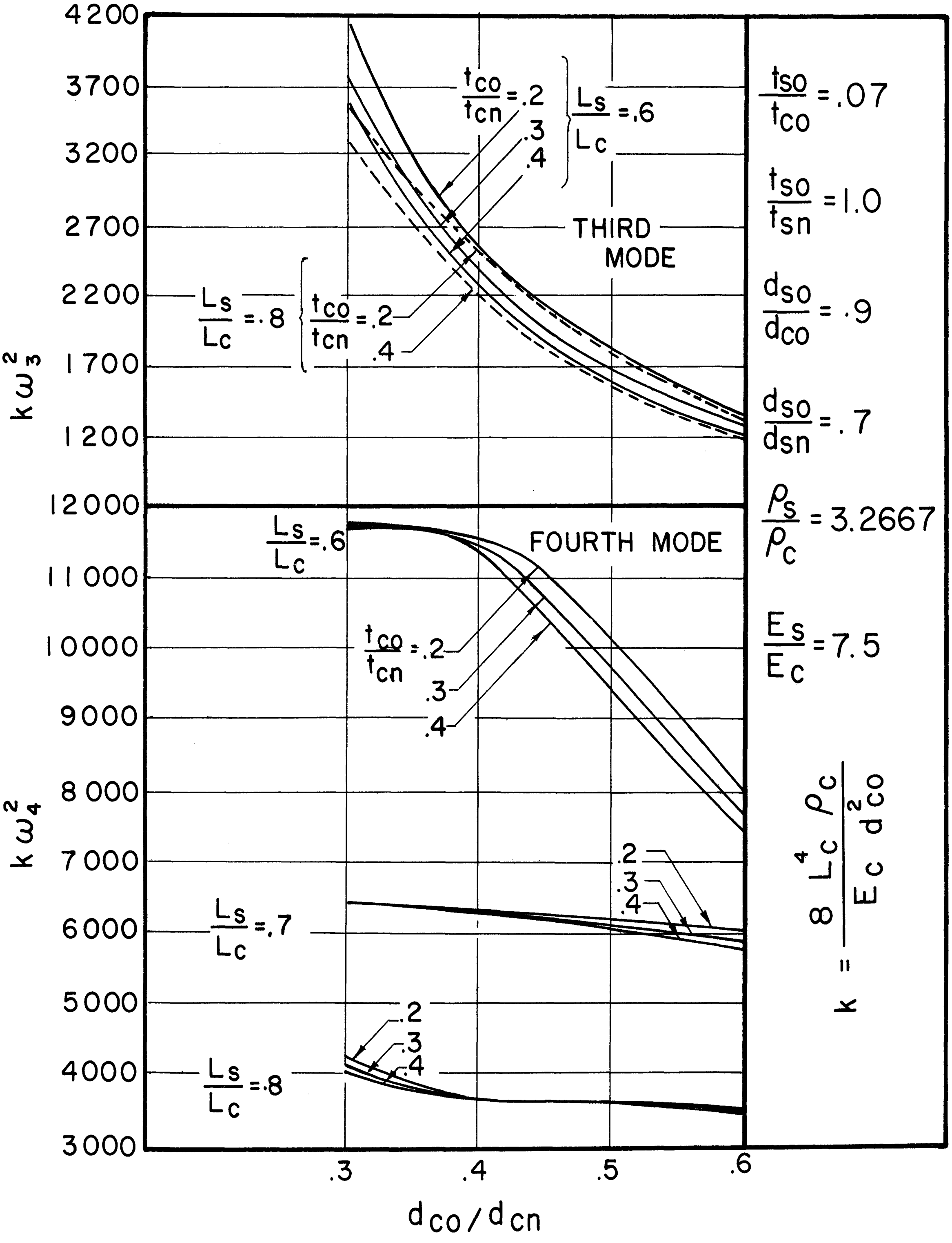

Figure 9. Third and Fourth Mode Frequencies vs. d_{co}/d_{cn} Ratio
Linearly Tapered Chimneys - Cantilever Liner.

Perhaps one of the most important parameters of this structure is the L_s/L_c ratio. To show the variations of the frequency with respect to this ratio, Figures 10, 11 and 12 are given. These figures are drawn for a uniform chimney with a uniform liner. The rather sudden jump in the curves of these figures is accompanied by a complete change in the shape of the modes as illustrated in Figures 13 and 14.

The curves for Case III (no liner) are obtained by making $\rho_s = 0$ or $L_s = 0$. These curves $(\frac{L_s}{L_c} = 0)$ agree with the results given by Housner and Keightley in their recent paper[3].

Example 2: A 622-ft. Concrete Chimney

An actual concrete chimney with a height of 622 ft. and with a 1/4 in. thick Corten steel liner is used in this example. The dimensions and other data for both the concrete shell and the steel liner are summarized in Table 1.

As in Example 1, the solution is obtained for three cases:

Case I - The chimney with a simply supported liner

Case II - The chimney with a cantilever liner

Case III - The chimney with no liner

The natural frequencies, the mode shapes and the bending moments associated with the mode shapes are given in Tables 2, 3, 4 and 5 for the first four fundamental modes of vibration and for the three cases listed above. If the mode shapes are expressed in feet, then the moments associated with these mode shapes will have the ft-kips units.

---

[3] "Vibrations of Linearly Tapered Cantilever Beams", by G. W. Housner and W. O. Keightley, Proceedings, A.S.C.E., Vol. 88, No. EM2, April, 1962.

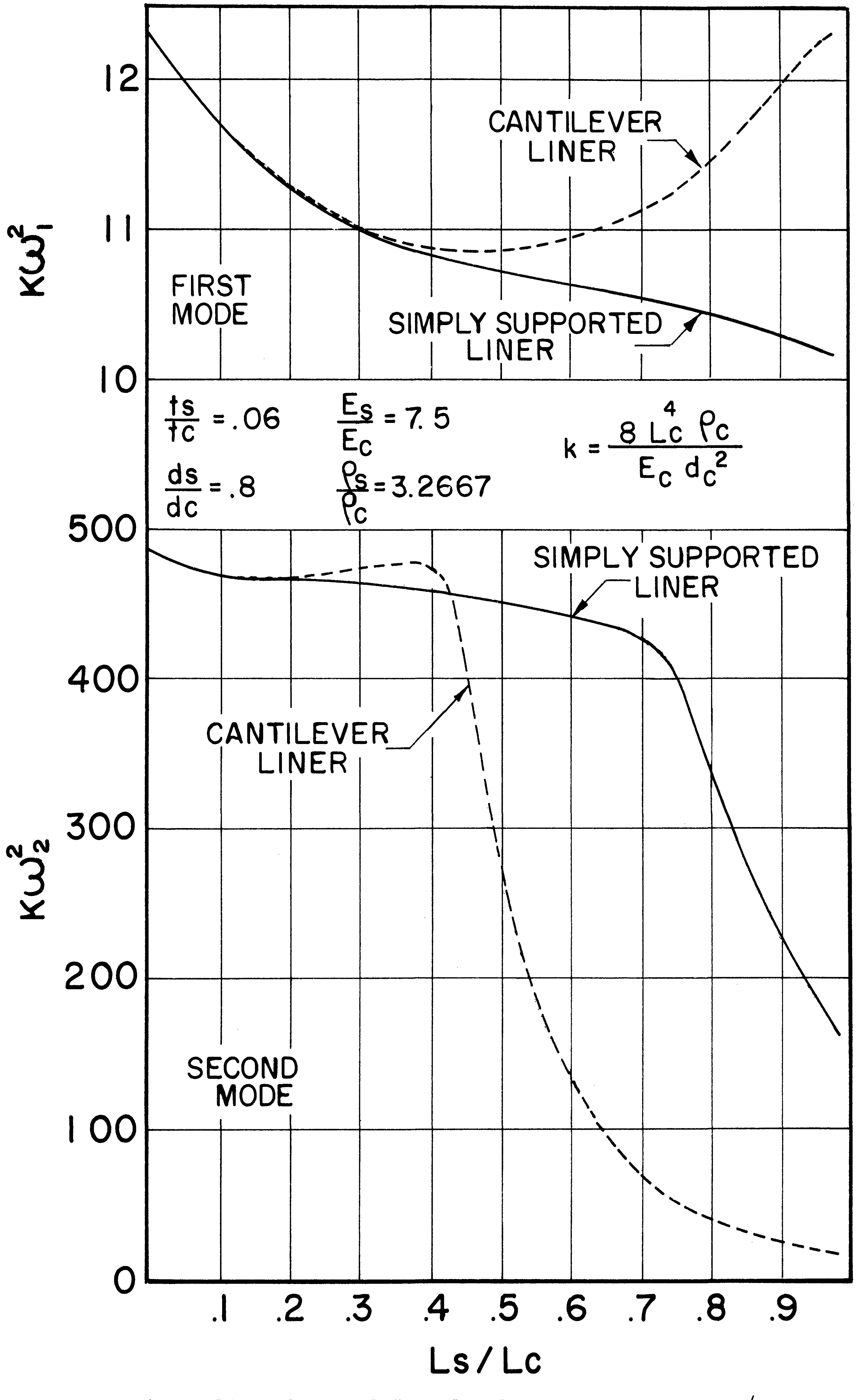

Figure 10. First and Second Mode Frequencies vs. Ls/Lc Ratio Uniform Chimneys.

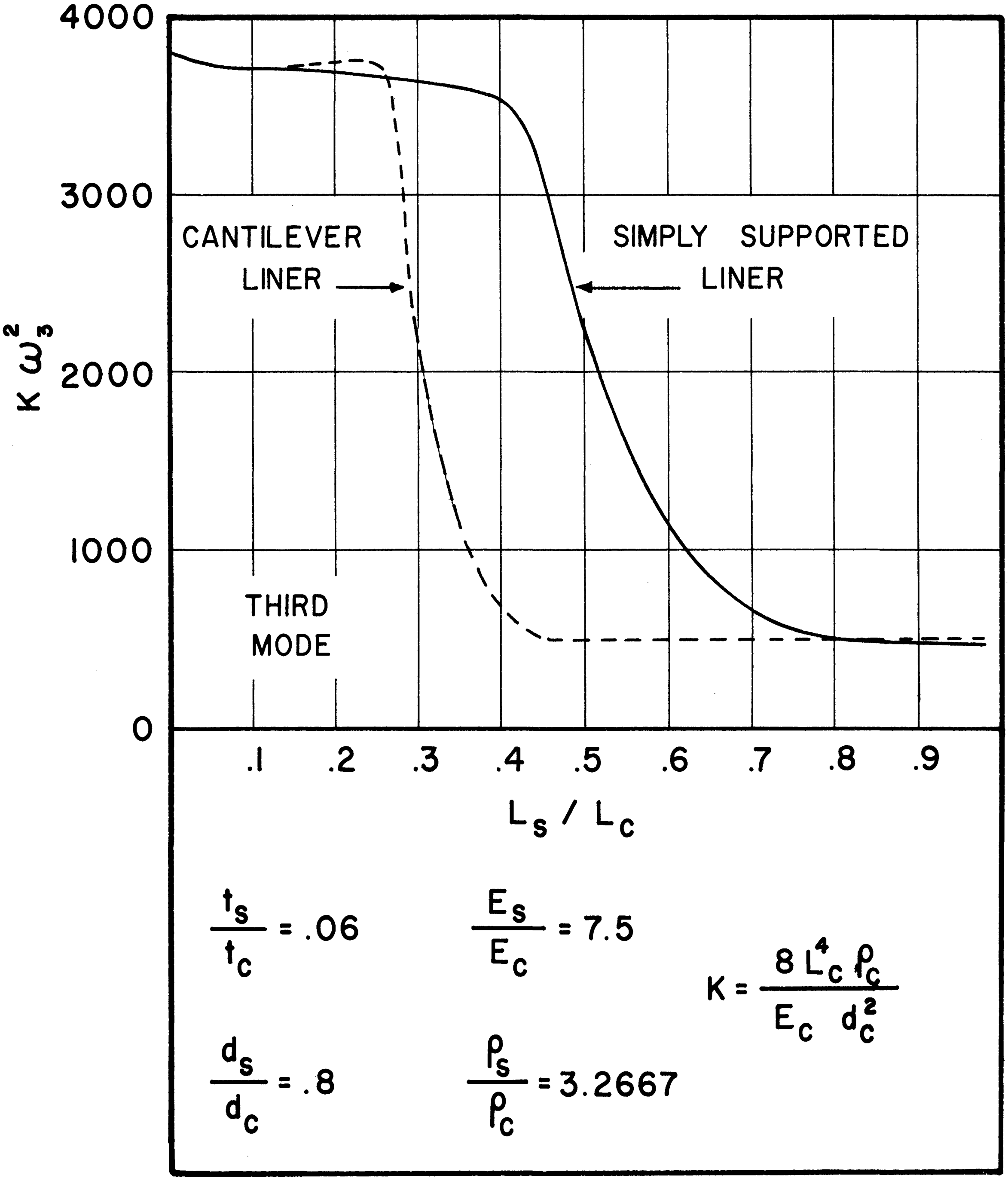

Figure 11. Third Mode Frequencies vs. Ls/Lc Ratio
Uniform Chimneys.

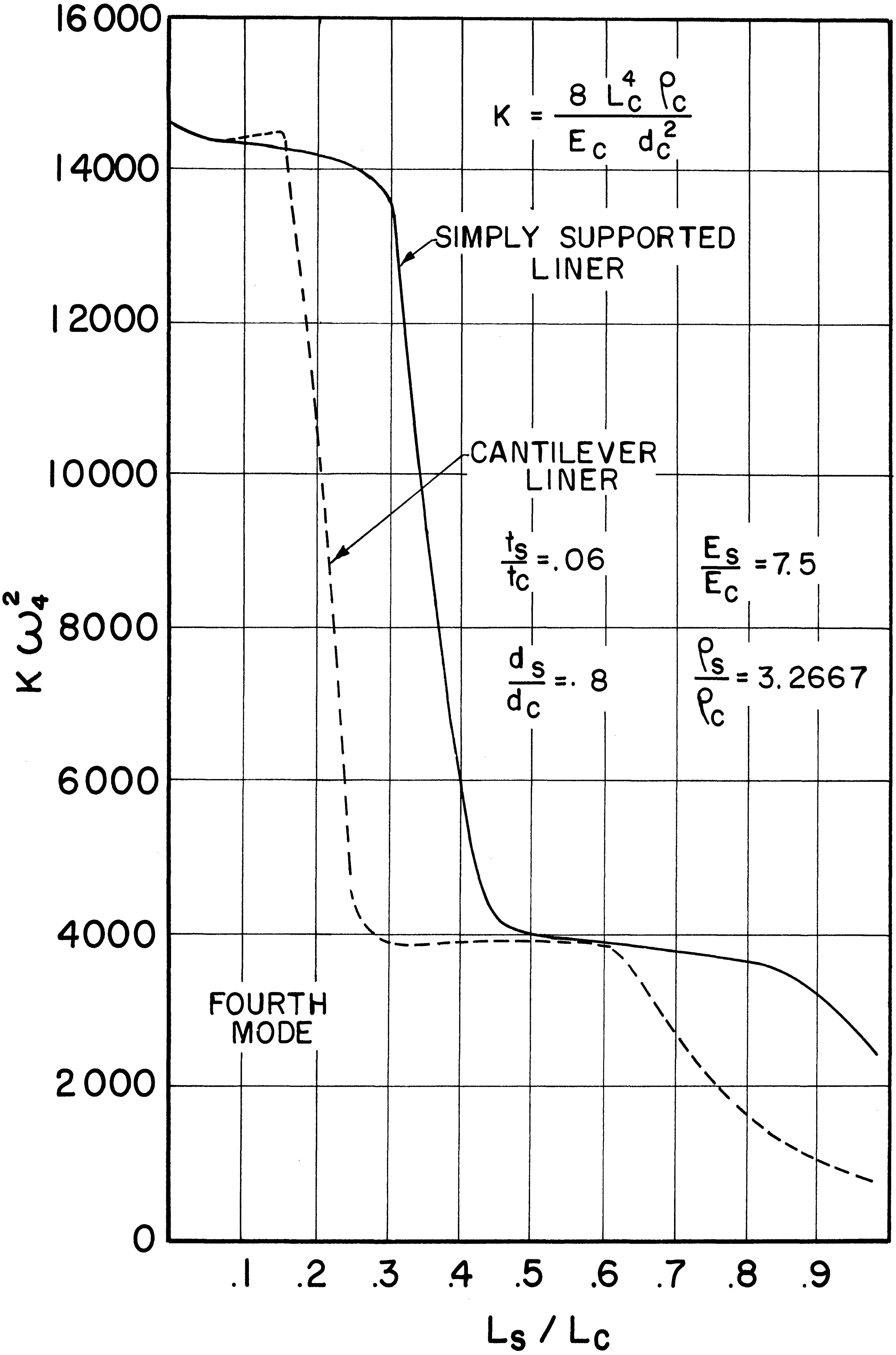

Figure 12. Fourth Mode Frequencies vs. Ls/Lc Ratio Uniform Chimneys.

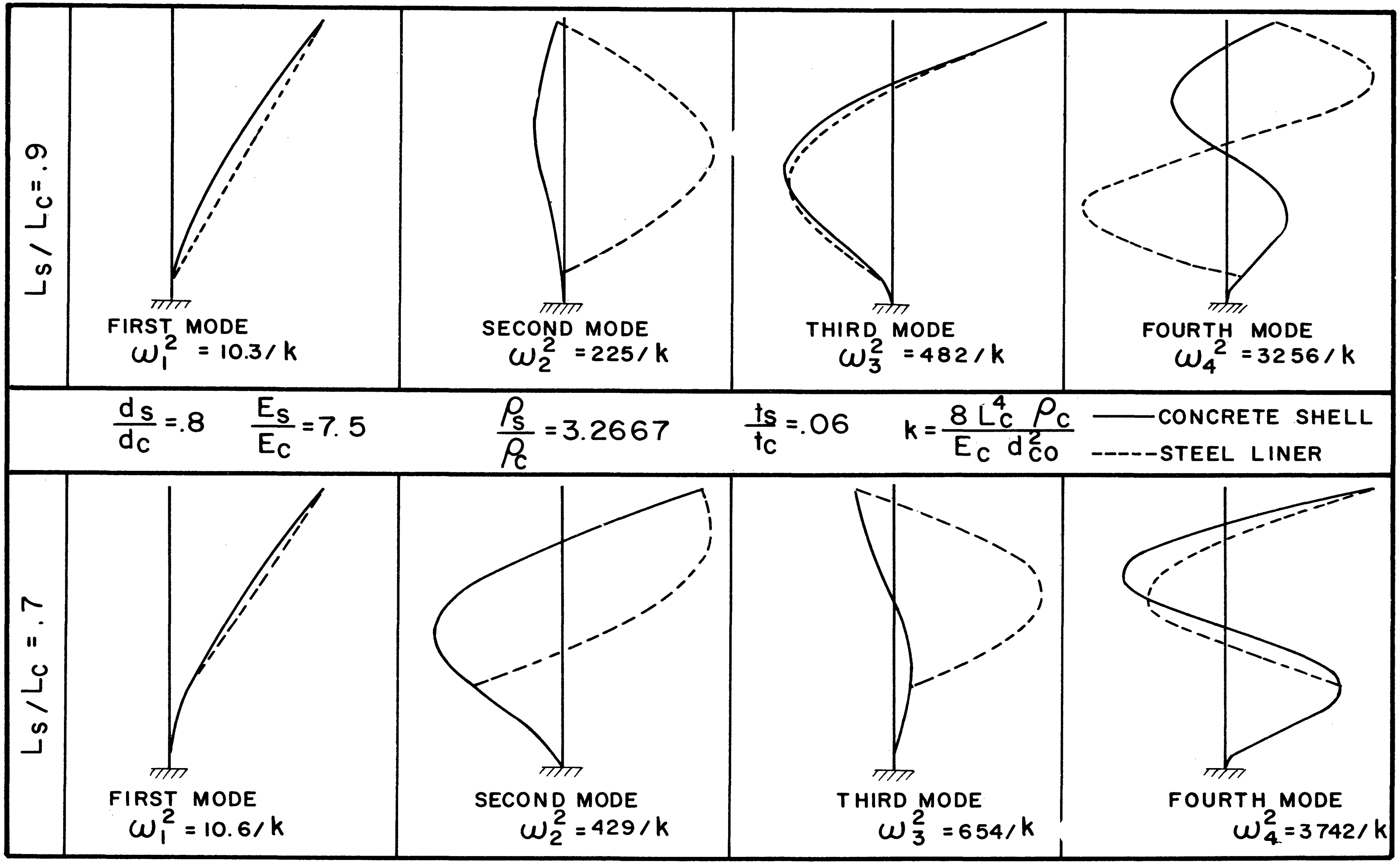

Figure 13. Mode Shapes vs. Ls/Lc Ratio
Uniform Chimneys - Simply Supported Liner.

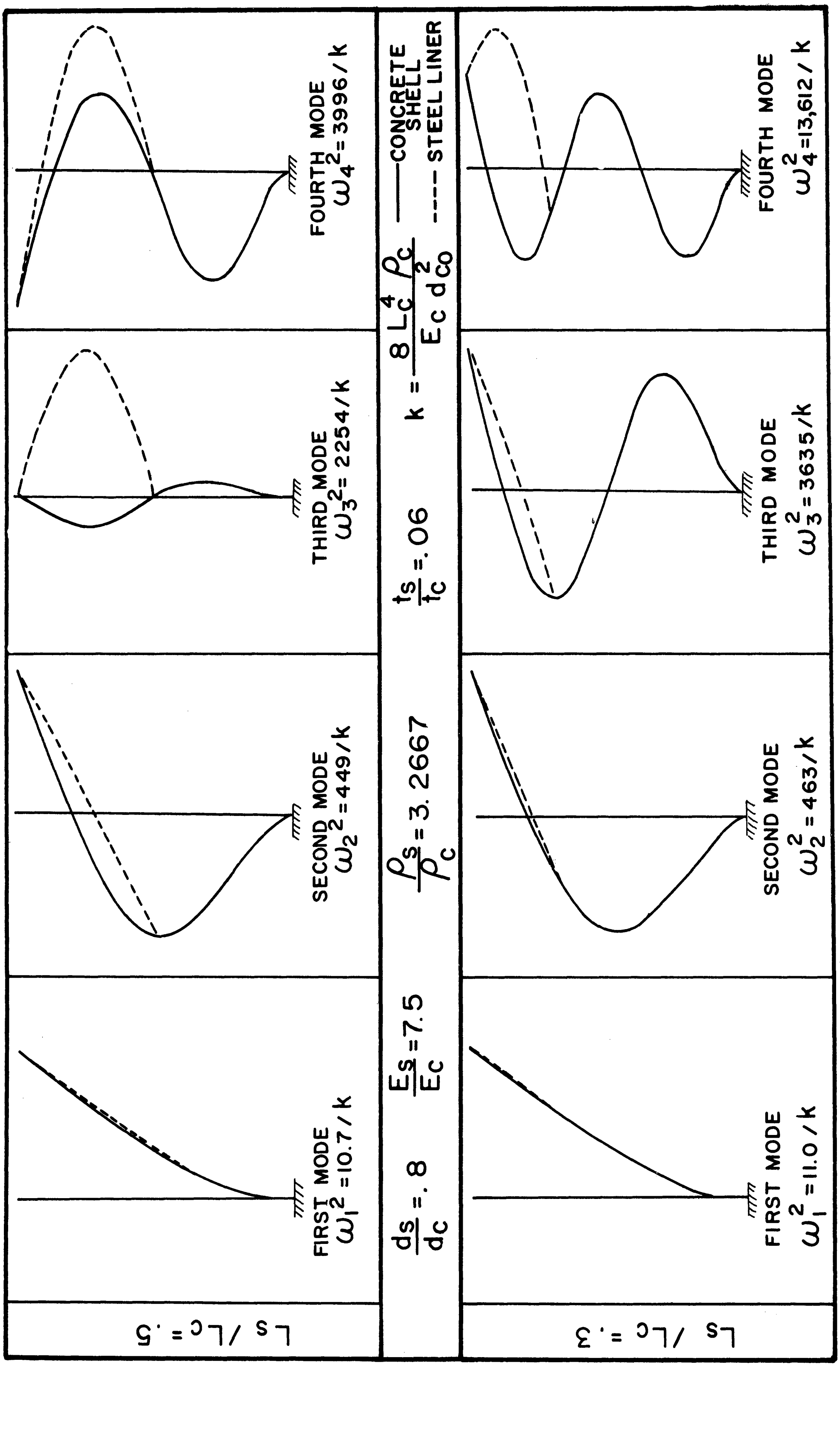

Figure 13. Continued.

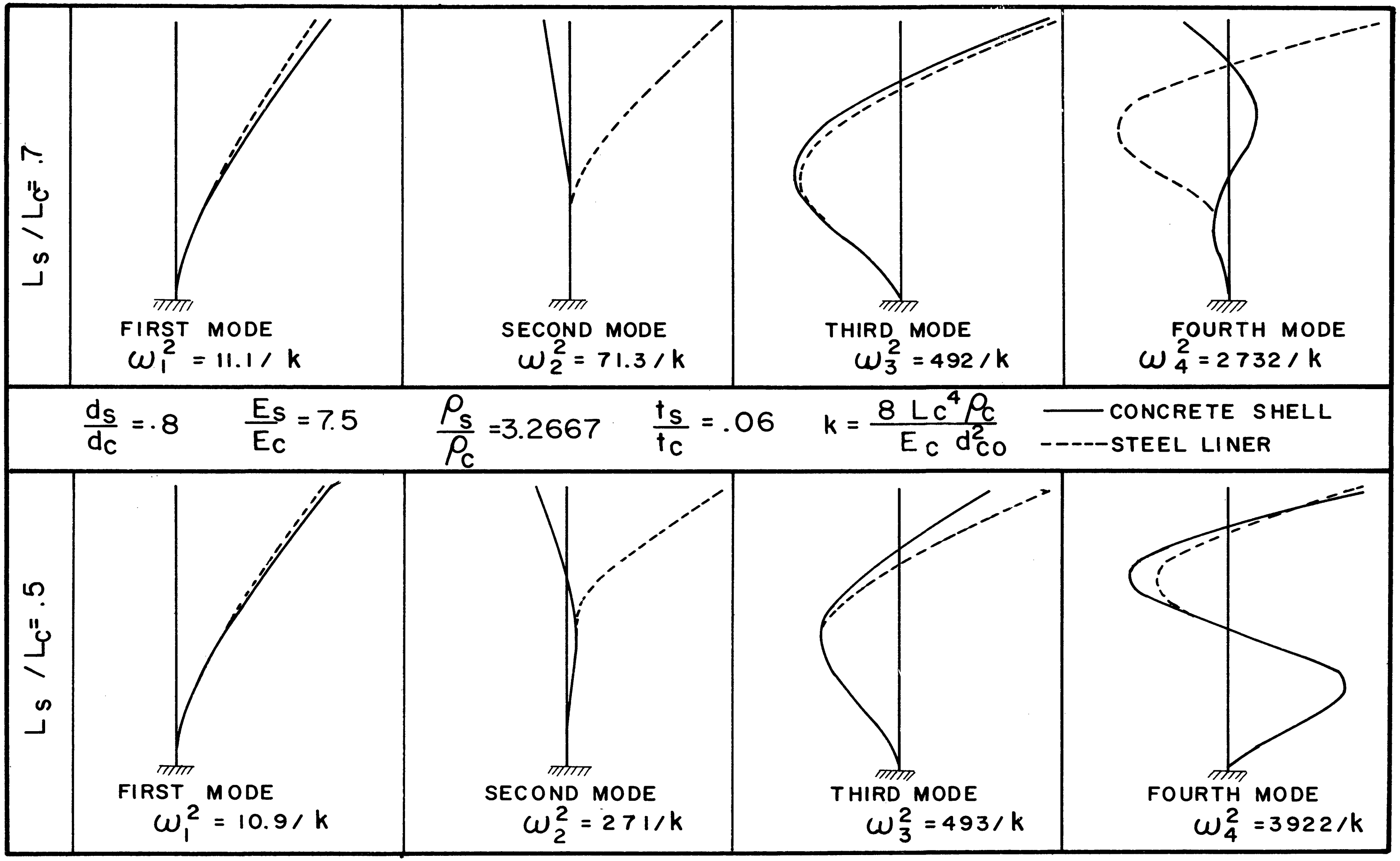

Figure 14. Mode Shapes vs. Ls/Lc Ratio
Uniform Chimneys - Cantilever Liner.

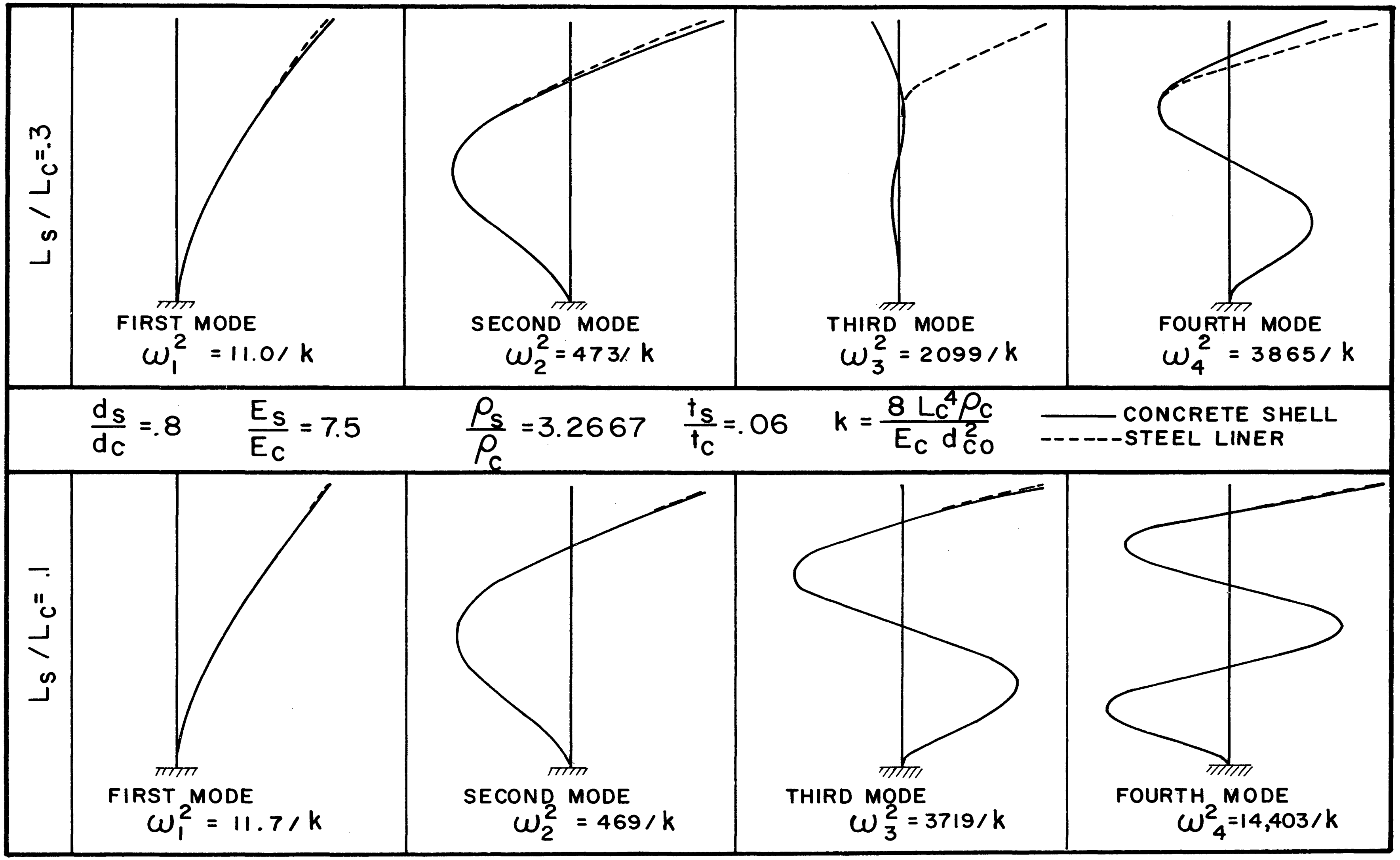

Figure 14. Continued.

TABLE 1 - DATA FOR THE CHIMNEY OF EXAMPLE 2

Concrete Shell			
Distance from top ft.	Outside Diameter ft.	Thickness ft.	Steel Liner
0	23.333	.5833	1/4" Corten Steel
38.46	24.813	.5833	Inside diam. (top) = 17.75 ft.
76.92	26.292	.5833	Inside diam. (bottom) = 26.0417 ft.
115.38	27.772	.5833	**Insulation**
153.85	29.252	.5900	2" Fiberglass insulation
192.31	30.731	.6434	wrapped around liner
230.77	32.211	.7121	
269.23	33.690	.7922	L_s = 500 ft.
307.69	35.170	.8723	L_c = 622 ft.
346.15	36.649	.9643	$E_s = 30 \times 10^6$ psi
384.62	38.129	1.0712	$E_c = 4 \times 10^6$ psi
423.08	39.608	1.6667	Weight of concrete = 150 psf
461.54	41.088	1.8333	Weight of steel = 490 psf
500.00	42.567	2.0000	Weight of insulation = 6 psf
540.67	44.132	1.5833	
581.33	45.696	2.5000	
622.00	47.260	2.0000	

TABLE 2 - FIRST MODE - 622' CHIMNEY WITH LINER - EXAMPLE 2

Distance from top (ft.)	Concrete Shell						Steel Liner			
	Y_c (ft.)			Moments (ft-k)			Y_s (ft.)		Moments (ft-k)	
	Case I	Case II	Case III	Case I	Case II	Case III	Case I	Case II	Case I	Case II
0	1.0000	1.0000	1.0000	0	0	0	1.0000	3.9800	0	0
38.46	.8826	.8924	.8872	2102	901	1035	.9649	3.5467	1117	430
76.92	.7669	.7855	.7753	5882	3540	4059	.9258	3.1154	2018	1678
115.38	.6552	.6810	.6662	11209	7802	8927	.8799	2.6898	2703	3669
153.85	.5496	.5809	.5620	17935	13554	15473	.8259	2.2754	3176	6323
192.31	.4524	.4871	.4650	25919	20668	23535	.7631	1.8786	3443	9558
230.77	.3649	.4009	.3767	35082	29082	33025	.6918	1.5057	3515	13288
269.23	.2876	.3233	.2981	45345	38735	43855	.6125	1.1631	3403	17428
307.69	.2207	.2546	.2297	56615	49548	55917	.5261	.8568	3124	21894
346.15	.1644	.1951	.1716	68774	61409	69068	.4337	.5922	2696	26609
384.62	.1183	.1447	.1238	81694	74197	83157	.3366	.3743	2144	31499
423.08	.0818	.1031	.0858	95263	87796	98046	.2360	.2072	1491	36505
461.54	.0524	.0679	.0550	109508	102267	113770	.1332	.0944	766	41578
500.00	.0294	.0387	.0308	124261	117411 164093	130123	.0294	.0387	0	46682
540.67	.0122	.0161	.0128	141017	185963	147828				
581.33	.0031	.0041	.0032	157910	208019	165704				
622.00	0	0	0	174855	230145	183644				

Case I - Simply Supported Liner ω = 2.494 radians/sec.

Case II - Cantilever Liner ω = 2.524 radians/sec.

Case III - No Liner ω = 2.708 radians/sec.

TABLE 3 - SECOND MODE - 622' CHIMNEY WITH LINER - EXAMPLE 2

Distance from top (ft.)	Concrete Shell: Y_c (ft.) Case I	Case II	Case III	Concrete Shell: Moments (ft-k) Case I	Case II	Case III	Steel Liner: Y_s (ft) Case I	Case II	Steel Liner: Moments (ft-k) Case I	Case II
0	1.0000	1.0000	1.0000	0	0	0	1.0000	-2.4746	0	0
38.46	1.3009	.8833	.6853	- 78772	1140	11874	- 2.7964	-2.1827	- 84609	- 339
76.92	1.5422	.7676	.3807	- 142784	4462	42681	- 6.2810	-1.8922	-165468	- 1315
115.38	1.6911	.6549	.1042	- 188269	9797	84672	- 9.2127	-1.6062	-237627	- 2864
153.85	1.7364	.5476	- .1253	- 212264	16950	129952	-11.4258	-1.3290	-296588	- 4916
192.31	1.6830	.4483	- .2946	- 212601	25732	171106	-12.8256	-1.0654	-338631	- 7399
230.77	1.5506	.3585	- .4007	- 186806	36032	201121	-13.3810	- .8204	-361063	- 10238
269.23	1.3633	.2792	- .4480	- 133140	47737	213606	-13.1156	- .5987	-362389	- 13360
307.69	1.1449	.2110	- .4458	- 51038	60713	203359	-12.0972	- .4048	-342393	- 16696
346.15	.9167	.1540	- .4060	58512	74793	167118	-10.4265	- .2427	-302147	- 20180
384.62	.6969	.1081	- .3411	193350	89801	103650	- 8.2268	- .1161	-243946	- 23757
423.08	.4991	.0728	- .2636	350954	105579	13044	- 5.6336	- .0277	-171184	- 27380
461.54	.3269	.0453	- .1847	533141	122139	-109190	- 2.7850	.0200	- 88184	- 31016
500.00	.1859	.0250	- .1122	734220	139270 104626	-257366	.1859	.0250	0	- 34644
540.67	.0782	.0104	- .0507	865453	119282	-432551				
581.33	.0200	.0026	- .0137	1001924	134089	-615904				
622.00	0	0	0	1140405	148953	-802580				

Case I - Simply Supported Liner $\omega = 6.095$ radians/sec.

Case II - Cantilever Liner $\omega = 2.843$ radians/sec.

Case III - No Liner $\omega = 9.519$ radians/sec.

TABLE 4 - THIRD MODE - 622' CHIMNEY WITH LINER - EXAMPLE 2

	Concrete Shell						Steel Liner			
	Y_c (ft.)			Moments (ft-k)			Y_s (ft.)		Moments (ft-k)	
Distance from top (ft.)	Case I	Case II	Case III	Case I	Case II	Case III	Case I	Case II	Case I	Case II
0	1.0000	1.0000	1.0000	0	0	0	1.000	.5800	0	0
38.46	.6858	.6863	.4684	11928	11768	61283	.7098	.4068	- 1228	825
76.92	.3817	.3826	- .0122	42570	42320	195747	.4249	.2370	- 4748	2969
115.38	.1057	.1069	- .3659	84289	84008	329278	.1562	.0774	- 9691	5909
153.85	- .1236	- .1223	- .5400	129302	129037	400659	- .0830	- .0643	- 15179	9120
192.31	- .2930	- .2917	- .5300	170294	170076	372937	- .2804	- .1810	- 20378	12105
230.77	- .3995	- .3982	- .3826	200343	200191	234168	- .4270	- .2675	- 24551	14425
269.23	- .4474	- .4462	- .1663	213128	213046	2711	- .5178	- .3212	- 27109	15729
307.69	- .4460	- .4449	.0519	203488	203469	- 275241	- .5523	- .3423	- 27646	15775
346.15	- .4068	- .4059	.2209	168163	168193	- 535841	- .5342	- .3334	- 25962	14440
384.62	- .3425	- .3417	.3116	105887	105949	- 713445	- .4707	- .2992	- 22066	11719
423.08	- .2653	- .2646	.3198	16694	16773	- 751194	- .3717	- .2461	- 16167	7720
461.54	- .1864	- .1859	.2729	- 103943	- 103858	- 572758	- .2486	- .1815	- 8642	2638
500.00	- .1136	- .1131	.1925	- 250441	- 250344 - 253607	- 179967	- .1136	- .1131	0	- 3263
540.67	- .0515	- .0512	.0997	- 433374	- 433556	411886				
581.33	- .0139	- .0138	.0295	- 624503	- 621682	1092431				
622.00	0	0	0	- 818975	- 813138	1812788				

Case I - Simply Supported Liner $\omega = 9.460$ radians/sec.

Case II - Cantilever Liner $\omega = 9.474$ radians/sec.

Case III - No Liner $\omega = 22.619$ radians/sec.

Table 5 - FOURTH MODE - 622' CHIMNEY WITH LINER - EXAMPLE 2

Distance from top (ft.)	Concrete Shell Y_c (ft.) Case I	Case II	Case III	Concrete Shell Moments (ft-k) Case I	Case II	Case III	Steel Liner Y_s (ft) Case I	Case II	Steel Liner Moments (ft-k) Case I	Case II
0	1.0000	1.0000	1.0000	0	0	0	1.0000	-13.1118	0	0
38.46	.3601	.5912	.2474	113132	25801	199432	1.3229	- 8.1722	50299	- 41236
76.92	- .1902	.2042	- .3438	278232	88428	543246	1.4660	- 3.4019	78361	-142728
115.38	- .5607	- .1249	- .5899	415934	164692	694655	1.3518	.8847	81002	-270367
153.85	- .7043	- .3635	- .4621	465752	232515	489129	.9958	4.3542	59416	-391364
192.31	- .6334	- .4947	- .1084	396693	273416	- 16627	.4781	6.7550	19384	-477309
230.77	- .4158	- .5251	.2467	205703	272403	- 600366	- .8831	7.9723	- 29807	-506893
269.23	- .1370	- .4761	.4373	- 77764	219863	- 967537	- .5889	8.0440	- 77283	-467746
307.69	.1256	- .3762	.4138	- 396224	113086	- 892767	- .9319	7.1493	-112816	-357032
346.15	.3169	- .2551	.2299	- 676617	- 42998	- 338477	- 1.0631	5.5786	-128870	-180718
384.62	.4094	- .1400	- .4309	- 847618	-237646	521801	- .9718	3.6932	-122054	48392
423.08	.4034	- .0520	- .1823	- 850015	-457238	1364232	- .6859	1.8854	- 93749	313642
461.54	.3351	.0030	- .2648	- 601953	-690920	1755998	- .2616	.5418	- 49888	598171
500.00	.2315	.0175	- .2473	- 111942	-924506 - 35547	1403311	.2315	.0175	0	888958
540.67	.1177	.0093	- .1565	549605	31221	228421				
581.33	.0345	.0028	- .0518	1307643	101280	- 1437443				
622.00	0	0	0	2108492		- 3352439				

Case I - Simply Supported Liner $\omega = 21.675$ radians/sec.

Case II - Cantilever Liner $\omega = 14.295$ radians/sec.

Case III - No Liner $\omega = 43.098$ radians/sec.

It is interesting to note that the first mode frequency is almost the same for the three cases, while the second mode frequency differs from one case to another. Note also that the third mode frequency for Cases I and II are very close to the second mode frequency of Case III and that the fourth mode frequency of Case I is close to the third mode frequency of Case III. As in the case of frequencies also notice the similarities between the mode shapes in as far as the concrete shell is concerned.

A study of the mode shapes for Case II indicates that the steel liner will have appreciable movement at the top. In fact, the movement of the steel liner at the top is 3.98 times the movement of the concrete shell if the chimney is vibrating in the first mode. If we arbitrarily assign a first mode top displacement for the concrete shell of 0.85 feet, which could be caused by earthquake or wind forces, then the movement of the liner at the top will be equal to (0.85) (3.98) = 3.38 ft. The relative movement of 2.53 ft. between the concrete shell and the liner is excessive and perhaps prohibitive.

In the case of the simple supported liner (case I), we note excessive movements in the liner at about mid-height due to the second mode. If we arbitrarily assume a top displacement of 0.15 ft. at the top of the concrete shell when vibrating in the second mode we get a movement in the liner of (.15) (13.38) = 2.0 ft. at about mid-height, and the relative movement between the liner and the concrete shell will be about 2.23 ft.

It will be instructive to obtain approximate values for the maximum stresses in the steel liner. This can be roughly done by assuming a reasonable top displacement of the concrete shell due to seismic forces. Let us assume a one foot displacement at the top of the concrete shell and consider that the first mode contribution to this movement is .85 ft. and the second mode contribution is .15 ft. and that the effect of the higher modes is negligible.

Based on the above assumptions and using the information in Tables 1 through 3, we can compute stresses in the liner. By ignoring the normal forces and considering only the stresses due to the bending moments, we find that the maximum stress in the liner for Case I is about 28,000 psi which is mostly contributed by the second mode and the maximum stress in the liner for Case II is about 18,000 psi contributed mostly by the first mode.

The above rough computations are intended to merely give an idea of maximum stresses in the liner of this example and should not be used for any other purpose. More research should be done on this type of a problem to determine the response to earthquake and wind forces. It should be mentioned here that once the modes are determined then the response can be computed.

SUMMARY AND CONCLUSIONS

An important element in the design of tall reinforced concrete chimneys is their response to earthquake and wind forces. The response can be computed if the fundamental modes of vibration are determined. For example, the response to earthquakes can be approximately obtained by using average velocity spectrum curves[4] or it can be obtained for any specific earthquake by finding the maximum response of the structure to the earthquake considering any number of modes desired. (In most practical problems the three fundamental modes are sufficient.) The study in this paper dealt primarily with the first part of the response problem, namely, the determination of the modes of vibration. The Stodola Method as used in this paper will not only give the frequencies but also the mode shapes and the shears and moments associated with each mode shape.

The effect of the addition of an independent steel liner to a concrete chimney has been illustrated by some curves presented in this paper. The results seem to indicate that for each mode of the unlined chimney there is a mode of near equal frequency for the same chimney with steel liner and that the mode shape of the concrete shell is nearly the same for both cases. The lined chimney, however, could have additional frequencies spaced between those frequencies described above, depending primarily on the L_s/L_c ratio. This ratio was found to be one of the major parameters of the problem.

One of the things that should be emphasized is that the second mode (for instance) of the lined chimney can have a completely different

---

4 "Behavior of Structures During Earthquakes," by G. W. Housner, Proceedings, ASCE, Vol. 85 No. EM4, October, 1959.

configuration and physical significance than the second mode of the same unlined chimney and that the major parameter effecting the configuration is perhaps the L_s/L_c ratio.

The additional frequencies and the different mode shapes that are obtained as a result of using the steel liner should have a marked effect on the response of steel lined chimneys to earthquake and wind forces.

No final conclusions can be reached about the displacements or the stresses in the liner due to earthquake or wind forces until more work has been done on this part of the problem. However, some rough preliminary work indicates that excessive movements and stresses in the liner can take place in both cases (simply supported liner or cantilever liner). These excessive movements and stresses can perhaps be reduced by using a liner fixed at the bottom and supported at the top or a liner supported at more than two points. Further study of this subject is warranted.

APPENDIX

NOTATION

The following symbols have been adopted for use in this paper.

d	=	mean diameter
E	=	modulus of elasticity
L	=	length
I	=	moment of inertia
M	=	bending moment
m	=	mass intensity per unit length
t	=	time or thickness
x	=	distance along chimney or liner
Y	=	mode shape
z	=	displacements in the chimney or liner
ρ	=	mass density
ω	=	natural undamped frequency in radians/second

Subscripts:

First Subscripts

C designates the concrete shell
S designates the steel liner

Second Subscripts

0 designates the top of concrete shell or liner
n designates the bottom of concrete shell or liner