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RESPONSE OF REINFORCED CONCRETE CHIMNEYS TO EARTHQUAKES

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RESPONSE OF REINFORCED CONCRETE CHIMNEYS
TO EARTHQUAKES

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SYNOPSIS

The response of eight actual reinforced concrete chimneys to accelerograms of three actual earthquakes are computed. This response includes the determination of the maximum bending moments and the maximum shears at different sections of the chimney. The ratios of maximum base shear to total weight are plotted against the first mode periods for the eight chimneys. For every chimney, the average of the maximum base shear due to the three earthquakes is distributed along the height according to three arbitrary rules. The forces obtained from any one rule are then used to compute bending moments at the different sections of the chimney. These bending moments are then compared to the average of the maximum bending moments due to the three earthquakes. The three rules used in this study for the distribution of the base shear are: intensity of lateral force at any level is proportional to the product of the weight intensity at that level times (1) distance of that level from the base, (2) square of the distance of that level from the base, and (3) cube of the distance of that level from the base.

All numerical computations were made on the IBM 7090 computer at the University of Michigan.

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INTRODUCTION*

The ever-increasing demand for air pollution control in the last decade or two has led to the construction of tall reinforced concrete chimneys. Many chimneys that are 800 ft. in height or higher have already been built or are to be built in the near future. With the increase of the height of concrete chimneys, their response to lateral forces such as wind and earthquakes becomes more and more important. The purpose of this paper is to furnish numerous results of response of chimneys to actual earthquakes and to compare these results with values obtained using certain simplified rules that can be adapted to design office procedures.

The study is based on an elastic response and on a damping coefficient of .05 of critical damping.

*Notation: The letter symbols adopted for use in this paper are defined where they first appear and are arranged alphabetically in Appendix 1.

METHOD OF SOLUTION

The response of a chimney to earthquake forces is computed by the modal analysis techniques. This requires that the natural modes of vibration be obtained and then the response of the chimney in the different modes be computed. The total response of the chimney is then obtained by the instantaneous combination of the responses in the different modes.

Modes of Vibration

The basic differential equation for the free vibration of a chimney (ignoring shear deformations and rotary inertia effects) with zero damping is given by:

$$\frac{\partial^2}{\partial x^2}\left(EI\,\frac{\partial^2 y}{\partial x^2}\right) + m\,\frac{\partial^2 y}{\partial t^2} = 0 \qquad (1)$$

in which y(x,t) are the displacements in the chimney, m(x) represents the mass per unit length, x refers to the distance along the chimney, and t is the time.

The method of separation of variables will give the following frequency equation:

$$\frac{d^2}{dx^2}\left(EI\,\frac{d^2\phi}{dx^2}\right) = m\,\omega^2\,\phi \qquad (2)$$

where ϕ (x) is a mode shape and ω is the frequency.

Equation 2 can only be satisfied for certain values of ω and ϕ which are the natural frequencies of the chimney and the mode shapes respectively. The solution of Equation 2, however, can only be achieved numerically because of the variations of the moment of inertias of the chimney. The numerical solution used in this paper is the well-known Stodola process. This process has been briefly outlined by the author in another paper[2] and will be presented in this paper under the computer solution.

Response of Chimney to Earthquakes

If a chimney is subjected to a base acceleration, a, then the basic differential equation with zero damping is:

$$m \left(\frac{\partial^2 y}{\partial t^2} + a\right) + \frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 y}{\partial x^2}\right) = 0 \tag{3}$$

or

$$m \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 y}{\partial x^2}\right) = -\, m\, a \tag{4}$$

where a is acceleration of the earthquake, and the other symbols are the same as in Equation 1.

Let

$$y(x,t) = \sum_{i=1}^{\infty} \phi_i (x) \cdot q_i (t) \tag{5}$$

where ϕ_i is the shape of the i^{th} mode (dimensionless) and q_i is the displacement in the i^{th} mode.

Substitution of Equation 5 into Equation 4 yields

$$\sum_{i=1}^{\infty} m\phi_i \ddot{q}_i + \sum_{i=1}^{\infty} \frac{\partial^2}{\partial x^2} (EI\, \phi_i'' \, q_i) = -\, ma \tag{6}$$

where $\ddot{q} = \frac{d^2 q}{dt^2}$ and $\phi'' = \frac{d^2 \phi}{dx^2}$

Multiplying Equation 6 by ϕ_j and integrating along the full height of the chimney yields

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2 "Vibrations of Steel-Lined Concrete Chimneys", by W.S. Rumman, Journal of the Structural Division, ASCE, Vol. 89, No. ST5, Proc. Paper 3661, October, 1963, pp. 35-63.

$$\sum_{i=1}^{\infty} \ddot{q}_i \int_0^L m\phi_i \phi_j dx + \sum_{i=1}^{\infty} q_i \int_0^L \phi_j \frac{d^2}{dx^2} (EI\ \phi_i'')\ dx = -a\int_0^L m\phi_j dx \quad (7)$$

The following two important orthogonality relationships will be used

$$\int_0^L m\ \phi_i\ \phi_j\ dx = 0 \qquad i \neq j \qquad (8)$$

$$\int_0^L EI\ \phi_i''\ \phi_j''\ dx = 0 \qquad i \neq j \qquad (9)$$

It can be shown (see Appendix II) that

$$\int_0^L \phi_j \frac{d^2}{dx^2} (EI\ \phi''_i)\ dx = \int_0^L EI\ \phi''_i\ \phi_j''\ dx \qquad (10)$$

and

$$\int_0^L \phi_j \frac{d^2}{dx^2} (EI\ \phi_j'')\ dx = \omega_j^2 \int_0^L m\ \phi_j^2\ dx \qquad (11)$$

Using the relationships of Equations 8, 9 and 10, it is seen that all the terms on the left hand side of Equation 7 will vanish except the j^{th} term (that is when i = j) and thus Equation 7 yields

$$\ddot{q}_j \int_0^L m\ \phi_j^2\ dx + q_j \int_0^L \phi_j \frac{d^2}{dx^2} (EI\ \phi_j'')\ dx = -a\int_0^L m\phi_j dx \qquad (12)$$

The use of Equation 11 will then reduce Equation 12 to the following form:

$$\ddot{q}_j \int_0^L m\ \phi_j^2\ dx + \omega_j^2\ q_j \int_0^L m\ \phi_j^2\ dx = -a \int_0^L m\ \phi_j\ dx \qquad (13)$$

or

$$\ddot{q}_j + \omega_j^2\ q_j = \frac{-a \int_0^L m\ \phi_j\ dx}{\int_0^L m\ \phi_j^2\ dx} \qquad (14)$$

Equation 14 is the differential equation of motion, in any mode j, of the undamped chimney when subjected to the earthquake acceleration a. The damping can be introduced by writing Equation 14 in the form:

$$\ddot{q}_j + 2\beta\omega_j\dot{q}_j + \omega_j^2 q_j = \frac{-a \int_0^L m \phi_j dx}{\int_0^L m \phi_j^2 dx} \tag{15}$$

where β is the fraction of critical damping and ω_j is the frequency of the chimney in the j^{th} mode.

COMPUTER SOLUTION

Determination of Modes

The procedure for finding the first mode using the Stodola methods is as follows:

1. Assume any deflected shape for the chimney. The specific shape assumed is a zero deflection at the bottom and unity at all the other stations.
2. Compute the values of the intensity of the dynamic loading $m \omega^2 \phi$ with ω considered equal to 1.
3. Find the values of the bending moments M at the different stations assuming a second degree variation in the loading, $m \phi$. Compute the values of $\frac{M}{EI}$.
4. Assuming a second degree variation in the values of $\frac{M}{EI}$ and using Newmark's Numerical Procedure[3], calculate the displacements at all

3 "Numerical Procedure for Computing Deflections,Moments, and Buckling Loads," by N.M. Newmard, Transactions, ASCE, Vol. 108, 1943.

stations of the chimney beginning with zero displacement and zero slope at the bottom.

5. Normalize the computed shape by making the maximum displacement equal to +1.

6. Compare the normalized shape of Step 5 with the assumed shape of Step 1. The comparison is made on the basis of the difference between the absolute values of the two shapes at all the stations.

   (i) If this difference does not exceed 0.000001 at any location, then the computed shape is considered equal to the assumed shape and the square of the frequency is obtained by finding the ratio between the maximum value of the assumed shape and the corresponding value of the computed shape of Step 4.

$$\omega^2 = \frac{\text{Maximum value of assumed } \phi}{\text{Corresponding value of computed } \phi}$$

   Note that the maximum value of assumed ϕ is always equal to +1.

   (ii) If the difference exceeds 0.000001 then the process is repeated by assuming the normalized shape of Step 5 as the deflected shape and repeating the process beginning with Step 2. The process is thus repeated until convergence is obtained.

A higher mode frequency is obtained as follows:

a. Assume any shape ; in this case, the same shape as in Step 1 is asaumed.

b. Complete Steps 2, 3 and 4 as for the first mode.

c. Purify the computed shape from the lower modes according to the following equation[1]

$$\phi_p = \phi - \frac{\int m\phi\ \phi_1\ dx}{\int m\ \phi_1^2\ dx}\phi_1 - \frac{\int m\phi\ \phi_2\ dx}{\int m\ \phi_2^2\ dx}\phi_2 \cdots\cdots\cdots$$

$$- \frac{\int m\phi\ \phi_{(i-1)}\ dx}{\int m\ \phi_{(i-1)}^2\ dx}\phi_{(i-1)}$$

where ϕ is the computed shape

and ϕ_p is the purified shape.

d. Normalize (c) in the same manner as in the first mode and compare the normalized shape with the assumed shape. The comparison test is identical to that of the first mode.

I. If the comparison test is not satisfied, then repeat the process beginning with (b) and using as the assumed shape the normalized shape of Step (d).

II. If the comparison test is satisfied, the process is stopped and the square of the frequency is obtained by finding the ratio between the maximum value of the assumed shape and the corresponding value of the computed-purified shape of Step (c).

Therefore for the higher mode

$$\omega^2 = \frac{\text{Maximum value of Assumed } \phi}{\text{Corresponding value of Computed-Purified } \phi}$$

<u>Response to Earthquakes</u>

The response of a chimney to an earthquake involves the determination of the maximum bending moments and the maximum shears at different sections along the chimney. These maximum moments and shears are calculated at each section by a combination of as many modes as desired. In other words, for any one section along the chimney the bending moment (or shear) is computed for each mode individually at short intervals* of time and then combined at each interval of time by obtaining their algebraic sum. The absolute maximum of these combined values is then taken as the maximum bending moment (or shear) at the particular section during the duration of the earthquake.

The above procedure requires that the response of the chimney be obtained individually and simultaneously for the different modes. The response of the chimney in any mode due to an earthquake will involve the solution of Equation 15.

Although the acceleration, a , of the earthquake can be expressed as a system of straight lines, yet the solution of Equation 15 for the duration of the earthquake will require a prohibitive amount of time unless a high speed computer is used. The IBM 7090 computer was therefore utilized in the solution of Equation 15 using a third order Runge-Kutta process. This process will be described briefly.

Equation 15 has the form

$$\ddot{x}_1 + C\dot{x}_1 + Kx_1 = -f(t) \tag{16}$$

Let

$$\dot{x}_1 = x_2 \tag{17}$$

then Equation 16 will take the form

$$\dot{x}_2 + Cx_2 + Kx_1 = -f \tag{18}$$

or

$$\dot{x}_2 = -(f + Kx_1 + Cx_2) \tag{18a}$$

If at any time, τ , x_1 and x_2 are known, which in the physical problem represent the displacement and velocity, respectively, then the

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* Interval did not exceed one twentieth of the period of the highest mode.

value of x_1 and x_2 can be computed at the next time $(\tau + h)$ in this manner:

If at $t = \tau$, $x_1 = x_1(\tau)$ and $x_2 = x_2(\tau)$

then at $t = (\tau + h)$

$$x_1(\tau + h) = x_1(\tau) + \frac{1}{4} k_{01} + \frac{3}{4} k_{21}$$

$$x_2(\tau + h) = x_2(\tau) + \frac{1}{4} k_{02} + \frac{3}{4} k_{22}$$

where:

$$k_{01} = h[x_2 (\tau)]$$

$$k_{02} = h[- f(\tau) - Kx_1(\tau) - Cx_2(\tau)]$$

$$k_{11} = h[x_2(\tau) + \frac{k_{02}}{3}]$$

$$k_{12} = h[- f(\tau + \frac{h}{3}) - K\{x_1(\tau) + \frac{k_{01}}{3}\} - C \{x_2(\tau) + \frac{k_{02}}{3}\}]$$

$$k_{21} = h[x_2(\tau) + \frac{2}{3} k_{12}]$$

$$k_{22} = h[- f(\tau + \frac{2h}{3}) - K \{x_1(\tau) + \frac{2}{3} k_{11}\} - C \{x_2(\tau) + \frac{2}{3} k_{12}\}]$$

The initial displacement and velocity are required for this process and are taken as zero.

SCOPE OF THE STUDY

The study reported herein was made on eight reinforced concrete chimneys ranging in height from 352 ft. to 825 ft. . The main dimensions and data of these chimneys are given in Table 1.

TABLE 1

DIMENSIONS AND DATA OF EIGHT CHIMNEYS

Chimney No.	Height (ft.)	Top Outside Diam. (ft.)	Bottom Outside Diam. (ft.)	Total Wt. (Kips)	Remarks
1	352	23.58	30.90	4532	Corbel supported brick lining
2	450	16.33	35.79	6743	Corbel supported brick lining
3	534	18.67	35.03	8374	Independent liner
4	622	23.33	47.26	12526	Independent liner
5	707	19.98	69.14	26236	Corbel supported brick lining
6	800	36.56	65.00	25033	Independent liner
7	800	29.31	52.90	12768	This is the concrete liner of Chimney #6
8	825	25.00	63.96	22970	Supported steel liner

To eliminate any unusual peaks in the response, it was felt at the outset that the best way to represent the results was in terms of the average response of a certain number of earthquakes. Initially seven earthquakes were used but this idea was abandoned in favor of three earthquakes that were found to give approximately the same average response as the seven earthquakes. Table 2 lists the seven earthquakes with the three at the top of the list being those that were finally used for the study.

The accelerograms of the earthquakes of Table 2 that were obtained by the U.S. Coast and Geodetic Survey have been reduced to punch card form for the purpose of using them in high speed computers[4].

[4] "Integrated Velocity and Displacement of Strong Earthquake Ground Motion," G.V. Berg and G.W. Housner, Bulletin of the Seismological Society of America, Vol. 51, No. 2, pp. 175-189, April 1961.

TABLE 2

LIST OF EARTHQUAKES

Designation	Location	Date	Direction
A	El Centro, Cal.	May 18, 1940	West
B	Olympia, Wash.	April 13, 1949	N 10° W
C	Taft, Cal.	July 21, 1952	S 21° W
D	Taft, Cal.	July 21, 1952	N 69° W
E	Olympia, Wash.	April 13, 1949	N 80° E
F	El Centro, Cal.	Dec. 30, 1934	South
G	El Centro, Cal.	Dec. 30, 1934	West

For each of chimneys 2, 6 and 7 the average of the maximum bending moments due to the three earthquakes (A, B and C) and also due to the seven earthquakes were computed based on four modes response. The results are given in Table 3 for the sake of comparison.

A study was made to determine the number of modes that should be incorporated in the solution to give good results. For this purpose responses were obtained by incorporating three modes, four modes and five modes of vibration. Table 4 gives the values of bending moments in chimneys 6 and 7 due to earthquakes A and B considering three, four and five modes of vibration. The values of Table 4 indicate that higher modes can be ignored and that three or four modes of vibration are sufficient to give good results for any practical problem. The results in this paper will be based on a four mode response.

TABLE 3

AVERAGE MOMENTS DUE TO THREE EARTHQUAKES (A, B, C) vs. SEVEN EARTHQUAKES (Ft-Kips) (β= .05)

Chimney #2		Chimney #6		Chimney #7	
Aver. of 3 Earthquakes	Aver. of 7 Earthquakes	Aver. of 3 Earthquakes	Aver. of 7 Earthquakes	Aver. of 3 Earthquakes	Aver. of 7 Earthquakes
0	0	0	0	0	0
1,076	1,076	9,618	10,542	6,225	5,865
3,832	3,849	32,145	34,975	20,343	19,162
8,277	8,388	59,729	64,215	35,803	33,857
13,192	13,346	90,014	94,050	47,805	45,344
18,058	18,044	122,594	121,682	54,435	52,537
22,475	22,289	154,877	144,397	59,638	58,541
26,066	25,515	182,725	166,540	64,940	63,124
29,221	28,325	205,129	184,002	68,977	67,599
31,250	29,731	220,130	196,808	71,658	70,760
33,688	31,133	224,216	215,164	71,921	73,016
35,220	31,088	240,161	239,427	75,658	77,705
37,434	33,216	288,206	280,106	82,429	86,793
40,632	36,451	336,398	327,248	99,371	101,179
45,702	41,510	401,852	397,339	122,301	124,607
52,511	48,311	492,352	492,465	165,613	162,088
61,234	56,749	591,318	599,278	220,357	206,913
73,454	68,015				
89,432	82,014				
109,493	98,400				
129,900	116,122				

Note: Values listed at equal intervals of height. Bottom values are for base of chimney.

TABLE 4

MOMENTS (ft-Kips) DUE TO THREE, FOUR AND FIVE MODE RESPONSE (β= .05)

Chimney #6						Chimney #7					
Earthquake A			Earthquake B			Earthquake A			Earthquake B		
Three Modes	Four Modes	Five Modes	Three Modes	Four Modes	Five Modes	Three Modes	Four Modes	Five Modes	Three Modes	Four Modes	Five Modes
0	0	0	0	0	0	0	0	0	0	0	0
9,422	10,623	10,826	8,179	9,077	8,920	5,776	6,009	6,554	6,551	7,737	8,486
33,468	37,196	37,048	27,169	27,991	28,717	19,900	20,183	21,373	21,898	25,054	26,786
67,829	73,477	70,773	48,354	47,805	49,560	37,428	36,994	37,858	39,515	43,407	44,863
109,574	113,518	113,962	65,716	65,413	65,100	54,048	52,833	52,814	53,703	56,240	56,052
154,333	154,530	157,867	86,522	86,486	86,350	66,746	66,585	67,682	60,661	60,824	59,398
199,187	197,207	197,695	110,009	110,986	110,294	74,794	77,179	78,684	61,201	63,221	64,827
241,599	240,307	240,221	132,851	132,428	132,200	82,411	83,331	83,607	66,829	70,084	70,649
276,473	276,099	276,247	156,732	157,190	156,826	93,809	89,728	89,346	69,344	71,238	70,106
298,393	298,408	298,607	180,898	181,564	180,783	100,665	99,086	96,926	70,613	70,533	72,581
301,629	301,416	301,577	200,400	199,617	198,731	98,567	99,936	98,259	73,679	73,890	73,483
334,829	350,121	350,669	212,328	211,947	212,121	104,671	111,189	111,320	76,764	74,609	74,652
359,943	390,607	395,184	233,101	241,719	242,037	122,808	131,039	133,716	68,913	70,556	70,051
413,324	428,839	431,560	260,419	270,995	271,650	144,220	147,075	151,139	93,809	95,166	94,601
492,949	493,307	495,959	323,242	323,189	323,002	171,538	172,520	172,774	120,126	119,955	119,938
580,076	601,566	602,032	393,099	404,526	405,391	230,216	241,352	242,574	156,378	156,803	158,415
677,791	725,271	726,881	465,223	493,293	496,417	291,170	313,320	317,841	210,633	213,804	217,829

Note: Values listed at equal intervals of height

The main investigation of the study will therefore be based on the eight chimneys listed in Table 1 and on using the average response of the three earthquakes listed at the top of Table 2 considering four modes of vibration. It should be mentioned again that the study is based on an elastic response using a damping coefficient of 5% of critical.

MAIN RESULTS OF THE STUDY

Shear Distribution Due to Earthquakes

For each of the eight chimneys listed in Table 1, the maximum shearing forces at different heights of a chimney were computed for earthquakes A, B, and C of Table 2. The distribution of the shearing forces along the chimney follows a certain pattern. This is illustrated in Figure 1 where the average of the maximum shears due to the three earthquakes are plotted for chimneys 1, 4 and 8.

Base Shear

The maximum shearing force that is transmitted at the base of a chimney during an earthquake is used by many engineers as the starting point in the earthquake design of chimneys. For this reason the maximum base shear for each of the eight chimneys was computed for earthquakes A, B, and C. The average of these maximum base shears are plotted in Figure 2 as base shear over total weight versus the first mode period of the chimney. It should be mentioned again that the maximum base shears are based on a four mode response. The first mode period was selected as the abscissa in Figure 2, because it was felt that the period is the one variable that can best represent the different parameters prescribing the chimney.

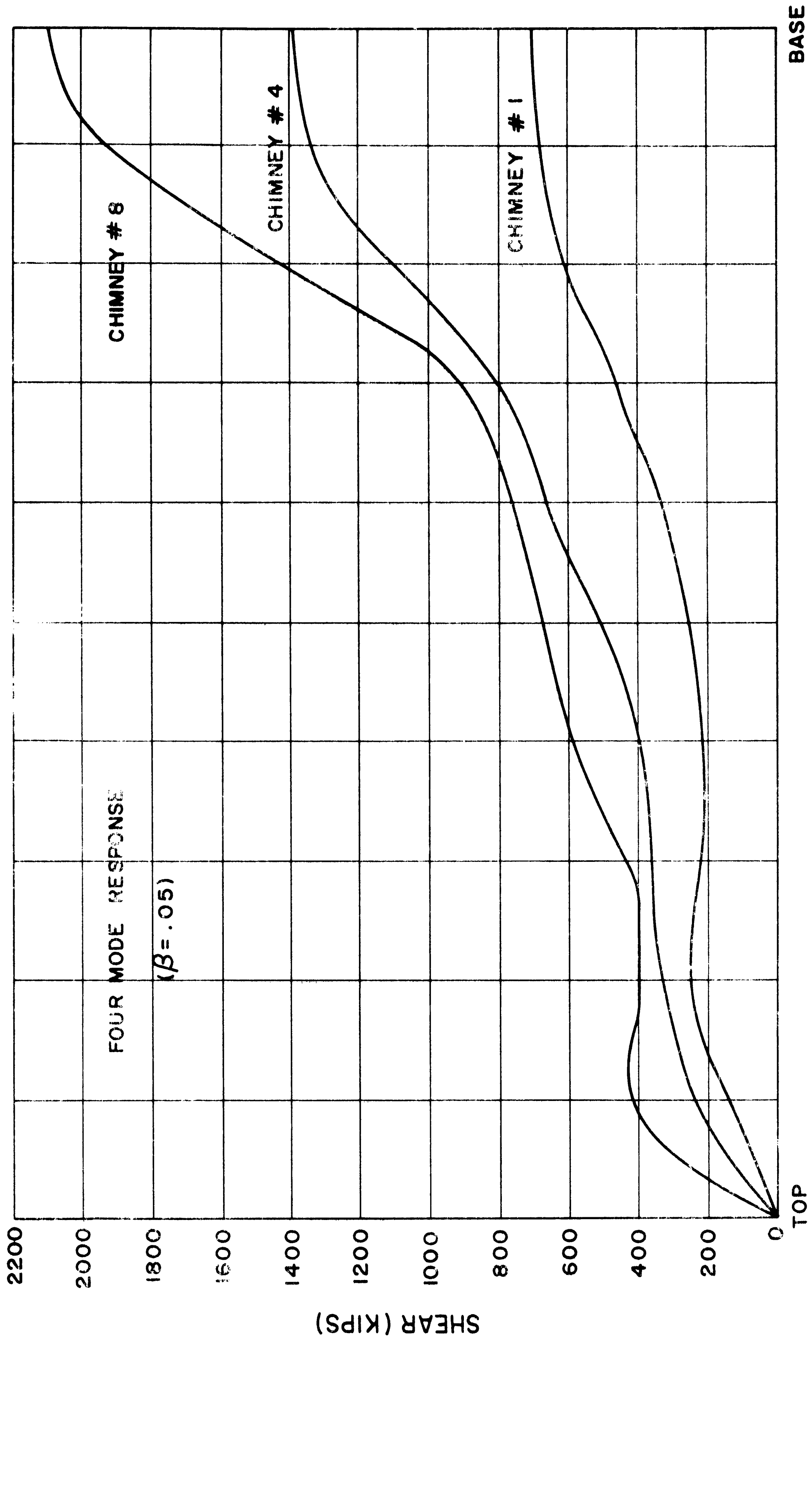

Figure 1. Average of Maximum Shears Due to Earthquakes A, B and C.

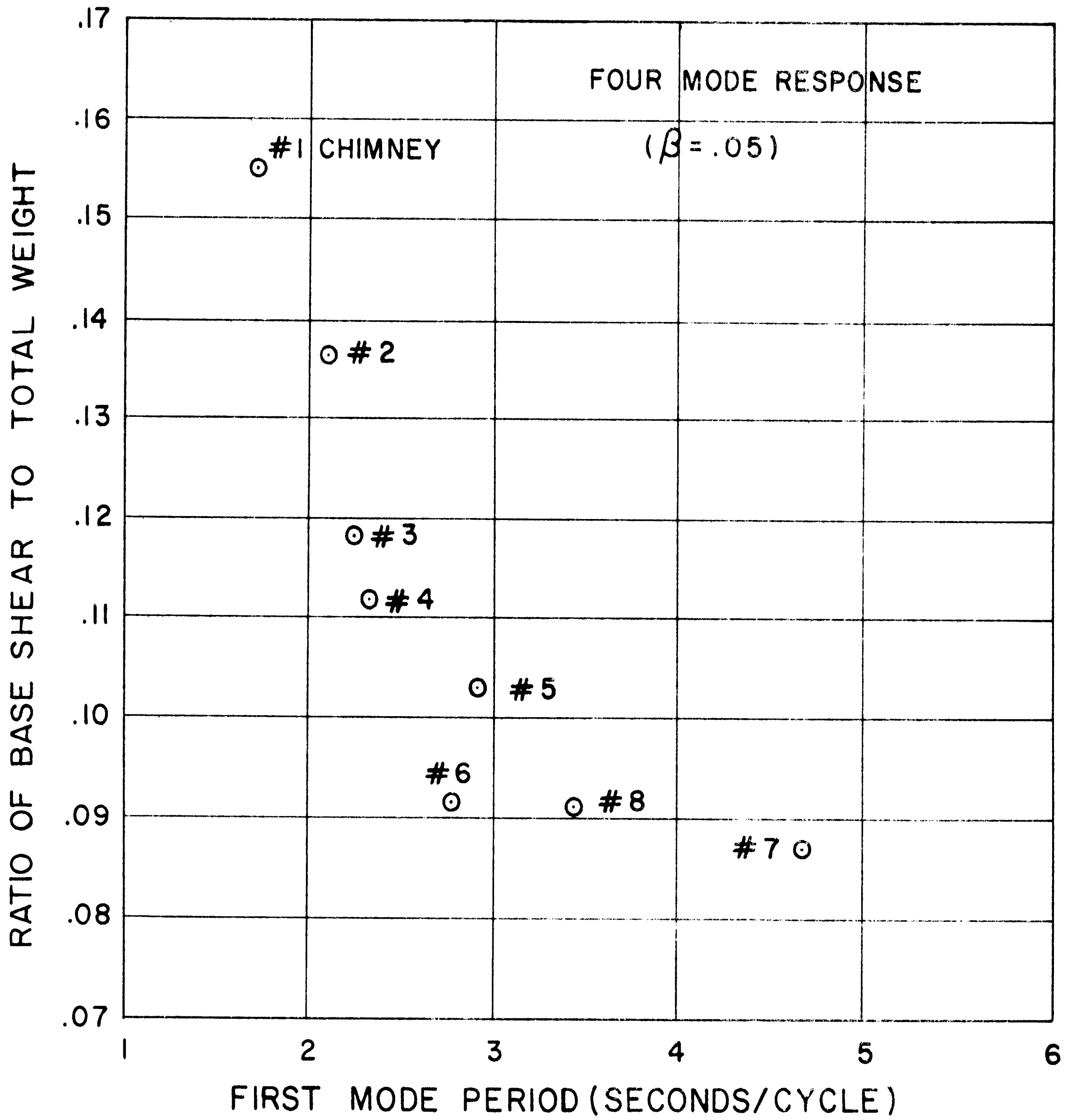

Figure 2. Base Shear/Weight Ratio vs. First Mode Period (Base shear is the average of maximum base shears due to Earthquakes A, B and C).

Maximum Bending Moment Curves

Figure 3 shows graphically the variation, with respect to height, of the maximum bending moments along a chimney due to earthquake forces. These bending moment curves which are the average of the maximum moment curves due to earthquakes A,B, and C, are normalized by making the base moment equal to 1.

Arbitrary Rules for Distribution of the Base Shear

Three arbitrary rules are used in this study to determine approximate values for the maximum bending moments due to earthquakes.

Rule 1: Distribute the base shear along the height of the chimney so that the force at any level is proportional to the product of the weight intensity at that level times the height of that level from the base. The total distributed load is made equal to the base shear.

For each chimney the base shear as obtained from the average of the maximum base shears of the three earthquakes, was used and distributed in the manner explained. The bending moments are then computed on the basis of a cantilever beam loaded by these distributed forces.

These bending moments at different levels are then corrected by a variable multiplier so that the corrected bending moment curve coincides with the bending moment curve obtained from the average of the maximum moments of the three earthquakes.

Figure 4 shows the variable multiplier for each of the eight chimneys.

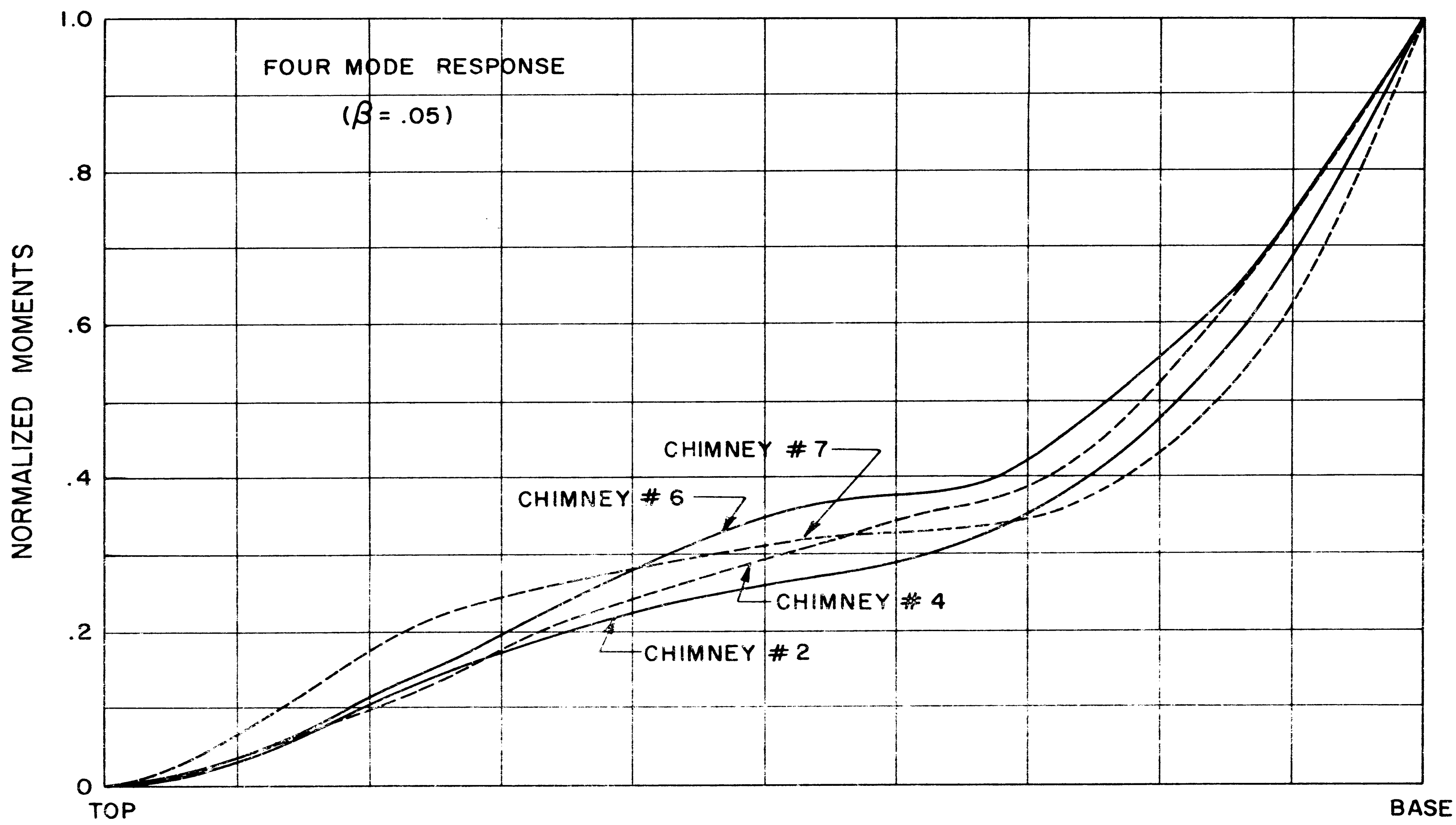

Figure 3. Normalized Average Bending Moments Due to Earthquakes A, B and C.

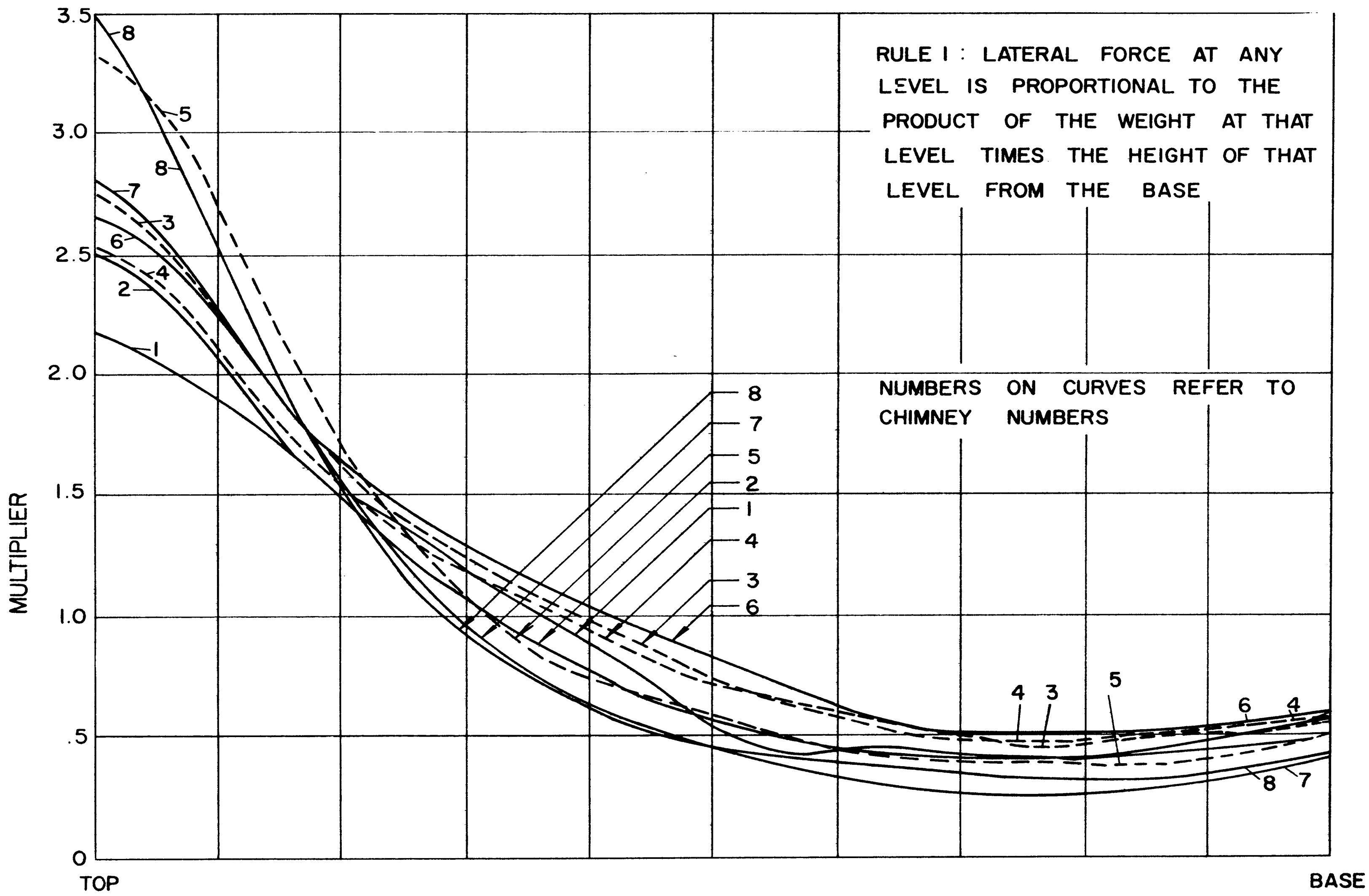

Figure 4. Variable Multiplier for Rule 1.

Rule 2: Distribute the base shear along the height of the chimney so that the force at any level is proportional to the product of the weight intensity at that level times the square of the height of that level from the base. The total distributed force is made equal to the base shear.

Using the distributed forces, a bending moment curve and a variable multiplier are obtained as in the first rule. The variable multiplier for each of the eight chimneys is plotted in Figure 5.

Rule 3: Distribute the base shear along the height of the chimney so that the force at any level is proportional to the product of the weight intensity at that level times the cube of the height of that level from the base. The total distributed force is again made equal to the base shear.

As in Rule 1 a variable multiplier is obtained for each chimney and is plotted in Figure 6.

The ideal rule for distributing the base shear is one that will give the same variable multiplier for all chimneys. This of course can never be achieved due to the many variables of the problem. A study of Figures 4, 5, and 6 shows a certain pattern for the variable multiplier and although this multiplier is not the same for all chimneys, one can nevertheless use a reasonable value for the multiplier for preliminary design.

The curves of Figure 5 (Rule 2) show the least deviation between maximum and minimum values in most regions of the chimneys.

It is interesting to note that the higher chimneys, as compared to chimneys of intermediate height, have larger multipliers in the top quarter, and smaller values in the bottom three quarters of the chimney.

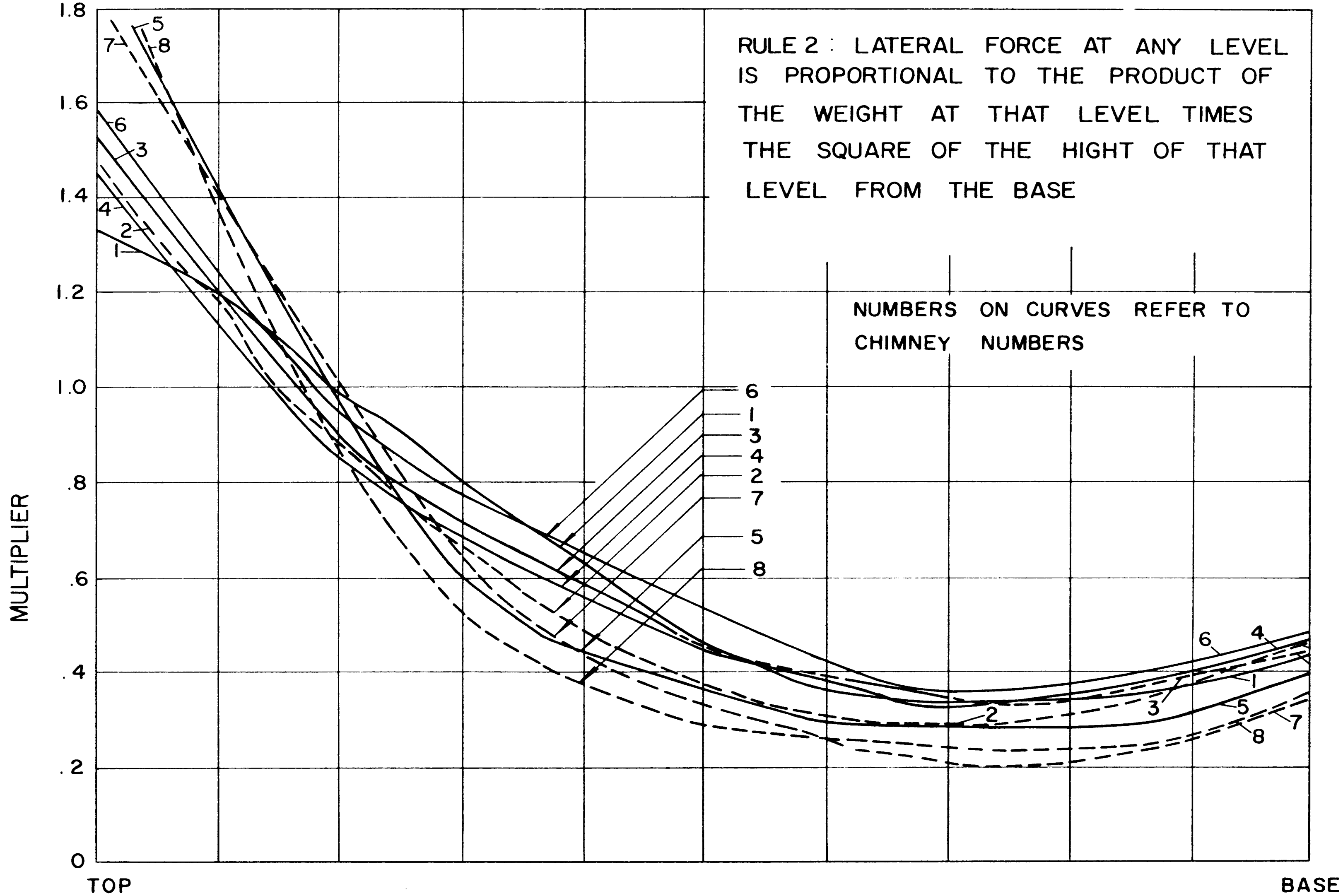

Figure 5. Variable Multiplier for Rule 2.

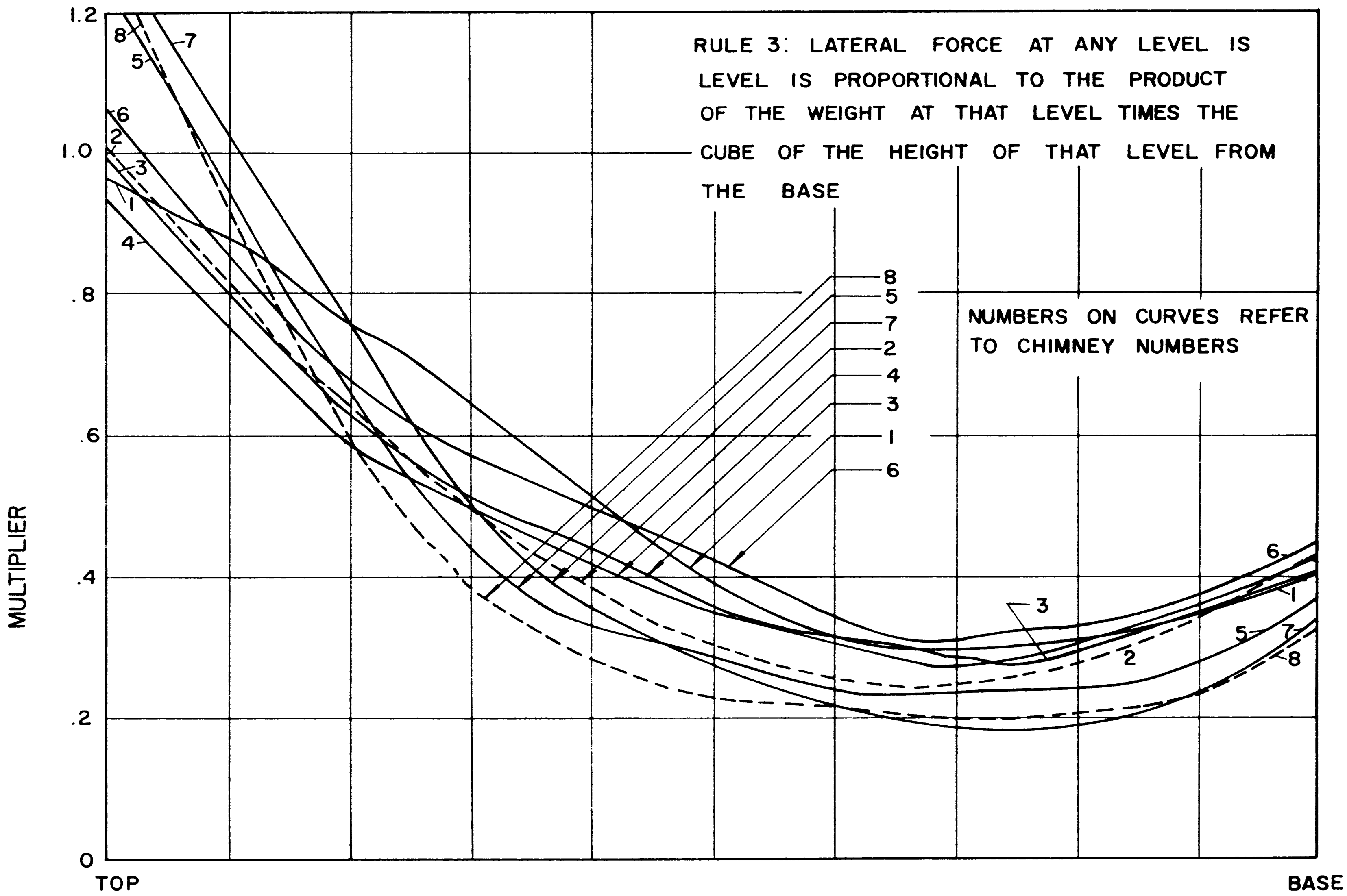

Figure 6. Variable Multiplier for Rule 3.

Use of Average Spectrum Curves

In using the spectrum techniques to compute a response of a chimney to an earthquake one can only determine the maximum response of a chimney in each mode separately. If the absolute values of the maximum responses in the different modes are added together one obtains an upper limit for the response. This upper limit can be much higher than the actual response which is the maximum of the algebraic sum of the responses of the different modes.

One can combine the maximum responses in the separate modes by taking the square root of the sum of the squares of the responses of the different modes. This method has been found to give good results for the eight chimneys studied in this paper. Table 5 compares the actual maximum bending moments in chimneys 1, 3, 4, 5 and 8 to the square root of the sum of the squares of the individual maximum modal bending moments. The same comparison is made for shears and is given in Table 6.

APPROXIMATE BENDING MOMENTS IN REINFORCED CONCRETE CHIMNEYS DUE TO EARTHQUAKES

The results included in this paper can be used to calculate approximate bending moments in reinforced concrete chimneys. The following steps can be followed:

1. Determine the period of the first mode of the chimney using a numerical procedure. The computations involved can be easily performed by hand.
2. Establish a value for the ratio of the base shear to the total weight of the chimney. This is the most important step and one that requires a careful study of the location of the chimney and

TABLE 5.

ACTUAL MAXIMUM BENDING MOMENTS COMPARED TO THE SQUARE ROOT OF THE SUM OF THE SQUARES OF THE INDIVIDUAL MODAL MOMENTS. (ft-k)
- AVERAGE OF EARTHQUAKES A, B AND C, FOUR MODES, $\beta = .05$ -

Chimney #1		Chimney #3		Chimney #4		Chimney #5		Chimney #8	
Actual	$\sqrt{\text{Sum of Squares}}$	Actual	$\sqrt{\text{Sum of Squares}}$	Actual	$\sqrt{\text{Sum of Squares}}$	Actual	$\sqrt{\text{Sum of Squares}}$	Actual	$\sqrt{\text{Sum of Squares}}$
0	0	0	0	0	0	0	0	0	0
1,125	1,192	1,700	1,633	3,961	4,036	8,888	8,242	6,983	5,933
4,016	4,174	5,920	5,691	13,075	13,428	31,383	28,622	22,569	19,587
9,100	9.100	11,319	10,984	23,797	24,599	55,475	50,259	39.838	35,661
14,605	14,312	17,002	16,593	35,683	35,424	73,974	67,322	55,166	50,306
19,655	18,941	23,023	21,996	48,431	45,248	86,358	79,287	65.260	61,823
23,936	23,025	29,277	27,028	59,969	53,978	98,997	89,300	73,727	70,482
26,149	25,631	35,374	31,599	68,894	61,512	117,855	101,866	81,881	77,621
27,569	27,649	40,837	36,010	77,097	67,924	135,247	117,845	88.606	84,616
28,310	28,462	45,076	39,835	85,412	73,980	145,934	135,065	95,466	92,295
29,901	29,768	47,895	43.153	94,201	81,180	157,354	152,010	101,903	100,598
33,377	31,938	51,230	46,102	99,525	91,547	185,907	169,185	115,048	109,385
38,735	36.332	56.717	49,141	117,026	107,467	216,593	192,056	127.368	118,485
44,225	43,579	62,064	52,953	145,328	131,831	246,359	232.453	138,970	127,733
51,744	53,468	66,336	55,340	178,147	165,287	293,879	300,073	150,610	137,686
63,084	65,354	69,513	66,068	220.192	205,713	380,350	387,852	166,562	150,982
74,908	78,383	82,017	76.632	263,631	250,553	493,666	488,403	187,893	172,680
		97,162	90,680					212,534	207,277
		114,232	108,205					246,208	255,776
		132,250	128,521					317,887	316,689
		153,428	150,588					402,828	385,393

Note: Values listed at equal intervals of height

TABLE 6.

ACTUAL MAXIMUM SHEARS COMPARED TO THE SQUARE ROOT OF THE SUM OF THE SQUARES OF THE INDIVIDUAL MODAL SHEARS. (k)
- AVERAGE OF EARTHQUAKES A, B and C, FOUR MODES, $\beta = .05$ -

Chimney #1		Chimney #3		Chimney #4		Chimney #5		Chimney #8	
Actual	$\sqrt{\text{Sum of Squares}}$	Actual	$\sqrt{\text{Sum of Squares}}$	Actual	$\sqrt{\text{Sum of Squares}}$	Actual	$\sqrt{\text{Sum of Squares}}$	Actual	$\sqrt{\text{Sum of Squares}}$
0	0	0	0	0	0	0	0	0	0
51	54	64	61	102	104	201	186	169	144
132	136	158	152	235	243	509	463	378	332
232	226	204	200	296	295	558	509	434	396
251	240	233	219	341	308	513	472	398	383
233	228	253	225	358	320	612	517	395	365
209	213	259	232	367	334	766	639	396	393
211	195	256	241	395	352	770	754	480	449
238	204	273	249	450	394	916	835	560	502
276	266	304	260	552	470	954	877	611	545
314	311	331	283	656	559	1,004	911	654	589
404	404	373	326	723	659	1,042	1,049	696	637
467	470	440	381	843	820	1,313	1,369	735	684
586	576	494	441	1,027	1,016	1,940	1,926	788	738
641	626	541	505	1,242	1,193	2,326	2,281	847	832
687	668	593	576	1,343	1,283	2,519	2,463	990	996
700	680	666	665	1,388	1,326	2,658	2,592	1,285	1,220
		823	801					1,563	1,450
		912	887					1,808	1,675
		968	944					2,004	1,868
		985	961					2,075	1,941

Note: Values listed at equal intervals of height except top and bottom values which are at 1/2 interval from the top and base of chimney respectively.

also of the earthquake risk that the owner is willing to take. Figure 2, which is based on the average response of earthquakes A, B and C can be used as a guide to establish such value. It is instructive to mention that the accelerogram of the south component of the May 18, 1940 El-Centro earthquake, which many researchers and engineers use in their earthquake investigations, will give about 2.5 times the values listed in Figure 2 for chimneys that have periods of 2.5-3.0 seconds.

3. The value of the base shear established in step 2 can be distributed along the height of the chimney according to any of the three rules considered in this study. These distributed forces can then be applied to the chimney as lateral forces to compute bending moments at different heights of the chimney.

4. The moments of step 3 can now be corrected to obtain earthquake moments using the coefficients in Figures 4, 5, or 6 depending on which rule was used to distribute the base shear. The values of these figures are computed for different height chimneys and the designer can use coefficients that will apply best to the chimney under consideration. These coefficients are multiplied by the moments of step 3 to give approximate earthquake bending moments for the chimney.

BENDING MOMENTS DETERMINED BY SPECTRUM TECHNIQUES

The response of a chimeny to the accelerogram of any earthquake can be determined, in any mode, by the use of spectrum curves. If a one degree of freedom system, whose period is T, is subjected to an earthquake, then the maximum relative displacement of the system during the duration of the earthquake, gives one point on the displacement spectrum curve for the earthquake. By using different systems with different values of T, one obtains a displacement spectrum curve whose ordinates are maximum displacement and whose abscissas are the periods. A family of such curves can be obtained, each with a specific value of the damping ratio β . Spectrum curves can also be obtained for acceleration of the system as well as for the velocity of the system[5].

To obtain a spectrum curve for any earthquake one has to solve equation (15) with the value of the quotient

$$\frac{\int_0^L m \phi_j \, dx}{\int_0^L m \phi_j^2 \, dx} = \alpha_j, \text{equal to unity.}$$

Once the solution of equation (15) is known with $\alpha_j = 1$, then the response of a chimney, in any mode, can be obtained by multiplying the results by the factor α_j .

The following procedure can be used to determine the response of a chimney to earthquakes using the spectrum techniques.

1. Calculate the mode shapes for the first three modes of vibration and also the shears and bending moments associated with each mode shape. The procedure of calculating the modes by the Stodola method has been outlined in this paper. The actual performance of the calculations will normally require a computer. For each mode evaluate the value of the factor α_j.

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5 Spectrum Analysis of Strong-Motion Earthquakes by J.L. Alford, G.W. Housner, and R.R. Martel, California Institute of Technology, August 1951.

2. Using the values of the three periods computed, determine the three spectrum values corresponding to the three periods. An average spectrum curve for the seven earthquakes of table 2 is given in Figure 7. This average spectrum curve is plotted as ωS_D (velocity spectrum) versus the period, where S_D is the displacement spectrum and ω is the frequency in radians/second[6].
3. The shears and bending moments in any mode, j , are then obtained by multiplying the modal shears or moments obtained in step 1 by the factor $\alpha_j S_{D_j}$.
4. The shears or moments for the different modes obtained in step 3 are then combined using the square root of the sum of the squares rule to give the total response of the chimney. This response which is the average response due to the seven earthquakes listed in Table 2, should be multiplied by whatever factor is necessary to meet the specific design requirements. For purposes of comparison, the spectrum of the May 18, 1940, El-Centro earthquake is also given in Figure 7.

It should be mentioned that the use of three modes is sufficient for design purposes. However, one can use any number of modes and combine them by the square root of the sum of the squares method which was found to give good results (see Table 5).

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6 The values of Figure 7 were obtained by using a computer program written by Professor G.V. Berg at the University of Michigan. The program solves Equation 15 with $\alpha_j = 1$, using a fourth order Runge-Kutta process.

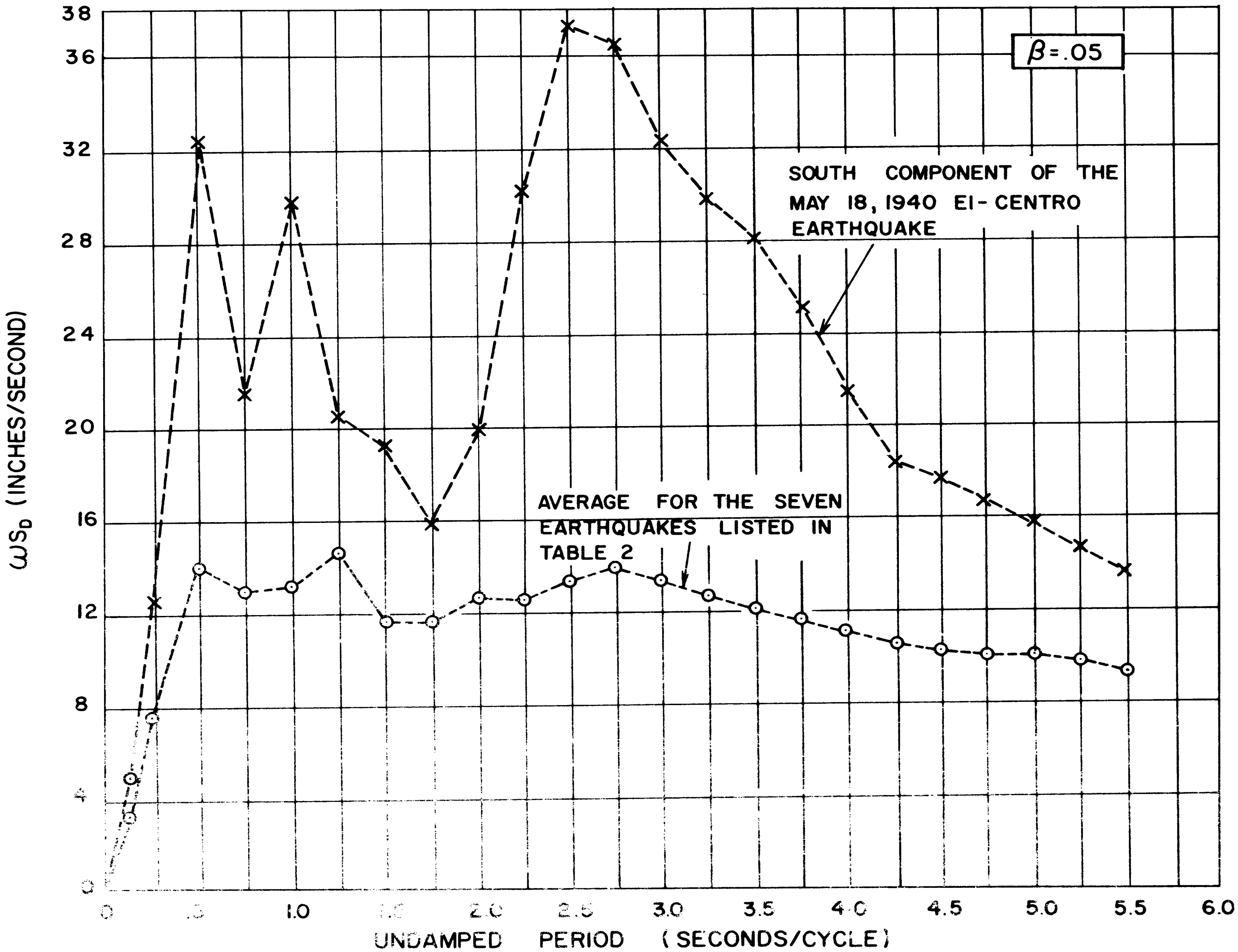

Figure 7. Linear Response Spectra.

SUMMARY AND CONCLUSIONS

The information presented in this paper should enable the designer to estimate the distribution and magnitude of the forces generated in a reinforced concrete chimney during earthquakes. Two procedures are given to estimate the maximum bending moments. Procedure I utilizes Figure 2 to establish the base shear, distributes the base shear according to rule 1, 2, Or 3 and corrects the bending moments obtained from such distribution using the curves of Figure 2, 3, or 4. Procedure II employs the spectrum techniques. Procedure I, which can be easily adapted to hand computations, is normally the less accurate one. On the other hand, procedure II is not well adapted to design office procedures and will normally require a computer to establish the modal characteristics of the chimney.

Although the judgment and experience of the designer is of primary importance in establishing zone and risk coefficients, it is hoped that the results and methods presented in this paper will aid him in arriving at a sound earthquake resistant design.

APPENDIX I

NOTATION

The following symbols have been adopted for use in this paper:

a	=	acceluation due to earthquake
E	=	modulus of elasticity
I	=	moment of inertia
L	=	length of chimney
m	=	mass per unit length
M	=	bending moment
$q_i(t)$	=	displacement in the i^{th} mode
S_D	=	displacement spectrum
t	=	time
x	=	distance along the chimney
$y(x,t)$	=	displacement in the chimney
β	=	fraction of critical damping
$\phi_i(x)$	=	shape of the i^{th} mode
ω	=	natural undamped frequency in radians per second

APPENDIX II

DERIVATION OF EQUATIONS 9, 10 & 11

In deriving Equations 9, 10 & 11, the following orthogonality relationship will be used

$$\int_0^L m\phi_i\phi_j \, dx = 0 \quad i \neq j \tag{a}$$

The frequency equation (Equation 2) written for the i^{th} mode is:

$$\frac{d^2}{dx^2}\left(EI \frac{d^2\phi_i}{dx^2}\right) = m\omega_i^2 \phi_i \tag{b}$$

Multiplying both sides of equation (b) by ϕ_j and integrating along the height of the chimney yields

$$\int_0^L \phi_j \frac{d^2}{dx^2}\left(EI \frac{d^2\phi_i}{dx^2}\right) dx = \omega_i^2 \int_0^L m\phi_i\phi_j dx \tag{c}$$

The left hand side of Equation (c) can be integrated by parts by setting

$$u = \phi_j \qquad du = \phi_j' \, dx$$

and

$$v = \frac{d}{dx}\left(EI \, \phi_i''\right) \qquad dv = \frac{d^2}{dx^2}\left(EI \, \phi_i''\right) dx$$

then

$$\int_0^L \phi_j \frac{d^2}{dx^2}\left(EI \frac{d^2\phi_i}{dx^2}\right)dx = \left[\phi_j \frac{d}{dx}\left(EI\phi_i''\right)\right]_0^L - \int_0^L \phi_j' \frac{d}{dx}\left(EI\phi_i''\right)dx \tag{d}$$

Noting that $\frac{d}{dx}\left(EI\phi_i''\right)$ = shear

= 0 at $x = L$

and ϕ_j = Mode shape

= 0 at $x = 0$

$$\therefore \left[\phi_j \frac{d}{dx} (EI\, \phi_i'') \right]_o^L = 0$$

Equation (d) will therefore reduce to:

$$\int_o^L \phi_j \frac{d^2}{dx^2} \left(EI \frac{d^2\phi_i}{dx^2}\right)dx = - \int_o^L \phi_j' \frac{d}{dx} (EI\, \phi_i'')\, dx$$

$$= \left[-\phi_j' \, EI\, \phi_i''\right]_o^L + \int_o^L EI\, \phi_i'' \phi_j''\, dx \qquad (e)$$

Noting again that $EI\, \phi_i'' =$ Moment

$= 0$ at $x = L$

and $\phi_j' =$ Slope

$= 0$ at $X = 0$

$$\therefore \left[- \phi_j' EI \phi_i''\right]_o^L = 0$$

Equation (e) will then reduce to:

$$\int_o^L \phi_j \frac{d^2}{dx^2} \left(EI \frac{d^2\phi_i}{dx^2}\right)dx = \int_o^L EI\, \phi_i'' \phi_j''\, dx \qquad (f)$$

The left hand side of Equation (f) is equal to the right hand side of Equation (c) which is equal to zero, when $i \neq j$, by virtue of the orthogonality relationship of Equation (a)

$$\therefore \int_o^L \phi_j \frac{d^2}{dx^2} \left(EI \frac{d^2\phi_i}{dx^2}\right)dx = \int_o^L EI\, \phi_i'' \phi_j''\, dx = 0, \; i \neq j \qquad (g)$$

Also using Equations (f) and (c)

$$\int_o^L \phi_j \frac{d^2}{dx^2} \left(EI \frac{d^2\phi_i}{dx^2}\right)dx = \int_o^L EI\, (\phi_j'')\, dx \qquad i = j \quad (h)$$

$$= \omega_i^2 \int_o^L m\, \phi_j^2\, dx \qquad i = j \quad (h)$$

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