The Schlieren Method In Flow Observation Of Rarefied Gases

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SUMMARY

An analysis is made showing the possible sensitivity limitations of the Schlieren optical system in observing low density shock phenomena. Shock wave thickness is estimated at higher Mach numbers as equal to the mean free path of air before the shock. A practical, very sensitive Schlieren system is assumed and its sensitivity calculated, showing an observable density limitation for the system. Applied to hypersonic wind tunnels, it is concluded that, although strong shocks may be observed, the Schlieren method may be limited in observing small changes in weak shock waves.

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Introduction

In the study of spersonic flow phenomena, it is indeed important to be able to observe what happens in a flowing stream, when physical disturbances are created either by confining the stream to a flow in a duct, or around a body. The variation of density in a flowing gas is a basic property which is utilized in flow observations by the Schlieren optical method. It depends on the deflection of a light beam when passing through a gas medium, where there is a density gradient. Shock waves (at normal densities) have density gradients of a very high value, which increase with the increase of Mach number. The density gradient through a shock is a function of the thickness of the shock wave, which in turn is a function of the initial Mach number and the initial density before the shock. In observing shock waves, it may be that through a combination of these factors a limitation may be reached whereby the Schlieren method will not be sensitive enough to measure the density variation. An effort is made in this analysis to show what the limitations may be at low density supersonic air flows.

The relationships for pressure gradients across shock waves may be found by calculations of the thickness of the shock, which is done in Part I.

The Schlieren method has certain practical sensitivity restrictions which limit the minimum observable density gradients. Assuming a highly sensitive Schlieren system with practical limitations for its components, the minimum density gradients that it can distinguish satisfactorily may be calculated. Part II presents the calculations for a parallel-beam two-mirror Schlieren system and for a coincidence system.

The criteria developed is next applied to the observation of flow in hypersonic wind tunnels in Part III which gives the ultimate limitation of the density gradient in the test section. Practical limitations of observing weak shock waves are also discussed.

Part I Pressure Gradients Across Shock Waves

In problems of very high altitude flight as well as in problems of shock wave discontinuities, the air medium and the accompanying aerodynamic flight phenomena must be considered from the viewpoint of the molecular kinetic theory. The discussion below will attempt to consider the thickness of a normal shock wave from this viewpoint.

The basic Boltzmann equation in the kinetic theory of gases is used to derive the general hydrodynamic equations of motion in terms of average values of the variables. In the zero approximation of the distribution function, the equations of motion reduce to the ideal hydrodynamic equations. In supersonic flows these equations predict discontinuities (shock waves) of infinitely small thickness (the effects of viscosity and heat transfer are neglected). Higher approximations to the distribution function will take into account first and higher order terms involving viscosity and heat transfer. These additional terms in the equations of motion increase in importance as the density of the medium decreases appreciably, or as the velocity of the medium changes appreciably in a distance comparable to the mean free path, as in a strong shock wave. It is then obvious that in a hydrodynamic treatment of shock waves, and especially at low densities, the higher approximations must be included.

Figure 1 presents the calculated thickness of a shock wave considering first order effects of viscosity and heat transfer in the equations of motion and the thickness with second order effects utilizing results from Reference 1.[1] These results show an asymptotic approach of the shock wave thickness to the mean free path of the air before the shock. Solutions with the higher approximations show a greater thickness, and it is possible that the series of approximations will not give a convergence for shock waves of high intensity; where the deviations from thermal equilibrium may be quite large.

It can be expected though that the hydrodynamic equations may yield a satisfactory solution if proper coefficients for viscosity,

1. *It should be noted that the mathematical solution of the equation for thickness gives an infinite value for the shock transition zone with an asymptotic approach to conditions before and after the shock and a definition of thickness has to be made.*

heat transfer are included with the effect of temperature and pressure taken accurately into account, as well as a proper model used in the kinetic theory, taking into account the variation of specific heats.....

It may be seen from existing solutions of the approximate equations of motion for shock wave thickness, and from pure physical reasoning, that the thickness should be at least of the order of the mean free path before the shock at the higher Mach numbers.

Figure 2 shows the ratio of the shockwave thickness and the free-stream mean free path for various Mach numbers and altitudes. It is seen that there is relatively little effect of altitude on this ratio.

Based upon the above reasoning, the following assumptions are made in calculating the density gradient across the shock:

1. Thickness of shock = mean free path of free stream.
2. Density gradient constant across shock wave
3. Normal shock wave.

Figure 3 presents calculations of $\frac{\Delta\rho}{\Delta x}$ through a normal shock wave for various Mach numbers and altitudes.

Part II Sensitivity Limits for the Schlieren Method

The Schlieren optical method for flow observation has certain practical limits of sensitivity.

Calculations are here made for an assumed Schlieren apparatus of rather high sensitivity and density limitations shown for normal shock waves.

In a two-dimensional flow, the radius of curvature of a light ray in passing through a variable density field is constant and equal to:

$$\frac{1}{R} = \frac{\text{grad } n}{n} \sin \phi$$

where n = index of refraction of the gas

ϕ = angle between grad n and the direction of the light ray.

For air: $n = 1 + .000294 \frac{\rho}{\rho_o}$

$\rho_o = .002378$ (mass density at sea level in slugs)

therefore $\text{grad } n = \frac{.000294}{.002378} \text{grad } \rho = .1236 \text{ grad } \rho$

Our interest lies in determining the minimum angular deflection of the light ray that can be measured by the Schlieren method. The angular deflection is given by:

$$\epsilon = \frac{\alpha \cdot a}{f} = \frac{\int ds}{R}$$

where: α - % change of brightness of the image that can be satisfactorily detected - function of photographic or viewing technique used - sensitivity factor.

a - normal width of light source image perpendicular to knife edge

f - focal length of mirror

s - width of test section

It is apparent that the sensitivity of the Schlieren method depends on a and f. Theoretically a could be made equal to zero and the sensitivity would be infinite. Practically the sensitivity is limited by:

1. Necessary image brightness for good definition of the image.

2. The defraction of light occuring at the Schlieren stop, which increases with contraction of stop (factor a).

The imposed sensitivity limits on any Schlieren arrangement are quite arbitrary, and depend in general on the quality and accuracy of measurements demanded of the equipment.

Let us assume arbitrary practical limits for two types of Schlieren apparatus and calculate ϵ_{min}.

1. Parallel beam two mirror system - practical for rather fairly large density gradients in the flow

   $a = 0.1$ cm

   $f = 500$ cm

   $\alpha = 10\%$

   $$\epsilon_{min} = \frac{.01}{500} \times .10 = 2 \times 10^{-6}$$

2. Coincidence system - suitable method for very small deflections and practical for low density gradients.

   $a = 6 \times 10^{-3}$ cm

   $f = 500$ cm

   $\alpha = 5\%$

   $$\epsilon_{min} = \frac{6 \times 10^{-3}}{500} \times .05 = .6 \times 10^{-6}$$

The minimum density gradients measurable with the above systems, assuming a test section width of 20 cm (s = 20 cm) are calculated below.

$$\text{grad } \rho = \frac{\delta \rho}{\delta x} = \frac{\Delta \rho}{\Delta x} = \frac{\rho_o}{.000294} \quad \frac{1}{R} \quad \frac{1}{\sin \phi}$$

$$R = \frac{s}{\epsilon_{min}}$$

$$\phi = 1.0 \text{ for normal shock wave}$$

$$\text{grad } \rho = \frac{.002378}{.000294} \times \frac{\epsilon_{min}}{20}$$

1. $\left(\frac{\Delta \rho}{\Delta x}\right)_{min} = 8.09 \times 10^{-7}$ slugs / cm (parallel beam-two mirror)

2. $\left(\frac{\Delta \rho}{\Delta x}\right)_{min} = 2.43 \times 10^{-7}$ slugs / cm (coincidence)

These are the minimum density gradients that the assumed Schlieren systems can satisfactorily distinguish.

Figure 3 presents the values of $\frac{\Delta \rho}{\Delta x}$ across the shock for various Mach number and altitudes and the constant values for the Schlieren system. The intersection gives the critical densities for various Mach numbers above which the system will not be able to distinguish the density gradient image of the shock wave.

Figure 4 presents the critical densities as a function of Mach number. At higher Mach numbers there is a little variation of critical altitude as the curve asymptotically approaches a value of maximum density rise across the shock.

The presented results are as valid as the assumptions made. The error involved should probably be not too large for the given range of Mach numbers. Of course, only actual experiments could in any way verify these predictions.

Part III Observation of Flow in Hypersonic Tunnels with Schlieren Apparatus

The previously calculated results may be applied to conditions in hypersonic wind tunnels.

The density limitations in observation of normal shocks at higher Mach numbers ($M \gtrsim 4.0$) may be summarized:

Coincidence system - $\rho > 6.5 \times 10^{-7}$

Two mirror parallel - $\rho > 2.8 \times 10^{-7}$

These results show the ultimate density limitations for normal shocks, where the density gradients are highest, and for assumptions which do seem conservative. It may be said with some assurance that the Schlieren method of the given sensitivity will not distinguish any disturbance whatsoever below the given density values. The actual experimental restrictions may vary appreciably.

Let us evaluate the conditions in a wind tunnel test section. Figure 5 shows the calculated densities as a function of dry air conditions at the inlet, that is, pressure and temperature. The range of initial pressures: 14.7 to 500 psia, temperature: 460° to 960° R and Mach number 4, 7, 10.

It can be seen that the above limiting values of density fall outside the range of wind tunnel test chamber densities. Therefore, it may be said, that strong normal shock waves for all wind tunnel conditions assumed, can be observed with the given Schlieren apparatus.

It is usually desirable to observe more than just strong shock waves in wind tunnels. Therefore, some practical density limitation should be made for the Schlieren method, which would give information of most use. It is quite apparent that only experimental studies of this problem can give any concrete results.

In supersonic wind tunnels it is advantageous to use the parallel beam system of Schlieren in order to get precise experimental results. Especially, in wind tunnel nozzle calibration, the use of the Schlieren method in recognizing flow irregularities due to inexactness of nozzle wall design is quite important. It is then possible to correct the flow in eliminating the disturbances and producing a parallel and uniform air stream. In order to recognize these small disturbances, it is necessary to make the optical apparatus extremely sensitive.

Some experimental data of this sort is available from the experiences in the German Kochel tunnel. Their Schlieren system of comparable sensitivity to the assumed one here, showed that Mach waves were not visually detected above a Mach number of about 3.0. Since it is a blow-down tunnel with the tank pressure atmospheric, the corresponding density in the test section was $\sim 2 \times 10^{-4}$ (slugs). This value of ρ shows the order of lower density limitations for Schlieren of extremely high sensitivity.

It should be remembered that these values are arbitrary and empirical, and will vary with the intensity of weak shocks (Mach waves) and with the sensitivity of the optical apparatus.

Conclusions and Remarks

The presented analysis shows qualitatively that using the Schlieren method for supersonic flow observation, three density regions may be distinguished: 1. where the Schlieren is useful for observing weak Mach waves and all finite shock waves, in the above case for $\rho \gtrsim 2 \times 10^{-4}$; 2. where only more intense shock waves may be distinguished; $2 \times 10^{-4} \gtrsim \rho \gtrsim 30 \times 10^{-7}$; 3. where the Schlieren method may not show any disturbances, for $\rho \lesssim 3 \times 10^{-7}$.

It can, therefore, be concluded that for the assumptions made in this analysis, the Schlieren method may have limited applications for observing flow in hypersonic tunnels, especially in utilizing it for tunnel calibration at low densities, where only Mach waves may appear.

A word of caution about the shock wave phenomena should be said. The internal molecular structure is still a big unknown. We can only more or less intuitively approach such problems as the thickness of the shock wave, velocity distribution, viscosity and heat transfer effects across it. Mathematical analysis, at least that of continum mechanics, may not give a satisfactory answer. As already pointed out, a study from the molecular point of view, both theoretical (direct solution of the Boltzman equation) and experimental should cast important light on this rather complex problem. This is expecially true in the treatment of physical phenomena at low densities and at high speeds. The importance and need of basic work in this field, that of theoretically evolving the basic laws, and experimentally proving them for better understanding and predicting the "misbehavior" of low density flow phenomena will be progressively greater. The low density gases exhibit novel boundary properties (temperature jump, etc.) which may be also true of the behavior of shock waves at these densities and new problems involving probably the effect of dissociation, molecular wall effect and interference etcetera, on the shock may appear.

From the interpretation of optical images, the Schlieren method may prove of value as a tool in the analysis of low density shock wave phenomena.

An investigation of other optical methods, the shadowgraph, interferometer, for similar applications may prove to be very useful. Other than optical methods should also warrant investigation.

Distribution

Distribution of this report is made in accordance with ANAF-GM Mailing List No. 5 dated February 1948, to include Part A, Part B, and Part C.

REFERENCES

1. Thomas, L. H., Note on Becker's Theory of the Shock Front, Journal of Chemical Physics, Vol. 12, No. 11, pp. 449 to 453 November 1944

2. Durand, W. F., Aerodynamic Theory Vol. 3, pp. 221

3. Schardin, "The Toepler Schlieren Method" 1934

THICKNESS OF SHOCK WAVE

ALTITUDE — 100,000 FT.
$\lambda = .512 \times 10^{-3}$ CM

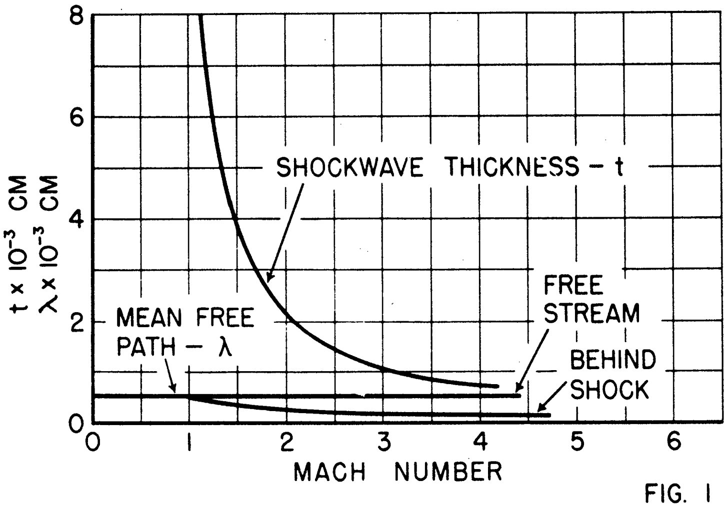

FIG. 1

RATIO OF SHOCK WAVE THICKNESS TO MEAN FREE PATH VS MACH NUMBERS

t - SHOCKWAVE THICKNESS
λ - MEAN FREE PATH - FREE STREAM
h - ALTITUDE

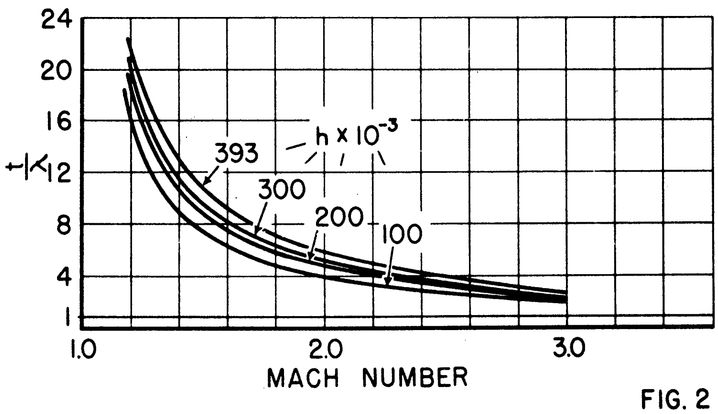

FIG. 2

FREE STREAM DENSITY
VS
DENSITY GRADIENT ACROSS SHOCK

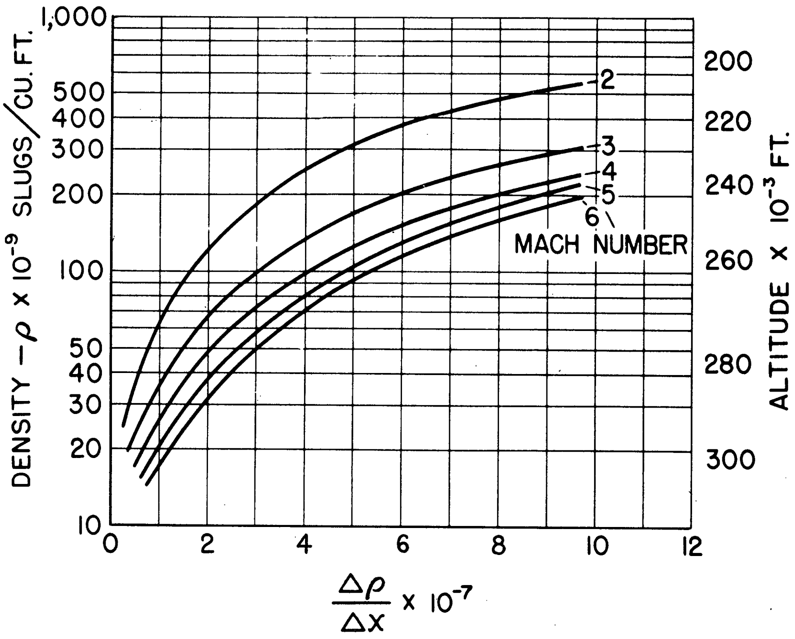

FIG. 3

CRITICAL DENSITY
VS
MACH NUMBER

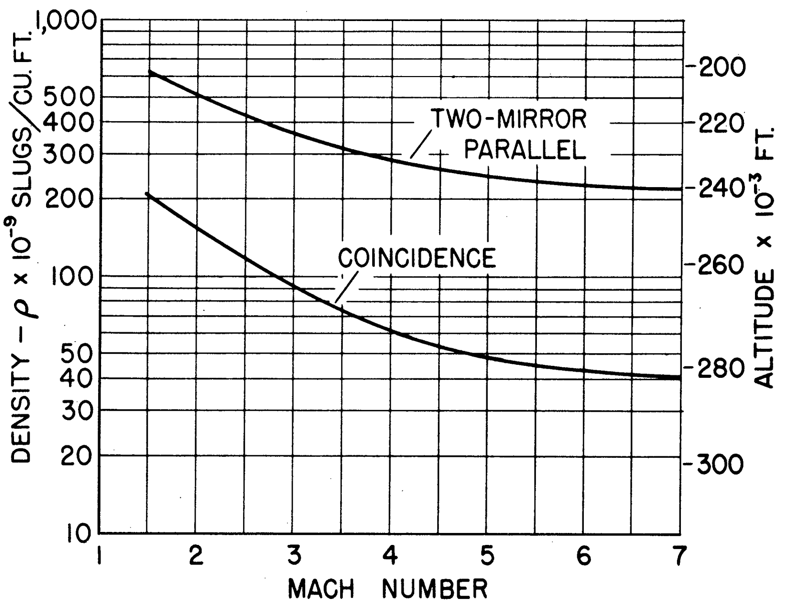

FIG. 4

WIND TUNNEL DENSITIES AS A FUNCTION OF INITIAL CONDITIONS AND MACH NUMBER

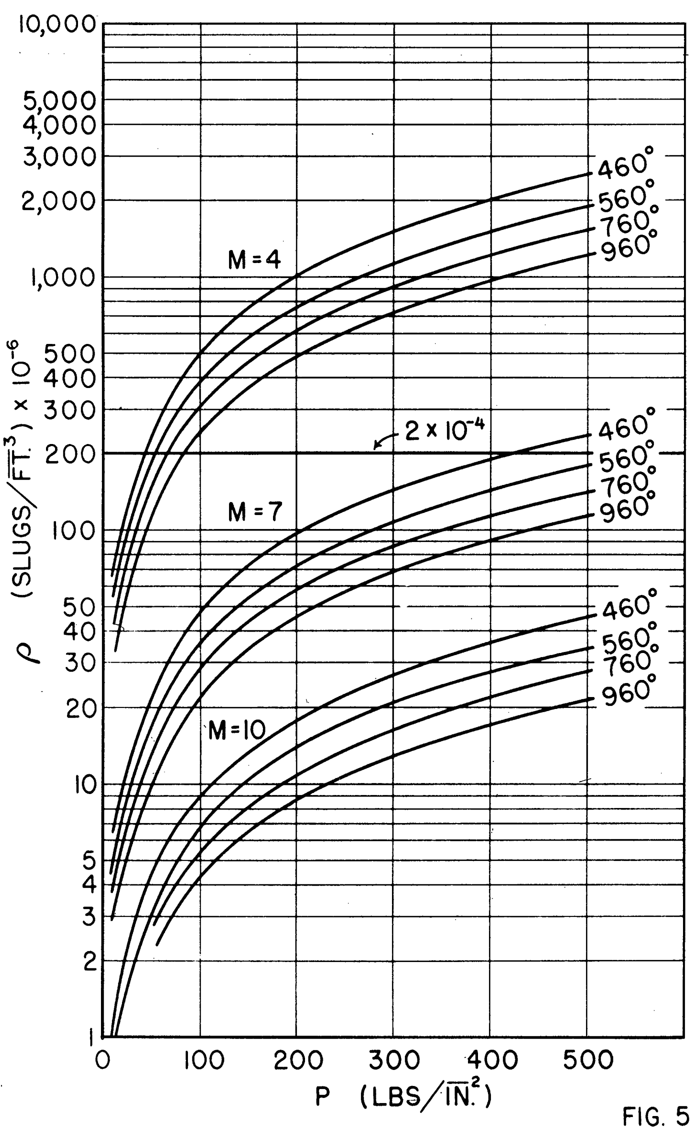

FIG. 5