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CONSIDERATIONS FOR THE ATTAINMENT OF A STANDING DETONATION WAVE

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ABSTRACT

This report outlines the considerations that are essential to experimentally attain a standing detonation wave. A comprehensive analysis of the stabilization of the detonation wave on a wedge is included. A method for the graphical construction of the detonation polar is presented.

OBJECT

The object of this research program is to experimentally stabilize a gaseous detonation wave.

NOMENCLATURE

a	speed of sound
C_p	specific heat at constant pressure
h	enthalpy
M	Mach number
P	pressure
Q	heat release
T	temperature
u,v	component velocities
V	velocity
x,y	coordinate axes
β	wave angle
γ	ratio of specific heats
δ	wedge or deflection angle
ρ	density
λ	ratio of stagnation enthalpies
μ	Mach angle

Subscripts

0	stagnation conditions
1	initial condition
2	conditions behind wave
J	Chapman-Jouguet condition
n	normal component
t	tangential component
T	tangent
$*$	conditions at sonic velocity
s	shock wave

SUMMARY

Experimental studies of detonation waves have hitherto been confined to unsteady one-dimensional flow and normal wave conditions. The significant information deduced is rather limited due to the singularity of the wave formation. The extension of the experimental work to the inclusion of standing detonation waves, stabilized in a channel, would immeasurably add to a better understanding of this complex phenomenon, in terms of the wave structure and wave stability.

Experimental considerations of stabilizing and maintaining a detonation wave in a duct are described. These include particularly the supersonic flow of a combustible gaseous mixture over a wedge. The experimental requirement of generating the relatively high Mach numbers ($M = 4.5$ and higher) and the proper pressures and temperatures for the gaseous mixture at the test section are rather severe and demanding in apparatus. The practical requirements and limitations for a blow-down wind tunnel system are discussed.

The theoretical considerations include a discussion of the steady two-dimensional supersonic flow over a wedge wherein a stabilized nonadiabatic wave is generated, with energy addition at the wave front.

The idealized conservation equations for nonadiabatic waves lead to the polar diagram, which describes the various regimes of steady supersonic flow with oblique waves. The possible solutions and the flow conditions behind the stabilized wave are considered and discussed in terms of initial flow conditions, energy release, and wedge angle.

A graphical solution for the nonadiabatic wave polar is presented which permits a simple construction of the diagram.

I. INTRODUCTION

Gaseous flow problems, involving steady one-dimensional flow of a perfect gas, are quite numerous insofar as their analytical formulation is concerned. However, only a few that represent physically possible processes have been completely solved. Of particular importance are the flow processes which involve sudden changes of physical parameters in a short distance, processes which involve flow discontinuities. The physical flow discontinuities may include:

(a) contact discontinuities (P = const., V = const.) and
(b) shock waves (T_o = const.).

The above two involve problems of steady or unsteady adiabatic flow, and have been dealt with to a considerable degree.

(c) Flame fronts (P = const. Q = finite)
(d) Detonation fronts (Q = finite)

The latter two involve flow problems of nonadiabatic processes, with energy addition at the front. A considerable amount of work has been done, both experimental and theoretical, on these flow processes.

The above often-experienced discontinuities show an order of increasing complexity, with the detonation wave being the most complex. A detonation wave may be described as an adiabatic shock wave plus a non-adiabatic flame front, a combination of two discontinuities. This description is fairly well substantiated, at least in a macroscopic sense.

The detonation wave may be analyzed quite readily if the analysis is confined to an idealized hydrodynamic system. Three important aspects must be considered in its general analysis, dictated by:

(a) laws of conservation,
(b) stability considerations, and
(c) mechanism of chemical reactions.

The hydrodynamic detonation wave theory does not explicitly introduce the chemical reactions by means of which chemical energy is released, and does not satisfactorily demonstrate the validity of the stable Chapman-Jouguet detonation velocity. However, calculations based on this theory are in very good agreement with existing experimental results for plane detonation waves.

Recognizing the theoretical limitations of detonation theories, a worthwhile effort would be to stabilize experimentally the detonation wave in a channel flow. Compared with the unsteady flow, plane wave, experimental approach of many investigators, usually conducted in flame or shock tubes, the problem of generating a stationary detonation wave is a formidable one. However it is entirely plausible, although demanding in experimental apparatus, and could shed considerable light on the structure, stability, and mechanism of energy release of a detonation wave. A wind tunnel of this type would also extend the experimental considerations of plane waves to those of diagonal or oblique detonation waves, generated by suitable geometrical shapes. The generation of strong detonation waves seens possible in steady channel flow in contrast to the unsteady flow experiments, which, to the authors' knowledge, have not realized it. Considerable information could also be gained on the deflagration portion of the wave in studies involving gaseous mixtures where the combustion and shock zones form distinctly separate regions.

Oblique detonation waves can be readily analyzed if the assumptions made lead to a purely aerothermodynamical process. The velocities associated with a steady oblique wave can be readily derived and conveniently represented in a velocity vector plane (hodograph).

Reference 9 describes geometrical properties of stationary detonation waves through a polar representation of the velocities. The derivation of the polar equation is described in the present paper, and the treatment extended to the inclusion of a graphical solution for the polar diagram.

II. PROPERTIES OF DETONATION WAVES

The detonation wave may be treated, as previously mentioned, as a shock wave followed by a combustion front or zone. The shock wave itself then is a special and limiting case of the detonation wave. Many properties of detonation waves can be and have been predicted by an idealized theoretical treatment, and also can be measured experimentally in shock or flame tubes generating one-dimensional unsteady flows. Let us consider a detonation wave traversing a constant area tube filled with a gaseous combustible mixture(Fig. 1). Assuming idealized conditions, the conven-

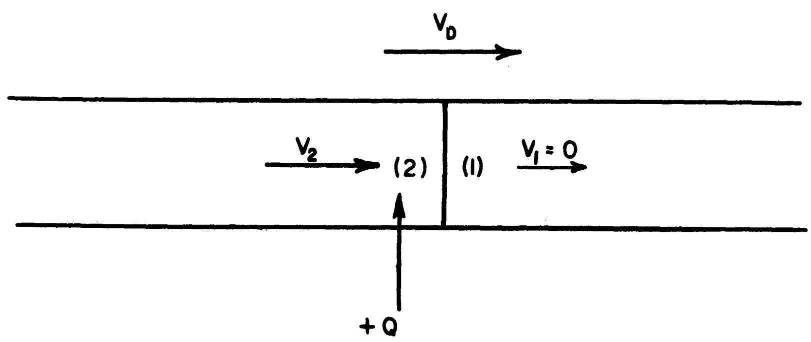


Fig. 1. Detonation wave in a constant area duct.

tional conservation equations can be readily formulated and solved for the various parameters. The following idealizations are usually assumed in an idealized hydrodynamic treatment:

1. Perfect fluid
2. No change in the molecular weight of the gas,
   (R = const., γ = const.)
3. Energy addition occurs instantaneously and only across the the wave
4. The wave is a mathematical discontinuity, i.e., of infinitesimal thickness; chemical reactions, heat conduction, and viscosity effects are neglected
5. The wave is stable.

The process is usually described, after eliminating the velocity terms, in the form of the Hugoniot curve (Fig. 2) where P_1 and v_1 represent

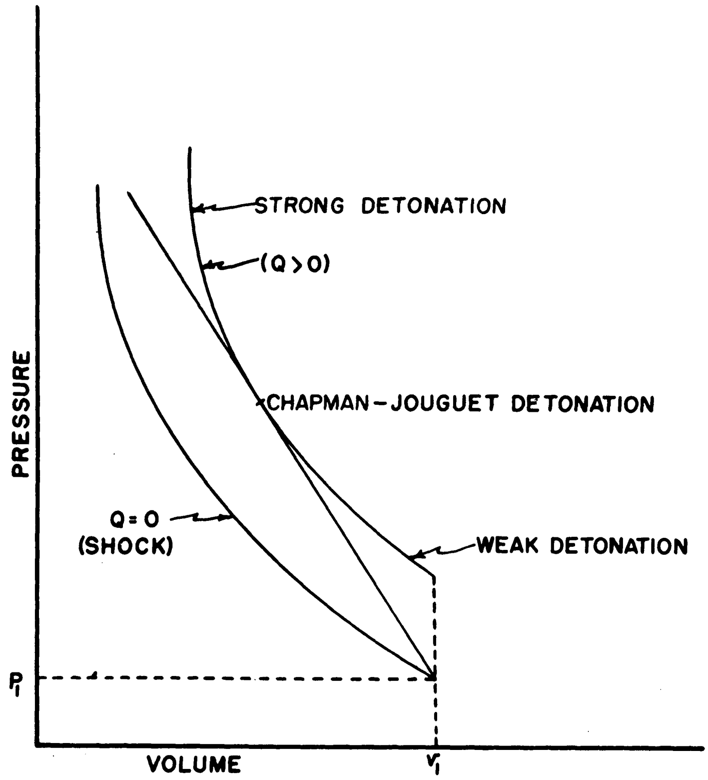


Fig. 2. Hugoniot curve, detonation branch.

the initial conditions of the mixture. In the case of a noncombustible mixture, the heat release is zero and the curve represents an adiabatic shock wave process. The shock curve passes through the initial state, while the nonadiabatic curve does not. There is also a deflagration branch to the Hugoniot curve, which we shall neglect in this paper.

Hydrodynamic theory predicts three types of detonations, which are classified according to the velocity of the burned gases relative to the wave front, i.e., $V_D - V_2^1$. This classification is:

(a)	subsonic relative velocity	strong detonation
(b)	supersonic relative velocity	weak detonation
(c)	sonic relative velocity	Chapman-Jouguet detonation

The weak detonation may be ruled out on the basis of entropy considerations. Strong detonation waves are theoretically possible, but have never been, to the authors' knowledge, observed as a stable state. In the type of experiment depicted here, the Chapman-Jouguet detonation wave is the only one generated in a gaseous mixture. This is not too surprising, when it is realized that strong detonation waves have subsonic flow velocities behind the front, and rarefaction waves can overtake the front and weaken it until the Chapman-Jouguet conditions are reached. Detonation waves generated in shock or flame tubes are normally influenced by the rarefaction waves, unless extreme driving pressures are used. A typical pressure distribution of such a wave is shown in Fig. 3, where

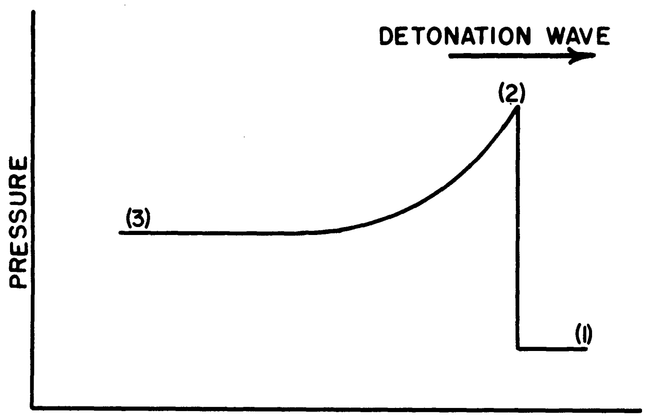


Fig. 3. Pressure distribution across a detonation wave.

point (2) corresponds to the Chapman-Jouguet state, and point (3) to the plateau pressure established by the rarefaction waves.

A very convenient classification of detonation waves in terms of heat release has been proposed by Morrison and Adamson.[2] They introduce a number, F, defined by the equation:

$$F = 1 + \sqrt{1 - \frac{2(\gamma_2 + 1)\, M_1^2}{(M_1^2 - 1)^2} \frac{Q}{C_p T_1}} \quad .$$

The changes across the detonation wave may then be written simply in terms of F and are as follows:

$$\frac{P_2 - P_1}{P_1} = \frac{\gamma F}{\gamma + 1} (M_1^2 - 1) \quad ,$$

$$\frac{\rho_2}{\rho_1} = \frac{1}{1 - \frac{F}{(\gamma + 1)} \frac{(M_1^2 - 1)}{M_1^2}} \quad ,$$

$$\frac{T_2}{T_1} = \left[1 + \frac{\gamma F}{\gamma + 1} (M_1^2 - 1)\right] \left[1 - \frac{F}{\gamma + 1} \frac{(M_1^2 - 1)}{M_1^2}\right] \quad , \text{ and}$$

$$M_2 = \sqrt{\frac{(\gamma + 1 - F)(M_1^2 - 1) + (\gamma + 1)}{\gamma F (M_1^2 - 1) + (\gamma + 1)}} \quad ,$$

where M_2 is the Mach number of the burned gases relative to the wave front. It is interesting to note the significance of F:

$F = 2 \longrightarrow$ shock wave,
$F = 1 \longrightarrow$ Chapman-Jouguet detonation, and
$1 < F < 2 \longrightarrow$ strong detonation wave.

Spark schlieren photographs of ethane-oxygen Chapman-Jouguet type detonations are shown in Fig. 4. The 25% mixture is near stoichiometric, while the 6% mixture is close to the lean limit of detonation. The density gradients indicated are normal to the wave front.

A number of investigators have measured the velocity of normal gaseous detonation waves.[3,4] These velocities are ordinarily in the range of about 4,000 to 12,000 feet per second. For our purposes the Mach numbers of detonation are more meaningful. Some typical ranges of the Mach number are shown in Fig. 5, where the data were taken from Reference 3. As can be seen, these Mach numbers are in the hypersonic range. Consequently, in order to stabilize a detonation wave one is faced with the high ratios of stagnation temperature to static temperature and stagnation pressure to static pressure. Hence, it becomes important to consider the effect of temperature and pressure on the Mach number of detonation. Information of this type is quite limited. However, Cannon and Jewell[5] have measured the effect of temperature on the Mach number of detonation for a few gaseous mixtures. Their results agree quite well with theoretical considerations, which predict that the Mach number varies inversely as the square root of the static temperature.[3] The effect of pressure on detonation Mach number was measured in this laboratory for a hydrogen-air-helium mixture. The results are shown in Fig. 6. The original data were subject to appreciable scatter and the curve represents average values. The effect

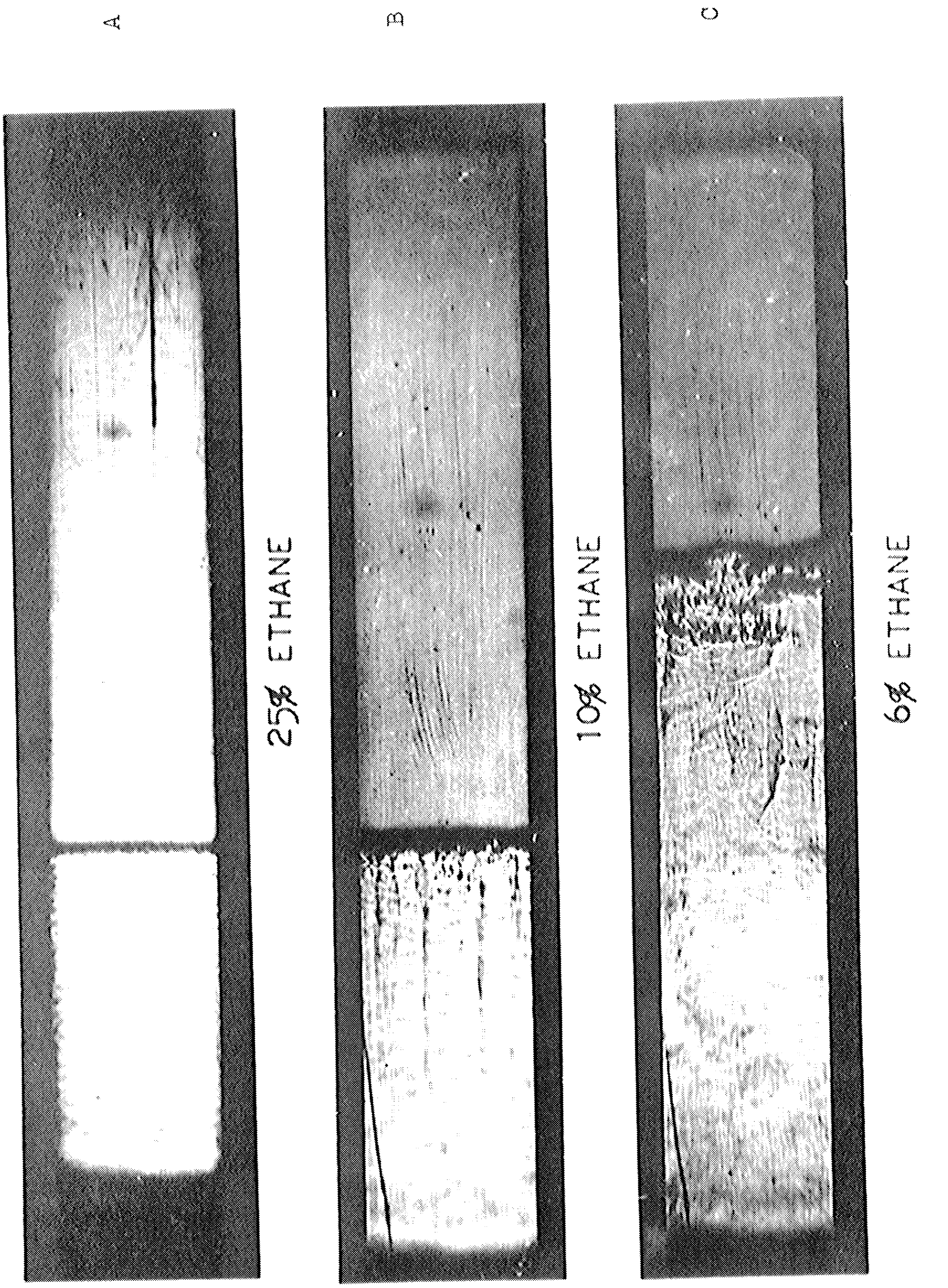


Fig. 4. Ethane - oxygen detonations.

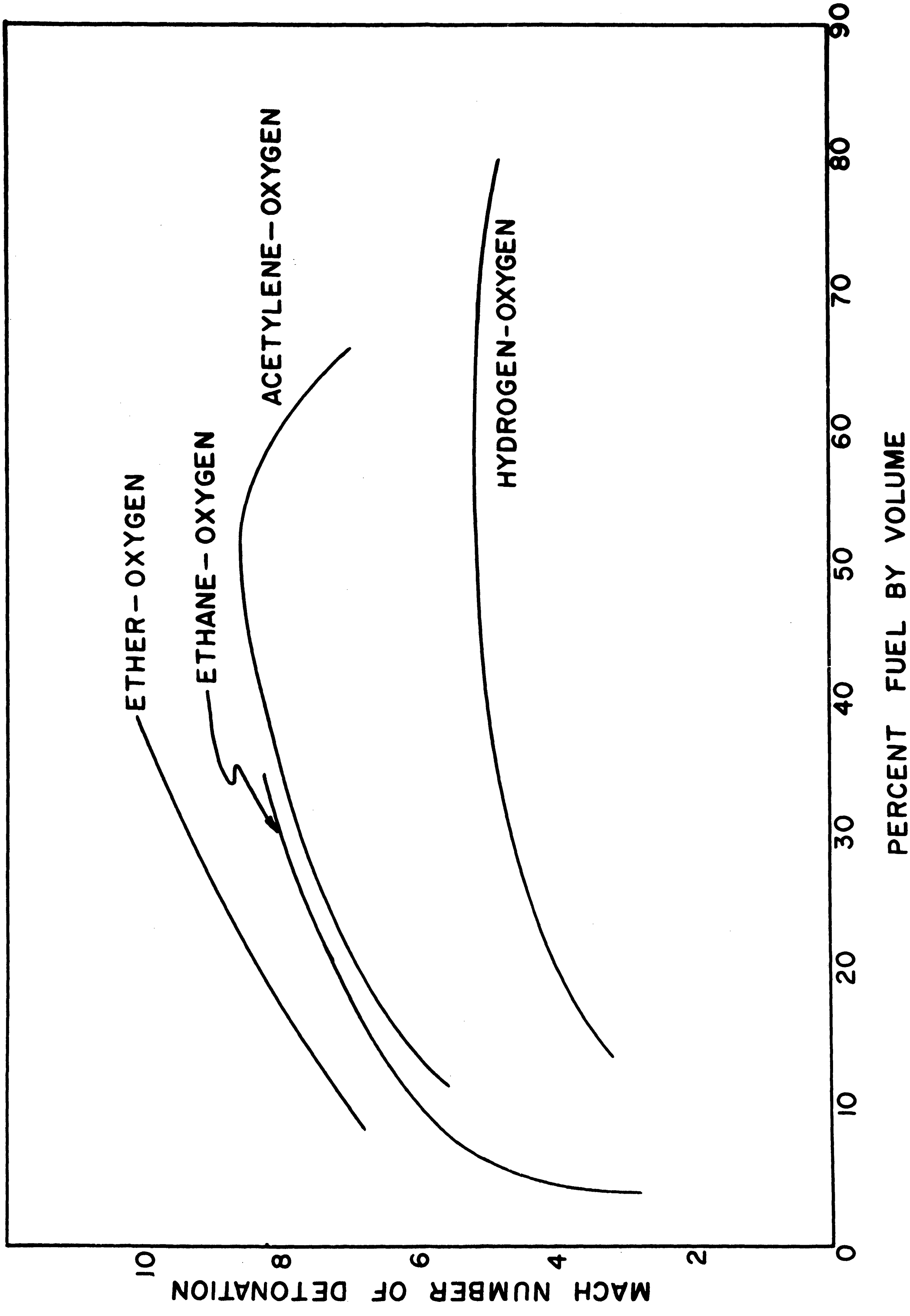


Fig. 5. Mach number of detonation of various gaseous mixtures.

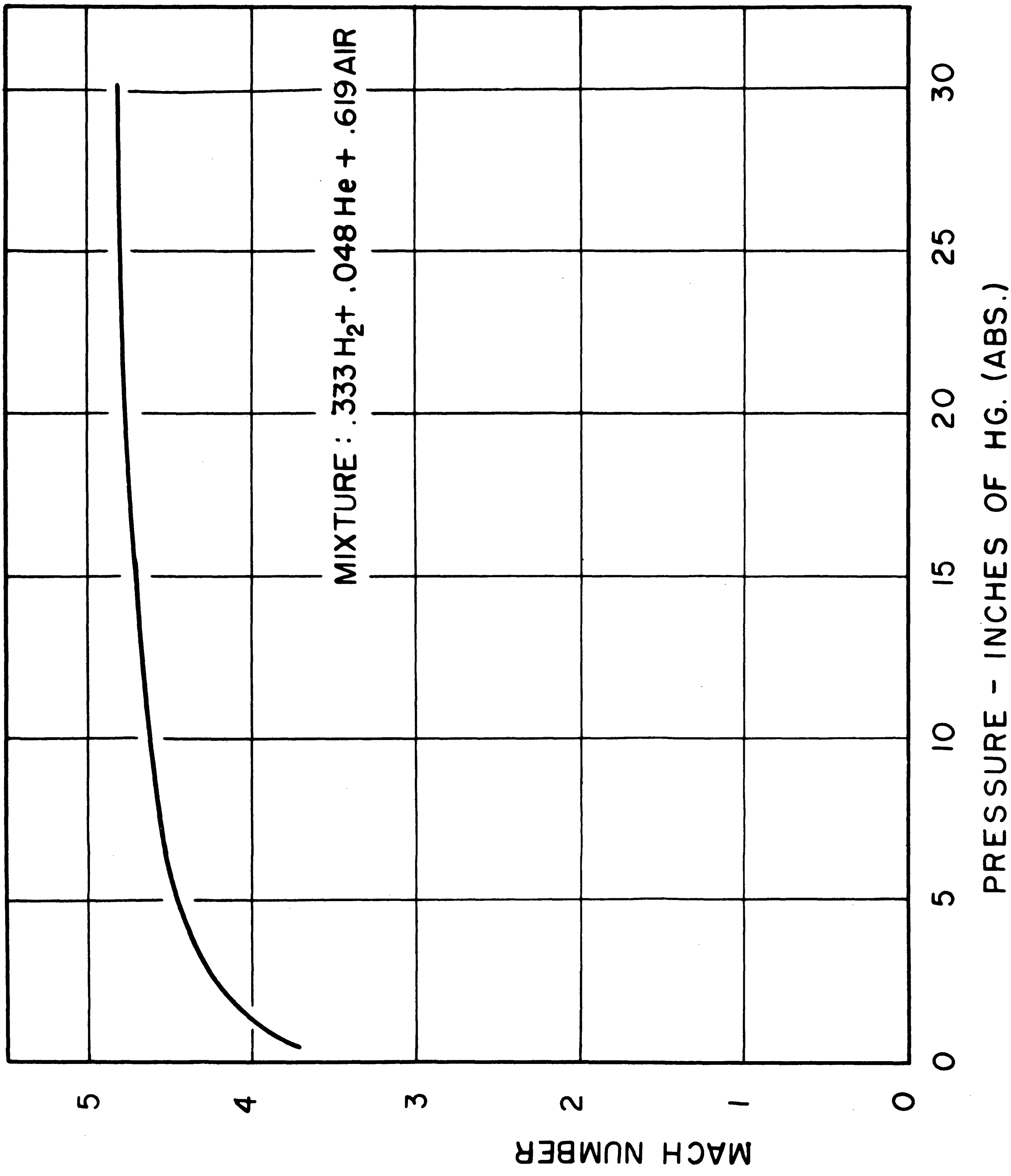


Fig. 6. Effect of pressure on detonation Mach number.

of pressure is small until rather low pressures are reached. It is felt that at these very low pressures the wave structure is probably tending toward the type shown in Fig. 4-C. Some additional information on detonation at low pressures is given in References 6 and 7.

III. CONDITIONS TO BE MET IN STABILIZING A DETONATION WAVE

It is anticipated that if a detonatable gaseous mixture could be accelerated to a high Mach number without premature burning and at the right conditions of pressure and temperature, a detonation wave could be stabilized in a channel or on some suitable body in the stream. Although a stability analysis has not been undertaken, there is reason to believe that such a wave could be stabilized just as shock waves are stabilized.

The achievement of high Mach number flows requires high pressure and temperature ratios. If the tests are solely of an aerodynamic nature, extreme stagnation conditions can usually be avoided by testing at very low pressures and temperatures. However, with the added complexity of chemical reaction, the freedom of choice for the static testing conditions is more restricted and consequently more demanding in experimental apparatus. Consider the expansion of a detonatable gaseous mixture to a supersonic Mach number and assume that a normal Chapman-Jouguet detonation is stabilized in the test section. For a given stagnation temperature, the variation of static temperature with Mach number may be readily plotted as shown in Fig. 7. The Mach number of the Chapman-Jouguet detonation may be introduced into this plot by assuming the inverse square root variation with static temperature and neglecting the effects of pressure. It is apparent that the intersection of the two curves represents the minimum Mach number at which the wave could be stabilized. Further, this minimum Mach number decreases as higher stagnation temperatures are utilized.

The above analysis is restricted to a normal Chapman-Jouguet detonation. Experimentally, it should be easier to stabilize a detonation wave in a high velocity flow over wedge or cone. For this case the normal component of the attached wave would be the Mach number of detonation and hence the free-stream Mach number must be increased. On Fig. 7 this represents an area to the right of the point of intersection. With some restriction on the static pressure that can be tolerated, the higher Mach numbers demand excessive stagnation pressure. This combination of high stagnation temperature and high stagnation pressure is difficult to obtain experimentally and is beyond the capabilities of conventional heat exchangers. Consequently, any minimization of the design Mach number can result in great simplification. The selection of the fuel is thus all-

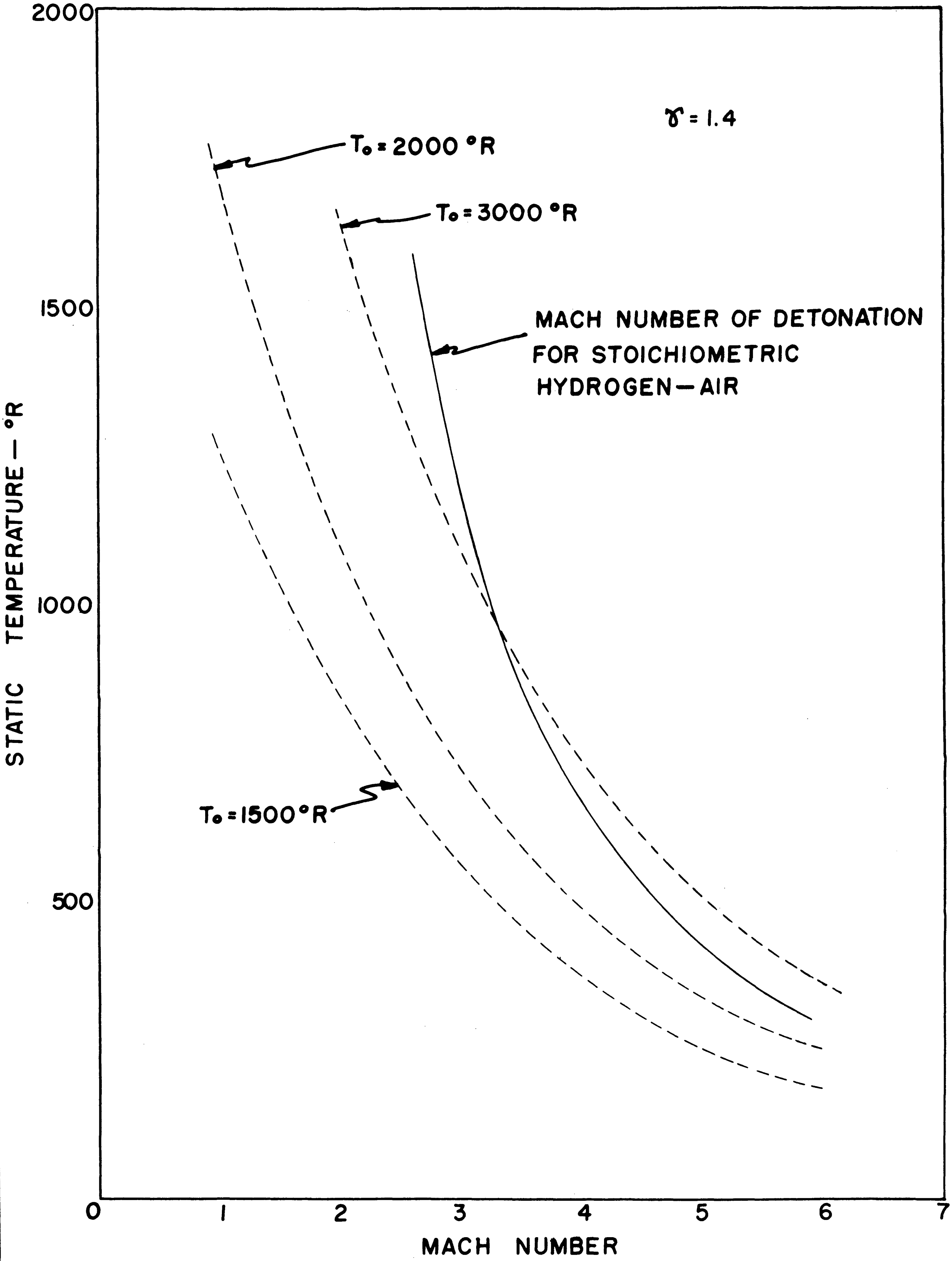


Fig. 7. Isentropic nozzle expansion.

important. Information available indicates that most Mach numbers of detonation are close to five and higher. Lower Mach number detonations appear to be of the type shown in Fig. 4-C. It is felt that this type would not readily lend itself to practical analysis if stabilized on a wedge and hence has not been considered for initial experimentation.

In view of the above demands for high stagnation temperature, it becomes impossible to mix the fuel and oxidant under stagnation conditions. One is then forced to mix the two streams only after the static temperatures have been reduced. A possible cycle is as shown in Fig. 8. Such a cycle involves heating the fuel and oxidant to high temperatures while at high pressure, separate expansion of the streams to a low supersonic Mach number, supersonic mixing, further expansion to the design Mach number, and stabilization on a suitable wedge or cone.

Calculations have been made for stabilizing a hydrogen-air detonation. These calculations indicate a stagnation temperature of 2800°R and a stagnation pressure of about 1000 psi. Such conditions are beyond the capabilities of conventional heat exchangers for the mass flow rates required. An extrapolation of existing pebble-type heat exchangers to the higher pressures and temperatures has been considered for short blow-down runs. Also calculations have been made on the possibility of adding excess hydrogen and oxygen to either or both the oxidant and fuel in pressure reservoirs. The mixture would then be burned to give the stagnation temperatures and pressures required. Although excess water vapor would be present, the calculations indicate that no condensation would occur throughout the cycle.

Supersonic mixing is necessitated in order that the static temperature be reduced to a point where deflagration will not occur. Little is known about supersonic mixing and nothing at all for the range of pressures and temperatures of interest here. Figure 9 is a spark schlieren photograph of the mixing of two supersonic airstreams. A few serious problems arise in the case of gases subject to reaction. Burning may be established prematurely by shock waves that are generated as well as by the boundary layers that are developed. Further, the rate of mixing is evidently rather slow.[8] Mathematical analysis reveals that the mixing should take place in a divergent channel in order to prevent temperature and pressure rise through the mixing zone. Conceivably the divergent mixing nozzle can be extended to yield further expansion of the mixed gases to the desired Mach number.

Stabilization of the detonation wave on a wedge involves a number of interesting concepts and it is planned to discuss this in detail.

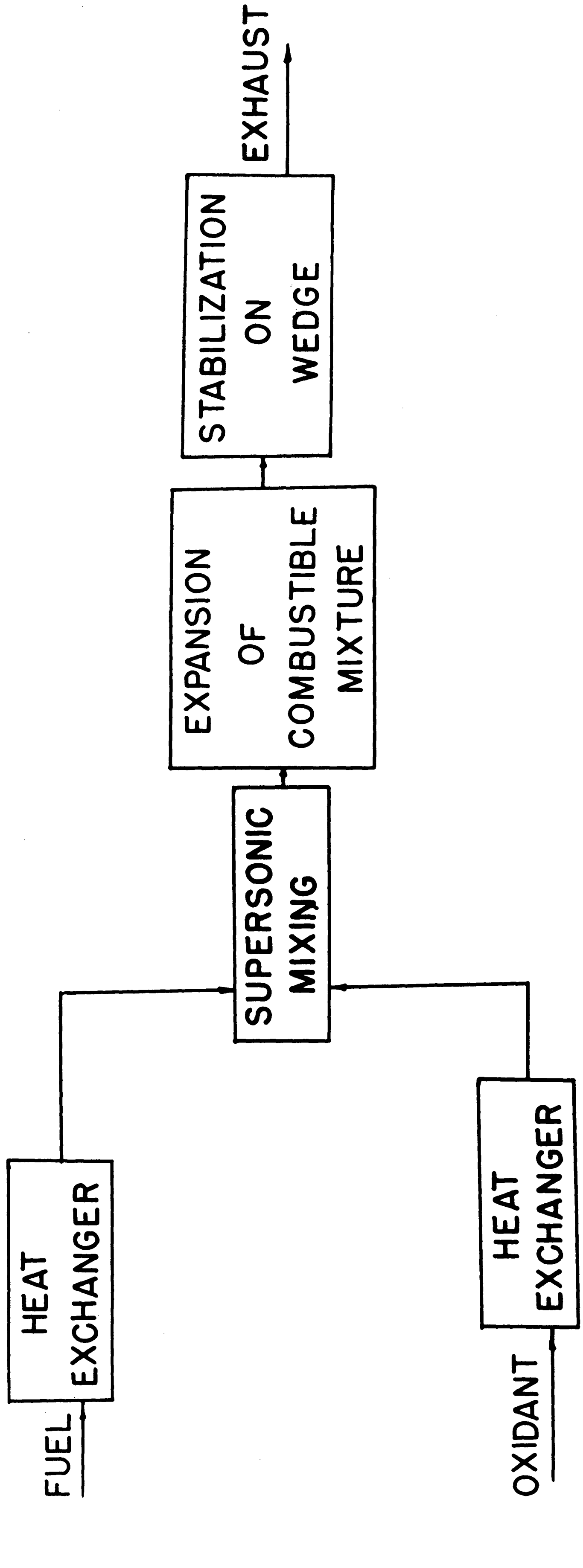


Fig. 8. Proposed experimental arrangement.

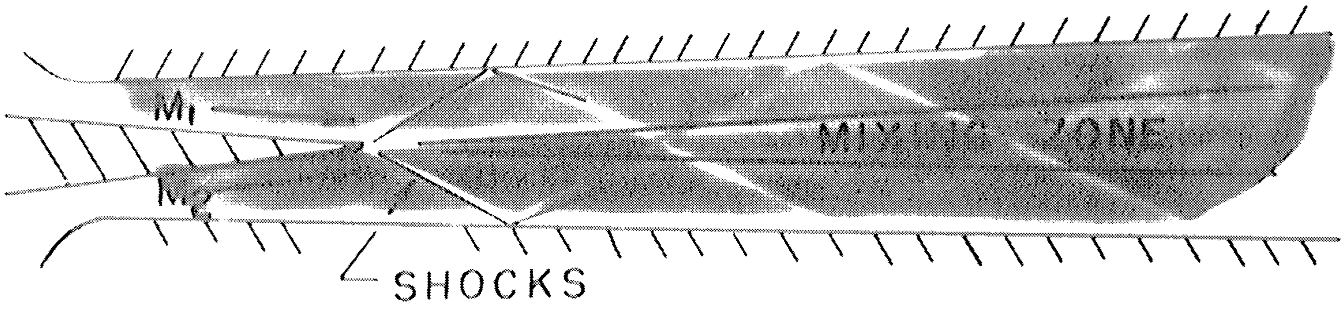


Fig. 9. Supersonic mixing.

IV. THE NONADIABATIC SHOCK-WAVE POLAR DIAGRAM

The extension of the conventional theoretical treatment of normal detonation waves, which results in the Hugoniot curve, can be readily made to a three-dimensional one by the use of the conservation equations, if the transition at each point of the wave is assumed to take place instantaneously, and consideration is given to an arbitrary small extent of this point. In general, the wave will be curved, but for some conditions, for example, a wedge in a uniform two-dimensional flow, it may be considered as a straight oblique wave.

The theoretical prediction of flow and wave conditions for stationary oblique nonadiabatic shock waves may be made by solving the conventional conservation equations. Two-dimensional flows can be interpreted conveniently in terms of the polar (hodograph) diagram; the three-dimensional symmetrical (conical) flows could be analyzed by extending the Taylor-Maccoll theory.

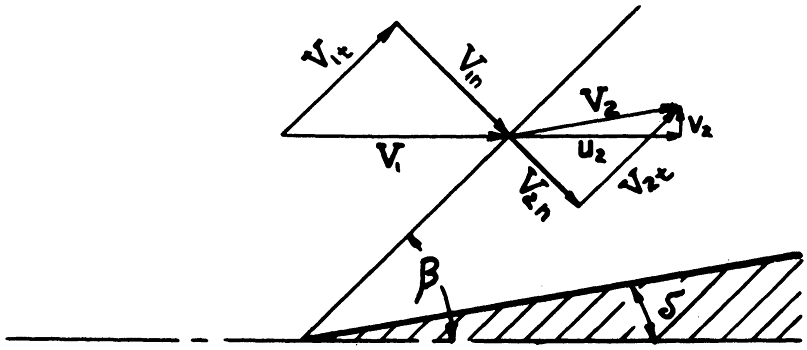
Assuming the idealizations previously given, the conservation equations for a steady diagonal two-dimensional nonadiabatic wave generated by a wedge of semiangle δ in a stream of initial velocity V_1 are:

mass	$\rho_1 V_{1n} = \rho_2 V_{2n}$
normal momentum	$P_1 + \rho_1 V_{1n}^2 = P_2 + \rho_2 V_{2n}^2$
tangential momentum	$\rho_1 V_{1n} V_{1t} = \rho_2 V_{2n} V_{2t}$
energy, front of wave	$1/2(V_{1n}^2 + V_{1t}^2) + h_1 = h_o$
energy, back of wave	$1/2(V_{2n}^2 + V_{2t}^2) + h_2 = \lambda h_o$

where λ = ratio of total enthalpies across the wave, i.e.,

$$\lambda = \frac{T_{o_2}}{T_{o_1}} = 1 + \frac{Q}{C_p T_{o_1}} \quad .$$

Figure 10 illustrates the conditions pertaining to the above



CONSERVATION EQUATIONS

MASS	$\rho_1 V_{1n} = \rho_2 V_{2n}$
NORMAL MOMENTUM	$P_1 + \rho_1 V_{1n}^2 = P_2 + \rho_2 V_{2n}^2$
TANGENTIAL MOMENTUM	$\rho_1 V_{1n} V_{1t} = \rho_2 V_{2n} V_{2t}$
ENERGY — FRONT OF WAVE	$\frac{1}{2}(V_{1n}^2 + V_{1t}^2) + h_1 = h_0$
ENERGY — BACK OF WAVE	$\frac{1}{2}(V_{2n}^2 + V_{2t}^2) + h_2 = \lambda h_0$

Fig. 10. Velocity diagram for nonadiabatic shock stabilization.

equations. Solving the above equations for the pertinent parameters yields the following results:

(a) $V_{1t} = V_{2t}$, i.e., the tangential velocity is constant across the wave, similar to a shock wave. It should be noted that nonadiabatic considerations involving stability in unsteady flow conditions will probably alter this result.

(b)
$$V_{1n} V_{2n} = \frac{V_{2n} - \lambda V_{1n}}{V_{2n} - V_{1n}} \, a_*^2 - \frac{\gamma - 1}{\gamma + 1} V_t^2 \quad .$$

This is the more general form of the Prandtl equation for a nonadiabatic wave, which simplifies for an adiabatic normal shock wave to $V_{1n} \cdot V_{2n} = a_*^2$.

Solving the above equations for v_2 in terms of V_1, u_2, and λ leads to the nonadiabatic wave polar equation:

$$v_2^2 = (V_1 - u_2)^2 \frac{V_1 u_2 - \dfrac{\lambda V_1 - u_2}{V_1 - u_2} a_*^2}{\dfrac{2}{\gamma+1} V_1^2 - V_1 u_2 + a_*^2} .$$

This equation simplifies to the familiar adiabatic shock wave polar equation for $\lambda = 1$, which can be considered as a special case of the nonadiabatic polar.

The polar equation represents the velocities associated with compression discontinuity waves in the velocity vector plane (hodograph plane). For any given velocity ahead of the wave, and a specified energy input at the wave, all (mathematically) possible velocity vectors behind it are given by a single curve. Figure 11 represents plots of the polar

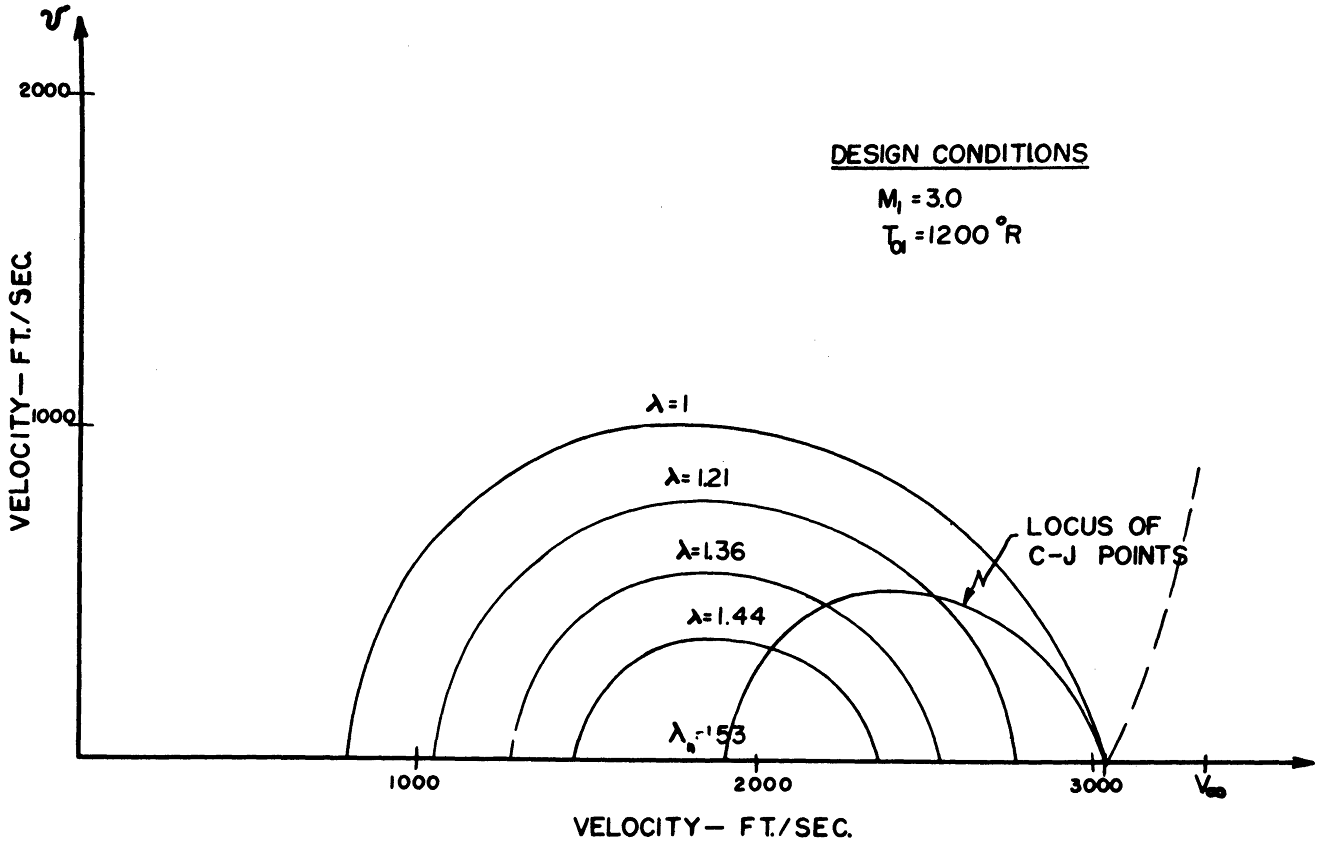


Fig. 11. Nonadiabatic shock polars.

equation for various values of λ.

Figure 12 illustrates some of the information which can be extracted from the polar diagram. Points P and Q on the u-axis ($v = 0$),

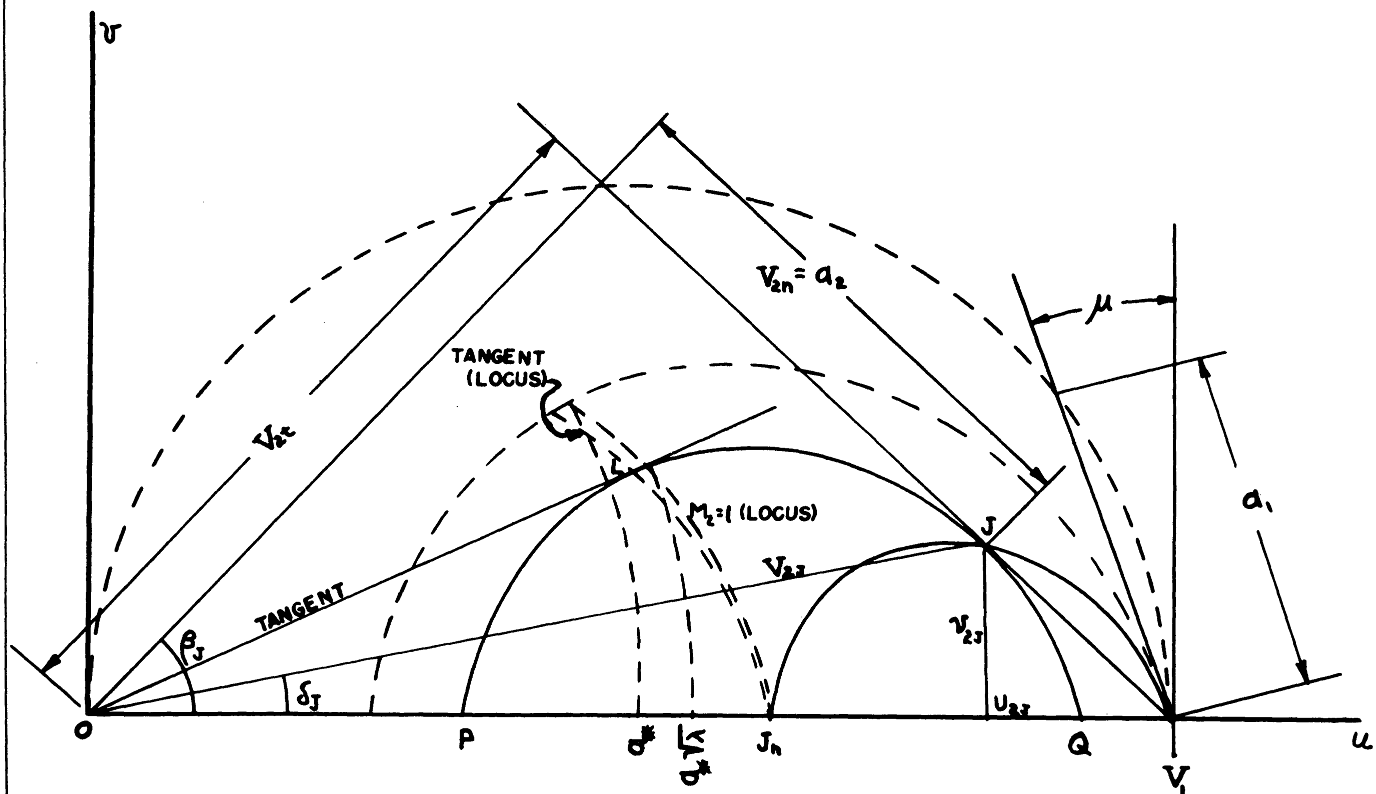


Fig. 12. Details of nonadiabatic shock polar.

represent a normal wave solution. The values of $\bar{u}_2$ are given by:

$$\bar{u}_{2n} = \frac{1}{2V_1} \left[(V_1^2 + a_*^2) \pm \sqrt{(V_1^2 + a_*^2)^2 - 4\lambda V_1 a_*^2} \right] .$$

A singular solution for u_2 at $v = 0$ occurs for a particular value of $\lambda = \lambda_n$ (point J_n):

$$u_{2J} = a_* \sqrt{\lambda_n} = \frac{V_1^2 + a_*^2}{2V_1} ,$$

which can be shown to be also (using the energy equation)

$$u_{2J} = a_2 ,$$

which is the condition for a Chapman-Jouguet wave, wherein the velocity in back of the wave is equal to the velocity of sound.

The above equations indicate the singularity of the Chapman-Jouquet condition in contrast to the two normal solutions for the more general case, i.e., point Q where $u_{2n} > u_{2J}$, weak wave; point P where $u_{2n} < u_{2J}$, strong wave.

Oblique wave solutions are given by the intersection of a line emanating from the origin at the wedge semiangle, δ, with the polar. Similar to normal waves, a singular (Chapman-Jouguet) condition exists for oblique waves, which corresponds (for a given λ and V_1) to the tangent point of a line from V_1 to the polar, where $V_{2n} = a_2$

Three regions of flow may be differentiated:

1. Solutions given by part of polar J-Q, for wedge angles $\delta < \delta_J$ where δ_J is the angle corresponding to the Chapman-Jouguet condition. These solutions are the weak waves, and are ruled out by entropy considerations.

2. Solutions given by L-J for wedge angles $\delta_T > \delta > \delta_J$, where δ_T is given by the tangent to the polar from point 0, and is the maximum angle at which the wave is still attached. This region corresponds to the strong waves, which are physically possible.

3. Solutions given by P-L, which region corresponds to detached waves, and are not covered by this analysis.

The flow conditions at wedge angle $\delta < \delta_J$, where the weak wave polar solution is ruled out, are illustrated on Fig. 13. The first stable

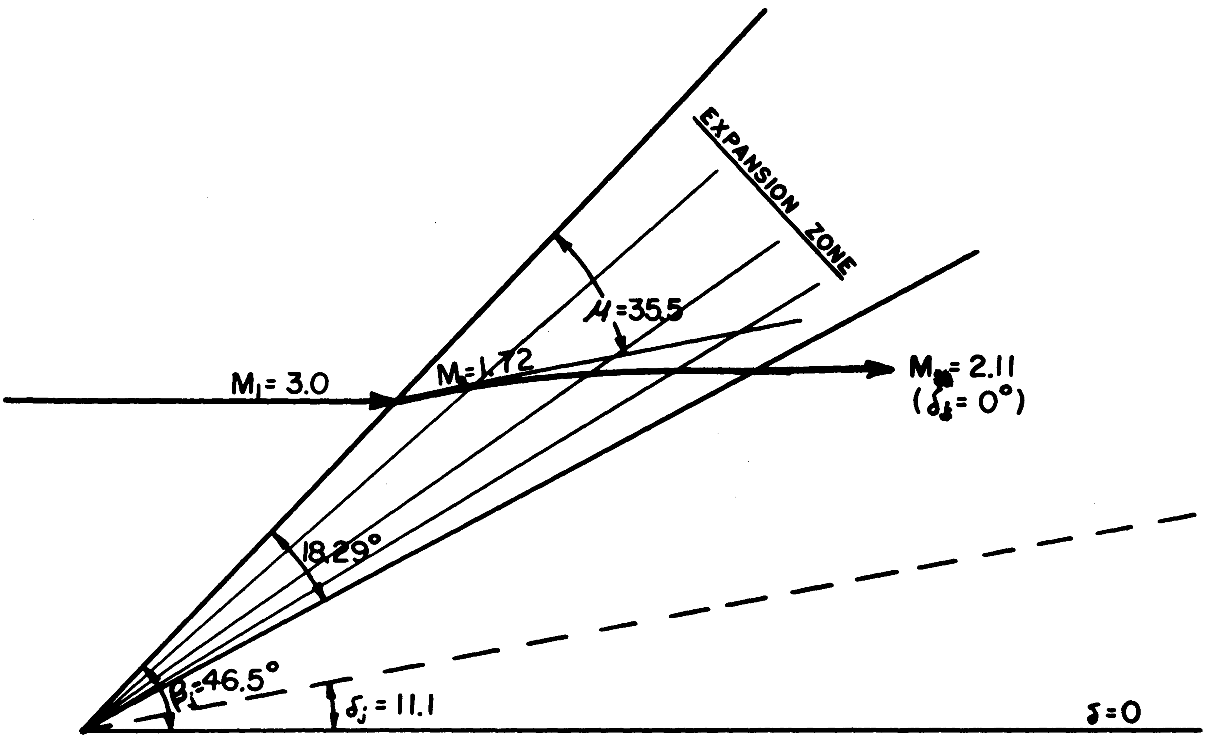


Fig. 13. Flow field for $\delta < \delta_J$.

wave angle is $\beta = \beta_J$; therefore, the wave will establish itself at this angle for any wedge $\delta < \delta_J$. Further expansion must occur behind the wave, until the flow becomes parallel to the wedge, the maximum expansion occurring for $\delta = 0$, as illustrated.

V. APPENDIX

Geometrical Construction of the Nonadiabatic Shock Polar

The nonadiabatic shock polar may also be considered in terms of the velocities normal to the wave front and certain convenient relationships may be derived, which will lead to a graphical construction of the polar.

If into the energy equation in front of wave:

$$\frac{a_1}{\gamma-1} + \frac{V_{1n}^2}{2} + \frac{V_t^2}{2} = \frac{\gamma+1}{2(\gamma-1)} a_*^2 ,$$

substitution is made for V from Prandtl's equation

$$V_t = \frac{\gamma+1}{\gamma-1} \frac{V_{2n} - \lambda V_{1n}}{V_{2n} - V_{1n}} a_*^2 - \frac{\gamma+1}{\gamma-1} V_{1n}V_{2n} .$$

Then the velocity a_1, may be expressed as:

$$a_1^2 = \frac{\gamma+1}{2} V_{1n}V_{2n} - \frac{\gamma-1}{2} V_{1n}^2 - \frac{\gamma+1}{2} a_*^2 (\lambda-1) \frac{V_{1n}}{V_{1n} - V_{2n}} .$$

The polar curve may be derived as follows:

From Fig. 14, a_1 is laid out as $\overline{V_1B}$

The similarity of the triangles (OAV_1) and (CDV_1) is evident, and so:

$$\frac{\overline{OV_1}}{\overline{DV_1}} = \frac{\overline{AV_1}}{\overline{CV_1}} ;$$

then

$$a_1^2 = \overline{BV_1}^2 = \overline{OV_1} \cdot \overline{CV_1} = \overline{DV_1} \cdot \overline{AV_1} = V_{1n} (V_{1n} - \overline{AD}) .$$

Substituting into the equation for a_1, the expression:

$$a_1^2 = V_{1n} (V_{1n} - \overline{AD}) \text{ and } V_1V_2 = V_{1n} - V_{2n}$$

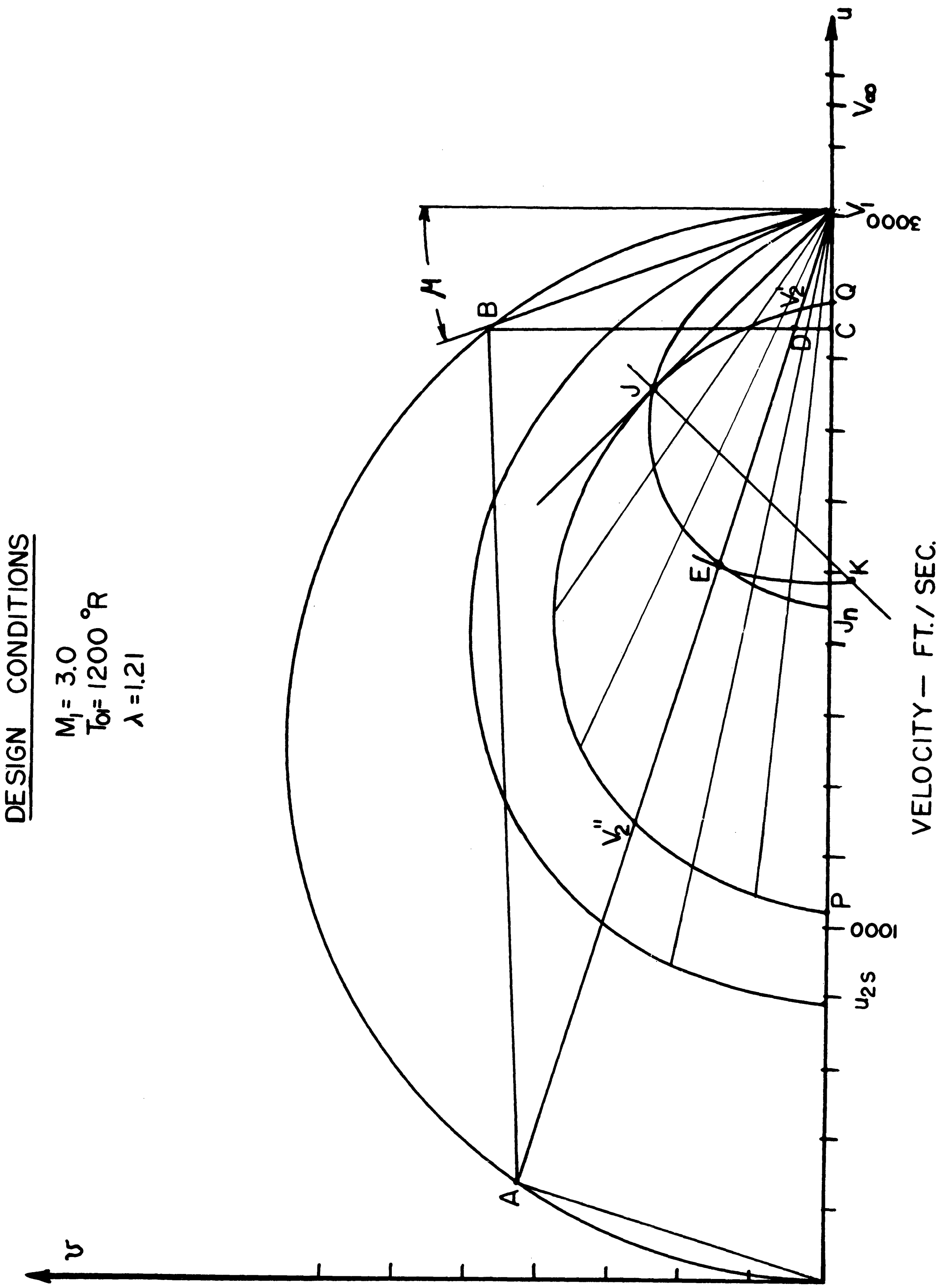


Fig. 14. Geometrical construction of nonadiabatic shock polar.

yields

$$\overline{V_1V_2}^2 - \frac{2}{\gamma+1}\overline{AD} \cdot \overline{V_1V_2} + a_*^2(\lambda-1) = 0 ,$$

which is the equation of the polar in terms of $\overline{V_1V_2}$ and $\overline{AD}$, with their meanings represented in the graph.

Solving for the double root of $\overline{V_1V_2}$,

$$\overline{V_1V_2} = \frac{\overline{AD}}{\gamma+1} \pm \sqrt{\left(\frac{\overline{AD}}{\gamma+1}\right)^2 - a_*^2(\lambda-1)} .$$

The nonadiabatic polar can now be plotted in terms of vector $\overline{V_1V_2}$ defined by the above equation. For any arbitrary line $\overline{AV_1}$ emanating from V_1, the distance $\overline{AD}$ is established by the intersection of $\overline{AV_1}$ and $\overline{BC}$. Two polar values may be evaluated, both lying on the line $\overline{AV_1}$.

$\overline{V_1V_2}$ has a singular value when the line $\overline{AV_1}$ is tangent to the polar. This occurs when:

$$\overline{AD} = a_*(\gamma+1)\cdot\sqrt{\lambda-1}$$

and

$$\overline{V_1V_2} = V_{1n} - a_*\sqrt{\lambda-1} ;$$

and from Prandtl's equation:

$$V_{2n} = \lambda a_*^2 - \frac{\gamma-1}{\gamma+1} V_t^2 .$$

If this value is now substituted into the energy equation behind the wave:

$$V_{2n} = a_2 .$$

Similarly as in the previous derivation, the single root solution gives the Chapman-Jouguet condition, which is unique for any given λ.

From the singular solution for $\overline{V_1V_2} = \overline{AD}/\gamma+1$, it is evident that the value $AD/\gamma+1$ is the distance from V_1 to the intersection of the particular line $\overline{AV_1}$ with the Chapman-Jouguet curve. The corresponding λ may be found from

$$\lambda = \left[\frac{\overline{AD}}{(\gamma+1)a_*}\right]^2 + 1 .$$

Then the equation for $\overline{V_1V_2}$ may be written as

$$\overline{V_1V_2} = \overline{V_1E} \pm \sqrt{(\overline{V_1E})^2 - a_*^2(\lambda-1)} ,$$

and for any $\overline{AV_1}$, the points V_2' and V_2'' on the polar may be determined.

The relationship to the shock polar may be found if $\lambda = 1$. Then

$$(\overline{V_1V_2})_s = \overline{2V_1E} \quad .$$

The Chapman-Jouguet curve is then half the distance from V_1 to the shock polar, on a line emanating from V_1.

The above relations lead to a graphical construction of the non-adiabatic shock polar diagram.

Following is the procedure for graphical construction of nonadiabatic shock-wave polar diagram (Fig. 14).

1. Specify initial upstream conditions.

		Example
V_1	= velocity	V_1 = 3030 ft/sec
T_{O_1}	= total temperature	T_{O_1} = 1200°R
a_1	= velocity of sound	a_1 = 1010 ft/sec
a_*	= reference velocity (M = 1)	$a_* \doteq$ 1550 ft/sec
λ	= total temperature ratio across wave	λ = 1.21

2. Calculate the following values:

V_{2s}	$= \dfrac{a_s^2}{V_1}$	V_{2s} = 790 ft/sec
$V_{2\infty}$	$= \dfrac{2V_1}{\gamma+1} + \dfrac{a_*^2}{V_1}$	$V_{2\infty}$ = 3315 ft/sec
λ_n	$= \left(\dfrac{V_1^2 + a_*^2}{2V_1a_*}\right)^2$	λ_n = 1.525
V_{2n}	$= a_*\sqrt{\lambda_n}$	V_{2n} = 1915 ft/sec
$\overline{J_nV_1}$	$= a_*\sqrt{\lambda-1}$	$\overline{J_nV_1}$ = 710 ft/sec

3. Construct shock-wave polar diagram.

4. Construct Chapman-Jouguet curve; for various straight lines emanating from V_1 measure 1/2 the distance between shock polar and V_1.

5. Measure off $a_*\sqrt{\lambda-1}$ to the intersection with the Chapman-Jouguet curve—this is the tangent point for the detonation polar; draw line connecting this point with V_1.

6. Draw line from point J (tangent point) perpendicular downward to $\overline{JV_1}$ (line $\overline{JK}$).

7. For an arbitrary line emanating from V_1 and intersecting the C-J curve (point E), measure off $\overline{EV_1}$ from V_1 to intersect line $\overline{JK}$ (point K).

8. Measure off $\overline{JK}$ on radial line $\overline{V_1D}$ to each side of point E; these two points are on the polar diagram, V_2' and V_2''.

9. Continue procedure (7) and (8) for other radial lines emanating from V_1.

10. Check points on axis:

$$V_{2n} = a_*(\sqrt{\lambda_n} \pm \sqrt{\lambda_n - \lambda}) \qquad V_{2n}' = 1045 \text{ ft/sec}$$
$$V_{2n}'' = 2785 \text{ ft/sec}$$

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