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DEPARTMENT OF AERONAUTICAL ENGINEERING
UNIVERSITY OF MICHIGAN

DEVELOPMENT OF THE MULTIPLEXED MULTIPLIER

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DEVELOPMENT OF THE MULTIPLEXED MULTIPLIER

1. INTRODUCTION

The proposed multiplexed multiplier consists of a wide-band function generator and an electronic commutator. Original plans called for the production of a final unit capable of delivering continuously logarithms of twenty channels and antilogarithms of ten channels. During the present report period, the goal has been modified to the construction of a prototype laborabory model of only three channels. The principles of operation, as well as the development work preceding the period covered by this report, are described in detail in the previous progress reports dated July 29, 1951, and September 29, 1951.

2. SUMMARY

2.1. Phototube Output as a Function of Spot Position

The output of the 1P21 multiplier phototube was found to vary excessively as the spot was deflected from the center of the cathode-ray tube. Considerably better uniformity has been achieved by means of a new optical system.

2.2. Spot-Diameter Measurements

A dynamic method of obtaining phototube output as a function of spot position as the spot was deflected perpendicularly past the edge of a mask was evolved. The resulting curve was photographed directly from the face of a calibrated cathode-ray oscillograph. Phototube sensitivity in terms of current output per unit spot deflection past the edge of the mask was obtained from the tangent to the curve. The sensitivity at several points of the cathode-ray tube was determined.

2.3. Deflection Amplifiers

The first models of the deflection amplifiers were completed and installed. Closed-loop operation of the function generator over a limited frequency range was achieved. Several methods of extending this range are discussed.

2.4. Multiplier Phototube Power Supply

An improved power supply for the multiplier phototube was designed and constructed.

2.5. Cathode-Ray Tube Accelerating Power Supply

A VR-tube regulated supply for the cathode-ray tube accelerating voltage was constructed and installed.

2.6. "Millisec" Relay Tests

The life test of the Stevens-Arnold "Millisec" relay was concluded after approximately 650 million cycles. The relay sustained no apparent damage.

2.7. Appendix I - The Closed-Loop Operation of the Function Generator

The closed-loop operation of the function generator is analyzed, and the gain equations derived for the conditions of an auxiliary approximate logarithmic input to the deflection amplifier from a source external to the loop and use of only a portion of the deflection-amplifier output for feedback purposes.

2.8. Appendix II - A Note on Necessary Spot Positioning Accuracy

The necessary spot-positioning accuracy of the function generator is analyzed and a maximum spot-positioning error determined. From this value, the minimum necessary open-loop gain is calculated. Positioning errors introduced by phototube noise current and non-uniform output as a function of spot position are compared with the derived criteria.

3. PHOTOTUBE OUTPUT AS A FUNCTION OF SPOT POSITION

Reference to Appendix I of this report will confirm the fact that any variation of phototube output as the spot is deflected in the absence of a mask will result in a source of error in the determination of the final spot position. It is desirable, therefore, to realize constant phototube output with varying spot position.

Figs. 1 and 2 are plots of phototube output as a function of spot position for two widely different intensities as the spot is deflected along the vertical and horizontal axes respectively. All curves are reduced to a reference value of unity at zero deflection and pertain to the function generator as it existed at the beginning of this report period. Data from which the curves were plotted were obtained through point-by-point meter readings as the spot was deflected. Random fluctuations of output as the spot is deflected are hidden by this method, and the peak value of these fluctuations is unknown. The difference in the shape of the curves for the two different intensities is probably due to the saturation limiting action of the photo-

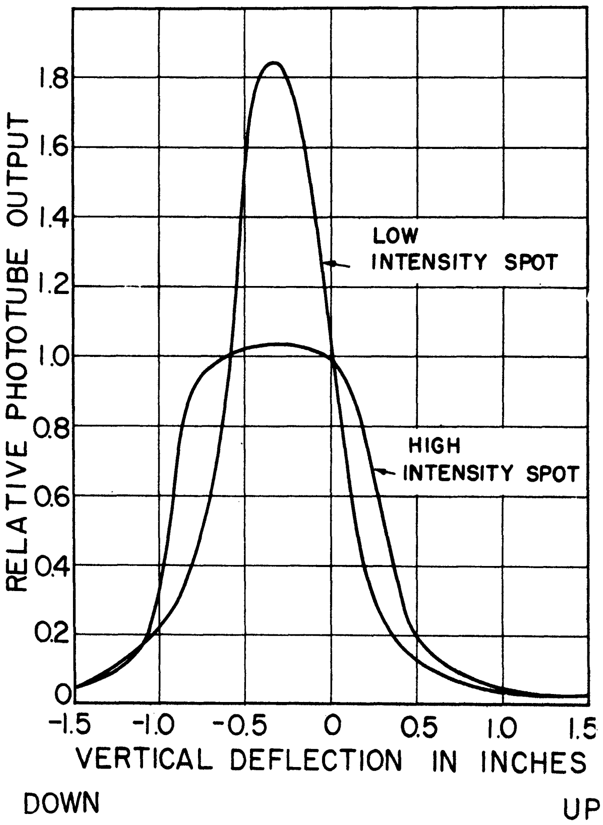


Figure 1

Phototube Output as a Function of Vertical Spot Position

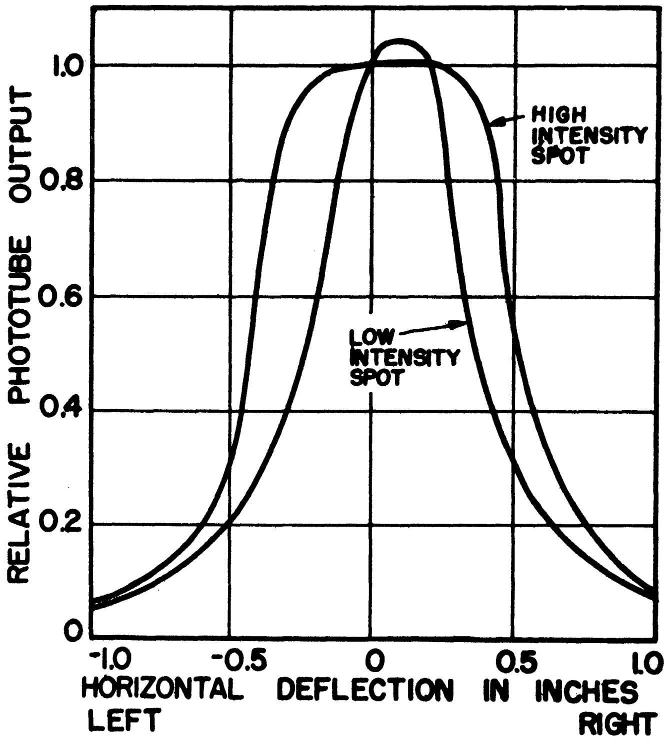


Figure 2

Phototube Output as a Function of Horizontal Spot Position

From an inspection of Figs. 1 and 2, and a study of Appendix I of this report, we may conclude that the wide variation in phototube output will result in excessive spot-positioning errors unless high values of S_p' are obtained, where

$$S_p' = S_p/i_c ,$$

S_p = phototube sensitivity in terms of current output per unit spot displacement perpendicularly past the edge of a mask, and

i_c = phototube current obtained when the spot is centered on the mask edge.

Hence, attention was directed toward methods of achieving a uniform output.

The optical system of the function generator as it existed when the data for Figs. 1 and 2 were obtained is shown diagrammatically in Fig. 3. The spot of light on the phosphor face of the cathode-ray tube is focused in the plane of the mask by means of an f/4.5, 152-mm Kodak Ektar lens. Light transmitted past this plane is diffused by a piece of frosted glass. A portion of the diffused light falls on the cathode of the phototube, resulting in an output signal. The nonuniform response obtained is due to either, or a combination of

a) nonuniform diffusion of light at the frosted surface, resulting in decreasing phototube output as the angle between the normal to the photocathode and the line joining the photocathode and spot image increases, and

b) decreasing phototube output as the angle of a constant beam of incident light deviates from the normal to the photocathode.

The evolution of the present optical system, shown diagrammatically in Fig. 4, involved trials with several condensing lenses, including Hartley field lenses (a Fresnel type of lens). The basic philosophy throughout was to focus an image of the object lens on the photocathode. Since light comes from the entire area of the object lens, its image is an illuminated circular area which, ideally, remains in a fixed position independently of the spot position. Hence, phototube output should remain constant.

The defects of the original system, itemized as (a) and (b) above, might have been minimized by simply moving the phototube to a greater distance from the ground-glass surface. It was felt, however, that the decreased optical efficiency would require unreasonable spot intensities.

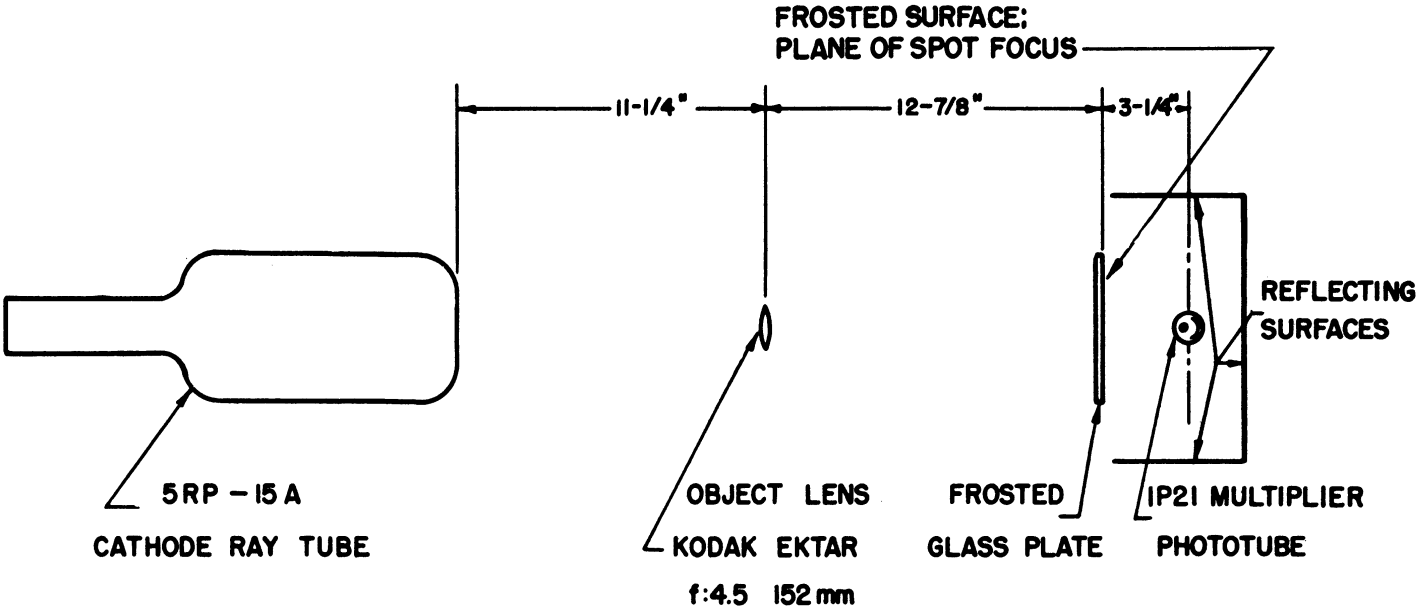


Figure 3. Original Function-Generator Optical System

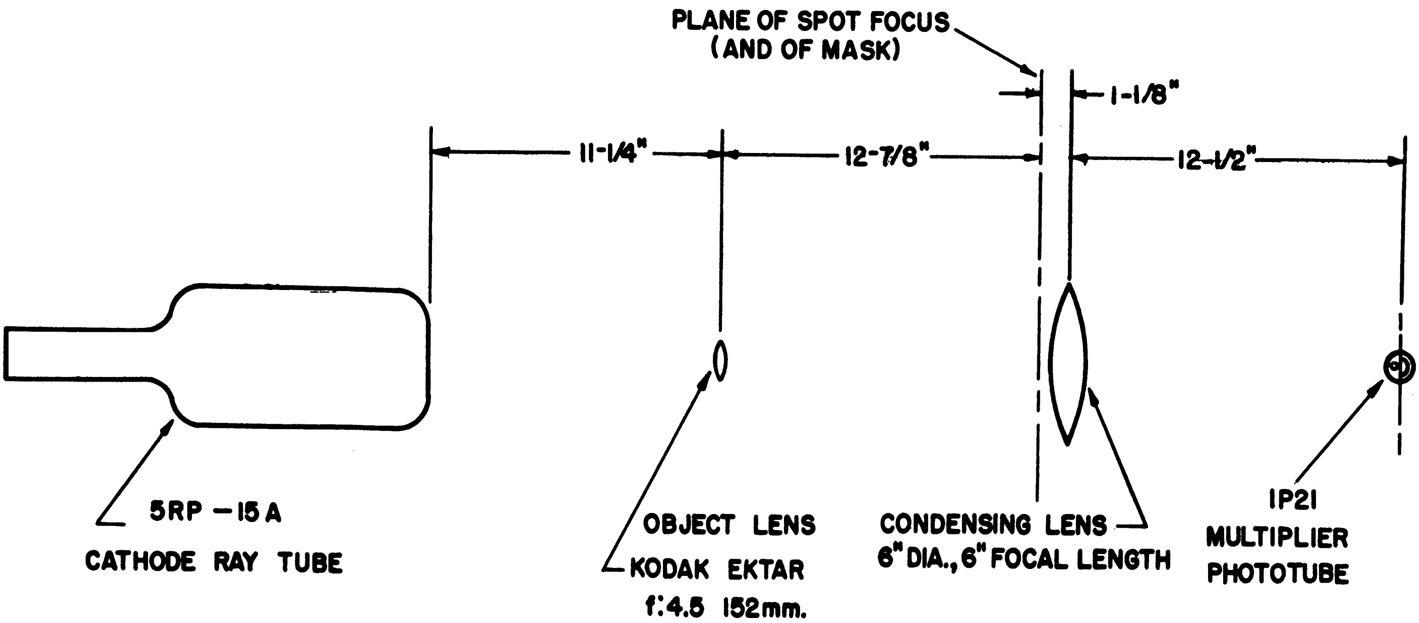


Figure 4. Present Function-Generator Optical System

The advantage of the present system lies in the fact that all the available light is directed toward the phototube. It should be noted that the available light floods the photocathode, thereby reducing the saturation effect which might exist if the light were concentrated on a smaller area.

Each trial of this system showed great improvements in phototube output as a function of spot position. The present arrangement, superior to all others tried, utilizes two 12-inch focal length, 6-inch diameter condensing lenses mounted back-to-back, giving the equivalent of a 6-inch focal length lens. These are placed directly behind the plane of the mask. The initial adjustment of the photocell is at the image of the object lens. Final adjustment is made empirically by setting the phototube at a position where the variations in output with changing spot position are at a minimum.

Rather than attempt a point-by-point evaluation of the present system such as produced the data for Figs. 1 and 2, a dynamic system, shown diagrammatically in Fig. 5, was used to display phototube output as a function of spot position on the screen of a cathode-ray oscillograph as the spot was cyclically swept parallel to one axis across the useful tube area. The sweep line was deflected along the opposite axis by means of the battery-potentiometer combination shown in Fig. 5, and readings were taken at approximately ten positions of the line across the useful tube area. A second set of readings was taken by reversing the cathode-ray tube axes. The divider at the H-axis input of the cathode-ray oscillograph merely serves to prevent overloading of the H-axis amplifier.

Each reading consisted of two components -- the indication of the vacuum tube voltmeter across the phototube output and an observation of the maximum positive and negative excursions on the face of the calibrated cathode-ray oscillograph. Fig. 6 is an oscillogram showing a typical presentation on the oscillograph. Results of this survey showed the outside limits of phototube output for all observed spot positions were -2.4 volts and -5.6 volts, respectively, or $\pm$40 per cent about a mean value of -4.0 volts. Comparison of these results with Figs. 1 and 2 indicates a major improvement, which would have appeared even more striking had data been obtained by the same method for both optical systems. While greater perfection is desirable, work on this phase of the project was suspended at this point to concentrate on other equally pressing problems.

The random excursions seen in Fig. 6 are probably due to either a nonuniformity in the cathode-ray tube phosphor which intensity modulates the spot as it moves across the surface, or to a shift in light intensity across a non-homogeneous photocathode. An attempt to isolate and correct the cause will be made when work on the function-generator optical system is resumed.

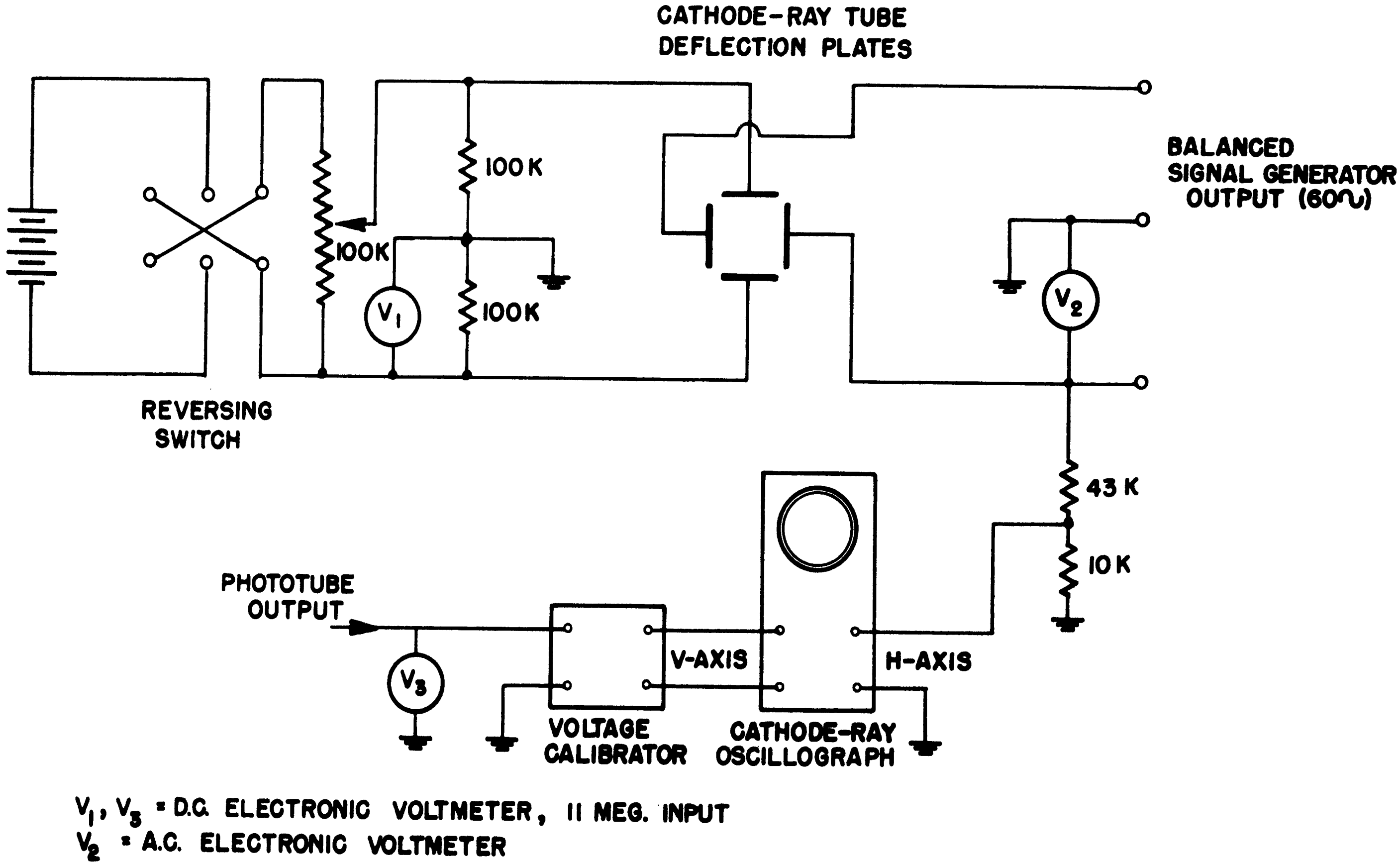


Figure 5. Dynamic Method of Displaying Phototube Output as a Function of Spot Position

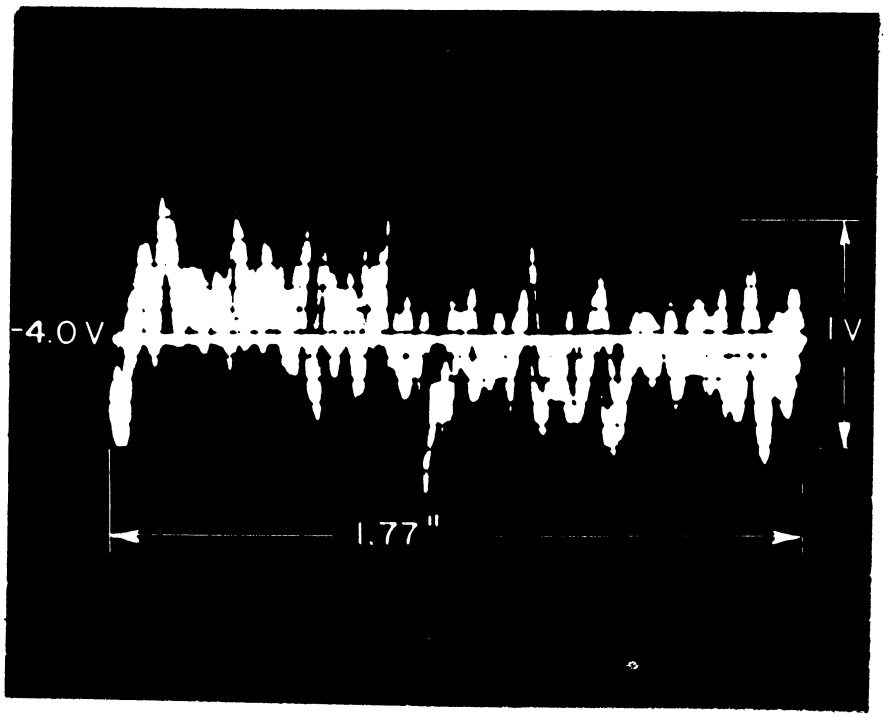


Figure 6. Typical Oscillogram - Phototube Output as a Function of Spot Position

4. SPOT-DIAMETER MEASUREMENTS

The value of phototube sensitivity, S_p, in terms of current output per unit spot displacement perpendicularly past the edge of a mask is inversely proportional to spot diameter if a circular, uniformly illuminated spot of fixed total intensity is assumed. From Appendix I of this report, it becomes obvious that it is desirable to achieve the maximum value of S_p/i_c and, therefore, the minimum spot diameter obtainable throughout the region bounding the logarithmic curve. From visual observations it appeared that serious defocusing of the spot occurred when it was deflected toward the edge of the screen under conditions of a grounded second anode and balanced deflection voltages around ground potential. It was found that focus could be improved by raising the second anode above ground potential and making the necessary focus voltage and intensity control settings. However, it was necessary to vary all these quantities to maintain optimum focus throughout the range of possible deflections. The complexity of the circuitry necessary to accomplish this type of focus compension was considered prohibitive. Instead, it was decided to restrict deflections to an area of the tube where defocusing effects were small enough to be tolerable. Following inquiries of the manufacturer's representative and further observations, this region was set as a circular area 2-1/2 inches in diameter, centered at the electrically neutral point. Thus, the lengths of the coordinates of the logarithmic curve are limited to 1.77 inches.

The next step to be undertaken was a determination of the spot diameter and the value of S_p directly. Actually, the latter is preferable since it eliminates any need for assumptions regarding spot shape or illumination gradients throughout the spot. Figs. 7 and 8 are graphs of the voltage measured across a 40-k resistor due to phototube current as the spot is deflected horizontally past a vertical straight-edge mask in a number of small steps. The two figures are for two widely different intensities, as will be seen from the values of the ordinates. In each case, the mask edge passed close to the electrical center of the tube, and the horizontal deflection path was approximately 3/8 inch below the electrical center to be at the peak of Fig. 1.

By drawing tangents to the curves of Figs. 7 and 8, the following values of S_p were determined:

For low intensity $S_p = 1.45 \times 10^{-3}$ amp/in., $S_p/i_c = 145$

For high intensity $S_p = 20.2 \times 10^{-3}$ amp/in., $S_p/i_c = 162$

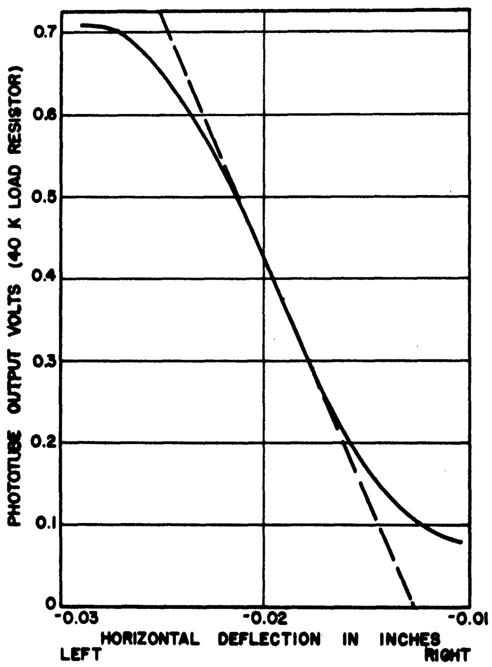


Figure 7

Phototube Output vs. Horizontal Spot Deflection Past a Vertical Mask Edge: Low Intensity

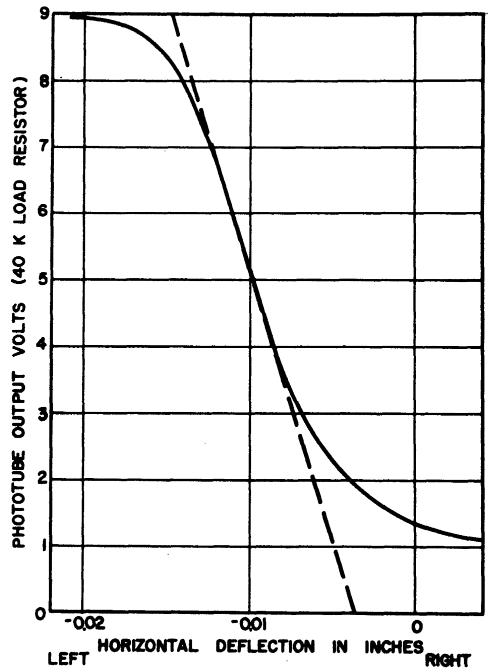


Figure 8

Phototube Output vs. Horizontal Spot Deflection Past a Vertical Mask Edge: High Intensity

As might be expected, higher values of S_p are realized with higher intensities. As can be seen from Equation (7) of Appendix I to this report, higher values of S_p ease the loop gain requirements. However, higher spot intensity will be accompanied by proportional increases in phototube noise current, i_n, and spot-centered current, i_c. Increasing spot intensity, therefore, will result in no improvement in errors due to i_n or to variations in output with spot position (see Appendix II). In subsequent work, therefore, the spot intensity was adjusted so that the spot-centered current, i_c, equalled the rated phototube output of 0.1 ma. When passed through a 40-k load resistor, this current gives an output of 4 volts. It can be noted from Fig. 8 that no saturation effects of the phototube exist at this value.

Next, the value of S_p throughout the useful tube area for both horizontal and vertical deflections was investigated. These measurements were made following installation of the latest optical system, described in a previous section of this report. As a representative sampling, nine points were selected as follows: each corner of the 1.77-inch square, the center of each side, and the center of the square. Rather than point-by-point determination of data

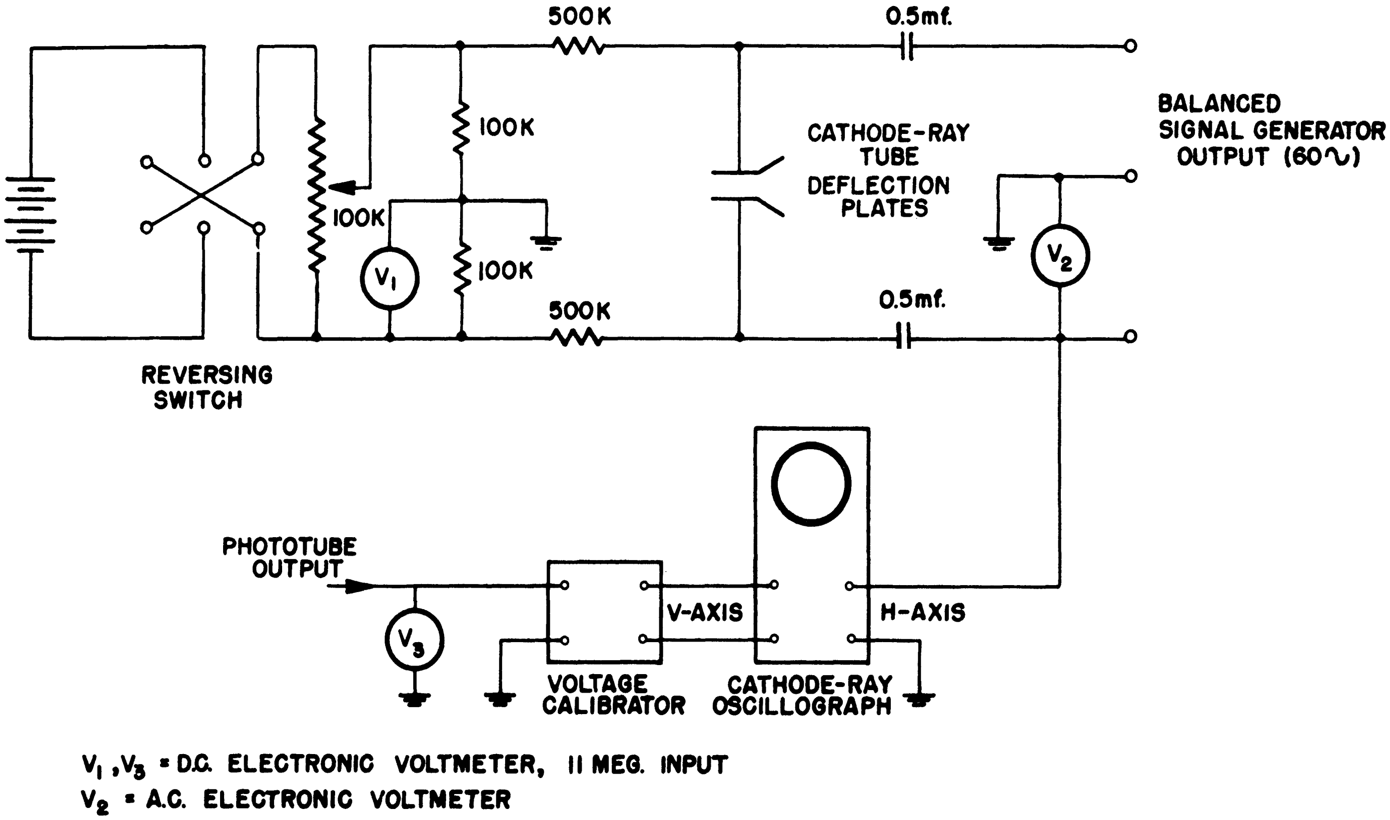


Figure 9. Circuit for Dynamic Measurements of Phototube Sensitivity, S_p

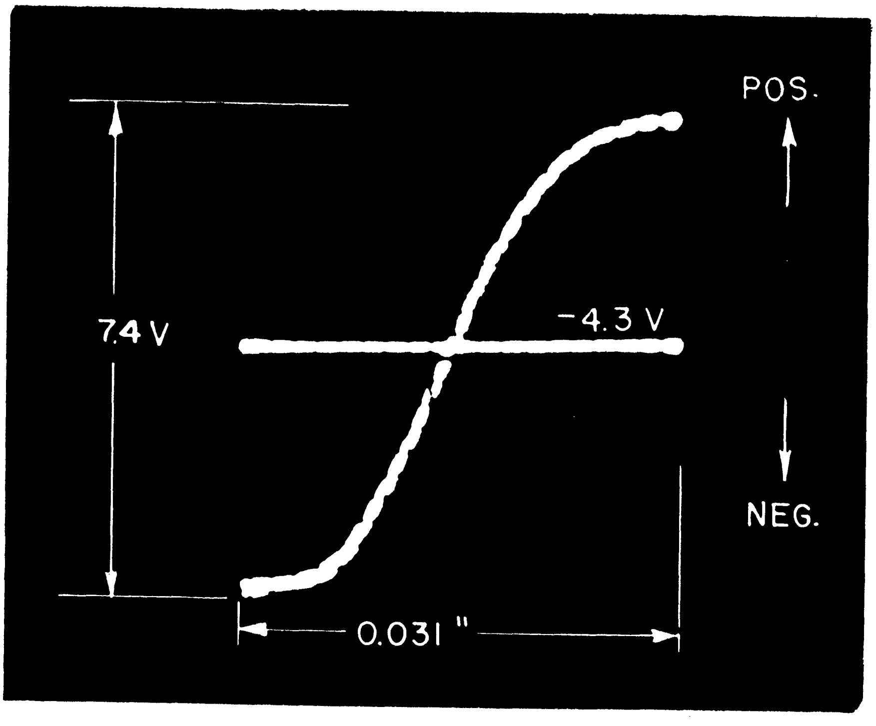


Figure 10. Typical Oscillogram - Phototube Output as Spot is Deflected Past Mask Edge

such as resulted in the curves of Figs. 7 and 8, the circuit of Fig. 9 was used to present, directly on the face of a calibrated cathode-ray oscillograph, a plot of phototube output as the spot was deflected past a mask edge. Horizontal and vertical straight-edge masks were prepared with edges passing close to the center and to points 0.88 inch from the center on each side. Steady-state deflections to position the spot at the desired edge were produced by the battery source of Fig. 9, and incremental deflections back and forth across the edge by the sinusoidal signal generator. A similar battery source, not shown in the diagram, was used to produce steady-state deflections along the other axis. A typical oscillogram taken in this manner is shown in Fig. 10.

Values of S_p were determined from the slope of the tangent to the curve as before. Values obtained in this manner varied between the limits

$$24.3 \times 10^{-3} \geq S_p \geq 10.6 \times 10^{-3} \text{ amp/in.}$$

$$236 \geq S_p/i_c \geq 103$$

for the different positions. In all cases, the average value of the output, equal to i_c, was kept very nearly constant. Paradoxically, some of the higher values of S_p determined by this method occurred at the corners of the square. This suggests that at least part of the "defocusing" observed visually was actually due to spot motion as a result of hum pickup. Further study of this problem with a view toward extending the useful tube area is indicated. A mu-metal shield for the cathode-ray tube is currently on order.

Higher values of S_p for the same total illumination would ease the amplifier requirements and reduce the errors due to noise currents and nonuniform output with spot position. This can be obtained with a spot of smaller diameter. Inquiries have been made of DuMont about such a tube.

5. DEFLECTION AMPLIFIERS

During the period covered by this report, the first models of the deflection and phase inverter amplifiers were completed and tested. These amplifiers are almost idential, the only difference being in the method of shaping the frequency-response characteristic. Both amplifiers were constructed on the same chassis and are shown in Fig. 11. The wiring diagram, exclusive of feedback and input resistors, is given in Fig. 12.

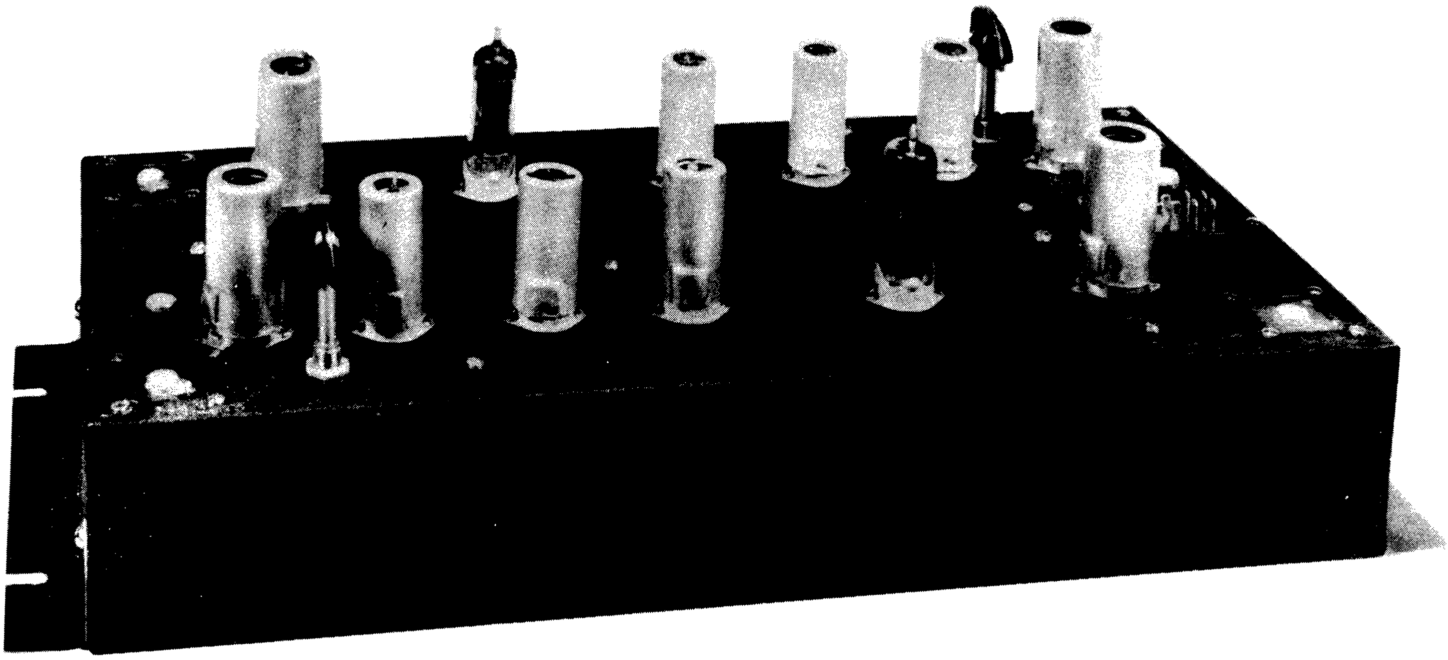
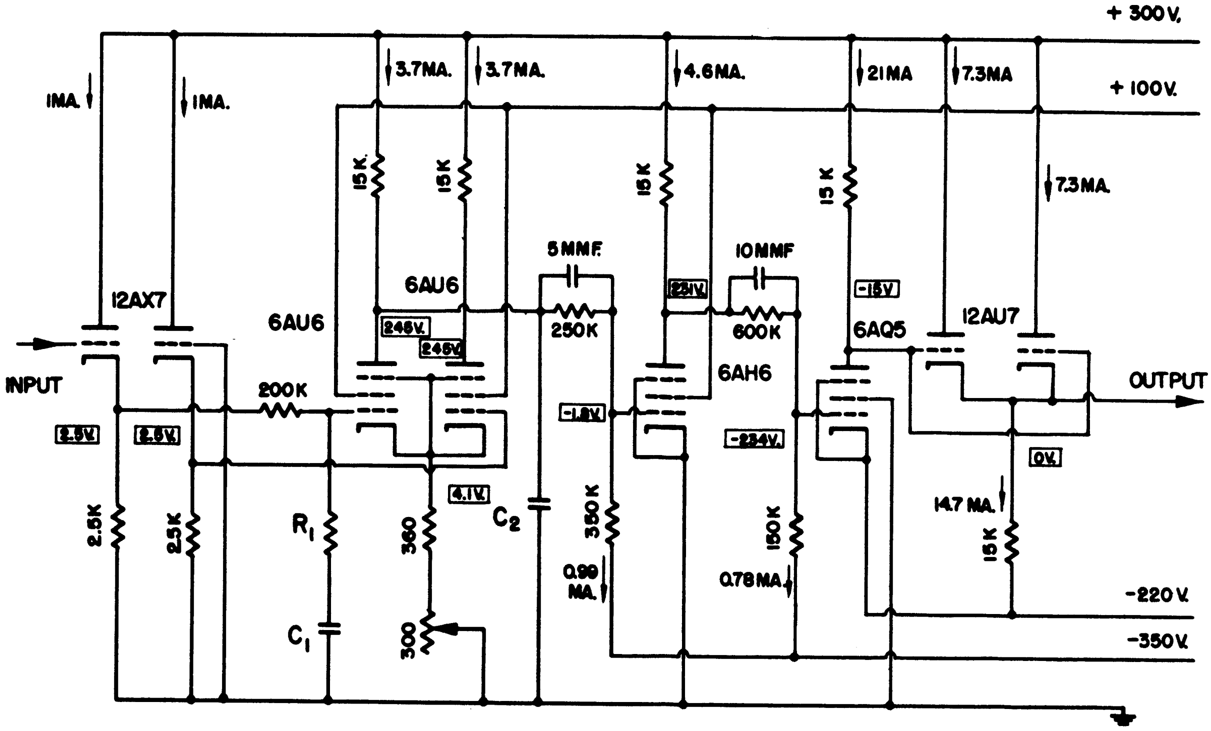


Figure 11. Deflection and Phase-Inverter Amplifier Chassis



	C_1	R_1	C_2
DEFLECTION AMPLIFIER	0.01 MF.	0	50 MMF.
PHASE INVERTER AMPLIFIER	0.004 MF.	1K	0

Figure 12. Circuit Diagram: Deflection and Phase-Inverter Amplifiers

Using the hookup of Fig. 13, closed-loop operation of the function generator was achieved. However, because of the inadequate frequency range of the amplifiers, reasonable square-wave operation was limited to frequencies below 1 kc.

Three expedients to increase the operating frequency are under consideration:

a) the provision of an approximate logarithmic input from a source external to the closed loop,

b) the use of only a portion of the amplifier output for feedback to the input, and

c) improved physical layout of the amplifiers proper.

The first two possibilities are treated in detail in Appendices I and II of this report. Briefly stated, the use of an auxiliary logarithmic input, accurate to 10 per cent, to the deflection amplifier reduces the loop gain requirements by 20 db. Thus the open-loop frequency-response characteristic

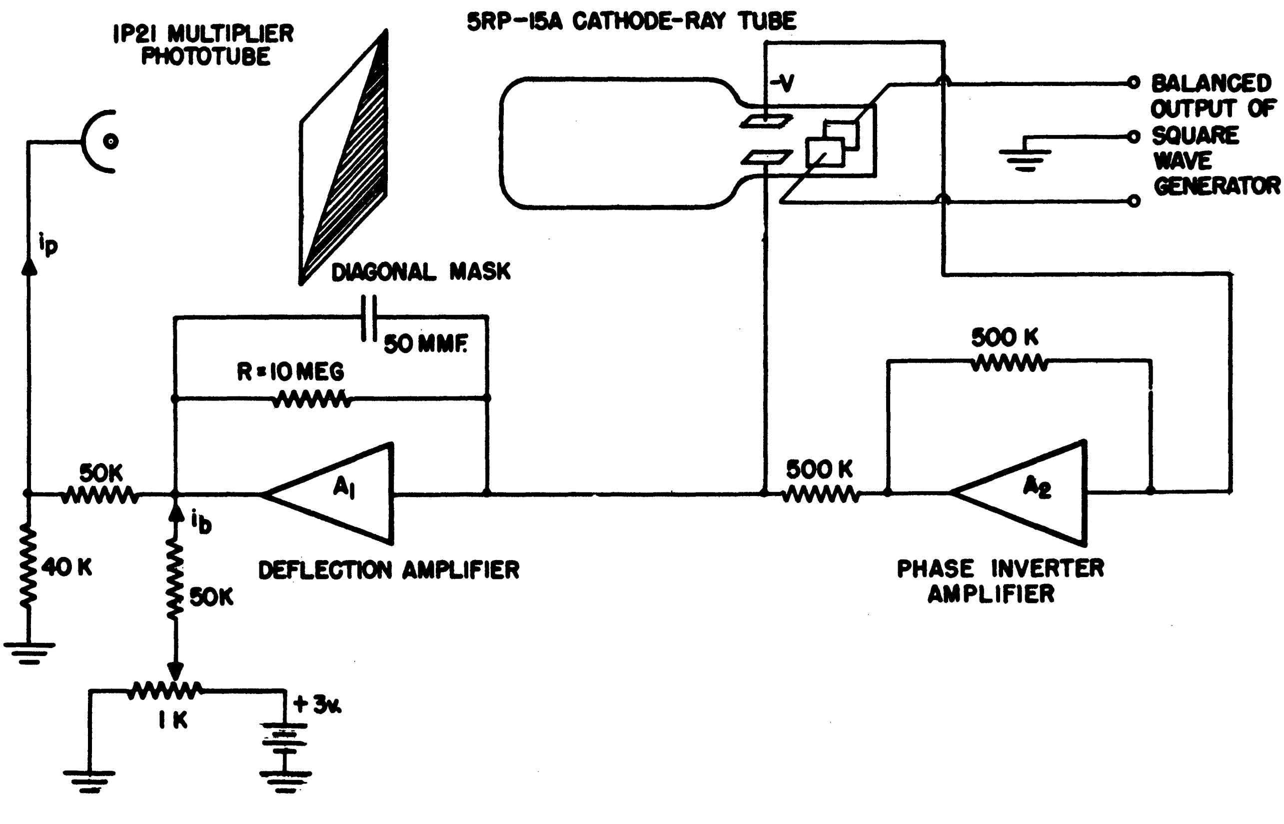


Figure 13. Function Generator Closed-Loop Diagram

can be shaped to 0-db gain about two octaves earlier, while meeting the condition of less than 12-db/octave fall-off required for stable closed-loop operation. The reduction in the controlled-frequency range eases the design requirements of the amplifier considerably. Feeding back only a portion of the amplifier output reduces the value of R (Fig. 13) required for a given open-loop gain. Hence, phase shifts introduced by the shunt capacity at the amplifier input occur at higher frequencies.

With the exception of the frequency-shaping components, the amplifier design of Fig. 12 is believed to be adequate for our purpose. Cathode followers are utilized at both the input and output. Thus a low output impedance of the order of 250 ohms is obtained and the input capacity is essentially reduced to the grid-plate capacity of the 12AX7 input tube (approx. 2.0 mmf). It will be noted that provision is made for a drift-stabilizing input should one prove to be necessary. Initial balance is obtained manually by means of the 300-ohm variable resistor in the common cathode circuit of the 6AU6 stages.

6. MULTIPLIER PHOTOTUBE POWER SUPPLY

The dynodes and the anode of the photomultiplier tube each had separate variable supplies. The regulation of the dynode supply was poor and the ripple content high. This condition was corrected by means of the power supply shown schematically in Fig. 14, which supplies current for both the anode and the dynodes. The output voltage is equally divided between the photomultiplier elements, so that the 1P21 is operated with 100 volts per dynode stage and 100 volts between dynode No. 9 and anode. From no load to a 10-ma load, the RMS ripple content measured 0.005 per cent and the output resistance 40 ohms.

7. CATHODE-RAY TUBE ACCELERATING POWER SUPPLY

Fig. 15 is a diagram of the accelerating supply for the cathode-ray tube which was constructed when the current capacity of the 20-kv regulated supply was found inadequate to provide the accelerating as well as the post-acceleration power. It will be noted the only regulation is obtained by means of VR tubes. This supply will eventually have to be replaced by one affording better regulation similar to the multiplier phototube supply of the previous section.

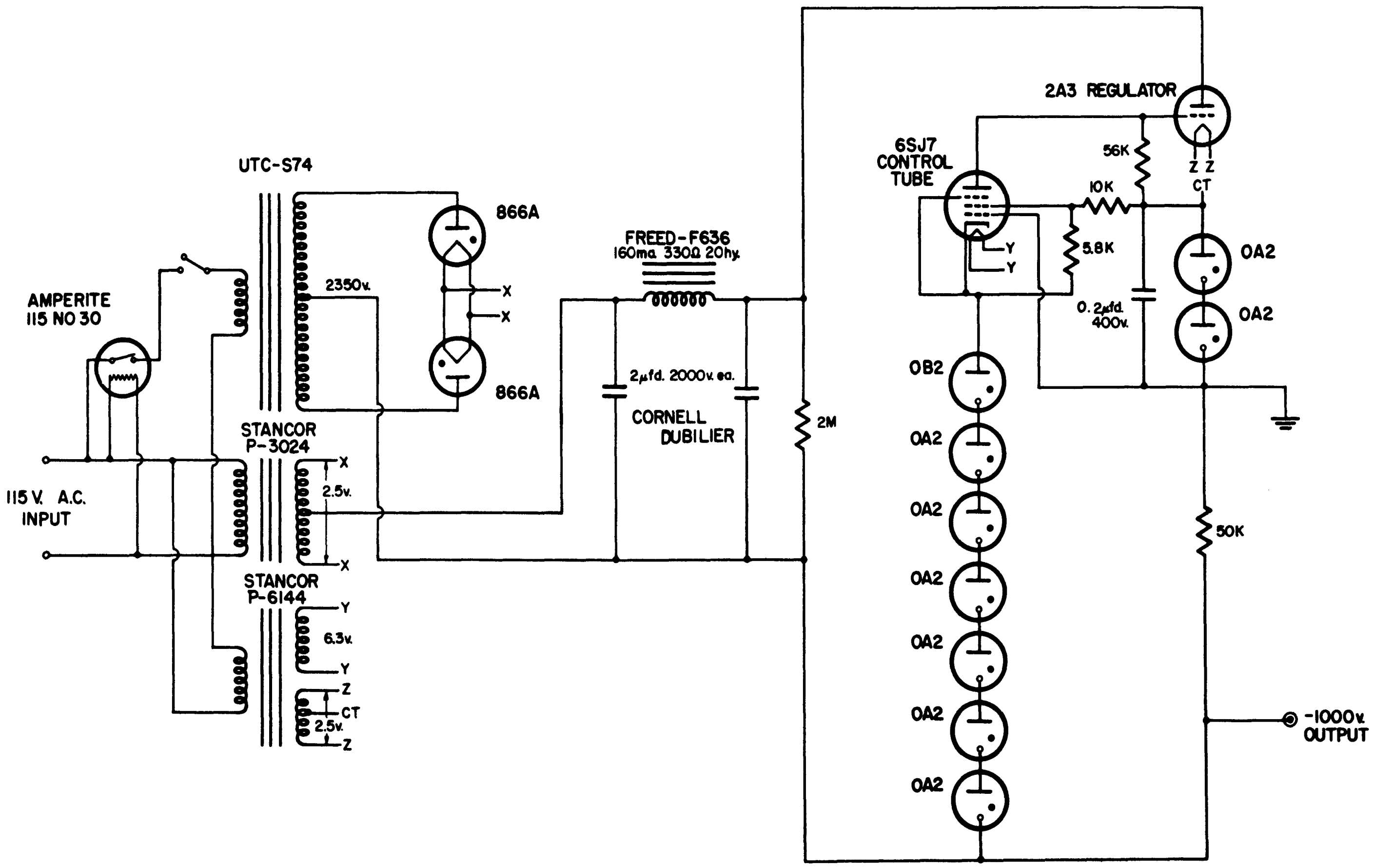


Figure 14. Multiplier Phototube Power Supply

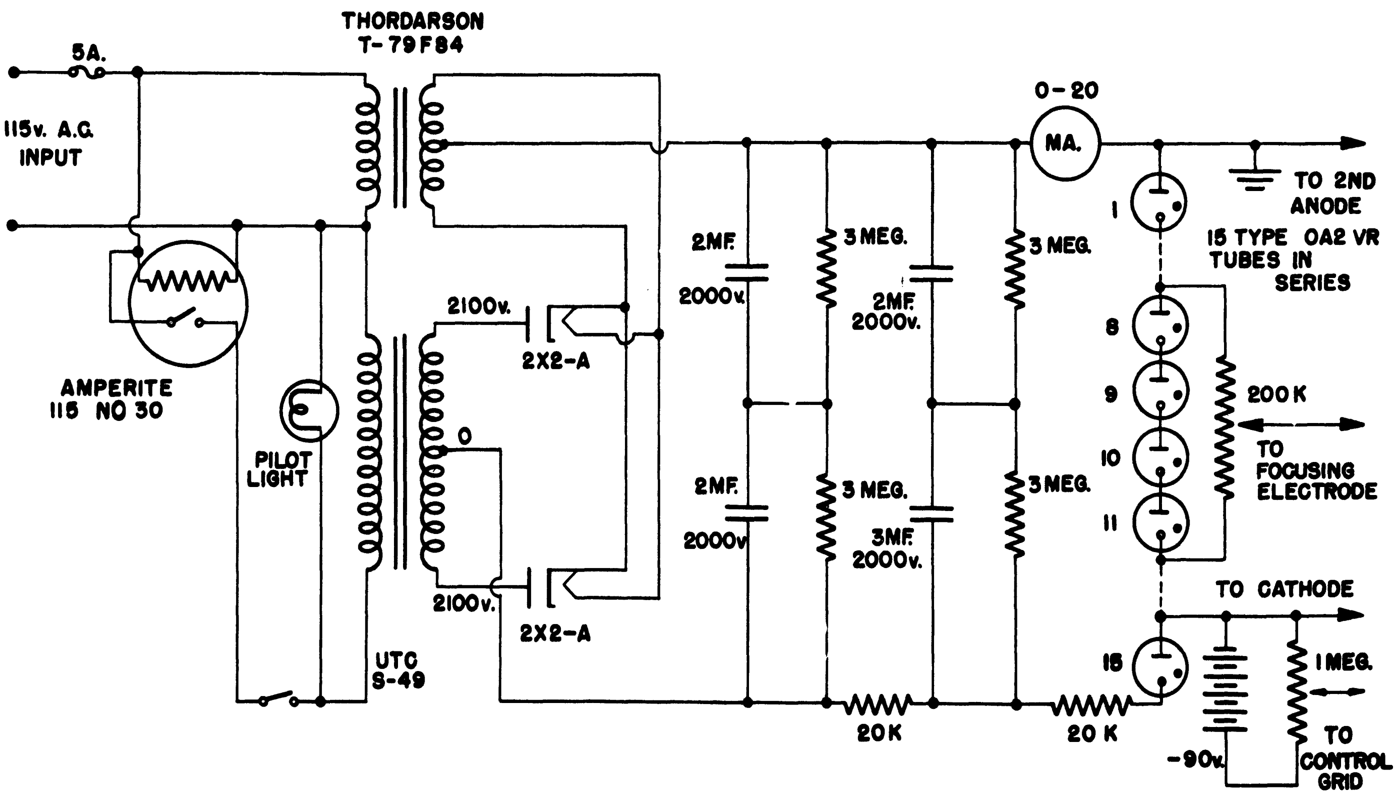


Figure 15. Cathode-Ray Tube Accelerating Power Supply

8. "MILLISEC" RELAY TESTS

During the period of the previous report, a Stevens-Arnold "Millisec" relay was started on a life test in which it was operated at a rate of 40 operations per second with an "on" period of 0.8 millisecond. Power supply failure during the present report period brought this test to a close after the relay had undergone more than 650 million cycles without apparent damage. These relays are under consideration for use in the sign-changing circuits of the multiplier.

9. FUTURE PROGRAM

Work on the function generator will be discontinued temporarily and development of the electronic-commutator circuits will be undertaken.

APPENDIX I

APPENDIX I

THE CLOSED-LOOP OPERATION OF THE FUNCTION GENERATOR

Two possibilities for easing the design requirements of the deflection-amplifier feedback loop have been suggested: a change in the method of feedback and the use of an approximate logarithm generator as an auxiliary deflection-amplifier input. Both these modifications reduce the magnitude of the feedback resistance necessary for a given accuracy. Stability of the feedback amplifier is thereby more readily achieved.

To illustrate the manner in which these suggested modifications affect the feedback loop design and also to take account of the dependence of phototube output on spot position, it is only necessary to elaborate somewhat on the derivation presented as Appendix III of Progress Report No. 2, September 29, 1951. With the exception of the auxiliary input represented by the voltage $-zV_y$ in series with R/a, and the feedback tap at ae_o instead of at the full output, Fig. A is an exact reproduction of the open loop of the function generator detailed in the previous report. The quantity $-zV_y$ represents the output of the approximate logarithm generator. The factor z, a measure of the degree by which the output approximates the voltage corresponding to the true logarithm, need not be constant but is assumed to be close to unity within fixed limits at all times.

Assuming A_1 is large enough to make the voltage e' at the amplifier input negligible, the output voltage e_o will be given by

$$e_o \quad = \quad - \, (i_p + i_b) \, \frac{R}{a} + zV_y \tag{1}$$

The total current output, i_p, of the phototube is given by

$$i_p \quad = \quad (S_p \Delta y + i_N + i_c) \, (w) \tag{2}$$

where Δy = an incremental displacement of the CRT spot center from the logarithmic mask edge along the axis driven by the phototube and deflection amplifiers (hereafter called the y axis);

S_p = phototube sensitivity in terms of current output per unit spot displacement past the edge of a mask parallel to the x axis;

i_N = noise component of phototube current;

i_c = phototube current obtained when spot is centered on mask edge; and

w = a function of spot position, the ratio of phototube output current referred to that obtained at zero deflection under conditions of complete spot visibility.

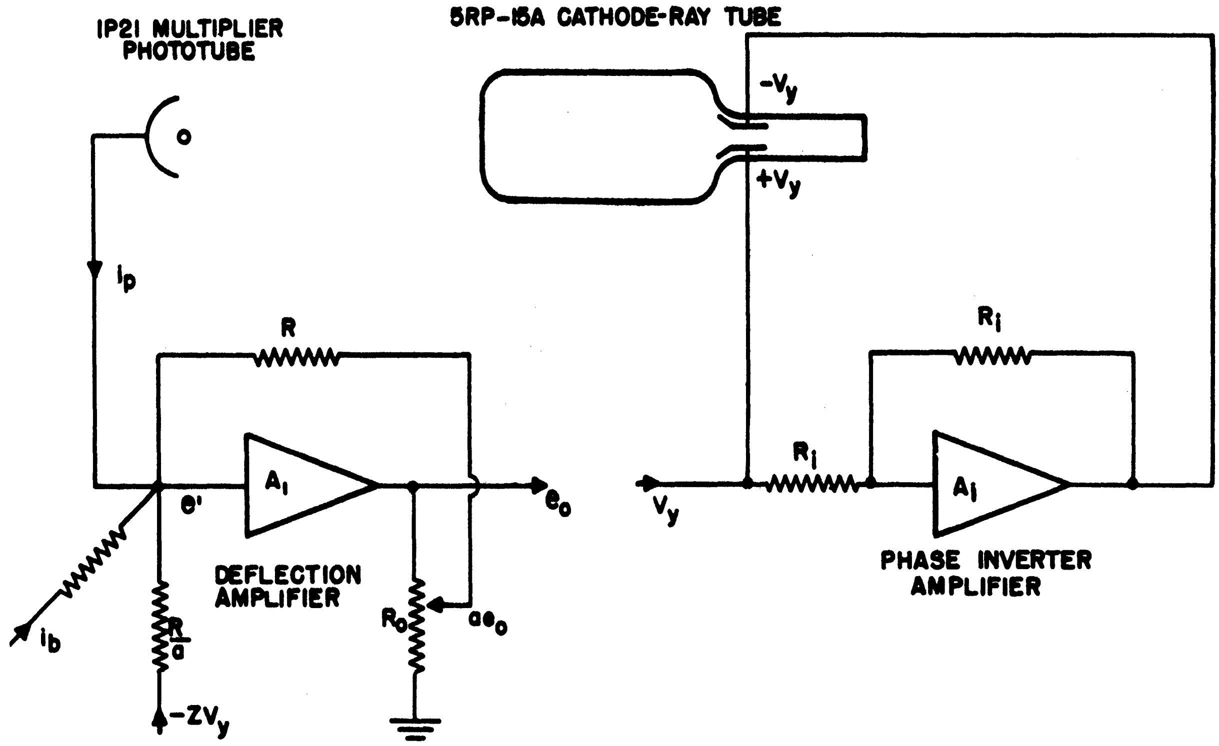


Figure A. The Open Loop of the Function Generator

The relations

$$\Delta y = y - y_s \tag{3}$$

$$y_s = \frac{V_y}{E_d} \tag{4}$$

$$e_o = V_y \tag{5}$$

where y = the mask ordinate,

y_s = the spot center ordinate, and

E_d = the deflection factor in volts per inch,

together with Equations (1) and (2), yields the relation

$$y_s = \frac{-\dfrac{S_pRw}{aE_d(1-z)}}{1-\dfrac{S_pRw}{aE_d(1-z)}}\,y - \frac{\left[(i_N + i_c)w + i_b\right]\dfrac{R}{a}}{E_d(1-z)\left[1-\dfrac{S_pRw}{aE_d(1-z)}\right]} \tag{6}$$

Since, ideally, $y_s = y$, $S_pRw/aE_d(1-z) \gg 1$ and Equation (6) can be rewritten as

$$y_s = \frac{-\dfrac{S_pRw}{aE_d(1-z)}}{1-\dfrac{S_pRw}{aE_d(1-z)}}\,y + \frac{i_N}{S_p} + \frac{i_cw + i_b}{S_pw}\ . \tag{7}$$

Comparing Equation (7) with its counterpart, Equation (11) of the previous report,

$$y_s = \frac{-\frac{S_p R}{E_d}}{1 - \frac{S_p R}{E_d}} y + \frac{i_N}{S_p} + \frac{i_c + i_b}{S_p} \quad .$$

we note that the coefficient of y can be made sufficiently close to unity with smaller values of the feedback resistance, R, since a is less than unity and z approximates unity. It will be noted, also, that the function w should be ideally constant in the region bounding the edge of the mask, so that the error due to the last term can be eliminated by setting $i_b = - i_c w$. A large value of S_p/i_c will help in minimizing this error as well as that due to the noise component of phototube current, i_N.

The auxiliary input, $- zV_y$, may be obtained with the use of a logarithmic attenuator manufactured by Kalbfell Laboratories, Inc. and sold under the name of "Logaten." Since the manufacturer claims a linearity within 1 db, the maximum absolute value of the quantity (1-z) in Equation (7) should be 0.12. One such unit is on order at present.

APPENDIX II

APPENDIX II

A NOTE ON NECESSARY SPOT POSITIONING ACCURACY

It is of fundamental interest to determine the greatest tolerable error in spot position consistent with the required system accuracy.

The function generator is required to deliver at its output either a logarithm of an input number or an antilogarithm, depending on the channel being sampled. Actually, our interest is centered on obtaining a given accuracy in the antilogarithm outputs, and the logarithm outputs must have errors below the maximum value which will just meet the required antilogarithm accuracy.

Because of the nature of the apparatus, let us assume the logarithmic curve is plotted with coordinates of equal length. Let the length of either coordinate be ℓ and the number of decades n, where n is any positive integer. It is readily apparent that, in this case, the logarithms are plotted to a scale expanded by a factor N_{max}/n relative to the number axis, where N_{max} is the full-scale value of N. If the numbers, N, are plotted along the x-axis and the logarithms along the y-axis we have

$$x = N \tag{1}$$

$$y = \frac{N_{max}}{n} \log N \quad , \tag{2}$$

and the geometric slope of the resultant curve at any point is

$$\tan \theta = \frac{dy}{dx} = 0.4343 \frac{N_{max}}{nN} = c \frac{N_{max}}{N} \quad , \tag{3}$$

where $c = 0.4343/n$, and

θ = angle made by the curve with the x-axis.

Now let us assume that we require the final antilogarithm output to be accurate within 0.1 per cent of full-scale. Since the number scale is linear, spot positioning along the x-axis must then be within 0.001 ℓ of the true value for the case where the logarithm input is precisely correct.

As demonstrated in Appendix III of the previous report, the spot-positioning accuracy along the y-axis is proportional to $\cos \theta$. In a similar manner, spot-positioning accuracy along the x-axis can be shown to be proportional to $\sin \theta$. Hence, the greatest spot-positioning error along the x-axis can be expected at full-scale, where θ is a minimum. At this point,

$$\tan \theta_m = c \tag{4}$$

$$\sin \theta_m = \frac{c}{\sqrt{1 + c^2}} = \frac{0.4343}{\sqrt{n^2 + 0.1886}} , \tag{5}$$

where θ_m = angle made by the curve with the x-axis at full-scale.

Now let us define $\Delta x'$ as the greatest equivalent positioning error we can tolerate at a mask edge parallel to the y-axis to obtain the required antilogarithm accuracy for the case when the logarithm inputs are precisely correct. Then, to obtain N within 0.1 per cent of full-scale,

$$\Delta x' = 0.001\, \ell \sin \theta_m = 0.001\, \ell \frac{0.4343}{\sqrt{n^2 + 0.1886}} . \tag{6}$$

During the period when the function generator is determining logarithms, its required positioning accuracy is fixed by the maximum error tolerable in the antilogarithm outputs. Let us assume, for the moment, that, given a logarithm, the function generator will deliver the antilogarithm precisely. In this case, the error along the number (x) axis due to the error in the logarithm is

$$\Delta x = \Delta y / \tan \theta , \tag{7}$$

where Δy is the positioning error along the y-axis. Hence, for an accuracy of 0.1 per cent of full-scale in the antilogarithm determination,

$$\Delta y = \Delta x \tan \theta = 0.001\, \ell\, c \frac{N_{max}}{N} . \tag{8}$$

It is easily apparent that the most stringent requirements occur at full-scale, where Δy must be smallest. At full-scale,

$$\Delta y = 0.001 \, \ell \, c \quad . \tag{9}$$

Let us define $\Delta y'$ as the greatest equivalent positioning error we can tolerate at a mask edge parallel to the x-axis. Since the spot-positioning accuracy along the y-axis is proportional to cos θ, at full-scale

$$\Delta y' = 0.001 \, \ell \, c \cos \theta_m = 0.001 \, \ell \frac{c}{\sqrt{1 + c^2}}$$

$$= 0.001 \, \ell \frac{0.4343}{\sqrt{n^2 + 0.1886}} \quad , \tag{10}$$

which is precisely the condition imposed on positioning along the x-axis (see Equation (6) above). It can be shown that if the function generator is capable of the positioning accuracy of Equation (10), it will provide antilogarithm outputs accurate within 0.1 per cent of full-scale over the entire range between the limits

$$0.001 \, N_{max} \leq N \leq N_{max} \quad .$$

Unfortunately, when input and output terminals are reversed for the determination of logarithms and antilogarithms, the errors $\Delta x'$ and $\Delta y'$ are cumulative. Hence, assuming operation of equal accuracy along either axis,

$$\Delta x'' = \Delta y'' = \frac{0.001 \, \ell}{2} \frac{0.4343}{\sqrt{n^2 + 0.1886}} \quad , \tag{11}$$

for an overall accuracy of 0.1 per cent of full-scale. It should be noted, however, that the foregoing figures are based on operation at the upper end of the scale, where the most rigid requirements exist.

Because of problems of spot focus and uniform phototube output sensitivity outlined in previous sections of this report, let us restrict our useful area to a circle 2-1/2 inches in diameter centered at the electrically neutral point on the cathode-ray tube. The length of the coordinate axes are thereby limited to 1.77 inches. Since the present goal is to

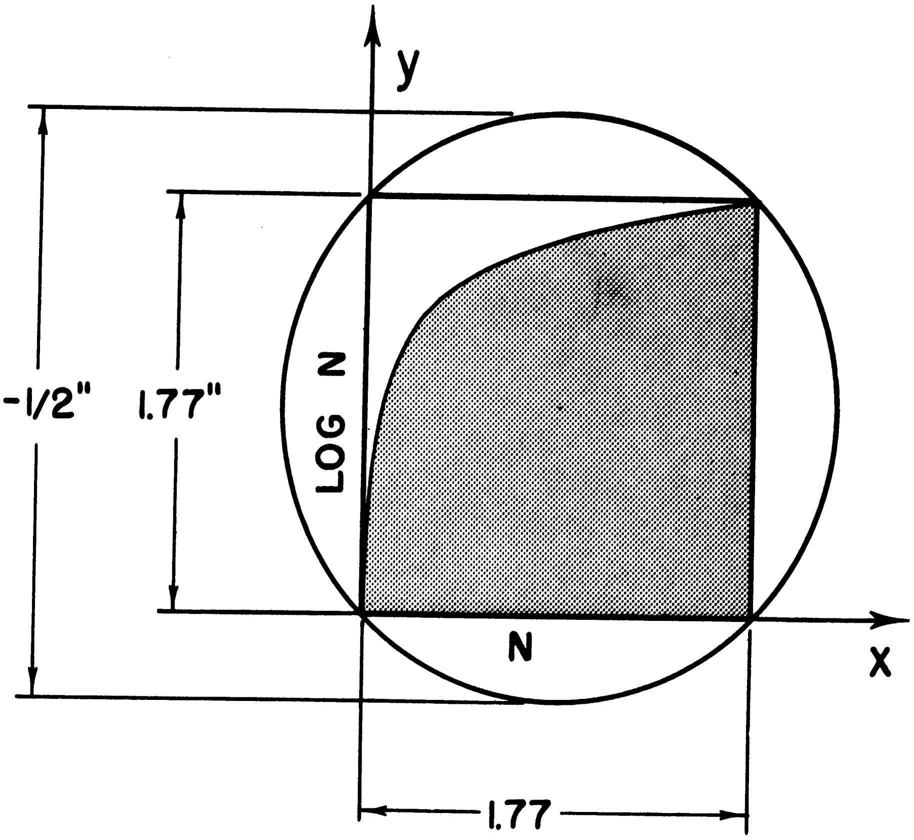


Figure B. Useful Tube Area

achieve operation over three decades, let us assume n = 3. Substituting these values in (11), we have

$$\Delta x'' = \Delta y'' = 0.000127'' \tag{12}$$

From (11) it can be noted that Δx" (or Δy") is approximately inversely proportional to n if $n \geq 1$. Thus, the positioning requirements can be eased by a factor of three if three function generators were employed, one for each decade.

To see what the foregoing result means in terms of the function-generator loop performance, let us rewrite Equation (7) of Appendix I as

$$y_s = \frac{-k}{1 - k} y + \frac{i_N}{S_p} + \frac{i_c w + i_b}{S_p w} \quad , \tag{13}$$

where

$$k = \frac{S_p Rw}{aE_d(1-z)} \quad , \qquad (14)$$

and investigate it term by term.

a) <u>The term, (-k/1-k)y.</u> If we neglect all other terms, Equation (13) becomes

$$y_s = \frac{-k}{1 - k} y \quad , \qquad (15)$$

from which

$$k = \frac{y}{\Delta y'} + 1 \quad .$$

Since no loop output is required for the condition where the spot comes to rest at the electrical center of either axis, the effective value of y at full-scale, where k is a maximum, is 1.77/2, or 0.885 inch. Hence

$$k = \frac{0.885}{0.000127} + 1 = 6970 \quad . \qquad (16)$$

Now, from recent measurement:

S_p (min) = 10.6×10^{-3} ampere/inch (page 11)

w = 0.6 (page 6)

E_d = 80 volts/inch;

also, assume (1-z) = 0.12 (page 21)

Substituting these values into (14)

$$R = 10.7 \times 10^6 \, a \quad .$$

If a is made equal to 0.1, the feedback resistor, R, falls in the neighborhood of one megohm.

b) The term, i_N/S_p. From recent measurements

$$i_N = 0.0038 \times 10^{-3} \text{ ampere peak}$$

Thus, the error in y_s introduced by this term is 0.00036 inch. By comparison with Equation (12) this random error is seen to be about three times the maximum positioning error required for 0.1 per cent accuracy in antilogarithm determinations. An improvement may be realized by obtaining a cathode-ray tube having smaller spot size, thus increasing the value of S_p/i_N. The noise current may also be reduced by eliminating any component that may be due to intensity modulation of the spot.

c) The term, $(i_c w + i_b)/S_p w$. Let

$$i_b = - i_c \quad .$$

Then the above term reduces to

$$\frac{i_c(w - 1)}{S_p w} \quad .$$

Now, if $i_c = 0.1 \times 10^{-3}$ amperes, the maximum rating of the 1P21 phototube, we see the positioning error introduced by this term is

$$\frac{0.1 \times 10^{-3} \; (0.6 - 1)}{10.6 \times 10^{-3} \times 0.6} = 0.0064 \text{ in.},$$

which is a factor of 50 higher than allowed by Equation (12). The best possibility for realizing this factor lies in obtaining a value of w much closer to unity by means of a better optical system. The value of S_p may also be increased with smaller spot size on a better cathode-ray tube. A reduction in i_c would be accompanied by a proportional reduction in S_p; hence, no gain can be realized in this direction.