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TECHNICAL REPORT

SOME METHODS IN PHOTOGRAMMETRIC REDUCTION FOR EXTERIOR ORIENTATION

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ABSTRACT

This report summarizes briefly the methods and procedures used by this project in obtaining a complete photogrammetric reduction of an aerial photograph. A complete reduction must contain all six coordinates necessary to orient a rigid body (the camera) in space. The method of obtaining each of these six coordinates is given along with an example. A discussion of photogrammetric errors and their effect on the above six elements is given with actual values as found in practice. The working procedures used at this project and the application of automatic computers to these procedures are also given.

TABLE OF CONTENTS

SOME METHODS IN PHOTOGRAMMETRIC REDUCTION FOR EXTERIOR ORIENTATION

INTRODUCTION

The purpose of this report is to outline the photogrammetric reduction process as developed and used at this project for Wright Air Development Center. During the past two years both Wright Air Development Center and this project have received requests from other governmental agencies and industrial groups working on government contracts for information on the reduction process. It is our hope that the following report will be sufficiently complete to enable those interested to determine the value of this method and the application with respect to their problem.

To keep this report from becoming too cumbersome complete derivations will be omitted and a knowledge of standard photogrammetric terms will be assumed. Such information if desired can be obtained from the references in the bibliography. An excellent standard reference in the general field of photogrammetry is the Manual of Photogrammetry published by the American Society of Photogrammetry. No attempt will be made here to duplicat information available in such sources.

The specific problem being dealt with here is that of finding the position and orientation of an aerial camera in space at the time of exposure with highest possible accuracy. This is to be done by using metrical data obtained from the photograph and from the object space of the area included in that photograph.

In this report the position of the camera and that of the aircraft will be used interchangeably. This is done with the assumption that the defining point of location for the aircraft is the lens of the camera and that the fiducial axes of the camera are parallel to the longitudinal and transverse axes of the aircraft. The physical achieving of the latter situation is a problem in instrumentation and will not be dealt with here.

The equipment needed for this evaluation consists of: an accurately calibrated precision aerial camera with development facilities, a precision linear measuring device with accuracy of at least 0.005 mm such as a comparator, a photogrammetric range suitable for all flight patterns desired, and computation facilities. The above are of course only the principal items.

THE POSITION OF AN AIRCRAFT IN SPACE

In order to define completely the position of an aircraft in space, six independent coordinates are needed. Of these three are linear coordinates and three are angular coordinates.

The linear coordinates are called the space coordinates and consist of the east-west coordinate, the north-south coordinate, and the altitude coordinate. They will be referred to hence as X_N, Y_N, and H, respectively. All three are referred to a fixed origin in the ground datum plane.

The angular coordinates will be called pitch, roll, and azimuth of the heading or P, R, and α_H, respectively. Taken in the order, α_H, P, and R, these angles form an Euler-type set of coordinates. The angle α_H is measured clockwise, as seen from above, about the true vertical axis from North to the vertical plane of the longitudinal axis of the aircraft in the direction of heading. Thus, for an aircraft heading East, $\alpha_H = 90°$; South, $\alpha_H = 180°$; West, $\alpha_H = 270°$; North, $\alpha_H = 0°$; etc. The angle P is measured in the vertical plane through the heading axis, from the true horizontal to the nose end of the longitudinal axis of the aircraft. The sign of P is positive nose up and negative nose down. Thus for level flight $P = 0°$, for a 45° climb $P = 45°$, for a 45° dive $P = -45°$, etc. The angle R is measured about the longitudinal axis of the aircraft in the plane perpendicular to that axis, from the horizontal to the transverse axis of the aircraft. The sign of R is positive left wing up and negative left wing down. It should be noted that the plane in which R is measured is inclined to the vertical by the angle P.

The angles α_H and P can be considered as spherical angles, azimuth and altitude, defining the position of the longitudinal axis of the aircraft. The angle R specifies a rotation about that axis. (See Fig. 1.)

These six coordinates X_N, Y_N, H, P, R, and α_H specify completely the position of the aircraft. The following will describe the methods used to solve for these coordinates using photogrammetric procedures.

The Calculation of P, R, and H

Ideally, the photograph should be considered as an exact reproduction of a truly flat terrain. This is, of course, not the case. However, in the following the above condition will be assumed and the deviations will be discussed later.

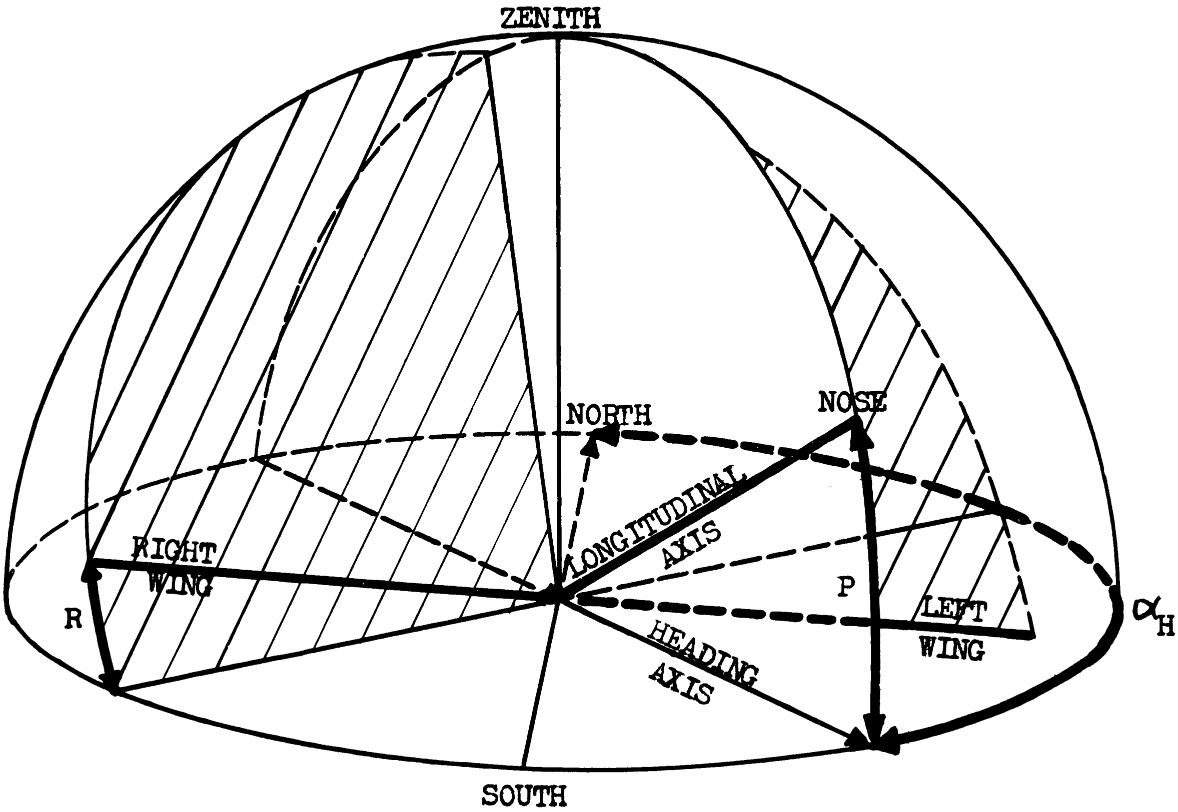
ZENITH
NORTH
NOSE
LONGITUDINAL
AXIS
RIGHT
WING
R
P
LEFT
WING
HEADING
AXIS
α_H
SOUTH

Fig. 1

Consider four photogrammetric targets in the datum plane forming the corners of a quadrilateral such that no one target lies within the triangle formed by the other three. Designate them in cyclic rotation as A, B, C, and D with datum plane coordinates X_A, Y_A, X_B, Y_B, etc. These four targets will be reproduced on the photograph in approximately the same configuration. They shall be called a, b, c, and d with photographic coordinates referred to the fiducial axes of x_a, y_a, x_b, y_b, etc. The areas inclosed by triangles ABC and abc are

$$\mathcal{A}\,ABC = 1/2 \left| (X_A-X_B)(Y_A-Y_C) - (X_A-X_C)(Y_A-Y_B) \right| \tag{1}$$

and

$$\mathcal{A}\,abc = 1/2 \left| (x_a-x_b)(y_a-y_c) - (x_a-x_c)(y_a-y_b) \right| . \tag{2}$$

Similarly for triangles acd, bcd, abd, ACD, BCD, and ABD. (Note: All areas are to be taken as positive.)

The constants K_1 and K_2 are defined

$$K_1 = \frac{\mathcal{A}abc \times \mathcal{A}ACD}{\mathcal{A}acd \times \mathcal{A}ABC} \tag{3}$$

$$K_2 = \frac{\mathcal{A}bcd \times \mathcal{A}ABD}{\mathcal{A}abd \times \mathcal{A}BCD} . \tag{4}$$

If x_n and y_n are designated as the photographic coordinates of the nadir point on the photograph, two equations follow with f being the calibrated focal length of the camera:

$$(x_b-K_1x_d)x_n+(y_b-K_1y_d)y_n = f^2(K_1-1) \tag{5}$$

$$(x_c-K_2x_a)x_n+(y_c-K_2y_a)y_n = f^2(K_2-1) \tag{6}$$

These equations can be solved easily for x_n and y_n. If the x axis on the photograph is assumed to be the transverse axis of the aircraft, then

$$R = \tan^{-1}\left(\frac{x_n}{f}\right) , \tag{7}$$

$$P = \tan^{-1}\left(\frac{y_n}{\sqrt{f^2+x_n^2}}\right) . \tag{8}$$

These last two equations are written without regard to sign as this depends on the choice of axes on the photograph and their relation to the aircraft axes. If the positive y axis were toward the nose and the positive x axis toward the left wing then the equations are correct with respect to sign.

If N_a is defined as

$$x_a x_n + y_a y_n + f^2 = N_a \quad , \tag{9}$$

similarly

$$x_b x_n + y_b y_n + f^2 = N_b \quad , \tag{10}$$

and the same for N_c and N_d.

Also defining f' as

$$\sqrt{x_n^2 + y_n^2 + f^2} = f' \quad , \tag{11}$$

then

$$H^2 = \frac{N_a N_b N_c \, A_{ABC}}{f f'^3 \, A_{abc}} \tag{12}$$

and

$$H^2 = \frac{N_a N_c N_d \, A_{ACD}}{f f'^3 \, A_{acd}} \tag{13}$$

the same for bcd and abd.

These four equations in H^2 serve as a check on the formation and solution of Equations 5 and 6, as well as solving for the altitude H.

In order to give this solution the greatest possible strength the areas should be as large as possible. An ideal situation is to have the four target images in the extreme corners of the photograph. More practical arrangements will be discussed under Photogrammetric Range.

The forgoing was merely the statement of the solution for P, R, and H. A more complete discussion and derivations of all formulae can be found in references 1, 2, and 3.

The Calculation of X_n and Y_n

The method is that of position circles. Knowing the distance on the ground from each control point to the nadir point, an equation may be written for the circle with center at that control point and passing through the nadir point on the ground

$$(X_A-X_N)^2 + (Y_A-Y_N)^2 = R_{AN}^2 \qquad (14)$$

$$(X_B-X_N)^2 + (Y_B-Y_N)^2 = R_{BN}^2 \qquad (15)$$

$$(X_C-X_N)^2 + (Y_C-Y_N)^2 = R_{CN}^2 \qquad (16)$$

$$(X_D-X_N)^2 + (Y_D-Y_N)^2 = R_{DN}^2 \qquad (17)$$

These circles intersect at the nadir point. Squaring and subtracting equation 14 from 16 and 15 from 17, respectively,

$$(X_C-X_A)X_N + (Y_C-Y_A)Y_N = 1/2(R_{AN}^2-R_{CN}^2+X_C^2+Y_C^2-X_A^2-Y_A^2) \qquad (18)$$

$$(X_D-X_B)X_N + (Y_D-Y_B)Y_N = 1/2(R_{BN}^2-R_{DN}^2+X_D^2+Y_D^2-X_B^2-Y_B^2) \qquad (19)$$

Equations 18 and 19 may be solved directly for X_N and Y_N the desired coordinates.

There remains to be found only the radii of the circles in order to be able to form Equations 18 and 19. Considering a cross section of the geometry in plane LNA (see Fig. 2), at once from analytic geometry the following is obtained,

$$\cos M_A = \frac{N_a}{f'L_a} , \qquad (20)$$

where N_a and f' are defined in Equations 9 and 11; respectively, and

$$\sqrt{x_a^2+y_a^2+f^2} = L_a , \qquad (21)$$

also

$$R_{AN} = H \tan M_a . \qquad (22)$$

Repeating this for R_{BN}, R_{CN}, and R_{DN}, Equations 18 and 19 may be formed and solved.

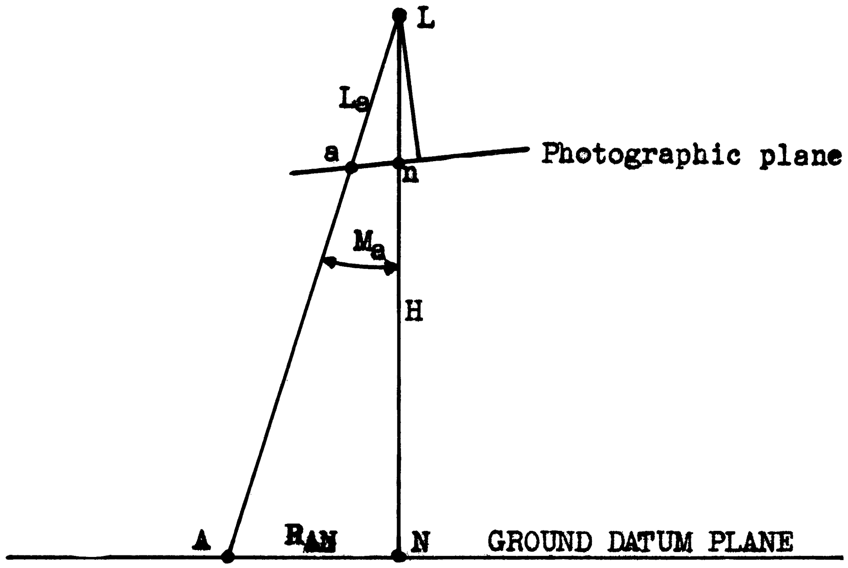
L
Le
a
n
Photographic plane
Ma
H
A
RAN
N
GROUND DATUM PLANE

Fig. 2

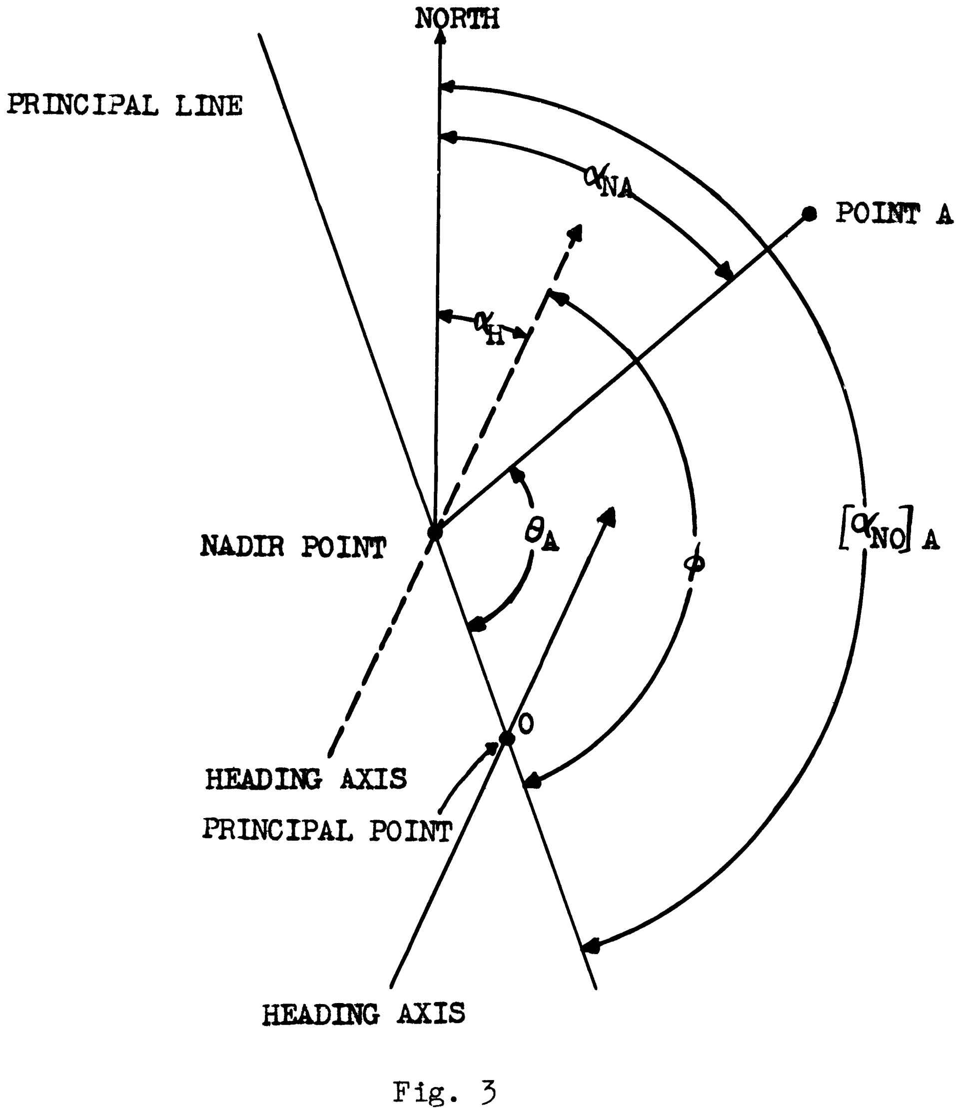
NORTH
PRINCIPAL LINE
POINT A
α_{NA}
α_H
NADIR POINT
θ_A
ϕ
$[\alpha_{NO}]_A$
O
HEADING AXIS
PRINCIPAL POINT
HEADING AXIS

Fig. 3

The Calculation of α_H

As mentioned previously this angle is measured about the true vertical, or equivalent, in a horizontal plane. To achieve this, project the photograph orthogonally to the ground datum plane which is of course horizontal. This projection might appear as in Fig. 3.

Before proceeding with the analysis there are several points that should be mentioned. First, if the projection is considered from a physical point of view, it is equivalent, ignoring tilt, to a contact print of the negative. Therefore, it is necessary analytically to reverse the direction of one of the photographic axes in as much as these axes are conventionally chosen on the negative with the emulsion up. This is also equivalent to viewing the negative with the emulsion down; Fig. 3 represents such a configuration. Second, the effect such a projection has on angles is as follows: Equivalent angles such that the vertex lies on the principal line and one leg of which is the principal line are related by

$$\tan \psi = \tan \psi' \sec t \quad , \tag{23}$$

where ψ' is the angle on the photograph and ψ is the equivalent angle as projected. Sec t is defined

$$\sec t = \frac{f'}{f} \quad . \tag{24}$$

In Fig. 3 angles ϕ and θ_A satisfy the conditions of Equation 23.

Now referring to Fig. 3, the azimuth α_{NA} is the basis for the calculation. Knowing the coordinates of the nadir point and the control point A, it can be easily found

$$\alpha_{NA} = \tan^{-1}\left(\frac{X_A - X_N}{Y_A - Y_N}\right) \quad . \tag{25}$$

Now employing the formula for the tangent of the angle between two lines in a plane and using relation 23,

$$\theta_A = \tan^{-1} \frac{|y_a x_n - x_a y_n|\ f'}{(f'^2 - N_a)} \ ^{*} \quad , \tag{26}$$

* Note that the sign of θ_A is determined by the denominator.

also

$$\phi = \tan^{-1}\left(\frac{x_n f'}{y_n f}\right)^{*} \quad . \tag{27}$$

Now from Fig. 3,

$$\alpha_H = \alpha_{NA} + \theta_A - \phi \quad . \tag{28}$$

In practice the signs used in Equation 28 might not be known. Therefore, it is simpler to first find α_{NO} by

$$\left[\alpha_{NO}\right]_A = \alpha_{NA} \pm \theta_A \quad . \tag{29}$$

If this is repeated for points B, C, and D, eight possible solutions for α_{NO} can be obtained. However, four of these will be correct and the other four extraneous. Thus, selecting the four that give the closest agreement,**

$$\alpha_{NO} = \frac{\left[\alpha_{NO}\right]_A + \left[\alpha_{NO}\right]_B + \left[\alpha_{NO}\right]_C + \left[\alpha_{NO}\right]_D}{4} \tag{30}$$

Then

$$\alpha_H = \alpha_{NO} \pm \phi \quad . \tag{31}$$

The sign to be used in Equation 31 can be found by physical examination of the photograph in comparison with the known configuration of the ground control points.

This completes the solution for position in space.

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* The assumption is made here that the positive y axis is the heading axis.

** In the ideal case these four angles will agree exactly. However, in a practical case they will differ by a slight amount. In this case only the mean as given in Equation 30 is significant. The reason for this situation is that using four control points for a solution is an overdetermination as three points are actually sufficient. The differences are the residuals of a meaning out process for the four point solution. Hence, the mean result is significant.

THE DEVIATIONS

Previously, the ideal situation was considered where all rays from the object space were straight lines and registered on the photograph in the corresponding position. Actually the rays are deviated by two effects. One of these is the lens distortion. This is a physical property of the lens which causes it to distort the angular field of the object space. The other effect is refraction in the atmosphere which has the same effect, but on a smaller scale. The lens distortion amounts to a maximum of about 0.150 mm on the photograph for a six-inch metrogon lens. The refraction amounts to a maximum of about 0.024 mm on the photograph for the same focal length and altitude of 30,000 feet.

In addition there is a deviation caused by the fact that the actual terrain is on a curved surface. This arises due to the fact that the ground points are considered to be on a plane surface with the same spacing as on the actual curved surface. The effect of this is to compress the field on the photograph. This causes a systematic displacement with a maximum value of about 0.110 mm on the photograph for a six-inch lens at 30,000 feet.

The relief of the control point also causes a displacement on the photograph, but this will be discussed under a separate heading.

The effects of the lens, atmosphere, curvature, and relief all cause displacements from the ideal position on the photograph. Their more precise effects will be discussed in the appendix or under separate headings. These effects are systematic and can be eliminated if the proper parameters are known.

Other types of deviation are caused by the film itself, the camera, and the methods of measuring the photographic coordinates. These deviations, although they also can be considered as displacements on the film, are not in the nature of systematic angular deviations. (The most serious of these is called residual film distortion.) They are caused by physical changes in the film base and emulsion due to changes in temperature and humidity as well as effects caused by the development and drying processes. Inasmuch as this is uniform expansion or contraction it can be eliminated by ratioing all measured distances back to the size of the frame at the time of exposure. Due to local stresses and irregularities, these effects cannot be entirely ratioed out. The remaining displacements are called residual film distortion. In carefully handled and processed film this effect can be held to an average of about 0.015 mm on the film. For the purpose of ratioing aerial cameras are provided with fixed markers at a known separation. By measuring this separation as registered on the film in conjunction with measuring the control points, the magnitude of the coefficient of linear expansion can be found. Most films have a different ratio across the film from the ratio along the strip. Thus, it is necessary to ratio each coordinate direction separately. The camera is provided with markers in both directions for this purpose. Therefore, the major portion of the film distortion can be ratioed out; leaving only the residual film distortion. The residual film distortion will be regarded as random in nature. The correct procedures for processing and handling the film should be obtained from the manufacturer of the type of film used.

Most of these film effects could be eliminated by the use of glass plates instead of film. However, in most cases this is not practical with present equipment, though not completely excluded.

The camera itself causes displacements on the photograph in addition to the effect of the lens. By far the most serious of these effects is caused by vacuum failure. This causes the film to bulge and distort as it is not held flat on the platen of the camera, which is the function of the vacuum system. This failure has a serious effect on the accuracy of the entire solution and every precaution should be taken to insure the proper function of the vacuum system.

There is a possibility that the platen itself is not sufficiently flat. The use of a well-calibrated modern camera will eliminate this source of error.

A type of displacement might be caused by poorly defined or inaccurately located fiducial marks in the camera. A well-made modern camera will have sharply defined fiducial marks forming orthogonal axes to a very few seconds of arc. These axes should intersect at the principal point to within a few microns on the photograph. Failure to do so has the effect of translating the measured coordinates. This translation has negligible effect on P, R, α_H, and H, but does shift the ground nadir point (X_N,Y_N) slightly. This shift amounts to $\Delta p\, H/f$ feet on the ground, where Δp is the linear shift of the fiducial axes origin from the principal point on the photograph. The value of Δp is given with the camera calibration and is usually less than 0.050 mm. If necessary the correction for this effect can be applied by using geometrical considerations. In general the effects caused by these considerations are much below the effects of the film itself and may be neglected if the camera calibration indicates that the camera meets these requirements.

Finally, the camera is subject to the effects of temperature and pressure. This is a somewhat incompletely investigated topic. The effect is appearantly very small and may contribute a very slight systematic error to the solution, principally to the altitude H. To actually eliminate this slight error it would be necessary to calibrate the camera at some standard temperature (already a practice) and pressure and then maintain these conditions for some time prior to and during exposure. This is not now a practical procedure. Fortunately this is almost a negligible effect.

In measuring the photograph there are several sources of deviation. The greatest contribution to this is operator error. This human element is naturally very complex (see reference 4). However, this error can be held to satisfactory limits by a slight amount of practice and experience. The measuring equipment should be of considerable precision with a linear accuracy of at least 0.005 mm. The Geartner Scientific Corporation, Chicago, Illinois and Mr. David W. Mann of Lincoln, Massachusetts both produce very excellent comparators for this purpose.

The measuring device should be equipped with glass flats between which the frame of film is rigidly clamped. The measurements should be made as parallel to the glass surfaces as possible. Both of the above companies provide such equipment. While measuring, the film fiducial axes should be as nearly perpendicular to the line of measurement as possible. Failure to meet the above conditions produces systematic deviations, some of which could have serious effects on the results. However, with good equipment and reasonably experienced operators the measuring error can easily be held to less than 0.005 mm providing the photogrammetric targets are sharply defined on the photograph. On a film base this error is much less than the residual film error and will have no serious effect on the overall accuracy. Target definitions will be discussed under Photogrammetric Range.

Before leaving the subject of deviation the effect of the focal length should be discussed. An actual camera has no fixed focal length, but does have a range of focal lengths varying with position in the focal plane. Each focal length is associated with one or more differential annular rings in the focal plane centered at the principal point. The scale of all distances between points on that ring is constant and equal to H/f for a vertical photograph. Because of the lens distortion, the scale and consequently f changes with each annular region. Also each focal length has an associated distortion curve. By displacing each point on the photograph as indicated by that distortion curve the scale is made constant over the photograph for that focal length. The normal practice is to use what is called the calibrated focal length and its corresponding distortion curve. The calibrated focal length might be selected in several ways. The most common way is to select that focal length which makes the maximum positive distortion equal to the negative distortion at 45°, or a radial distance of f from the principal point. This method of selecting a focal length is used with the metrogon lens. The focal length selected in this way is thought to give a good average scale over the photographic frame for purposes where accuracy requirements do not require the use of the distortion curve. The only reason the calibrated focal length is used for our purposes is that it is the only focal length for which we have a distortion curve available.

Now any source of deviation whose first order effect is to cause a linear expansion or contraction in the photograph, the camera, or even the comparator has the same effect as changing the focal length used in the solution. The systematic deviations in general have such an effect. It also can be shown that small variations in the focal length have only second order effects on P, R, X_N, Y_N, and α_H, but first order effects on H. Hence, if H is a critical value in the solution, all known systematic deviations should be removed as accurately as possible.

The effect of random deviations on the solution is to give a normal error distribution in all six elements. Here H and α_H are less sensitive to random deviations, and hence, give relatively stronger values than P, R, X_N, and Y_N. There is relation between errors in the latter four elements. If the photographic nadir point is in error by Δs or

$$\sqrt{\Delta x_n^2+\Delta y_n^2}$$

and the ground nadir point in error by ΔS or

$$\sqrt{\Delta X_N^2+\Delta Y_N^2} \quad ,$$

then

$$\Delta S = H\Delta s/f \quad . \tag{32}$$

This is only an approximate relation.

Tilt is the angle between the normal to the photograph and the normal to the ground datum plane or analytically

$$t = \tan^{-1}\left(\frac{\sqrt{x_n^2+y_n^2}}{f}\right) , \tag{33}$$

or in terms of P and R

$$t = \tan^{-1}\sqrt{\tan^2 R + \tan^2 P \sec^2 R} \quad . \tag{34}$$

Errors in tilt also follow the normal error distribution.

The tilt components are defined with the positive y axis, the heading axis, and the positive x axis for the left wing

$$t_x = \tan^{-1} x_n/f = R \qquad (35)$$

$$t_y = \tan^{-1} y_n/f = \tan^{-1}(\tan P \sec R) \qquad (36)$$

It has been found by this project that the error in tilt components in practice for film negatives is about 15 seconds of arc. This assumes that all precautions are taken and systematic errors eliminated within reason. Noting the relations in Equations 35 and 36, this yields the following errors in the six elements.

$$\Delta R \simeq 15 \text{ seconds}$$
$$\Delta P \simeq 15 \text{ seconds}$$
$$\Delta \alpha_H < 15 \text{ seconds}$$
$$\sqrt{\Delta X_N^2 + \Delta Y_N^2} = \Delta S \simeq 0.00010 \text{ H feet}$$
$$\Delta H < 0.00010 \text{ H feet}$$

For glass plates the tilt error is considerably less with corresponding elements as above. This indicates about what accuracies can be expected from this method.

To sum the deviations that can be expected:

Systematic

- Lens distortion
- Earth curvature
- Atmospheric refraction
- Relief
- Linear effects of temperature, humidity, and pressure on the film, camera, and comparator systems.

Random

- Residual film distortion
- Human and mechanical errors in measurement
- Residual effects in the above systematic deviations
- Errors in the ground survey and reduction

The last item will be discussed under Photogrammetric Range and Relief will be discussed as the next topic.

RELIEF

There are two approaches to the incorporation of this element into the four point solution given earlier. The choice of these two methods depends on several factors, such as proposed altitudes, magnitudes of the reliefs, computation facilities, and overall accuracy desired.

Relief means the elevation of the control point target over the datum plane elevation used in the photogrammetric range. The physical effect effect is illustrated in Fig. 4.

Method A Vector Relief

This is an approximation method and does not give an exact solution. To apply this method the tilt problem is worked ignoring the reliefs. This gives values t_x', t_y', and H'. Now the assumption is made that these values apply most nearly to a false datum plane that passes as closely as possible through the four control targets in space. As four points are used, no such true plane exists. This false datum plane is called the substitute slant plane and is found as follows: Consider two of the problem targets at opposite corners of the ground quadrilateral. Fig. 5 is a vertical cross section through those two targets (A and C in Fig. 5). Now the substitute slant plane is parallel to the space line $\overline{AC}$. In the same way this plane is also parallel to the space line $\overline{BD}$, and AC and $\overline{BD}$ are skew lines in space not parallel. Such lines can always be joined by a mutual perpendicular. The substitute slant plane is perpendicular to this line and passes through its midpoint. This completely determines the substitute slant plane in space.

Now if the tilt computed is with respect to this plane and the angle between the substitute slant plane and the ground datum plane is added, the correct tilt should be obtained. This is done as follows: Consider Fig. 4 which is a cross section in the plane LAC.

In this plane clearly

$$\Delta t_{ac} = \frac{|h_A - h_C|}{\overline{A'C'}} \quad , \qquad (37)$$

similarly

$$\Delta t_{bd} = \frac{|h_B - h_D|}{\overline{B'D'}} \quad . \qquad (38)$$

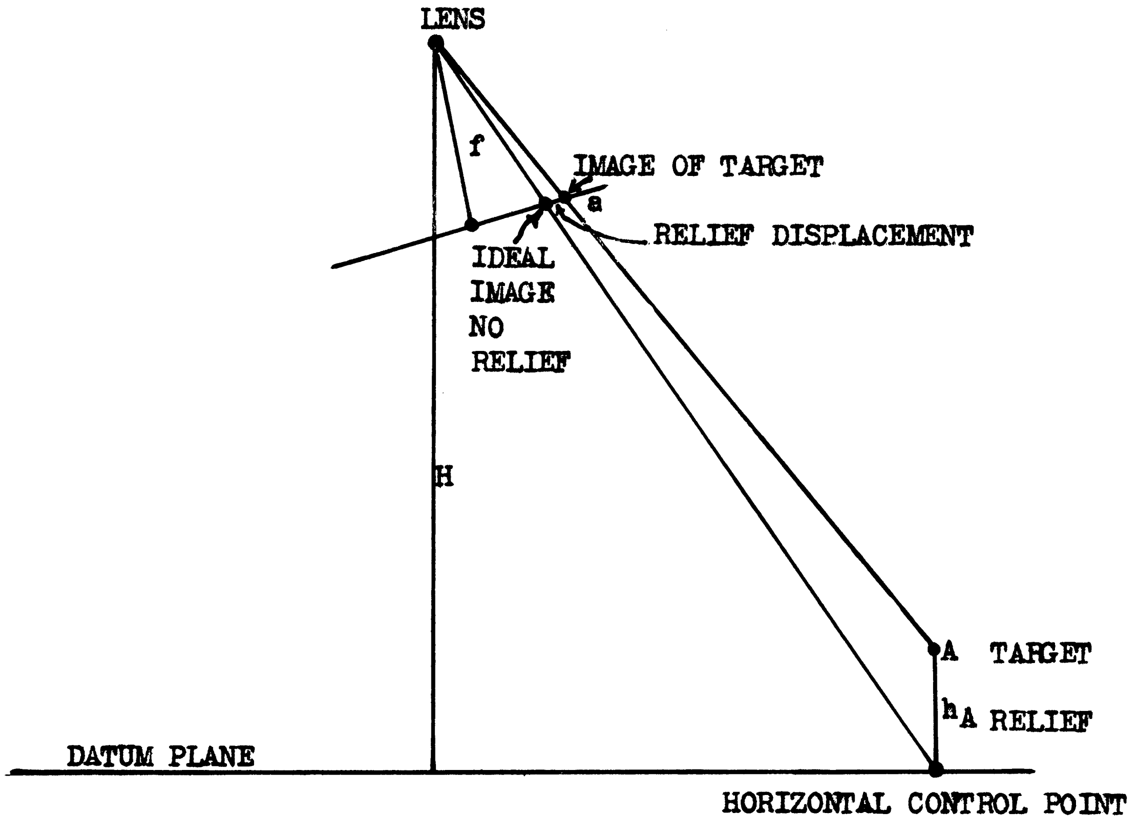

LENS
f
IMAGE OF TARGET
a
RELIEF DISPLACEMENT
IDEAL
IMAGE
NO
RELIEF
H
A TARGET
hA RELIEF
DATUM PLANE
HORIZONTAL CONTROL POINT

Fig. 4

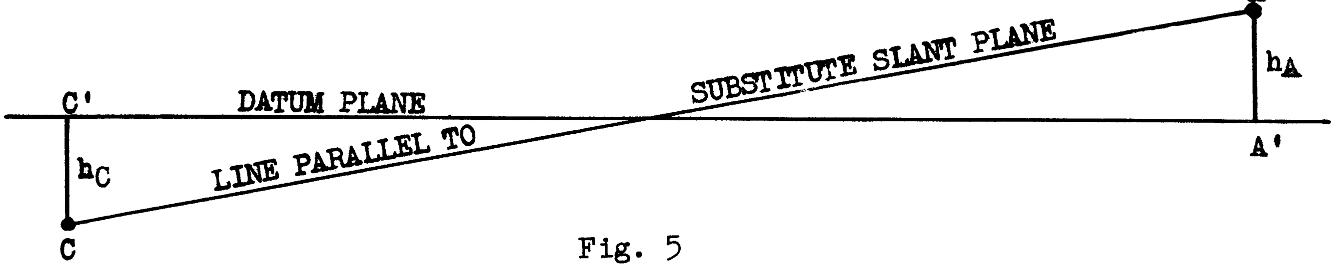

A
SUBSTITUTE SLANT PLANE
hA
C'
DATUM PLANE
LINE PARALLEL TO
hC
A'
C

Fig. 5

Inasmuch as Δt_{ac} and Δt_{bd} are small angles they may be summed as vectors Δt_{ac} along line $\overline{A'C'}$ and Δt_{bd} along line $\overline{B'D'}$. This sum Δt is the angle between the substitute slant plane and the ground datum plane.

It is now necessary to project this vector into the component planes of the photographic coordinates in order to apply the corrections to t_x' and t_y'. To do this consider Fig. 7 which represents the traces of planes LAC and LBD on the photograph. The projection on the x component plane is

$$\Delta t_x = \Delta t_{ac} \cos \xi + \Delta t_{bd} \cos \eta \quad , \tag{39}$$

and on the y plane

$$\Delta t_y = \Delta t_{bd} \sin \eta - \Delta t_{ac} \sin \xi \quad . \tag{40}$$

Now assume the following conventions: vectors Δt_{ac} and Δt_{bd} are directed toward the lower of the two reliefs in question and Δt_{ac} and Δt_{bd} are positive if they are directed toward increasing x and y values respectively. Then

$$t_x = t_x' + \Delta t_x \quad , \tag{41}$$

$$t_y = t_y' + \Delta t_y \quad , \tag{42}$$

$$H = H' \frac{h_A+h_B+h_C+h_D}{4} \quad . \tag{43}$$

now define

$$x_n = f \tan t_x \quad , \tag{44}$$

$$y_n = f \tan t_y \quad , \tag{45}$$

then proceed with the calculation for X_N, Y_N, and α_H using the above corrected values with only one change. Equation 22 becomes

$$R_{AN} = (H-h_A) \tan M_A \quad . \tag{46}$$

The reader might note that in the foregoing several approximations were used. All these contribute to the error in the method. The author is not able to place a rigorous upper limit on the flight, terrain conditions for a given error in this method. However, it can be safely stated from experience that if the reliefs are less than about 25 feet the error of the method is appreciably less than the random error of 15 seconds in t_x and t_y for altitudes over 10,000 feet. This forms a somewhat serious limitation on this method, although the above restrictions could probably be relaxed somewhat with no serious effect on results.

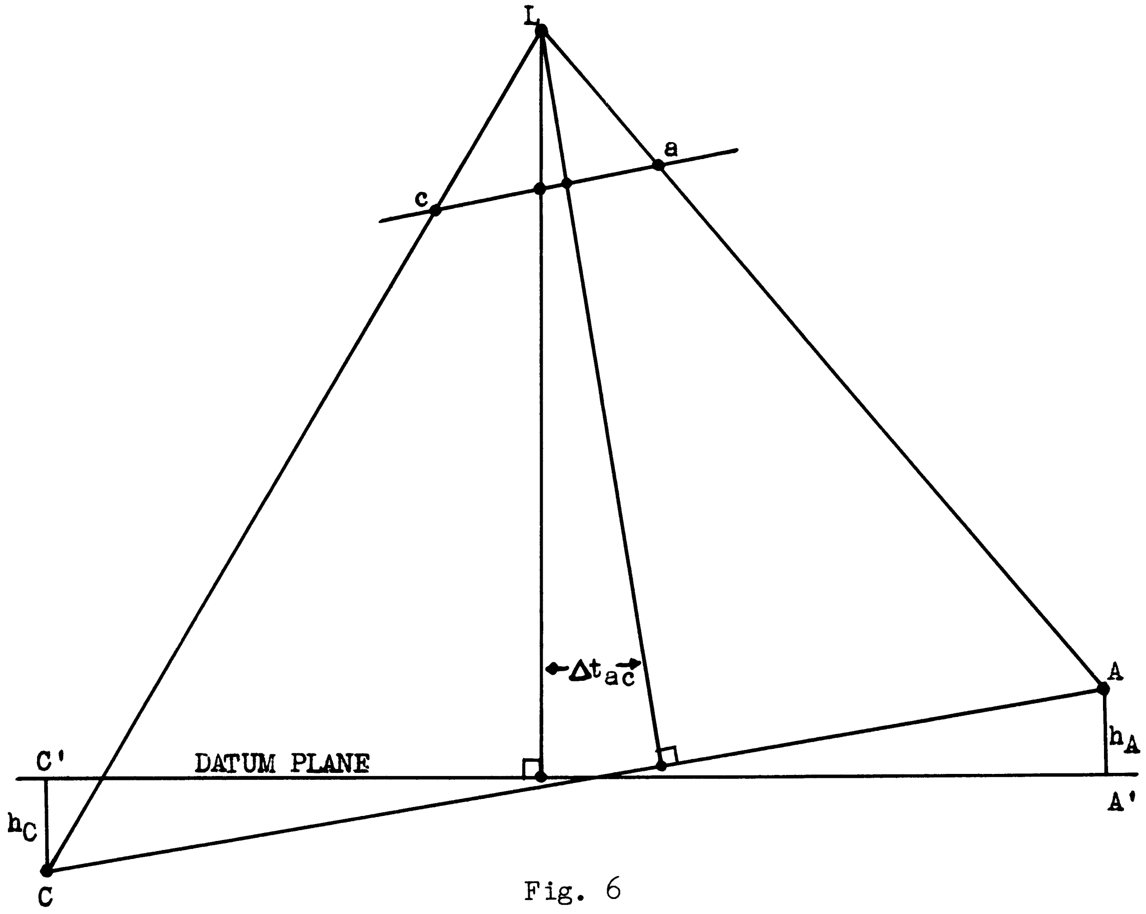

L
a
c
A
Δt_{ac}
C'
DATUM PLANE
h_A
A'
h_C
C

Fig. 6

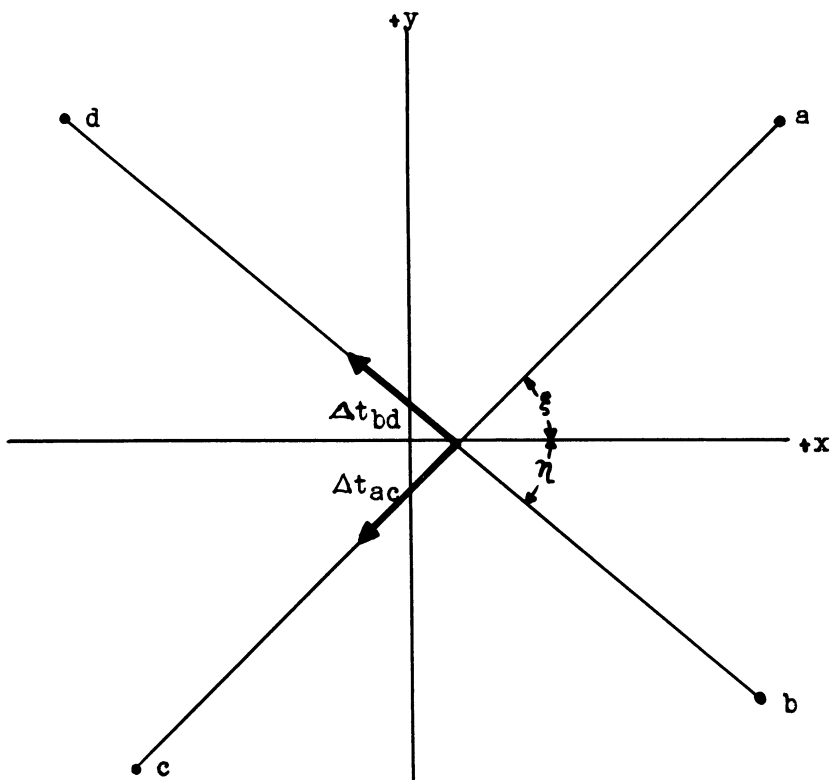

+y
d
a
Δt_{bd}
ξ
+x
η
Δt_{ac}
b
c

Fig. 7

The method of vector relief was adopted basically from a method developed by Dr. Traenkle of the Wright Field Air Development Center.

Method B Iterative Procedure

Unlike the vector relief method there are no approximations used with this method. The solution obtained is precise. Hence, there are no restrictions as with the vector relief method. As will be seen the computations are more complex.

Refer back to Fig. 4 which shows geometrically the displacement on the photograph due to relief when the position in space of the photograph is known. Analytically this displacement can be expressed componentwise as

$$\Delta x_a = - \frac{x_a - x_n}{1 + \frac{(H-h_A)f'^2}{h_A N_a}} , \tag{47}$$

$$\Delta y_a = - \frac{y_a - y_n}{1 + \frac{(H-h_A)f'^2}{h_A N_a}} \tag{48}$$

where all symbols used have been previously identified. From Fig. 4 it is obvious that if Δx and Δy were known, point a could be displaced to its ideal position and the relief h_A ignored. In Equations 47 and 48 the unknowns are H, x_n, and y_n. Thus, by guessing at H by rough scale or altimeter reading as a starting point and assuming that $x_n = y_n = 0$, Equations 47 and 48 become

$$\Delta x_a = - \frac{h_A x_a}{H} , \tag{49}$$

$$\Delta y_a = - \frac{h_A y_a}{H} \tag{50}$$

Repeating this for points b, c, and d the photographic coordinates can be corrected by these values and x_n, y_n, and H can be determined. Now discarding the first solution, start over applying Equations 47 and 48 with our new values for x_n, y_n, and H and repeat the entire procedure until the values of x_n, y_n, and H no longer vary to the accuracy desired. It should be emphasized here that in Equations 47 and 48 the coordinates x_a and y_a (they also appear in N_a) refer always to the original values and not to any values obtained during the previous iteration.

Now taking the final corrected coordinates as used in the final solution in the iterative procedure (not those used in Equations 47 and 48) with the final values for x_n, y_n, and H the remainder of the solution proceeds as previously indicated.

This method involes more complicated computational techniques and is not at all suited to hand computation. However, in the more recently developed electronic computers the method causes no difficulties. This project has had the vector-relief method programmed on an IBM punch card computer (CPC-2) and the iterative procedure programmed on an electronic computer (MIDAC). MIDAC (Michigan Digital Automatic Computer) is the property of the University of Michigan. Results have been very successful with both of these computers.

The iteration closes with great rapidity and in general the first trial is as good as the vector-relief method itself. The second iteration is usually sufficicent to exceed the random elements by a large amount which is, of course, sufficient. Tests made during this project show that for reliefs as great as 0.2 H the iteration closes to less than 3 seconds of arc in only five iterations. Hence, with automatic computers available this method is much superior to the vector method.

THE PHOTOGRAMMETRIC RANGE

The types of target, their distribution, extent of the range, relief considerations, and the calculation of horizontal control will be discussed under this topic.

In general the photogrammetric range is an array of clearly marked points on the ground, the geodetic coordinates of which have been located with high accuracy by triangulation or traverse.

There are two types of photogrammetric targets. One is the specially constructed panel and the other is the natural terrain feature. The latter type consists mainly of road intersections, building features, hill tops, and even foot-path crossings. Examples of the use of this type of target are the ranges at Brookville, Ohio and Phoenix, Arizona, although the latter range does contain some panel targets. This type of target though usable is not efficient. Many of these targets are very difficult to find and measure in the comparator. Some of them may be altered by construction or even obliterated by the property owner. As each target is different in nature it is virtually impossible to set a uniform altitude limit for range use. Thus, while a road intersection may be fine at 30,000 feet a neighboring foot-path crossing would be completely useless. These same points at 10,000 feet might be reversed as while the foot-path crossing may be sharply defined, the roadway would probably appear broad and hazy. This is the type of difficulty to be expected with this type of target.

The other type of target is the panel. These panels are uniform in size and appearance in one range. However, they may take several forms. One form is that of a circular disk; another is a nine square checker board pattern, and a third is that of a cross. The function of these panels is to form a clearly defined image on the photograph. In order to do this the panel must have dimensions that will overcome the attenuating factors such as halation, resolution, and image motion. If the target has a pattern such as a checker board then each segment must meet the above requirements. In addition these panels must contrast well with the surrounding terrain.

The circular disk, providing contrast is obtained, best satisfies all these conditions as none of the above effects vary the symmetry of the target outline. The range at McClure, Ohio is of this type. It consists of 6-foot concrete disks in the center of the range and 12-foot disks along the outer edge. The reason for the two different sizes of targets in this range has no relation to the problem at hand. This range is excellent up to 15,000 feet and usable up to at least 20,000 feet.

The cross satisfies all the conditions that the disk does and is correspondingly useful. It is not quite as easy to measure on the comparator as the symmetry is not as easily distinguished as that of a disk. The range at Phoenix, Arizona, contains a few cross panels which are 48 feet long and about 10 feet wide. These crosses are excellent up to at least 20,000 feet and usable probably to 30,000 feet.

The checker-board pattern satisfies all the conditions of the cross except the possibility of image motion effects. This pattern appears as a cross on the photograph. The most useful aspect of this pattern is that contrast is always achieved as the target contains both dark and light elements. However, these targets must be physically large as each segment of the pattern must have the same dimensional requirements as that of the entire disk target. Also if the image motion is large enough to superimpose one segment on the next the entire pattern might well disappear. The range at Eglin Air Force Base, Florida, is of this type and the targets are 30 feet square with 10-foot checkers. At 10,000 feet image motion difficulty has been encountered. At 20,000 feet the range is excellent and probably usable to 30,000 feet.

In conclusion it is recommended that the panel target be used in this type of reduction. The disk type is probably the best but the other types are completely usable. The panel should have such contrast and dimensions as to form a clear image at the highest altitude desired. Unfortunately, due to varying resolutions of different lens systems, it is not possible here to fix target dimensions that will cover all siuations. It can be said, however, that for the metrogon lens the target elements should subtend an angle of about 0.0003 radians or greater at the highest altitude to be used. It is best to determine the target dimensions experimentally with the equipment to be used to insure the best results.

The distribution of targets must be such that sufficient images fall in one photographic frame to provide data for a predetermined number of solutions. On this project it has been a practice to work three independent solutions requiring twelve separate control targets on the photograph. In order to get strong solutions these targets must be well spread or distributed around the outer edges of the frame.

Most photogrammetric ranges are laid out in a rectangular grid. The ranges at Eglin Air Force Base, Florida, Phoenix, Arizona, and Brookville, Ohio, are such ranges. The first is a 10,000-foot grid and the other two 5,000-foot grids. The McClure, Ohio range contains a 5,000-foot grid, but in addition there are many targets interspersed in this pattern.

A typical aerial camera records a ground area of approximately 3H/2 feet square. This assumes a six-inch lens and a nine by nine-inch frame. Assuming a grid pattern of targets an ideal minimum arrangment on the photograph would appear as in Fig. 8. To insure the appearance of at least the arrangement of Fig. 8 on the photograph, the ground area must obviously be at least 4S square where S is the grid separation. This corresponds to an altitude of 8S/3 feet. Also from Fig. 8 it can be seen that if the position of the aircraft can be controlled accurately an area of only 3S feet square or an altitude of 2S feet would be sufficient. Thus, the minimum altitude for range use lies between 8S/3 and 2S feet. For S equal to 10,000 feet this corresponds to a range of from about 20,000 to 26,666 feet. This is merely an example of the type of reasoning that can be applied and it, of course, must be modified to fit the camera-range combination that is to be used as well as the experimental conditions.

The distribution of the targets gives a minimum working altitude. The maximum working altitude is found from a consideration of the target dimensions and the extent of the range.

The extent of the range, providing the targets are usable, determines the upper limit of the working altitude as well as restricting the pattern of flight. Using the same reasoning as before (six-inch lens and nine by nine-inch frame), a range whose minimum dimension is K feet should not be used at altitudes over 8K/9 feet. This assures that the range covers at least 3/4 of the frame width. Much less than this weakens the solution too much. The maximum dimension of the range is also the maximum length of usable flight path in one pass over the range.

All the above assumed that the tilt of the camera was low, say less that 10°. If the tilt is high, then the ground area included in one frame increases greatly and the targets may be much farther from the camera than before at a given altitude. This much more complex problem will not be discussed here as this is essentially a low tilt reduction.

The foregoing also assumed a six-inch lens and nine by nine-inch frame. There are of course a wide range of cameras available with different dimensions. Each could be considered in the same manner with the ground area covered being WH/f feet square, where f is the focal length, W frame width, and H altitude.

If the vector-relief method in calculation is desired, then the target reliefs must be considered. These should be of the order of about 25 feet or less from the datum plane. Only the McClure, Ohio, range is completely satisfactory in this respect. With the iteration procedure the reliefs are of no special interest.

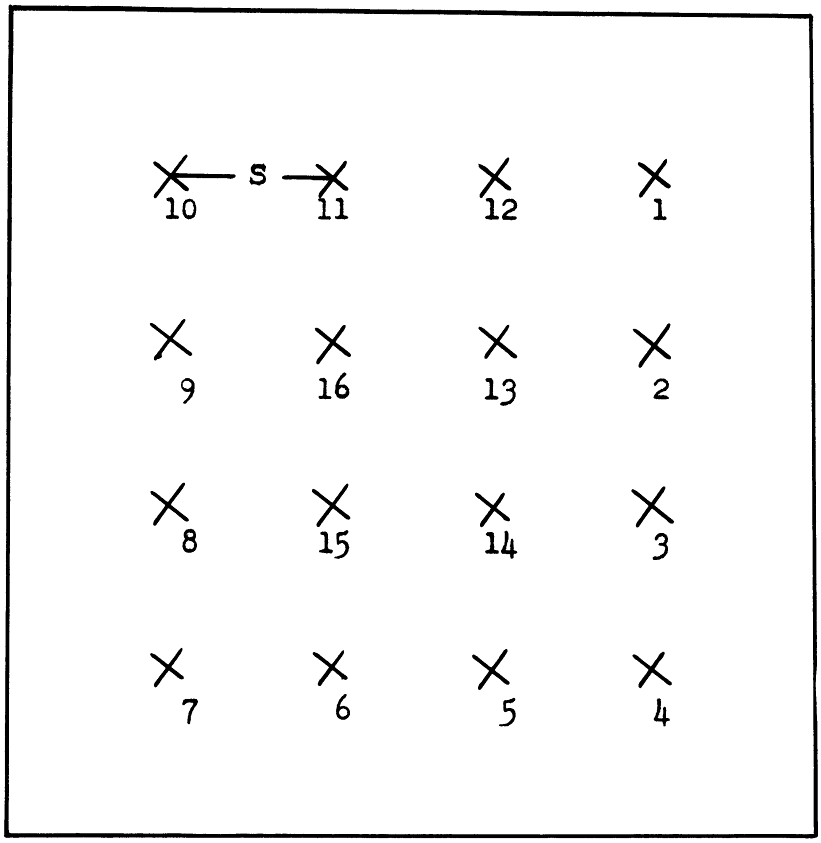


Fig. 8

To sum the data for the three principal ranges now in use that are known to the author:

Phoenix, Arizona. This range has external dimensions of 150,000 by 170,000 feet. The targets are terrain features in a 5,000-foot grid. The lower altitude limit is from 14,000 to 10,000 feet. The upper limit is difficult to fix with this type of target, but is probably around 30,000 feet in most sections of the range. There are unfortunately many gaps in the control points which would restrict the flight patterns somewhat. However, this range permits much more maneuvering than the other three. The reliefs are quite high and the iteration procedure should be used with this range.

Eglin Air Force Base, Florida. This range has external dimensions of 30,000 by 90,000 feet. The targets are panels in a 10,000-foot grid. These panels are checker board and are 30 feet square with 10-foot checkers. The lower altitude limit is from 27,000 to 20,000 feet and the upper limit is about 27,000 feet. This range is highly restricted as to altitude and flight patterns. The reliefs are moderate and lie on the borderline for the vector relief method.

McClure, Ohio. This range has external dimensions of 25,000 by 25,000 feet. The targets are panels in a 5,000-foot grid with many more interspersed in this pattern. These panels are six and twelve-foot disks. The lower altitude limit is about 7,000 feet and the upper limit is about 20,000 feet. This lower altitude range permits a slight amount of maneuvering.

The above altitude considerations assumed a six-inch lens with a nine by nine-inch frame and the inclusion of sufficient targets for three independent solutions. The altitudes should, of course, be modified for other conditions.

The method and the necessary parameters for the calculation of the horizontal control are given in references 5 and 6. Only the accuracy will be discussed here.

Most ranges are surveyed by intersections from first order triangulation stations. The position accuracy of such points varies slightly with the distances involved. At the McClure, Ohio, range this error is about four inches and probably is of this order of magnitude in most such photogrammetric ranges. The calculation procedures give an accuracy of one part in a hundred thousand, providing the origin for calculation is within 18.6 miles of the point in question. These errors are much smaller than the film errors mentioned before, and hence, have negligable effect on the overall accuracy of the solution.

The target elevations are surveyed from sea level to a very high degree of accuracy. Errors in elevation have even less effect on the overall accuracy than horizontal errors and may also be neglected.

In discussing the ground datum plane there are several points that should be mentioned briefly. In effect the computation locates the control point on a flat surface such that planar distances equal the equivalent geodetic values with an accuracy of one part in one hundred thousand within 18.6 miles of the origin. This means that this plane can be considered tangent to the earths surface anywhere within 18.6 miles of the origin without appreciable error in horizontal position. As such an area includes all present ranges it is a valid assumption that the ground planar coordinates lie in a plane that is tangent to the earths surface at the ground nadir point for each photograph considered. The actual determination of such a plane is done by the curvature correction which will be considered in the appendix. Although distances in the plane are true, azimuths are not unless corrected. This correction is

$$\Delta\alpha = \Delta\lambda'' \sin \phi_m \quad . \tag{51}$$

Hence, if it is desired to determine the true azimuth about some point in the plane, the correction is found by Equation 51, where $\Delta\lambda''$ is the difference in seconds in longitude between the point in question and the origin of calculation and ϕ_m is their mean latitude. $\Delta\alpha$ is in seconds and is positive east and negative west of the origin.

Also the coordinates as computed in reference 6 are at sea level scale and should be corrected to some mean elevation for the range. This correction factor is $1 + h/R$, where h is the elevation desired and R is 20,906,000 feet. The only reason for this is to keep the reliefs as small as possible and to reduce the computed altitude to the datum plane reference. On this project the mean elevation of all the control points was used.

The origin of the calculation is usually chosen as some point near the center of the range to hold the calculation error as low as possible. It is common practice to later translate the origin so that all coordinates are positive. The elevations are from sea level and should be translated also to the mean elevation. This gives both positive and negative values for elevation from the ground plane.

In conclusion it can be said that the random errors introduced by the ground control points are dominated by the film errors to such a degree that they may be considered as second order.

In order to insure this situation great care should be taken both in the survey and in computing the ground coordinate system.

PROCEDURE

Following are the procedures used on this project. Inasmuch as this project has no direct experience beyond this point it will be assumed that the procedure starts with the exposed and developed film. It will be assumed also that automatic computation is available. If hand computation is used much of the procedure is best developed by the computers themselves.

First, the frames are identified and cut to fit the measuring apparatus. The identification is usually done by flight and run numbers in conjunction with the film-counter number which ordinarily appears on each frame.

Next, the control points on the photograph are identified and marked on the frame. This identification is most easily done with a map or mosaic of the range for comparison. At this point the most desirable configurations for solutions are selected and only the control points to be used are selected. These points are recorded on the data sheet for measuring in groups of four for each solution desired. For example, in Fig. 8 the points might be grouped 1-4-7-10,2-5-8-11, and 3-6-9-12. This gives three well-spread configurations made up of four independent control points in each case. Here points 13-14-15-16 are not used. On a less ideal photograph this system has to be compromised as such symmetric groupings may not be possible. It may be necessary to include one or more control points in one solution, however, this should be avoided as much as possible. On this project the following guides are used for each frame when three solutions are desired: Only one point is to be used more than once and then in two, but never three solutions. Thus, there must be a minimum of eleven usable control points. The areas inclosed by each group are held as equal and as large as possible. Each group is selected to be as close to a parallelogram as possible. Any grouping that has one or more triangular areas that is very much smaller than the others is to be avoided. If a frame that does not meet these conditions is evaluated, the solution should be expected to be accordingly weaker.

Following the group selection the frame is placed in the comparator for measuring. The frame should always be placed in a consistent manner. On this project the frame is placed emulsion up so that the positive y axis is to the right. The "y" coordinates are then read and the frame rotated 90° so that the positive x axis is to the right for the "x" coordinates. The reason for such a consistent orientation is that from frame-to-frame the coordinate sign as determined by the computer will be consistent. The frame is now measured as follows: First the frame is adjusted until the comparator reads the same at both the upper and lower fiducial marks. This assures that the reference axis is normal to the line of measurement and this reading is recorded as the axis reading. Next, the reading is taken at each of the control points desired and recorded in the appropriate position on the data sheet. Also, the reading is taken on the two ratioing markers and the ratio recorded. This ratio is the known distance between these markers divided by the measured distance. The frame is then rotated to the other axis and the procedure repeated. An experienced operator can get sufficient accuracy (0.005 mm) with one set of readings with clear control points.

The data sheet used for measuring should provide space for two coordinates for each control point. In addition space is required for the two axes and the two ratios. If the sheet is also used to code the computer it should provide space for three datum plane coordinates for each control point.

The frame is now ready for computation. The computer then forms and solves the necessary equations for P, R, α_H, X_N, Y_N, and H. In doing this the computer must also remove the systematic deviations in order to get the necessary photographic coordinates for the solution. The details of the operation of the computer in getting a solution from the photographic coordinates will not be discussed here. They are very straight forward and can be developed as needed. The method in getting the photographic coordinates from the comparator data will be discussed next.

With reasonably experienced personnel the entire procedure including one set of measurements takes about fifteen minutes exclusive of computer time. For efficiency these operations can done on a mass production basis if a large number of photographs is to be evaluated.

OBTAINING PHOTOGRAPHIC COORDINATES FROM THE COMPARATOR DATA

This is actually part of the operation of the automatic computer. With automatic computation the following should be programmed on the computer.

In order to get the photographic coordinates it is first necessary to remove the systematic deviations. Any thermal-pressure effects on the camera-measuring system that are known can be removed by ratioing the focal length of the camera. In general, these will not be known and no provision can therefore be made. However, as mentioned before this is of no significance unless H is a critical value. The other deviations to be removed are lens distortion, earth curvature, and atmospheric refraction. Each of these are considered to be functions of the distance on the photograph from the principal point to the control point in question. This will be called the radial distance r. Also each of these deviations is considered to be along this radial line. The magnitudes of these deviations are a known empirical or analytical function of r with f and H parameters. The three deviations if summed can be tabulated or plotted with r as argument and f and H parameters. The three deviations are discussed in more detail in the appendix. It has been our experience that the simplest way to express the deviation as a function of r is with a polynomial in r. This polynomial is found by the method of least squares from tabulated data. It has also been found that a cubic gives sufficient accuracy for our purposes.

Therefore, if f and H are known the deviation as a function of r is

$$D = C_1r^3 + C_2r^2 + C_3r + C_4 \quad , \qquad (52)$$

where D is the deviation in millimeters along the radial line and C_1, C_2, C_3, and C_4 are known constants.

Now the step-by-step procedure taken by the computer for one of four control points is:

1. Obtain the ratioed, measured coordinates (x_p', y_p') ,

$$x_p' = (x_A - x_p'')R_x \qquad (53)$$

$$y_p' = (y_A - y_p'')R_y \quad , \qquad (54)$$

where (x_p'', y_p'') are the comparator readings of the control point, (x_A, y_A) are the corresponding axes readings, and R_x and R_y are the ratios in x and y, respectively.

2. Find the radial distance r_p ,

$$r_p = \sqrt{x_p'^2+y_p'^2} \quad . \tag{55}$$

3. Form and solve Equation 52 for D.

4. Find the coordinate components for D or D_x and D_y,

$$D_x = D\,(x_p'/r_p) \tag{56}$$

$$D_y = D\,(y_p'/r_p) \quad . \tag{57}$$

5. Obtain photographic coordinates (x_p,y_p),

$$x_p = x_p' + D_x \tag{58}$$

$$y_p = y_p' + D_y \quad . \tag{59}$$

This is of course repeated for the other three control points in the solution. At this point the main part of the solution is done by the computer. The constants C_1, C_2, C_3, and C_4 along with f may be considered as parameters of the solution.

INITIAL DATA FOR COMPUTATION

Parameters

For a given camera f is fixed and if H is known then C_1, C_2, C_3, and C_4 can be found. As these values are not at all critical in H it is usually possible to use one average value for the entire series of flights. For example, if H is taken as 25,000 feet then the computed constants are sufficiently accurate (within the film errors) from 22,500 to 27,500 feet or a 5,000-foot range. Thus, if C_1,C_2,C_3 and C_4 are found for 5,000-foot intervals in the range of altitudes to be used, sufficient accuracy will be obtained. These parameters are best stored in the computer and applied as long as the altitude does not vary beyond the above limits. For this purpose the altimeter reading is accurate enough for H.

Frame Data

For each frame there are four input items for the computer. These are the two ratios and the two axes readings. There is no point in repeating these for each problem in the frame, although it may be simpler to do so from the point of view of the programmer of the computer.

Problem Data

For each problem there are twenty input items consisting of eight photographic coordinates and twelve ground coordinates. If an electronic computer is used, it is possible with the large storage available to store the entire system of ground coordinates and refer to them by code numbers. This would save considerable transcription.

There are in addition to these input items a guessed value for H in the case of the iteration procedure. Some example data sheets are included in the appendix.

CONCLUSION

It is hoped that the foregoing will answer the major questions that might arise in the consideration or application of these procedures. There are of course many questions left unanswered, but it is hoped that these are of secondary importance and that most of them can be worked out independently.

The procedures given are based on those in present use on this project and naturally depend on the equipment possessed by this project. It should be reemphasized that much of the text refers to specific conditions and the use of certain equipment. In these cases it is meant merely to give examples and modification may well be necessary.

It should also be mentioned that this reduction process is essentially a low-tilt method (10° or less). By slight extensions of reasoning it can be made applicable to low-oblique photography by reviewing all conditions that depend on a low-tilt consideration.

In all, over 500 frames have been subject to this analysis in one form or another by this project with success in almost every case.

APPENDIX

The inclusions in this appendix are first brief discussions and examples of the three deviations, curvature, refraction, and lens distortion. This is followed by an example of a composite or sum curve of these deviations along with a sample cubic equivalent.

Next, is a typical frame taken from a run over the McClure, Ohio, area. Then follows a sample reduction using this frame by both the vector-relief and iterative process.

Curvature

The curvature correction is probably the least understood of the three systematic deviations and will be discussed here in more detail. Consider Fig. 9. As noted under Photogrammetric Range the plane of the horizontal control may be considered tangent to the earth (approximated by a sphere in Fig. 9) at the ground nadir point O. The point P on the spherical surface is computed to be at P' in the tangent plane such that $\overline{OP'} = \overset{\frown}{OP}$. Inasmuch as ψ will not exceed five minutes under any practical circumstance, this is certainly true to a very high degree of accuracy. The physical effect of the point P actually being on the spherical surface is to compress the photograph in that region by the amount pp' as is obvious from Fig. 9. Now the distance PP' is

$$PP' = OP'^2/2R \tag{60}$$

and as $\overline{PC}$ is practically parallel to $\overline{OC}$

$$PP'' = OP'^3/2RH \quad . \tag{61}$$

Ratioing this value to the photograph gives

$$pp' = Hr^3/2Rf^2 \quad . \tag{62}$$

For a six-inch lens with R equal to 20,906,000 feet this becomes

$$pp' = 1.051408\ 10^{-12}\ Hr^3\ \text{mm} \quad . \tag{63}$$

Now the tangent plane and spherical surface in Fig. 9 are at the elevation of the target in question. The correction for a different elevation would change pp' very slightly. However, for normal reliefs in a photogrammetric range this difference is small enough to neglect. Also, Fig. 9

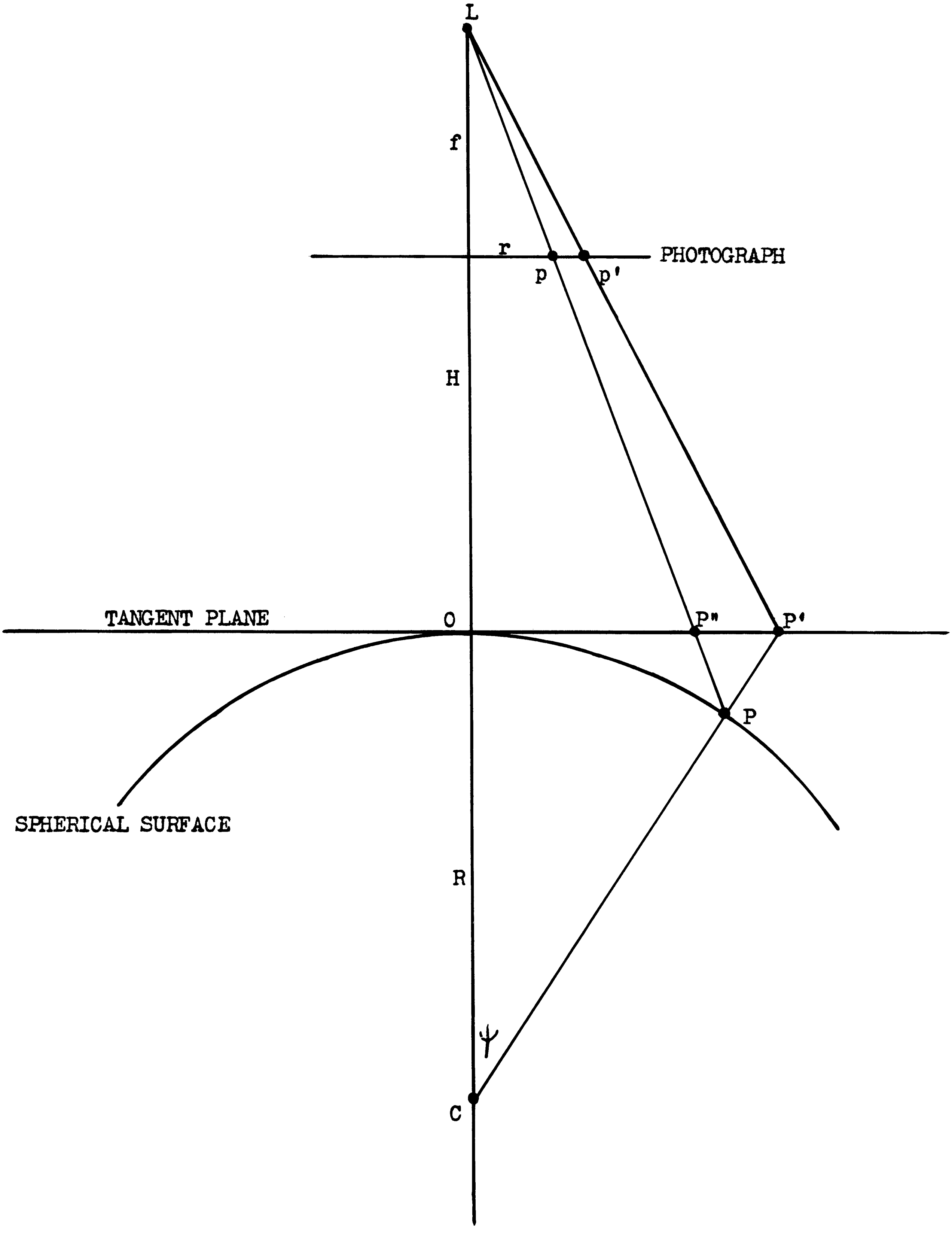

L
f
r
PHOTOGRAPH
p
p'
H
TANGENT PLANE
O
P"
P'
P
SPHERICAL SURFACE
R
ψ
C

Fig. 9

assumes no tilt which can only be neglected for low tilt such as less than 10°. A plot of Equation 63 appears in Fig. 10 for varying altitudes. This plot is a radial correction on the photograph against radial distance from the principal point. The correction is outward from the principal point.

Refraction

This effect is discussed thoroughly in reference 7. The plot in Fig. 11 for a six-inch lens shows the radial correction against radial distance from the principal point for varying altitudes. This correction is toward the principal point.

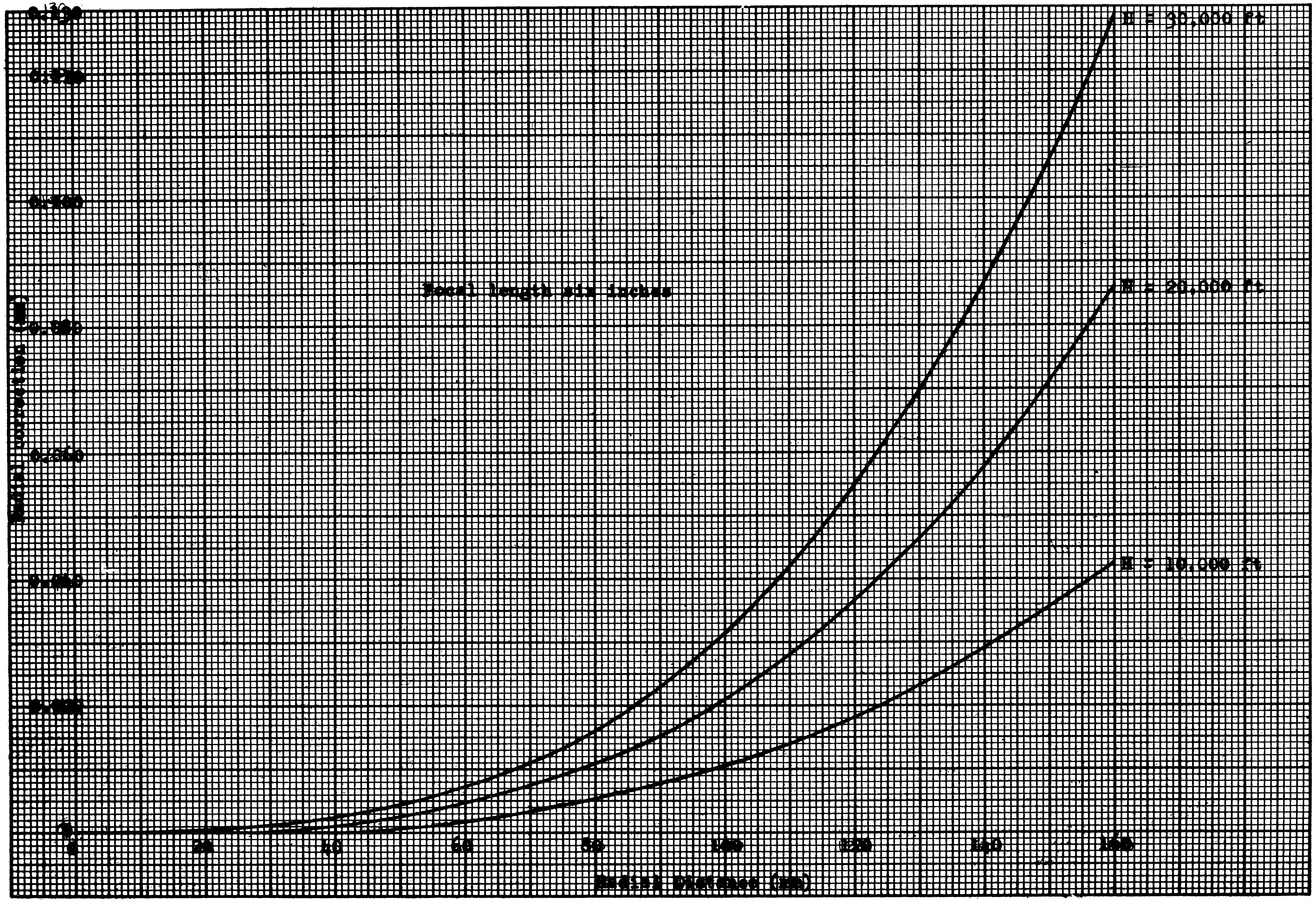
Lens Distortion

This effect is discussed in any good reference on photogrammetry. A typical distortion curve is shown in Fig. 12. This is for a six-inch metrogon lens. The distortion correction is negative above the axis and positive below. The curve is for T-11 51-193.

Composite Curve and Cubic

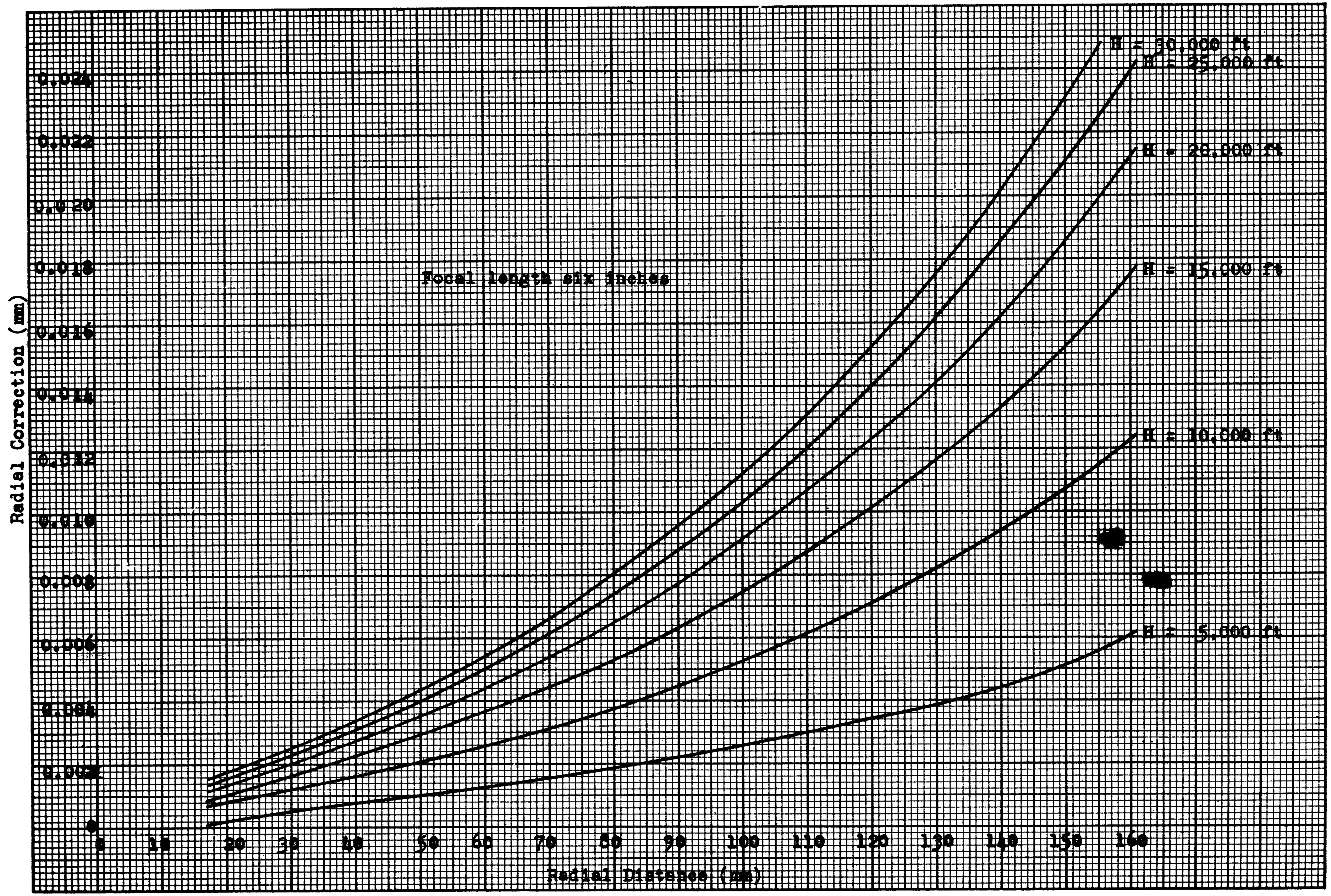
The following curves (fig. 13) are the sum of the preceding three for H = 10,000 feet and the corresponding cubic curve. This curve is positive beneath the axis and negative above it. The solid curve is the tabulated sum curve and the dashed line curve is the cubic computed to fit this curve by least squares. These curves are the ones to be used in the elimination of the systematic deviations as discussed under Obtaining the Photographic Coordinates. It can be seen from the curve that for radial distances less than about 30 mm the cubic curve deviated badly. This curve is to be applied for radial distances greater than 30 mm only. However, a radial distance this small is never found and for ordinary use this deviation is of no serious consequence. It should of course be kept in mind for exceptional circumstances. The cubic otherwise never deviates from the composite curve by more than 0.010 mm. This is sufficient accuracy for most purposes. If a better fit is desired it can of course be obtained by the use of a higher order polynomial. The cubic illustrated has the following coefficients:

$$
\begin{aligned}
C_1 &= 7.5233 \times 10^{-7} \\
C_2 &= -1.6253 \times 10^{-4} \\
C_3 &= 9.0578 \times 10^{-3} \\
C_4 &= -1.5699 \times 10^{-1}
\end{aligned}
$$



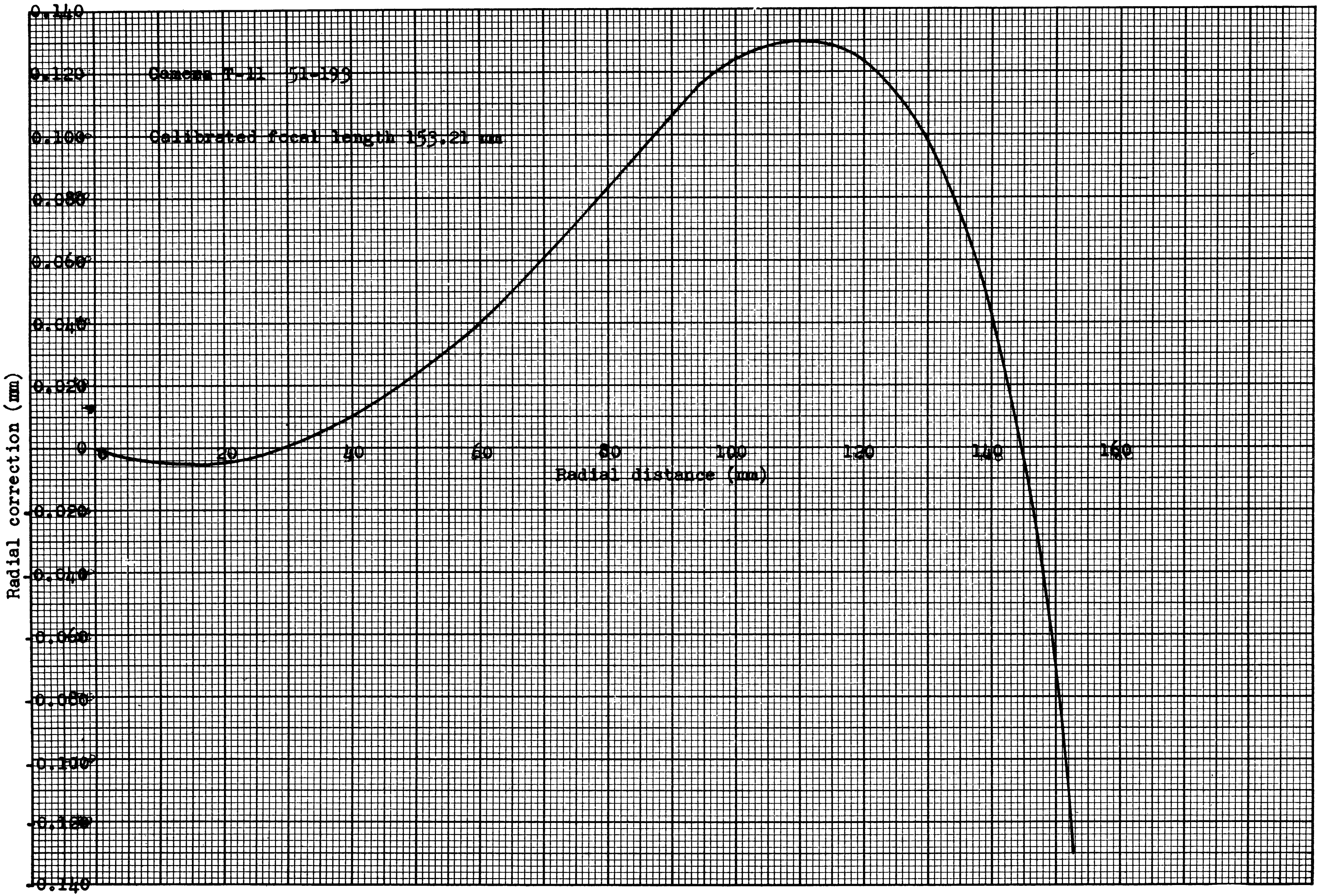
CURVATURE CORRECTION CURVE

Fig. 10



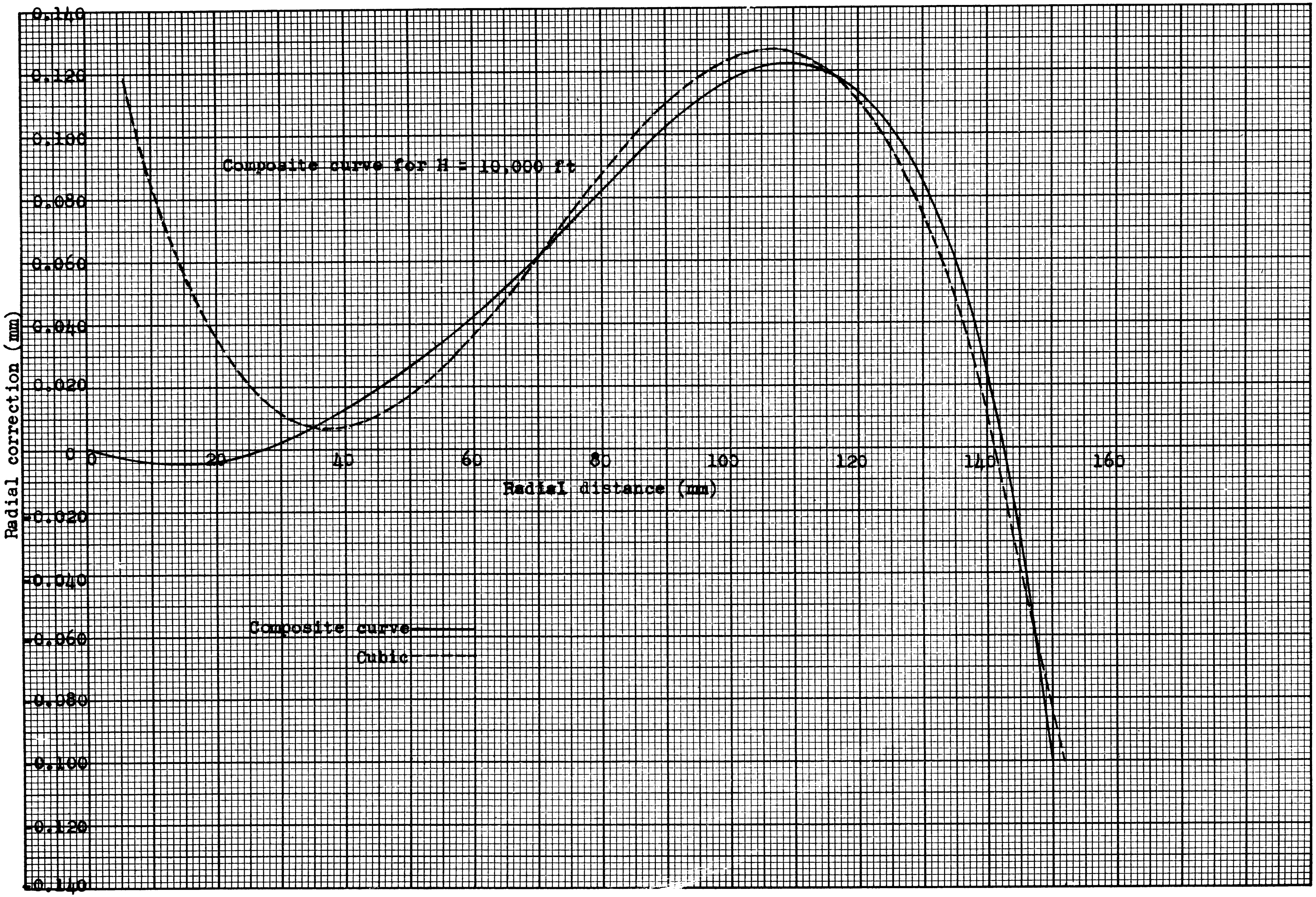
REFRACTION CORRECTION CURVE

Fig. 11



LENS DISTORTION CURVE

Fig. 12



COMPOSITE CURVE AND CUBIC

Fig. 13

It should be noted that in this curve and the ones for lens distortion and refraction a positive correction is towards the principal point. The curvature curve has the opposite sense.

A Typical Frame

Fig. 14 illustrates a typical frame taken from a run over the McClure, Ohio, area. This gives a practical illustration of the selection of control points on a frame, as well as showing a portion of one of the photogrammetric ranges in present use. Here the problems are grouped as follows: 14-49-47-9, 13-52-50-15, and 12-34-51-30. There are in addition to the twelve control points selected thirty more on this frame. This shows the very dense control in this area. These targets are all six-foot disks which in general are very sharp on this frame at about 10,000 feet. The east-north arrows shown are for the purpose of facilitating the location of control by the comparator operator. The following numerical examples are from data taken from this frame.

Numerical Example

Vector Relief. This problem is 14-49-47-9 from the frame illustrated. The initial data are given on the following data sheet. This sheet is for IBM computation with the CPC-2. This machine program is only set up to solve for t_x and t_y. However, here the entire solution will be obtained. The symbols used in the text will be followed as closely as possible and appropriate equation numbers will be referred to.

The first step is to obtain the photographic coordinates. This is done as follows: For point a using Equations 53 and 54

$$x'_a = 112.483 \quad *$$

$$y'_a = 99.247 \quad .$$

From Equation 55,

$$r_a = 150.008 \quad .$$

From Equation 52 using the given coefficients

$$D = 0.084$$

* In all cases photographic distances are given in millimeters and ground distances in feet. The solution will have corresponding units.

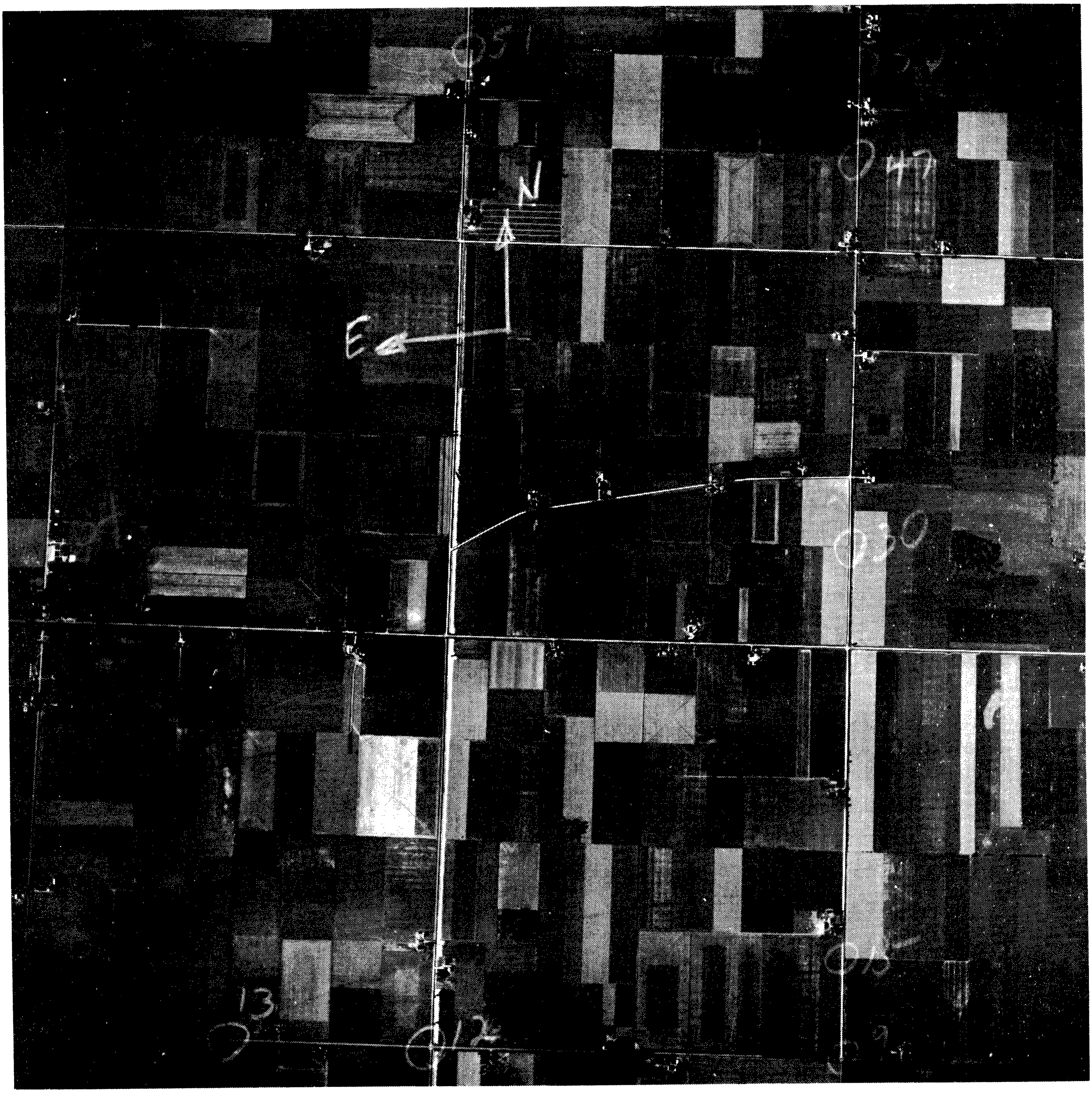


Fig. 14

INITIAL IBM CPC-2 DATA

Photo No. 16	CODE	00 - Ground Coord.	Date 12-12-52
Run No. 16		40 - Axes, Ratios, and Cubic	Control 14-49-47-9
Camera T-11 51-193		50 - Photo Coord.	Prod. No. 1
		80 - Elevations and f	

Code	X_A	Y_A	X_B	Y_B	X_C	Y_C	X_D	Y_D
	(x Coord.) x axis	(y Coord.) x axis	R_x	R_y	C_1	C_2	C_3	C_4
	x_a	y_a	x_b	y_b	x_c	y_c	x_d	y_d
	Z_A	Z_B	Z_C	Z_D	f (xxx,xx000)			
00	19,061.59	3,446.72	19,051.22	15,319.10	8,464.52	15,406.24	8,403.34	3,485.84
40	130.116	133.051	1.0029271	1.0029572	00075233	-00162530	00090578	-00015699
50	17.961	34.097	32.836	221.364	196.332	213.567	189.496	25.360
80	696.12	683.68	684.87	694.54	153.21000			

From Equations 56 and 57,

$$D_x = 0.063$$

$$D_y = 0.056$$

and the desired coordinates from Equations 58 and 59 are

$$x_a = 112.546$$

$$y_a = 99.303$$

Repeating for points b, c, and d, the coordinates are

$$x_b = 97.518$$

$$y_b = -88.531$$

$$x_c = -66.329$$

$$y_c = -80.656$$

$$x_d = -59.505$$

$$y_d = 107.921$$

Now using Equations 1 and 2, solve for the eight areas necessary to find K_1 and K_2 in Equations 3 and 4. For this purpose, use the photographic coordinates and the datum plane coordinates given on the data sheet.

$$\begin{aligned}
A_{abc} &= 15{,}447.19145 \\
A_{acd} &= 16{,}251.83533 \\
A_{bcd} &= 15{,}475.75736 \\
A_{abd} &= 16{,}223.26942 \\
A_{ABC} &= 62{,}844{,}210.85 \\
A_{ACD} &= 63{,}526{,}498.33 \\
A_{BCD} &= 63{,}101{,}514.95 \\
A_{ABD} &= 63{,}269{,}194.23 \\
K_1 &= 0.960808321 \\
K_2 &= 0.956458319
\end{aligned}$$

Using f from the data sheet, form Equations 5 and 6

$$\begin{aligned}
154.6908991\, x_n - 192.2223948\, y_n &= -\,919.958199 \\
-173.9745580\, x_n - 175.6351805\, y_n &= -1{,}022.067119 \quad .
\end{aligned}$$

Solving for x_n and y_n the following is obtained.

$$x_n = 0.575591594$$
$$y_n = 5.249112525$$

Now from Equations 9, 10, and 11

$$N_a = 24{,}059.33725$$
$$N_b = 23{,}064.72546$$
$$N_c = 23{,}011.75327$$
$$N_d = 24{,}005.54299$$
$$f' = 153.3009739 \quad .$$

Using these values, find the four check numbers H^2 from Equations 12 and 13.

$$H^2 = 94{,}118{,}620.31$$
$$H^2 = 94{,}118{,}620.36$$
$$H^2 = 94{,}118{,}620.33$$
$$H^2 = 94{,}118{,}620.33$$

This agreement indicates that the solution is consistent with the initial data. From this H is found to be

$$H = 9{,}701.475$$

Now refering to the topic Method A Vector Relief,

$$t'_x = 0.2153°$$
$$t'_y = 1.9622°$$
$$H' = 9{,}701.475 \text{ ft.}$$

From Equations 37 and 38,

$$\Delta t_{ac} = 0.0007040$$
$$\Delta t_{bd} = 0.0006822 \quad .$$

From Equations 39 and 40,

$$\Delta t_x = -0.0000704 = -0.0040°$$
$$\Delta t_y = -0.0010322 = -0.0591° \quad .$$

From Equations 41, 42, and 43, noting datum plane elevation 686.87 ft,

$$t_x = 0.2113°$$
$$t_y = 1.9031°$$
$$H = 9{,}704.71 \text{ ft.}$$

This is as far as the CPC-2 was programmed, but the solution proceeds as follows From Equations 44 and 45,

$$x_n = 0.5650232$$
$$y_n = 5.0908006$$

Using these values and Equations 20 and 21, find tan M_a which is

$$\tan M_a = 0.9328033 \quad .$$

Now apply Equation 46 and not Equation 22 to find R_{AN}. Noting again that the elevation of the McClure range datum plane is 686.87 feet, find that h_A is 9.25 feet above the datum plane. Using this value from Equation 46, $R_{AN} = 9{,}043.96$. In the same way

$$R_{BN} = 8{,}689.37$$
$$R_{CN} = 7{,}025.31$$
$$R_{DN} = 7{,}376.91$$

Now Equations 18 and 19 can be formed,

$$-10{,}597.07\ X_N + 11{,}959.52\ Y_N = -16{,}892{,}765.2$$
$$-10{,}647.88\ X_N - 11{,}833.26\ Y_N = -246{,}885{,}127.4 \quad .$$

Solving for X_N, Y_N

$$X_N = 12{,}473.32$$
$$Y_N = 9{,}639.84$$

Before proceeding with the solution for azimuth, we note the positon of the photograph in the aircraft. The positive y axis is in the nose and the positive x axis is toward the left wing. Hence, due to the reversal in the lens system the negative y axis becomes the heading axis in an orthogonal projection. Now for point A find the azimuth α_{NA}. This is done with Equation 25.

$$\alpha_{NA} = 133.2292°$$

From Equation 26,

$$\theta_A = 136.3920° \quad .$$

From Equation 29,

$$\left[\alpha_{NO}\right]_A = 269.6212° \text{ or } 356.8372° \quad .$$

In the same way

$$[\alpha_{NO}]_B = 101.5434° \text{ or } 356.8430°$$
$$[\alpha_{NO}]_C = 356.8330° \text{ or } 293.5530°$$
$$[\alpha_{NO}]_D = 356.8382° \text{ or } 70.1196°$$

From Equation 30,

$$\alpha_{NO} = 356.8379° \quad .$$

From Equation 27,

$$\phi = 6.3368° \quad .$$

Finally, from Equation 31 and by examining the photograph to determine sign

$$\alpha_H = 3.1747° \quad .$$

Now noting Equations 7 and 8, write the final solution.

$$P = 1.9031°$$
$$R = 0.2113°$$
$$H = 9{,}704.71 \text{ ft}$$
$$X_N = 12{,}473.32 \text{ ft}$$
$$Y_N = 9{,}639.84 \text{ ft}$$
$$\alpha_H = 3.1747° \quad .$$

Numerical Example Iteration

This is the same problem as in the previous example. The initial data is given on the following data sheet. This sheet is for MIDAC and includes some additional input items. H_G is the starting value for the iteration in altitude, the item titled No. of solutions merely indicates the amount of the solution to be printed out, i.e., all six items or just t_x and t_y and the item No. of iterations $.2^{-44}$ tells the computer how many iterations to make. The second column locates the decimal and the third is a binary scale factor used by the computer. With MIDAC the parameters C_1, C_2, C_3, C_4, and f are listed separately.

The first step is to compute the photographic coordinates which is identical to the preceding example and will not be repeated. Next the iteration is started by applying Equations 49 and 50 to the photographic

INITIAL MIDAC DATA

Photo 16 — Prob 1
Run 1 — Control Point 14-49-47-9

CONTROL COMBINATION			EXPLANATION
.17961	2d	-9b	x_a
.34097	2d	-9b	y_a
.130116	3d	-9b	x_o
.133051	3d	-9b	y_o
.32836	2d	-9b	x_b
.221364	3d	-9b	y_b
.10029271	1d	-1b	R_x
.10029572	1d	-1b	R_y
.196332	3d	-9b	x_c
.213567	3d	-9b	y_c
.1	5d	-16b	H_G
.1	1d	-44b	no. of solutions
.189496	3d	-9b	x_d
.2536	2d	-9b	y_d
.925	1d	-14b	h_A
.1906159	5d	-16b	X_A
.344672	4d	-16b	Y_A
.1906122	5d	-16b	X_B
-.319	1d	-14b	h_B
.153191	5d	-16b	Y_B
.846452	4d	-16b	X_C
.1540624	5d	-16b	Y_C
-.2	1d	-14b	h_C
.840334	4d	-16b	X_D
.348584	4d	-16b	Y_D
.3	1d	-44b	no. of iterations $.2^{-44}$
.767	1d	-14b	h_D

coordinates. The corrected coordinates become

$$x_a = 112.442$$
$$y_a = 99.211$$
$$x_b = 97.549$$
$$y_b = -88.503$$
$$x_c = -66.342$$
$$y_c = -80.672$$
$$x_d = -59.459$$
$$y_d = 107.838 \quad .$$

Using these coordinates, solve for x_n,y_n, and H as before getting

$$x_n = 0.5647140$$
$$y_n = 5.1088577$$
$$H = 9{,}704.13 \quad .$$

The second iteration can now be started using the above values along with the original photographic coordinates and getting the new corrected coordinates by applying Equations 47 and 48.

$$x_a = 112.437$$
$$y_a = 99.211$$
$$x_b = 97.549$$
$$y_b = -88.561$$
$$x_c = -66.343$$
$$y_c = -80.673$$
$$x_d = -59.457$$
$$y_d = 107.838$$

Solving for x_n,y_n, and H,

$$x_n = 0.5649228$$
$$y_n = 5.1029126$$
$$H = 9{,}704.17 \quad .$$

Starting the third iteration, it is found that the new corrected coordinates are identical to the last ones to the accuracy being carried. Hence, no new results will be obtained by reworking the solution and the iteration is closed. To proceed with the rest of the solution, use the last set of corrected coordinates with the last solution. The only change is in Equation 46 with Equation 22 being substituted. Otherwise the calculation is the same as before so the results will be given directly.

$$P = 1.9076°$$
$$R = 0.2113°$$
$$H = 9{,}704.17 \text{ ft}$$
$$X_N = 12{,}473.42 \text{ ft}$$

$$Y_N = 9{,}638.55 \text{ ft.}$$
$$\alpha_H = 3.1744°$$

It is of interest to compare the results between the two methods for this type of relief. Note the following differences:

$$\Delta P = 0.0045° = 16.2''$$
$$\Delta R = 0.0000°$$
$$\Delta H = 0.54 \text{ ft.}$$
$$\Delta X_N = 0.10 \text{ ft.}$$
$$\Delta Y_N = 1.29 \text{ ft.}$$
$$\Delta \alpha_H = 0.0003° = 1.08''$$

The mean difference in tilt components is about eight seconds which is within the specified error of this method. The other differences are extremely small and of no special significance.

There are two other problems on the frame illustrated and to give some idea of what might be expected from such a frame these problems have been worked. The mean result of all three problems followed by the probable errors of the mean are given below. For these other two problems the method of iteration was used.

P	=	1.9067°	PEM	=	0.0007°
R	=	0.2075°	PEM	=	0.0085°
H	=	9,703.86 ft	PEM	=	0.28 ft
X_N	=	12,473.11 ft	PEM	=	1.74 ft
Y_N	=	9,639.09 ft	PEM	=	0.38 ft
α_H	=	3.1734°	PEM	=	0.0014°

Here the mean error in tilt component is 0.0046° or 16.56 seconds which is about what was to be expected. The error in ground position is 1.76 ft. Checking relation 32, $\Delta s/f = 0.000149$ which in Equation 32 gives $\Delta S = 1.45$ ft this is a fair agreement. The relatively higher accuracy of H and α_H should be noted. This is also in agreement with the text.

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