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Progress Report 2

FURTHER STUDIES OF TRANSISTOR MEASUREMENT TECHNIQUES AT HIGH FREQUENCIES

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ABSTRACT

Studies related to measurements of the external properties of transistors have led to techniques for the prediction of current-gain 3-db cutoff frequencies, and of internal properties associated with a suggested equivalent circuit.

OBJECTIVE

The purpose of this investigation is to find methods for determining the internal properties of transistors designed for operation in the VHF and UHF frequency ranges through measurement of their external properties.

1. INTRODUCTION

This progress report, to be read in conjunction with an earlier one dated February, 1958, refers to a later phase of a task under Contract No. DAI-49-186-502-ORD(P)-194 whose purpose is to investigate instrumentation requirements and devise suitable instrumentation techniques for the experimental study and evaluation of very-high-frequency semiconductor devices.

A particular objective is to evaluate very-high-frequency transistors in terms of measurements of their externally measurable parameters as represented, for example, by an equivalent circuit. Work has been continued which enables the understanding of limitations of transistors, limitations which become apparent at frequencies lower than the VHF range. Mainly, however, investigations reported here were confined to frequencies above 100 mc.

This report summarizes efforts to obtain information from impedance measurements of a transistor in the grounded collector configuration and from direct current-gain measurements of the transistor in the grounded base configuration.

2. RX-METER MEASUREMENTS OF TRANSISTORS IN THE GROUNDED COLLECTOR CONFIGURATION

The input impedance of a grounded collector configuration is

$$Z_{i_{GC}} \approx r'_b + \frac{r_\ell(1+\beta)}{1+j\frac{\omega}{\omega_o}} , \qquad r_c \gg r_\ell(1+\beta), \tag{1}$$

where β is the grounded emitter current gain, and

$$\omega_o = \frac{1}{r_\ell \, C_c(1+\beta)} .$$

The synthesis of Eq.(1) is illustrated in Fig. 1.

The measurement could be useful in obtaining a direct indication of the quantity $r_\ell(1+\beta)$ at low frequencies where $\omega \ll \omega_o$ in Eq. (1). Due to the limitation of ω_o, however, measurements are generally not practical in the frequency range above 40 mc.

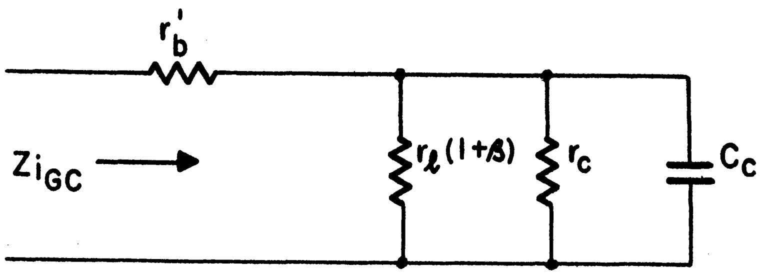

Fig. 1. Equivalent circuit for the input impedance of the grounded collector configuration.

The output impedance of the grounded collector configuration is:

$$Z_o = r_e + \frac{Z_c}{1+\beta}\left(\frac{r_b' + Z_g}{r_b'+Z_g+Z_c}\right) . \tag{2}$$

If $r_b' + Z_g \ll Z_c$,

$$Z_o \approx r_e + \frac{r_b'+Z_g}{1+\beta} .$$

If, in addition, $r_e \ll r_b' \ll r_g$,

$$Z_o \approx \frac{r_g}{1+\beta} . \tag{3}$$

The condition that $r_e \ll r_b'$ is easily met when high emitter current is used. ($I_e \approx 10$ ma for the GA 53233-type transistor.) r_g can be selected so that $r_b' \ll r_g$, and a value of 430 ohms was chosen for a test. The limiting factor of difficulty is again the collector-base capacitance which prevents the condition $r_b'+Z_g \ll Z_c$ from being valid at higher frequencies. The requirements here are not as stringent as in the case of input impedance, however, because of the low value of $r_b'+Z_g$.

The frequency at which the grounded collector current gain β drops to its 3-db down value can be measured practically when testing the GA 53233 transistor and limiting the frequency of measurement to approximately 120 mc.

The circuit used is shown in Fig. 2.

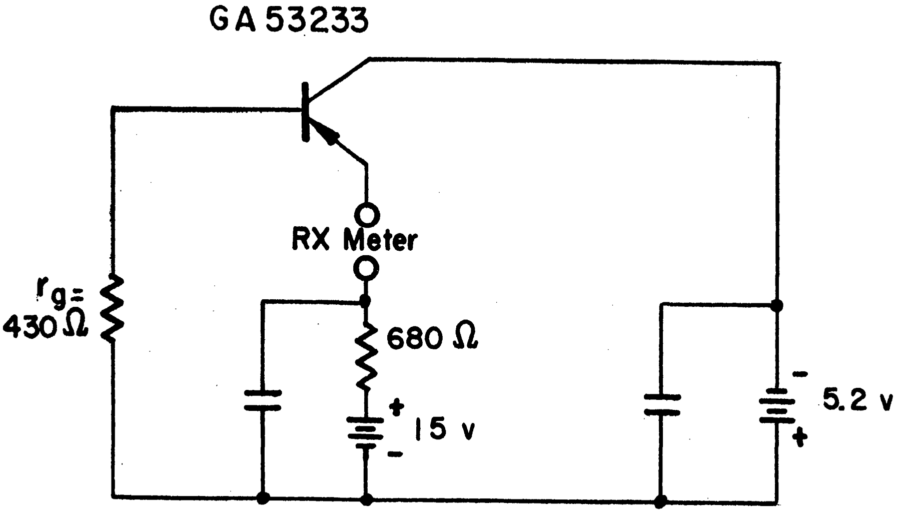

Fig. 2. Circuit used for measurement of input impedance of the grounded collector configuration.

To enable measurements to be of use simply and expeditiously, it was decided to determine the relationship between the real part of the complex quantity $(1+\beta)$ and the frequency of operation, thus avoiding the need for calculation of impedance from the parallel combination of resistance and capacitance from data obtained with the RX meter.

Under the assumptions that $\alpha = a_o/1+j(\omega/\omega_\alpha)$ and $\beta = \alpha/1-\alpha$, $b_o = a_o/1-a_o$, the relationship between f_α and f_β is first obtained:

$$1-\alpha = 1 - \frac{a_o}{1+j\gamma}, \quad \gamma = \frac{\omega}{\omega_\alpha}$$

$$= \frac{(1-a_o) + j\gamma}{1+j\gamma}$$

$$\frac{1}{1-\alpha} = \frac{1+j\gamma}{(1-a_o) + j\gamma}$$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{1+j\gamma}{(1-a_o)+j\gamma} \times \frac{a_o}{1+j\gamma} = \frac{a_o}{(1-a_o)+j\gamma} \times \frac{(1-a_o)-j\gamma}{(1-a_o)-j\gamma} = \frac{a_o[(1-a_o)-j\gamma]}{1-2a_o+a_o^2+\gamma^2}$$

$$\beta = \frac{a_o[(1-a_o) - j\gamma]}{1 + \gamma^2 + a_o(a_o-2)} \tag{4}$$

$$\text{Arg } \beta = \text{arc tan } (-\gamma/1-a_o)$$

at ω_β, arg β = 45° and tan (arg β) = $-1 = -\gamma/1-a_o$.

Then $\gamma = \omega_\beta/\omega_\alpha = 1-a_o$, $\omega_\alpha = \omega_\beta/1-a_o$

$$\therefore\ f_\alpha = (1+b_o)\, f_\beta\ . \tag{5}$$

Obtaining a relationship for the quantity (1+β):

$$1+\beta = \frac{1}{1-\alpha} = \frac{1+j\gamma}{(1-a_o)+j\gamma} \cdot \frac{(1-a_o)-j\gamma}{(1-a_o)-j\gamma} = \frac{1-a_o+\gamma^2+j\gamma[(1-a_o)-1]}{1-2a_o+a_o^2+\gamma^2}$$

$$1+\beta = \frac{(1+\gamma^2-a_o) - j\gamma a_o}{1+\gamma^2-2a_o+a_o^2}$$

$$\text{Re}[1+\beta] = \frac{1+\gamma^2-a_o}{1+\gamma^2-2a_o+a_o^2}$$

at f = f_β (frequency at which the gain drops 3 db down)

$$\gamma = 1 - a_o,\ (\omega = \omega_\beta)$$

$$\gamma^2 = 1 - 2a_o + a_o^2\ .$$

Then at f_β, $\text{Re}[1+\beta] = \dfrac{1+(1-2a_o+a_o^2)-a_o}{1+(1-2a_o+a_o^2)-2a_o+a_o^2}$

$$= \frac{2-2a_o+a_o^2-a_o}{2-4a_o+2a_o^2} = \frac{2-a_o-(2a_o-a_o^2)}{2(1-2a_o+a_o^2)}$$

$$\text{Re}[1+\beta] = \frac{(2-a_o)(1-a_o)}{2(1-a_o)^2} = \frac{2-a_o}{2(1-a_o)}$$

$$= \frac{1}{1-a_o} - \frac{a_o}{2(1-a_o)} = 1 + b_o - 1/2\ b_o$$

$$\therefore\ \text{at } f = f_\beta,\ \text{Re}[1+\beta] = 1 + b_o/2 \tag{6}$$

If the real part of $(1+\beta)$ can be determined experimentally, it is simple to determine at which frequency Eq. (6) is valid. This frequency can then be multiplied by $(1+b_0)$ to predict a value for f_α.

Experimentally, a value of $1+b_0 = 4.2$ was determined for a GA 53233-type transistor, and the frequency at which $\mathrm{Re}[1+\beta] = 1 + b_0/2$ was 110 mc. Thus an $f_\alpha \approx 460$ mc was predicted.

3. MEASUREMENT OF CURRENT GAIN

A very simple but effective technique allowing determination of f_α is illustrated in Fig. 3.

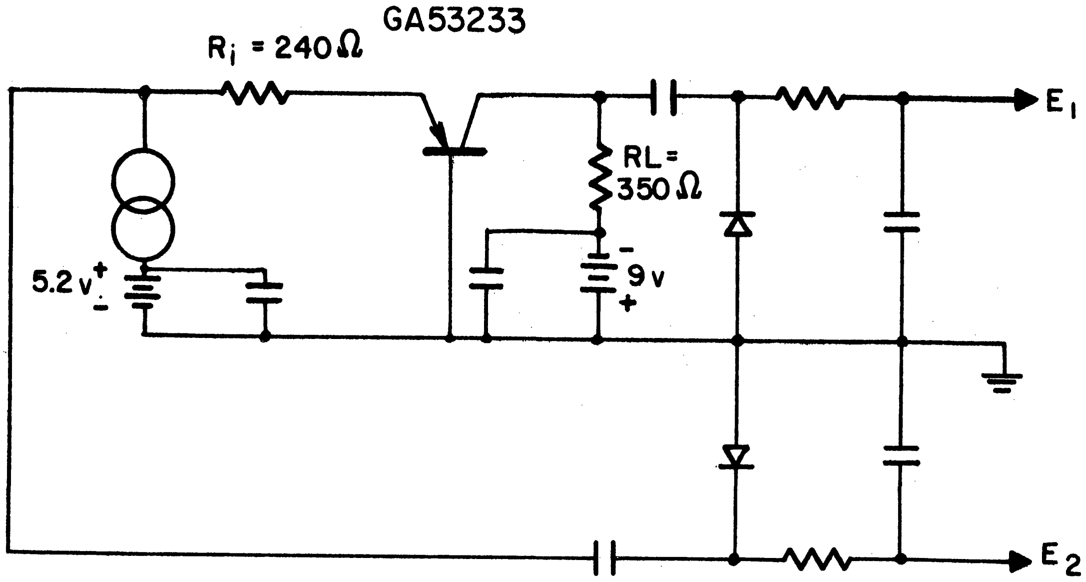

Fig. 3. A circuit enabling determination of f_α.

The load resistor was chosen so that $R_L = \sqrt{2}\,Ri$, i.e., $E_L = \sqrt{2}\,E_2$ at frequencies much lower than f_α assuming $\alpha = 1$. Thus if the frequency of the signal generator is increased until $E_1 = E_2$, f_α can be determined directly.

Testing GA 53233 transistors, apparent f_α's were in the range of from 370 mc to 420 mc. Inaccuracies here are undoubtedly due to the limitations of lumped circuit components. The resistance values are not constant with frequency and the inherent shunt capacitance across the resistors and the diodes cannot be neglected in the range of interest.

To allow the same basic idea as was illustrated in Fig. 3, the possibility of using distributed elements in the form of transmission lines (or filters terminated by their iterative impedances) was considered. Associated microwave equipment is shown in Fig. 4. This system has not been actually constructed and is therefore no more than a point of study.

The directional couplers have a dual purpose in the above arrangement: 1) each can be used individually in conjunction with the ratio meter to allow adjustment of the matching stubs to eliminate standing waves; and 2) in the setup as shown, the reading of the meter is more accurate by virtue of being independent of reflected waves that might exist due to imperfect matching.

The ratio meter acts as the instrument for comparison of the two voltages which allow indication of the magnitude of current gain. The detectors supplying the ratio meter should be a matched pair (so that their response characteristics are essentially the same).

The matching stubs not only eliminate standing waves but also act as d-c short circuits for the transistor biasing currents. The stub across the emitter-to-base region in Fig. 2 is primarily intended to permit d-c biasing current while affording a very high impedance at RF frequencies. If desired, the stub could be tuned slightly inductive to resonate with the intrinsic capacitance between the emitter and base.

The actual impedance from emitter to base is very low compared to the characteristic impedance of the input 50Ω transmission lines, and so the current from the signal generator essentially will divide evenly between the shunt to RF ground and the transistor input junction. Therefore the characteristic impedance of the load transmission line should be twice that of the input line to develop equal voltages to the ratio meter (assuming unity current gain).

4. CONCLUSIONS

Several methods of determining transistor parameters have been devised in conjunction with RX-meter impedance measurements. The highest frequency of operation considered pertaining to these measurements was 250 mc. Above 250 mc the grounded base current gain was measured by a voltage-magnitude comparison technique. A system avoiding the use of lumped circuit components has been proposed, and if proven experimentally feasible, it is suggested that the same general technique applied to a transistor in the grounded emitter configuration be considered.

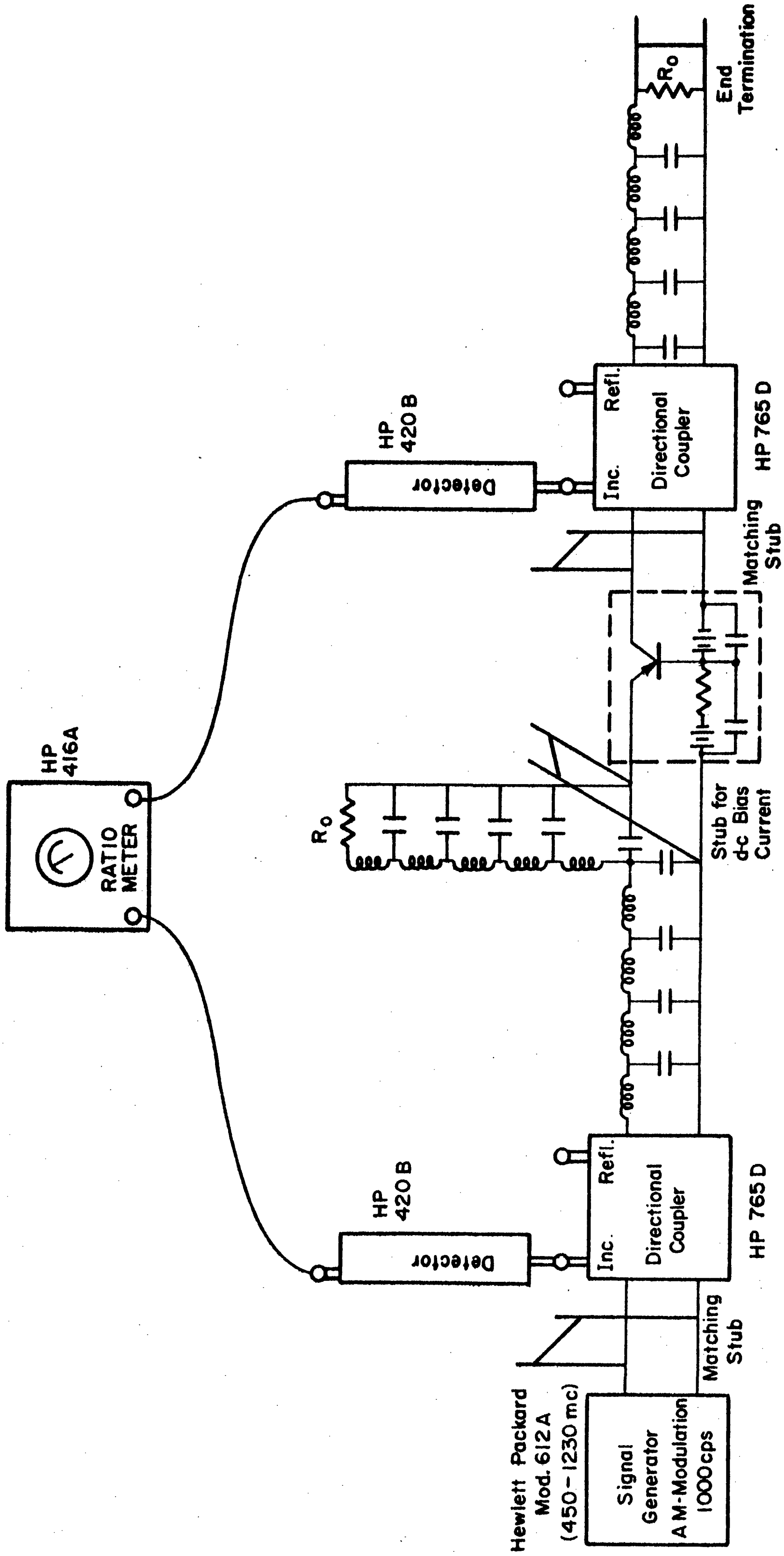

Fig. 4. A system applicable for measurement of f_α.