A Picturebook of Tetrahedral Packings

by

Elizabeth R. Chen

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Mathematics)
in the University of Michigan
2010

Doctoral Committee:

Professor Jeffrey C. Lagarias, Chairperson
Professor Alexandre I. Barvinok
Professor Igor Kriz
Professor G. Peter Scott
Associate Professor Ramón Satyendra
Professor Richard Pollack, New York University

Acknowledgments

Thank you to all those (and only those) who deserve thanks! O guilty ones, you know who y'all are!

Alexandre Barvinok
Anna Banana Chen
Berit Stensønes
Bob Messer
Dad
Dave Askins
Denise Lee
Divine Goddess
Liz Beazley
Liz Cole
Igor Kriz
Jeff Lagarias
John-Erik Fornæss
Karen Rhea
Kenneth Chang
Marion Law
Marjorie Senechal
Martha O'Kennon
Martin Henk
Math Dept Staff
Michael Engel
Michael Zieve
Mom
Naomi André
Pedagoguery Software
Peter Scott
Rackham Dissertations Office
Ramón Satyendra
Ricky Pollack
Sharon Glotzer
Springer Verlag
VTeX Typesetting
Women's Studies Dept

& many other Supportive Friends!!! (I'm sorry if I've forgotten you!)

Table of contents

Chapter 1
Introduction

Abstract

We explore many different packings of regular tetrahedra, with various clusters & lattices & symmetry groups. We construct a dense packing of regular tetrahedra, with packing density $D >$ .7786157.

History

The problem of packing by regular tetrahedra is very old.

(1925) Dirk Struik gives a very detailed history of this problem. [15]

(1981) Marjorie Senechal gives a very nice overview of this problem. [14]

(≈330 BC) Ἀριστοτέλης (Aristotelēs, 384-322 BC) mistakenly believed that regular tetrahedra (pyramids) tile space perfectly. He said ("Περὶ ᾿ουρανοῦ", book 3, part 8): "῾όλως δὲ τὸ πειρᾶσθαι τὰ ῾απλᾶ σώματα σχηματίζειν ᾿αλογόν ᾿έστι, πρῶτον μὲν ῾ότι συμβήσεται μὴ ᾿αναπληροῦσθαι τὸ ῾όλον. ᾿εν μὲν γὰρ τοῖς ᾿επιπέδοις τρία σχήματα δοκεῖ συμπληροῦν τὸν τόπον, τρίγωνον καὶ τετράγωνον καὶ ῾εξάγωνον, ᾿εν δὲ τοῖς στερεοῖς δύο μόνα, πυραμὶς καὶ κύβος. ᾿ανάγκη δὲ πλείω τούτων λαμβάνειν διὰ τὸ πλείω τὰ στοιχεῖα ποιεῖν." [1]

(1986) William Guthrie translates Aristotelēs ("On heavenly bodies"): "This attempt to assign geometrical figures to the simple bodies is on all counts irrational. In the first place, the whole of space will not be filled up. Among surfaces it is agreed that there are three figures which fill the place that contains them – the triangle, the square, and the hexagon; among solids only two, the pyramid and the cube. But they need more than these, since they hold that the elements are more." [1]

This is a very understandable mistake, because tetrahedra almost tile space locally, but not quite.

The cluster $\mathbf{E}_5$ (consisting of 5 tetrahedra joined symmetrically about an edge) has total solid angle $5 \cdot 2\cos^{-1}\frac{1}{3} \approx 12.309594173408$ about the edge, and local density (solid angle fraction) $D \approx .979566380077$.

The cluster $\mathbf{V}_{20}$ (consisting of 20 tetrahedra joined symmetrically about a vertex) has total solid angle $20 \cdot (-\pi + 3\cos^{-1}\frac{1}{3}) \approx 11.025711968651$ about the vertex, and local density (solid angle fraction) $D \approx .877398280459$.

أبو الوليد محمد ابن احمد ابن رشد (Abū al-Walīd Muhammad ibn Ahmad ibn Rushd, aka Averroës, 1126-1198) wrote a commentary on Aristotelēs and gave remarks on trihedral angles. He mistakenly asserted that 12 tetrahedra fill space about a vertex. (In the cluster $\mathbf{V}_{20}$, 20 tetrahedra touch at a point.) He said: "Cum aliquis punctus fuerit positus comunis octō cubis, implēbunt locō necessāriō: et similiter, cum fuerit comunis duodecim pȳramidibus, angulī enim sex pȳramidālēs sunt aequālēs quattuor cubōrum, quum angulus pȳramidis solidus est ex duōbus rectīs, et angulus cubī ex tribus. Erunt igitur trēs angulī pȳramidis aequālis duōbus angulis cubōrum, cum sit aequālēs sex angulī rectīs."

Johannes Müller (aka Regiomontanus, 1436-1476) reportedly wrote a lost manuscript ("Dē quinque solidis, quae vulgō regulāria dīcuntur, quae vidēlicet eōrum locum implēant, et quae nōn, contrā commentātorem Aristotelis Averroëm").

(1529) Franciscus Maurolycus (1494-1575) said (same title: "On the five like-sided bodies, that are usually called regular, and which of them fill their natural place, and which do not, in contradiction to the commentator on Aristotelēs, Averroës") that either a cube, or a combination of 6 regular tetrahedra together with 8 regular octahedra can fill space.

(1896) Hermann Minkowski (1864-1909) invented the geometry of numbers, which studies lattice packings of convex bodies. He showed that the densest lattice packing of any convex body must satisfy certain constraints. [13]

(1900) David Hilbert (1862-1943) posed the problem of dense packing of regular tetrahedra, as the 18th of his famous list of problems. He said ("Mathematische Probleme", p.25): "Ich weise auf die hiermit in Zusammenhang stehende, für die Zahlentheorie wich-tige und vielleicht auch der Physik und Chemie einmal Nutzen bringende Frage hin, wie man unendlich viele Körper von der gleichen vorgeschriebenen Gestalt, etwa Kugeln mit gegebenem Radius oder reguläre Tetraeder mit gegebener Kante (bez. in vorgeschriebener Stellung) im Raume am dichtesten einbetten, d.h. so lagern kann, daß das Verhältnis des erfüllten Raumes zum nichter-füllten Raume möglichst groß ausfällt." [9]

(1902) Mary Winston Newson translates Hilbert ("Mathematical problems"): "I point out the following question, related to the preceding one, and important in number theory and perhaps sometimes useful in physics and chemistry: How can one arrange most densely in space an infinite number of solids of an given form, e.g. spheres with given radii or regular tetrahedra with given edges (or in prescribed position), that is, how can one so fit them together that the ratio of the filled to the unfilled space is as large as possible?" [9]

(1904) Hermann Minkowski posed the problem of densest lattice packing of translates of regular tetrahedra, as a special case of the geometry of numbers. He mistakenly believed that the densest lattice packing of the regular tetrahedra had density $D = \frac{1}{3} \approx .333333333333$. [12]

(1962) Helmut Grömer constructed a lattice packing of the single tetrahedron $\mathbf{B}_1$ with density $D = \frac{18}{49} \approx .367346938775$. [6]

(1970) Douglas Hoylman proved that Grömer's packing was the densest lattice packing. [10]

(1985) Andrew Hurley constructed the tetrahelix, which allows tetrahedra in different orientations. [11]

I constructed a packing and calculated its density $D = \sqrt{\frac{50000}{177417}} \approx .531273435694$.

(2000) Ulrich Betke & Martin Henk developed an efficient computer algorithm to compute the densest lattice packing of any convex body, and they applied it to the Archimedean solids. [2, 3]

I used Betke & Henk's program to calculate the packing density of the convex hull of $\mathbf{V}_{20}$, $D \approx .716796401602$.

(2006) John Conway & Salvatore Torquato used Betke & Henk's program to calculate the packing density of $\mathbf{V}_{20}$ inscribed in a regular icosahedron, $D > .7165598$. They wiggled the tetrahedra to slightly improve the packing density $D > .717455$. They remark that further small improvements are possible, and conclude: "It is very difficult to say exactly how dense such "reformed Scottish" packings can be (especially because they will involve slight changes to the lattice), but we suspect it will be less than 72%." They said (in their abstract): "Our results suggest that the regular tetrahedron may not be able to pack as densely as the sphere, which would contradict a conjecture of Ulam. The regular tetrahedron might even be the convex body having the smallest possible packing density." [5]

(2008) In my paper ("A dense packing of regular tetrahedra"), we constructed a dense packing of regular tetrahedra with packing density $D \approx .778615700855$, which beats the densest sphere packing! [4]

(2009) Salvatore Torquato & Yang Jiao used a computer program to compress my packing to improve the packing density $D > .782$. [16]

The analogous problem of packing by spheres was also very challenging.

(1611) Johannes Kepler (1571-1630) conjectured that the densest packing is the hexagonal close-packing (HCP) or face-centered cubic (FCC) lattice, with density $D = \pi/\sqrt{18} \approx .740480489693$.

(1831) Carl Friedrich Gauss (1777-1855) proved that the FCC is the densest lattice packing.

(1972) Stanislaw Ulam conjectured that the sphere has the minimal packing density among all 3-dimensional convex bodies.

(2005) Thomas Hales & Samuel Ferguson proved that the HCP or FCC is the densest packing in general. [7, 8]

Main result

We include the full-color version of my paper "A Dense Packing of Regular Tetrahedra" (ch.7).

We construct a 2-parameter family of clusters of regular tetrahedra. Each cluster (9 tetrahedra) is the union of 1 'central' tetrahedron, 4 'upper' tetrahedra attached to an edge of the central tetrahedron, and 4 'lower' tetrahedra attached to the opposite edge of the central tetrahedron. The 4 upper tetrahedra can rotate about its attached edge, by an angle parametrized by u, and the 4 lower tetrahedra can rotate about its attached edge, by an angle parametrized by v. Clusters have 2 orientations: 'positive' and 'negative' (scalar mult. by -1).

For each parameter value $\langle u, v \rangle$, we construct an optimized packing of clusters, which is crystallographic. The clusters pack in layers with alternating orientations. The full symmetry group of the packing acts transitively on all clusters. The direct symmetry group, which is also the translation symmetry group, acts transitively on a coset. There are 2 cosets, which correspond to the 2 orientations = 2 translation classes. So the fundamental domain (under the lattice of translations) contains 2 clusters = 18 tetrahedra.

Since every cluster is equivalent to all other clusters, we restrict our attention to the cluster at the origin and its immediate neighbors. Also, since the cluster (9 tetrahedra) is extremely non-convex, we think of it as the union of (the convex hull of) the 'upper half-cluster' (5 tetrahedra) and (the convex hull of) the 'lower half-cluster' (5 tetrahedra). In order to check that clusters don't overlap in the packing, it's convenient to check that half-clusters don't overlap.

Each cluster has transverse edge-to-edge intersections with neighboring clusters in the same layer, and partial face-to-face intersections with neighboring clusters in adjacent layers. For intersecting edges, the separating plane contains the union of the edges, and for non-intersecting edges, the separating plane is between and parallel to both edges. For intersecting faces, the separating plane contains the union of the faces, and for non-intersecting faces, the separating plane is between and parallel to both faces.

The intersections determine equations in terms of the cluster coordinates and lattice vectors, which we solve in terms of $\langle u,v\rangle$ and optimize over all packings in the family. The maximum occurs at $\langle u,v\rangle \approx \langle -.034789016702, +.089604971413\rangle$, with packing density $D \approx .778615700855$.

Summary

In this thesis, we limit ourselves to periodic packings. (It is too difficult to compute general packings. And even if we find a locally dense packing, there is no guarantee that we can extend it globally.)

One difficulty in understanding packings by regular tetrahedra is that in small regions regular tetrahedra can pack quite densely. ($\mathbf{E}_5$ has solid angle fraction $D \approx .979566380077$ about an edge. $\mathbf{V}_{20}$ has solid angle fraction $D \approx .877398280459$ about a vertex.) In order to get a good global packing density, you have to consider larger clusters of tetrahedra.

We systematically examine clusters and packings of various types. For convex clusters we use Betke & Henk's program to find the densest lattice packing (ch.2). For general clusters and packings, we examine lattices with various symmetry groups: (ch.3) "cubic", (ch.4) "square", (ch.5-7) "oblique", (ch.8) "rectangular", (ch.9) "trigonal". We construct a planar hexagonal packing of Hurley's tetrahelix (ch.10). We generalize the cubic packings to a 4-parameter family (ch.11-12).

As a joke, we end with the least dense packing of regular tetrahedra.

We do not address the problem of finding an upper bound on density of packings. No explicit upper bounds on the density are known for this problem.

2 Densest lattice packings

We construct various 'locally dense' convex clusters. We compute their densest lattice packings, using Betke & Henk's (FORTRAN) program for computations. We feed to the program the numerical coordinates (vertices and faces) of the difference body. The program spits out the numerical coordinates (lattice vectors) of the densest lattice packing.

The program also gives us the incidence relations between lattice vectors and faces of the difference body. (Saying that a cluster and its neighbor intersect, is equivalent to saying that the neighbor's center lives in a face of the cluster's difference body.) We translate these incidence relations into intersection equations, which we can write in closed algebraic form. We solve the intersection equations in closed algebraic form, and verify that the roots match the program's numeric output.

According to Minkowski's theory, any convex body has a densest lattice packing, which satisfies one of 3 types of lattices. Betke & Henk's program tries all 3 types. For each type, it tries all (intelligent) combinations of incidence relations between faces and lattice vectors. For each combination of faces, it optimizes the lattice vectors. Then it gathers all valid packings and chooses the packing with the maximum packing density.

3 Cubic packings

We construct various 'locally dense' clusters with tetrahedral symmetry. We compute their packings with cubic symmetry.

The clusters have full symmetry group $\mathbf{S}_4$, direct symmetry group $\mathbf{A}_4 = \mathbf{Z}_2^2 \rtimes \mathbf{Z}_3$.

$$\mathbf{Z}_2^2 = \left\{ \begin{bmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & +1 \end{bmatrix}, \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{bmatrix} \right\}$$

$$\mathbf{Z}_3 = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \right\} \qquad \text{rotation by } \{-\tfrac{2}{3}\pi, 0, +\tfrac{2}{3}\pi\} \text{ about } \langle 1,1,1 \rangle$$

We start with the simple cubic lattice $2a\mathbf{Z}^3$, which has cubic symmetry. The lattice vectors partition $\mathbf{R}^3$ into cubes of length/width/height $2a$. The lattice basis is $\{\langle 2a,0,0\rangle, \langle 0,2a,0\rangle, \langle 0,0,2a\rangle\}$.

We partition the lattice vectors into an 'even coset' ($x+y+z=(4k)a$) and an 'odd coset' ($x+y+z=(4k+2)a$). The 'even coset' is a sublattice of index 2 (and is isomorphic to what most people would call the "face-centered cubic lattice"). We consider the 'even coset' the basic building block of our lattices. The lattice basis is $\{\langle 0,2a,2a\rangle, \langle 2a,0,2a\rangle, \langle 2a,2a,0\rangle\}$.

The simple cubic lattice (what most people would consider the basic building block) consists of 2 cosets of the 'even coset'.

We can insert xyz-body-centered translates of the simple cubic lattice, at offset $\langle a,a,a\rangle$. We partition these translates also into 2 cosets ($x+y+z=(4k+1)a$) and ($x+y+z=(4k+3)a$). So the xyz-body-centered cubic lattice consists of 4 cosets of the 'even coset'.

The simple cubic lattice has point group $\mathbf{S}_4 \rtimes \mathbf{Z}_2$. We consider these 2 ways to construct finite-index superlattices, which preserve the point group $\mathbf{S}_4 \rtimes \mathbf{Z}_2$.

For each coset (of the 'even coset'), we can choose whether to place 'positive clusters', 'negative clusters', or nothing. There are 7 types of lattices which are 'uniform' (cluster-transitive). (Between any two clusters, there is an isometry which also preserves intersection incidence relations between neighbors. So when we write the intersection equations, we need to consider only the cluster at the origin.)

4 Square packings

We construct various clusters and packings with 'square' symmetry. The lattice basis is $\{\langle a-b, a+b, 2c\rangle, \langle a+b, -a+b, 2c\rangle, \langle 2a, 2b, 0\rangle\}$. The symmetry group is $\mathbf{Z}_2^2$

$$\mathbf{Z}_2^2 = \left\{ \begin{bmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & +1 \end{bmatrix}, \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{bmatrix} \right\}$$

We start with the 'simple' square lattice, which has square symmetry. (The lattice points form square prisms of length/width $\sqrt{2a^2+2b^2}$ and height $2c$.) The lattice basis is $\{\langle a+b, -a+b, 0\rangle, \langle a-b, a+b, 0\rangle, \langle 0,0,2c\rangle\}$. This is similar to the 'simple' cubic lattice, but with additional degrees of freedom. We can rotate/dilate the xy-plane about the z-axis, and we can dilate along the z-axis.

We can insert no additional cosets, or we can insert a coset of (xyz)'body-centered' translates (at offset $\langle a,b,c\rangle$), a coset of (xy)'face-centered' translates (at offset $\langle a,b,0\rangle$), a coset of (z)'edge-centered' translates (at offset $\langle 0,0,c\rangle$). So there are 4 basic lattice types which preserve the square symmetry group. We can also partition the lattice vectors into an 'even' coset and an 'odd' coset. (These are the rotated/dilated versions of the analogous cosets in the cubic case.) Taking into account these cosets, there are 11 types of packings.

(Most people would take the 'simple' square lattice as their basic lattice, but we will not follow this convention!) The 'simple' square lattice consists of 2 translates of the 'even' coset. The 'even' coset is a lattice in its own right, and we take this as our basic lattice. The lattice basis is $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$.

5 Oblique$_u$ packings of $\mathbf{B}_u^9$

We start with the 'square packing' $Q_3^+\,P_{xy}$ of $\mathbf{B}^9$, which has packing density $D\approx .776114181859$. This packing occurs in layers, and each cluster touches 2 clusters in the above layer (partial face-to-face), and 2 clusters in the below layer (partial face-to-face).

We can shift adjacent layers along a line segment, which preserves the above face-to-face intersections. At the endpoints of this line segment, each cluster touches 3 clusters in the above layer (partial face-to-face), and 3 clusters in the below layer (partial face-to-face). We call this the 'oblique packing' of $\mathbf{B}^9$.

The cluster $\mathbf{B}^9$ is the union of 3 subclusters, $\mathbf{B}^9=\mathbf{B}^1\cup\mathbf{E}^{4+}\cup\mathbf{E}^{4-}$. We can rotate the subclusters by a small angle (parametrized by u). We call this new cluster $\mathbf{B}_u^9=\mathbf{B}^1\cup\mathbf{E}_u^{4+}\cup\mathbf{E}_u^{4-}$, and the new packing the 'oblique$_u$ packing'.

Going from the 'square packing' to the 'oblique$_u$ packing', we relax symmetries in order to increase packing density. The cluster $\mathbf{B}_u^9$ has symmetry group $\mathbf{Z}_2$, and the 'oblique$_u$ packing' has symmetry group 1.

$$\mathbf{Z}_2=\left\{\begin{bmatrix}+1&0&0\\0&+1&0\\0&0&+1\end{bmatrix},\begin{bmatrix}+1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}\right\}$$

6 Oblique$_{u,v}$ packings of $\mathbf{B}_{u,v}^9$

The cluster $\mathbf{B}^9$ is the union of 3 subclusters, $\mathbf{B}^9=\mathbf{B}^1\cup\mathbf{E}^{4+}\cup\mathbf{E}^{4-}$. We can rotate the subclusters independently by small angles (parametrized by $\langle u,v\rangle$). We call this new cluster $\mathbf{B}_{u,v}^9=\mathbf{B}^1\cup\mathbf{E}_u^{4+}\cup\mathbf{E}_v^{4-}$, and the new packing the 'oblique$_{u,v}$ packing'.

We relax symmetries further to improve the packing density further. The cluster $\mathbf{B}_{u,v}^9$ has symmetry group 1, and the 'oblique$_{u,v}$ packing' has symmetry group 1.

7 Optimal oblique$_{u,v}$ packing of $\mathbf{B}_{u,v}^9$

We construct the optimal 'oblique$_{u,v}$ packing' of $\mathbf{B}_{u,v}^9$. We include all the fine details from my paper.

8 $\mathbf{Rect}_u$ packings of $\mathbf{E}_u^{15}$

We construct the cluster $\mathbf{E}_u^{15}$ and its packing with 'rectangular' symmetry. The lattice basis is $\{\langle 0,2b,2c\rangle, \langle 2a,0,2c\rangle, \langle 2a,2b,0\rangle\}$.

9 Trigonal packings

We construct various clusters and packings with trigonal (or triangular) symmetry. The symmetry group is $\mathbf{Z}_3$ (or $\mathbf{S}_3$). (We do not include the packings in this paper, because there are too many possibilities, and the packing densities are low.)

$$\mathbf{Z}_3 = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \right\} \qquad \text{rotation by } \{-\tfrac{2}{3}\pi, 0, +\tfrac{2}{3}\pi\} \text{ about } \langle 1,1,1\rangle$$

10 Hexagonal packing of the tetrahelix

We construct the tetrahelix and bounding spirals (of Andrew Hurley). We compute the densest planar hexagonal packing.

11 Cubic family spacings

We generalize the 'cubic packings'.

We construct various clusters with 'cubic' symmetry. We compute spacings in the $\langle 2,2,0\rangle$, $\langle 1,1,+1\rangle$, $\langle 2,0,0\rangle$, $\langle 1,1,-1\rangle$ directions, between all combinations of clusters.

12 Cubic family packings

All packings with 'cubic' symmetry are members of a 4-parameter family of clusters. We use a computer program to calculate spacings between all combinations of clusters. We give all packings with packing density $D \geq \frac{4}{7}$. We also give the packing with minimum packing density.

Chapter 2
Densest lattice packings

Construct a convex cluster $\mathbf{X}$, which is highly symmetric and relatively dense

$\mathbf{B}_1$ is the convex hull of 1 tetrahedron glued along the tetrahedron with vertices $\mathbf{Z}_2^2\langle 1,1,1\rangle$ with the symmetries of a regular tetrahedron

$\mathbf{B}_1 =$ (1 body)
1 tetrahedron: $\{\langle +1,+1,+1\rangle,\langle +1,-1,-1\rangle,\langle -1,+1,-1\rangle,\langle -1,-1,+1\rangle\}$

$\mathbf{F}_2$ is the convex hull of 2 tetrahedra glued along the triangle with vertices $\mathbf{Z}_3\langle \frac{4}{3},-\frac{2}{3},-\frac{2}{3}\rangle$ with the symmetries of a regular triangle

$\mathbf{F}_2 =$ (2 bodies)
1 tetrahedron: $\{\langle +\frac{4}{3},+\frac{4}{3},+\frac{4}{3}\rangle,\langle +\frac{4}{3},-\frac{2}{3},-\frac{2}{3}\rangle,\langle -\frac{2}{3},+\frac{4}{3},-\frac{2}{3}\rangle,\langle -\frac{2}{3},-\frac{2}{3},+\frac{4}{3}\rangle\}$
1 tetrahedron: $\{\langle -\frac{4}{3},-\frac{4}{3},-\frac{4}{3}\rangle,\langle +\frac{4}{3},-\frac{2}{3},-\frac{2}{3}\rangle,\langle -\frac{2}{3},+\frac{4}{3},-\frac{2}{3}\rangle,\langle -\frac{2}{3},-\frac{2}{3},+\frac{4}{3}\rangle\}$

$\mathbf{E}_4$ is the convex hull of 4 tetrahedra glued along the line segment with vertices $\langle 0,0,\pm\sqrt{2}\rangle$ with the symmetries of a square

$\mathbf{E}_4 =$ (4 bodies)
2 tetrahedra: $\pm\{\langle 0,0,+\sqrt{2}\rangle,\langle 0,0,-\sqrt{2}\rangle,\langle 2,+\sqrt{2},0\rangle,\langle 2,-\sqrt{2},0\rangle\}$
2 tetrahedra: $\pm\{\langle 0,0,+\sqrt{2}\rangle,\langle 0,0,-\sqrt{2}\rangle,\langle +\sqrt{2},2,0\rangle,\langle -\sqrt{2},2,0\rangle\}$

$\mathbf{E}_5$ is the convex hull of 5 tetrahedra glued along the line segment with vertices $\langle 0,0,\pm\sqrt{2}\rangle$ with the symmetries of a regular pentagon

$\mathbf{E}^5 =$ (5 bodies)
5 tetrahedra: $\mathbf{Z}_5\{\langle 0,0,+\sqrt{2}\rangle,\langle 0,0,-\sqrt{2}\rangle,\langle 2,+\sqrt{2},0\rangle,\langle 2,-\sqrt{2},0\rangle\}$

$$\mathbf{Z}_5 = \{R_{-\frac{4}{5}\pi},R_{-\frac{2}{5}\pi},R_0,R_{+\frac{2}{5}\pi},R_{+\frac{4}{5}\pi}\},\ R_\theta = \begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\mathbf{E}_6$ is the convex hull of 6 tetrahedra glued to the hexagon with vertices $\mathbf{S}_3\langle 0,-2,+2\rangle$ with the symmetries of a regular hexagon

$\mathbf{E}_6 =$ (6 bodies)
6 tetrahedra: $\mathbf{S}_3\{\langle +2,-2,0\rangle,\langle +2,0,-2\rangle,$
$\langle 2-\sqrt{6},-1,-1\rangle,\langle 2-\frac{1}{3}\sqrt{6},-1+\frac{2}{3}\sqrt{6},-1+\frac{2}{3}\sqrt{6}\rangle\}$

$\mathbf{V}_{20}$ is the convex hull of 20 tetrahedra glued along the origin with the symmetries of a regular icosahedron

$\mathbf{V}_{20} =$ (20 bodies)
8 tetrahedra: $\pm\mathbf{Z}_2^2\{\langle 0,0,0\rangle,\langle q_0,q_1,q_2\rangle,\langle q_2,q_0,q_1\rangle,\langle q_1,q_2,q_0\rangle\}$
12 tetrahedra: $\mathbf{A}_4\{\langle 0,0,0\rangle,\langle 0,s_1,s_2\rangle,\langle r_1,r_2,r_0\rangle,\langle -r_1,r_2,r_0\rangle\}$

$P=\frac{1}{2}(1+\sqrt{5}),\ Q=\frac{1}{2}(-1+\sqrt{5})$
$\langle q_0,q_1,q_2\rangle = \frac{1}{3}\langle 4-P^2\sqrt{2},4+Q^2\sqrt{2},4+\sqrt{10}\rangle \approx \langle .099180275452, 1.513393837825, 2.387425886723\rangle$
$\langle r_0,r_1,r_2\rangle = \frac{1}{3}\langle 4Q-P\sqrt{2},\sqrt{2},4P+Q\sqrt{2}\rangle \approx \langle .061296781243, 1.414213562373, 2.448722667966\rangle$
$\langle 0,s_1,s_2\rangle = \frac{1}{3}\langle 0,4P-2Q\sqrt{2},4Q+2P\sqrt{2}\rangle \approx \langle .000000000000, 1.574690619068, 2.349542392514\rangle$

Construct the difference body $\mathbf{X} - \mathbf{X}$

Construct the various types of lattices

basis $G_6^- \{a, b, c\}$ (Minkowski case #1)
$a, b, c, a-b, b-c, c-a$ live in faces of the difference body

basis $G_6^+ \{a, b, c\}$ (Minkowski case #2)
$a, b, c, a+b, b+c, c+a$ live in faces of the difference body
$a+b+c$ does not live in any face of the difference body

basis $G_7^+ \{a, b, c\}$ (Minkowski case #3)
$a, b, c, a+b, b+c, c+a$ live in faces of the difference body
$a+b+c$ lives in a face of the difference body

Write the intersection equations

Write the free variables

Write the (common denominator) Wronskian W in terms of the free variables

Write the lattice vectors a, b, c in terms of the free variables

Write the lattice volume $V = \det(a, b, c)$ in terms of the free variables

Write the equations of minimum lattice volume V

Evaluate the free variables which minimize the lattice volume V

Evaluate the lattice vectors a, b, c

Evaluate the lattice volume $V = \det(a, b, c)$

Evaluate the packing density D

Note: Each tetrahedron has volume $\frac{8}{3}$

G_6^- (polar = maximal)

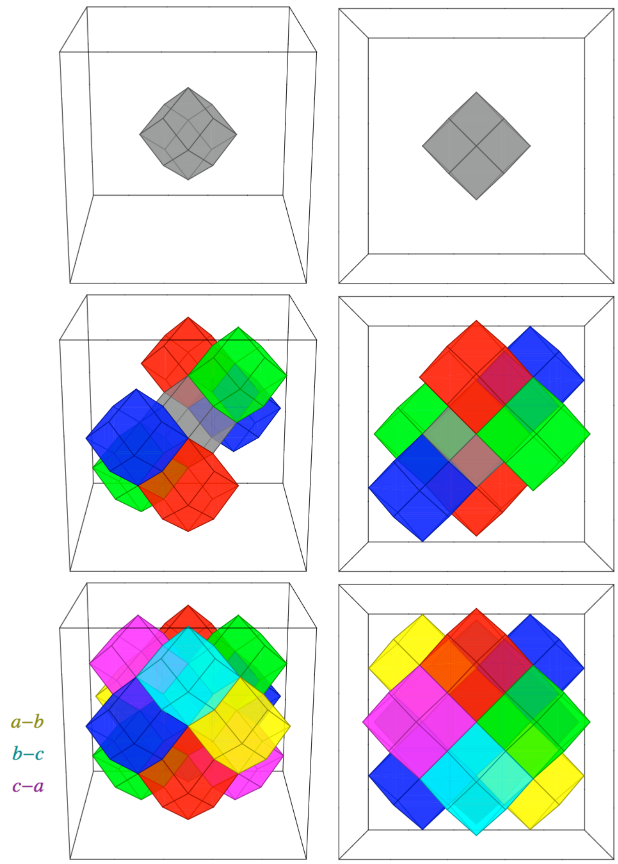
$a = \langle 0,2a,2a \rangle$

$b = \langle 2a,0,2a \rangle$

$c = \langle 2a,2a,0 \rangle$

$V = 16a^3$

basis $G_6^-\{a,b,c\}$



G_6^- (equatorial = minimal)

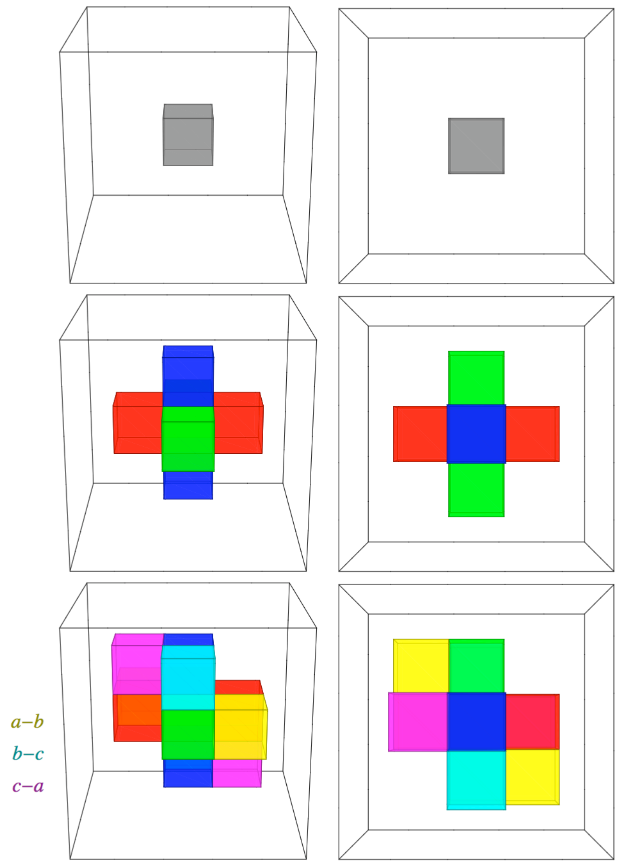
$a = \langle 2a,0,0 \rangle$

$b = \langle 0,2a,0 \rangle$

$c = \langle 0,0,2a \rangle$

$V = 8a^3$

basis $G_6^-\{a,b,c\}$



G_7^+ (equatorial = maximal)

$a = \langle -a,+a,+a \rangle$

$b = \langle +a,-a,+a \rangle$

$c = \langle +a,+a,-a \rangle$

$V = 4a^3$

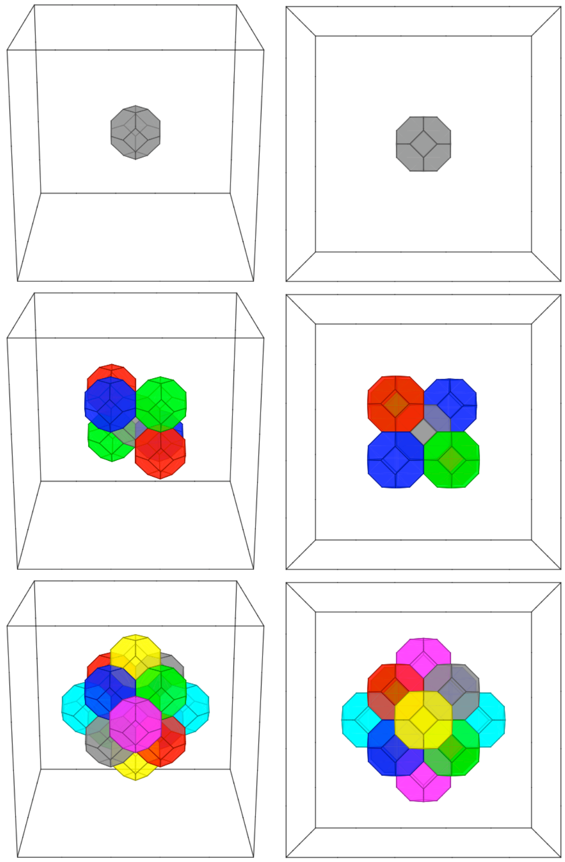
basis $G_7^+\{a,b,c\}$

$a+b$

$b+c$

$c+a$

$a+b+c$



G_7^+ (polar = mimimal)

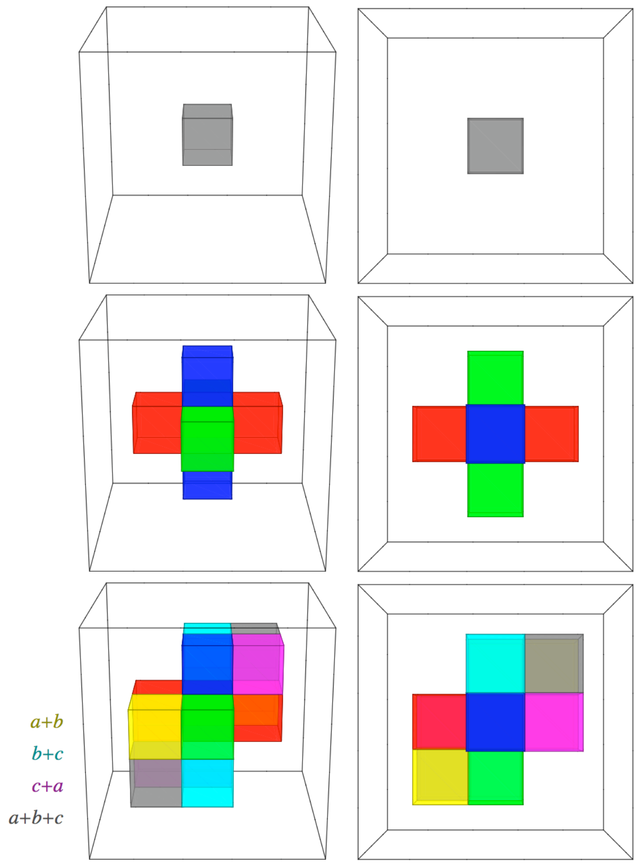
$a = \langle 2a,0,0 \rangle$

$b = \langle 0,2a,0 \rangle$

$c = \langle 0,0,2a \rangle$

$V = 8a^3$

basis $G_7^+\{a,b,c\}$



2.1 $\mathbf{B}_1$

$\mathbf{B}_1$ G_7^+

1 coset

1 tetrahedron

$U = \frac{8}{3}$

2.666666666667

$a = \langle +2, -\frac{1}{3}, -\frac{1}{3} \rangle$

+2.000000000000
−0.333333333333
−0.333333333333

$b = \langle -\frac{1}{3}, +2, -\frac{1}{3} \rangle$

−0.333333333333
+2.000000000000
−0.333333333333

$c = \langle -\frac{1}{3}, -\frac{1}{3}, +2 \rangle$

−0.333333333333
−0.333333333333
+2.000000000000

$V = \frac{196}{27}$

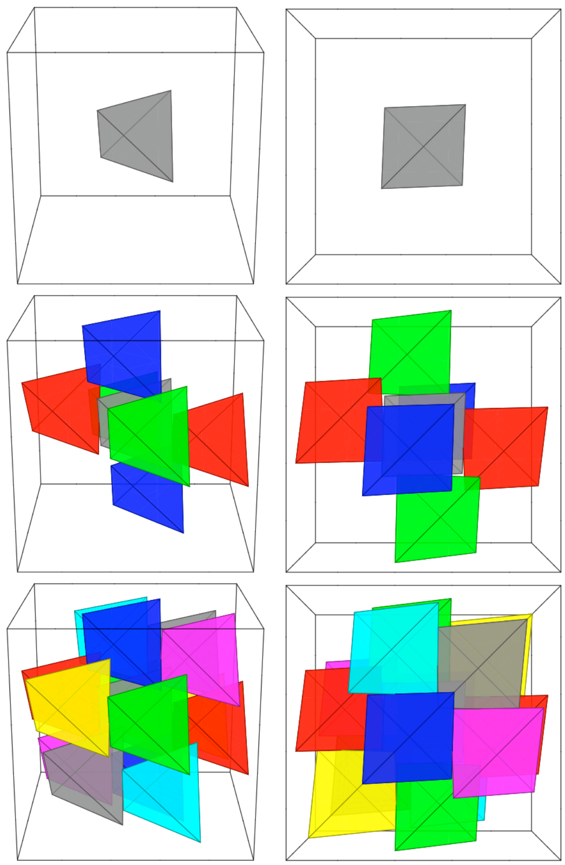
7.259259259259

$D = \frac{18}{49}$

.367346938776

basis $G_7^+\{a, b, c\}$

$a+b$
$b+c$
$c+a$
$a+b+c$



There are 4 vertices and 4 faces

4 points: $\langle x, y, z \rangle = \mathbf{Z}_2^2 \langle 1, 1, 1 \rangle$
4 triangles: $\langle x, y, z \rangle \cdot \mathbf{Z}_2^2 \langle -1, -1, -1 \rangle = 1$

1 tetrahedron has volume $1 \cdot \frac{8}{3} \approx 2.666666666667$

$\mathbf{B}_1$ has volume $4 \cdot \frac{2}{3} = \frac{8}{3} \approx 2.666666666667$

The relative density of tetrahedra is 1

4 triangles: $\frac{1}{6} \det \begin{bmatrix} +1 & -1 & -1 \\ -1 & -1 & +1 \\ -1 & +1 & -1 \end{bmatrix} = \frac{2}{3}$

The difference body has 12 vertices and 14 faces

12 points: $\langle x, y, z \rangle = \mathbf{Z}_3 \langle 0, \pm 2, \pm 2 \rangle$
6 squares: $\langle x, y, z \rangle \cdot \mathbf{Z}_3 \langle \pm 2, 0, 0 \rangle = 4$
8 triangles: $\langle x, y, z \rangle \cdot \langle \pm 1, \pm 1, \pm 1 \rangle = 4$

The lattice basis is $G_7^+\{a,b,c\}$

The intersections are $\begin{cases} a\cdot\langle 1,0,0\rangle = 2 \\ b\cdot\langle 0,1,0\rangle = 2 \\ c\cdot\langle 0,0,1\rangle = 2 \\ (a+b)\cdot\langle 1,1,-1\rangle = 4 \\ (b+c)\cdot\langle -1,1,1\rangle = 4 \\ (c+a)\cdot\langle 1,-1,1\rangle = 4 \\ (a+b+c)\cdot\langle 1,1,1\rangle = 4 \\ \text{minimum lattice volume} \end{cases}$

The lattice vectors are

$$\begin{cases} a = \langle +2, -\frac{1}{3}, -\frac{1}{3}\rangle \approx \langle +2.000000000000, -0.333333333333, -0.333333333333\rangle \\ b = \langle -\frac{1}{3}, +2, -\frac{1}{3}\rangle \approx \langle -0.333333333333, +2.000000000000, -0.333333333333\rangle \\ c = \langle -\frac{1}{3}, -\frac{1}{3}, +2\rangle \approx \langle -0.333333333333, -0.333333333333, +2.000000000000\rangle \end{cases}$$

The lattice volume is $V = \frac{196}{27} \approx 7.259259259259$

The packing density is $D = \frac{18}{49} \approx .367346938776$

2.2 $\mathbf{F}_2$

$\mathbf{F}_2$ G_6^-

1 coset

2 tetrahedra

$U = \frac{16}{3}$

5.333333333333

$a = ?$

+1.522166537942

+1.804226470292

−0.673606991766

$b = ?$

+2.378957802040

−0.258946730413

+0.153735740613

$c = ?$

+1.007860776779

−0.541695100368

−2.150148040951

$V = ?$

11.174896750106

$D = ?$

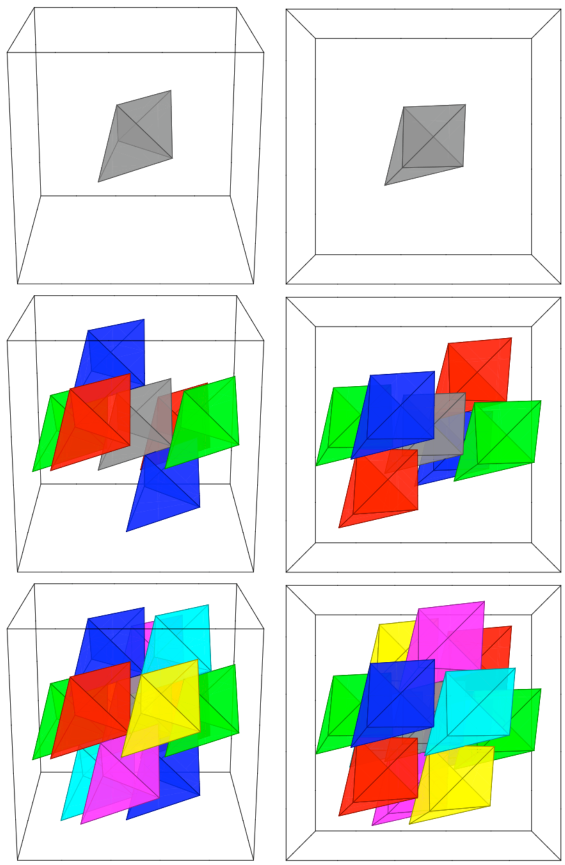
.477260188850

basis $G_6^-\{a,b,c\}$

$a-b$

$b-c$

$c-a$



There are 5 vertices and 6 faces

2 points (axial): $\langle x,y,z\rangle = \pm\langle \frac{4}{3}, \frac{4}{3}, \frac{4}{3}\rangle$
3 points: $\langle x,y,z\rangle = \mathbf{Z}_3\langle \frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}\rangle$
3 triangles: $\langle x,y,z\rangle \cdot \mathbf{Z}_3\langle -1,1,1\rangle = \frac{4}{3}$
3 triangles: $\langle x,y,z\rangle \cdot \mathbf{Z}_3\langle -5,1,1\rangle = 4$

2 tetrahedra have volume $2 \cdot \frac{8}{3} = \frac{16}{3} \approx 5.333333333333$

$\mathbf{F}_2$ has volume $3 \cdot \frac{8}{9} + 3 \cdot \frac{8}{9} = \frac{16}{3} \approx 5.333333333333$

The relative density of tetrahedra is 1

3 triangles: $\frac{1}{6}\det\begin{bmatrix} +\frac{4}{3} & +\frac{4}{3} & +\frac{4}{3} \\ -\frac{2}{3} & +\frac{4}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & +\frac{4}{3} \end{bmatrix} = \frac{8}{9}$

3 triangles: $\frac{1}{6}\det\begin{bmatrix} -\frac{4}{3} & -\frac{4}{3} & -\frac{4}{3} \\ -\frac{2}{3} & -\frac{2}{3} & +\frac{4}{3} \\ -\frac{2}{3} & +\frac{4}{3} & -\frac{2}{3} \end{bmatrix} = \frac{8}{9}$

The difference body has 20 vertices and 24 faces

2 axial points: $\langle x,y,z\rangle = \pm\langle \frac{8}{3}, \frac{8}{3}, \frac{8}{3}\rangle$
6 inner points: $\langle x,y,z\rangle = \pm\mathbf{Z}_3\langle \frac{8}{3}, \frac{2}{3}, \frac{2}{3}\rangle$
6 inner points: $\langle x,y,z\rangle = \pm\mathbf{Z}_3\langle 0,2,2\rangle$
6 outer points: $\langle x,y,z\rangle = \mathbf{S}_3\langle 0,-2,+2\rangle$
12 inner rhombi: $\langle x,y,z\rangle \cdot \pm\mathbf{S}_3\langle -1,+1,3\rangle = 8$
6 outer triangles: $\langle x,y,z\rangle \cdot \pm\mathbf{Z}_3\langle -1,1,1\rangle = 4$
6 outer triangles: $\langle x,y,z\rangle \cdot \pm\mathbf{Z}_3\langle -5,1,1\rangle = 12$

The lattice basis is $G_6^-\{a,b,c\}$

$$\text{The intersections are } \begin{cases} a\cdot\langle 1,1,-1\rangle = 4 \\ b\cdot\langle 5,-1,-1\rangle = 12 \\ c\cdot\langle 1,-1,-3\rangle = 8 \\ (a-b)\cdot\langle -1,5,-1\rangle = 12 \\ (b-c)\cdot\langle 1,-1,3\rangle = 8 \\ (c-a)\cdot\langle 1,-3,-1\rangle = 8 \\ \text{minimum lattice volume} \end{cases}$$

$$\text{The lattice vectors are } \begin{cases} a \approx \langle +1.522166537942, +1.804226470292, -0.673606991766\rangle \\ b \approx \langle +2.378957802040, -0.258946730413, +0.153735740613\rangle \\ c \approx \langle +1.007860776779, -0.541695100368, -2.150148040951\rangle \end{cases}$$

$$0 = 1200a_x^8 + 5280a_x^7 - 282792a_x^6 - 1740592a_x^5 + 8608315a_x^4 + 28031220a_x^3 - 153829320a_x^2 + 209739600a_x - 90306000$$
$$0 = 150a_y^8 - 3780a_y^7 + 34854a_y^6 - 136051a_y^5 + 137160a_y^4 + 479590a_y^3 - 1370800a_y^2 + 1144581a_y - 255704$$
$$0 = 400a_z^8 + 4480a_z^7 - 116936a_z^6 - 1570936a_z^5 - 3793495a_z^4 + 8716250a_z^3 + 25877610a_z^2 + 5846802a_z - 4564983$$

$$0 = 7873200b_x^8 - 201553920b_x^7 + 2210847048b_x^6 - 13617169200b_x^5 + 51631877655b_x^4 - 123621683760b_x^3 \\ +182752855545b_x^2 - 152655852814b_x + 55202193950$$
$$0 = 984150b_y^8 - 16927380b_y^7 + 92252034b_y^6 - 154495917b_y^5 - 185974650b_y^4 + 668567925b_y^3 \\ -7404060b_y^2 - 635472038b_y - 151821276$$
$$0 = 7873200b_z^8 - 116523360b_z^7 - 411908328b_z^6 + 3058210080b_z^5 + 3848025555b_z^4 - 14143542840b_z^3 \\ -17802243240b_z^2 - 3472745632b_z + 1003619440$$

$$0 = 39858075c_x^8 - 1923816420c_x^7 + 5284413108c_x^6 + 44902320516c_x^5 - 82199853000c_x^4 - 305176248000c_x^3 \\ -66160877760c_x^2 + 265439385600c_x + 146680064000$$
$$0 = 39858075c_y^8 - 871563240c_y^7 - 513253908c_y^6 + 61877333376c_y^5 - 28323457920c_y^4 - 842892402240c_y^3 \\ -1422041080320c_y^2 - 660861198336c_y - 69364154368$$
$$0 = 39858075c_z^8 + 499554540c_z^7 + 932265612c_z^6 - 13165206372c_z^5 - 86460129000c_z^4 - 222121083600c_z^3 \\ -275505973920c_z^2 - 156776641536c_z - 32634809344$$

The lattice volume is $V \approx 11.174896750106$

$$0 = 358722675V^8 - 347945051520V^7 + 94215240337032V^6 - 11187174899713392V^5 \\ +669928638620640960V^4 - 21450619419797145600V^3 + 368350804479038238720V^2 \\ -3176271015792315465728V + 10756169493618646581248$$

The packing density is $D \approx .477260188850$

$$0 = 30773647766929569D^8 - 48466049435307548D^7 + 29976465208255065D^6 - 9310164678731400D^5 \\ +1550760737547780D^4 - 138113270366832D^3 + 6203472614784D^2 - 122186465280D + 671846400$$

2.3 $\mathbf{E}_4$

$\mathbf{E}_4$ G_7^+

1 coset

4 tetrahedra

$U = \frac{32}{3}$

10.666666666667

$a = ?$

−0.025765265330

+2.209200951184

−1.266286151160

$b = ?$

−1.870758393460

+1.635540344736

+1.376070633654

$c = ?$

+3.117483108257

−0.882516891743

+0.624033678663

$V = ?$

16.364564161886

$D = ?$

.651814894741

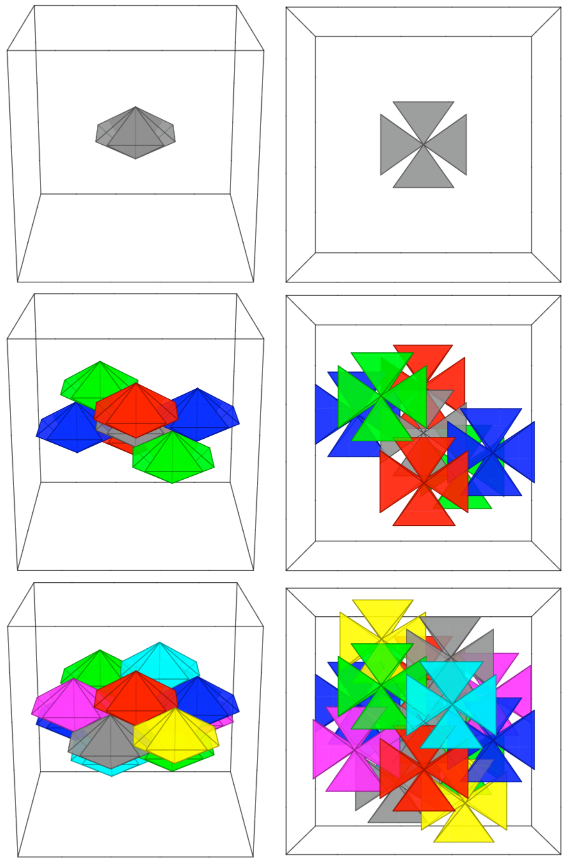
basis $G_7^+\{a,b,c\}$

$a{+}b$

$b{+}c$

$c{+}a$

$a{+}b{+}c$



There are 10 vertices and 16 faces

2 points (axial): $\langle x,y,z\rangle = \langle 0,0,\pm\sqrt{2}\rangle$
4 points: $\langle x,y,z\rangle = \langle \pm 2,\pm\sqrt{2},0\rangle$
4 points: $\langle x,y,z\rangle = \langle \pm\sqrt{2},\pm 2,0\rangle$
4 triangles: $\langle x,y,z\rangle \cdot \langle \pm 1,0,\pm\sqrt{2}\rangle = 2$
4 triangles: $\langle x,y,z\rangle \cdot \langle 0,\pm 1,\pm\sqrt{2}\rangle = 2$
8 triangles (slivers): $\langle x,y,z\rangle \cdot \langle \pm(2-\sqrt{2}),\pm(2-\sqrt{2}),\pm\sqrt{2}\rangle = 2$

4 tetrahedra have volume $4\cdot\frac{8}{3} = \frac{32}{3} \approx 10.666666666667$

$\mathbf{E}_4$ has volume $8\cdot\frac{4}{3} + 8\cdot\frac{1}{3}\sqrt{2} = \frac{1}{3}(32+8\sqrt{2}) \approx 14.437902832995$

The relative density of tetrahedra is $\frac{1}{7}(8-2\sqrt{2}) \approx .738796125036$

8 triangles: $\frac{1}{6}\det\begin{bmatrix} 0 & 0 & \sqrt{2} \\ 2 & -\sqrt{2} & 0 \\ 2 & +\sqrt{2} & 0 \end{bmatrix} = \frac{4}{3}$

8 triangles (slivers): $\frac{1}{6}\det\begin{bmatrix} 0 & 0 & \sqrt{2} \\ 2 & \sqrt{2} & 0 \\ \sqrt{2} & 2 & 0 \end{bmatrix} = \frac{1}{3}\sqrt{2}$

The difference body is $2\mathbf{E}_4$

The lattice basis is $G_7^+\{a,b,c\}$

The intersections are $\begin{cases} a\cdot\langle 0,1,-\sqrt{2}\rangle = 4 \\ b\cdot\langle -2+\sqrt{2},2-\sqrt{2},\sqrt{2}\rangle = 4 \\ c\cdot\langle 1,0,\sqrt{2}\rangle = 4 \\ (a+b)\cdot\langle 0,1,\sqrt{2}\rangle = 4 \\ (b+c)\cdot\langle 2-\sqrt{2},2-\sqrt{2},\sqrt{2}\rangle = 4 \\ (c+a)\cdot\langle 1,0,-\sqrt{2}\rangle = 4 \\ (a+b+c)\cdot\langle 0,1,\sqrt{2}\rangle = 4 \\ \text{minimum lattice volume} \end{cases}$

The lattice vectors are $\begin{cases} a \approx \langle -0.025765265330,+2.209200951184,-1.266286151160\rangle \\ b \approx \langle -1.870758393460,+1.635540344736,+1.376070633654\rangle \\ c \approx \langle +3.117483108257,-0.882516891743,+0.624033678663\rangle \end{cases}$

$$0 = 2883a_x^4 + (-94668+79974\sqrt{2})a_x^3 - (-287304+384508\sqrt{2})a_x^2 + (-8012736+5959112\sqrt{2})a_x + (9364736-6614192\sqrt{2})$$
$$0 = 93a_y^4 - (3232-1706\sqrt{2})a_y^3 + (29336-19260\sqrt{2})a_y^2 - (100992-70904\sqrt{2})a_y - (-115968+83440\sqrt{2})$$
$$0 = 93a_z^4 + (1706-872\sqrt{2})a_z^3 + (-260+606\sqrt{2})a_z^2 + (-644+592\sqrt{2})a_z + (-416+300\sqrt{2})$$

$$0 = 2883b_x^4 + (6136+11850\sqrt{2})b_x^3 + (7344+41552\sqrt{2})b_x^2 + (-163744+174352\sqrt{2})b_x + (-140448+126304\sqrt{2})$$
$$0 = 961b_y^4 - (26996-14116\sqrt{2})b_y^3 - (85152+18496\sqrt{2})b_y^2 - (813696-471232\sqrt{2})b_y + (-299136+609280\sqrt{2})$$
$$0 = 2883b_z^4 - (148120-94558\sqrt{2})b_z^3 - (278312-171840\sqrt{2})b_z^2 + (-2595680+2011232\sqrt{2})b_z - (8397216-5762272\sqrt{2})$$

$$0 = 2883c_x^4 - (25816+13544\sqrt{2})c_x^3 + (40768+88896\sqrt{2})c_x^2 - (-376960+400768\sqrt{2})c_x + (-929536+702464\sqrt{2})$$
$$0 = 2883c_y^4 + (20312-13544\sqrt{2})c_y^3 - (-7744+73632\sqrt{2})c_y^2 - (-201984+339712\sqrt{2})c_y - (-316416+345088\sqrt{2})$$
$$0 = 2883c_z^4 - (-13544+10145\sqrt{2})c_z^3 - (-3872+36816\sqrt{2})c_z^2 + (169856-50496\sqrt{2})c_z - (-79104+86272\sqrt{2})$$

The lattice volume is $V \approx 16.364564161886$

$$0 = 24935067V^4 - (95063108864-63362895264\sqrt{2})V^3 + (-874563881984+893456785408\sqrt{2})V^2$$
$$-(138718501666816-90579218546688\sqrt{2})V + (-2889280359235584+2107901352083456\sqrt{2})$$

The packing density is $D \approx .651814894741$

$$0 = 57280578849693D^4 - (21479020933356+34824904072173\sqrt{2})D^3 + (15003005751570+8930240557404\sqrt{2})D^2$$
$$-(973080415656+2234460122906\sqrt{2})D + (99195568974+72369049716\sqrt{2})$$

2.4 $\mathbf{E}_5$

$\mathbf{E}_5\ G_6^-$

1 coset

5 tetrahedra

$U = \frac{40}{3}$

13.333333333333

$a = ?$

+1.762915609592

+2.245606548163

+1.103562870315

$b = ?$

+1.432724855460

−1.898223619474

+1.390815543708

$c = ?$

+3.994958242085

−0.369606202651

+0.321259556765

$V = ?$

19.059009547662

$D = ?$

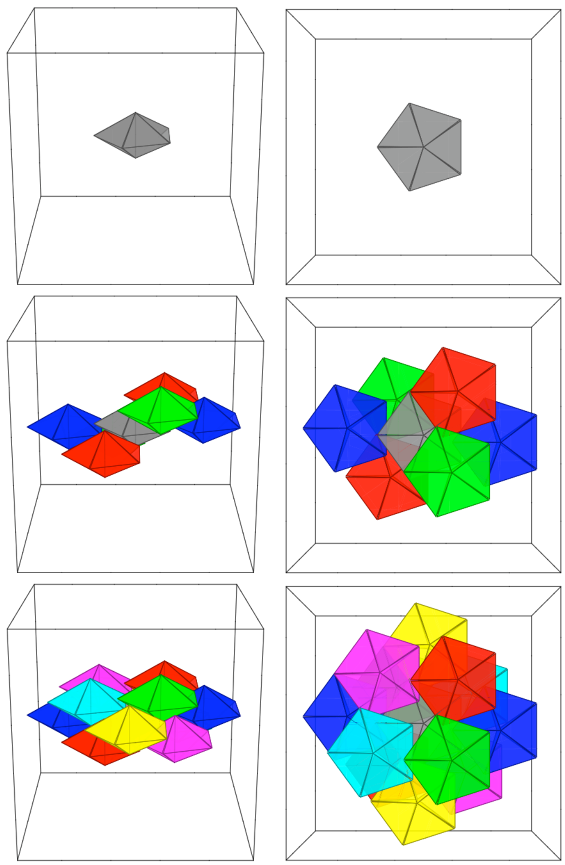
.699581649298

basis $G_6^-\{a,b,c\}$

$a-b$

$b-c$

$c-a$



$R = \sqrt{5+\sqrt{5}},\ S = \sqrt{5-\sqrt{5}}$

There are 12 vertices and 20 faces

2 points (axial): $\langle x,y,z\rangle = \langle 0,0,\pm\sqrt{2}\rangle$
10 points: $\langle x,y,z\rangle = \mathbf{Z}_5\langle 2,\pm\sqrt{2},0\rangle$
10 triangles: $\langle x,y,z\rangle \cdot \mathbf{Z}_5\langle 1,0,\pm\sqrt{2}\rangle = 2$
10 triangles (slivers): $\langle x,y,z\rangle \cdot \mathbf{Z}_5\langle -1,0,\pm\frac{1}{4}\sqrt{2}(1+\sqrt{5}+S)\rangle = \frac{1}{2}(1+\sqrt{5}+S)$

$$\mathbf{Z}_5 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \theta = \{-\tfrac{4}{5}\pi, -\tfrac{2}{5}\pi, 0, +\tfrac{2}{5}\pi, +\tfrac{4}{5}\pi\}$$

5 tetrahedra have volume $5\cdot\frac{8}{3} = \frac{40}{3} \approx 13.333333333333$

$\mathbf{E}_5$ has volume $10\cdot\frac{4}{3} + 10\cdot\frac{1}{6}(2-2\sqrt{5}+R) = \frac{1}{3}(50-10\sqrt{5}+5R) \approx 13.696430154760$

The relative density of tetrahedra is $\frac{1}{1205}(1480+360\sqrt{5}-312R-164S) \approx .973489674512$

$$\text{10 triangles: } \tfrac{1}{6}\det\begin{bmatrix} 0 & 0 & \sqrt{2} \\ 2 & -\sqrt{2} & 0 \\ 2 & +\sqrt{2} & 0 \end{bmatrix} = \tfrac{4}{3}$$

$$\text{10 triangles (slivers): } \tfrac{1}{6}\det\begin{bmatrix} 0 & 0 & \sqrt{2} \\ -\frac{1}{2}(1+\sqrt{5}+S) & +\frac{1}{4}\sqrt{2}(-1-\sqrt{5}+2S) & 0 \\ -\frac{1}{2}(1+\sqrt{5}+S) & -\frac{1}{4}\sqrt{2}(-1-\sqrt{5}+2S) & 0 \end{bmatrix} = \tfrac{1}{6}(2-2\sqrt{5}+R)$$

The difference body has 52 vertices and 60 faces

2 axial points: $\langle x,y,z\rangle = \langle 0,0,\pm 2\sqrt{2}\rangle$
40 inner points: $\langle x,y,z\rangle = \mathbf{Z}_5\langle \pm 2, \pm\sqrt{2}, \pm\sqrt{2}\rangle$
10 outer points: $\langle x,y,z\rangle = \mathbf{Z}_5\langle \pm\frac{1}{2}\sqrt{2}(-1+\sqrt{5}+2R), 0, 0\rangle$
20 inner rhombi: $\langle x,y,z\rangle \cdot \mathbf{Z}_5\langle 0, \pm 1, \pm\frac{1}{4}(-1+\sqrt{5}+2R)\rangle = \frac{1}{2}\sqrt{2}(-1+\sqrt{5}+2R)$
20 inner triangles (slivers): $\langle x,y,z\rangle \cdot \mathbf{Z}_5\langle \pm 1, 0, \pm\frac{1}{4}\sqrt{2}(1+\sqrt{5}+S)\rangle = 1+\sqrt{5}+S$
20 outer triangles: $\langle x,y,z\rangle \cdot \mathbf{Z}_5\langle \pm 1, 0, \pm\sqrt{2}\rangle = \frac{1}{2}(5+\sqrt{5}+S)$

$$\mathbf{Z}_5 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \theta = \{-\tfrac{4}{5}\pi, -\tfrac{2}{5}\pi, 0, +\tfrac{2}{5}\pi, +\tfrac{4}{5}\pi\}$$

The lattice basis is $G_6^-\{a,b,c\}$

The intersections are

$$\begin{cases} a\cdot\langle \frac{1}{4}\sqrt{2}\,S, \frac{1}{4}(1+\sqrt{5}), \frac{1}{4}(-1+\sqrt{5}+2R)\rangle = \frac{1}{2}\sqrt{2}(-1+\sqrt{5}+2R) \\ b\cdot\langle \frac{1}{4}\sqrt{2}\,S, -\frac{1}{4}(1+\sqrt{5}), \frac{1}{4}(-1+\sqrt{5}+2R)\rangle = \frac{1}{2}\sqrt{2}(-1+\sqrt{5}+2R) \\ c\cdot\langle 1, 0, \sqrt{2}\rangle = \frac{1}{2}(5+\sqrt{5}+S) \\ (a-b)\cdot\langle \frac{1}{4}(-1+\sqrt{5}), \frac{1}{4}\sqrt{2}\,R, -\sqrt{2}\rangle = \frac{1}{2}(5+\sqrt{5}+S) \\ (b-c)\cdot\langle -\frac{1}{4}\sqrt{2}\,R, \frac{1}{4}(-1+\sqrt{5}), \frac{1}{4}(-1+\sqrt{5}+2R)\rangle = \frac{1}{2}\sqrt{2}(-1+\sqrt{5}+2R) \\ (c-a)\cdot\langle \frac{1}{4}(1+\sqrt{5}), -\frac{1}{4}\sqrt{2}\,S, -\sqrt{2}\rangle = \frac{1}{2}(5+\sqrt{5}+S) \\ \text{minimum lattice volume} \end{cases}$$

$$\text{The lattice vectors are } \begin{cases} a \approx \langle +1.762915609592, +2.245606548163, +1.103562870315\rangle \\ b \approx \langle +1.432724855460, -1.898223619474, +1.390815543708\rangle \\ c \approx \langle +3.994958242085, -0.369606202651, +0.321259556765\rangle \end{cases}$$

The lattice volume is $V \approx 19.059009547662$

The packing density is $D \approx .699581649298$

$$\begin{aligned}
0 = {} & 5999158588055988398350863380269576133507283377003858844460a_x^8 \\
& +(-7142070415269653230435156054928930124249453450709925784720 \\
& \quad -6333684947898560365958531913682743050199666310685392799880\sqrt{5} \\
& \quad -2978772507220534459918229869958120043990453132642353232336R \\
& \quad +6520767213441398539927871164820089971093877655377917756 48S)a_x^7 \\
& +(1124552636069954787995476371877726391025528573044421382516 20 \\
& \quad +2112281950874102786203312428637346231256032934783775757 2392\sqrt{5} \\
& \quad +7784425915295528917126212082524811090978232716409460420408R \\
& \quad +2903078182232395539364718986752787815521953369922973295371 6S)a_x^6 \\
& +(1340519674680441505699289916299225455497406474728617338236 80 \\
& \quad -8981470933658130471825273977969559400567387220403614479590 8\sqrt{5} \\
& \quad -1223090141768682383832503828045290856620967764242415814839 8R \\
& \quad +1169296897110033809156804868724431709856335971968883731380 74S)a_x^5 \\
& +(-3035108292786467297675446839488645250106108848091538368375 45 \\
& \quad -5652110572251895363438437657235481617021793106107898930735 83\sqrt{5} \\
& \quad -6628194013001669366345495334650710709413572659237851503155 42R \\
& \quad -5264089736072706872750652316743300556432372014762480219138 4S)a_x^4 \\
& +(-1073372166179877968496707694299951674095151109710947625072 40 \\
& \quad -6283517187960027954657446480520520048095848075192464445460 24\sqrt{5} \\
& \quad -5157512707356941812613535598583300156435045123190941435788 78R \\
& \quad +1636655694685575875671287538775926426628449337152999831605 80S)a_x^3 \\
& +(7730190549409737462734762725495882807051877056567738162176455 \\
& \quad +3578484191415284824976641411474363471635814485000631929116589\sqrt{5} \\
& \quad +4103886850843582293925038550148221090390430630341403367353796R \\
& \quad +2435395849145534176985193670223412228881776667899184137872734S)a_x^2 \\
& +(-10764677872525395452703365376730952033013121607265267050984390 \\
& \quad -3381322672797684563320509290694329800801668400337081853210730\sqrt{5} \\
& \quad -4207403314687689806751080537318703080521636762508367218154228R \\
& \quad -3791328517315856321383206810858539503397810156024060731436462S)a_x \\
& +(4992982377903152419630293542794001706661904522181831506247 70 \\
& \quad +1644650972947339004843909568099540746930548768615324864034690\sqrt{5} \\
& \quad +1595085137889028778851783288123620818168639744047174469443712R \\
& \quad -1956925265245943505584184733889067661711119153055725181046 04S)
\end{aligned}$$

$$
\begin{aligned}
0 = {} & 441083773226128551607068623616867435306371537172 0a_y^8 \\
& +\sqrt{2}(487630711440543976112424521389387468077847752318 0 \\
& \quad -96048479906435915155722514805134195188895611282 0\sqrt{5} \\
& \quad -76720122976620199100513910153049075692826695464 64R \\
& \quad -21339052601625929030755453696311213288955984415 28S)a_y^7 \\
& +(-14766017185348738325103829504019245512698475339990 \\
& \quad -669741538997499737749613870018504294426783004384 18\sqrt{5} \\
& \quad -101123698709804093395170104289824733980827342058 544R \\
& \quad -123242018094244136612107586704834604628488377600 08S)a_y^6 \\
& +\sqrt{2}(192355971739974416543873892262356909374182113461 4480 \\
& \quad +257875837106600857407949631267032354188694562915 748\sqrt{5} \\
& \quad +411630239252347608491229342417465777689232083493 56R \\
& \quad +755126925976897743031785831261074710983695699711 812S)a_y^5 \\
& +(-692366014833463159851316329002147431200584017870 5870 \\
& \quad -527997587368888302877198467137272854208649503089 0506\sqrt{5} \\
& \quad -587034217222355554139984624783589035747854307039 0804R \\
& \quad -181292897544751901943961051377462870816384895975 2568S)a_y^4 \\
& +\sqrt{2}(482798263746859363119705478906646527400883146210 00805 \\
& \quad -571635155143099444582236569773328964126866482629 2241\sqrt{5} \\
& \quad -572757926317567417073619151535865400881020489411 330R \\
& \quad +223456862104923135487713768060808674447563081371 60716S)a_y^3 \\
& +(-195135856565410603236670558298482493981440175955 34835 \\
& \quad -165704820310480757606399878805943825999678319185 53627\sqrt{5} \\
& \quad -180733461551321404736257765762255653660268990851 81084R \\
& \quad -464980193191456018958816185010558095926581866138 7258S)a_y^2 \\
& +\sqrt{2}(562973449786222553461771281997088087103714251661 413710 \\
& \quad -254769658337745454668770546759665622216009518239 621500\sqrt{5} \\
& \quad -190090716109778601239005703822733560552468254013 595492R \\
& \quad +303580015777397753821836387905082759500031467939 890264S)a_y \\
& +(-717356238200081783025557374633086433026316842844 734700 \\
& \quad +343015437168807401726989829701218510530121936284 436056\sqrt{5} \\
& \quad +260135556785401732368274837514015221854220886606 185828R \\
& \quad -391036067304599250385111198380242194682092823880 980216S)
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & 299957929402799419917543169013478806675364168850 192942230a_z^8 \\
& +\sqrt{2}(-807428824685319383082394867324542561814077004195 024222350 \\
& \quad +131981963808579913763959039460236437066187527237 139825560\sqrt{5} \\
& \quad +169677202011054011478050385893025812320107138543 222201894R \\
& \quad -290188536612024444161809656287429787170636029355 870349972S)a_z^7 \\
& +(-657982358499605611837724149907009702454978847510 9956369780 \\
& \quad -671237358803560492779568869015737902961312025732 8962697367\sqrt{5} \\
& \quad -482038573158702341665516416760239940905092966323 7829729840R \\
& \quad +152093206914694484318458978672716494318468303751 483974370S)a_z^6 \\
& +\sqrt{2}(434578066528388959341321245511764510557718413840 48492464835 \\
& \quad +296418783025409038230344926387785377385305663715 71248306031\sqrt{5} \\
& \quad +236192532372604727056375640793813304583755664873 82302553164R \\
& \quad +611314921100646068939243999226438136590819804597 45069185 08S)a_z^5 \\
& +(-187143497776307635230456143348470901392431686966 854476295330 \\
& \quad -123534912116148122061985349691077455455671862214 6199777978154\sqrt{5} \\
& \quad -108349944010689847462984131803674258655765237670 307260499376R \\
& \quad -338453433511418943645759181187426740725837072952 75136406232S)a_z^4 \\
& +\sqrt{2}(166634434455186449502879911357870542182259192571 909490850060 \\
& \quad +131250779102387107029623626006566263487204390675 454919450968\sqrt{5} \\
& \quad +121030702019410176986760242904076382422913823928 309291836928R \\
& \quad +276443951678926095135104608471365362099334561837 064585434 00S)a_z^3 \\
& +(-799708467755388495583225301361901756548866502979 03826403560 \\
& \quad -141713570241184994951204294167149227736452108595 802845040608\sqrt{5} \\
& \quad -130971778556543322971970921895930889035724313136 486222837944R \\
& \quad +712593841159725930650663430719762304885947172005 08990883 44S)a_z^2 \\
& +\sqrt{2}(-395659578570767756087113250403410733501631394007 21322425760 \\
& \quad +407902990576075385519752885566746865222593125079 36356208496\sqrt{5} \\
& \quad +356740151959668835284141532763704467319622352459 28989209424R \\
& \quad -265679073292329495567252173885327917700231943515 47596541744S)a_z \\
& +(167907903384835945546428124413321969537054153658 34010464160 \\
& \quad -748654986258266663627350125823174629302841278436 84910719 68\sqrt{5} \\
& \quad -638768042284814624868085683486030262007175495539 06918678 40R \\
& \quad +851736328247087387355245838728304380403809653294 4576039616S)
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & 20761615633138747697541218789464642427216522240 b_x^8 \\
& +(-29948291661441842847150131389841487614095860240 \\
& \quad -95360102009212589837224870770623188086981199 20\sqrt{5} \\
& \quad -3263116572906746549019777943475732431448221008R \\
& \quad -54228818574424654943529012429796615320219430 56S)b_x^7 \\
& +(-189271606521736502909230625160649672490512504080 \\
& \quad -1047267329005372039848647531468301013478099476 38\sqrt{5} \\
& \quad -118530236658208329781216019867132919869059074696R \\
& \quad -565846993815519565571824676145512785564263176 52S)b_x^6 \\
& +(311726667486653334648453290019644256786073715 9590 \\
& \quad +1464036785143762430161650167795747740057861690650\sqrt{5} \\
& \quad +1563874240809889469721050857436191219584118892080R \\
& \quad +908377603169651299115091595212644903620989833070S)b_x^5 \\
& +(-1871313487168948696710052944663476482529572343 2980 \\
& \quad -8591250086468875152704784233895176977343068039662\sqrt{5} \\
& \quad -9685425911264303388574759147513959691500998507844R \\
& \quad -5800983631721666286347332059498252770764664622488S)b_x^4 \\
& +(11417350601475476262115710289184309645719095357 8410 \\
& \quad +51718629729364785513423885460781944134333569327550\sqrt{5} \\
& \quad +59979886680479104070804641255374032011399267326244R \\
& \quad +36522073656585930121692564085703638636057525688874S)b_x^3 \\
& +(-33618665346281993202224208263564130179130718605 8805 \\
& \quad -151765863786241393957277098609321093390352823451933\sqrt{5} \\
& \quad -177610280951022752531691427454936861993091508973672R \\
& \quad -108589955566451294660913662646431378594480806446892S)b_x^2 \\
& +(34278735148900274607542911346709288355929411830 3830 \\
& \quad +155280789454493865346042206942855830292270599052610\sqrt{5} \\
& \quad +181297972570223256538959963205592169632008745952892R \\
& \quad +110401075260419889889424701645702896741414563961268S)b_x \\
& +(-75143705491347748638315812583914193375712613714595 \\
& \quad -34607428485550393953257380957595778712266508204991\sqrt{5} \\
& \quad -38990899323098957498174996166726890602758430731780R \\
& \quad -23264666800460873461340331635142795469509622084088S)
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & 26195969507461671437185311702056201409648 0 b_y^8 \\
& +\sqrt{2}(27203630410473797516257895481477162243324 00 \\
& \quad -247080954715373694188109618378428739565680\sqrt{5} \\
& \quad +136457368265717066738045848891725083287296 0R \\
& \quad +211444170260968560876209675620722012402720 0S)b_y^7 \\
& +(94653275378027709580380624813367356442431360 \\
& \quad +1169104089878670683240395412921149407223914 0\sqrt{5} \\
& \quad +267752906580435481411293721447266942529531 2R \\
& \quad +2709782771059580214106354362303065448016946 4S)b_y^6 \\
& +\sqrt{2}(2432532104562476325599064581118557894062076 00 \\
& \quad -834522781740601228066571444913304097410926 80\sqrt{5} \\
& \quad +210861745455330365033863236641903162509107 2R \\
& \quad +161105642886471431426590488415330007249233744S)b_y^5 \\
& +(3497139279607305259936071356085997068040096 95 \\
& \quad -1487072623082875265477390944129544323996776775\sqrt{5} \\
& \quad -1574736481946947016904472304746098002928189100R \\
& \quad +3928990775525925036090321845627198518214163 00S)b_y^4 \\
& +\sqrt{2}(-3793297489447770260986432444349169157752113920 \\
& \quad -6641803964587532236797703901650493321616049322\sqrt{5} \\
& \quad -6816212871605772469023189363531968692335068458R \\
& \quad -10177679549017244170670805988108344938647134 6S)b_y^3 \\
& +(-12998805088894963638849141342711377878229804140 \\
& \quad -25166189511751687102977223178750295534883201740\sqrt{5} \\
& \quad -25528494039582836909652311618842976623619246992R \\
& \quad +30973532205305217169833724392571620919784341 6S)b_y^2 \\
& +\sqrt{2}(-442244120524346108827983303761643001571352262 0 \\
& \quad -19444301030877587995661289438311082934162519556\sqrt{5} \\
& \quad -19723409585255331010790348667309856602945089306R \\
& \quad +2329379266131893188127205494855358726070852528S)b_y \\
& +(-23829812980215364513581450626735903158702195460 \\
& \quad +2612466528532923248013722942292881759076956566\sqrt{5} \\
& \quad +3337801997092969262572092239924472546368803 88R \\
& \quad -10824021555357955210079385635000147661238939344S)
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & 436535231983573332580765744101443280639540 b_z^8 \\
& +\sqrt{2}(1595632354626986872131938488732724324418 0 \\
& \quad +1481728416216953624959580847899080959351960\sqrt{5} \\
& \quad +776753944201403768650149282402843960427740 R \\
& \quad -6796944908499072438439871526449268119182 80 S) b_z^7 \\
& +(2522717444100775133015288002500091771698300 0 \\
& \quad -6766751454500209801453630126549818352854960\sqrt{5} \\
& \quad -5144536231315498712047939660150272039763524 R \\
& \quad +11800483707440083222873138445158203287840892 S) b_z^6 \\
& +\sqrt{2}(-8144394722680456645340424088902137284934276 0 \\
& \quad +26588247542608897849913513261766472302851630\sqrt{5} \\
& \quad +1920119175871828768312511490077882206408100 2 R \\
& \quad -40508967211982296443232821878073767497433536 S) b_z^5 \\
& +(26653162678396239297007465856672900420903667 5 \\
& \quad -146298310205515289156257374241866824662145200\sqrt{5} \\
& \quad -11609118144388214203778249951838941786446660 2 R \\
& \quad +148945080681790932779893810816461251087551006 S) b_z^4 \\
& +\sqrt{2}(-37054402786723748889355923972372205879304978 0 \\
& \quad +174363088575488401742612632203041196106059756\sqrt{5} \\
& \quad +124343237509553729631255891580127108125478640 R \\
& \quad -205840650135580057763472919207789001407506760 S) b_z^3 \\
& +(61198101367497778158725331212018837174483061 0 \\
& \quad -227284800949796490986177199976176499273186602\sqrt{5} \\
& \quad -167596304154758278646408484434881395462708728 R \\
& \quad +312853897802387906306973265419903860140108224 S) b_z^2 \\
& +\sqrt{2}(-17711407907250718970879444424622148585768722 0 \\
& \quad +99259063254489928424760432373037897657201132\sqrt{5} \\
& \quad +76465464320497007560918282650609644819750760 R \\
& \quad -101093035863672742153235144852451819776064520 S) b_z \\
& +(4171775453663902062358218686473978382875469 0 \\
& \quad -13357941332465779975920863124434464235402326\sqrt{5} \\
& \quad -8934539090526729410603758990017650135762848 R \\
& \quad +21090484921323007377491768505937151082309784 S)
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & 79840659023137874349597815367572823068619370574999838599544 00 c_x^8 \\
& +(-208244368532588661212851605918914426496731704029252225992145600 \\
& \quad -634355058951654347459929164374618753484220284678132078464256 00\sqrt{5} \\
& \quad -148280185726285568412407040925151759901563161200521012993635 20 R \\
& \quad -335588219998209430392184807046709310229533334153860219391654 40 S) c_x^7 \\
& +(3079934326848412470984733485894031319066553548713486248851702400 \\
& \quad +1174184192226782255618115298117353225416126203031990992713996960\sqrt{5} \\
& \quad +658067124762758007090748886917019677732145956476608759734649280 R \\
& \quad +575618817822126914993537718769467196739992225280059639287108000 S) c_x^6 \\
& +(-25303878748181244739872692554006236045075417206062854032486970800 \\
& \quad -10557793901693140467184467596504049292219633198628055026046103120\sqrt{5} \\
& \quad -8013614401462858101936142987997824956603111175014960385238343360 R \\
& \quad -5583146447703426771397233140676159056999278915560542531164750080 S) c_x^5 \\
& +(125145855677527601482788631676099031946304482790645633005710446080 \\
& \quad +54282274577445604162845013177130644405352886016582394711561401520\sqrt{5} \\
& \quad +48388041300549280600473745266624815221730748710330498056892038000 R \\
& \quad +31305828920488155202679965647187259031029672275629477598184006000 S) c_x^4 \\
& +(-373101250891860839183427674839614262822859530154137368017710534840 \\
& \quad -164552740741591590411300968595603898315053711685627479125495655000\sqrt{5} \\
& \quad -161126460285373412934118222259556614876991314423340279755314671088 R \\
& \quad -101496182142194631031796890389447148337824291672571640209749441296 S) c_x^3 \\
& +(613082352990206788970910852196207142611376132409849861495378079460 \\
& \quad +272227123517235357171199899966746967592900499299744024928233360202 0\sqrt{5} \\
& \quad +281080292884766121882636467145672702389184658426597143033327583928 R \\
& \quad +175302814817765345637987871735806830495984273465972936816991405696 S) c_x^2 \\
& +(-395859428498856973996569513464052200654874898942972683483975476960 \\
& \quad -176156217096215293520317643070703322452688950732129203841176501320\sqrt{5} \\
& \quad -183457850815417262224732891908062488404024505597808579547024601888 R \\
& \quad -114112613609596827927369439410598064939592182480408708708032466576 S) c_x \\
& +(-72601747199659903832720534926628514243933968563810421062577925995 \\
& \quad -32639774778044252847896811790720071290009555367228954199540725525\sqrt{5} \\
& \quad -42918056807128256366823682735684131263900234926263940854785172368 R \\
& \quad -26382435376264239689436096500951460787041005064589454356182524396 S)
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & 1996016475578446858739945384189320576715484264374995964988600c_y^8 \\
& +\sqrt{2}(1221678383871635457221185452151802220945098171936519866870470 0 \\
& \quad -40611848786759715941269458189842379086133327369248306481789 00\sqrt{5} \\
& \quad -40815951072412643142804066283230059500529406675066446996589 60R \\
& \quad +56251955236462208871329841472026531723220394747559706748458 80S)c_y^7 \\
& +(484751477988950284499613317307742240891988653937971400151214 50 \\
& \quad -13588972189216152789443628304363676206487334270955927997707293 0\sqrt{5} \\
& \quad -12114097169264436727605062212226874320810457703732232869885416 0R \\
& \quad +56055069309639571736243990710158761056982521538541706011324960S)c_y^6 \\
& +\sqrt{2}(1256797059373463844035597630470291350753215533539502491590021300 \\
& \quad -15649586722094673569189403200634541041827628165916286161211834 0\sqrt{5} \\
& \quad -742785360039529968043783283934188236531033487696914482591064 0R \\
& \quad +592712451788110024412825573352038793179149764017142112436135320S)c_y^5 \\
& +(848938725131351610712346101089641358205108614478360051353242939 5 \\
& \quad -79387659288339654008466452885426287402492342699604301736446705\sqrt{5} \\
& \quad +76744871453537393434179258080762630599370275869964075746634052 0R \\
& \quad +369621346983396117055847123216384602290314640754350801228020266 0S)c_y^4 \\
& +\sqrt{2}(-131003107646309872980297654283099185586448951181965222640166232 65 \\
& \quad -1193250999257972053999375101108820283858815885985786463214786259 5\sqrt{5} \\
& \quad -130080657724609274216806889278006693017130715392692920665322330 26R \\
& \quad -299049637249451002886997840652108402193984035014878442490857759 2S)c_y^3 \\
& +(-45677096936035094950041637833630948649909421057057994593537869 45 \\
& \quad -91698972910923287437888793510134771106833067653358783730107783 50\sqrt{5} \\
& \quad -93325032031761039988952243967077972049336391656125713687275215 36R \\
& \quad +15667138143926102183222300733658790170758558286897240803006408 8S)c_y^2 \\
& +\sqrt{2}(19546689279457046201005572884972435729774745947558615238452616 75 \\
& \quad +75225712830434742683218119682463798424639212491150189383388987 85\sqrt{5} \\
& \quad +75242650744436239713554199779530103765351718078492666531792152 18R \\
& \quad -87627064481737946423178277723418033587922483434117247089947207 4S)c_y \\
& +(-24975756879841329771103161364466004186939855177568636412195841 215 \\
& \quad +15744375863540669255811698771397264511419651340479599724768230 195\sqrt{5} \\
& \quad +12759290323328617922274481423616296454587085430286701166047412 141R \\
& \quad -14486586891852954708055760434149345352283075482942447953539075 408S)
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & 4990041188946117146849863460473301441788710660937489912471 50c_z^8 \\
& +\sqrt{2}(15175953276972785160517492244927743862341550899766421497830 50 \\
& \quad +98435132143469640644230594657602331628044625743166476270650 0\sqrt{5} \\
& \quad +46337558039464240128877200289109924969238487875162816560511 0R \\
& \quad +50704949705181040605604829926306306109549537043315203104620S)c_z^7 \\
& +(-95514598302272746848881955974023784092004547677091193174058150 \\
& \quad -46292672489353486103342303006243523105711178287855495219468 45\sqrt{5} \\
& \quad -33977411019550088404034936247351486896446868005275133741374 60R \\
& \quad -18038796526617306787644870896940059171020013943983388846664 50S)c_z^6 \\
& +\sqrt{2}(28539735655425616791961967037670162792559399494114424689151 00 \\
& \quad +10213657635584735004826647820356579995629629802590944219181 10\sqrt{5} \\
& \quad +91403385214639895892129125687812673926101169436567990594364 0R \\
& \quad +77686673414338164327109628364387580579114014850632888680562 0S)c_z^5 \\
& +(14425308933683214088799935806482579080290796714768427353085845 \\
& \quad +69491590910253902696374725216831900561018955976606596434141 55\sqrt{5} \\
& \quad +64383960863342659439266632450834105534052512135773607264047 00R \\
& \quad +35652123614473769507094624376395420574303732997660034527959 00S)c_z^4 \\
& +\sqrt{2}(-41968399509933022012605884607239186563446238197460425212715720 \\
& \quad -18566987708690983417564339113545875230586356769610078064797620\sqrt{5} \\
& \quad -18774213418262087064921740259224265391481056915723498121572264R \\
& \quad -11770891538064957060480997447743624477723130597559685063132808S)c_z^3 \\
& +(53765495823195583561226182603402348284472627636830804377408 30 \\
& \quad +19821775718051098246647691002839238442200676149420195689046 90\sqrt{5} \\
& \quad +29026817399965808428726643155761723877963629486339671536767 76R \\
& \quad +21449849243264023395509497241001190457489077224854598997359 52S)c_z^2 \\
& +\sqrt{2}(48284992601418990946482368147965581623626750933734428169520280 \\
& \quad +21524739309126063689128275226512717495440049994279274228840440\sqrt{5} \\
& \quad +25595533985945116224516412912443326244669113684875035823458048R \\
& \quad +15876238086000742353948234547030698253347752464925627845805376S)c_z \\
& +(-22454777348736115243652097036919761876064136739458840521143660 \\
& \quad -98920260815486849918648102699047563831043348923683938994146 20\sqrt{5} \\
& \quad -10184153693736443856623557879873231244374661897739319022325888R \\
& \quad -64188874556380283772298976698739000985554814913618513534126 56S)
\end{aligned}
$$

2.5 $\mathbf{E}_6$

$\mathbf{E}_6$ G_6^-

1 coset

6 tetrahedra

$U = 16$

16.000000000000

$a = \langle +\frac{2}{5}, +\frac{2}{5}, +\frac{14}{5} \rangle$

+0.400000000000

+0.400000000000

+2.800000000000

$b = \langle +\frac{6}{5}, -\frac{14}{5}, +\frac{14}{5} \rangle$

+1.200000000000

−2.800000000000

+2.800000000000

$c = \langle +\frac{16}{5}, -\frac{4}{5}, 0 \rangle$

+3.200000000000

−0.800000000000

0.000000000000

$V = \frac{672}{25}$

26.880000000000

$D = \frac{25}{42}$

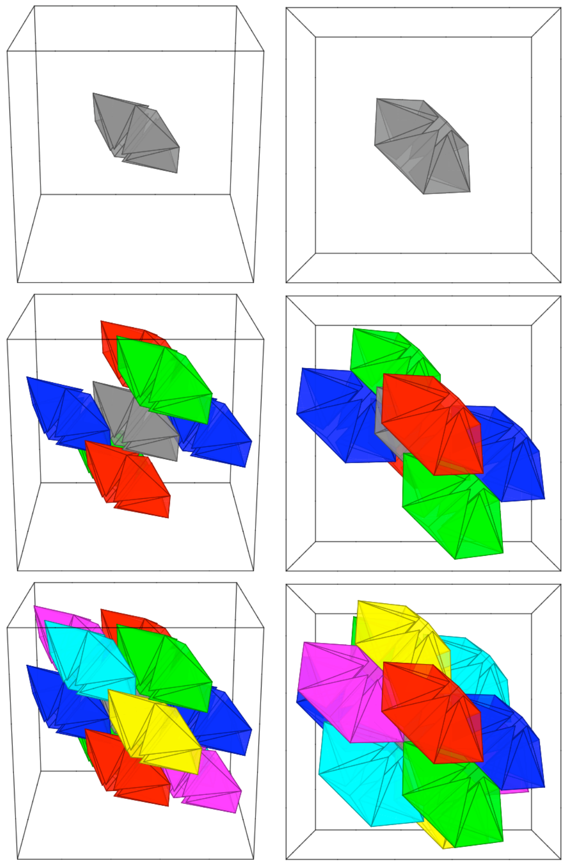
.595238095238

basis $G_6^-\{a,b,c\}$

$a-b$

$b-c$

$c-a$



There are 18 vertices and 26 faces

- 6 inner points: $\langle x,y,z \rangle = \pm\mathbf{Z}_3\langle -2+\sqrt{6}, 1, 1 \rangle$
- 6 inner points: $\langle x,y,z \rangle = \pm\mathbf{Z}_3\langle 2-\frac{1}{3}\sqrt{6}, -1+\frac{2}{3}\sqrt{6}, -1+\frac{2}{3}\sqrt{6} \rangle$
- 6 outer points: $\langle x,y,z \rangle = \mathbf{S}_3\langle 0, -2, +2 \rangle$
- 2 axial hexagons: $\langle x,y,z \rangle \cdot \pm\langle 1,1,1 \rangle = \sqrt{6}$
- 6 triangles: $\langle x,y,z \rangle \cdot \pm\mathbf{Z}_3\langle 0,1,1 \rangle = 2$
- 6 triangles: $\langle x,y,z \rangle \cdot \pm\mathbf{Z}_3\langle 4,1,1 \rangle = 6$
- 12 triangles (slivers): $\langle x,y,z \rangle \cdot \pm\mathbf{S}_3\langle 2+\frac{1}{6}\sqrt{6}, 1+\frac{1}{6}\sqrt{6}, \frac{1}{6}\sqrt{6} \rangle = 4$

6 tetrahedra have volume $6 \cdot \frac{8}{3} = 16 \approx 16.000000000000$

$\mathbf{E}_6$ has volume

$$2(-12+5\sqrt{6}) + 6 \cdot \tfrac{2}{3}\sqrt{6} + 6 \cdot \tfrac{2}{3}\sqrt{6} + 12(-\tfrac{4}{3}+\tfrac{2}{3}\sqrt{6}) = -40+26\sqrt{6} \approx 23.686733312363$$

The relative density of tetrahedra is $\frac{1}{307}(80+52\sqrt{6}) \approx .675483604641$

$$\text{2 hexagons (axial): } 6\cdot\frac{1}{6}\det\begin{bmatrix} +\frac{1}{3}\sqrt{6} & +\frac{1}{3}\sqrt{6} & +\frac{1}{3}\sqrt{6} \\ 2-\frac{1}{3}\sqrt{6} & \frac{2}{3}\sqrt{6}-1 & \frac{2}{3}\sqrt{6}-1 \\ +1 & +1 & \sqrt{6}-2 \end{bmatrix} = 5\sqrt{6}-12$$

$$\text{6 triangles: } \frac{1}{6}\det\begin{bmatrix} \sqrt{6}-2 & +1 & +1 \\ -2 & +2 & 0 \\ -2 & 0 & +2 \end{bmatrix} = \frac{2}{3}\sqrt{6}$$

$$\text{6 triangles: } \frac{1}{6}\det\begin{bmatrix} 2-\frac{1}{3}\sqrt{6} & \frac{2}{3}\sqrt{6}-1 & \frac{2}{3}\sqrt{6}-1 \\ +2 & -2 & 0 \\ +2 & 0 & -2 \end{bmatrix} = \frac{2}{3}\sqrt{6}$$

$$\text{12 triangles (slivers): } \frac{1}{6}\det\begin{bmatrix} +1 & +1 & \sqrt{6}-2 \\ 2-\frac{1}{3}\sqrt{6} & \frac{2}{3}\sqrt{6}-1 & \frac{2}{3}\sqrt{6}-1 \\ +2 & 0 & -2 \end{bmatrix} = \frac{2}{3}\sqrt{6}-\frac{4}{3}$$

The difference body is $2\mathbf{E}_6$

The lattice basis is $G_6^-\{a,b,c\}$

$$\text{The intersections are } \begin{cases} a\cdot\langle 1,1,4\rangle = 12 \\ b\cdot\langle 1,0,1\rangle = 4 \\ c\cdot\langle 4,1,1\rangle = 12 \\ (a-b)\cdot\langle 1,4,1\rangle = 12 \\ (b-c)\cdot\langle -1,-1,0\rangle = 4 \\ (c-a)\cdot\langle 0,-1,-1\rangle = 4 \\ \text{minimum lattice volume} \end{cases}$$

The lattice vectors are

$$\begin{cases} a = \langle +\frac{2}{5},+\frac{2}{5},+\frac{14}{5}\rangle \approx \langle +0.400000000000,+0.400000000000,+2.800000000000\rangle \\ b = \langle +\frac{6}{5},-\frac{14}{5},+\frac{14}{5}\rangle \approx \langle +1.200000000000,-2.800000000000,+2.800000000000\rangle \\ c = \langle +\frac{16}{5},-\frac{4}{5},0\rangle \approx \langle +3.200000000000,-0.800000000000,0.000000000000\rangle \end{cases}$$

The lattice volume is $V = \frac{672}{25} \approx 26.880000000000$

The packing density is $D = \frac{25}{42} \approx .595238095238$

2.6 $\mathbf{V}_{20}$

$\mathbf{V}_{20}$ G_6^-

1 coset

20 tetrahedra

$U = \frac{160}{3}$

53.333333333333

$a = ?$

−2.448722667966

+2.284560963913

+3.148865237924

$b = ?$

+2.448722667966

+2.284560963913

+3.148865237924

$c = ?$

0.000000000000

+4.840583005102

+0.271461077276

$V = ?$

74.405135435359

$D = ?$

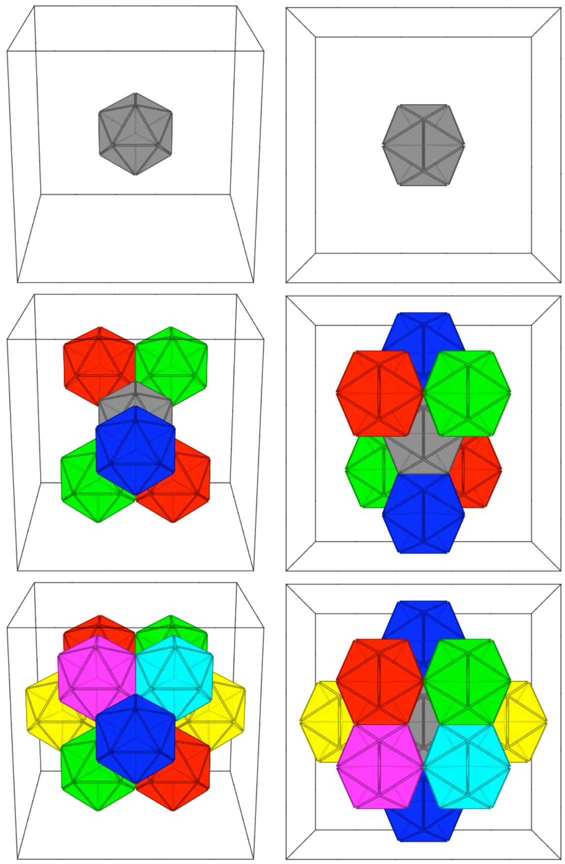
.716796401502

basis $G_6^-\{a,b,c\}$

$a-b$

$b-c$

$c-a$



$$P = \tfrac{1}{2}(1+\sqrt{5}),\ Q = \tfrac{1}{2}(-1+\sqrt{5})$$

$$\langle q_0, q_1, q_2\rangle = \tfrac{1}{3}\langle 4-P^2\sqrt{2}, 4+Q^2\sqrt{2}, 4+\sqrt{10}\rangle \approx \langle 0.099180275452, 1.513393837825, 2.387425886723\rangle$$
$$\langle r_0, r_1, r_2\rangle = \tfrac{1}{3}\langle 4Q-P\sqrt{2}, \sqrt{2}, 4P+Q\sqrt{2}\rangle \approx \langle 0.061296781243, 1.414213562373, 2.448722667966\rangle$$
$$\langle 0, s_1, s_2\rangle = \tfrac{1}{3}\langle 0, 4P-2Q\sqrt{2}, 4Q+2P\sqrt{2}\rangle \approx \langle 0.000000000000, 1.574690619068, 2.349542392514\rangle$$

There are 60 vertices and 62 faces

24 points: $\langle x,y,z\rangle = \mathbf{Z}_3\langle \pm q_0, \pm q_1, \pm q_2\rangle$
24 points: $\langle x,y,z\rangle = \mathbf{Z}_3\langle \pm r_0, \pm r_1, \pm r_2\rangle$
12 points: $\langle x,y,z\rangle = \mathbf{Z}_3\langle 0, \pm s_1, \pm s_2\rangle$
8 triangles: $\langle x,y,z\rangle \cdot \langle \pm 1, \pm 1, \pm 1\rangle = 4$
12 triangles: $\langle x,y,z\rangle \cdot \mathbf{Z}_3\langle 0, \pm P, \pm Q\rangle = 4$
6 rectangles (slivers): $\langle x,y,z\rangle \cdot \mathbf{Z}_3\langle \pm 2, 0, 0\rangle = 2r_2$
24 rectangles (slivers): $\langle x,y,z\rangle \cdot \mathbf{Z}_3\langle \pm 1, \pm Q, \pm P\rangle = 2r_2$
12 pentagons (specks): $\langle x,y,z\rangle \cdot \mathbf{Z}_3\langle 0, \pm Q, \pm 1\rangle = s_2\sqrt{2}$

20 tetrahedra have volume $20 \cdot \frac{8}{3} = \frac{160}{3} \approx 53.333333333333$

$\mathbf{V}_{20}$ has volume

$$\begin{aligned}
&8 \cdot \tfrac{8}{3} + 12 \cdot \tfrac{8}{3} + 6 \cdot \tfrac{1}{27}(-32\sqrt{5} + 56\sqrt{2}) + 24 \cdot \tfrac{1}{27}(-32\sqrt{5} + 56\sqrt{2}) \\
&\quad + 12 \cdot \tfrac{1}{81}(100 + 44\sqrt{5} + 40\sqrt{2} - 80\sqrt{10}) \\
&\quad = \tfrac{1}{27}(1840 - 784\sqrt{5} + 1840\sqrt{2} - 320\sqrt{10}) \approx 62.116548487140
\end{aligned}$$

The relative density of tetrahedra is

$$\tfrac{1}{719}(-12305 + 5487\sqrt{5} + 8970\sqrt{2} - 3805\sqrt{10}) \approx .858601043237$$

8 triangles: $\frac{1}{6} \det \begin{bmatrix} q_0 & q_1 & q_2 \\ q_2 & q_0 & q_1 \\ q_1 & q_2 & q_0 \end{bmatrix} = \frac{8}{3}$

12 triangles: $\frac{1}{6} \det \begin{bmatrix} 0 & s_1 & s_2 \\ +r_1 & r_2 & r_0 \\ -r_1 & r_2 & r_0 \end{bmatrix} = \frac{8}{3}$

6 rectangles (slivers): $2 \cdot \frac{1}{6} \det \begin{bmatrix} r_2 & r_0 & r_1 \\ r_2 & -r_0 & r_1 \\ r_2 & r_0 & -r_1 \end{bmatrix} = \frac{1}{27}\left(56\sqrt{2} - 32\sqrt{5}\right)$

24 rectangles (slivers): $2 \cdot \frac{1}{6} \det \begin{bmatrix} s_1 & s_2 & 0 \\ q_1 & q_2 & q_0 \\ q_2 & q_0 & q_1 \end{bmatrix} + \frac{1}{6} \det \begin{bmatrix} s_1 & s_2 & 0 \\ q_2 & q_0 & q_1 \\ r_2 & r_0 & r_1 \end{bmatrix} = \frac{1}{27}\left(56\sqrt{2} - 32\sqrt{5}\right)$

12 pentagons (specks): $5 \cdot \frac{1}{6} \det \begin{bmatrix} s_1 & s_2 & 0 \\ r_1 & r_2 & -r_0 \\ r_1 & r_2 & +r_0 \end{bmatrix} + 2 \cdot \frac{1}{6} \det \begin{bmatrix} s_1 & s_2 & 0 \\ r_1 & r_2 & r_0 \\ q_1 & q_2 & q_0 \end{bmatrix}$
$= \frac{1}{81}\left(100 + 44\sqrt{5} + 40\sqrt{2} - 80\sqrt{10}\right)$

The difference body is $2\mathbf{V}_{20}$

The lattice basis is $G_6^- \{a, b, c\}$

$$\text{The intersections are } \begin{cases} a \cdot \langle -1, 1, 1 \rangle = 8 \\ b \cdot \langle 1, 1, 1 \rangle = 8 \\ c \cdot \langle 0, P, Q \rangle = 8 \\ (a-b) \cdot \langle -1, 0, 0 \rangle = 2r_2 \\ (b-c) \cdot \langle 1, -1, 1 \rangle = 8 \\ (c-a) \cdot \langle 1, 1, -1 \rangle = 8 \\ \text{minimum lattice volume} \end{cases}$$

$$\text{The lattice vectors are } \begin{cases} a \approx \langle -2.448722667966, +2.284560963913, +3.148865237924 \rangle \\ b \approx \langle +2.448722667966, +2.284560963913, +3.148865237924 \rangle \\ c \approx \langle 0.000000000000, +4.840583005102, +0.271461077276 \rangle \end{cases}$$

$$\begin{aligned}
a &= \langle -\tfrac{1}{3}(4P + \sqrt{2}\,Q), \tfrac{1}{12}(80 - 24\sqrt{5} + 3\sqrt{2} - \sqrt{10}), \tfrac{1}{12}(8 + 16\sqrt{5} - \sqrt{2} - \sqrt{10}) \rangle \\
b &= \langle \tfrac{1}{3}(4P + \sqrt{2}\,Q), \tfrac{1}{12}(80 - 24\sqrt{5} + 3\sqrt{2} - \sqrt{10}), \tfrac{1}{12}(8 + 16\sqrt{5} - \sqrt{2} - \sqrt{10}) \rangle \\
c &= \langle 0, \tfrac{1}{15}(130 - 26\sqrt{5} + 5\sqrt{2} - 2\sqrt{10}), \tfrac{1}{30}(-140 + 68\sqrt{5} - 5\sqrt{2} + \sqrt{10}) \rangle
\end{aligned}$$

The lattice volume is $V \approx 74.405135435359$

$$V = \tfrac{1}{135}(-5080 + 6544\sqrt{5} - 6385\sqrt{2} + 3011\sqrt{10})$$

The packing density is $D \approx .716796401502$

$$D = \tfrac{1}{10403529603}(-12681540400 + 9187359120\sqrt{5} + 10911615300\sqrt{2} - 5007833780\sqrt{10})$$

Chapter 3
Cubic packings

Construct a cluster $\mathbf{X}$ with 'cubic' symmetry $\mathbf{S}_4$

$$\mathbf{Z}_3 = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \right\} \qquad \text{rotation by } \{-\tfrac{2}{3}\pi, 0, +\tfrac{2}{3}\pi\} \text{ about } \langle 1,1,1 \rangle$$

$$\mathbf{Z}_2^2 = \left\{ \begin{bmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & +1 \end{bmatrix}, \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{bmatrix} \right\}$$

$\mathbf{B}_1 = $ (1 body)
1 tetrahedron: $\{\langle +1,+1,+1\rangle, \langle +1,-1,-1\rangle, \langle -1,+1,-1\rangle, \langle -1,-1,+1\rangle\}$

$\mathbf{B}_5 = \mathbf{B}_1 \cup$ (4 bodies)
4 tetrahedra: $\mathbf{Z}_2^2\{\langle +1,-1,-1\rangle, \langle -1,+1,-1\rangle, \langle -1,-1,+1\rangle, \langle -\tfrac{5}{3},-\tfrac{5}{3},-\tfrac{5}{3}\rangle\}$

$\mathbf{B}_{17} = \mathbf{B}_5 \cup$ (12 bodies)
12 tetrahedra: $\mathbf{A}_4\{\langle -1,+1,-1\rangle, \langle -1,-1,+1\rangle, \langle -\tfrac{5}{3},-\tfrac{5}{3},-\tfrac{5}{3}\rangle, \langle -\tfrac{31}{9},-\tfrac{1}{9},-\tfrac{1}{9}\rangle\}$

$\mathbf{B}^*_{17} = \mathbf{B}_5 \cup$ (12 bodies)
12 tetrahedra: $\mathbf{A}_4\{\langle -1,+1,-1\rangle, \langle -1,-1,+1\rangle, \langle -1-\sqrt{6},0,0\rangle, \langle -1-\tfrac{1}{3}\sqrt{6},-\tfrac{2}{3}\sqrt{6},-\tfrac{2}{3}\sqrt{6}\rangle\}$

$\mathbf{B}_{41} = \mathbf{B}_{17} \cup$ (24 bodies)
12 tetrahedra: $\mathbf{A}_4\{\langle -1,-1,+1\rangle, \langle -\tfrac{5}{3},-\tfrac{5}{3},-\tfrac{5}{3}\rangle, \langle -\tfrac{31}{9},-\tfrac{1}{9},-\tfrac{1}{9}\rangle, \langle -\tfrac{83}{27},-\tfrac{77}{27},+\tfrac{13}{27}\rangle\}$
12 tetrahedra: $\mathbf{A}_4\{\langle -1,+1,-1\rangle, \langle -\tfrac{5}{3},-\tfrac{5}{3},-\tfrac{5}{3}\rangle, \langle -\tfrac{31}{9},-\tfrac{1}{9},-\tfrac{1}{9}\rangle, \langle -\tfrac{83}{27},+\tfrac{13}{27},-\tfrac{77}{27}\rangle\}$

$\mathbf{B}_{57} = \mathbf{B}_{41} \cup$ (16 bodies)
4 tetrahedra: $\mathbf{Z}_2^2\{\langle 1,1,1\rangle, \langle 1,3,3\rangle, \langle 3,1,3\rangle, \langle 3,3,1\rangle\}$
12 tetrahedra: $\mathbf{A}_4^4\{\langle 1,1,1\rangle, \langle 3,1,3\rangle, \langle 3,3,1\rangle, \langle \tfrac{11}{3},\tfrac{1}{3},\tfrac{1}{3}\rangle\}$

$\mathbf{V}_{20} = $ (20 bodies)
8 tetrahedra: $\pm\mathbf{Z}_2^2\{\langle 0,0,0\rangle, \langle q_0,q_1,q_2\rangle, \langle q_2,q_0,q_1\rangle, \langle q_1,q_2,q_0\rangle\}$
12 tetrahedra: $\mathbf{A}_4\{\langle 0,0,0\rangle, \langle 0,s_1,s_2\rangle, \langle r_1,r_2,r_0\rangle, \langle -r_1,r_2,r_0\rangle\}$

$P = \tfrac{1}{2}(1+\sqrt{5}),\ Q = \tfrac{1}{2}(-1+\sqrt{5})$
$\langle q_0,q_1,q_2\rangle = \tfrac{1}{3}\langle 4-P^2\sqrt{2}, 4+Q^2\sqrt{2}, 4+\sqrt{10}\rangle \approx \langle .099180275452, 1.513393837825, 2.387425886723\rangle$
$\langle r_0,r_1,r_2\rangle = \tfrac{1}{3}\langle 4Q-P\sqrt{2}, \sqrt{2}, 4P+Q\sqrt{2}\rangle \approx \langle .061296781243, 1.414213562373, 2.448722667966\rangle$
$\langle 0,s_1,s_2\rangle = \tfrac{1}{3}\langle 0, 4P-2Q\sqrt{2}, 4Q+2P\sqrt{2}\rangle \approx \langle .000000000000, 1.574690619068, 2.349542392514\rangle$

Note: $-\mathbf{V}_{20} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \mathbf{V}_{20}$

Construct a packing with ‘cubic’ symmetry $\mathbf{S}_4$

$C_2^=$ (pure 2-direction) face-centered cubic
There is 1 coset $\{+\mathbf{X}_A\langle 0,0,0\rangle, *, *, *\}$

$C_1^=$ (pure 1-direction) simple cubic
There are 2 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle, *, +\mathbf{X}_C\langle 2a,0,0\rangle, *\}$

$C_1^{\neq}$ (mixed 1-direction) simple cubic
There are 2 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle, *, -\mathbf{X}_C\langle 2a,0,0\rangle, *\}$

$C_3^=$ (pure 3-direction) body-centered cubic
There are 4 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle, +\mathbf{X}_B\langle a,a,+a\rangle, +\mathbf{X}_C\langle 2a,0,0\rangle, +\mathbf{X}_D\langle a,a,-a\rangle\}$

$C_3^{\neq}$ (mixed 3-direction) body-centered cubic
There are 4 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle, -\mathbf{X}_B\langle a,a,+a\rangle, +\mathbf{X}_C\langle 2a,0,0\rangle, -\mathbf{X}_D\langle a,a,-a\rangle\}$

C_3^+ (positive 3-direction) diamond-crystal cubic
There are 2 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle, -\mathbf{X}_B\langle a,a,+a\rangle, *, *\}$

C_3^- (negative 3-direction) diamond-crystal cubic
There are 2 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle, *, *, -\mathbf{X}_D\langle -a,-a,-a\rangle\}$

The lattice basis is $\{\langle 0,2a,2a\rangle, \langle 2a,0,2a\rangle, \langle 2a,2a,0\rangle\}$

Write the intersection equations

Evaluate the lattice vector a

Note: a is the minimum spacing between neighboring clusters

Evaluate the lattice volume $V = \det \begin{bmatrix} 0 & 2a & 2a \\ 2a & 0 & 2a \\ 2a & 2a & 0 \end{bmatrix} = 16a^3$

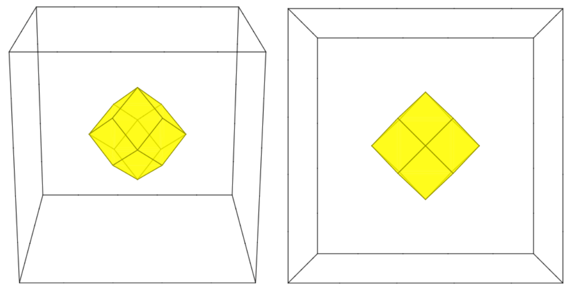
Evaluate the packing density D

Note: Each tetrahedron has volume $\frac{8}{3}$

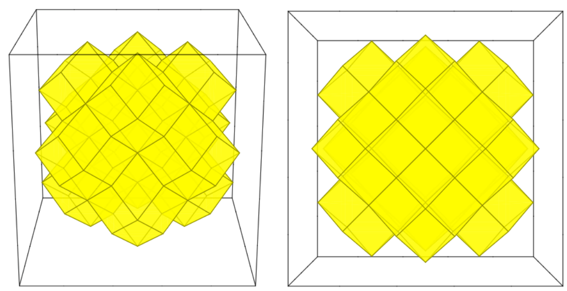
$\mathbf{X}\ C_2^{=}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset

1 0-neighbor $+\mathbf{X}_A\langle 0,0,0\rangle$



12 1-neighbors $+\mathbf{X}_A\langle 2a,2a,0\rangle$

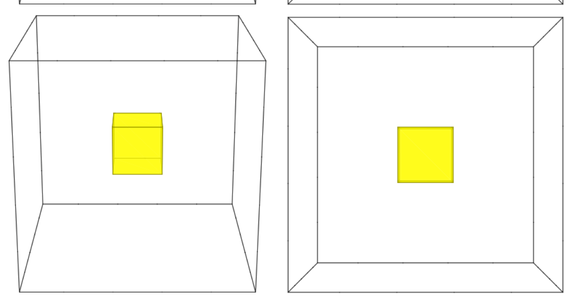


$\mathbf{X}\ C_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

$\mathbf{X}\ C_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets

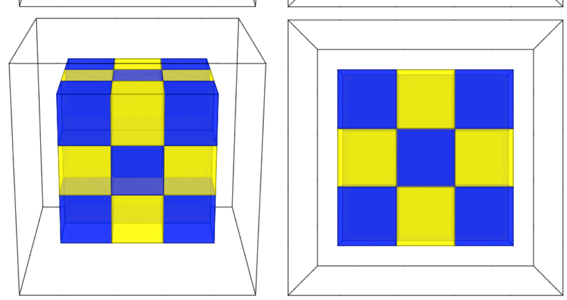
1 0-neighbor $+\mathbf{X}_A\langle 0,0,0\rangle$



6 1-neighbors $\pm\mathbf{X}_C\langle 2a,0,0\rangle$

12 1-neighbors $+\mathbf{X}_A\langle 2a,2a,0\rangle$

8 1-neighbors $\pm\mathbf{X}_C\langle 2a,2a,2a\rangle$

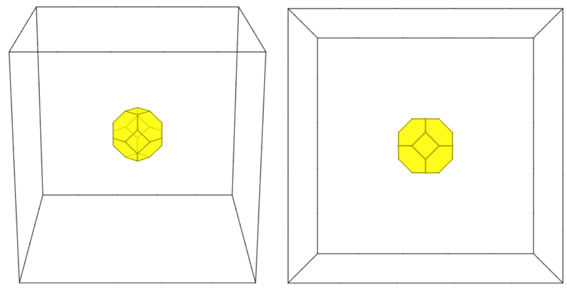


$\mathbf{X}\ C_3^{=}\ \{+\mathbf{X}_A, +\mathbf{X}_B, +\mathbf{X}_C, +\mathbf{X}_D\}$

$\mathbf{X}\ C_3^{\neq}\ \{+\mathbf{X}_A, -\mathbf{X}_B, +\mathbf{X}_C, -\mathbf{X}_D\}$

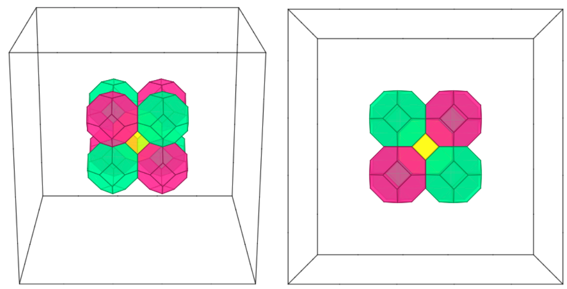
4 cosets

1 0·neighbor $+\mathbf{X}_A\langle 0,0,0\rangle$



4 1·neighbors $\pm\mathbf{X}_B\langle +a,a,a\rangle$

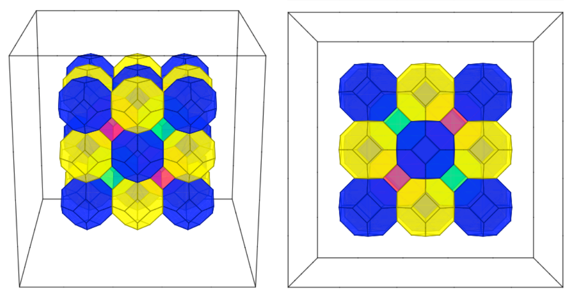
4 1·neighbors $\pm\mathbf{X}_D\langle -a,a,a\rangle$



6 2·neighbors $+\mathbf{X}_C\langle 2a,0,0\rangle$

12 2·neighbors $+\mathbf{X}_A\langle 2a,2a,0\rangle$

8 2·neighbors $+\mathbf{X}_C\langle 2a,2a,2a\rangle$



12 3·neighbors $\pm\mathbf{X}_B\langle -3a,a,a\rangle$

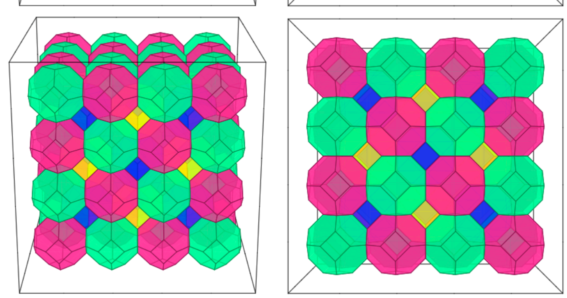
12 3·neighbors $\pm\mathbf{X}_D\langle +3a,a,a\rangle$

12 3·neighbors $\pm\mathbf{X}_B\langle +3a,3a,a\rangle$

12 3·neighbors $\pm\mathbf{X}_D\langle -3a,3a,a\rangle$

4 3·neighbors $\pm\mathbf{X}_B\langle -3a,3a,3a\rangle$

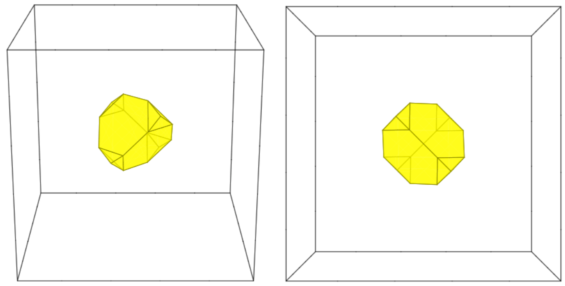
4 3·neighbors $\pm\mathbf{X}_D\langle +3a,3a,3a\rangle$



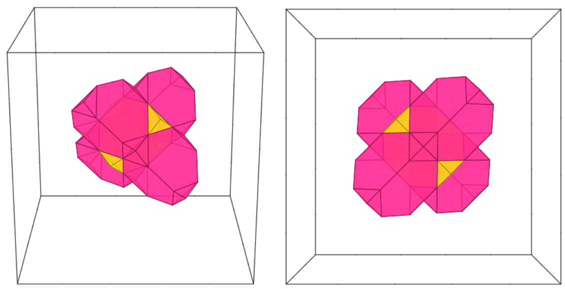
$\mathbf{X}\ C_3^+\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

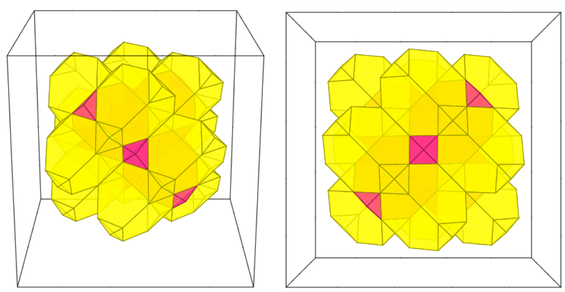
1 0·neighbor $+\mathbf{X}_A\langle 0,0,0\rangle$



4 1·neighbors $-\mathbf{X}_B\langle +a,a,a\rangle$



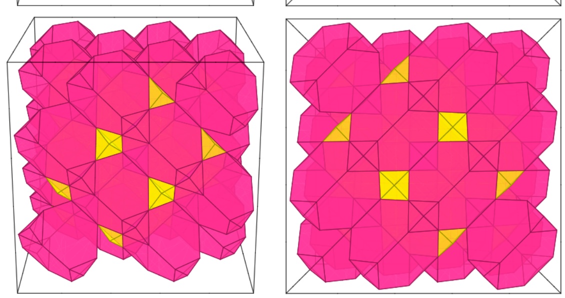
12 2·neighbors $+\mathbf{X}_A\langle 2a,2a,0\rangle$



12 3·neighbors $-\mathbf{X}_B\langle -3a,a,a\rangle$

12 3·neighbors $-\mathbf{X}_B\langle +3a,3a,a\rangle$

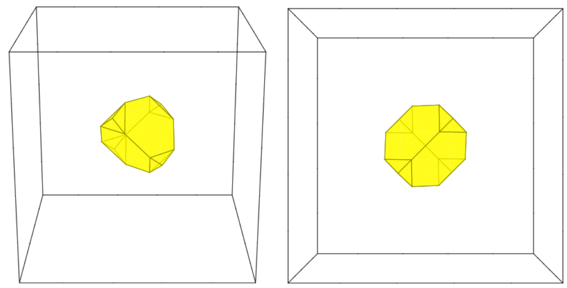
4 3·neighbors $-\mathbf{X}_B\langle -3a,3a,3a\rangle$



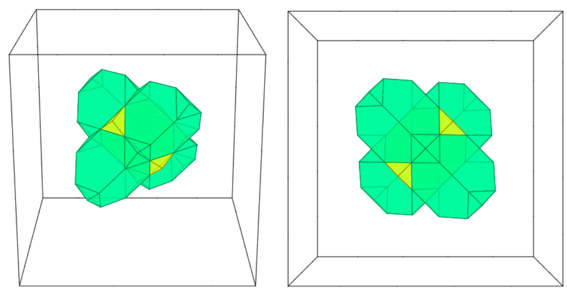
$\mathbf{X}\ C_3^-\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

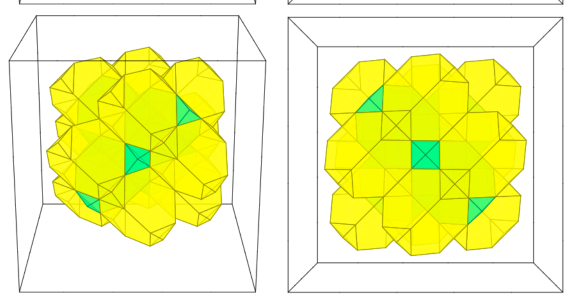
1 0-neighbor $+\mathbf{X}_A\langle 0,0,0\rangle$



4 1-neighbors $-\mathbf{X}_D\langle -a,a,a\rangle$



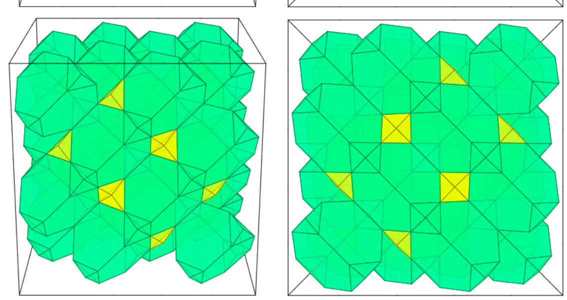
12 2-neighbors $+\mathbf{X}_A\langle 2a,2a,0\rangle$



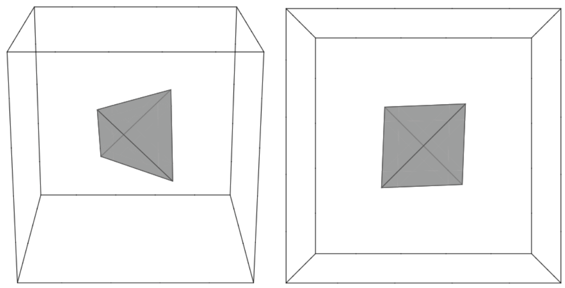
12 3-neighbors $-\mathbf{X}_D\langle +3a,a,a\rangle$

12 3-neighbors $-\mathbf{X}_D\langle -3a,3a,a\rangle$

4 3-neighbors $-\mathbf{X}_D\langle +3a,3a,3a\rangle$



$\mathbf{B}_1$



n cosets

1 tetrahedron

$U = \frac{8}{3}$

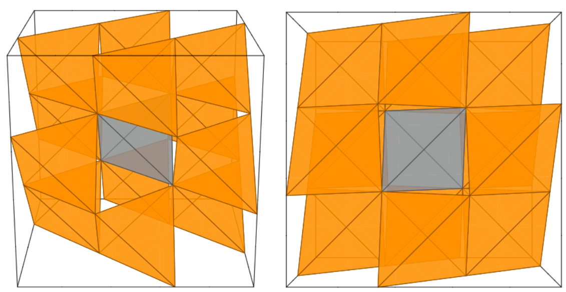
2.666666666667

$V = 16a^3$

$D = n{\cdot}U/V$

basis $\{\langle 0,2a,2a\rangle,\langle 2a,0,2a\rangle,\langle 2a,2a,0\rangle\}$

$\mathbf{B}_1$ $C_2^{=}$ $\{+\mathbf{X}_A,*,*,*\}$



1 coset

1 tetrahedron

$a = 1$

1.000000000000

$V = 16$

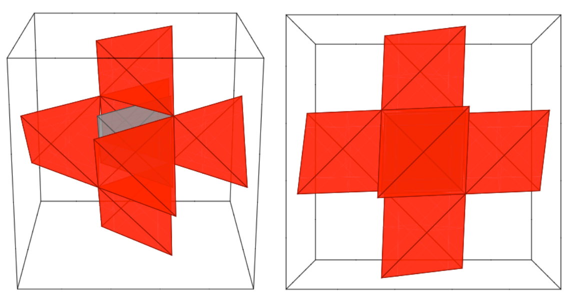
16.000000000000

$D = \frac{1}{6}$

.166666666667

$V_A\langle +1,+1,+1\rangle = V_A\langle -1,-1,+1\rangle + \langle 2a,2a,0\rangle$

$\mathbf{B}_1$ $C_1^{=}$ $\{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$



2 cosets

2 tetrahedra

$a = 1$

1.000000000000

$V = 16$

16.000000000000

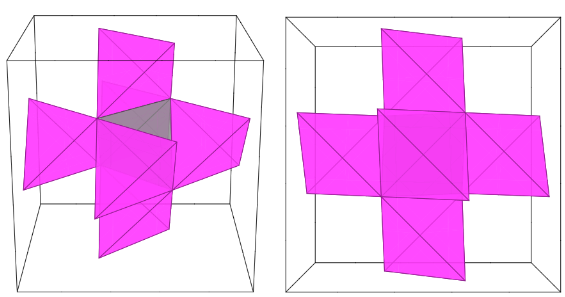
$D = \frac{1}{3}$

.333333333333

$V_A\langle +1,+1,+1\rangle = V_A\langle -1,-1,+1\rangle + \langle 2a,2a,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +1,-1,-1\rangle] = E_C[\langle -1,+1,-1\rangle,\langle -1,-1,+1\rangle] + \langle 2a,0,0\rangle$

$\mathbf{B}_1$ $C_1^{\neq}$ $\{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$



2 cosets

2 tetrahedra

$a = 1$

1.000000000000

$V = 16$

16.000000000000

$D = \frac{1}{3}$

.333333333333

$V_A\langle +1,+1,+1\rangle = V_A\langle -1,-1,+1\rangle + \langle 2a,2a,0\rangle$

$V_A\langle +1,+1,+1\rangle = V_C\langle -1,+1,+1\rangle + \langle 2a,0,0\rangle$

$V_A\langle +1,+1,+1\rangle = V_C\langle -1,-1,-1\rangle + \langle 2a,2a,2a\rangle$

$\mathbf{B}_1\ C_3^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

4 tetrahedra

$a = \frac{4}{3}$

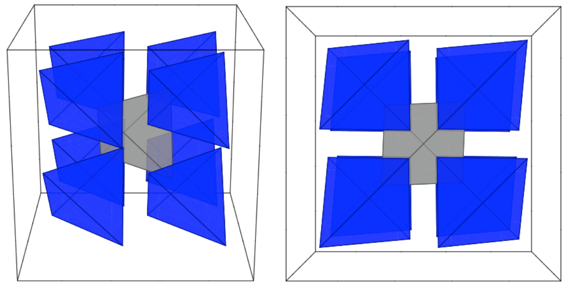
1.333333333333

$V = \frac{1024}{27}$

37.925925925926

$D = \frac{9}{32}$

.281250000000



$V_A\langle+1,+1,+1\rangle = F_B[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle,\langle-1,-1,+1\rangle] + \langle a,a,a\rangle$

$F_A[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle,\langle-1,-1,+1\rangle] = V_D\langle+1,+1,+1\rangle - \langle a,a,a\rangle$

$\mathbf{B}_1\ C_3^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

4 tetrahedra

$a = 2$

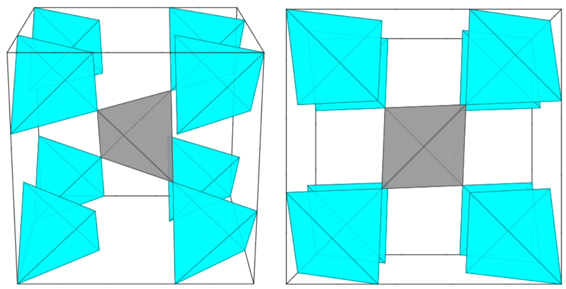
2.000000000000

$V = 128$

128.000000000000

$D = \frac{1}{12}$

.083333333333



$V_A\langle+1,+1,+1\rangle = V_B\langle-1,-1,-1\rangle + \langle a,a,a\rangle$

$\mathbf{B}_1\ C_3^{+}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

2 tetrahedra

$a = 2$

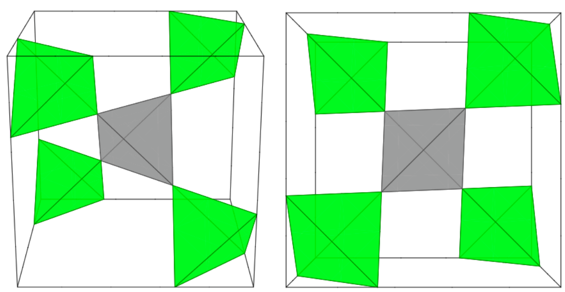
2.000000000000

$V = 128$

128.000000000000

$D = \frac{1}{24}$

.041666666667



$V_A\langle+1,+1,+1\rangle = V_B\langle-1,-1,-1\rangle + \langle a,a,a\rangle$

$\mathbf{B}_1\ C_3^{-}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

2 tetrahedra

$a = 1$

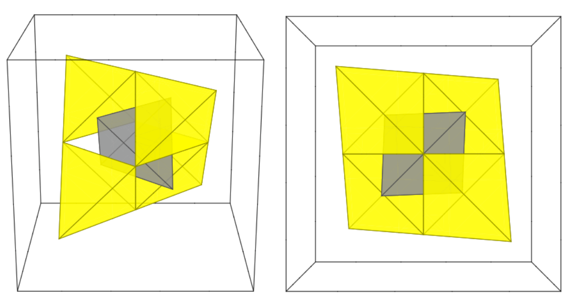
1.000000000000

$V = 16$

16.000000000000

$D = \frac{1}{3}$

.333333333333



$V_A\langle+1,+1,+1\rangle = V_A\langle-1,-1,+1\rangle + \langle 2a,2a,0\rangle$

B_5

n cosets

5 tetrahedra

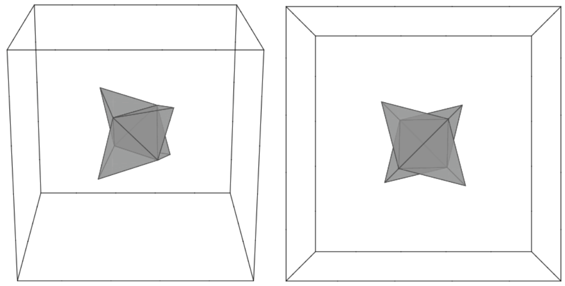
$U = \frac{40}{3}$

13.333333333333

$V = 16a^3$

$D = n{\cdot}U/V$

basis $\{\langle 0,2a,2a\rangle,\langle 2a,0,2a\rangle,\langle 2a,2a,0\rangle\}$



B_5 $C_2^{=}$ $\{+\mathbf{X}_A,*,*,*\}$

1 coset

5 tetrahedra

$a = \frac{5}{3}$

1.666666666667

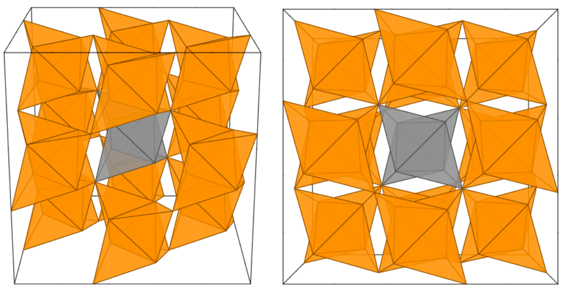
$V = \frac{2000}{27}$

74.074074074074

$D = \frac{9}{50}$

.180000000000

$V_A\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle = V_A\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle + \langle 2a,2a,0\rangle$



B_5 $C_1^{=}$ $\{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets

10 tetrahedra

$a = \frac{5}{3}$

1.666666666667

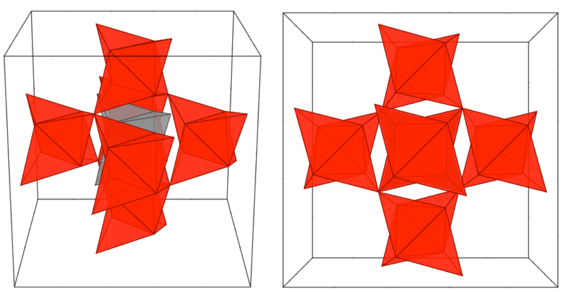
$V = \frac{2000}{27}$

74.074074074074

$D = \frac{9}{25}$

.360000000000

$V_A\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle = V_A\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle + \langle 2a,2a,0\rangle$



B_5 $C_1^{\neq}$ $\{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets

10 tetrahedra

$a = \frac{5}{3}$

1.666666666667

$V = \frac{2000}{27}$

74.074074074074

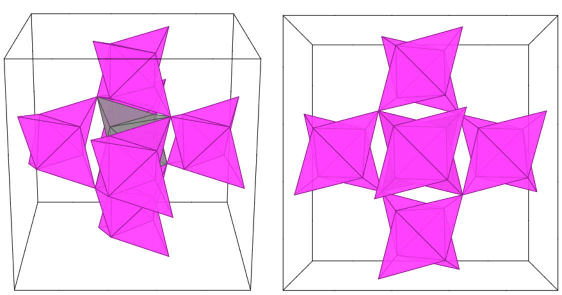
$D = \frac{9}{25}$

.360000000000

$V_A\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle = V_A\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle + \langle 2a,2a,0\rangle$

$V_A\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle = V_C\langle -\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle + \langle 2a,0,0\rangle$

$V_A\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle = V_C\langle +\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle - \langle 2a,2a,2a\rangle$



$\mathbf{B}_5\ C_3^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

20 tetrahedra

$a = \frac{8}{3}$

2.666666666667

$V = \frac{8192}{27}$

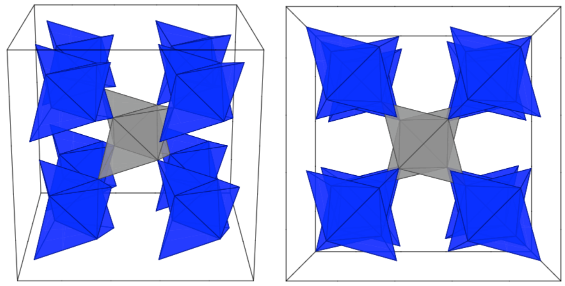
303.407407407407

$D = \frac{45}{256}$

.175781250000

$V_A\langle+1,+1,+1\rangle = V_B\langle-\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle + \langle a,a,a\rangle$

$V_A\langle-\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle = V_D\langle+1,+1,+1\rangle - \langle a,a,a\rangle$



$\mathbf{B}_5\ C_3^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

20 tetrahedra

$a = \frac{10}{3}$

3.333333333333

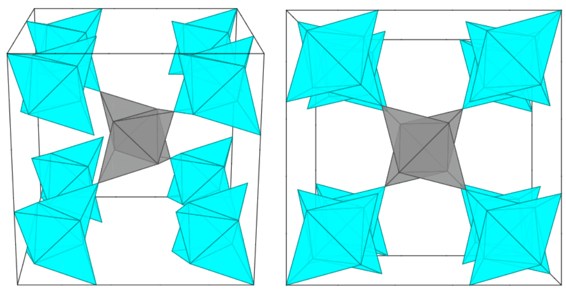
$V = \frac{16000}{27}$

592.592592592593

$D = \frac{9}{100}$

.090000000000

$V_A\langle-\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle = V_D\langle+\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle - \langle a,a,a\rangle$



$\mathbf{B}_5\ C_3^{+}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

10 tetrahedra

$a = 2$

2.000000000000

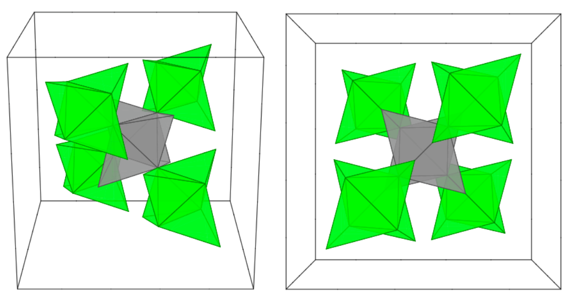
$V = 128$

128.000000000000

$D = \frac{5}{24}$

.208333333333

$V_A\langle+1,+1,+1\rangle = V_B\langle-1,-1,-1\rangle + \langle a,a,a\rangle$



$\mathbf{B}_5\ C_3^{-}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

10 tetrahedra

$a = \frac{10}{3}$

3.333333333333

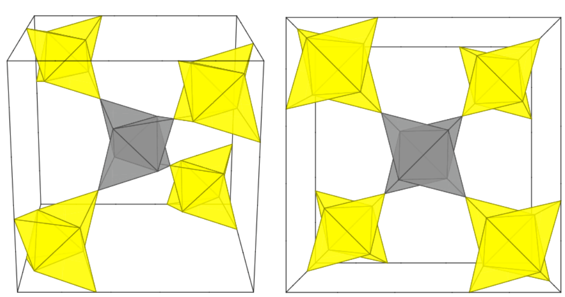
$V = \frac{16000}{27}$

592.592592592593

$D = \frac{9}{200}$

.045000000000

$V_A\langle-\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle = V_D\langle+\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle - \langle a,a,a\rangle$



$\mathbf{B}_{17}$

n cosets

17 tetrahedra

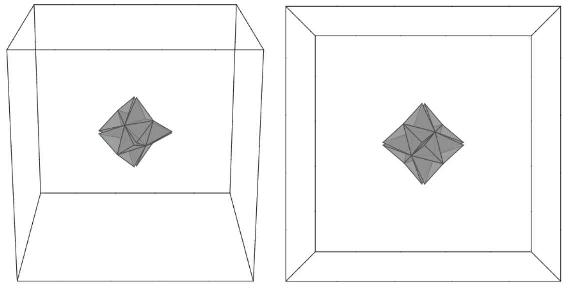
$U = \frac{136}{3}$

45.333333333333

$V = 16a^3$

$D = n{\cdot}U/V$

basis $\{\langle 0,2a,2a\rangle,\langle 2a,0,2a\rangle,\langle 2a,2a,0\rangle\}$



$\mathbf{B}_{17}\ C_2^{=}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset

17 tetrahedra

$a = \frac{16}{9}$

1.777777777777

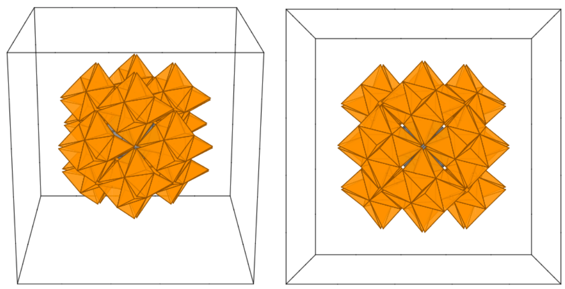
$V = \frac{65536}{729}$

89.898491083676

$D = \frac{4131}{8192}$

.504272460938

$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_A\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle 2a,2a,0\rangle$



$\mathbf{B}_{17}\ C_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets

34 tetrahedra

$a = \frac{16}{5}$

3.200000000000

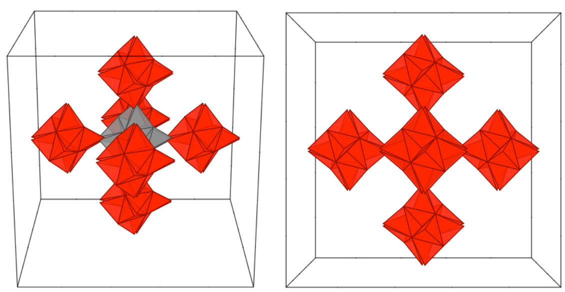
$V = \frac{65536}{125}$

524.288000000000

$D = \frac{2125}{12288}$

.172932942708

$E_A[\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle,\langle +1,+1,+1\rangle] = E_C[\langle -\frac{31}{9},+\frac{1}{9},+\frac{1}{9}\rangle,\langle -1,+1,-1\rangle] + \langle 2a,0,0\rangle$



$\mathbf{B}_{17}\ C_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets

34 tetrahedra

$a = \frac{31}{9}$

3.444444444444

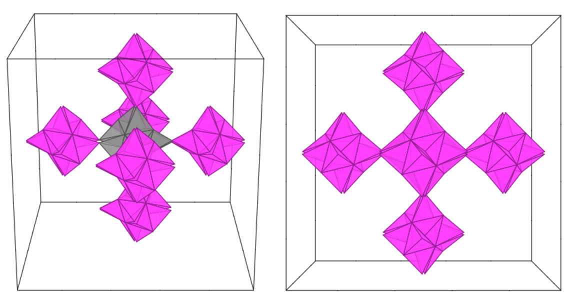
$V = \frac{476656}{729}$

653.849108367627

$D = \frac{4131}{29791}$

.138666040079

$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_C\langle -\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle + \langle 2a,0,0\rangle$



$\mathbf{B}_{17}\ C_3^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = \frac{16}{5}$

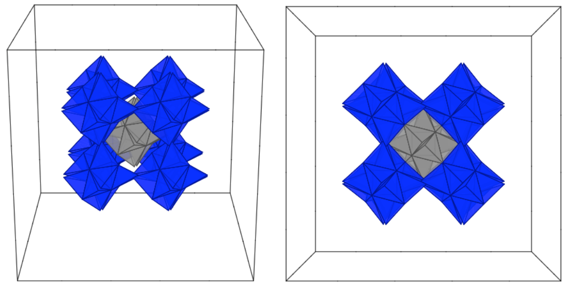
3.200000000000

$V = \frac{65536}{125}$

524.288000000000

$D = \frac{2125}{6144}$

.345865885417



$E_A[\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle,\langle +1,+1,+1\rangle] = E_C[\langle -\frac{31}{9},+\frac{1}{9},+\frac{1}{9}\rangle,\langle -1,+1,-1\rangle] + \langle 2a,0,0\rangle$

$\mathbf{B}_{17}\ C_3^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = \frac{10}{3}$

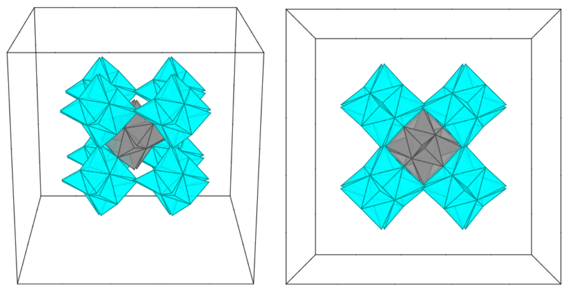
3.333333333333

$V = \frac{16000}{27}$

592.592592592593

$D = \frac{153}{500}$

.306000000000



$V_A\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle = V_D\langle +\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle - \langle a,a,a\rangle$

$\mathbf{B}_{17}\ C_3^{+}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

34 tetrahedra

$a = 2$

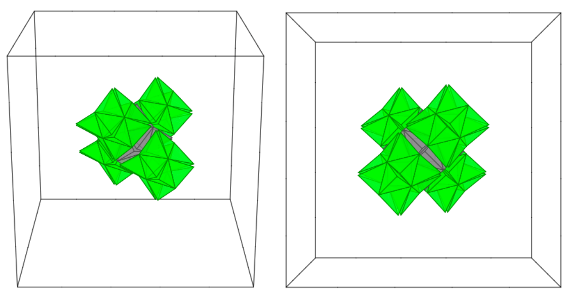
2.000000000000

$V = 128$

128.000000000000

$D = \frac{17}{24}$

.708333333333



$V_A\langle +1,+1,+1\rangle = V_B\langle -1,-1,-1\rangle + \langle a,a,a\rangle$

$\mathbf{B}_{17}\ C_3^{-}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

34 tetrahedra

$a = \frac{10}{3}$

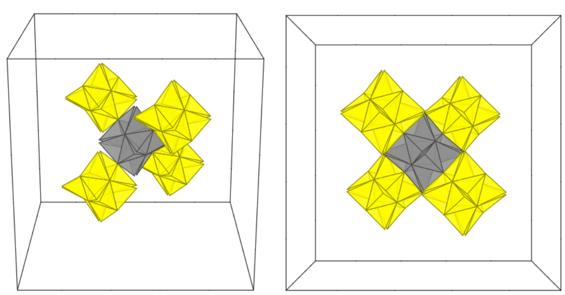
3.333333333333

$V = \frac{16000}{27}$

592.592592592593

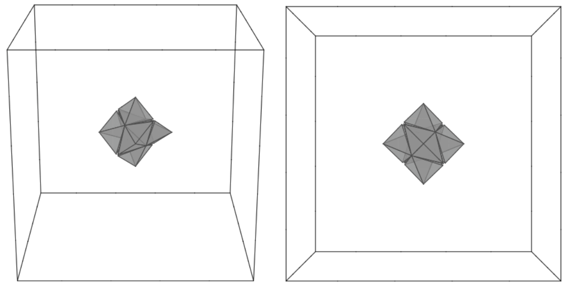
$D = \frac{153}{1000}$

.153000000000



$V_A\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle = V_D\langle +\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle - \langle a,a,a\rangle$

$\mathbf{B}^{*}_{17}$



n cosets

17 tetrahedra

$U = \frac{136}{3}$

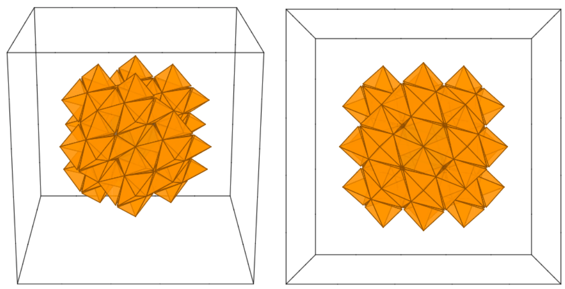
45.333333333333

$V = 16a^3$

$D = n \cdot U / V$

basis $\{\langle 0,2a,2a\rangle,\langle 2a,0,2a\rangle,\langle 2a,2a,0\rangle\}$

$\mathbf{B}^{*}_{17}\ C_2^{=}\ \{+\mathbf{X}_A,*,*,*\}$



1 coset

17 tetrahedra

$a = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

$V = 38+18\sqrt{6}$

82.090815370097

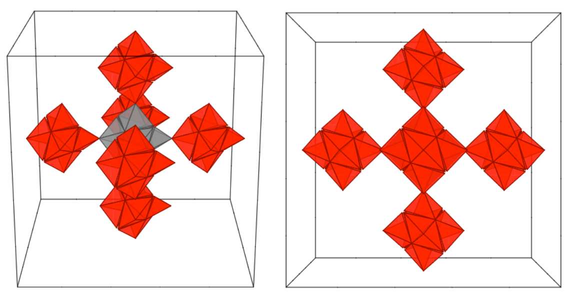
$D = -\frac{1292}{375}+\frac{204}{125}\sqrt{6}$

.552233926889

$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle 0,-1-\sqrt{6},0\rangle + \langle 2a,2a,0\rangle$

$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle -1-\sqrt{6},0,0\rangle + \langle 4a,0,0\rangle$

$\mathbf{B}^{*}_{17}\ C_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$



2 cosets

34 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$V = 304+144\sqrt{6}$

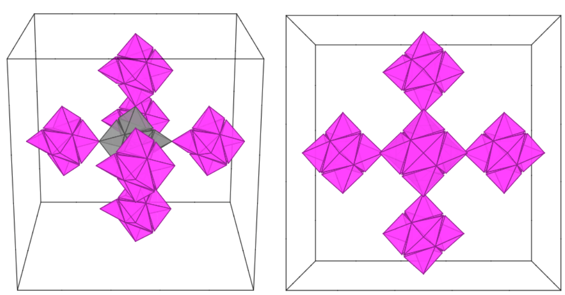
656.726522960778

$D = -\frac{323}{375}+\frac{51}{125}\sqrt{6}$

.138058481722

$V_A\langle +1+\sqrt{6},0,0\rangle = V_C\langle -1-\sqrt{6},0,0\rangle + \langle 2a,0,0\rangle$

$\mathbf{B}^{*}_{17}\ C_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$



2 cosets

34 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$V = 304+144\sqrt{6}$

656.726522960778

$D = -\frac{323}{375}+\frac{51}{125}\sqrt{6}$

.138058481722

$V_A\langle +1+\sqrt{6},0,0\rangle = V_C\langle -1-\sqrt{6},0,0\rangle + \langle 2a,0,0\rangle$

$\mathbf{B}^{*}_{17}\ C_3^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

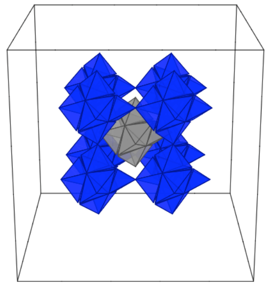
$V = 304+144\sqrt{6}$

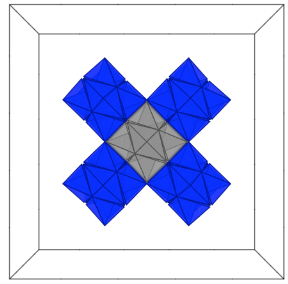
656.726522960778

$D = -\frac{646}{375}+\frac{102}{125}\sqrt{6}$

.276116963444

$V_A\langle +1+\sqrt{6},0,0\rangle = V_C\langle -1-\sqrt{6},0,0\rangle + \langle 2a,0,0\rangle$





$\mathbf{B}^{*}_{17}\ C_3^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

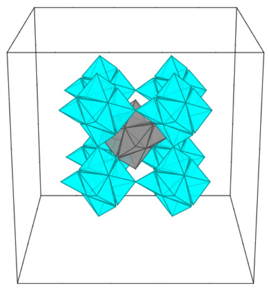
$V = 304+144\sqrt{6}$

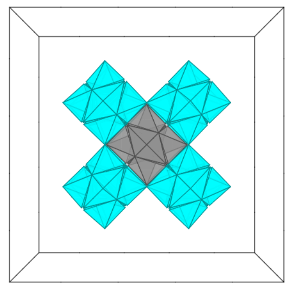
656.726522960778

$D = -\frac{646}{375}+\frac{102}{125}\sqrt{6}$

.276116963444

$V_A\langle +1+\sqrt{6},0,0\rangle = V_C\langle -1-\sqrt{6},0,0\rangle + \langle 2a,0,0\rangle$





$\mathbf{B}^{*}_{17}\ C_3^{+}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

34 tetrahedra

$a = 2$

2.000000000000

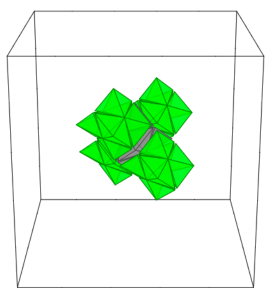
$V = 128$

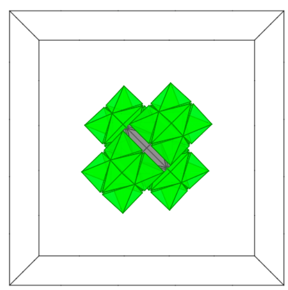
128.000000000000

$D = \frac{17}{24}$

.708333333333

$V_A\langle +1,+1,+1\rangle = V_B\langle -1,-1,-1\rangle + \langle a,a,a\rangle$





$\mathbf{B}^{*}_{17}\ C_3^{-}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

34 tetrahedra

$a = \frac{10}{3}$

3.333333333333

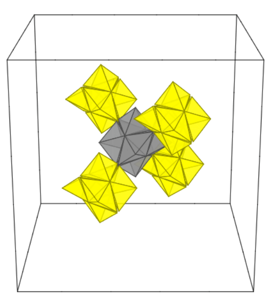
$V = \frac{16000}{27}$

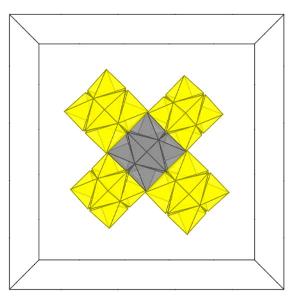
592.592592592593

$D = \frac{153}{1000}$

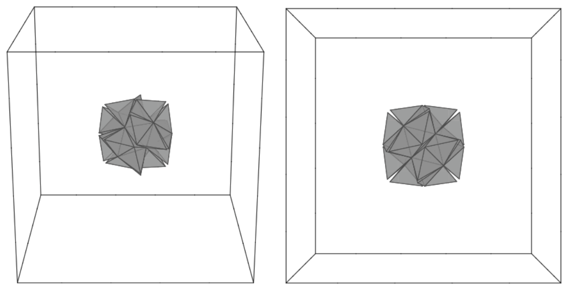
.153000000000

$V_A\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle = V_D\langle +\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle - \langle a,a,a\rangle$





$\mathbf{B}_{41}$



n cosets

41 tetrahedra

$U = \frac{328}{3}$

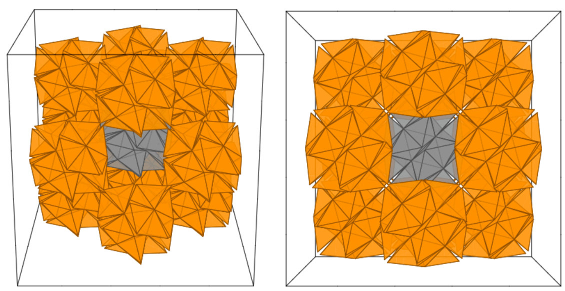
109.333333333333

$V = 16a^3$

$D = n{\cdot}U/V$

basis $\{\langle 0,2a,2a\rangle,\langle 2a,0,2a\rangle,\langle 2a,2a,0\rangle\}$

$\mathbf{B}_{41}\ C_2^{=}\ \{+\mathbf{X}_A,*,*,*\}$



1 coset

41 tetrahedra

$a = \frac{80}{27}$

2.962962962963

$V = \frac{8192000}{19683}$

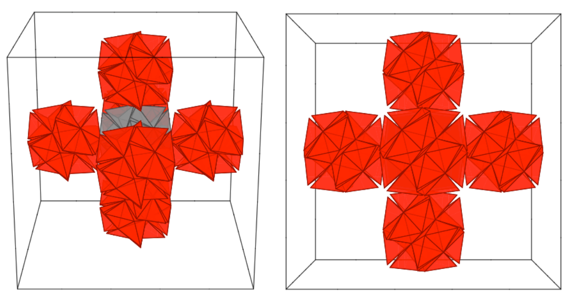
416.196717979983

$D = \frac{269001}{1024000}$

.262696289063

$V_A\langle +\frac{83}{27},+\frac{77}{27},+\frac{13}{27}\rangle = V_A\langle -\frac{77}{27},-\frac{83}{27},+\frac{13}{27}\rangle + \langle 2a,2a,0\rangle$

$\mathbf{B}_{41}\ C_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$



2 cosets

82 tetrahedra

$a = \frac{27}{8}$

3.375000000000

$V = \frac{19683}{32}$

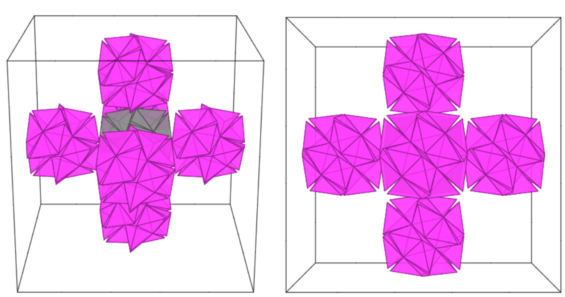
615.093750000000

$D = \frac{20992}{59049}$

.355501363275

$E_A[\langle +\frac{83}{27},+\frac{77}{27},+\frac{13}{27}\rangle,\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle] = E_C[\langle -\frac{83}{27},+\frac{77}{27},-\frac{13}{27}\rangle,\langle -\frac{31}{9},+\frac{1}{9},+\frac{1}{9}\rangle] + \langle 2a,0,0\rangle$

$\mathbf{B}_{41}\ C_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$



2 cosets

82 tetrahedra

$a = \frac{31}{9}$

3.444444444444

$V = \frac{476656}{729}$

653.849108367627

$D = \frac{9963}{29791}$

.334429861368

$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_C\langle -\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle + \langle 2a,0,0\rangle$

$\mathbf{B}_{41}\ C_3^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

164 tetrahedra

$a = \frac{7084}{2025}$

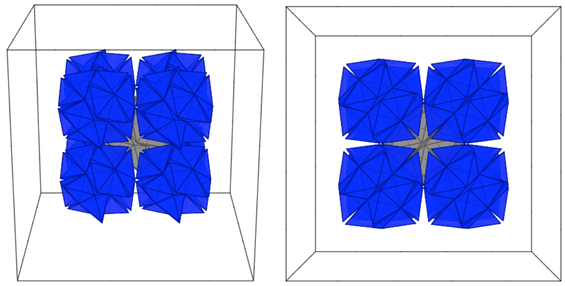
3.498271604938

$V = \frac{5687948299264}{8303765625}$

684.984205495805

$D = \frac{113484796875}{177748384352}$

.638457543728



$V_A\langle +\frac{83}{27},+\frac{77}{27},+\frac{13}{27}\rangle = F_B[\langle +\frac{13}{27},-\frac{77}{27},-\frac{83}{27}\rangle,\langle -\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle,\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle a,a,a\rangle$

$\mathbf{B}_{41}\ C_3^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

164 tetrahedra

$a = \frac{246}{71}$

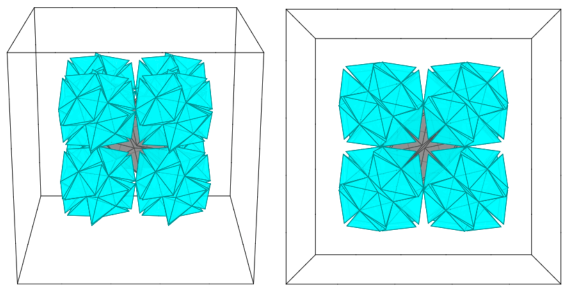
3.464788732394

$V = \frac{238190976}{357911}$

665.503368155771

$D = \frac{357911}{544644}$

.657146686643



$E_A[\langle +\frac{83}{27},+\frac{77}{27},+\frac{13}{27}\rangle,\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle] = E_B[\langle -\frac{13}{27},-\frac{77}{27},-\frac{83}{27}\rangle,\langle +\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle] + \langle a,a,a\rangle$

$\mathbf{B}_{41}\ C_3^{+}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

82 tetrahedra

$a = \frac{246}{71}$

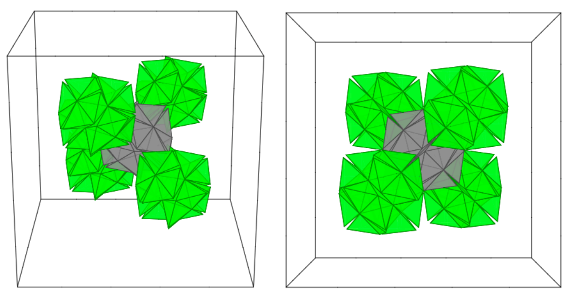
3.464788732394

$V = \frac{238190976}{357911}$

665.503368155771

$D = \frac{357911}{1089288}$

.328573343322



$E_A[\langle +\frac{83}{27},+\frac{77}{27},+\frac{13}{27}\rangle,\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle] = E_B[\langle -\frac{13}{27},-\frac{77}{27},-\frac{83}{27}\rangle,\langle +\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle] + \langle a,a,a\rangle$

$\mathbf{B}_{41}\ C_3^{-}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

82 tetrahedra

$a = \frac{10}{3}$

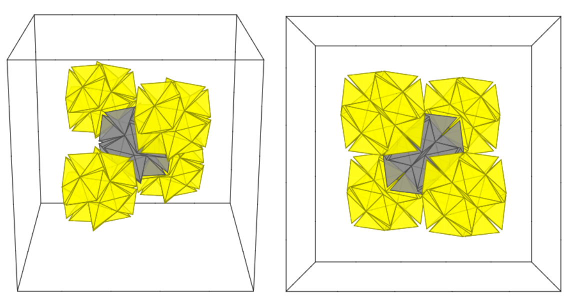
3.333333333333

$V = \frac{16000}{27}$

592.592592592593

$D = \frac{369}{1000}$

.369000000000



$V_A\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle = V_D\langle +\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle - \langle a,a,a\rangle$

$\mathbf{B}_{57}$

n cosets

57 tetrahedra

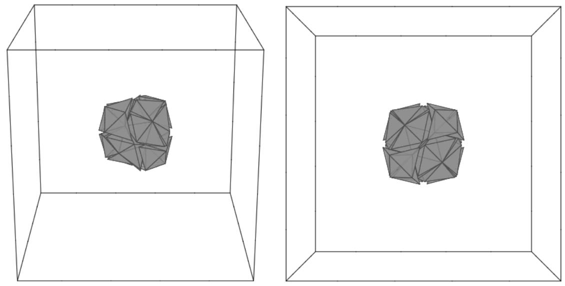
$U = 152$

152.000000000000

$V = 16a^3$

$D = n{\cdot}U/V$

basis $\{\langle 0,2a,2a\rangle,\langle 2a,0,2a\rangle,\langle 2a,2a,0\rangle\}$



$\mathbf{B}_{57}\ C_2^{=}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset

57 tetrahedra

$a = 3$

3.000000000000

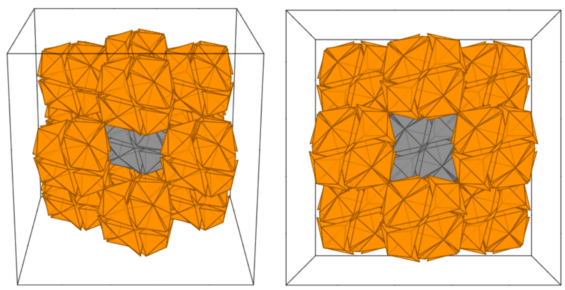
$V = 432$

432.000000000000

$D = \frac{19}{54}$

.351851851852

$V_A\langle +3,+3,+1\rangle = V_A\langle -3,-3,+1\rangle + \langle 2a,2a,0\rangle$



$\mathbf{B}_{57}\ C_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets

114 tetrahedra

$a = \frac{24}{7}$

3.428571428571

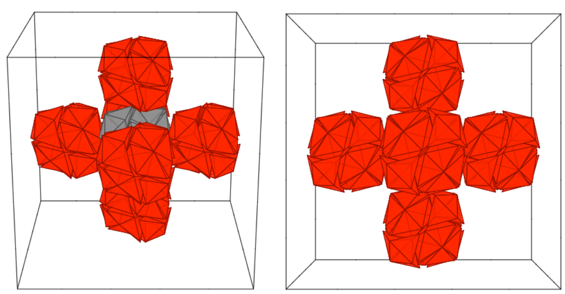
$V = \frac{221184}{343}$

644.851311953353

$D = \frac{6517}{13824}$

.471426504630

$V_A\langle +\frac{11}{3},+\frac{1}{3},+\frac{1}{3}\rangle = E_C[\langle -\frac{31}{9},+\frac{1}{9},+\frac{1}{9}\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle 2a,0,0\rangle$



$\mathbf{B}_{57}\ C_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets

114 tetrahedra

$a = \frac{11}{3}$

3.666666666667

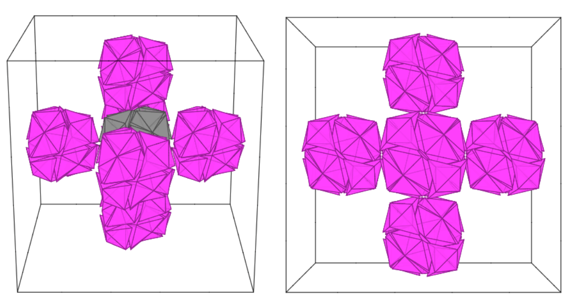
$V = \frac{21296}{27}$

788.740740740741

$D = \frac{513}{1331}$

.385424492863

$V_A\langle +\frac{11}{3},+\frac{1}{3},+\frac{1}{3}\rangle = V_C\langle -\frac{11}{3},+\frac{1}{3},+\frac{1}{3}\rangle + \langle 2a,0,0\rangle$



$\mathbf{B}_{57}\ C_3^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

228 tetrahedra

$a = \frac{376}{93}$

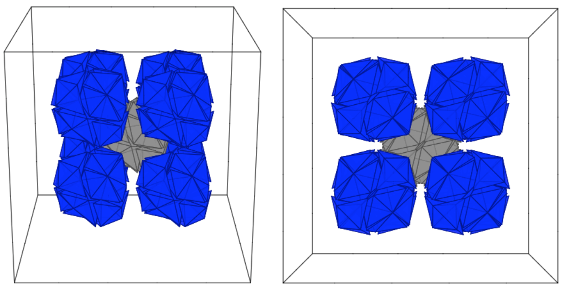
4.043010752688

$V = \frac{850518016}{804357}$

1057.388716701664

$D = \frac{15282783}{26578688}$

.575001407142



$E_A[\langle +3,+3,+1\rangle,\langle +3,+1,+3\rangle] = E_B[\langle +\frac{13}{27},-\frac{77}{27},-\frac{83}{27}\rangle,\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle a,a,a\rangle$

$E_A[\langle +\frac{13}{27},-\frac{77}{27},-\frac{83}{27}\rangle,\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] = E_D[\langle +3,+3,+1\rangle,\langle +3,+1,+3\rangle] - \langle a,a,a\rangle$

$\mathbf{B}_{57}\ C_3^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

228 tetrahedra

$a = \frac{14}{3}$

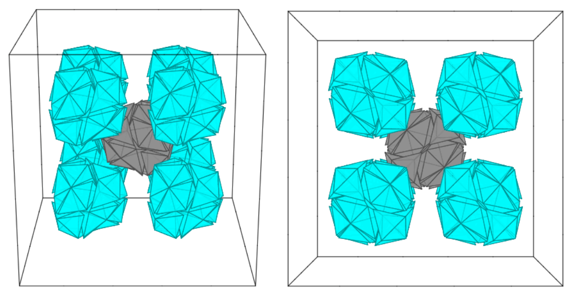
4.666666666667

$V = \frac{43904}{27}$

1626.074074074074

$D = \frac{513}{1372}$

.373906705539



$F_A[\langle +3,+3,+1\rangle,\langle +3,+1,+3\rangle,\langle +1,+3,+3\rangle] = F_B[\langle -1,-3,-3\rangle,\langle -3,-1,-3\rangle,\langle -3,-3,-1\rangle] + \langle a,a,a\rangle$

$\mathbf{B}_{57}\ C_3^{+}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

114 tetrahedra

$a = \frac{14}{3}$

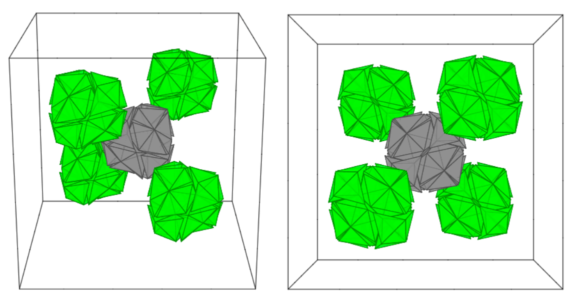
4.666666666667

$V = \frac{43904}{27}$

1626.074074074074

$D = \frac{513}{2744}$

.186953352770



$F_A[\langle +3,+3,+1\rangle,\langle +3,+1,+3\rangle,\langle +1,+3,+3\rangle] = F_B[\langle -1,-3,-3\rangle,\langle -3,-1,-3\rangle,\langle -3,-3,-1\rangle] + \langle a,a,a\rangle$

$\mathbf{B}_{57}\ C_3^{-}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

114 tetrahedra

$a = \frac{10}{3}$

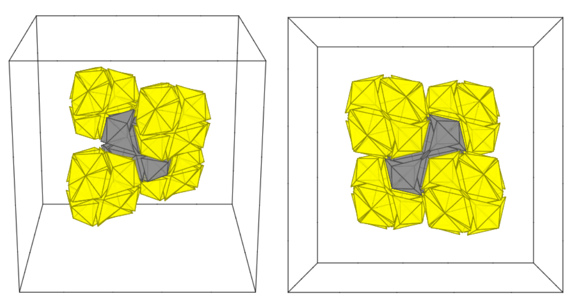
3.333333333333

$V = \frac{16000}{27}$

592.592592592593

$D = \frac{513}{1000}$

.513000000000



$V_A\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle = V_D\langle +\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle - \langle a,a,a\rangle$

$\mathbf{V}_{20}$

n cosets

20 tetrahedra

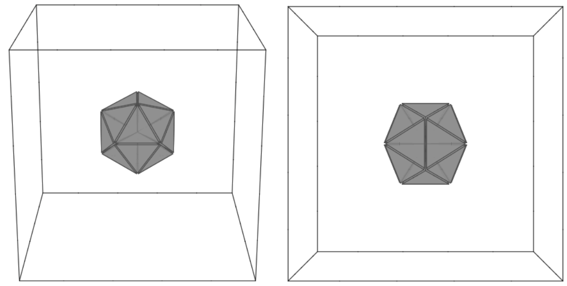
$U = \frac{160}{3}$

53.333333333333

$V = 16a^3$

$D = n{\cdot}U/V$

basis $\{\langle 0,2a,2a\rangle,\langle 2a,0,2a\rangle,\langle 2a,2a,0\rangle\}$



$\mathbf{B}_1\ C_2^{=}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset

20 tetrahedra

$a = \frac{4}{5}\sqrt{5}$

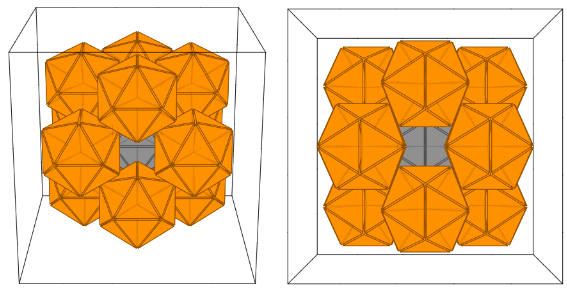
1.788854382000

$V = \frac{1024}{25}\sqrt{5}$

91.589344358391

$D = \frac{25}{96}\sqrt{5}$

.582309369141



$F_A[\langle +s_1,+s_2,0\rangle,\langle +r_2,+r_0,+r_1\rangle,\langle +r_2,+r_0,-r_1\rangle] = F_A[\langle -s_1,-s_2,0\rangle,\langle -r_2,-r_0,+r_1\rangle,\langle -r_2,-r_0,-r_1\rangle] + \langle 2a,2a,0\rangle$

$\langle 0,s_1,s_2\rangle = \langle 0,\frac{2}{3}+\frac{2}{3}\sqrt{5}+\frac{1}{3}\sqrt{2}-\frac{1}{3}\sqrt{10},-\frac{2}{3}+\frac{2}{3}\sqrt{5}+\frac{1}{3}\sqrt{2}+\frac{1}{3}\sqrt{10}\rangle$

$\langle$.000000000000,1.574690619068,2.349

$\mathbf{V}_{20}\ C_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets

40 tetrahedra

$a = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

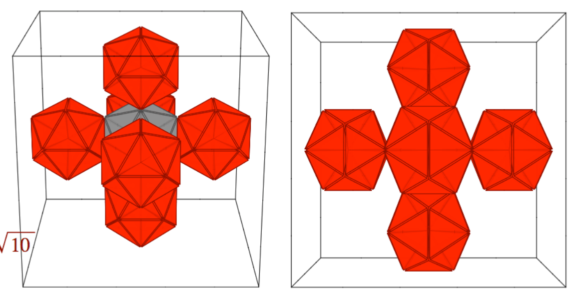
2.448722667966

$V = \frac{1856}{27}+\frac{1216}{27}\sqrt{5}+\frac{320}{27}\sqrt{2}+\frac{416}{27}\sqrt{10}$

234.930166934414

$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5}+\frac{15350}{3}\sqrt{2}-\frac{6865}{3}\sqrt{10}$

.454035631348



$V_A\langle +r_2,+r_0,+r_1\rangle = V_C\langle -r_2,+r_0,+r_1\rangle + \langle 2a,0,0\rangle$

$\langle r_0,r_1,r_2\rangle = \langle -\frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}-\frac{1}{6}\sqrt{10},\sqrt{2},\frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}\rangle$

$\langle$.061296781243,1.414213562373,2.448

$\mathbf{V}_{20}\ C_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets

40 tetrahedra

$a = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

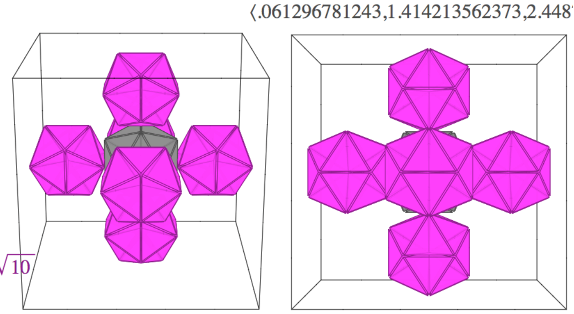
2.448722667966

$V = \frac{1856}{27}+\frac{1216}{27}\sqrt{5}+\frac{320}{27}\sqrt{2}+\frac{416}{27}\sqrt{10}$

234.930166934414

$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5}+\frac{15350}{3}\sqrt{2}-\frac{6865}{3}\sqrt{10}$

.454035631348



$E_A[\langle +r_2,+r_0,+r_1\rangle,\langle +r_2,+r_0,-r_1\rangle] = E_C[\langle -r_2,+r_1,+r_0\rangle,\langle -r_2,-r_1,+r_0\rangle] + \langle 2a,0,0\rangle$

$\mathbf{V}_{20}\ C_3^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{8}{3}$

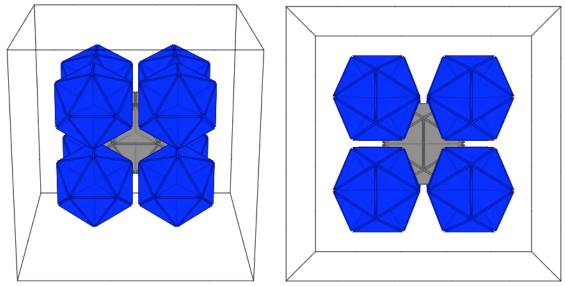
2.666666666667

$V = \frac{8192}{27}$

303.407407407407

$D = \frac{45}{64}$

.703125000000



$F_A[\langle+q_2,+q_0,+q_1\rangle,\langle+q_1,+q_2,+q_0\rangle,\langle+q_0,+q_1,+q_2\rangle] = F_B[\langle-q_0,-q_1,-q_2\rangle,\langle-q_1,-q_2,-q_0\rangle,\langle-q_2,-q_0,-q_1\rangle] + \langle a,a,a\rangle$

$F_A[\langle-q_2,-q_0,-q_1\rangle,\langle-q_1,-q_2,-q_0\rangle,\langle-q_0,-q_1,-q_2\rangle] = F_D[\langle+q_0,+q_1,+q_2\rangle,\langle+q_1,+q_2,+q_0\rangle,\langle+q_2,+q_0,+q_1\rangle] - \langle a,a,a\rangle$

$\mathbf{V}_{20}\ C_3^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{8}{3}$

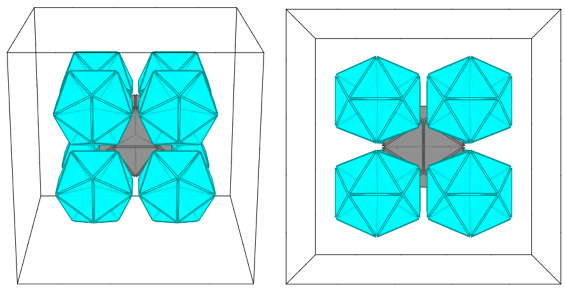
2.666666666667

$V = \frac{8192}{27}$

303.407407407407

$D = \frac{45}{64}$

.703125000000



$F_A[\langle+q_2,+q_0,+q_1\rangle,\langle+q_1,+q_2,+q_0\rangle,\langle+q_0,+q_1,+q_2\rangle] = F_B[\langle-q_0,-q_2,-q_1\rangle,\langle-q_1,-q_0,-q_2\rangle,\langle-q_2,-q_1,-q_0\rangle] + \langle a,a,a\rangle$

$F_A[\langle-q_2,-q_0,-q_1\rangle,\langle-q_1,-q_2,-q_0\rangle,\langle-q_0,-q_1,-q_2\rangle] = F_D[\langle+q_0,+q_2,+q_1\rangle,\langle+q_1,+q_0,+q_2\rangle,\langle+q_2,+q_1,+q_0\rangle] - \langle a,a,a\rangle$

$\mathbf{V}_{20}\ C_3^{+}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

40 tetrahedra

$a = \frac{8}{3}$

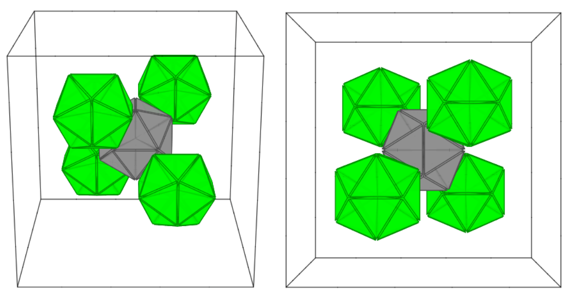
2.666666666667

$V = \frac{8192}{27}$

303.407407407407

$D = \frac{45}{128}$

.351562500000



$F_A[\langle+q_2,+q_0,+q_1\rangle,\langle+q_1,+q_2,+q_0\rangle,\langle+q_0,+q_1,+q_2\rangle] = F_B[\langle-q_0,-q_2,-q_1\rangle,\langle-q_1,-q_0,-q_2\rangle,\langle-q_2,-q_1,-q_0\rangle] + \langle a,a,a\rangle$

$\langle q_0,q_1,q_2\rangle = \langle \frac{4}{3}-\frac{1}{2}\sqrt{2}-\frac{1}{6}\sqrt{10},\frac{4}{3}+\frac{1}{2}\sqrt{2}-\frac{1}{6}\sqrt{10},\frac{4}{3}+\frac{1}{3}\sqrt{10}\rangle$

$\langle .099180275452, 1.513393837825, 2.387425886723\rangle$

$\mathbf{V}_{20}\ C_3^{-}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

40 tetrahedra

$a = \frac{8}{3}$

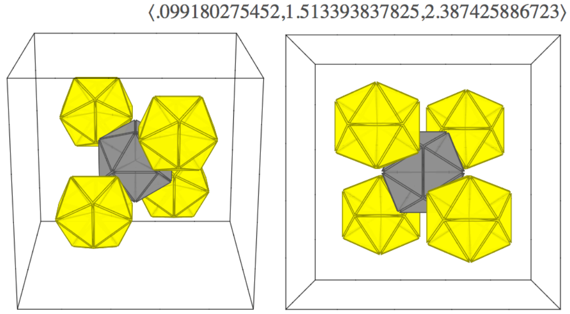
2.666666666667

$V = \frac{8192}{27}$

303.407407407407

$D = \frac{45}{128}$

.351562500000



$F_A[\langle-q_2,-q_0,-q_1\rangle,\langle-q_1,-q_2,-q_0\rangle,\langle-q_0,-q_1,-q_2\rangle] = F_D[\langle+q_0,+q_2,+q_1\rangle,\langle+q_1,+q_0,+q_2\rangle,\langle+q_2,+q_1,+q_0\rangle] - \langle a,a,a\rangle$

Chapter 4
Square packings

Construct a cluster $\mathbf{X}$ with 'square' symmetry $\mathbf{Z}_2^2$

$$\mathbf{Z}_2^2 = \left\{ \begin{bmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & +1 \end{bmatrix}, \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{bmatrix} \right\}$$

$\mathbf{B}_1 = (1 \text{ body})$
1 tetrahedron: $\{\langle +1,+1,+1\rangle, \langle +1,-1,-1\rangle, \langle -1,+1,-1\rangle, \langle -1,-1,+1\rangle\}$

$\mathbf{B}_5 = \mathbf{B}_1 \cup (4 \text{ bodies})$
4 tetrahedra: $\mathbf{Z}_2^2\{\langle +1,-1,-1\rangle, \langle -1,+1,-1\rangle, \langle -1,-1,+1\rangle, \langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle\}$

$\mathbf{B}_9 = \mathbf{B}_5 \cup (4 \text{ bodies})$
4 tetrahedra: $\mathbf{A}_4\{\langle +1,-1,-1\rangle, \langle -1,+1,-1\rangle, \langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle, \langle -\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle\}$

$\mathbf{B}_9^* = \mathbf{B}_5 \cup (4 \text{ bodies})$
4 tetrahedra: $\mathbf{A}_4\{\langle +1,-1,-1\rangle, \langle -1,+1,-1\rangle, \langle 0,0,-1-\sqrt{6}\rangle, \langle -\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6}\rangle\}$

$\mathbf{B}^9 = \mathbf{B}_1 \cup (8 \text{ bodies})$
4 tetrahedra: $\mathbf{A}_4\{\langle +1,-1,-1\rangle, \langle -1,+1,-1\rangle, \langle 0,0,-1-\sqrt{6}\rangle, \langle -\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6}\rangle\}$
4 tetrahedra: $\mathbf{A}_4\{\langle +1,-1,-1\rangle, \langle -1,+1,-1\rangle,$
$\langle -\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6}\rangle, \langle -\frac{4}{9}\sqrt{6},-\frac{4}{9}\sqrt{6},-1+\frac{7}{9}\sqrt{6}\rangle\}$

$\mathbf{B}_{13} = \mathbf{B}_5 \cup (8 \text{ bodies})$
4 tetrahedra: $\mathbf{A}_4\{\langle -1,+1,-1\rangle, \langle -1,-1,+1\rangle, \langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle, \langle -\frac{31}{9},-\frac{1}{9},-\frac{1}{9}\rangle\}$
4 tetrahedra: $\mathbf{A}_4\{\langle +1,-1,-1\rangle, \langle -1,-1,+1\rangle, \langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle, \langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle\}$

$\mathbf{B}_{13}^* = \mathbf{B}_5 \cup (8 \text{ bodies})$
4 tetrahedra: $\mathbf{A}_4\{\langle -1,+1,-1\rangle, \langle -1,-1,+1\rangle, \langle -1-\sqrt{6},0,0\rangle, \langle -1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle\}$
4 tetrahedra: $\mathbf{A}_4\{\langle +1,-1,-1\rangle, \langle -1,-1,+1\rangle, \langle 0,-1-\sqrt{6},0\rangle, \langle -\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle\}$

$\mathbf{B}_{17} = \mathbf{B}_5 \cup (12 \text{ bodies})$
12 tetrahedra: $\mathbf{A}_4\{\langle -1,+1,-1\rangle, \langle -1,-1,+1\rangle, \langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle, \langle -\frac{31}{9},-\frac{1}{9},-\frac{1}{9}\rangle\}$

$\mathbf{B}_{17}^* = \mathbf{B}_5 \cup (12 \text{ bodies})$
12 tetrahedra: $\mathbf{A}_4\{\langle -1,+1,-1\rangle, \langle -1,-1,+1\rangle, \langle -1-\sqrt{6},0,0\rangle, \langle -1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle\}$

$\mathbf{V}_{20} = (20 \text{ bodies})$
8 tetrahedra: $\pm\mathbf{Z}_2^2\{\langle 0,0,0\rangle, \langle q_0,q_1,q_2\rangle, \langle q_2,q_0,q_1\rangle, \langle q_1,q_2,q_0\rangle\}$
12 tetrahedra: $\mathbf{A}_4\{\langle 0,0,0\rangle, \langle 0,s_1,s_2\rangle, \langle r_1,r_2,r_0\rangle, \langle -r_1,r_2,r_0\rangle\}$

$P = \frac{1}{2}(1+\sqrt{5}),\ Q = \frac{1}{2}(-1+\sqrt{5})$
$\langle q_0,q_1,q_2\rangle = \frac{1}{3}\langle 4-P^2\sqrt{2}, 4+Q^2\sqrt{2}, 4+\sqrt{10}\rangle \approx \langle .099180275452, 1.513393837825, 2.387425886723\rangle$
$\langle r_0,r_1,r_2\rangle = \frac{1}{3}\langle 4Q-P\sqrt{2}, \sqrt{2}, 4P+Q\sqrt{2}\rangle \approx \langle .061296781243, 1.414213562373, 2.448722667966\rangle$
$\langle 0,s_1,s_2\rangle = \frac{1}{3}\langle 0, 4P-2Q\sqrt{2}, 4Q+2P\sqrt{2}\rangle \approx \langle .000000000000, 1.574690619068, 2.349542392514\rangle$

Note: $-\mathbf{V}_{20} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \mathbf{V}_{20}$

Construct a packing with 'square' symmetry $\mathbf{Z}_2^2$

$Q_2^=$ (pure 2-direction) face-centered square
There is 1 coset $\{+\mathbf{X}_A\langle 0,0,0\rangle,*,*,*\}$

$Q_1^=$ (pure 1-direction) simple square
There are 2 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle,*,+\mathbf{X}_C\langle 0,0,2c\rangle,*\}$

$Q_1^{\neq}$ (mixed 1-direction) simple square
There are 2 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle,*,-\mathbf{X}_C\langle 0,0,2c\rangle,*\}$

$Q_3^=$ (pure 3-direction) body-centered square
There are 4 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle,+\mathbf{X}_B\langle a,b,c\rangle,+\mathbf{X}_C\langle 0,0,2c\rangle,+\mathbf{X}_D\langle -a,-b,-c\rangle\}$
Note: $S_2^=\langle a,b,c\rangle$ is equivalent to $S_3^=\langle a+b,a-b,2c\rangle$

$Q_3^{\neq}$ (mixed 3-direction) body-centered square
There are 4 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle,-\mathbf{X}_B\langle a,b,c\rangle,+\mathbf{X}_C\langle 0,0,2c\rangle,-\mathbf{X}_D\langle -a,-b,-c\rangle\}$

Q_3^+ (positive 3-direction) diamond-crystal square
There are 2 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle,-\mathbf{X}_B\langle a,b,c\rangle,*,*\}$

Q_3^- (negative 3-direction) diamond-crystal square
There are 2 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle,*,*,-\mathbf{X}_D\langle -a,-b,-c\rangle\}$

$Q_F^=$ (pure xy-direction) face-centered square
There are 4 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle,+\mathbf{X}_B\langle a,b,0\rangle,+\mathbf{X}_C\langle 0,0,2c\rangle,+\mathbf{X}_D\langle a,b,2c\rangle\}$
Note: $S_1^=\langle a,b,c\rangle$ is equivalent to $S_F^=\langle a+b,a-b,c\rangle$

$Q_F^{\neq}$ (mixed xy-direction) face-centered square
There are 4 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle,-\mathbf{X}_B\langle a,b,0\rangle,+\mathbf{X}_C\langle 0,0,2c\rangle,-\mathbf{X}_D\langle a,b,2c\rangle\}$

$Q_E^=$ (pure z-direction) edge-centered square
There are 4 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle,+\mathbf{X}_B\langle 0,0,c\rangle,+\mathbf{X}_C\langle 0,0,2c\rangle,+\mathbf{X}_D\langle 0,0,-c\rangle\}$
Note: $S_1^=\langle a,b,c\rangle$ is equivalent to $S_E^=\langle a,b,2c\rangle$

$Q_E^{\neq}$ (mixed z-direction) edge-centered square
There are 4 cosets $\{+\mathbf{X}_A\langle 0,0,0\rangle,-\mathbf{X}_B\langle 0,0,c\rangle,+\mathbf{X}_C\langle 0,0,2c\rangle,-\mathbf{X}_D\langle 0,0,-c\rangle\}$

Each packing could have several variations

P_z linear close-packing (z-direction)

P_{xy} planar close-packing (xy-direction)

P^+ parallel orientation

$P^\times$ oblique orientation

The lattice basis is $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$

Write the intersection equations

Evaluate the lattice vectors a,b,c

Evaluate the lattice volume $V=\det\begin{bmatrix} a-b & a+b & 2c \\ a+b & -a+b & 2c \\ 2a & 2b & 0 \end{bmatrix}=8c(a^2+b^2)$

Evaluate the packing density D

Note: Each tetrahedron has volume $\frac{8}{3}$

$\mathbf{X}\ S_2^{=}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset

1 0·neighbor $+\mathbf{X}_A\langle 0,0,0\rangle$

4 1·neighbors $+\mathbf{X}_A\langle 2a,2b,0\rangle$

8 1·neighbors $+\mathbf{X}_A\langle a-b,b+a,2c\rangle$

$\mathbf{X}\ S_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

$\mathbf{X}\ S_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets

1 0·neighbor $+\mathbf{X}_A\langle 0,0,0\rangle$

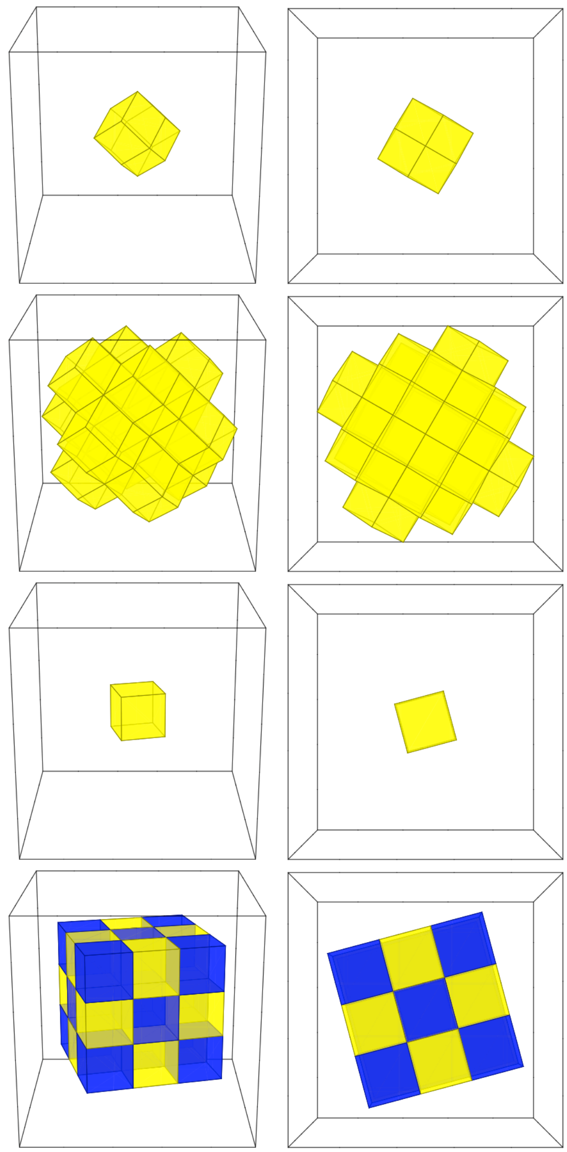
4 1·neighbors $\pm\mathbf{X}_C\langle a-b,b+a,0\rangle$

4 1·neighbors $+\mathbf{X}_A\langle 2a,2b,0\rangle$

2 1·neighbors $\pm\mathbf{X}_C\langle 0,0,2c\rangle$

8 1·neighbors $+\mathbf{X}_A\langle a-b,b+a,2c\rangle$

8 1·neighbors $\pm\mathbf{X}_C\langle 2a,2b,2c\rangle$

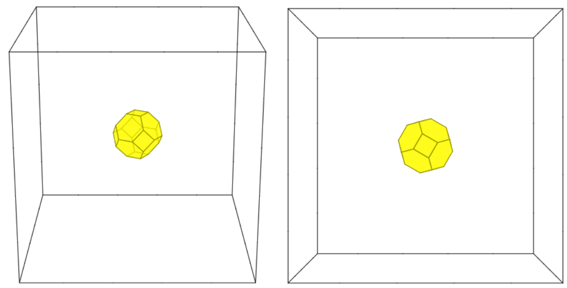


$\mathbf{X}\ S_3^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

$\mathbf{X}\ S_3^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

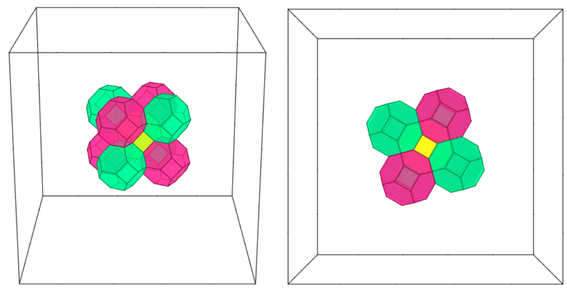
4 cosets

1 0·neighbor $+\mathbf{X}_A\langle 0,0,0\rangle$



4 1·neighbors $\pm\mathbf{X}_B\langle +a,b,c\rangle$

4 1·neighbors $\pm\mathbf{X}_D\langle -b,a,c\rangle$



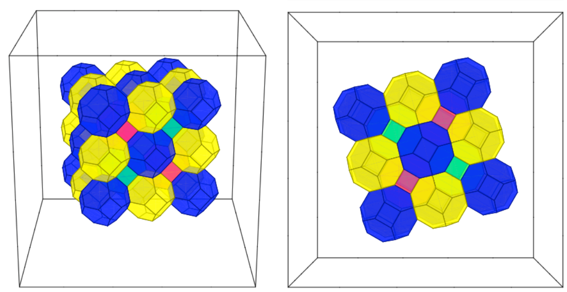
4 1·neighbors $\pm\mathbf{X}_C\langle a-b,b+a,0\rangle$

4 1·neighbors $+\mathbf{X}_A\langle 2a,2b,0\rangle$

2 1·neighbors $\pm\mathbf{X}_C\langle 0,0,2c\rangle$

8 1·neighbors $+\mathbf{X}_A\langle a-b,b+a,2c\rangle$

8 1·neighbors $\pm\mathbf{X}_C\langle 2a,2b,2c\rangle$



8 3·neighbors $\pm\mathbf{X}_B\langle a-2b,b+2a,c\rangle$

8 3·neighbors $\pm\mathbf{X}_D\langle 2a-b,2b+a,c\rangle$

4 3·neighbors $\pm\mathbf{X}_B\langle +3a,3b,c\rangle$

4 3·neighbors $\pm\mathbf{X}_D\langle -3b,3a,c\rangle$

4 3·neighbors $\pm\mathbf{X}_B\langle -b,a,3c\rangle$

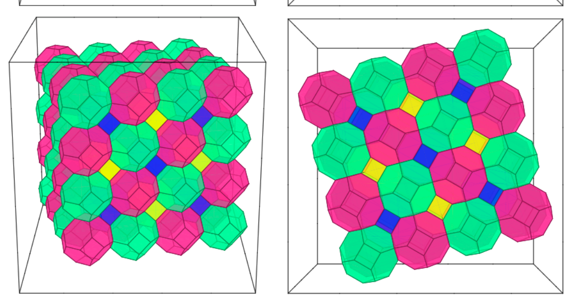
4 3·neighbors $\pm\mathbf{X}_D\langle +a,b,3c\rangle$

8 3·neighbors $\pm\mathbf{X}_B\langle 2a-b,2b+a,3c\rangle$

8 3·neighbors $\pm\mathbf{X}_D\langle a-2b,b+2a,3c\rangle$

4 3·neighbors $\pm\mathbf{X}_B\langle -3b,3a,3c\rangle$

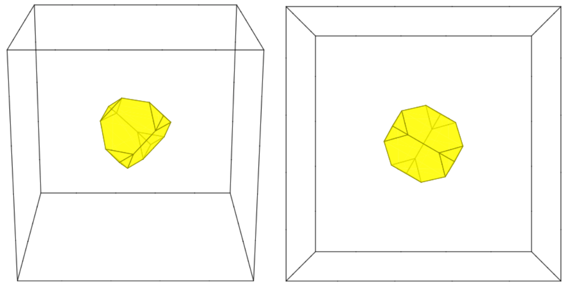
4 3·neighbors $\pm\mathbf{X}_D\langle +3a,3b,3c\rangle$



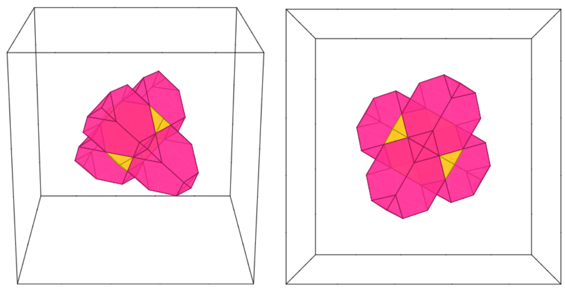
$\mathbf{X}\ S_3^+\ \{+\mathbf{X}_A, -\mathbf{X}_B, *, *\}$

2 cosets

1 0·neighbor $+\mathbf{X}_A\langle 0,0,0\rangle$

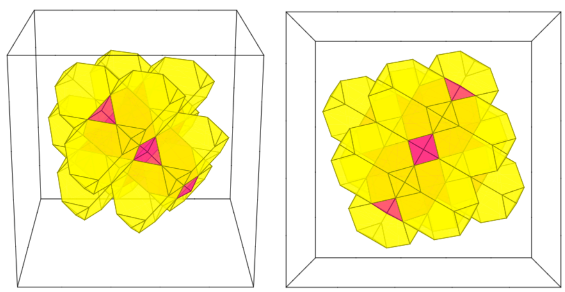


4 1·neighbors $-\mathbf{X}_B\langle +a,b,c\rangle$



4 2·neighbors $+\mathbf{X}_A\langle 2a,2b,0\rangle$

8 2·neighbors $+\mathbf{X}_A\langle a-b,b+a,2c\rangle$



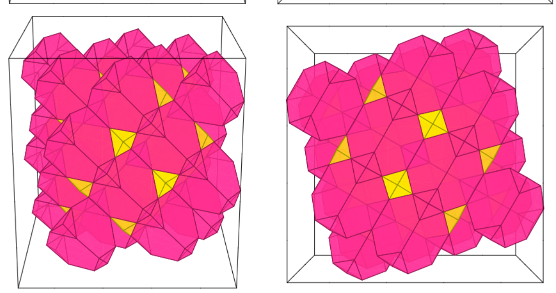
8 3·neighbors $-\mathbf{X}_B\langle a-2b,b+2a,c\rangle$

4 3·neighbors $-\mathbf{X}_B\langle +3a,3b,c\rangle$

4 3·neighbors $-\mathbf{X}_B\langle -b,a,3c\rangle$

8 3·neighbors$-\mathbf{X}_B\langle 2a-b,2b+a,3c\rangle$

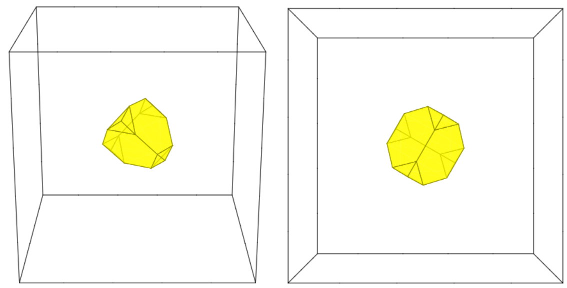
4 3·neighbors $-\mathbf{X}_B\langle -3b,3a,3c\rangle$



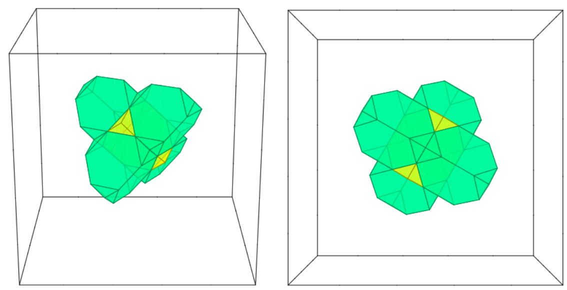
X S_3^- {+$\mathbf{X}_A$,∗,∗,−$\mathbf{X}_D$}

2 cosets

1 0·neighbor +$\mathbf{X}_A\langle 0,0,0\rangle$

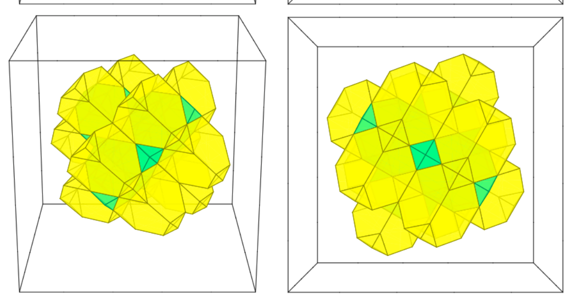


4 1·neighbors −$\mathbf{X}_D\langle -b,a,c\rangle$



4 2·neighbors +$\mathbf{X}_A\langle 2a,2b,0\rangle$

8 2·neighbors +$\mathbf{X}_A\langle a-b,b+a,2c\rangle$



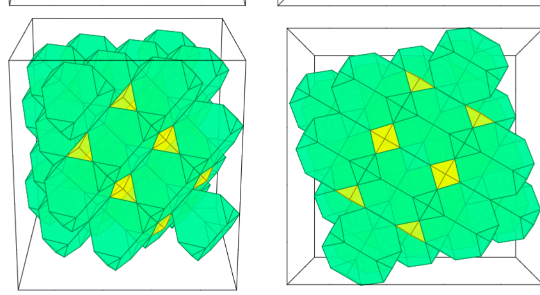
8 3·neighbors −$\mathbf{X}_B\langle 2a-b,2b+a,c\rangle$

4 3·neighbors −$\mathbf{X}_B\langle -3b,3a,c\rangle$

4 3·neighbors −$\mathbf{X}_B\langle +a,b,3c\rangle$

8 3·neighbors−$\mathbf{X}_B\langle a-2b,b+2a,3c\rangle$

4 3·neighbors −$\mathbf{X}_B\langle +3a,3b,3c\rangle$



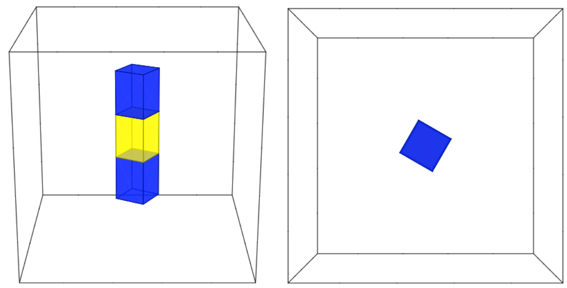
$\mathbf{X}\ S_{\mathrm{F}}^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

$\mathbf{X}\ S_{\mathrm{F}}^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

1 0·neighbor $+\mathbf{X}_A\langle 0,0,0\rangle$

2 0·neighbors $+\mathbf{X}_C\langle 0,0,2c\rangle$

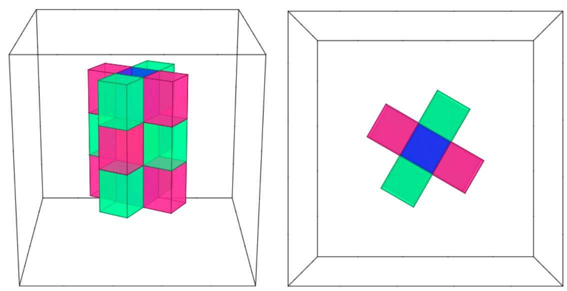


2 1·neighbors $\pm\mathbf{X}_B\langle +a,b,0\rangle$

2 1·neighbors $\pm\mathbf{X}_D\langle -b,a,0\rangle$

4 1·neighbors $\pm\mathbf{X}_B\langle -b,a,2c\rangle$

4 1·neighbors $\pm\mathbf{X}_D\langle +a,b,2c\rangle$

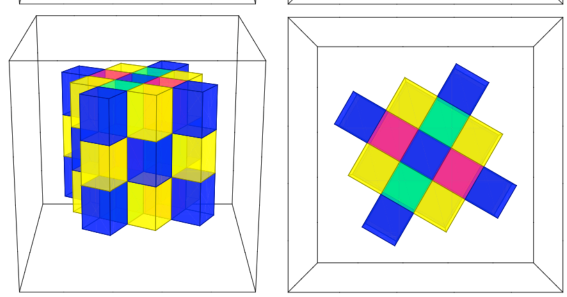


4 2·neighbors $+\mathbf{X}_A\langle 2a,2b,0\rangle$

4 2·neighbors $+\mathbf{X}_C\langle a-b,b+a,0\rangle$

8 2·neighbors $+\mathbf{X}_A\langle a-b,b+a,2c\rangle$

8 2·neighbors $+\mathbf{X}_C\langle 2a,2b,2c\rangle$



4 3·neighbors $\pm\mathbf{X}_B\langle a-2b,b+2a,0\rangle$

4 3·neighbors $\pm\mathbf{X}_D\langle 2a-b,2b+a,0\rangle$

2 3·neighbors $\pm\mathbf{X}_B\langle +3a,3b,0\rangle$

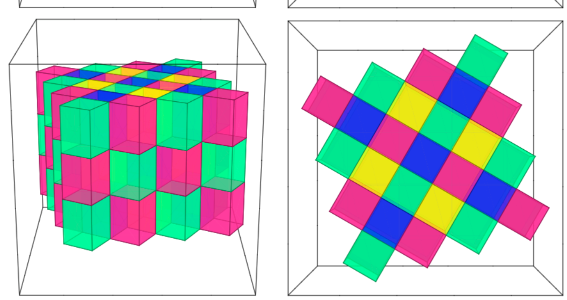
2 3·neighbors $\pm\mathbf{X}_D\langle -3b,3a,0\rangle$

8 3·neighbors $\pm\mathbf{X}_B\langle 2a-b,2b+a,2c\rangle$

8 3·neighbors $\pm\mathbf{X}_D\langle a-2b,b+2a,2c\rangle$

4 3·neighbors $\pm\mathbf{X}_B\langle -3b,3a,2c\rangle$

4 3·neighbors $\pm\mathbf{X}_D\langle +3a,3b,2c\rangle$



$\mathbf{X}\ S_{\mathrm{E}}^{=}\ \{+\mathbf{X}_A, +\mathbf{X}_B, +\mathbf{X}_C, +\mathbf{X}_D\}$

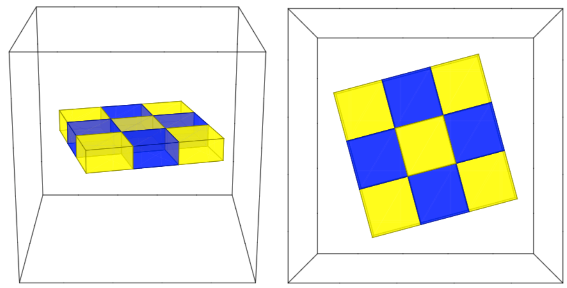
$\mathbf{X}\ S_{\mathrm{E}}^{\neq}\ \{+\mathbf{X}_A, -\mathbf{X}_B, +\mathbf{X}_C, -\mathbf{X}_D\}$

4 cosets

1 0-neighbor $+\mathbf{X}_A\langle 0,0,0\rangle$

4 0-neighbors $+\mathbf{X}_C\langle a-b, b+a, 0\rangle$

4 0-neighbors $+\mathbf{X}_A\langle 2a, 2b, 0\rangle$



1 1-neighbor $\pm\mathbf{X}_B\langle 0,0,+c\rangle$

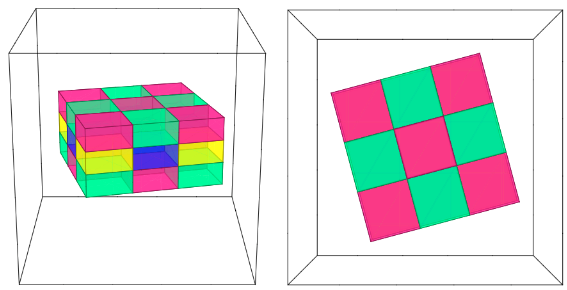
1 1-neighbor $\pm\mathbf{X}_D\langle 0,0,-c\rangle$

4 1-neighbors $\pm\mathbf{X}_B\langle a-b, b+a, -c\rangle$

4 1-neighbors $\pm\mathbf{X}_D\langle a-b, b+a, +c\rangle$

4 1-neighbors $\pm\mathbf{X}_B\langle 2a, 2b, +c\rangle$

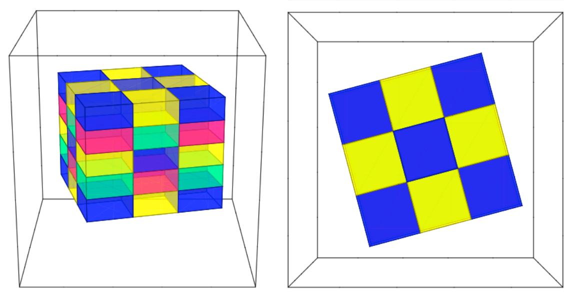
4 1-neighbors $\pm\mathbf{X}_D\langle 2a, 2b, -c\rangle$



2 2-neighbors $+\mathbf{X}_C\langle 0,0,2c\rangle$

8 2-neighbors $+\mathbf{X}_A\langle a-b, b+a, 2c\rangle$

8 2-neighbors $+\mathbf{X}_C\langle 2a, 2b, 2c\rangle$



1 3-neighbor $\pm\mathbf{X}_B\langle 0,0,-3c\rangle$

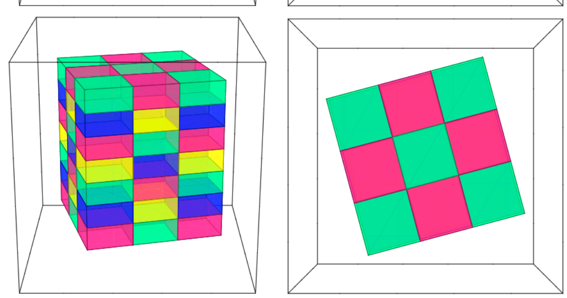
1 3-neighbor $\pm\mathbf{X}_D\langle 0,0,+3c\rangle$

4 3-neighbors $\pm\mathbf{X}_B\langle a-b, b+a, +3c\rangle$

4 3-neighbors $\pm\mathbf{X}_D\langle a-b, b+a, -3c\rangle$

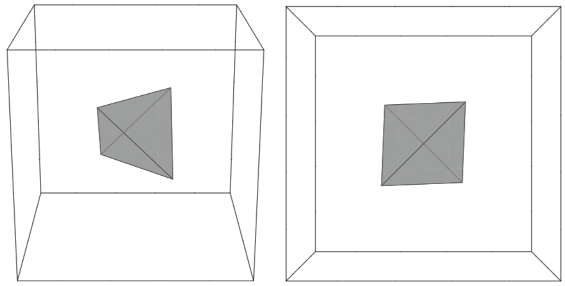
4 3-neighbors $\pm\mathbf{X}_B\langle 2a, 2b, -3c\rangle$

4 3-neighbors $\pm\mathbf{X}_D\langle 2a, 2b, +3c\rangle$



$\mathbf{B}_1$

n cosets
1 tetrahedron



$U = \frac{8}{3}$ 2.666666666667

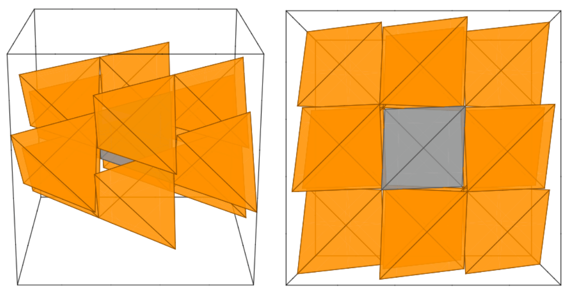
$V = 8c(a^2+b^2)$

$D = n\cdot U/V$

basis $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$

$\mathbf{B}_1\ Q_2^{=}\ P_z\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
1 tetrahedron



$a = 1$ 1.000000000000

$b = 1$ 1.000000000000

$c = \frac{1}{2}$ .500000000000

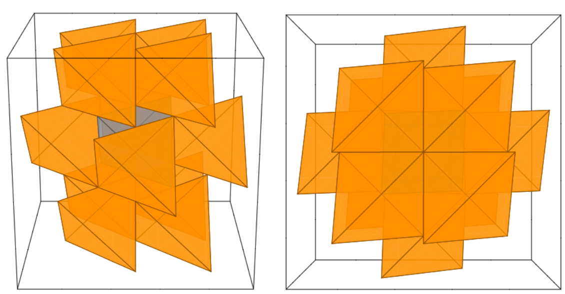
$V = 8$ 8.000000000000

$D = \frac{1}{3}$ .333333333333

$V_A\langle+1,+1,+1\rangle = V_A\langle-1,-1,+1\rangle + \langle 2a,2b,0\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-1,+1,-1\rangle] = E_A[\langle+1,-1,-1\rangle,\langle-1,-1,+1\rangle] + \langle a-b,a+b,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-1,-1,+1\rangle] = E_A[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle] + \langle 0,0,4c\rangle$

$\mathbf{B}_1\ Q_2^{=}\ P_{xy}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
1 tetrahedron



$a = 1$ 1.000000000000

$b = 0$ .000000000000

$c = 1$ 1.000000000000

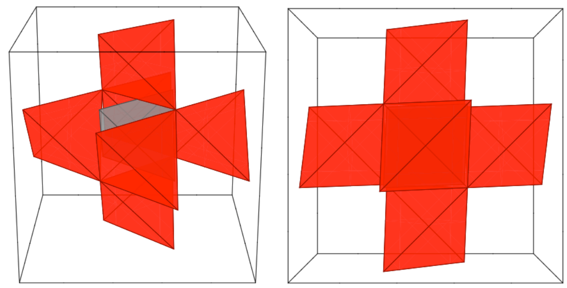
$V = 8$ 8.000000000000

$D = \frac{1}{3}$ .333333333333

$V_A\langle+1,+1,+1\rangle = V_A\langle-1,-1,+1\rangle + \langle 2a-2b,2a+2b,0\rangle$
$V_A\langle+1,+1,+1\rangle = E_A[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle] + \langle a-b,a+b,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+1,-1,-1\rangle] = E_A[\langle-1,+1,-1\rangle,\langle-1,-1,+1\rangle] + \langle 2a,2b,0\rangle$

B$_1$ $Q_1^{=}$ {+$\mathbf{X}_A$,∗,+$\mathbf{X}_C$,∗}

2 cosets
2 tetrahedra

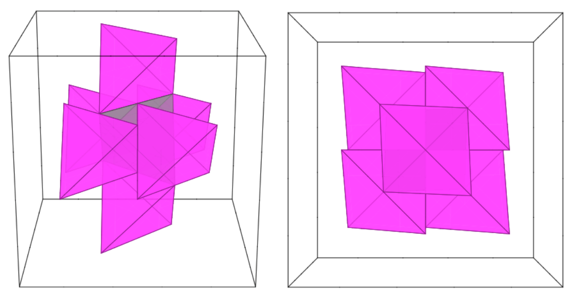


$a = 1$
1.000000000000
$b = 1$
1.000000000000
$c = 1$
1.000000000000
$V = 16$
16.000000000000
$D = \frac{1}{3}$
.333333333333

$V_A\langle+1,+1,+1\rangle = V_A\langle-1,-1,+1\rangle + \langle 2a,2b,0\rangle$
$V_A\langle+1,+1,+1\rangle = V_A\langle+1,-1,-1\rangle + \langle a-b,a+b,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-1,-1,+1\rangle] = E_C[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle] + \langle 0,0,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-1,+1,-1\rangle] = E_C[\langle+1,-1,-1\rangle,\langle-1,-1,+1\rangle] + \langle a-b,a+b,0\rangle$

B$_1$ $Q_1^{\neq}$ {+$\mathbf{X}_A$,∗,−$\mathbf{X}_C$,∗}

2 cosets
2 tetrahedra

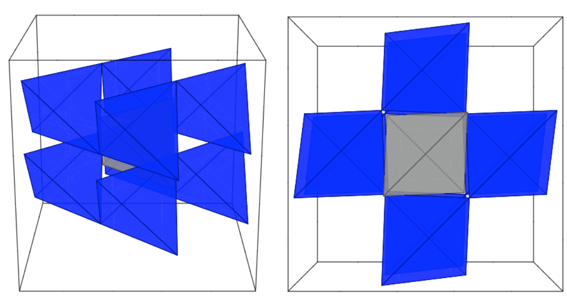


$a = 1$
1.000000000000
$b = 0$
.000000000000
$c = 1$
1.000000000000
$V = 8$
8.000000000000
$D = \frac{2}{3}$
.666666666667

$V_A\langle+1,+1,+1\rangle = V_A\langle-1,-1,+1\rangle + \langle 2a-2b,2a+2b,0\rangle$
$V_A\langle+1,+1,+1\rangle = E_A[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle] + \langle a-b,a+b,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+1,-1,-1\rangle] = E_A[\langle-1,+1,-1\rangle,\langle-1,-1,+1\rangle] + \langle 2a,2b,0\rangle$
$V_A\langle+1,+1,+1\rangle = V_C\langle+1,+1,-1\rangle + \langle 0,0,2c\rangle$
$V_A\langle+1,+1,+1\rangle = V_C\langle-1,-1,-1\rangle + \langle 2a-2b,2a+2b,2c\rangle$
$V_A\langle+1,+1,+1\rangle = E_C[\langle+1,-1,+1\rangle,\langle-1,+1,+1\rangle] + \langle a-b,a+b,0\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+1,-1,-1\rangle] = E_C[\langle+1,-1,+1\rangle,\langle-1,-1,-1\rangle] + \langle a-b,a+b,0\rangle$

B$_1$ $Q_3^{=}$ P_z {+$\mathbf{X}_A$,+$\mathbf{X}_B$,+$\mathbf{X}_C$,+$\mathbf{X}_D$}

4 cosets
4 tetrahedra



$a = 2$
2.000000000000
$b = 0$
.000000000000
$c = 1$
1.000000000000
$V = 32$
32.000000000000
$D = \frac{1}{3}$
.333333333333

B$_1$ $Q_2^{=}$ P_z

$\mathbf{B}_1\ Q_3^{=}\ P_{xy}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

4 tetrahedra

$a = 1$

1.000000000000

$b = 1$

1.000000000000

$c = 2$

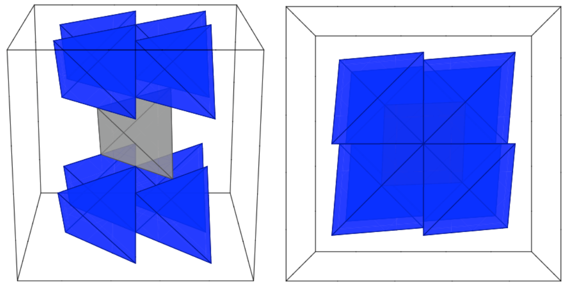
2.000000000000

$V = 32$

32.000000000000

$D = \frac{1}{3}$

.333333333333



$\mathbf{B}_1\ Q_2^{=}\ P_{xy}$

$\mathbf{B}_1\ Q_3^{\neq}\ P_z\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

4 tetrahedra

$a = \frac{3}{2}$

1.500000000000

$b = \frac{1}{2}$

.500000000000

$c = 1$

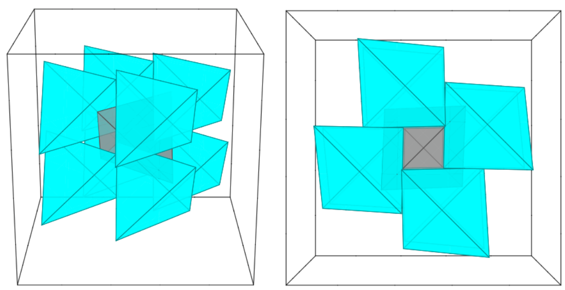
1.000000000000

$V = 20$

20.000000000000

$D = \frac{8}{15}$

.533333333333



$E_A[\langle+1,+1,+1\rangle,\langle-1,+1,-1\rangle] = E_C[\langle+1,-1,-1\rangle,\langle-1,-1,+1\rangle] + \langle a-b,a+b,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle-1,-1,+1\rangle] = E_C[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle] + \langle 0,0,2c\rangle$

$V_A\langle+1,+1,+1\rangle = F_B[\langle+1,+1,-1\rangle,\langle-1,+1,+1\rangle,\langle-1,-1,-1\rangle] + \langle a,b,c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle-1,-1,+1\rangle] = E_B[\langle-1,+1,+1\rangle,\langle-1,-1,-1\rangle] + \langle a,b,c\rangle$

$\mathbf{B}_1\ Q_3^{\neq}\ P_{xy}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

4 tetrahedra

$a = 1$

1.000000000000

$b = 1$

1.000000000000

$c = 2$

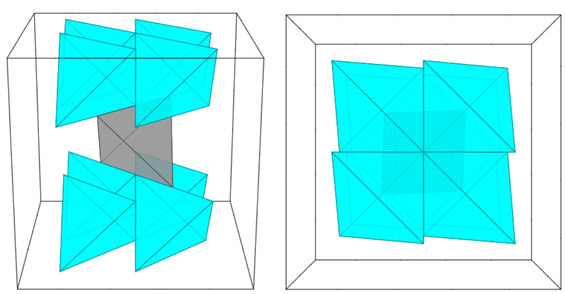
2.000000000000

$V = 32$

32.000000000000

$D = \frac{1}{3}$

.333333333333

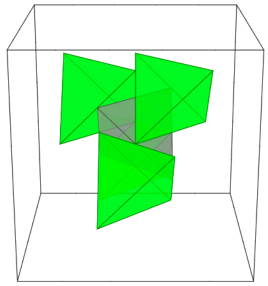
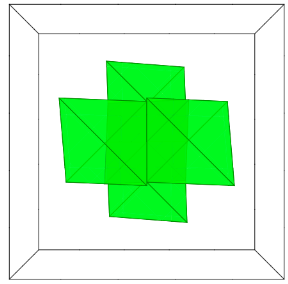


$V_A\langle+1,+1,+1\rangle = V_A\langle-1,-1,+1\rangle + \langle 2a,2b,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle-1,+1,-1\rangle] = E_C[\langle+1,-1,-1\rangle,\langle-1,-1,+1\rangle] + \langle a-b,a+b,0\rangle$

$V_A\langle+1,+1,+1\rangle = E_B[\langle+1,+1,-1\rangle,\langle-1,-1,-1\rangle] + \langle a,b,c\rangle$

$\mathbf{B}_1\ Q_3^+\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

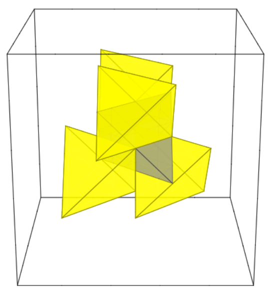
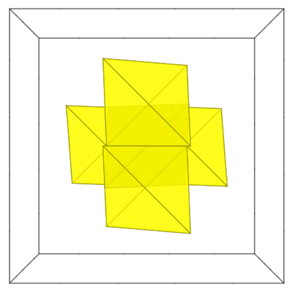



2 cosets
2 tetrahedra

$a = 1$
1.000000000000
$b = 0$
.000000000000
$c = 1$
1.000000000000
$V = 8$
8.000000000000
$D = \frac{2}{3}$
.666666666667

$V_A\langle+1,+1,+1\rangle = V_A\langle-1,-1,+1\rangle + \langle 2a-2b,2a+2b,0\rangle$
$V_A\langle+1,+1,+1\rangle = E_A[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle] + \langle a-b,a+b,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+1,-1,-1\rangle] = E_A[\langle-1,+1,-1\rangle,\langle-1,-1,+1\rangle] + \langle 2a,2b,0\rangle$
$V_A\langle+1,+1,+1\rangle = E_B[\langle+1,+1,-1\rangle,\langle-1,+1,+1\rangle] + \langle a,b,c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-1,-1,+1\rangle] = E_B[\langle-1,+1,+1\rangle,\langle-1,-1,-1\rangle] + \langle a,b,c\rangle$

$\mathbf{B}_1\ Q_3^-\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

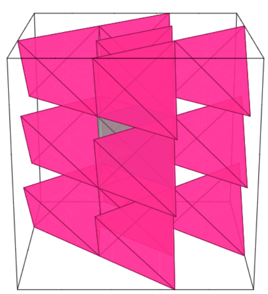
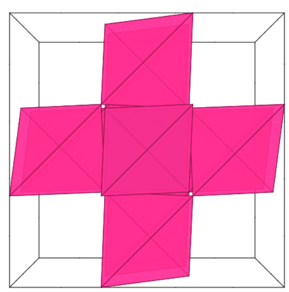



2 cosets
2 tetrahedra

$a = 1$
1.000000000000
$b = 0$
.000000000000
$c = 1$
1.000000000000
$V = 8$
8.000000000000
$D = \frac{2}{3}$
.666666666667

$V_A\langle+1,+1,+1\rangle = V_A\langle-1,-1,+1\rangle + \langle 2a-2b,2a+2b,0\rangle$
$V_A\langle+1,+1,+1\rangle = E_A[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle] + \langle a-b,a+b,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+1,-1,-1\rangle] = E_A[\langle-1,+1,-1\rangle,\langle-1,-1,+1\rangle] + \langle 2a,2b,0\rangle$
$V_A\langle+1,+1,+1\rangle = E_B[\langle+1,+1,-1\rangle,\langle+1,-1,+1\rangle] + \langle -b,a,c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-1,-1,+1\rangle] = E_B[\langle+1,-1,+1\rangle,\langle-1,-1,-1\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}_1\ Q_{\mathrm{F}}^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets
4 tetrahedra

$a = 2$
2.000000000000
$b = 0$
.000000000000
$c = 1$
1.000000000000
$V = 32$
32.000000000000
$D = \frac{1}{3}$
.333333333333

$\mathbf{B}_1\ Q_1^{=}$

$\mathbf{B}_1\ Q_{\mathrm{F}}^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

4 tetrahedra

$a = 1$

1.000000000000

$b = 1$

1.000000000000

$c = 1$

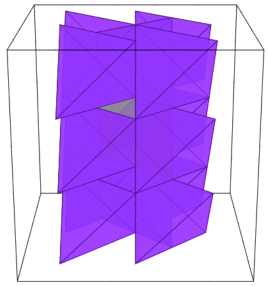
1.000000000000

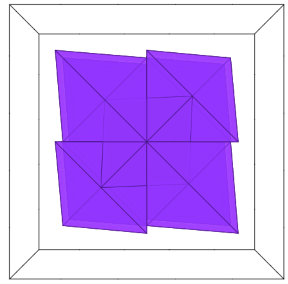
$V = 16$

16.000000000000

$D = \frac{2}{3}$

.666666666667





$V_A\langle+1,+1,+1\rangle = V_A\langle-1,-1,+1\rangle + \langle 2a,2b,0\rangle$
$V_A\langle+1,+1,+1\rangle = V_A\langle+1,-1,-1\rangle + \langle a-b,a+b,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-1,+1,-1\rangle] = E_C[\langle+1,-1,-1\rangle,\langle-1,-1,+1\rangle] + \langle a-b,a+b,0\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-1,-1,+1\rangle] = E_C[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle] + \langle 0,0,2c\rangle$
$V_A\langle+1,+1,+1\rangle = E_B[\langle+1,-1,+1\rangle,\langle-1,+1,+1\rangle] + \langle a,b,0\rangle$
$V_A\langle+1,+1,+1\rangle = E_D[\langle+1,+1,-1\rangle,\langle-1,-1,-1\rangle] + \langle a,b,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-1,+1,-1\rangle] = E_B[\langle-1,+1,+1\rangle,\langle-1,-1,-1\rangle] + \langle a,b,0\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-1,+1,-1\rangle] = E_D[\langle+1,+1,-1\rangle,\langle+1,-1,+1\rangle] + \langle -b,a,0\rangle$

$\mathbf{B}_1\ Q_{\mathrm{E}}^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

4 tetrahedra

$a = 1$

1.000000000000

$b = 1$

1.000000000000

$c = 2$

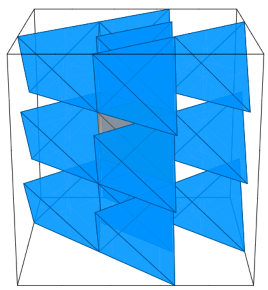
2.000000000000

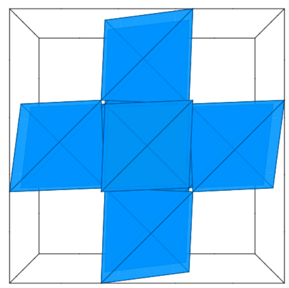
$V = 32$

32.000000000000

$D = \frac{1}{3}$

.333333333333





$\mathbf{B}_1\ Q_{1}^{=}$

$\mathbf{B}_1\ Q_{\mathrm{E}}^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

4 tetrahedra

$a = 1$

1.000000000000

$b = 1$

1.000000000000

$c = 2$

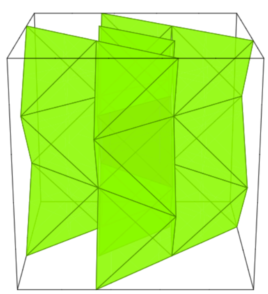
2.000000000000

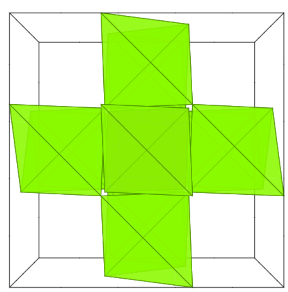
$V = 32$

32.000000000000

$D = \frac{1}{3}$

.333333333333





$V_A\langle+1,+1,+1\rangle = V_A\langle-1,-1,+1\rangle + \langle 2a,2b,0\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-1,+1,-1\rangle] = E_C[\langle+1,-1,-1\rangle,\langle-1,-1,+1\rangle] + \langle a-b,a+b,0\rangle$
$V_A\langle+1,+1,+1\rangle = V_B\langle+1,+1,-1\rangle + \langle 0,0,c\rangle$
$V_A\langle+1,+1,+1\rangle = V_B\langle-1,-1,-1\rangle + \langle 2a,2b,c\rangle$

$\mathbf{B}_5$

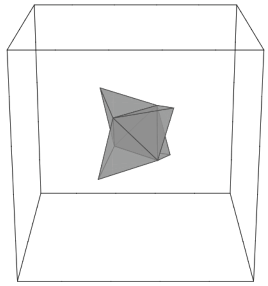
n cosets
5 tetrahedra

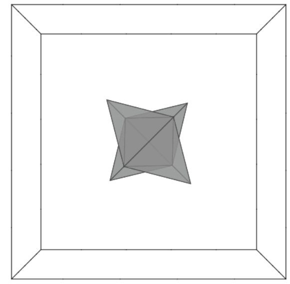
$U = \frac{40}{3}$ 13.333333333333

$V = 8c(a^2+b^2)$

$D = n{\cdot}U/V$

basis $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$





$\mathbf{B}_5\ Q_2^{=}\ P_z^{\mathrm{VF}}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
5 tetrahedra

$a = \frac{175}{104}$ 1.682692307692

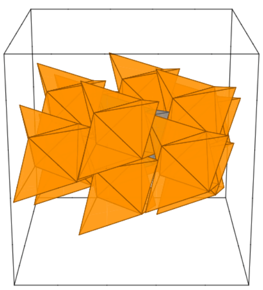
$b = \frac{175}{156}$ 1.121794871795

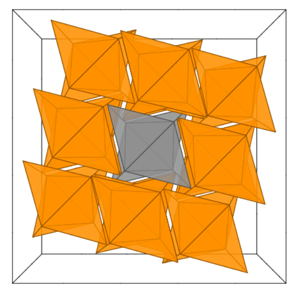
$c = \frac{5}{8}$ .625000000000

$V = \frac{153125}{7488}$ 20.449385683761

$D = \frac{19968}{30625}$ .652016326531

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle+1,-1,-1\rangle,\langle+\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,4c\rangle$
$V_A\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle = F_A[\langle-1,+1,-1\rangle,\langle-1,-1,+1\rangle,\langle-\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle a+b,-a+b,2c\rangle$
$a = \frac{3}{2}b$ (minimum lattice volume)





$\mathbf{B}_5\ Q_2^{=}\ P_z^{\mathrm{EE}}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
5 tetrahedra

$a = \frac{25}{16}$ 1.562500000000

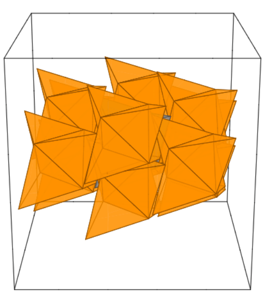
$b = \frac{5}{4}$ 1.250000000000

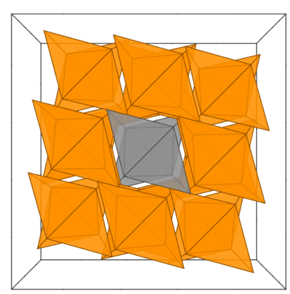
$c = \frac{5}{8}$ .625000000000

$V = \frac{5125}{256}$ 20.019531250000

$D = \frac{2048}{3075}$ .666016260163

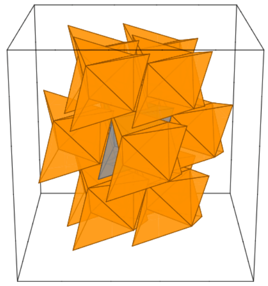
$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle+1,-1,-1\rangle,\langle+\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,4c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle+1,-1,-1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle a+b,-a+b,2c\rangle$

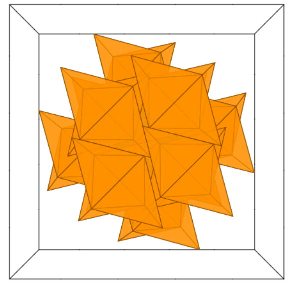




$\mathbf{B}_5\ Q_2^{=}\ P_{xy}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
5 tetrahedra





$a = \frac{30}{23}$ 1.304347826087

$b = \frac{5}{23}$ .217391304348

$c = \frac{95}{69}$ 1.376811594203

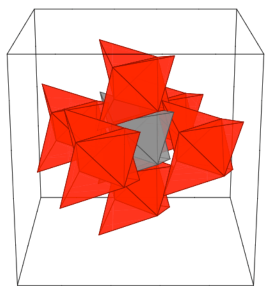
$V = \frac{703000}{36501}$ 19.259746308320

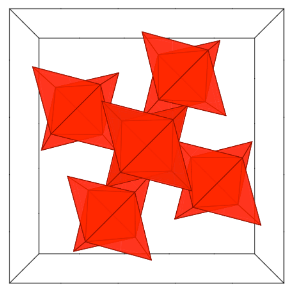
$D = \frac{12167}{17575}$ .692290184922

$V_A\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle = F_A[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle,\langle-\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle a+b,-a+b,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle 2a+2b,-2a+2b,0\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$

$\mathbf{B}_5\ Q_1^{=}\ P_z\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets
10 tetrahedra





$a = \frac{235}{108}$ 2.175925925926

$b = \frac{25}{36}$ .694444444444

$c = \frac{5}{4}$ 1.250000000000

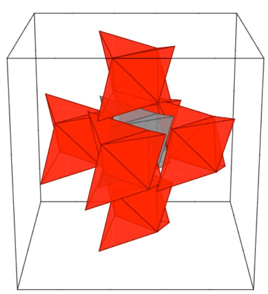
$V = \frac{152125}{2916}$ 52.169067215364

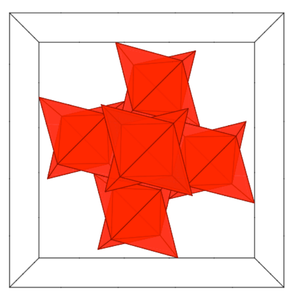
$D = \frac{15552}{30425}$ .511158586689

$V_A\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle = F_A[\langle-1,+1,-1\rangle,\langle-1,-1,+1\rangle,\langle-\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle a+b,-a+b,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle+1,-1,-1\rangle,\langle+\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$

$\mathbf{B}_5\ Q_1^{=}\ P_{xy}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets
10 tetrahedra





$a = \frac{35}{23}$ 1.521739130435

$b = \frac{25}{23}$ 1.086956521739

$c = \frac{107}{69}$ 1.550724637681

$V = \frac{1583600}{36501}$ 43.385112736637

$D = \frac{12167}{19795}$ .614650164183

$V_A\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle = F_A[\langle+1,-1,-1\rangle,\langle-1,+1,-1\rangle,\langle-\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle a+b,-a+b,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$

$\mathbf{B}_5\ Q_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

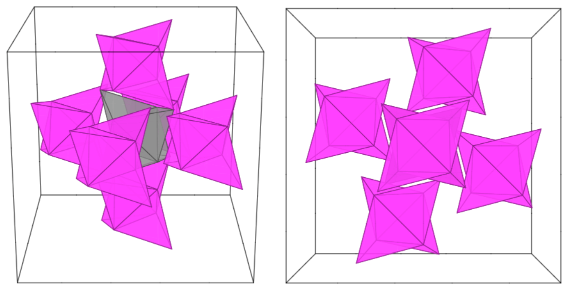
2 cosets
10 tetrahedra

$a = 2$
2.000000000000
$b = 1$
1.000000000000
$c = \frac{5}{3}$
1.666666666667
$V = \frac{200}{3}$
66.666666666667
$D = \frac{2}{5}$
.400000000000

$V_A\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle = V_C\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle + \langle 0,0,2c\rangle$

$E_A[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle -1,+1,+1\rangle,\langle -\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$

$a = 2b$ (minimum lattice volume)

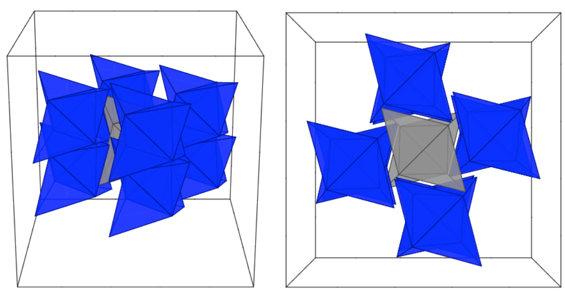


$\mathbf{B}_5\ Q_3^{=}\ P_z^{\mathrm{VF}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets
20 tetrahedra

$a = \frac{875}{312}$
2.804487179487
$b = \frac{175}{312}$
.560897435897
$c = \frac{5}{4}$
1.250000000000
$V = \frac{153125}{1872}$
81.797542735043
$D = \frac{19968}{30625}$
.652016326531

$\mathbf{B}_5\ Q_2^{=}\ P_z^{\mathrm{VF}}$

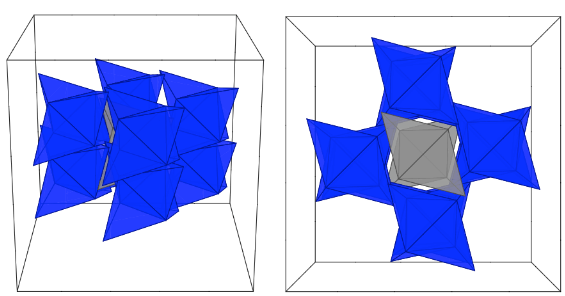


$\mathbf{B}_5\ Q_3^{=}\ P_z^{\mathrm{EE}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets
20 tetrahedra

$a = \frac{45}{16}$
2.812500000000
$b = \frac{5}{16}$
.312500000000
$c = \frac{5}{4}$
1.250000000000
$V = \frac{5125}{64}$
80.078125000000
$D = \frac{2048}{3075}$
.666016260163

$\mathbf{B}_5\ Q_2^{=}\ P_z^{\mathrm{EE}}$



$\mathbf{B}_5\ Q_3^{=}\ P_{xy}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

20 tetrahedra

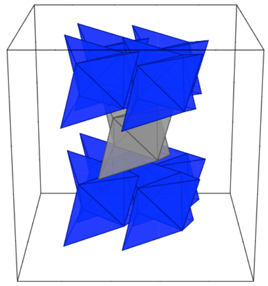
$a = \frac{35}{23}$ 1.521739130435

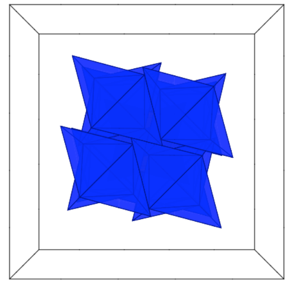
$b = \frac{25}{23}$ 1.086956521739

$c = \frac{190}{69}$ 2.753623188406

$V = \frac{2812000}{36501}$ 77.038985233281

$D = \frac{12167}{17575}$ .692290184922





$\mathbf{B}_5\ Q_2^{=}\ P_{xy}$

$\mathbf{B}_5\ Q_3^{\neq}\ P_z\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

20 tetrahedra

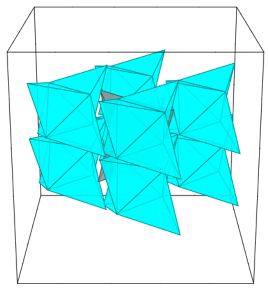
$a = \frac{215}{72}$ 2.986111111111

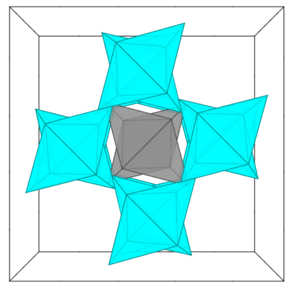
$b = \frac{5}{24}$ .208333333333

$c = \frac{5}{4}$ 1.250000000000

$V = \frac{116125}{1296}$ 89.602623456790

$D = \frac{13824}{23225}$ .595220667384





$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle+1,-1,-1\rangle,\langle+\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_D[\langle-1,-1,-1\rangle,\langle-\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}_5\ Q_3^{\neq}\ P_{xy}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

20 tetrahedra

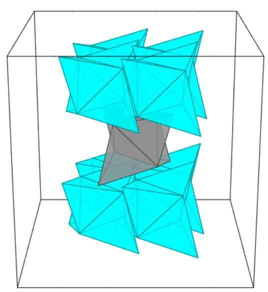
$a = \frac{35}{23}$ 1.521739130435

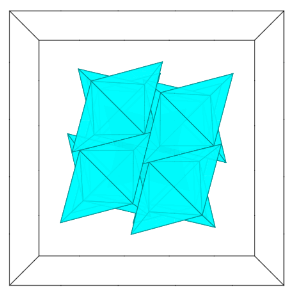
$b = \frac{25}{23}$ 1.086956521739

$c = \frac{194}{69}$ 2.811594202899

$V = \frac{2871200}{36501}$ 78.660858606614

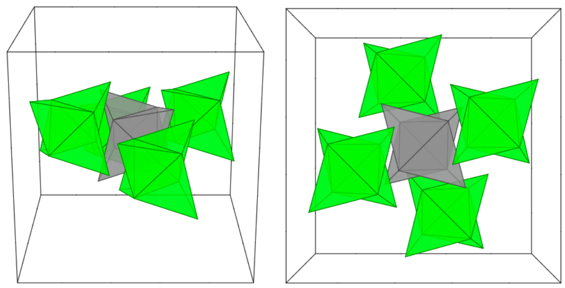
$D = \frac{12167}{17945}$ .678016160490





$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$
$V_A\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle = F_D[\langle+1,+1,-1\rangle,\langle-1,-1,-1\rangle,\langle-\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle -b,a,c\rangle$

B$_5$ Q_3^+ P_z $\{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$



2 cosets
10 tetrahedra

$a = \frac{45}{16}$ 2.812500000000

$b = \frac{15}{16}$ .937500000000

$c = \frac{5}{8}$ .625000000000

$V = \frac{5625}{128}$ 43.945312500000

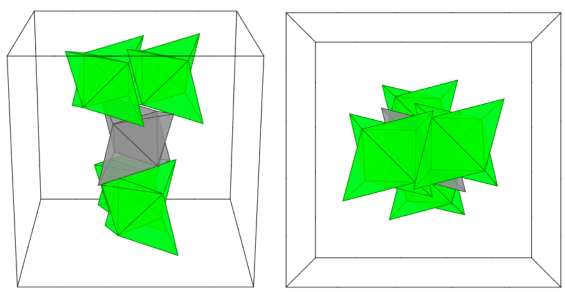
$D = \frac{2048}{3375}$ .606814814815

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle+1,-1,-1\rangle,\langle+\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,4c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_B[\langle-1,-1,-1\rangle,\langle-\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle a,b,c\rangle$

$a = 3b$ (minimum lattice volume)

B$_5$ Q_3^+ P_{xy} $\{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$



2 cosets
10 tetrahedra

$a = \frac{30}{23}$ 1.304347826087

$b = \frac{5}{23}$ .217391304348

$c = \frac{65}{23}$ 2.826086956522

$V = \frac{481000}{12167}$ 39.533163474973

$D = \frac{24334}{36075}$ .674539154539

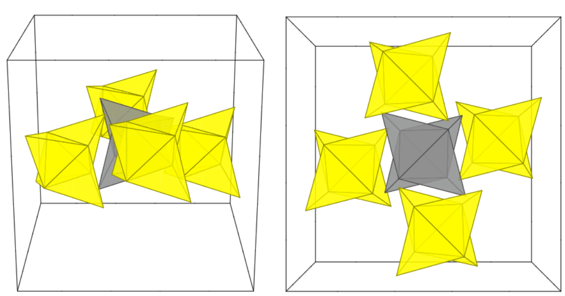
$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle 2a+2b,-2a+2b,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$

$V_A\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle = F_B[\langle+1,+1,-1\rangle,\langle-1,-1,-1\rangle,\langle+\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle -a-2b,2a-b,c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_B[\langle+1,+1,-1\rangle,\langle+\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle -a-2b,2a-b,c\rangle$

B$_5$ Q_3^- P_z $\{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$



2 cosets
10 tetrahedra

$a = \frac{1925}{624}$ 3.084935897436

$b = \frac{385}{624}$ .616987179487

$c = \frac{5}{8}$ .625000000000

$V = \frac{741125}{14976}$ 49.487513354701

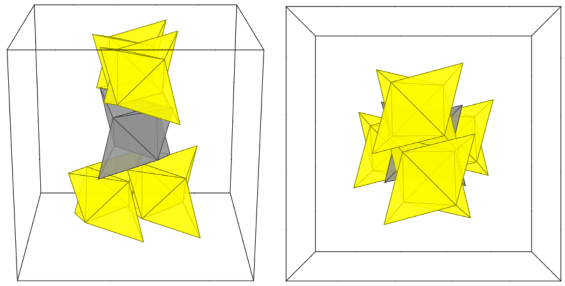
$D = \frac{79872}{148225}$ .538856468207

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle+1,-1,-1\rangle,\langle+\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,4c\rangle$

$V_A\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle = F_D[\langle+1,-1,+1\rangle,\langle-1,-1,-1\rangle,\langle-\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -b,a,c\rangle$

$a = 5b$ (minimum lattice volume)

$\mathbf{B}_5\ Q_3^{-}\ P_{xy}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$



2 cosets
10 tetrahedra

$a = \frac{30}{23}$ 1.304347826087

$b = \frac{5}{23}$ .217391304348

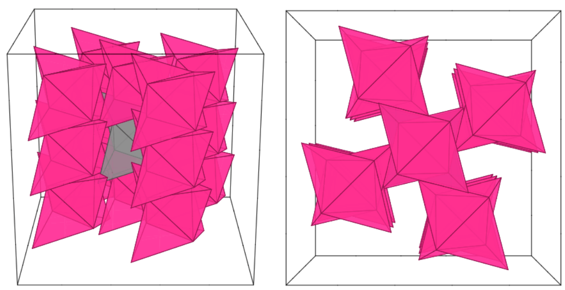
$c = \frac{205}{69}$ 2.971014492754

$V = \frac{1517000}{36501}$ 41.560505191639

$D = \frac{24334}{37925}$ .641634805537

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle 2a+2b,-2a+2b,0\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_D[\langle -1,-1,-1\rangle,\langle -\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}_5\ Q_{\mathrm{F}}^{=}\ P_z\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$



4 cosets
20 tetrahedra

$a = \frac{155}{54}$ 2.870370370370

$b = \frac{40}{27}$ 1.481481481481

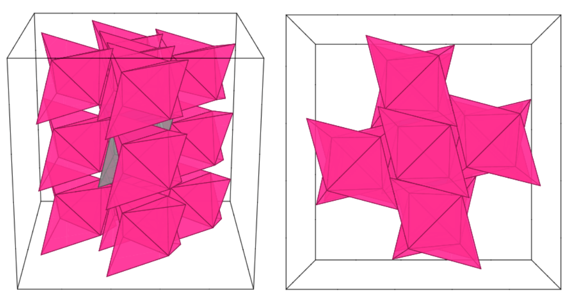
$c = \frac{5}{4}$ 1.250000000000

$V = \frac{152125}{1458}$ 104.338134430727

$D = \frac{15552}{30425}$ .511158586689

$\mathbf{B}_5\ Q_1^{=}\ P_z$

$\mathbf{B}_5\ Q_{\mathrm{F}}^{=}\ P_{xy}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$



4 cosets
20 tetrahedra

$a = \frac{60}{23}$ 2.608695652174

$b = \frac{10}{23}$ .434782608696

$c = \frac{107}{69}$ 1.550724637681

$V = \frac{3167200}{36501}$ 86.770225473275

$D = \frac{12167}{19795}$ .614650164183

$\mathbf{B}_5\ Q_1^{=}\ P_{xy}$

$\mathbf{B}_5\ Q_{\mathrm{F}}^{\neq}\ P_z^{\mathrm{VF}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets
20 tetrahedra

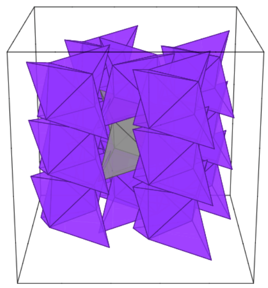
$a = \frac{45}{16}$ 2.812500000000

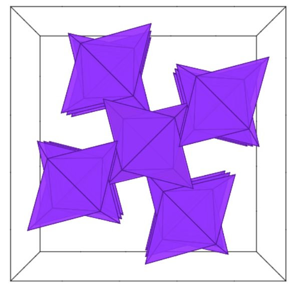
$b = \frac{25}{16}$ 1.562500000000

$c = \frac{5}{4}$ 1.250000000000

$V = \frac{6625}{64}$ 103.515625000000

$D = \frac{2048}{3975}$ .515220125786





$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle+1,-1,-1\rangle,\langle+\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,2c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_B[\langle-1,-1,-1\rangle,\langle-\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle a,b,0\rangle$

$V_A\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle = F_D[\langle+1,-1,+1\rangle,\langle-1,-1,-1\rangle,\langle+\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle -b,a,2c\rangle$

$\mathbf{B}_5\ Q_{\mathrm{F}}^{\neq}\ P_{xy}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets
20 tetrahedra

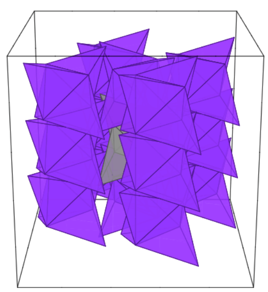
$a = 3$ 3.000000000000

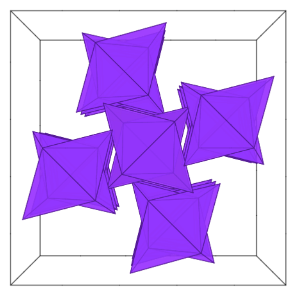
$b = 1$ 1.000000000000

$c = \frac{5}{4}$ 1.250000000000

$V = 100$ 100.000000000000

$D = \frac{8}{15}$ .533333333333





$E_A[\langle 1,1,1\rangle,\langle 5/3,-5/3,5/3\rangle] = E_C[\langle 1,-1,-1\rangle,\langle 5/3,5/3,-5/3\rangle] + \langle 0,0,2c\rangle$

$E_A[\langle 1,1,1\rangle,\langle 5/3,-5/3,5/3\rangle] = E_B[\langle -1,-1,-1\rangle,\langle -5/3,-5/3,5/3\rangle] + \langle a,b,0\rangle$

$a = 3b$ (minimum lattice volume)

$\mathbf{B}_5\ Q_{\mathrm{E}}^{=}\ P_z\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets
20 tetrahedra

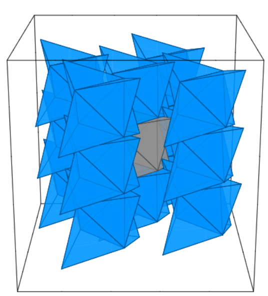
$a = \frac{235}{108}$ 2.175925925926

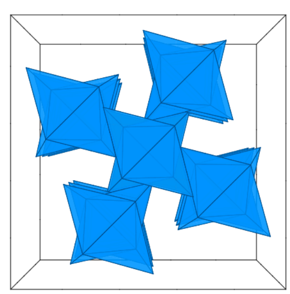
$b = \frac{25}{36}$ .694444444444

$c = \frac{5}{2}$ 2.500000000000

$V = \frac{152125}{1458}$ 104.338134430727

$D = \frac{15552}{30425}$ .511158586689



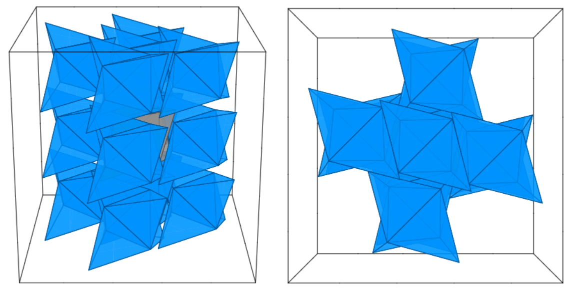


$\mathbf{B}_5\ Q_1^{=}\ P_z$

$\mathbf{B}_5\ Q_{\mathrm{E}}^{=}\ P_{xy}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

20 tetrahedra



$a = \frac{35}{23}$ 1.521739130435

$b = \frac{25}{23}$ 1.086956521739

$c = \frac{214}{69}$ 3.101449275362

$V = \frac{3167200}{36501}$ 86.770225473275

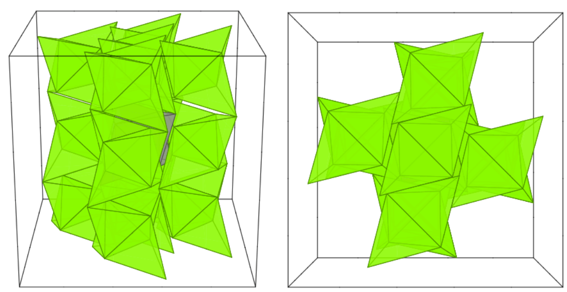
$D = \frac{12167}{19795}$ .614650164183

$\mathbf{B}_5\ Q_{1}^{=}\ P_{xy}$

$\mathbf{B}_5\ Q_{\mathrm{E}}^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

20 tetrahedra



$a = \frac{35}{23}$ 1.521739130435

$b = \frac{25}{23}$ 1.086956521739

$c = \frac{10}{3}$ 3.333333333333

$V = \frac{148000}{1587}$ 93.257718966604

$D = \frac{529}{925}$ .571891891892

$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle-2b,2a,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$

$V_A\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle = V_B\langle+\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle + \langle 0,0,c\rangle$

$\mathbf{B}_9$

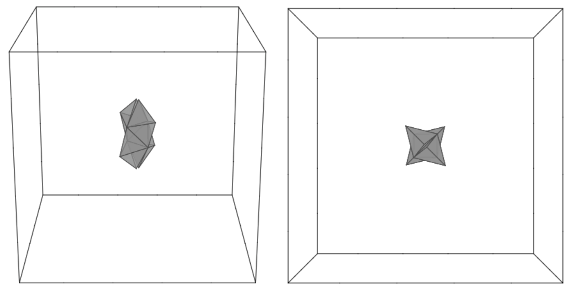
n cosets
9 tetrahedra

$U = 24$
24.000000000000

$V = 8c(a^2+b^2)$

$D = n{\cdot}U/V$

basis $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$



$\mathbf{B}_9\ Q_2^{=}\ P_z\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
9 tetrahedra

$a = \frac{996}{485}$
2.053608247423

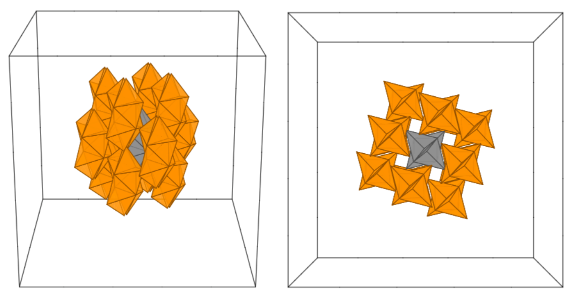
$b = \frac{1328}{1455}$
.912714776632

$c = \frac{8}{5}$
1.600000000000

$V = \frac{7054336}{109125}$
64.644545246277

$D = \frac{327375}{881792}$
.371261023008

$V_A\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle = F_A[\langle -1,+1,-1\rangle,\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle -\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle] + \langle a+b,-a+b,2c\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle +\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle] = E_A[\langle +1,-1,-1\rangle,\langle +\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle 0,0,4c\rangle$
$a = \frac{9}{4}b$ (minimum lattice volume)



$\mathbf{B}_9\ Q_2^{=}\ P_{xy}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
9 tetrahedra

$a = \frac{30}{23}$
1.304347826087

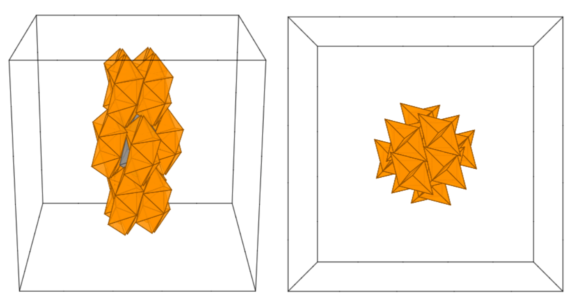
$b = \frac{5}{23}$
.217391304348

$c = \frac{435}{161}$
2.701863354037

$V = \frac{3219000}{85169}$
37.795442003546

$D = \frac{85169}{134125}$
.634997204101

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle 2a+2b,-2a+2b,0\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle,\langle +\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle] = E_A[\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle -\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle] + \{a+b,-a+b,2c\}$



$\mathbf{B}_9\ Q_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets
18 tetrahedra

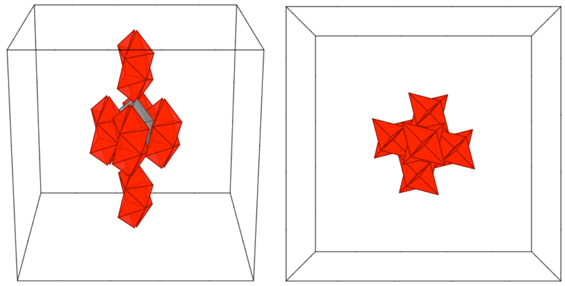
$a = \frac{35}{23}$ 1.521739130435

$b = \frac{25}{23}$ 1.086956521739

$c = \frac{16}{5}$ 3.200000000000

$V = \frac{47360}{529}$ 89.527410207940

$D = \frac{1587}{2960}$ .536148648649



$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle-2b,2a,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle] = E_C[\langle+1,-1,-1\rangle,\langle+\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle0,0,2c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$

$\mathbf{B}_9\ Q_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets
18 tetrahedra

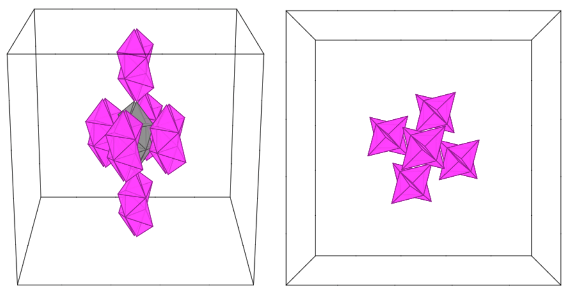
$a = 2$ 2.000000000000

$b = 1$ 1.000000000000

$c = \frac{31}{9}$ 3.444444444444

$V = \frac{1240}{9}$ 137.777777777778

$D = \frac{54}{155}$ .348387096774



$V_A\langle+\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle = V_C\langle+\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle + \langle0,0,2c\rangle$

$E_A[\langle+1,-1,-1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle-1,+1,+1\rangle,\langle-\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$

$a = 2b$ (minimum lattice volume)

$\mathbf{B}_9\ Q_3^{=}\ P_z\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets
36 tetrahedra

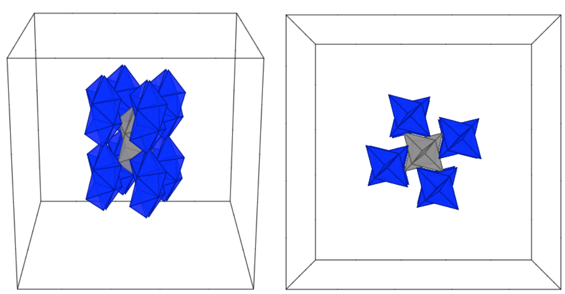
$a = \frac{4316}{1455}$ 2.966323024055

$b = \frac{332}{291}$ 1.140893470790

$c = \frac{16}{5}$ 3.200000000000

$V = \frac{28217344}{109125}$ 258.578180985109

$D = \frac{327375}{881792}$ .371261023008



$\mathbf{B}_9\ Q_2^{=}\ P_z$

$\mathbf{B}_9\ Q_3^{=}\ P_{xy}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

36 tetrahedra

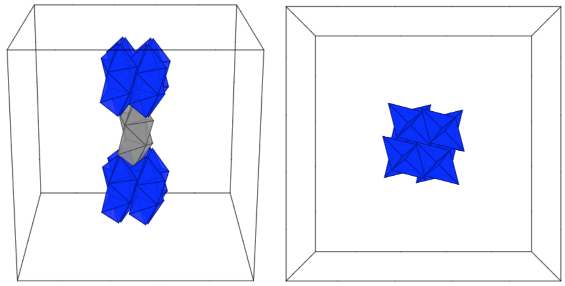
$a = \frac{35}{23}$ 1.521739130435

$b = \frac{25}{23}$ 1.086956521739

$c = \frac{870}{161}$ 5.403726708075

$V = \frac{12876000}{85169}$ 151.181768014184

$D = \frac{85169}{134125}$ .634997204101



$\mathbf{B}_9\ Q_2^{=}\ P_{xy}$

$\mathbf{B}_9\ Q_3^{\neq}\ P_z\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

36 tetrahedra

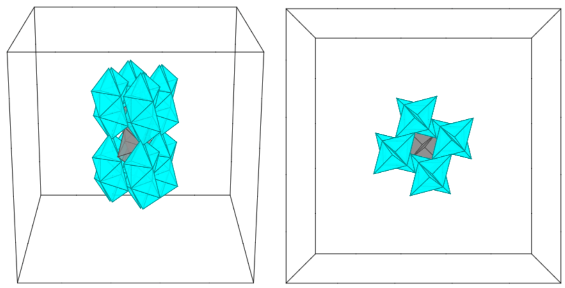
$a = \frac{47}{20}$ 2.350000000000

$b = \frac{59}{100}$ .590000000000

$c = \frac{16}{5}$ 3.200000000000

$V = \frac{469648}{3125}$ 150.287360000000

$D = \frac{18750}{29353}$ .638776274997



$E_A[\langle+1,+1,+1\rangle,\langle+\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle] = E_C[\langle+1,-1,-1\rangle,\langle+\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle 0,0,2c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-\frac{1}{9},+\frac{1}{9},+\frac{31}{9}\rangle] = E_D[\langle-1,-1,-1\rangle,\langle+\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle -b,a,c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-\frac{1}{9},+\frac{1}{9},+\frac{31}{9}\rangle] = E_D[\langle+\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle+\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}_9\ Q_3^{\neq}\ P_{xy}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

36 tetrahedra

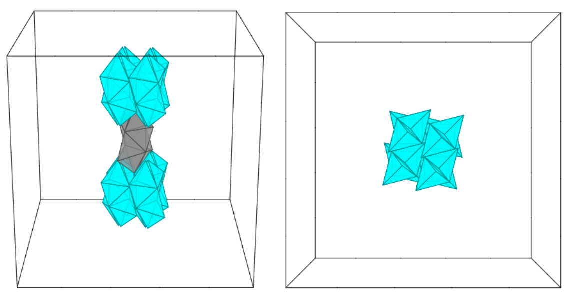
$a = \frac{35}{23}$ 1.521739130435

$b = \frac{25}{23}$ 1.086956521739

$c = \frac{820}{161}$ 5.093167701863

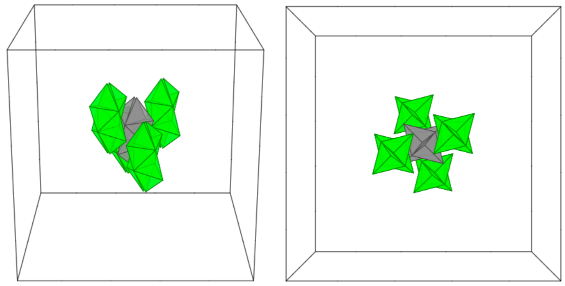
$V = \frac{12136000}{85169}$ 142.493160657047

$D = \frac{255507}{379250}$ .673716545814



$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$
$V_A\langle-\frac{1}{9},+\frac{1}{9},+\frac{31}{9}\rangle = F_D[\langle-1,-1,-1\rangle,\langle+\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle+\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle] + \langle -b,a,c\rangle$
$E_A[\langle+1,+1,+1\rangle,\langle-\frac{1}{9},+\frac{1}{9},+\frac{31}{9}\rangle] = E_D[\langle+\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle+\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle] + \langle -b,a,c\rangle$

B_9 Q_3^+ P_z $\{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$



2 cosets
18 tetrahedra

$a = \frac{63}{25}$ 2.520000000000

$b = \frac{21}{25}$ .840000000000

$c = \frac{8}{5}$ 1.600000000000

$V = \frac{56448}{625}$ 90.316800000000

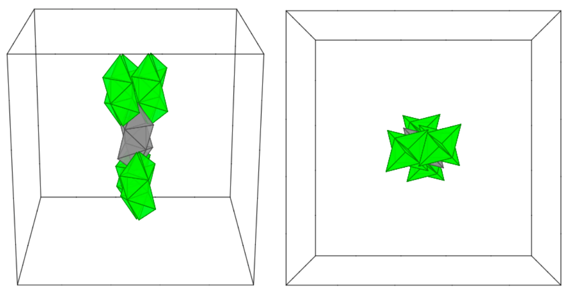
$D = \frac{625}{1176}$ .531462585034

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle] = E_A[\langle+1,-1,-1\rangle,\langle+\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle 0,0,4c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_B[\langle-1,-1,-1\rangle,\langle-\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle a,b,c\rangle$

$a = 3b$ (minimum lattice volume)

B_9 Q_3^+ P_{xy} $\{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$



2 cosets
18 tetrahedra

$a = \frac{30}{23}$ 1.304347826087

$b = \frac{5}{23}$ .217391304348

$c = \frac{105}{23}$ 4.565217391304

$V = \frac{777000}{12167}$ 63.861264074957

$D = \frac{24334}{32375}$ .751629343629

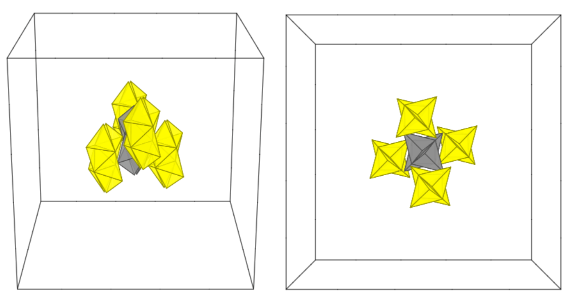
$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle 2a+2b,-2a+2b,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_B[\langle-1,-1,-1\rangle,\langle-\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle a,b,c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_B[\langle-\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle,\langle-\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle a,b,c\rangle$

B_9 Q_3^- P_z $\{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$



2 cosets
18 tetrahedra

$a = \frac{113}{39}$ 2.897435897436

$b = \frac{113}{195}$ .579487179487

$c = \frac{8}{5}$ 1.600000000000

$V = \frac{1634432}{14625}$ 111.756034188034

$D = \frac{43875}{102152}$ .429507009163

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle] = E_A[\langle+1,-1,-1\rangle,\langle+\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle 0,0,4c\rangle$

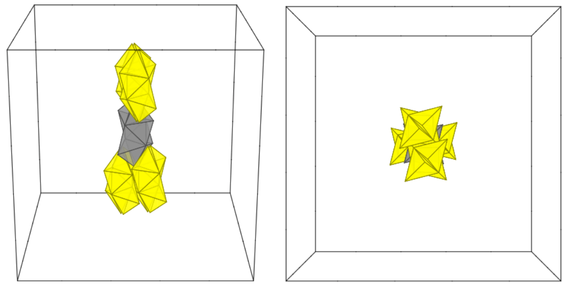
$V_A\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle = F_D[\langle+1,-1,+1\rangle,\langle-1,-1,-1\rangle,\langle-\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -b,a,c\rangle$

$a = 5b$ (minimum lattice volume)

$\mathbf{B}_9$ Q_3^- P_{xy} $\{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

18 tetrahedra



$a = \frac{30}{23}$ 1.304347826087

$b = \frac{5}{23}$ .217391304348

$c = \frac{785}{161}$ 4.875776397516

$V = \frac{5809000}{85169}$ 68.205567753525

$D = \frac{511014}{726125}$ .703754863143

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle 2a+2b,-2a+2b,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$

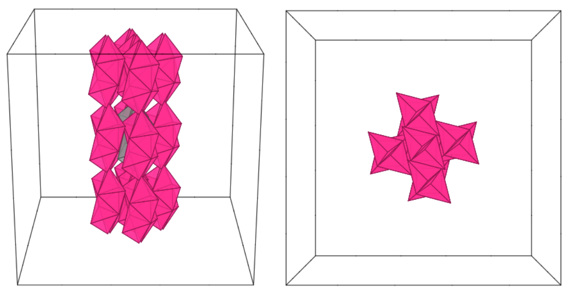
$V_A\langle-\frac{1}{9},+\frac{1}{9},+\frac{31}{9}\rangle = F_D[\langle-1,-1,-1\rangle,\langle+\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle+\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle] + \langle -b,a,c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle-\frac{1}{9},+\frac{1}{9},+\frac{31}{9}\rangle] = E_D[\langle+\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle+\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}_9$ $Q_F^=$ $\{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

36 tetrahedra



$a = \frac{60}{23}$ 2.608695652174

$b = \frac{10}{23}$ .434782608696

$c = \frac{16}{5}$ 3.200000000000

$V = \frac{94720}{529}$ 179.054820415879

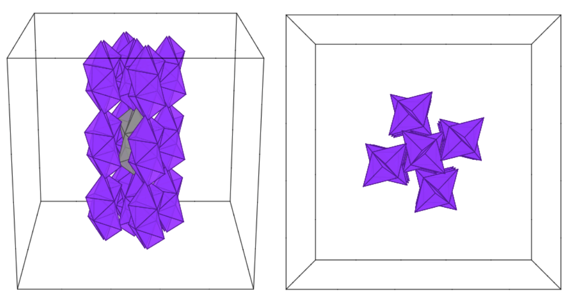
$D = \frac{1587}{2960}$ .536148648649

$\mathbf{B}_9$ $Q_1^=$

$\mathbf{B}_9$ $Q_F^{\neq}$ $\{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

36 tetrahedra



$a = 3$ 3.000000000000

$b = 1$ 1.000000000000

$c = \frac{16}{5}$ 3.200000000000

$V = 256$ 256.000000000000

$D = \frac{3}{8}$ .375000000000

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle] = E_C[\langle+1,-1,-1\rangle,\langle+\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle 0,0,2c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_B[\langle-1,-1,-1\rangle,\langle-\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle a,b,0\rangle$

$E_A[\langle-1,+1,-1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_D[\langle+1,-1,+1\rangle,\langle-\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -b,a,0\rangle$

$a = 3b$ (minimum lattice volume)

$\mathbf{B}_9\ \mathcal{Q}_{\mathrm{E}}^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

36 tetrahedra

$a = \frac{35}{23}$ 1.521739130435

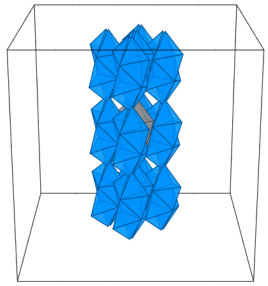
$b = \frac{25}{23}$ 1.086956521739

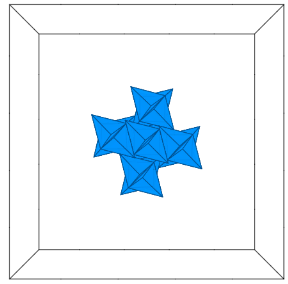
$c = \frac{32}{5}$ 6.400000000000

$V = \frac{94720}{529}$ 179.054820415879

$D = \frac{1587}{2960}$ .536148648649

$\mathbf{B}_9\ \mathcal{Q}_{1}^{=}$





$\mathbf{B}_9\ \mathcal{Q}_{\mathrm{E}}^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

36 tetrahedra

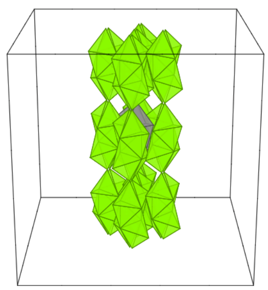
$a = \frac{35}{23}$ 1.521739130435

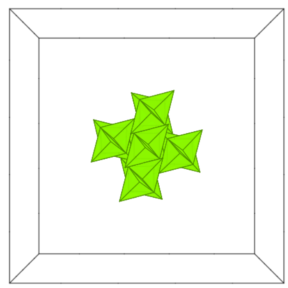
$b = \frac{25}{23}$ 1.086956521739

$c = \frac{62}{9}$ 6.888888888889

$V = \frac{917600}{4761}$ 192.732619197648

$D = \frac{14283}{28675}$ .498099389712





$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle -1,-1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$

$V_A\langle +\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle = V_B\langle +\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle + \langle 0,0,c\rangle$

$V_A\langle +\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle = V_D\langle +\frac{1}{9},+\frac{1}{9},+\frac{31}{9}\rangle + \langle 0,0,-c\rangle$

$\mathbf{B}_9^*$

n cosets
9 tetrahedra

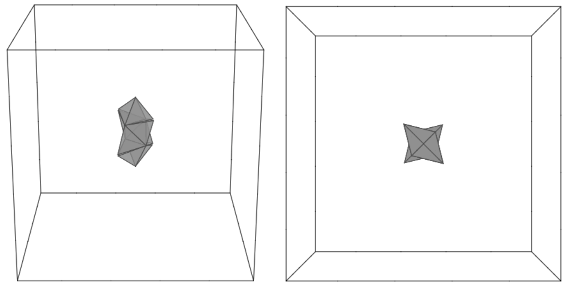
$U = 24$

24.000000000000

$V = 8c(a^2+b^2)$

$D = n \cdot U/V$

basis $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$



$\mathbf{B}_9^*\ Q_2^{=}\ P_z\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
9 tetrahedra

$a = \frac{2}{7}+\frac{5}{7}\sqrt{6}$ 2.035349816274

$b = \frac{5}{7}+\frac{1}{21}\sqrt{6}$ .830928082990

$c = \frac{1}{2}+\frac{1}{2}\sqrt{6}$ 1.724744871392

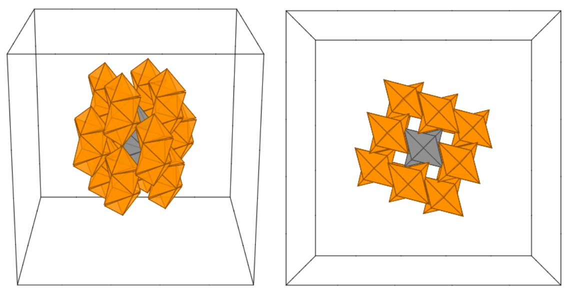
$V = \frac{548}{21}+\frac{116}{7}\sqrt{6}$ 66.686782404216

$D = -\frac{17262}{26645}+\frac{10962}{26645}\sqrt{6}$ .359891407783

$V_A\langle 0,0,+1+\sqrt{6}\rangle = V_A\langle 0,0,-1-\sqrt{6}\rangle + \langle 0,0,4c\rangle$

$V_A\langle +\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle = F_A[\langle -1,+1,-1\rangle,\langle 0,0,-1-\sqrt{6}\rangle,\langle -\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6}\rangle] + \langle a+b,-a+b,2c\rangle$

$a = \sqrt{6}\, b$ (minimum lattice volume)



$\mathbf{B}_9^*\ Q_2^{=}\ P_{xy}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
9 tetrahedra

$a = \frac{30}{23}$ 1.304347826087

$b = \frac{5}{23}$ .217391304348

$c = \frac{11}{46}+\sqrt{6}$ 2.688620177566

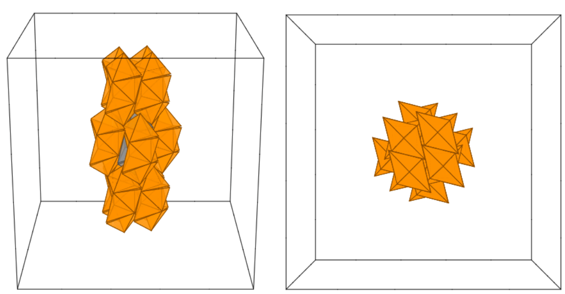
$V = \frac{40700}{12167}+\frac{7400}{529}\sqrt{6}$ 37.610187739106

$D = -\frac{803022}{11631875}+\frac{3358092}{11631875}\sqrt{6}$ .638124972055

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle 2a+2b,-2a+2b,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$

$E_A[\langle 0,0,+1+\sqrt{6}\rangle,\langle +\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] = E_A[\langle 0,0,-1-\sqrt{6}\rangle,\langle -\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6}\rangle] + \langle a+b,-a+b,2c\rangle$



$\mathbf{B}_9^* \, Q_1^= \, \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets

18 tetrahedra

$a = \frac{35}{23}$

1.521739130435

$b = \frac{25}{23}$

1.086956521739

$c = 1+\sqrt{6}$

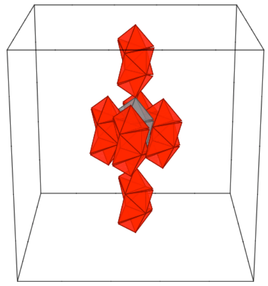
3.449489742783

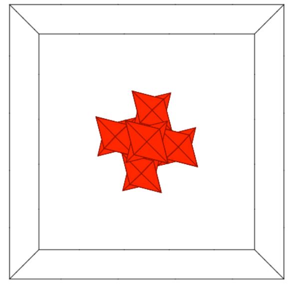
$V = \frac{14800}{529}+\frac{14800}{529}\sqrt{6}$

96.507463503197

$D = -\frac{1587}{4625}+\frac{1587}{4625}\sqrt{6}$

.497370858767





$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$

$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_C\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,2c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle -1,-1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$

$\mathbf{B}_9^* \, Q_1^{\neq} \, \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets

18 tetrahedra

$a = \frac{2848}{1565}+\frac{128}{1565}\sqrt{6}$

2.020149959793

$b = -\frac{164}{1565}+\frac{696}{1565}\sqrt{6}$

.984565406375

$c = 1+\sqrt{6}$

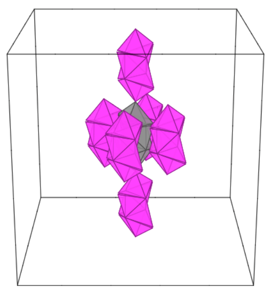
3.449489742783

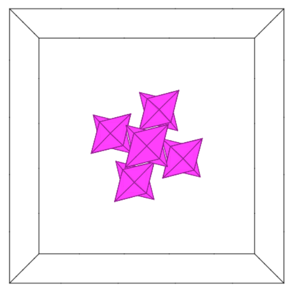
$V = \frac{14464}{313}+\frac{11904}{313}\sqrt{6}$

139.369731303805

$D = -\frac{339}{1000}+\frac{279}{1000}\sqrt{6}$

.344407638237





$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_C\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,2c\rangle$

$E_A[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle -1,+1,+1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle a+b,-a+b,0\rangle$

$b = (-\frac{1}{8}+\frac{1}{4}\sqrt{6}\,)a$ (minimum lattice volume)

$\mathbf{B}_9^* \, Q_3^= \, P_z \, \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

36 tetrahedra

$a = 1+\frac{16}{21}\sqrt{6}$

2.866277899263

$b = -\frac{3}{7}+\frac{2}{3}\sqrt{6}$

1.204421733284

$c = 1+\sqrt{6}$

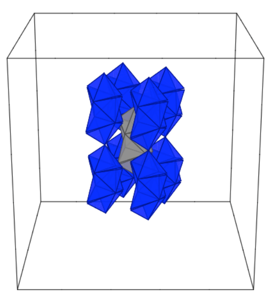
3.449489742783

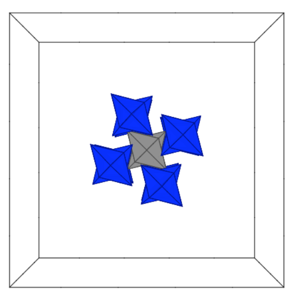
$V = \frac{2192}{21}+\frac{464}{7}\sqrt{6}$

266.747129616866

$D = -\frac{17262}{26645}+\frac{10962}{26645}\sqrt{6}$

.359891407783





$\mathbf{B}_9^* \, Q_2^= \, P_z$

$\mathbf{B}_9^* \, Q_3^{=} \, P_{xy} \, \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets
36 tetrahedra

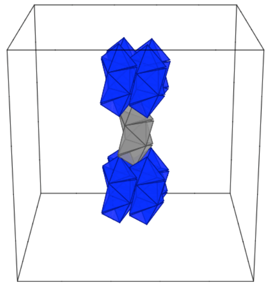
$a = \frac{35}{23}$
1.521739130435

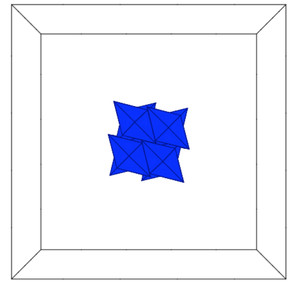
$b = \frac{25}{23}$
1.086956521739

$c = \frac{11}{23}+2\sqrt{6}$
5.377240355132

$V = \frac{162800}{12167}+\frac{29600}{529}\sqrt{6}$
150.440750956422

$D = -\frac{803022}{11631875}+\frac{3358092}{11631875}\sqrt{6}$
.638124972055





$\mathbf{B}_9^* \, Q_2^{=} \, P_{xy}$

$\mathbf{B}_9^* \, Q_3^{\neq} \, P_z \, \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets
36 tetrahedra

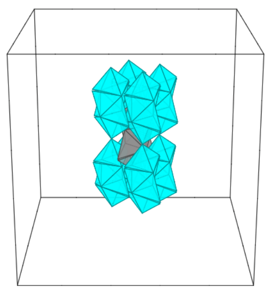
$a = \frac{48}{19}-\frac{2}{19}\sqrt{6}$
2.268474763918

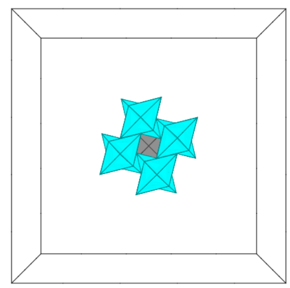
$b = \frac{46}{95}+\frac{6}{95}\sqrt{6}$
.638915141649

$c = 1+\sqrt{6}$
3.449489742783

$V = \frac{280352}{9025}+\frac{450272}{9025}\sqrt{6}$
153.272980106645

$D = -\frac{26283}{123125}+\frac{42213}{123125}\sqrt{6}$
.626333486393





$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_C\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,2c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle -1,-1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\,\rangle] = E_D[\langle -1,-1,-1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle -b,a,c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\,\rangle] = E_D[\langle 0,0,-1-\sqrt{6}\,\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}_9^* \, Q_3^{\neq} \, P_{xy} \, \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets
36 tetrahedra

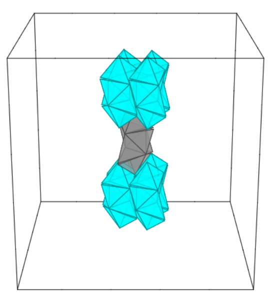
$a = \frac{35}{23}$
1.521739130435

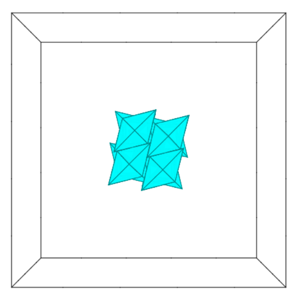
$b = \frac{25}{23}$
1.086956521739

$c = \frac{16}{23}+\frac{41}{23}\sqrt{6}$
5.062133889309

$V = \frac{236800}{12167}+\frac{606800}{12167}\sqrt{6}$
141.624917886154

$D = -\frac{584016}{4546375}+\frac{1496541}{4546375}\sqrt{6}$
.677846818433





$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle -1,-1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\,\rangle] = E_D[\langle -1,-1,-1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle -b,a,c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\,\rangle] = E_D[\langle 0,0,-1-\sqrt{6}\,\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}_9^* \; Q_3^+ \; P_z \; \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

18 tetrahedra

$a = \frac{11513}{7825}+\frac{3243}{7825}\sqrt{6}$ 2.486478624389

$b = \frac{15159}{7825}-\frac{3451}{7825}\sqrt{6}$ .856972638678

$c = \frac{1}{2}+\frac{1}{2}\sqrt{6}$ 1.724744871392

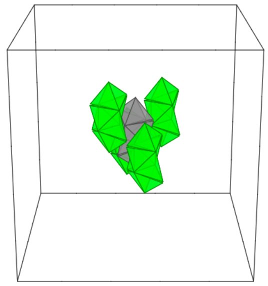
$V = \frac{162136}{7825}+\frac{238696}{7825}\sqrt{6}$ 95.440179379345

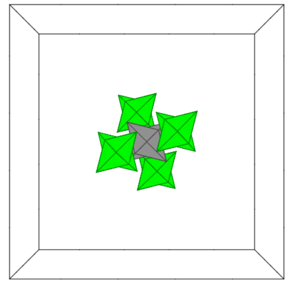
$D = -\frac{121602}{630125}+\frac{179022}{630125}\sqrt{6}$ .502932835124

$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_A\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,4c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_B[\langle -1,-1,-1\rangle,\langle -\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle a,b,c\rangle$

$b = (\frac{87}{25}-\frac{32}{25}\sqrt{6}\,)a$ (minimum lattice volume)





$\mathbf{B}_9^* \; Q_3^+ \; P_{xy} \; \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

18 tetrahedra

$a = \frac{30}{23}$ 1.304347826087

$b = \frac{5}{23}$ .217391304348

$c = \frac{67}{46}+\frac{57}{46}\sqrt{6}$ 4.491759029101

$V = \frac{247900}{12167}+\frac{210900}{12167}\sqrt{6}$ 62.833680180239

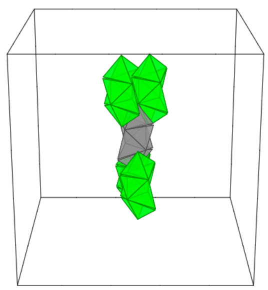
$D = -\frac{9782268}{13879625}+\frac{8322228}{13879625}\sqrt{6}$ .763921512512

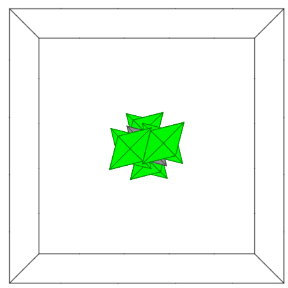
$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle 2a+2b,-2a+2b,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\,\rangle] = E_B[\langle -1,-1,-1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle a,b,c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\,\rangle] = E_B[\langle 0,0,-1-\sqrt{6}\,\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle a,b,c\rangle$





$\mathbf{B}_9^* \; Q_3^- \; P_z \; \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

18 tetrahedra

$a = \frac{485}{156}-\frac{5}{52}\sqrt{6}$ 2.873446499091

$b = \frac{97}{156}-\frac{1}{52}\sqrt{6}$ .574689299818

$c = \frac{1}{2}+\frac{1}{2}\sqrt{6}$ 1.724744871392

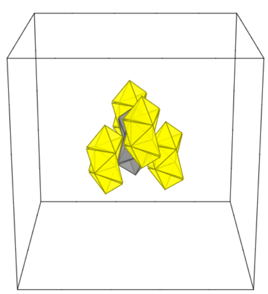
$V = \frac{5971}{234}+\frac{8881}{234}\sqrt{6}$ 118.482557289134

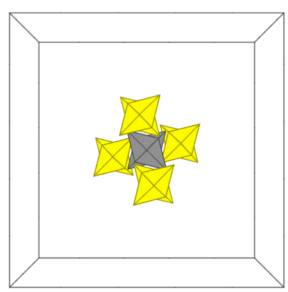
$D = -\frac{67066272}{437580125}+\frac{99751392}{437580125}\sqrt{6}$ .405122923561

$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_A\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,4c\rangle$

$V_A\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle = F_D[\langle +1,-1,+1\rangle,\langle -1,-1,-1\rangle,\langle -\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -b,a,c\rangle$

$a = 5b$ (minimum lattice volume)

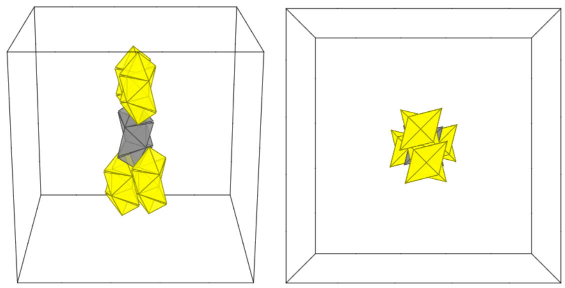




$\mathbf{B}_9^* \, Q_3^- \, P_{xy} \, \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

18 tetrahedra



$a = \frac{30}{23}$

1.304347826087

$b = \frac{5}{23}$

.217391304348

$c = \frac{57}{46}+\frac{67}{46}\sqrt{6}$

4.806865494923

$V = \frac{210900}{12167}+\frac{247900}{12167}\sqrt{6}$

67.241596715374

$D = -\frac{2774076}{7302875}+\frac{3260756}{7302875}\sqrt{6}$

.713843845844

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle 2a+2b,-2a+2b,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle-1,-1,+1\},\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle-2b,2a,0\rangle$

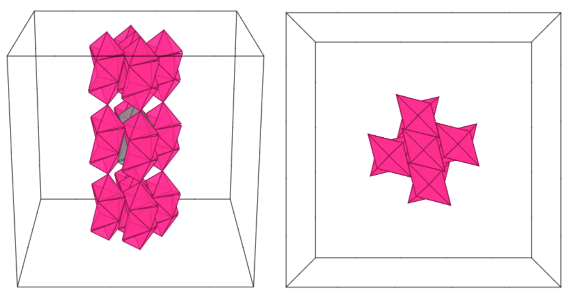
$E_A[\langle+1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\rangle] = E_D[\langle-1,-1,-1\rangle,\langle+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6}\rangle] + \langle-b,a,c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\rangle] = E_D[\langle 0,0,-1-\sqrt{6}\rangle,\langle+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6}\rangle] + \langle-b,a,c\rangle$

$\mathbf{B}_9^* \, Q_F^= \, \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

36 tetrahedra



$a = \frac{60}{23}$

2.608695652174

$b = \frac{10}{23}$

.434782608696

$c = 1+\sqrt{6}$

3.449489742783

$V = \frac{29600}{529}+\frac{29600}{529}\sqrt{6}$

193.014927006393

$D = -\frac{1587}{4625}+\frac{1587}{4625}\sqrt{6}$

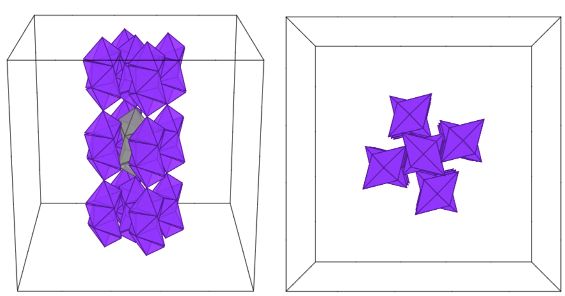
.497370858767

$\mathbf{B}_9^* \, Q_1^=$

$\mathbf{B}_9^* \, Q_F^{\neq} \, \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

36 tetrahedra



$a = \frac{2684}{1565}+\frac{824}{1565}\sqrt{6}$

3.004715366168

$b = \frac{3012}{1565}-\frac{568}{1565}\sqrt{6}$

1.035584553418

$c = 1+\sqrt{6}$

3.449489742783

$V = \frac{28928}{313}+\frac{23808}{313}\sqrt{6}$

278.739462607610

$D = -\frac{339}{1000}+\frac{279}{1000}\sqrt{6}$

.344407638237

$V_A\langle 0,0,+1+\sqrt{6}\rangle = V_C\langle 0,0,-1-\sqrt{6}\rangle + \langle 0,0,2c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] = E_B[\langle-1,-1,-1\rangle,\langle-\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle a,b,0\rangle$

$E_A[\langle-1,+1,-1\rangle,\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_D[\langle+1,-1,+1\rangle,\langle-\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] + \langle-b,a,0\rangle$

$b = (\frac{87}{25}-\frac{32}{25}\sqrt{6})a$ (minimum lattice volume)

$\mathbf{B}_9^* \; Q_{\mathrm{E}}^{=} \; \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

36 tetrahedra

$a = \frac{35}{23}$

1.521739130435

$b = \frac{25}{23}$

1.086956521739

$c = 2+2\sqrt{6}$

6.898979485566

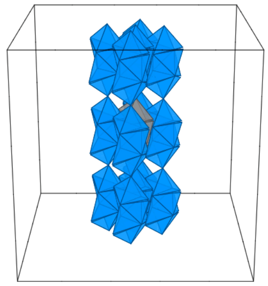
$V = \frac{29600}{529}+\frac{29600}{529}\sqrt{6}$

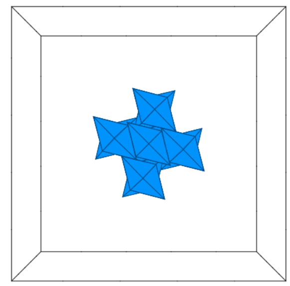
193.014927006393

$D = -\frac{1587}{4625}+\frac{1587}{4625}\sqrt{6}$

.497370858767

$\mathbf{B}_9^* \; Q_1^{=}$





$\mathbf{B}_9^* \; Q_{\mathrm{E}}^{\neq} \; \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

36 tetrahedra

$a = \frac{35}{23}$

1.521739130435

$b = \frac{25}{23}$

1.086956521739

$c = 2+2\sqrt{6}$

6.898979485566

$V = \frac{29600}{529}+\frac{29600}{529}\sqrt{6}$

193.014927006393

$D = -\frac{1587}{4625}+\frac{1587}{4625}\sqrt{6}$

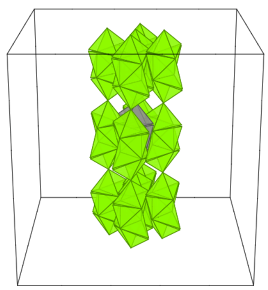
.497370858767

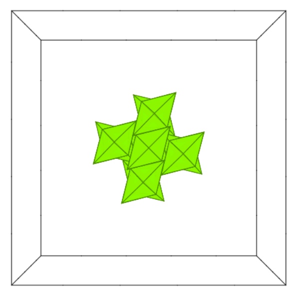
$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] + \langle -2b,2a,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle -1,-1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] + \langle a+b,-a+b,0\rangle$

$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_B\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,c\rangle$

$V_A\langle 0,0,-1-\sqrt{6}\,\rangle = V_D\langle 0,0,+1+\sqrt{6}\,\rangle + \langle 0,0,-c\rangle$





$\mathbf{B}^9$

n cosets
9 tetrahedra

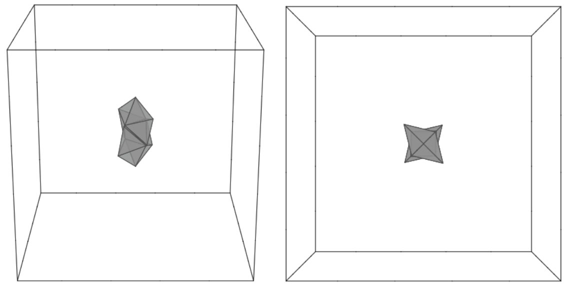
$U = 24$

24.000000000000

$V = 8c(a^2+b^2)$

$D = n{\cdot}U/V$

basis $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$



$\mathbf{B}^9\ Q_2^{=}\ P_z\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
9 tetrahedra

$a = \frac{2}{7}+\frac{5}{7}\sqrt{6}$

2.035349816274

$b = \frac{5}{7}+\frac{1}{21}\sqrt{6}$

.830928082990

$c = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

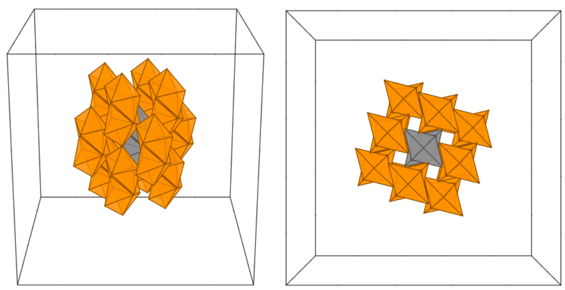
$V = \frac{548}{21}+\frac{116}{7}\sqrt{6}$

66.686782404216

$D = -\frac{17262}{26645}+\frac{10962}{26645}\sqrt{6}$

.359891407783

$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_A\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,4c\rangle$
$V_A\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle = F_A[\langle -1,+1,-1\rangle,\langle 0,0,-1-\sqrt{6}\,\rangle,\langle -\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle a+b,-a+b,2c\rangle$
$a = \sqrt{6}\,b$ (minimum lattice volume)



$\mathbf{B}^9\ Q_2^{=}\ P_{xy}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
9 tetrahedra

$a = -\frac{168}{71}+\frac{106}{71}\sqrt{6}$

1.290787503310

$b = -\frac{88}{71}+\frac{42}{71}\sqrt{6}$

.209557312632

$c = \frac{199}{71}-\frac{3}{71}\sqrt{6}$

2.699317334812

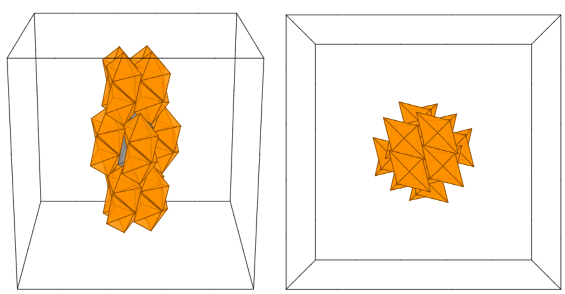
$V = \frac{187630208}{357911}-\frac{71203968}{357911}\sqrt{6}$

36.927668438630

$D = \frac{4397583}{13056080}+\frac{1668843}{13056080}\sqrt{6}$

.649919180245

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_A[\langle -1,-1,+1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle 2a+2b,-2a+2b,0\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle 0,0,+1+\sqrt{6}\,\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_A[\langle 0,0,-1-\sqrt{6}\,\rangle,\langle -\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle a+b,-a+b,2c\rangle$



$\mathbf{B}^9\ Q_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets
18 tetrahedra

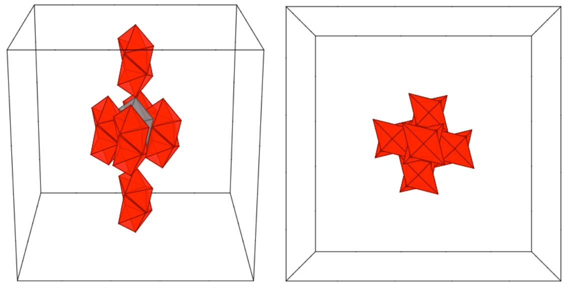
$a = -\frac{256}{71}+\frac{148}{71}\sqrt{6}$
1.500344815942

$b = -\frac{80}{71}+\frac{64}{71}\sqrt{6}$
1.081230190678

$c = 1+\sqrt{6}$
3.449489742783

$V = -\frac{2305280}{5041}+\frac{1135360}{5041}\sqrt{6}$
94.380613839776

$D = \frac{5403}{23440}+\frac{2661}{23440}\sqrt{6}$
.508579019008



$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_C\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,2c\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_C[\langle -1,-1,+1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle a+b,-a+b,0\rangle$

$\mathbf{B}^9\ Q_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets
18 tetrahedra

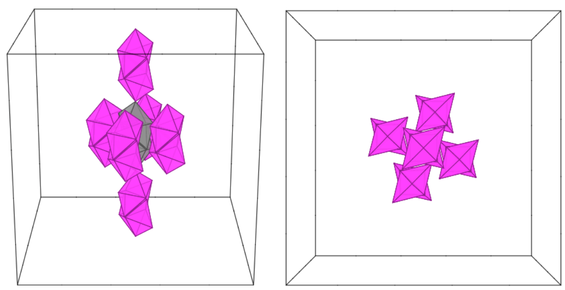
$a = \frac{62600}{76129}+\frac{35240}{76129}\sqrt{6}$
1.956153614729

$b = \frac{46600}{76129}+\frac{12126}{76129}\sqrt{6}$
1.003598388268

$c = 1+\sqrt{6}$
3.449489742783

$V = \frac{5010112}{76129}+\frac{2098112}{76129}\sqrt{6}$
133.318654168718

$D = -\frac{234849}{16820}+\frac{98349}{16820}\sqrt{6}$
.360039638108



$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_C\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,2c\rangle$
$E_A[\langle +\frac{4}{9}\sqrt{6}\,,-\frac{4}{9}\sqrt{6}\,,+1-\frac{7}{9}\sqrt{6}\,\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_C[\langle -1,+1,+1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle a+b,-a+b,0\rangle$
$b = (-\frac{1}{10}+\frac{1}{4}\sqrt{6}\,)a$ (minimum lattice volume)

$\mathbf{B}^9\ Q_3^{=}\ P_z\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

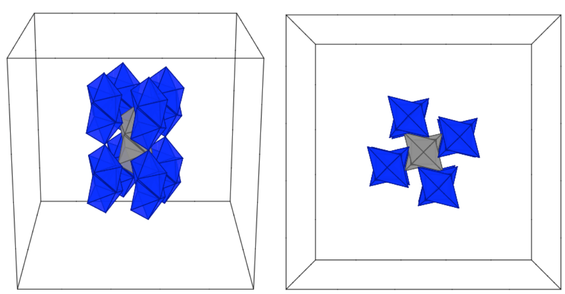
$a = 1+\frac{16}{21}\sqrt{6}$
2.866277899263

$b = -\frac{3}{7}+\frac{2}{3}\sqrt{6}$
1.204421733284

$c = 1+\sqrt{6}$
3.449489742783

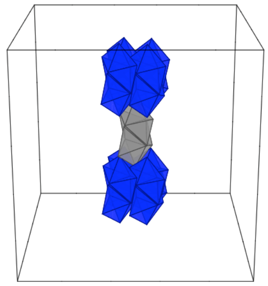
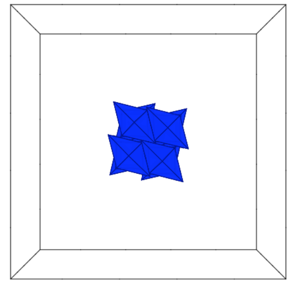
$V = \frac{2192}{21}+\frac{464}{7}\sqrt{6}$
266.747129616866

$D = -\frac{17262}{26645}+\frac{10962}{26645}\sqrt{6}$
.359891407783

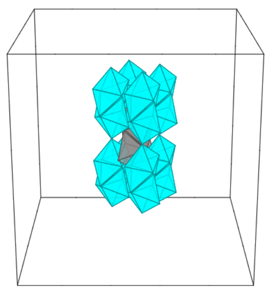
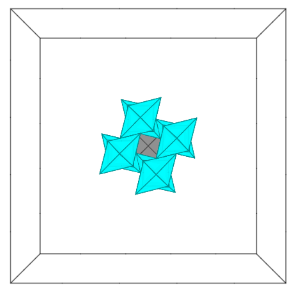


$\mathbf{B}^9\ Q_2^{=}\ P_z$

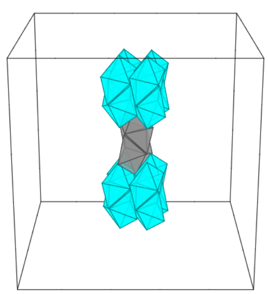
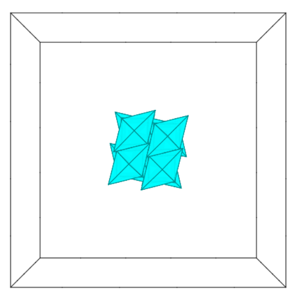
$\mathbf{B}^9\ Q_3^{=}\ P_{xy}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$
4 cosets
36 tetrahedra
$a = -\frac{256}{71}+\frac{148}{71}\sqrt{6}$
1.500344815942
$b = -\frac{80}{71}+\frac{64}{71}\sqrt{6}$
1.081230190678
$c = \frac{398}{71}-\frac{6}{71}\sqrt{6}$
5.398634669624
$V = \frac{750520832}{357911}-\frac{284815872}{357911}\sqrt{6}$
147.710673754518
$D = \frac{4397583}{13056080}+\frac{1668843}{13056080}\sqrt{6}$
.649919180245
$\mathbf{B}^9\ Q_2^{=}\ P_{xy}$

$\mathbf{B}^9\ Q_3^{\neq}\ P_{z}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$
4 cosets
36 tetrahedra
$a = -\frac{144}{23}+\frac{80}{23}\sqrt{6}$
2.259094757507
$b = -\frac{74}{23}+\frac{36}{23}\sqrt{6}$
.616592640878
$c = 1+\sqrt{6}$
3.449489742783
$V = -\frac{782560}{529}+\frac{352160}{529}\sqrt{6}$
151.327614023675
$D = \frac{14673}{48625}+\frac{6603}{48625}\sqrt{6}$
.634385208670
$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_C\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,2c\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_C[\langle -1,-1,+1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle a+b,-a+b,0\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\,\rangle] = E_D[\langle -1,-1,-1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle -b,a,c\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\,\rangle] = E_D[\langle 0,0,-1-\sqrt{6}\,\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle -b,a,c\rangle$

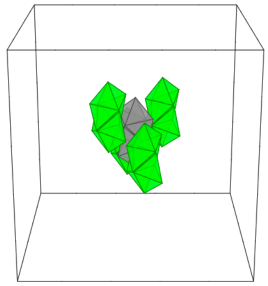
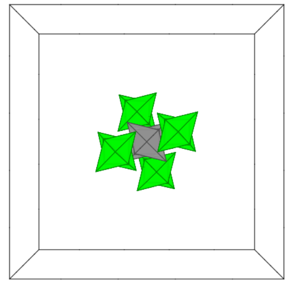



$\mathbf{B}^9\ Q_3^{\neq}\ P_{xy}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$
4 cosets
36 tetrahedra
$a = -\frac{256}{71}+\frac{148}{71}\sqrt{6}$
1.500344815942
$b = -\frac{80}{71}+\frac{64}{71}\sqrt{6}$
1.081230190678
$c = \frac{58}{71}+\frac{124}{71}\sqrt{6}$
5.094883494438
$V = -\frac{406204928}{357911}+\frac{186201088}{357911}\sqrt{6}$
139.399814901101
$D = \frac{2380107}{7336720}+\frac{545511}{3668360}\sqrt{6}$
.688666624616
$E_A[\langle +1,+1,+1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_C[\langle -1,-1,+1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle a+b,-a+b,0\rangle$
$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = E_D[\langle -1,-1,-1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle -b,a,c\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\,\rangle] = E_D[\langle 0,0,-1-\sqrt{6}\,\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}^9\ Q_3^+\ P_z\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets
18 tetrahedra

$a = \frac{94524}{76129}+\frac{40515}{76129}\sqrt{6}$ 2.545220309328

$b = \frac{17196}{76129}+\frac{18485}{76129}\sqrt{6}$ .820644142119

$c = \frac{1}{2}+\frac{1}{2}\sqrt{6}$ 1.724744871392

$V = \frac{3725232}{76129}+\frac{1546032}{76129}\sqrt{6}$ 98.677527959313

$D = -\frac{77609}{2645}+\frac{32209}{2645}\sqrt{6}$ .486432939623

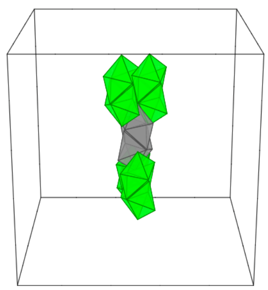
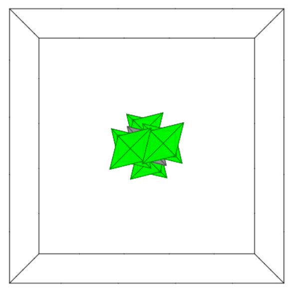
$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_A\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,4c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_B[\langle -\frac{4}{9}\sqrt{6}\,,-\frac{4}{9}\sqrt{6}\,,+1-\frac{7}{9}\sqrt{6}\,\rangle,\langle -\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle a,b,c\rangle$

$b = (\frac{91}{29}-\frac{100}{87}\sqrt{6}\,)a$ (minimum lattice volume)

$\mathbf{B}^9\ Q_3^+\ P_{xy}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets
18 tetrahedra

$a = -\frac{168}{71}+\frac{106}{71}\sqrt{6}$ 1.290787503310

$b = -\frac{88}{71}+\frac{42}{71}\sqrt{6}$ .209557312632

$c = -\frac{262}{71}+\frac{238}{71}\sqrt{6}$ 4.520824771583

$V = -\frac{730200320}{357911}+\frac{307139840}{357911}\sqrt{6}$ 61.846569901642

$D = \frac{1711407}{4477040}+\frac{719859}{4477040}\sqrt{6}$ .776114181859

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_A[\langle -1,-1,+1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle 2a+2b,-2a+2b,0\rangle$

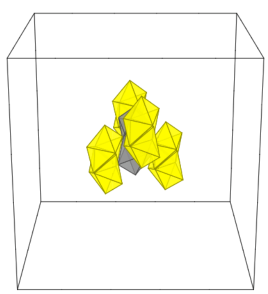
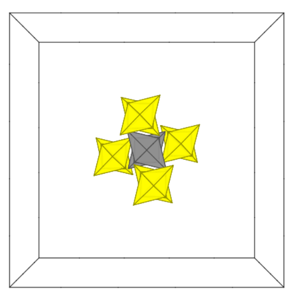
$E_A[\langle +1,+1,+1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] = E_A[\langle -1,-1,+1\rangle,\langle +\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle -2b,2a,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\,\rangle] = E_B[\langle -1,-1,-1\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle a,b,c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\,\rangle] = E_B[\langle 0,0,-1-\sqrt{6}\,\rangle,\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,\rangle] + \langle a,b,c\rangle$

$\mathbf{B}^9\ Q_3^-\ P_z\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets
18 tetrahedra

$a = \frac{217}{158}+\frac{146}{237}\sqrt{6}$ 2.882386086271

$b = -\frac{13}{158}+\frac{61}{237}\sqrt{6}$ .548180904261

$c = \frac{1}{2}+\frac{1}{2}\sqrt{6}$ 1.724744871392

$V = \frac{13714}{237}+\frac{5894}{237}\sqrt{6}$ 118.781825080017

$D = -\frac{39002616}{5090405}+\frac{16762536}{5090405}\sqrt{6}$ .404102226647

$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_A\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,4c\rangle$

$V_A\langle -\frac{2}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle = F_D[\langle +1,-1,+1\rangle,\langle -\frac{4}{9}\sqrt{6}\,,-\frac{4}{9}\sqrt{6}\,,+1-\frac{7}{9}\sqrt{6}\,\rangle,\langle -\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,,+1+\frac{1}{3}\sqrt{6}\,\rangle] + \langle -b,a,c\rangle$

$b = (\frac{79}{29}-\frac{30}{29}\sqrt{6}\,)a$ (minimum lattice volume)

$\mathbf{B}^9\ Q_3^-\ P_{xy}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

18 tetrahedra

$a = -\frac{168}{71}+\frac{106}{71}\sqrt{6}$

1.290787503310

$b = -\frac{88}{71}+\frac{42}{71}\sqrt{6}$

.209557312632

$c = \frac{78}{71}+\frac{108}{71}\sqrt{6}$

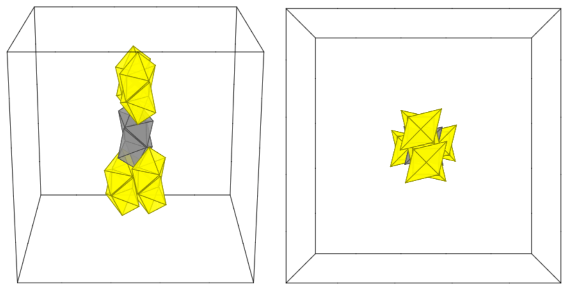
4.824575946769

$V = -\frac{151837440}{357911}+\frac{71631360}{357911}\sqrt{6}$

66.001999328350

$D = \frac{39541}{117200}+\frac{9327}{58600}\sqrt{6}$

.727250696774



$E_A[\langle+1,+1,+1\rangle,\langle+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle-\frac{2}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] + \langle 2a+2b,-2a+2b,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle-\frac{2}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] + \langle -2b,2a,0\rangle$

$V_A\langle 0,0,+1+\sqrt{6}\rangle = E_D[\langle-1,-1,-1\rangle,\langle+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6}\rangle] + \langle -b,a,c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle 0,0,+1+\sqrt{6}\rangle] = E_D[\langle 0,0,-1-\sqrt{6}\rangle,\langle+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6}\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}^9\ Q_F^=\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

$a = -\frac{336}{71}+\frac{212}{71}\sqrt{6}$

2.581575006620

$b = -\frac{176}{71}+\frac{84}{71}\sqrt{6}$

.419114625265

$c = 1+\sqrt{6}$

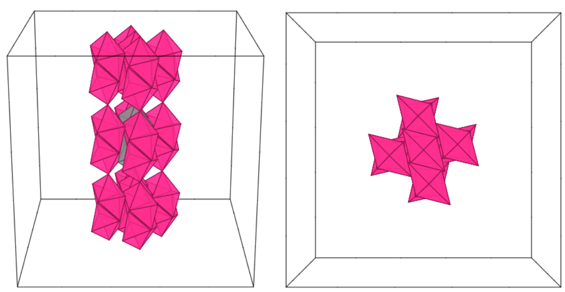
3.449489742783

$V = -\frac{4610560}{5041}+\frac{2270720}{5041}\sqrt{6}$

188.761227679551

$D = \frac{5403}{23440}+\frac{2661}{23440}\sqrt{6}$

.508579019008



$\mathbf{B}^9\ Q_1^=$

$\mathbf{B}^9\ Q_F^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

36 tetrahedra

$a = \frac{109200}{76129}+\frac{47366}{76129}\sqrt{6}$

2.958432806902

$b = \frac{16000}{76129}+\frac{23114}{76129}\sqrt{6}$

.953874422555

$c = 1+\sqrt{6}$

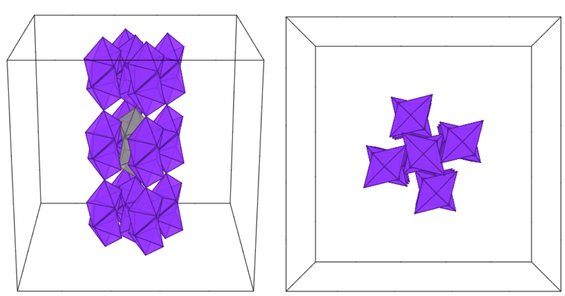
3.449489742783

$V = \frac{10020224}{76129}+\frac{4196224}{76129}\sqrt{6}$

266.637308337435

$D = -\frac{234849}{16820}+\frac{98349}{16820}\sqrt{6}$

.360039638108



$V_A\langle 0,0,+1+\sqrt{6}\rangle = V_C\langle 0,0,-1-\sqrt{6}\rangle + \langle 0,0,2c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] = E_B[\langle-\frac{4}{9}\sqrt{6},-\frac{4}{9}\sqrt{6},+1-\frac{7}{9}\sqrt{6}\rangle,\langle-\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] + \langle a,b,0\rangle$

$E_A[\langle-\frac{4}{9}\sqrt{6},+\frac{4}{9}\sqrt{6},+1-\frac{7}{9}\sqrt{6}\rangle,\langle-\frac{2}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] = E_D[\langle+1,-1,+1\rangle,\langle-\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] + \langle -b,a,($

$b = (\frac{91}{29}-\frac{100}{87}\sqrt{6})a$ (minimum lattice volume)

B[9] $Q^{=}_{\mathrm{E}}$ $\{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

36 tetrahedra

$a = -\frac{256}{71}+\frac{148}{71}\sqrt{6}$

1.500344815942

$b = -\frac{80}{71}+\frac{64}{71}\sqrt{6}$

1.081230190678

$c = 2+2\sqrt{6}$

6.898979485566

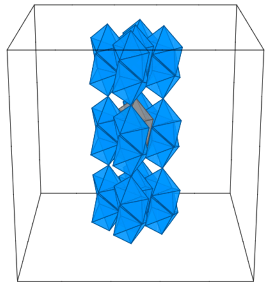
$V = -\frac{4610560}{5041}+\frac{2270720}{5041}\sqrt{6}$

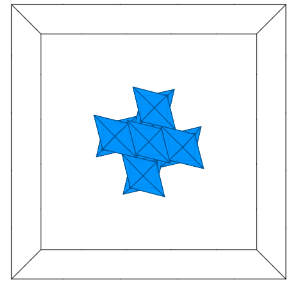
188.761227679551

$D = \frac{5403}{23440}+\frac{2661}{23440}\sqrt{6}$

.508579019008

B[9] $Q^{=}_{1}$





B[9] $Q^{\neq}_{\mathrm{E}}$ $\{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

36 tetrahedra

$a = -\frac{256}{71}+\frac{148}{71}\sqrt{6}$

1.500344815942

$b = -\frac{80}{71}+\frac{64}{71}\sqrt{6}$

1.081230190678

$c = 2+2\sqrt{6}$

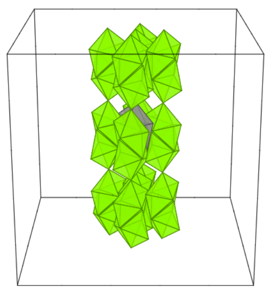
6.898979485566

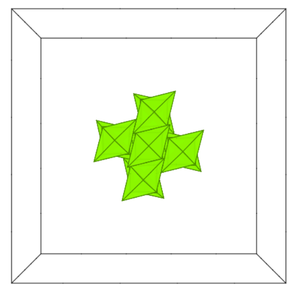
$V = -\frac{4610560}{5041}+\frac{2270720}{5041}\sqrt{6}$

188.761227679551

$D = \frac{5403}{23440}+\frac{2661}{23440}\sqrt{6}$

.498099389712





$E_A[\langle+1,+1,+1\rangle,\langle-\frac{2}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] = E_A[\langle-1,-1,+1\rangle,\langle+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] + \langle-2b,2a,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] = E_C[\langle-1,-1,+1\rangle,\langle-\frac{2}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle] + \langle a+b,-a+b,0\rangle$

$V_A\langle0,0,+1+\sqrt{6}\rangle = V_B\langle0,0,-1-\sqrt{6}\rangle + \langle0,0,c\rangle$

$V_A\langle0,0,-1-\sqrt{6}\rangle = V_D\langle0,0,+1+\sqrt{6}\rangle + \langle0,0,-c\rangle$

$\mathbf{B}_{13}$

n cosets
13 tetrahedra

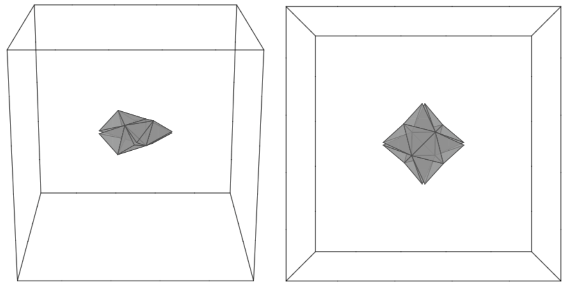
$U = \frac{104}{3}$

34.666666666667

$V = 8c(a^2+b^2)$

$D = n{\cdot}U/V$

basis $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$



$\mathbf{B}_{13}\ Q_2^{=}\ P_z\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
13 tetrahedra

$a = \frac{16}{5}$

3.200000000000

$b = 0$

.000000000000

$c = \frac{5}{8}$

.625000000000

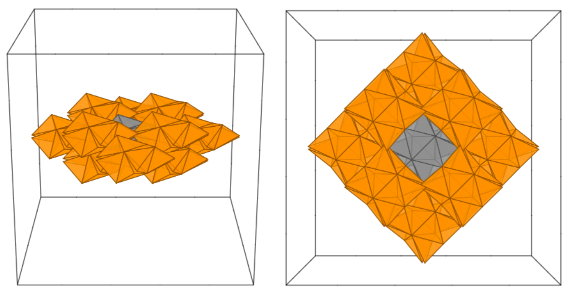
$V = \frac{256}{5}$

51.200000000000

$D = \frac{65}{96}$

.677083333333

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,4c\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle +\frac{31}{9},-\frac{1}{9},+\frac{1}{9}\rangle] = E_A[\langle -1,-1,+1\rangle,\langle -\frac{31}{9},+\frac{1}{9},+\frac{1}{9}\rangle] + \langle 2a,2b,0\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle] = E_A[\langle -1,+1,-1\rangle,\langle -\frac{31}{9},+\frac{1}{9},+\frac{1}{9}\rangle] + \langle 2a,2b,0\rangle$



$\mathbf{B}_{13}\ Q_2^{=}\ P_{xy}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
13 tetrahedra

$a = \frac{16}{9}$

1.777777777778

$b = \frac{16}{9}$

1.777777777778

$c = \frac{113}{72}$

1.569444444444

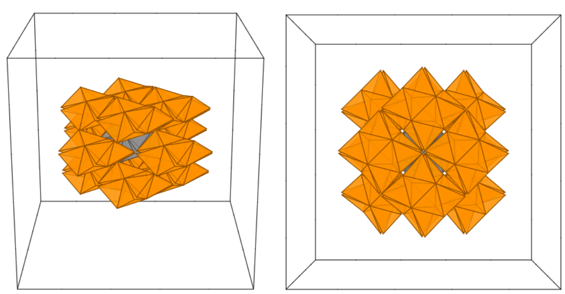
$V = \frac{57856}{729}$

79.363511659808

$D = \frac{3159}{7232}$

.436808628319

$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_A\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle 2a,2b,0\rangle$
$E_A[\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle,\langle -\frac{1}{9},+\frac{31}{9},+\frac{1}{9}\rangle] = E_A[\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle] + \langle a-b,a+b,2c\rangle$



$\mathbf{B}_{13}\ Q_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets

26 tetrahedra

$a = \frac{32}{9}$ 3.555555555556

$b = 0$ .000000000000

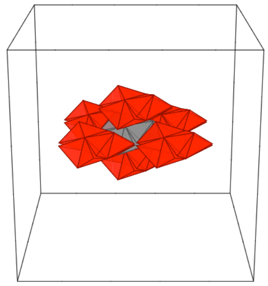
$c = \frac{5}{4}$ 1.250000000000

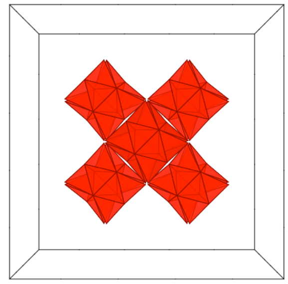
$V = \frac{10240}{81}$ 126.419753086420

$D = \frac{351}{640}$ .548437500000

$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_C\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle a-b,a+b,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,2c\rangle$





$\mathbf{B}_{13}\ Q_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets

26 tetrahedra

$a = \frac{17}{5}$ 3.400000000000

$b = 0$ .000000000000

$c = \frac{5}{3}$ 1.666666666667

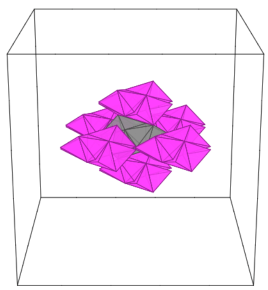
$V = \frac{2312}{15}$ 154.133333333333

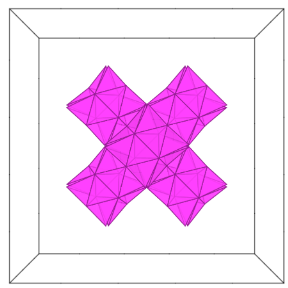
$D = \frac{130}{289}$ .449826989619

$V_A\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle = V_C\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle + \langle 0,0,2c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle] = E_C[\langle -1,-1,-1\rangle,\langle -\frac{1}{9},-\frac{31}{9},+\frac{1}{9}\rangle] + \langle a-b,a+b,0\rangle$

$b = 0$ (minimum lattice volume)





$\mathbf{B}_{13}\ Q_3^{=}\ P_z\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

52 tetrahedra

$a = \frac{16}{5}$ 3.200000000000

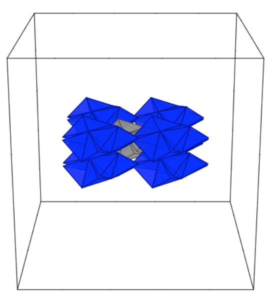
$b = \frac{16}{5}$ 3.200000000000

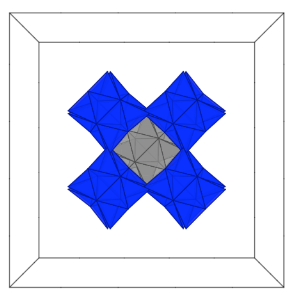
$c = \frac{5}{4}$ 1.250000000000

$V = \frac{1024}{5}$ 204.800000000000

$D = \frac{65}{96}$ .677083333333

$\mathbf{B}_{13}\ Q_2^{=}\ P_z$





$\mathbf{B}_{13}\ Q_3^{=}\ P_{xy}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

52 tetrahedra

$a = \frac{32}{9}$ 3.555555555556

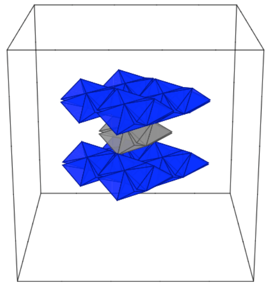
$b = 0$ .000000000000

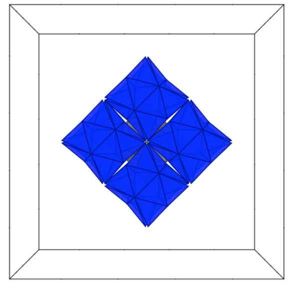
$c = \frac{113}{36}$ 3.138888888889

$V = \frac{231424}{729}$ 317.454046639232

$D = \frac{3159}{7232}$ .436808628319

$\mathbf{B}_{13}\ Q_2^{=}\ P_{xy}$





$\mathbf{B}_{13}\ Q_3^{\neq}\ P_z\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

52 tetrahedra

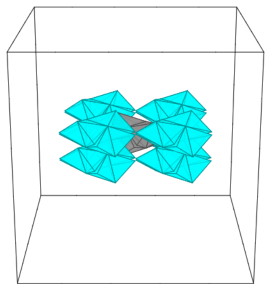
$a = \frac{195}{56}$ 3.482142857143

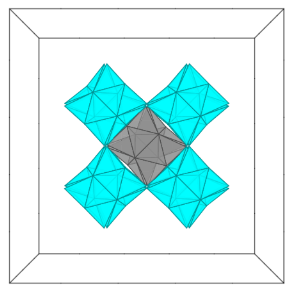
$b = \frac{195}{56}$ 3.482142857143

$c = \frac{5}{4}$ 1.250000000000

$V = \frac{190125}{784}$ 242.506377551020

$D = \frac{25088}{43875}$ .571806267806





$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,2c\rangle$

$E_A[\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle,\langle -\frac{1}{9},+\frac{31}{9},+\frac{1}{9}\rangle] = E_D[\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle +\frac{31}{9},-\frac{1}{9},-\frac{1}{9}\rangle] + \langle -b,a,c\rangle$

$a = b$ (minimum lattice volume)

$\mathbf{B}_{13}\ Q_3^{\neq}\ P_{xy}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

52 tetrahedra

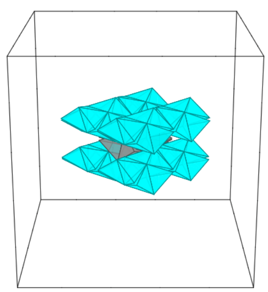
$a = \frac{32}{9}$ 3.555555555556

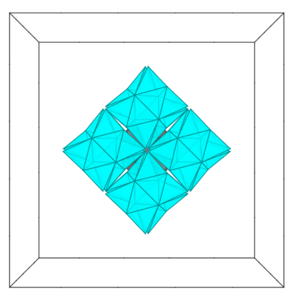
$b = 0$ .000000000000

$c = \frac{226}{117}$ 1.931623931624

$V = \frac{1851392}{9477}$ 195.356336393373

$D = \frac{41067}{57856}$ .709814021018





$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_C\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle a-b,a+b,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_B[\langle -1,-1,-1\rangle,\langle -\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle] + \langle a,b,c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_B[\langle -\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle,\langle -\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle] + \langle a,b,c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_D[\langle -1,-1,-1\rangle,\langle +\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle] + \langle -b,a,c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_D[\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle +\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}_{13}\ Q_3^+\ P_z\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets
26 tetrahedra

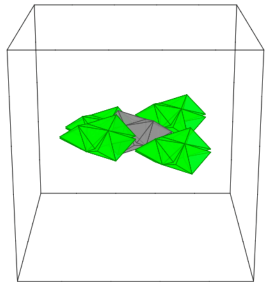
$a = \frac{55}{16}$ 3.437500000000

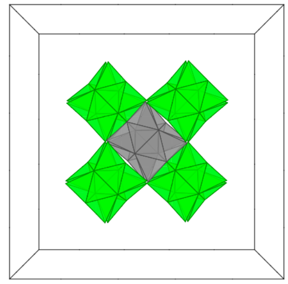
$b = \frac{55}{16}$ 3.437500000000

$c = \frac{5}{8}$ .625000000000

$V = \frac{15125}{128}$ 118.164062500000

$D = \frac{26624}{45375}$ .586754820937





$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,4c\rangle$
$E_A[\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle,\langle -\frac{1}{9},+\frac{31}{9},+\frac{1}{9}\rangle] = E_B[\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle +\frac{31}{9},-\frac{1}{9},-\frac{1}{9}\rangle] + \langle -b,a,3c\rangle$
$a = b$ (minimum lattice volume)

$\mathbf{B}_{13}\ Q_3^+\ P_{xy}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets
26 tetrahedra

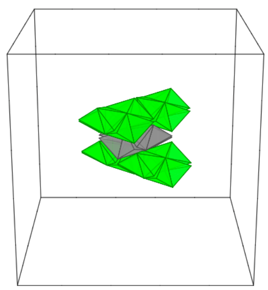
$a = \frac{16}{9}$ 1.777777777778

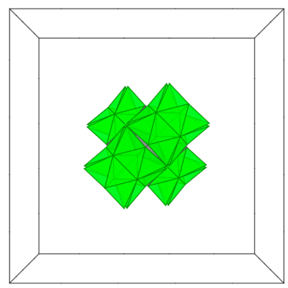
$b = \frac{16}{9}$ 1.777777777778

$c = \frac{58}{27}$ 2.148148148148

$V = \frac{237568}{2187}$ 108.627343392775

$D = \frac{9477}{14848}$ .638267780172





$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_A\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle 2a,2b,0\rangle$
$E_A[\langle 1,1,1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_B[\langle -1,-1,-1\rangle,\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle a,b,c\rangle$

$\mathbf{B}_{13}\ Q_3^-\ P_z\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets
26 tetrahedra

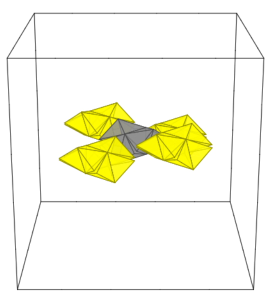
$a = \frac{395}{112}$ 3.526785714286

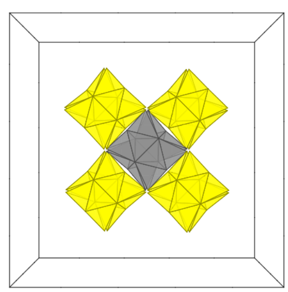
$b = \frac{395}{112}$ 3.526785714286

$c = \frac{5}{8}$ .625000000000

$V = \frac{780125}{6272}$ 124.382174744898

$D = \frac{1304576}{2340375}$ .557421780698





$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,4c\rangle$
$E_A[\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle,\langle -\frac{1}{9},+\frac{31}{9},+\frac{1}{9}\rangle] = E_D[\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle +\frac{31}{9},-\frac{1}{9},-\frac{1}{9}\rangle] + \langle -b,a,c\rangle$
$a = b$ (minimum lattice volume)

$\mathbf{B}_{13}\ Q_3^-\ P_{xy}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

26 tetrahedra

$a = \frac{16}{9}$

1.777777777778

$b = \frac{16}{9}$

1.777777777778

$c = \frac{122}{45}$

2.711111111111

$V = \frac{499712}{3645}$

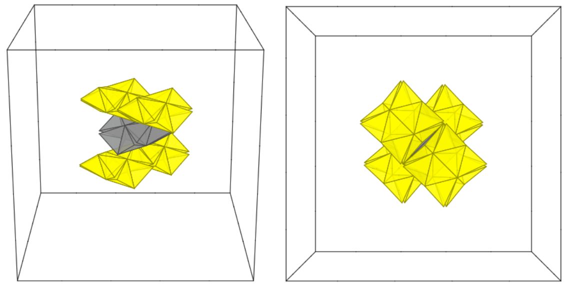
137.095198902606

$D = \frac{15795}{31232}$

.505731301230

$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_A\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle 2a,2b,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_D[\langle +1,+1,-1\rangle,\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle -b,a,c\rangle$



$\mathbf{B}_{13}\ Q_{\mathrm{F}}^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

52 tetrahedra

$a = \frac{32}{9}$

3.555555555556

$b = \frac{32}{9}$

3.555555555556

$c = \frac{5}{4}$

1.250000000000

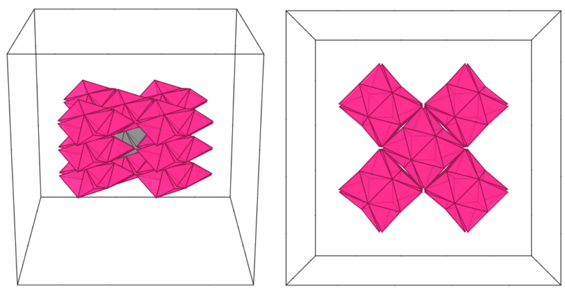
$V = \frac{20480}{81}$

252.839506172840

$D = \frac{351}{640}$

.548437500000

$\mathbf{B}_{13}\ Q_1^{=}$



$\mathbf{B}_{13}\ Q_{\mathrm{F}}^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

52 tetrahedra

$a = \frac{17}{5}$

3.400000000000

$b = \frac{17}{5}$

3.400000000000

$c = \frac{5}{4}$

1.250000000000

$V = \frac{1156}{5}$

231.200000000000

$D = \frac{520}{867}$

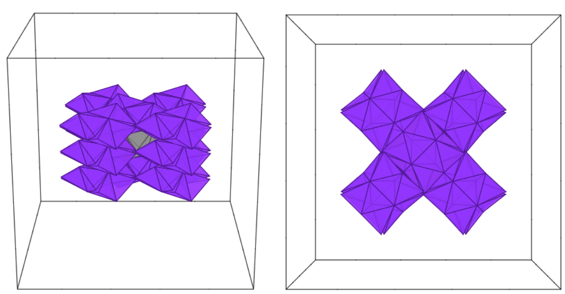
.599769319493

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,2c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{1}{9},+\frac{31}{9},-\frac{1}{9}\rangle] = E_B[\langle -1,-1,-1\rangle,\langle -\frac{31}{9},-\frac{1}{9},+\frac{1}{9}\rangle] + \langle a,b,0\rangle$

$E_A[\langle -1,+1,-1\rangle,\langle -\frac{1}{9},+\frac{31}{9},+\frac{1}{9}\rangle] = E_D[\langle +1,-1,+1\rangle,\langle +\frac{31}{9},-\frac{1}{9},-\frac{1}{9}\rangle] + \langle -b,a,0\rangle$

$a = b$ (minimum lattice volume)



$\mathbf{B}_{13}\ Q^{=}_{\mathrm{E}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

52 tetrahedra

$a = \frac{32}{9}$ 3.555555555556

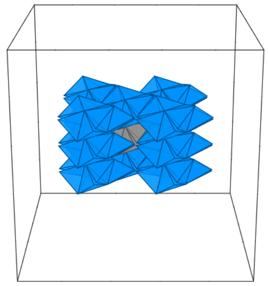
$b = 0$ .000000000000

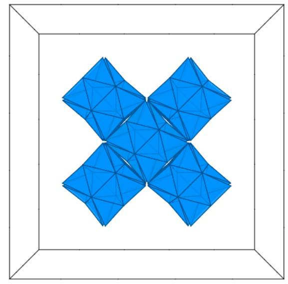
$c = \frac{5}{2}$ 2.500000000000

$V = \frac{20480}{81}$ 252.839506172840

$D = \frac{351}{640}$ .548437500000

$\mathbf{B}_{13}\ Q^{=}_{1}$





$\mathbf{B}_{13}\ Q^{\neq}_{\mathrm{E}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

52 tetrahedra

$a = \frac{32}{9}$ 3.555555555556

$b = 0$ .000000000000

$c = \frac{10}{3}$ 3.333333333333

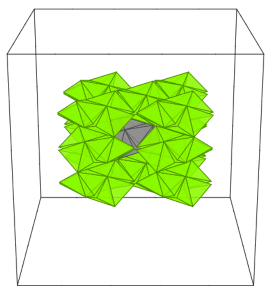
$V = \frac{81920}{243}$ 337.119341563786

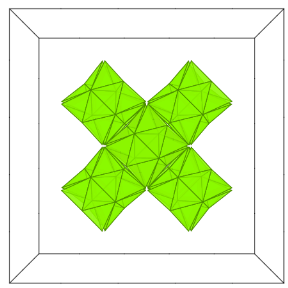
$D = \frac{1053}{2560}$ .411328125000

$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_C\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle a-b,a+b,0\rangle$

$V_A\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle = V_B\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle + \langle 0,0,c\rangle$

$V_A\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle = V_D\langle +\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle + \langle 0,0,-c\rangle$





$\mathbf{B}^*_{13}$

n cosets
13 tetrahedra

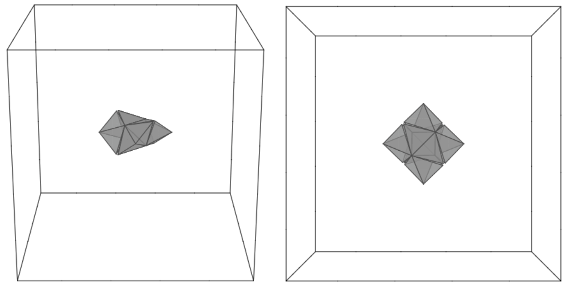
$U = \frac{104}{3}$

34.666666666667

$V = 8c(a^2+b^2)$

$D = n{\cdot}U/V$

basis $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$



$\mathbf{B}^*_{13}\ Q_2^{=}\ P_z\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
13 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 0$

.000000000000

$c = \frac{5}{8}$

.625000000000

$V = 35+10\sqrt{6}$

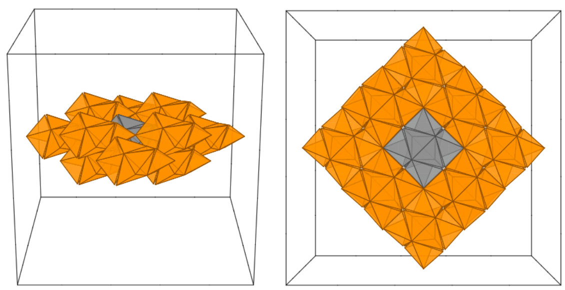
59.494897427832

$D = \frac{728}{375}-\frac{208}{375}\sqrt{6}$

.582683022670

$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle -1-\sqrt{6},0,0\rangle + \langle 2a,2b,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_A[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,4c\rangle$



$\mathbf{B}^*_{13}\ Q_2^{=}\ P_{xy}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
13 tetrahedra

$a = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

$b = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

$c = \frac{1}{2}+\frac{7}{15}\sqrt{6}$

1.643095213299

$V = \frac{182}{5}+\frac{256}{15}\sqrt{6}$

78.204624943500

$D = -\frac{4732}{1585}+\frac{6656}{4755}\sqrt{6}$

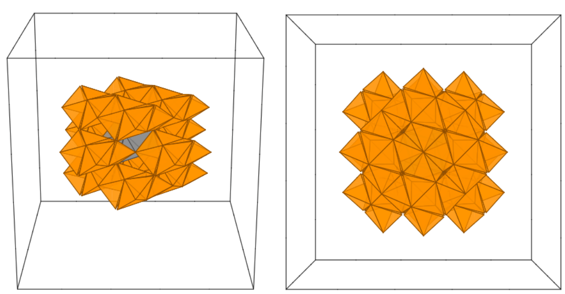
.443281541107

$V_A\langle 0,+1+\sqrt{6},0\rangle = V_A\langle 0,-1-\sqrt{6},0\rangle + \langle 2a-2b,2a+2b,0\rangle$

$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle 0,-1-\sqrt{6},0\rangle + \langle 2a,2b,0\rangle$

$V_A\langle +1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle = V_A\langle -\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle + \langle 2a,2b,0\rangle$

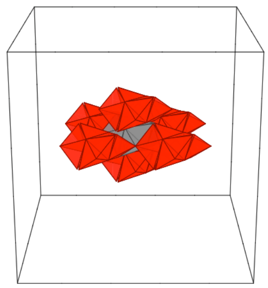
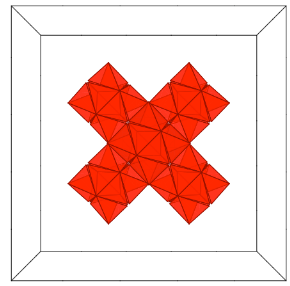
$V_A\langle -\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6}\rangle = F_A[\langle +1,-1,-1\rangle,\langle -1,+1,-1\rangle,\langle -\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle a-b,a+b,2c\rangle$



$\mathbf{B}^*_{13}\ Q_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets
26 tetrahedra

$a = 1+\sqrt{6}$
3.449489742783
$b = 0$
.000000000000
$c = \frac{5}{4}$
1.250000000000
$V = 70+20\sqrt{6}$
118.989794855664
$D = \frac{728}{375}-\frac{208}{375}\sqrt{6}$
.582683022670

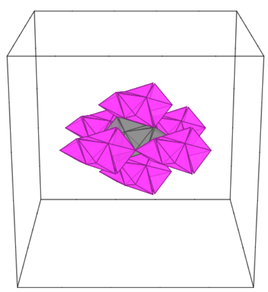
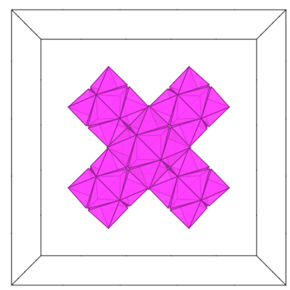



$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle -1-\sqrt{6},0,0\rangle + \langle 2a,2b,0\rangle$
$V_A\langle +1+\sqrt{6},0,0\rangle = V_C\langle 0,-1-\sqrt{6},0\rangle + \langle a-b,a+b,0\rangle$
$V_A\langle +1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle = V_C\langle -\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle + \langle a-b,a+b,0\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,2c\rangle$

$\mathbf{B}^*_{13}\ Q_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets
26 tetrahedra

$a = 1+\sqrt{6}$
3.449489742783
$b = 0$
.000000000000
$c = \frac{5}{3}$
1.666666666667
$V = \frac{280}{3}+\frac{80}{3}\sqrt{6}$
158.653059807551
$D = \frac{182}{125}-\frac{52}{125}\sqrt{6}$
.437012267002

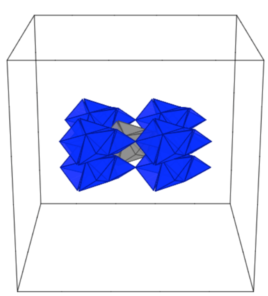
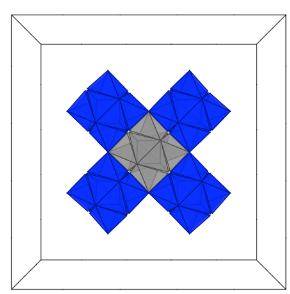



$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle -1-\sqrt{6},0,0\rangle + \langle 2a,2b,0\rangle$
$V_A\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle = V_C\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle + \langle 0,0,2c\rangle$
$V_A\langle +1+\sqrt{6},0,0\rangle = V_C\langle 0,-1-\sqrt{6},0\rangle + \langle a-b,a+b,0\rangle$

$\mathbf{B}^*_{13}\ Q_3^{=}\ P_z\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets
52 tetrahedra

$a = 1+\sqrt{6}$
3.449489742783
$b = 1+\sqrt{6}$
3.449489742783
$c = \frac{5}{4}$
1.250000000000
$V = 140+40\sqrt{6}$
237.979589711327
$D = \frac{728}{375}-\frac{208}{375}\sqrt{6}$
.582683022670

$\mathbf{B}^*_{13}\ Q_2^{=}\ P_z$

$\mathbf{B}^*_{13}\ Q^{=}_3\ P_{xy}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

52 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 0$

.000000000000

$c = 1+\frac{14}{15}\sqrt{6}$

3.286190426598

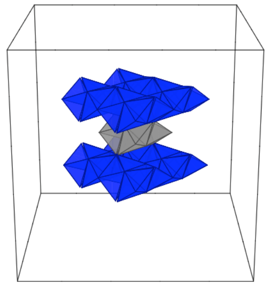
$V = \frac{728}{5}+\frac{1024}{15}\sqrt{6}$

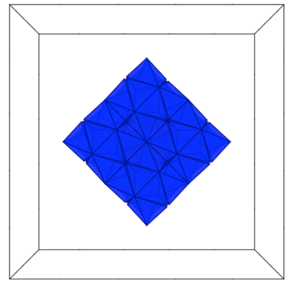
312.818499773998

$D = -\frac{4732}{1585}+\frac{6656}{4755}\sqrt{6}$

.443281541107

$\mathbf{B}^*_{13}\ Q^{=}_2\ P_{xy}$





$\mathbf{B}^*_{13}\ Q^{\neq}_3\ P_z\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

52 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 1+\sqrt{6}$

3.449489742783

$c = \frac{5}{4}$

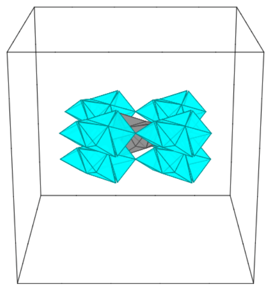
1.250000000000

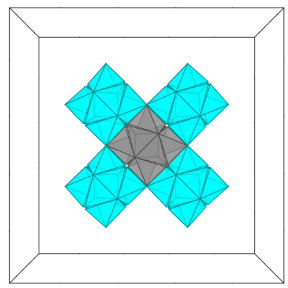
$V = 140+40\sqrt{6}$

237.979589711327

$D = \frac{728}{375}-\frac{208}{375}\sqrt{6}$

.582683022670





$V_A\langle 0,+1+\sqrt{6},0\rangle = V_C\langle 0,-1-\sqrt{6},0\rangle + \langle a-b,a+b,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,2c\rangle$

$E_A[\langle 0,+1+\sqrt{6},0\rangle,\langle -\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6}\rangle] = E_D[\langle +1+\sqrt{6},0,0\rangle,\langle +1+\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}^*_{13}\ Q^{\neq}_3\ P_{xy}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

52 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 0$

.000000000000

$c = 2$

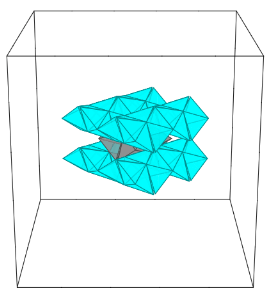
2.000000000000

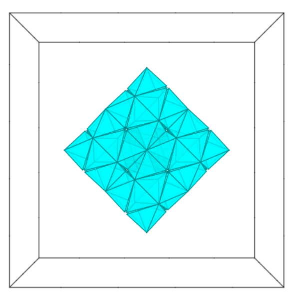
$V = 112+32\sqrt{6}$

190.383671769062

$D = \frac{182}{75}-\frac{52}{75}\sqrt{6}$

.728353778337





$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle -1-\sqrt{6},0,0\rangle + \langle 2a,2b,0\rangle$

$V_A\langle +1+\sqrt{6},0,0\rangle = V_C\langle 0,-1-\sqrt{6},0\rangle + \langle a-b,a+b,0\rangle$

$V_A\langle +1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle = V_C\langle -\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle + \langle a-b,a+b,0\rangle$

$V_A\langle +1,+1,+1\rangle = E_B[\langle -1-\sqrt{6},0,0\rangle,\langle -1-\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle] + \langle a,b,c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +1+\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+\frac{2}{3}\sqrt{6}\rangle] = E_B[\langle -1,-1,-1\rangle,\langle -1-\sqrt{6},0,0\rangle] + \langle a,b,c\rangle$

$V_A\langle +1,+1,+1\rangle = E_D[\langle 0,-1-\sqrt{6},0\rangle,\langle +\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle] + \langle -b,a,c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle -\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6}\rangle] = E_D[\langle -1,-1,-1\rangle,\langle 0,-1-\sqrt{6},0\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}^{*}_{13}\ Q^{+}_{3}\ P_{z}\ \{+\mathbf{X}_{A},-\mathbf{X}_{B},*,*\}$

2 cosets

26 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 1+\sqrt{6}$

3.449489742783

$c = \frac{5}{8}$

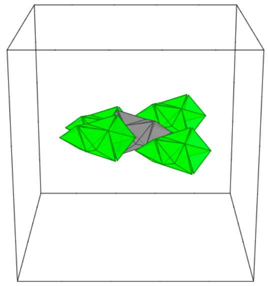
.625000000000

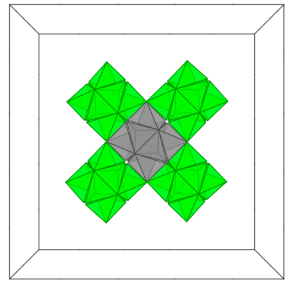
$V = 70+20\sqrt{6}$

118.989794855664

$D = \frac{728}{375}-\frac{208}{375}\sqrt{6}$

.582683022670





$E_{A}[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_{A}[\langle+1,-1,-1\rangle,\langle+\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,4c\rangle$

$E_{A}[\langle-\frac{2}{3}\sqrt{6},+1+\sqrt{2/3},+\frac{2}{3}\sqrt{6}\rangle,\langle 0,+1+\sqrt{6},0\rangle] = E_{B}[\langle+1+\sqrt{2/3},-\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle,\langle+1+\sqrt{6},0,0\rangle] + \langle -b,a,3c$

$a = b$ (minimum lattice volume)

$\mathbf{B}^{*}_{13}\ Q^{+}_{3}\ P_{xy}\ \{+\mathbf{X}_{A},-\mathbf{X}_{B},*,*\}$

2 cosets

26 tetrahedra

$a = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

$b = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

$c = -37+16\sqrt{6}$

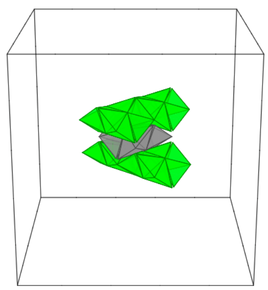
2.191835884531

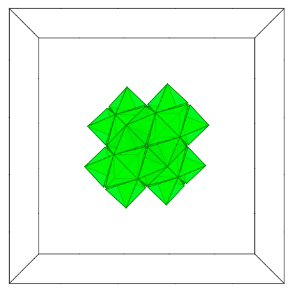
$V = -268+152\sqrt{6}$

104.322440903043

$D = \frac{3484}{12525}+\frac{1976}{12525}\sqrt{6}$

.664606126287





$V_{A}\langle 0,+1+\sqrt{6},0\rangle = V_{A}\langle 0,-1-\sqrt{6},0\rangle + \langle 2a-2b,2a+2b,0\rangle$

$V_{A}\langle+1+\sqrt{6},0,0\rangle = V_{A}\langle 0,-1-\sqrt{6},0\rangle + \langle 2a,2b,0\rangle$

$V_{A}\langle+1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle = V_{A}\langle-\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle + \langle 2a,2b,0\rangle$

$E_{A}[\langle+1,+1,+1\rangle,\langle+1+\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},+\frac{2}{3}\sqrt{6}\rangle] = E_{B}[\langle-1,-1,-1\rangle,\langle+\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle] + \langle a,b,c\rangle$

$\mathbf{B}^{*}_{13}\ Q^{-}_{3}\ P_{z}\ \{+\mathbf{X}_{A},*,*,-\mathbf{X}_{D}\}$

2 cosets

26 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 1+\sqrt{6}$

3.449489742783

$c = \frac{5}{8}$

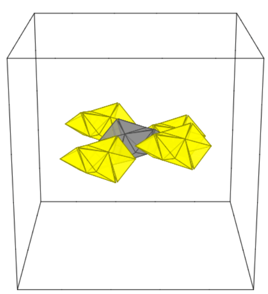
.625000000000

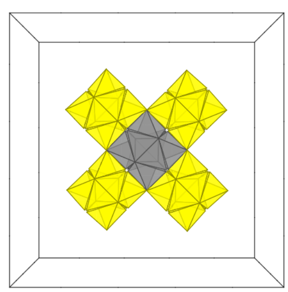
$V = 70+20\sqrt{6}$

118.989794855664

$D = \frac{728}{375}-\frac{208}{375}\sqrt{6}$

.582683022670





$E_{A}[\langle+1,+1,+1\rangle,\langle+\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_{A}[\langle+1,-1,-1\rangle,\langle+\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,4c\rangle$

$E_{A}[\langle-\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6}\rangle,\langle 0,+1+\sqrt{6},0\rangle] = E_{D}[\langle+1+\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle,\langle+1+\sqrt{6},0,0\rangle] + \langle -b,a,c\rangle$

$a = b$ (minimum lattice volume)

$\mathbf{B}^*_{13}\ Q^-_3\ P_{xy}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

26 tetrahedra

$a = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

$b = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

$c = \frac{11}{5}+\frac{1}{5}\sqrt{6}$

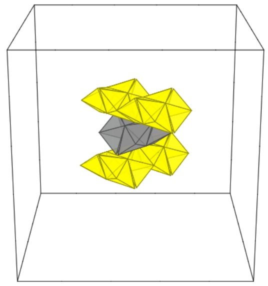
2.689897948557

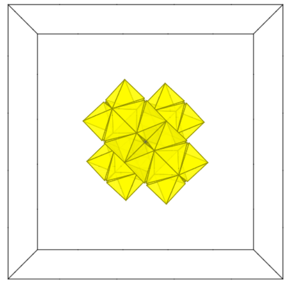
$V = \frac{356}{5}+\frac{116}{5}\sqrt{6}$

128.028162032570

$D = \frac{4628}{1725}-\frac{1508}{1725}\sqrt{6}$

.541547517613





$V_A\langle 0,+1+\sqrt{6},0\rangle = V_A\langle 0,-1-\sqrt{6},0\rangle + \langle 2a-2b,2a+2b,0\rangle$

$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle 0,-1-\sqrt{6},0\rangle + \langle 2a,2b,0\rangle$

$V_A\langle +1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle = V_A\langle -\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle + \langle 2a,2b,0\rangle$

$V_A\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle = F_D[\langle +1,+1,-1\rangle,\langle -1,-1,-1\rangle,\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle -b,a,c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle -\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle] = E_D[\langle +1,+1,-1\rangle,\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}^*_{13}\ Q^=_{\mathrm{F}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

52 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 1+\sqrt{6}$

3.449489742783

$c = \frac{5}{4}$

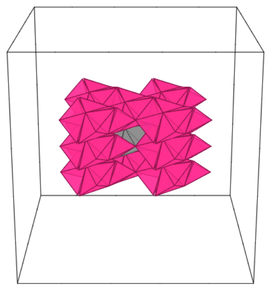
1.250000000000

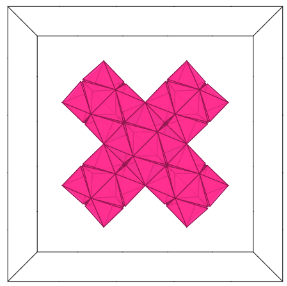
$V = 140+40\sqrt{6}$

237.979589711327

$D = \frac{728}{375}-\frac{208}{375}\sqrt{6}$

.582683022670





$\mathbf{B}^*_{13}\ Q^=_1$

$\mathbf{B}^*_{13}\ Q^{\neq}_{\mathrm{F}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

52 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 1+\sqrt{6}$

3.449489742783

$c = \frac{5}{4}$

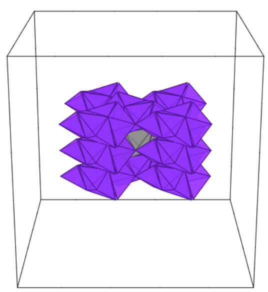
1.250000000000

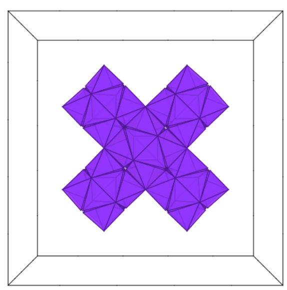
$V = 140+40\sqrt{6}$

237.979589711327

$D = \frac{728}{375}-\frac{208}{375}\sqrt{6}$

.582683022670





$V_A\langle 0,+1+\sqrt{6},0\rangle = V_C\langle 0,-1-\sqrt{6},0\rangle + \langle a-b,a+b,0\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle] = E_C[\langle +1,-1,-1\rangle,\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle] + \langle 0,0,2c\rangle$

$V_A\langle 0,+1+\sqrt{6},0\rangle = V_B\langle -1-\sqrt{6},0,0\rangle + \langle a,b,0\rangle$

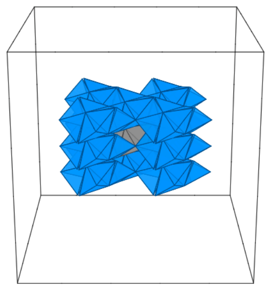
$V_A\langle 0,+1+\sqrt{6},0\rangle = V_D\langle +1+\sqrt{6},0,0\rangle + \langle -b,a,0\rangle$

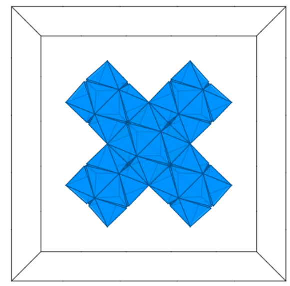
$\mathbf{B}^*_{13}\ Q^{=}_{\mathrm{E}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets
52 tetrahedra

$a = 1+\sqrt{6}$
3.449489742783
$b = 0$
.000000000000
$c = \frac{5}{2}$
2.500000000000
$V = 140+40\sqrt{6}$
237.979589711327
$D = \frac{728}{375}-\frac{208}{375}\sqrt{6}$
.582683022670

$\mathbf{B}^*_{13}\ Q^{=}_{1}$

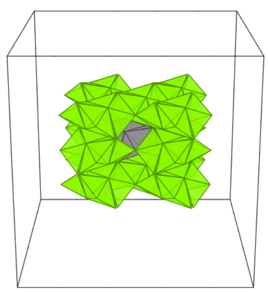


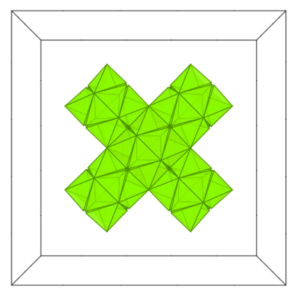


$\mathbf{B}^*_{13}\ Q^{\neq}_{\mathrm{E}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets
52 tetrahedra

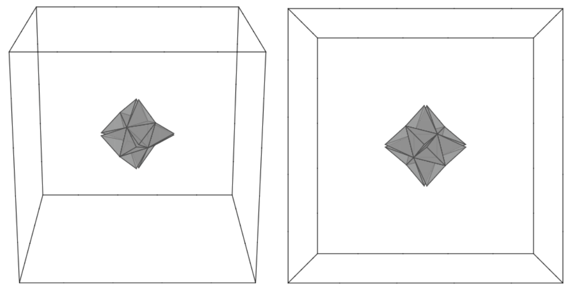
$a = 1+\sqrt{6}$
3.449489742783
$b = 0$
.000000000000
$c = \frac{10}{3}$
3.333333333333
$V = \frac{560}{3}+\frac{160}{3}\sqrt{6}$
317.306119615103
$D = \frac{182}{125}-\frac{52}{125}\sqrt{6}$
.437012267002





$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle -1-\sqrt{6},0,0\rangle + \langle 2a,2b,0\rangle$
$V_A\langle +1+\sqrt{6},0,0\rangle = V_C\langle 0,-1-\sqrt{6},0\rangle + \langle a-b,a+b,0\rangle$
$V_A\langle +1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle = V_C\langle -\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle + \langle a-b,a+b,0\rangle$
$V_A\langle +\frac{5}{3},-\frac{5}{3},+\frac{5}{3}\rangle = V_B\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle + \langle 0,0,c\rangle$
$V_A\langle +\frac{5}{3},+\frac{5}{3},-\frac{5}{3}\rangle = V_D\langle +\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle + \langle 0,0,-c\rangle$

$\mathbf{B}_{17}$



n cosets
17 tetrahedra

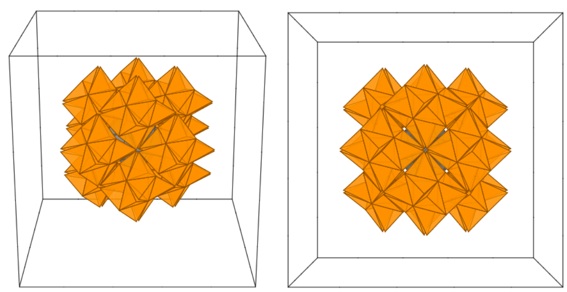
$U = \frac{136}{3}$
45.333333333333

$V = 8c(a^2+b^2)$

$D = n{\cdot}U/V$

basis $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$

$\mathbf{B}_{17}\ Q_2^{=}\ \{+\mathbf{X}_A,*,*,*\}$



1 coset
17 tetrahedra

$a = \frac{16}{9}$
1.777777777778

$b = \frac{16}{9}$
1.777777777778

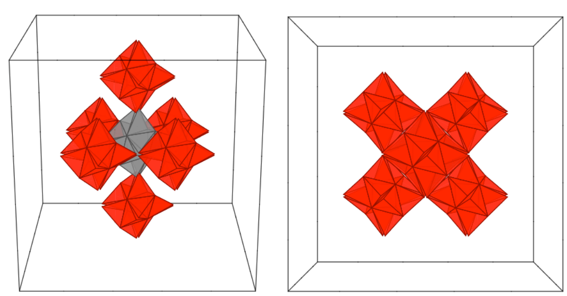
$c = \frac{16}{9}$
1.777777777778

$V = \frac{65536}{729}$
89.898491083676

$D = \frac{4131}{8192}$
.504272460938

$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_A\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle 2a,2b,0\rangle$
$V_A\langle -\frac{1}{9},+\frac{1}{9},+\frac{31}{9}\rangle = V_A\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle a-b,a+b,2c\rangle$

$\mathbf{B}_{17}\ Q_1^{=}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$



2 cosets
34 tetrahedra

$a = \frac{32}{9}$
3.555555555556

$b = 0$
.000000000000

$c = \frac{16}{5}$
3.200000000000

$V = \frac{131072}{405}$
323.634567901235

$D = \frac{2295}{8192}$
.280151367188

$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_C\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle a-b,a+b,0\rangle$
$E_A[\langle +1,+1,+1\rangle,\langle +\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle] = E_C[\langle +1,-1,-1\rangle,\langle +\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle 0,0,2c\rangle$

$\mathbf{B}_{17}\ Q_1^{\neq}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets
34 tetrahedra

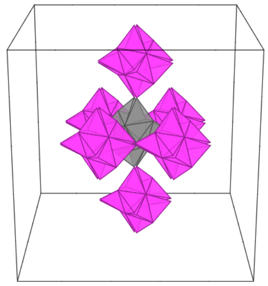
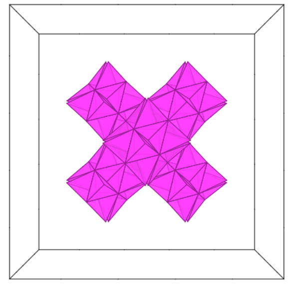
$a = \frac{17}{5}$ 3.400000000000

$b = 0$ .000000000000

$c = \frac{31}{9}$ 3.444444444444

$V = \frac{71672}{225}$ 318.542222222222

$D = \frac{150}{527}$ .284629981025

$V_A\langle +\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle = V_C\langle +\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle + \langle 0,0,2c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle] = E_C[\langle -1,-1,-1\rangle,\langle -\frac{1}{9},-\frac{31}{9},+\frac{1}{9}\rangle] + \langle a-b,a+b,0\rangle$

$b = 0$ (minimum lattice volume)

$\mathbf{B}_{17}\ Q_3^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets
68 tetrahedra

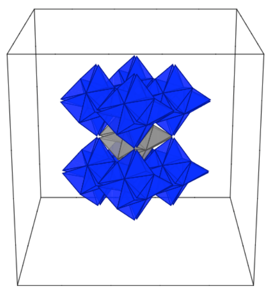
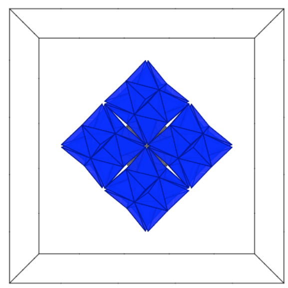
$a = \frac{32}{9}$ 3.555555555556

$b = 0$ .000000000000

$c = \frac{32}{9}$ 3.555555555556

$V = \frac{262144}{729}$ 359.593964334705

$D = \frac{4131}{8192}$ .504272460938

$\mathbf{B}_{17}\ Q_2^{=}$

$\mathbf{B}_{17}\ Q_3^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets
68 tetrahedra

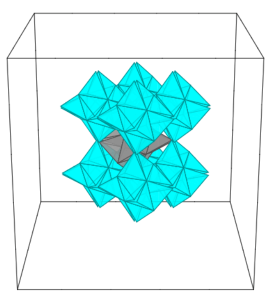
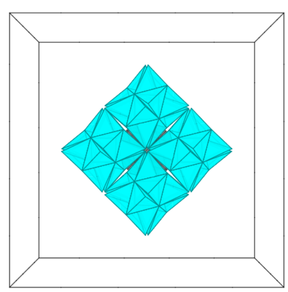
$a = \frac{32}{9}$ 3.555555555556

$b = 0$ .000000000000

$c = \frac{10}{3}$ 3.333333333333

$V = \frac{81920}{243}$ 337.119341563786

$D = \frac{1377}{2560}$ .537890625000

$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_C\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle a-b,a+b,0\rangle$

$V_A\langle +\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle = V_B\langle -\frac{31}{9},-\frac{1}{9},+\frac{1}{9}\rangle + \langle a,b,c\rangle$

$V_A\langle -\frac{1}{9},+\frac{1}{9},+\frac{31}{9}\rangle = V_D\langle -\frac{1}{9},-\frac{31}{9},+\frac{1}{9}\rangle + \langle -b,a,c\rangle$

$\mathbf{B}_{17}\ Q_3^+\ P_z\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

34 tetrahedra

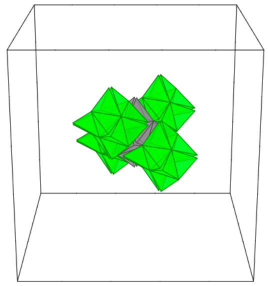
$a = \frac{23}{10}$ 2.300000000000

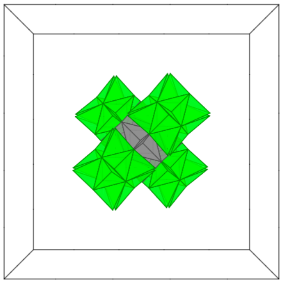
$b = \frac{23}{10}$ 2.300000000000

$c = \frac{8}{5}$ 1.600000000000

$V = \frac{16928}{125}$ 135.424000000000

$D = \frac{2125}{3174}$ .669502205419





$E_A[\langle+1,+1,+1\rangle,\langle+\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle] = E_A[\langle+1,-1,-1\rangle,\langle+\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle 0,0,4c\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{31}{9},-\frac{1}{9},+\frac{1}{9}\rangle] = E_B[\langle-1,-1,-1\rangle,\langle+\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle] + \langle a,b,c\rangle$

$a = b$ (minimum lattice volume)

$\mathbf{B}_{17}\ Q_3^+\ P_{xy}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

34 tetrahedra

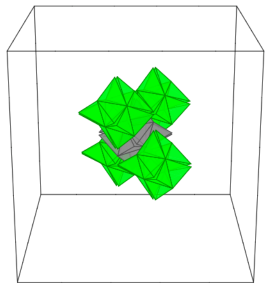
$a = \frac{16}{9}$ 1.777777777778

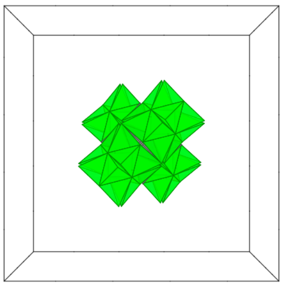
$b = \frac{16}{9}$ 1.777777777778

$c = \frac{23}{9}$ 2.555555555556

$V = \frac{94208}{729}$ 129.229080932785

$D = \frac{4131}{5888}$ .701596467391





$V_A\langle+\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_A\langle-\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle 2a,2b,0\rangle$

$E_A[\langle+1,+1,+1\rangle,\langle+\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle] = E_B[\langle-1,-1,-1\rangle,\langle+\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle] + \langle a,b,c\rangle$

$\mathbf{B}_{17}\ Q_3^-\ P_z\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

34 tetrahedra

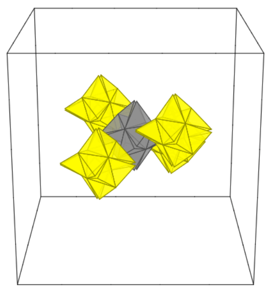
$a = \frac{121}{35}$ 3.457142857143

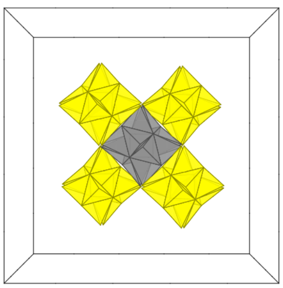
$b = \frac{121}{35}$ 3.457142857143

$c = \frac{8}{5}$ 1.600000000000

$V = \frac{1874048}{6125}$ 305.967020408163

$D = \frac{104125}{351384}$ .296328233500





$E_A[\langle+1,+1,+1\rangle,\langle+\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle] = E_A[\langle+1,-1,-1\rangle,\langle+\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle 0,0,4c\rangle$

$E_A[\langle-\frac{5}{3},+\frac{5}{3},+\frac{5}{3}\rangle,\langle-\frac{1}{9},+\frac{31}{9},+\frac{1}{9}\rangle] = E_D[\langle+\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle,\langle+\frac{31}{9},-\frac{1}{9},-\frac{1}{9}\rangle] + \langle -b,a,c\rangle$

$a = b$ (minimum lattice volume)

$\mathbf{B}_{17}\ Q_3^-\ P_{xy}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

34 tetrahedra

$a = \frac{16}{9}$ 1.777777777778

$b = \frac{16}{9}$ 1.777777777778

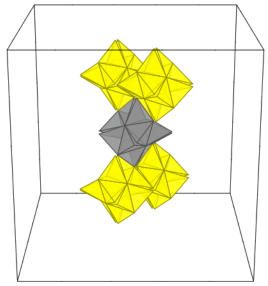
$c = \frac{46}{9}$ 5.111111111111

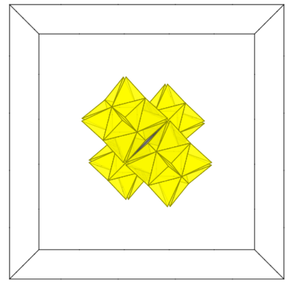
$V = \frac{188416}{729}$ 258.458161865569

$D = \frac{4131}{11776}$ .350798233696

$V_A\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_A\langle -\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle 2a,2b,0\rangle$

$V_A\langle -\frac{1}{9},+\frac{1}{9},+\frac{31}{9}\rangle = V_D\langle +\frac{5}{3},-\frac{5}{3},-\frac{5}{3}\rangle + \langle -b,a,c\rangle$





$\mathbf{B}_{17}\ Q_F^=\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = \frac{32}{9}$ 3.555555555556

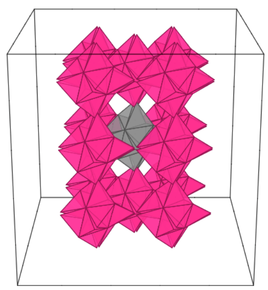
$b = \frac{32}{9}$ 3.555555555556

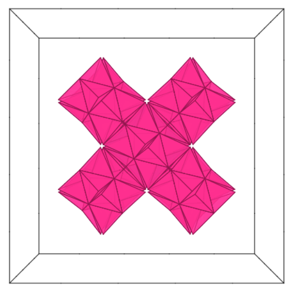
$c = \frac{16}{5}$ 3.200000000000

$V = \frac{262144}{405}$ 647.269135802469

$D = \frac{2295}{8192}$ .280151367188

$\mathbf{B}_{17}\ Q_1^=$





$\mathbf{B}_{17}\ Q_F^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = \frac{17}{5}$ 3.400000000000

$b = \frac{17}{5}$ 3.400000000000

$c = \frac{16}{5}$ 3.200000000000

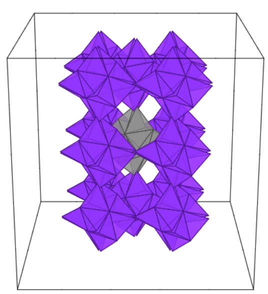
$V = \frac{73984}{125}$ 591.872000000000

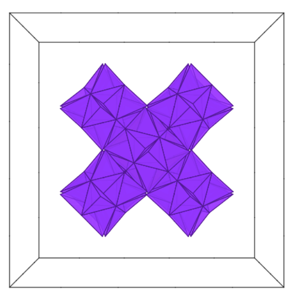
$D = \frac{125}{408}$ .306372549020

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle] = E_C[\langle +1,-1,-1\rangle,\langle +\frac{1}{9},+\frac{1}{9},-\frac{31}{9}\rangle] + \langle 0,0,2c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle] = E_B[\langle -1,-1,-1\rangle,\langle -\frac{1}{9},-\frac{31}{9},+\frac{1}{9}\rangle] + \langle a,b,0\rangle$

$a = b$ (minimum lattice volume)





$\mathbf{B}_{17}\ Q_{\mathrm{E}}^{=}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = \frac{32}{9}$ 3.555555555556

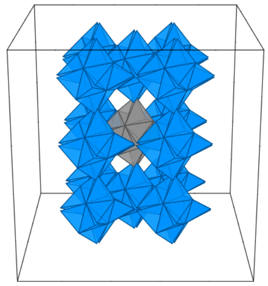
$b = 0$ .000000000000

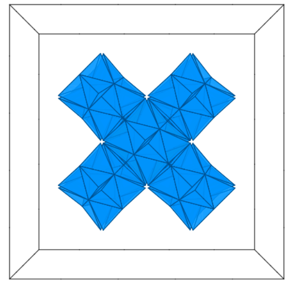
$c = \frac{32}{5}$ 6.400000000000

$V = \frac{262144}{405}$ 647.269135802469

$D = \frac{2295}{8192}$ .280151367188

$\mathbf{B}_{17}\ Q_{1}^{=}$





$\mathbf{B}_{17}\ Q_{\mathrm{E}}^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = \frac{32}{9}$ 3.555555555556

$b = 0$ .000000000000

$c = \frac{62}{9}$ 6.888888888889

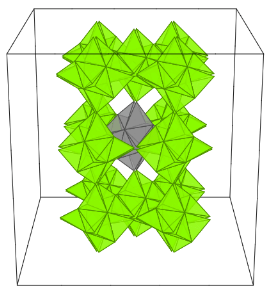
$V = \frac{507904}{729}$ 696.713305898491

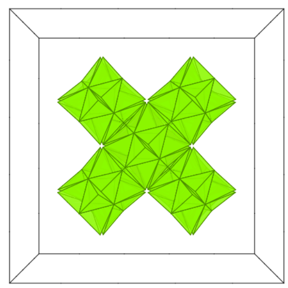
$D = \frac{4131}{15872}$ .260269657258

$V_A\langle+\frac{31}{9},+\frac{1}{9},-\frac{1}{9}\rangle = V_C\langle-\frac{1}{9},-\frac{31}{9},-\frac{1}{9}\rangle + \langle a-b,a+b,0\rangle$

$V_A\langle+\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle = V_B\langle+\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle + \langle 0,0,c\rangle$

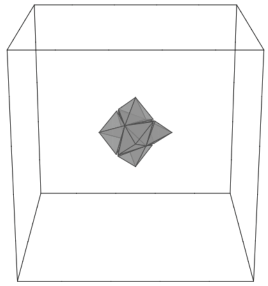
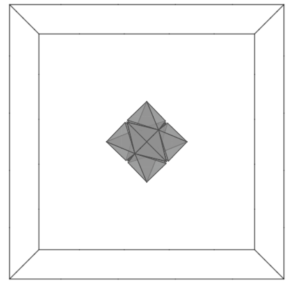
$V_A\langle+\frac{1}{9},-\frac{1}{9},-\frac{31}{9}\rangle = V_D\langle+\frac{1}{9},-\frac{1}{9},+\frac{31}{9}\rangle + \langle 0,0,-c\rangle$





$\mathbf{B}^*_{17}$

n cosets
17 tetrahedra

$U = \frac{136}{3}$
45.333333333333

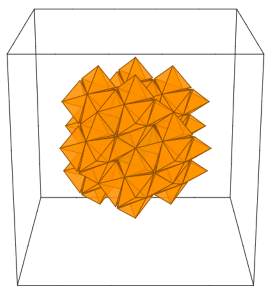
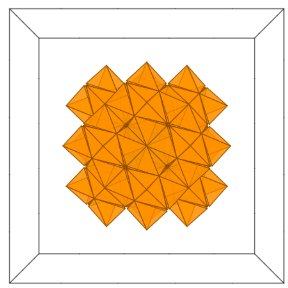
$V = 8c(a^2+b^2)$

$D = n \cdot U/V$

basis $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$

$\mathbf{B}^*_{17}\ Q_2^=\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
17 tetrahedra

$a = \frac{1}{2}+\frac{1}{2}\sqrt{6}$
1.724744871392

$b = \frac{1}{2}+\frac{1}{2}\sqrt{6}$
1.724744871392

$c = \frac{1}{2}+\frac{1}{2}\sqrt{6}$
1.724744871392

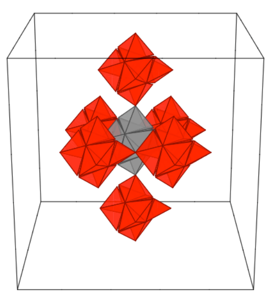
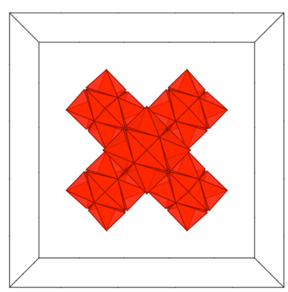
$V = 38+18\sqrt{6}$
82.090815370097

$D = -\frac{1292}{375}+\frac{204}{125}\sqrt{6}$
.552233926889

$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_A\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,4c\rangle$
$V_A\langle 0,+1+\sqrt{6}\,,0\rangle = V_A\langle 0,-1-\sqrt{6}\,,0\rangle + \langle 2a-2b,2a+2b,0\rangle$
$V_A\langle +1+\sqrt{6}\,,0,0\rangle = V_A\langle 0,-1-\sqrt{6}\,,0\rangle + \langle 2a,2b,0\rangle$
$V_A\langle +1+\frac{1}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,\rangle = V_A\langle -\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,\rangle + \langle 2a,2b,0\rangle$
$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_A\langle 0,-1-\sqrt{6}\,,0\rangle + \langle a-b,a+b,2c\rangle$

$\mathbf{B}^*_{17}\ Q_1^=\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets
34 tetrahedra

$a = 1+\sqrt{6}$
3.449489742783

$b = 0$
.000000000000

$c = 1+\sqrt{6}$
3.449489742783

$V = 152+72\sqrt{6}$
328.363261480389

$D = -\frac{646}{375}+\frac{102}{125}\sqrt{6}$
.276116963444

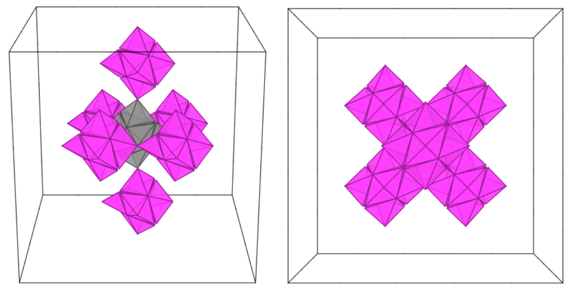
$V_A\langle +1+\sqrt{6}\,,0,0\rangle = V_A\langle -1-\sqrt{6}\,,0,0\rangle + \langle 2a,2b,0\rangle$
$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_C\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,2c\rangle$
$V_A\langle +1+\sqrt{6}\,,0,0\rangle = V_C\langle 0,-1-\sqrt{6}\,,0\rangle + \langle a-b,a+b,0\rangle$
$V_A\langle +1+\frac{1}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,\rangle = V_C\langle -\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,\rangle + \langle a-b,a+b,0\rangle$

$\mathbf{B}^*_{17}\ Q^{\neq}_1\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets
34 tetrahedra

$a = 1+\sqrt{6}$
3.449489742783
$b = 0$
.000000000000
$c = 1+\sqrt{6}$
3.449489742783
$V = 152+72\sqrt{6}$
328.363261480389
$D = -\frac{646}{375}+\frac{102}{125}\sqrt{6}$
.276116963444

$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle -1-\sqrt{6},0,0\rangle + \langle 2a,2b,0\rangle$
$V_A\langle 0,0,+1+\sqrt{6}\rangle = V_C\langle 0,0,-1-\sqrt{6}\rangle + \langle 0,0,2c\rangle$
$V_A\langle +1+\sqrt{6},0,0\rangle = V_C\langle 0,-1-\sqrt{6},0\rangle + \langle a-b,a+b,0\rangle$

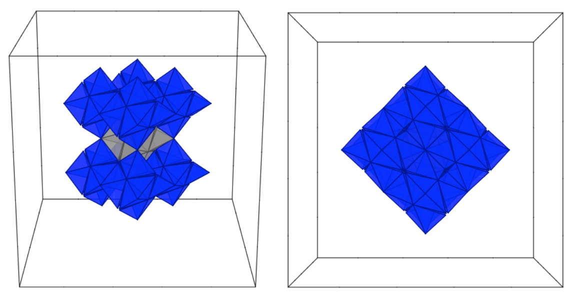


$\mathbf{B}^*_{17}\ Q^{=}_3\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets
68 tetrahedra

$a = 1+\sqrt{6}$
3.449489742783
$b = 0$
.000000000000
$c = 1+\sqrt{6}$
3.449489742783
$V = 152+72\sqrt{6}$
328.363261480389
$D = -\frac{1292}{375}+\frac{204}{125}\sqrt{6}$
.552233926889

$\mathbf{B}^*_{17}\ Q^{=}_2$

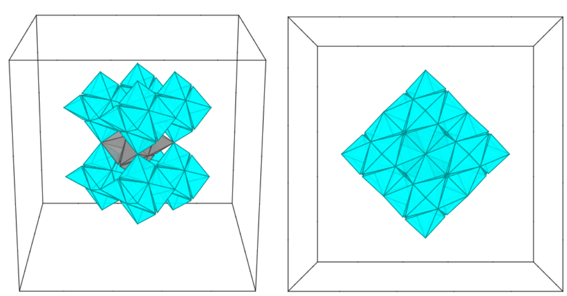


$\mathbf{B}^*_{17}\ Q^{\neq}_3\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets
68 tetrahedra

$a = 1+\sqrt{6}$
3.449489742783
$b = 0$
.000000000000
$c = 1+\sqrt{6}$
3.449489742783
$V = 152+72\sqrt{6}$
328.363261480389
$D = -\frac{1292}{375}+\frac{204}{125}\sqrt{6}$
.552233926889

$V_A\langle 0,0,+1+\sqrt{6}\rangle = V_A\langle 0,0,-1-\sqrt{6}\rangle + \langle 0,0,2c\rangle$
$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle -1-\sqrt{6},0,0\rangle + \langle 2a,2b,0\rangle$
$V_A\langle +1+\sqrt{6},0,0\rangle = V_C\langle 0,-1-\sqrt{6},0\rangle + \langle a-b,a+b,0\rangle$
$V_A\langle +1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle = V_C\langle -\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle + \langle a-b,a+b,0\rangle$
$V_A\langle 0,0,+1+\sqrt{6}\rangle = V_B\langle -1-\sqrt{6},0,0\rangle + \langle a,b,c\rangle$
$V_A\langle 0,0,+1+\sqrt{6}\rangle = V_D\langle 0,-1-\sqrt{6},0\rangle + \langle -b,a,c\rangle$



$\mathbf{B}^*_{17}\ Q^+_3\ P_z\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

34 tetrahedra

$a = -\frac{1}{4}+\sqrt{6}$

2.199489742783

$b = -\frac{1}{4}+\sqrt{6}$

2.199489742783

$c = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

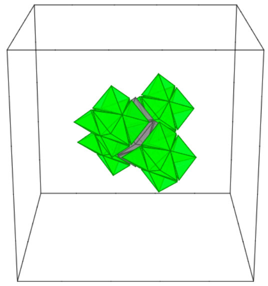
1.724744871392

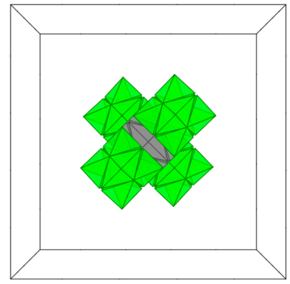
$V = \frac{49}{2}+\frac{89}{2}\sqrt{6}$

133.502293553851

$D = -\frac{26656}{135375}+\frac{48416}{135375}\sqrt{6}$

.679139393437





$V_A\langle 0,0,+1+\sqrt{6}\rangle = V_A\langle 0,0,-1-\sqrt{6}\rangle + \langle 0,0,4c\rangle$

$E_A[\langle +1,+1,+1\rangle,\langle +1+\sqrt{6},0,0\rangle] = E_B[\langle -1,-1,-1\rangle,\langle 0,-1-\sqrt{6},0\rangle] + \langle a,b,c\rangle$

$a = b$ (minimum lattice volume)

$\mathbf{B}^*_{17}\ Q^+_3\ P_{xy}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

34 tetrahedra

$a = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

$b = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

$c = -1+\frac{3}{2}\sqrt{6}$

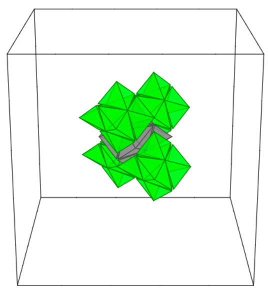
2.674234614175

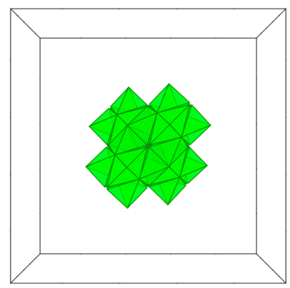
$V = 44+34\sqrt{6}$

127.282651254628

$D = -\frac{1496}{1875}+\frac{1156}{1875}\sqrt{6}$

.712325409417





$V_A\langle 0,+1+\sqrt{6},0\rangle = V_A\langle 0,-1-\sqrt{6},0\rangle + \langle 2a-2b,2a+2b,0\rangle$

$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle 0,-1-\sqrt{6},0\rangle + \langle 2a,2b,0\rangle$

$V_A\langle +1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle = V_A\langle -\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle + \langle 2a,2b,0\rangle$

$V_A\langle +1,+1,+1\rangle = E_B[\langle -1,-1,-1\rangle,\langle 0,0,-1\sqrt{6}\rangle] + \langle a,b,c\rangle$

$\mathbf{B}^*_{17}\ Q^-_3\ P_z\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

34 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 1+\sqrt{6}$

3.449489742783

$c = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

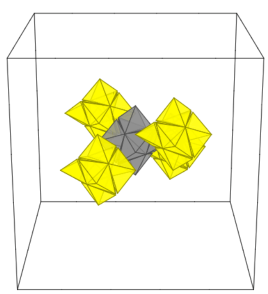
1.724744871392

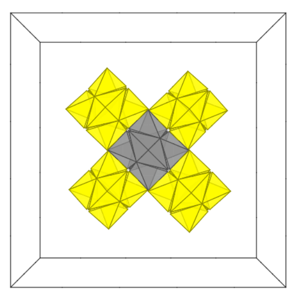
$V = 152+72\sqrt{6}$

328.363261480389

$D = -\frac{646}{375}+\frac{102}{125}\sqrt{6}$

.276116963444





$V_A\langle 0,0,+1+\sqrt{6}\rangle = V_A\langle 0,0,-1-\sqrt{6}\rangle + \langle 0,0,4c\rangle$

$E_A[\langle -\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6}\rangle,\langle 0,+1+\sqrt{6},0\rangle] = E_D[\langle +1+\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle,\langle +1+\sqrt{6},0,0\rangle] + \langle -b,a,c\rangle$

$a = b$ (minimum lattice volume)

$\mathbf{B}^*_{17}\ Q_3^-\ P_{xy}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$

2 cosets

34 tetrahedra

$a = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

$b = \frac{1}{2}+\frac{1}{2}\sqrt{6}$

1.724744871392

$c = \frac{3}{2}+\frac{3}{2}\sqrt{6}$

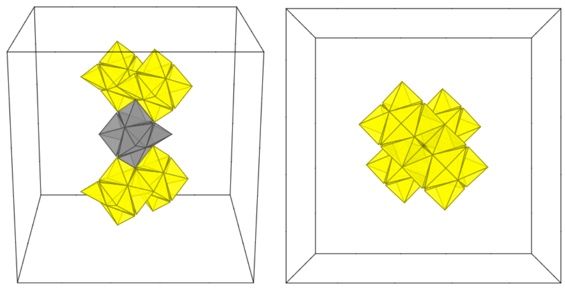
5.174234614175

$V = 114+54\sqrt{6}$

246.272446110292

$D = -\frac{2584}{1125}+\frac{136}{125}\sqrt{6}$

.368155951259



$V_A\langle 0,+1+\sqrt{6},0\rangle = V_A\langle 0,-1-\sqrt{6},0\rangle + \langle 2a-2b,2a+2b,0\rangle$

$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle 0,-1-\sqrt{6},0\rangle + \langle 2a,2b,0\rangle$

$V_A\langle +1+\frac{1}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle = V_A\langle -\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6},-\frac{2}{3}\sqrt{6}\rangle + \langle 2a,2b,0\rangle$

$V_A\langle -\frac{2}{3}\sqrt{6},+\frac{2}{3}\sqrt{6},+1+\frac{1}{3}\sqrt{6}\rangle = E_D[\langle 0,0,-1-\sqrt{6}\rangle,\langle +\frac{2}{3}\sqrt{6},-\frac{2}{3}\sqrt{6},-1-\frac{1}{3}\sqrt{6}\rangle] + \langle -b,a,c\rangle$

$\mathbf{B}^*_{17}\ Q_F^=\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 1+\sqrt{6}$

3.449489742783

$c = 1+\sqrt{6}$

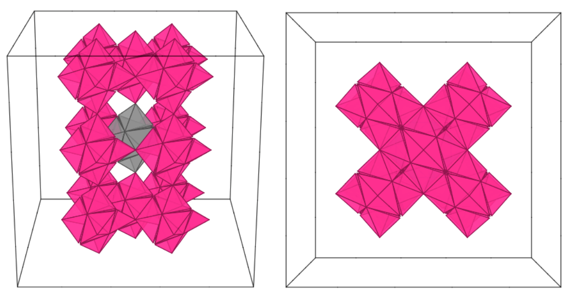
3.449489742783

$V = 304+144\sqrt{6}$

656.726522960778

$D = -\frac{646}{375}+\frac{102}{125}\sqrt{6}$

.276116963444



$\mathbf{B}^*_{17}\ Q_1^=$

$\mathbf{B}^*_{17}\ Q_F^{\neq}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 1+\sqrt{6}$

3.449489742783

$c = 1+\sqrt{6}$

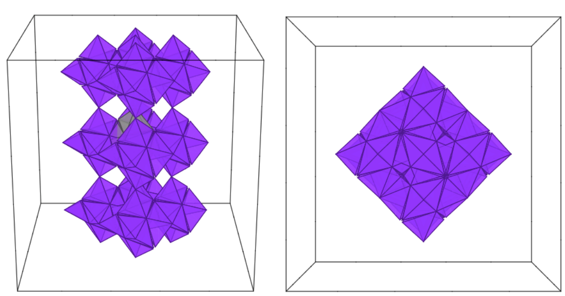
3.449489742783

$V = 304+144\sqrt{6}$

656.726522960778

$D = -\frac{646}{375}+\frac{102}{125}\sqrt{6}$

.276116963444



$V_A\langle +1+\sqrt{6},0,0\rangle = V_A\langle -1-\sqrt{6},0,0\rangle + \langle 2a,2b,0\rangle$

$V_A\langle 0,0,+1+\sqrt{6}\rangle = V_C\langle 0,0,-1-\sqrt{6}\rangle + \langle 0,0,2c\rangle$

$V_A\langle +1+\sqrt{6},0,0\rangle = V_B\langle 0,-1-\sqrt{6},0\rangle + \langle a-b,a+b,0\rangle$

$\mathbf{B}^*_{17}\ Q^{=}_{\mathrm{E}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 0$

.000000000000

$c = 2+2\sqrt{6}$

6.898979485566

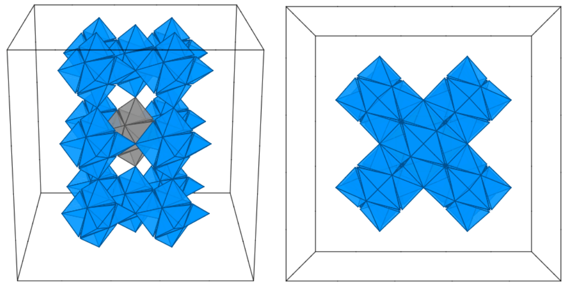
$V = 304+144\sqrt{6}$

656.726522960778

$D = -\frac{646}{375}+\frac{102}{125}\sqrt{6}$

.276116963444

$\mathbf{B}^*_{17}\ Q^{=}_{1}$



$\mathbf{B}^*_{17}\ Q^{\neq}_{\mathrm{E}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

68 tetrahedra

$a = 1+\sqrt{6}$

3.449489742783

$b = 0$

.000000000000

$c = 2+2\sqrt{6}$

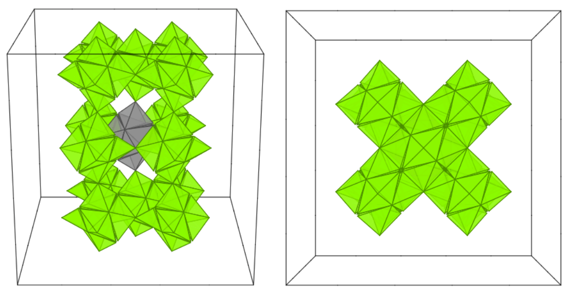
6.898979485566

$V = 304+144\sqrt{6}$

656.726522960778

$D = -\frac{646}{375}+\frac{102}{125}\sqrt{6}$

.276116963444



$V_A\langle +1+\sqrt{6}\,,0,0\rangle = V_A\langle -1-\sqrt{6}\,,0,0\rangle + \langle 2a,2b,0\rangle$

$V_A\langle +1+\sqrt{6}\,,0,0\rangle = V_C\langle 0,-1-\sqrt{6}\,,0\rangle + \langle a-b,a+b,0\rangle$

$V_A\langle +1+\frac{1}{3}\sqrt{6}\,,+\frac{2}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,\rangle = V_C\langle -\frac{2}{3}\sqrt{6}\,,-1-\frac{1}{3}\sqrt{6}\,,-\frac{2}{3}\sqrt{6}\,\rangle + \langle a-b,a+b,0\rangle$

$V_A\langle 0,0,+1+\sqrt{6}\,\rangle = V_B\langle 0,0,-1-\sqrt{6}\,\rangle + \langle 0,0,c\rangle$

$V_A\langle 0,0,-1-\sqrt{6}\,\rangle = V_D\langle 0,0,+1+\sqrt{6}\,\rangle + \langle 0,0,-c\rangle$

$\mathbf{V}_{20}$

n cosets

20 tetrahedra

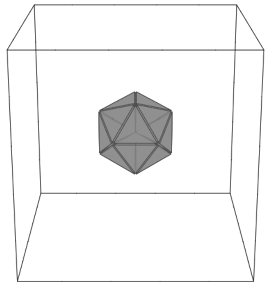
$U = \frac{160}{3}$

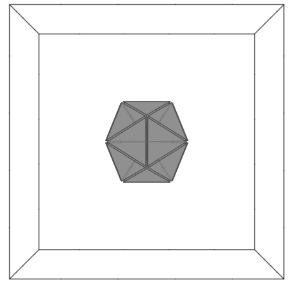
53.333333333333

$V = 8c(a^2+b^2)$

$D = n{\cdot}U/V$

basis $\{\langle a-b,a+b,2c\rangle,\langle a+b,-a+b,2c\rangle,\langle 2a,2b,0\rangle\}$





$\mathbf{V}_{20}\ Q_2^{=}\ P_z^{\mathrm{EE}}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset

20 tetrahedra

$a = \frac{11}{3}-\frac{1}{3}\sqrt{5}+\frac{1}{12}\sqrt{2}-\frac{1}{12}\sqrt{10}$

2.775638666017

$b = 0$

.000000000000

$c = \frac{1}{3}+\frac{1}{3}\sqrt{5}-\frac{1}{12}\sqrt{2}+\frac{1}{12}\sqrt{10}$

.635105682994

$V = -\frac{28}{27}+\frac{892}{27}\sqrt{5}-\frac{652}{27}\sqrt{2}+\frac{314}{27}\sqrt{10}$

75.461502909460

$D = -\frac{323107760}{105174723}+\frac{178984240}{105174723}\sqrt{5}+\frac{241153840}{105174723}\sqrt{2}-\frac{108726280}{105174723}\sqrt{10}$

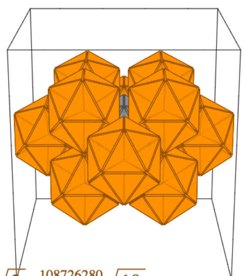
.706762140655

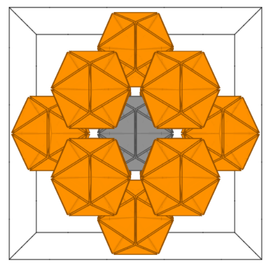
$V_A\langle r0,r1,r2\rangle = V_A\langle r0,r1,-r2\rangle + \langle 0,0,4c\rangle$

$E_A[\langle q0,q1,q2\rangle,\langle q2,q0,q1\rangle] = E_A[\langle -q0,-q1,-q2\rangle,\langle -q1,-q2,-q0\rangle] + \langle a-b,a+b,2c\rangle$

$E_A[\langle q0,q1,q2\rangle,\langle q2,q0,q1\rangle] = E_A[\langle -q1,-q2,-q0\rangle,\langle -q2,-q0,-q1\rangle] + \langle a-b,a+b,2c\rangle$

$b = 0$ (minimum lattice volume)





$\mathbf{V}_{20}\ Q_2^{=}\ P_z^{\mathrm{VF}}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset

20 tetrahedra

$a = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

2.448722667966

$b = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

2.448722667966

$c = \frac{1}{3}+\frac{1}{3}\sqrt{5}-\frac{1}{12}\sqrt{2}+\frac{1}{12}\sqrt{10}$

1.224361333983

$V = \frac{928}{27}+\frac{608}{27}\sqrt{5}+\frac{160}{27}\sqrt{2}+\frac{208}{27}\sqrt{10}$

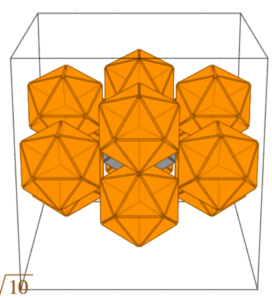
117.465083467207

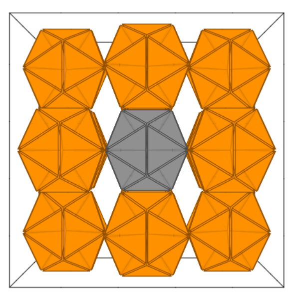
$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5}+\frac{15350}{3}\sqrt{2}-\frac{6865}{3}\sqrt{10}$

.454035631348

$V_A\langle r0,r1,r2\rangle = V_A\langle r0,r1,-r2\rangle + \langle 0,0,4c\rangle$

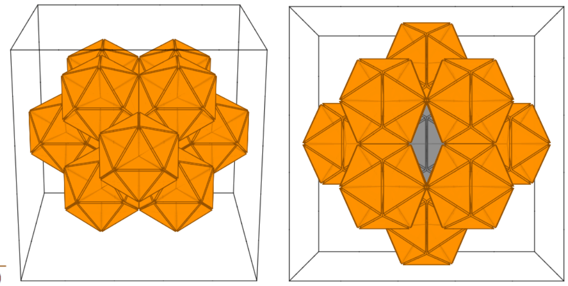
$V_A\langle r2,r0,r1\rangle = E_A[\langle -r2,r0,r1\rangle,\langle -r2,r0,-r1\rangle] + \langle a+b,-a+b,2c\rangle$





$\mathbf{V}_{20}\ Q_2^{=}\ P_{xy}^{\mathrm{EE}}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
20 tetrahedra

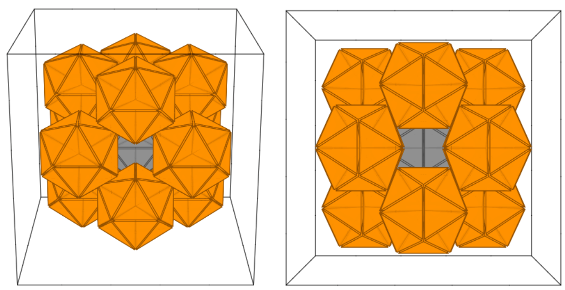


$a = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966
$b = 0$
.000000000000
$c = \frac{10}{3}-\frac{2}{3}\sqrt{5}+\frac{1}{6}\sqrt{2}-\frac{1}{6}\sqrt{10}$
1.551277332034
$V = \frac{1664}{27}+\frac{64}{27}\sqrt{5}+\frac{608}{27}\sqrt{2}-\frac{208}{27}\sqrt{10}$
74.414683080289
$D = \frac{4280}{39}-\frac{1900}{39}\sqrt{5}-\frac{3010}{39}\sqrt{2}+\frac{1345}{39}\sqrt{10}$
.716704434201

$V_A\langle \mathrm{r2,r0,r1}\rangle = V_A\langle -\mathrm{r2,r0,r1}\rangle + \langle 2a,2b,0\rangle$
$E_A[\langle \mathrm{q0,q1,q2}\rangle,\langle \mathrm{q2,q0,q1}\rangle] = E_A[\langle -\mathrm{q0},-\mathrm{q1},-\mathrm{q2}\rangle,\langle -\mathrm{q1},-\mathrm{q2},-\mathrm{q0}\rangle] + \langle a-b,a+b,2c\rangle$
$E_A[\langle \mathrm{q0,q1,q2}\rangle,\langle \mathrm{q2,q0,q1}\rangle] = E_A[\langle -\mathrm{q1},-\mathrm{q2},-\mathrm{q0}\rangle,\langle -\mathrm{q2},-\mathrm{q0},-\mathrm{q1}\rangle] + \langle a-b,a+b,2c\rangle$
$b = 0$ (minimum lattice volume)

$\mathbf{V}_{20}\ Q_2^{=}\ P_{xy}^{\mathrm{VF}}\ \{+\mathbf{X}_A,*,*,*\}$

1 coset
20 tetrahedra

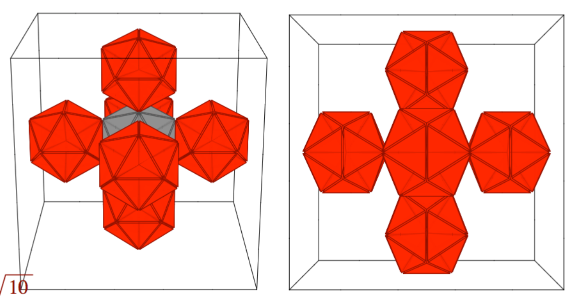


$a = \frac{4}{5}\sqrt{5}$
1.788854382000
$b = \frac{4}{5}\sqrt{5}$
1.788854382000
$c = \frac{4}{5}\sqrt{5}$
1.788854382000
$V = \frac{1024}{25}\sqrt{5}$
91.589344358391
$D = \frac{25}{96}\sqrt{5}$
.582309369141

$V_A\langle \mathrm{s1,s2},0\rangle = F_A[\langle -\mathrm{s1},-\mathrm{s2},0\rangle,\langle -\mathrm{r2},-\mathrm{r0},\mathrm{r1}\rangle,\langle -\mathrm{r2},-\mathrm{r0},-\mathrm{r1}\rangle] + \langle 2a,2b,0\rangle$
$E_A[\langle \mathrm{s1,s2},0\rangle,\langle \mathrm{r2,r0,r1}\rangle] = E_A[\langle -\mathrm{s1},-\mathrm{s2},0\rangle,\langle -\mathrm{r2},-\mathrm{r0},\mathrm{r1}\rangle] + \langle 2a,2b,0\rangle$
$V_A\langle -\mathrm{s1},\mathrm{s2},0\rangle = F_A[\langle \mathrm{s1},-\mathrm{s2},0\rangle,\langle \mathrm{r2},-\mathrm{r0},\mathrm{r1}\rangle,\langle \mathrm{r2},-\mathrm{r0},-\mathrm{r1}\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle -\mathrm{s1},\mathrm{s2},0\rangle,\langle -\mathrm{r2},\mathrm{r0},\mathrm{r1}\rangle] = E_A[\langle \mathrm{s1},-\mathrm{s2},0\rangle,\langle \mathrm{r2},-\mathrm{r0},\mathrm{r1}\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle 0,\mathrm{s1,s2}\rangle,\langle \mathrm{r1,r2,r0}\rangle] = E_A[\langle 0,-\mathrm{s1},-\mathrm{s2}\rangle,\langle \mathrm{r1},-\mathrm{r2},-\mathrm{r0}\rangle] + \langle a-b,a+b,2c\rangle$

$\mathbf{V}_{20}\ Q_1^{=}\ P^{\mathrm{EE}}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets
40 tetrahedra



$a = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966
$b = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966
$c = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966
$V = \frac{1856}{27}+\frac{1216}{27}\sqrt{5}+\frac{320}{27}\sqrt{2}+\frac{416}{27}\sqrt{10}$
234.930166934414
$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5}+\frac{15350}{3}\sqrt{2}-\frac{6865}{3}\sqrt{10}$
.454035631348

$V_A\langle \mathrm{r0,r1,r2}\rangle = V_C\langle \mathrm{r0,r1},-\mathrm{r2}\rangle + \langle 0,0,2c\rangle$
$V_A\langle \mathrm{r1,r2,r0}\rangle = V_C\langle \mathrm{r1},-\mathrm{r2},\mathrm{r0}\rangle + \langle a-b,a+b,0\rangle$

$\mathbf{V}_{20}\ Q_1^{=}\ P^{\mathrm{VF}}\ \{+\mathbf{X}_A,*,+\mathbf{X}_C,*\}$

2 cosets
40 tetrahedra

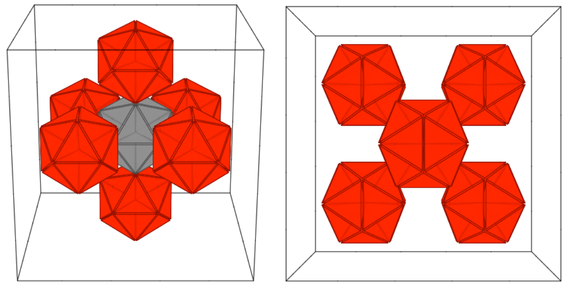
$a = \frac{8}{5}\sqrt{5}$
3.577708764000

$b = 0$
.000000000000

$c = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966

$V = \frac{1024}{15}+\frac{1024}{15}\sqrt{5}-\frac{256}{15}\sqrt{2}+\frac{256}{15}\sqrt{10}$
250.749201199692

$D = -\frac{25}{6}+\frac{25}{12}\sqrt{5}+\frac{275}{96}\sqrt{2}-\frac{125}{96}\sqrt{10}$
.425391850328



$V_A\langle \mathrm{r0},\mathrm{r1},\mathrm{r2}\rangle = V_C\langle \mathrm{r0},\mathrm{r1},-\mathrm{r2}\rangle + \langle 0,0,2c\rangle$
$V_A\langle \mathrm{s1},\mathrm{s2},0\rangle = F_C[\langle -\mathrm{s1},-\mathrm{s2},0\rangle,\langle -\mathrm{r2},-\mathrm{r0},\mathrm{r1}\rangle,\langle -\mathrm{r2},-\mathrm{r0},-\mathrm{r1}\rangle] + \langle a-b,a+b,0\rangle$
$E_A[\langle \mathrm{s1},\mathrm{s2},0\rangle,\langle \mathrm{r2},\mathrm{r0},\mathrm{r1}\rangle] = E_C[\langle -\mathrm{s1},-\mathrm{s2},0\rangle,\langle -\mathrm{r2},-\mathrm{r0},\mathrm{r1}\rangle] + \langle a-b,a+b,0\rangle$
$V_A\langle \mathrm{s1},-\mathrm{s2},0\rangle = F_C[\langle -\mathrm{s1},\mathrm{s2},0\rangle,\langle -\mathrm{r2},\mathrm{r0},\mathrm{r1}\rangle,\langle -\mathrm{r2},\mathrm{r0},-\mathrm{r1}\rangle] + \langle a+b,-a+b,0\rangle$
$E_A[\langle \mathrm{s1},-\mathrm{s2},0\rangle,\langle \mathrm{r2},-\mathrm{r0},\mathrm{r1}\rangle] = E_C[\langle -\mathrm{s1},\mathrm{s2},0\rangle,\langle -\mathrm{r2},\mathrm{r0},\mathrm{r1}\rangle] + \langle a+b,-a+b,0\rangle$

$\mathbf{V}_{20}\ Q_1^{\neq}\ P^{\mathrm{EE}}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets
40 tetrahedra

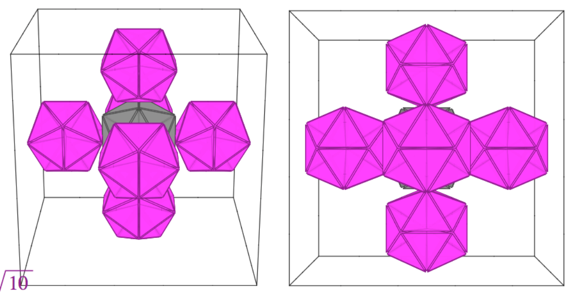
$a = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966

$b = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966

$c = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966

$V = \frac{1856}{27}+\frac{1216}{27}\sqrt{5}+\frac{320}{27}\sqrt{2}+\frac{416}{27}\sqrt{10}$
234.930166934414

$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5}+\frac{15350}{3}\sqrt{2}-\frac{6865}{3}\sqrt{10}$
.454035631348



$E_A[\langle \mathrm{r0},\mathrm{r1},\mathrm{r2}\rangle,\langle \mathrm{r0},-\mathrm{r1},\mathrm{r2}\rangle] = E_C[\langle \mathrm{r1},\mathrm{r0},-\mathrm{r2}\rangle,\langle -\mathrm{r1},\mathrm{r0},-\mathrm{r2}\rangle] + \langle 0,0,2c\rangle$
$E_A[\langle \mathrm{r1},\mathrm{r2},\mathrm{r0}\rangle,\langle -\mathrm{r1},\mathrm{r2},\mathrm{r0}\rangle] = E_C[\langle \mathrm{r0},-\mathrm{r2},\mathrm{r1}\rangle,\langle \mathrm{r0},-\mathrm{r2},-\mathrm{r1}\rangle] + \langle a-b,a+b,0\rangle$
$b = a$ (minimum lattice volume)

$\mathbf{V}_{20}\ Q_1^{\neq}\ P^{\mathrm{VF}}\ \{+\mathbf{X}_A,*,-\mathbf{X}_C,*\}$

2 cosets
40 tetrahedra

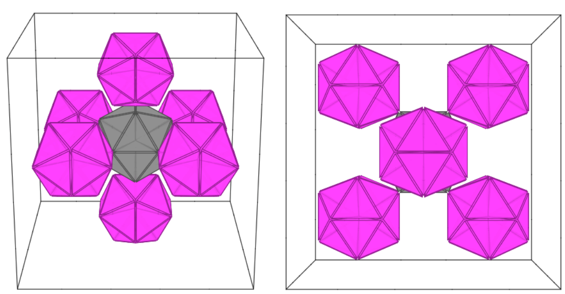
$a = \frac{4}{3}\sqrt{5}+\frac{2}{3}\sqrt{2}$
3.924233011582

$b = 0$
.000000000000

$c = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966

$V = \frac{2048}{27}+\frac{1280}{27}\sqrt{5}+\frac{928}{27}\sqrt{2}+\frac{608}{27}\sqrt{10}$
301.674889424605

$D = -\frac{95}{27}+\frac{55}{27}\sqrt{5}+\frac{205}{108}\sqrt{2}-\frac{115}{108}\sqrt{10}$
.353581522380



$E_A[\langle \mathrm{r0},\mathrm{r1},\mathrm{r2}\rangle,\langle \mathrm{r0},-\mathrm{r1},\mathrm{r2}\rangle] = E_C[\langle \mathrm{r1},\mathrm{r0},-\mathrm{r2}\rangle,\langle -\mathrm{r1},\mathrm{r0},-\mathrm{r2}\rangle] + \langle 0,0,2c\rangle$
$V_A\langle \mathrm{s1},\mathrm{s2},0\rangle = V_C\langle -\mathrm{s2},-\mathrm{s1},0\rangle + \langle a-b,a+b,0\rangle$

$\mathbf{V}_{20}\ Q_3^{=}\ P_z^{\mathrm{EE}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{11}{3}-\frac{1}{3}\sqrt{5}+\frac{1}{12}\sqrt{2}-\frac{1}{12}\sqrt{10}$

2.775638666017

$b = \frac{11}{3}-\frac{1}{3}\sqrt{5}+\frac{1}{12}\sqrt{2}-\frac{1}{12}\sqrt{10}$

2.775638666017

$c = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

2.448722667966

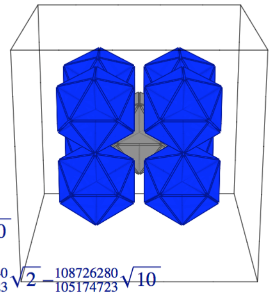
$V = -\frac{112}{27}+\frac{3568}{27}\sqrt{5}-\frac{2608}{27}\sqrt{2}+\frac{1256}{27}\sqrt{10}$

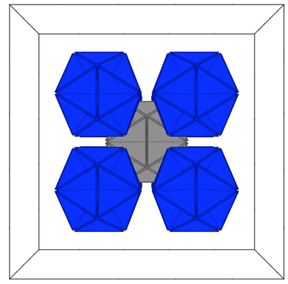
301.846011637841

$D = -\frac{323107760}{105174723}+\frac{178984240}{105174723}\sqrt{5}+\frac{241153840}{105174723}\sqrt{2}-\frac{108726280}{105174723}\sqrt{10}$

.706762140655

$\mathbf{V}_{20}\ Q_2^{=}\ P_z^{\mathrm{EE}}$





$\mathbf{V}_{20}\ Q_3^{=}\ P_z^{\mathrm{VF}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{4}{3}+\frac{4}{3}\sqrt{5}-\frac{1}{3}\sqrt{2}+\frac{1}{3}\sqrt{10}$

4.897445335931

$b = 0$

.000000000000

$c = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

2.448722667966

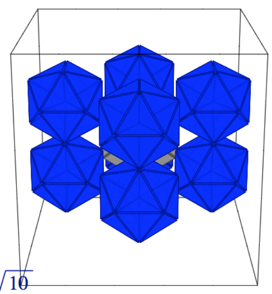
$V = \frac{3712}{27}+\frac{2432}{27}\sqrt{5}+\frac{640}{27}\sqrt{2}+\frac{832}{27}\sqrt{10}$

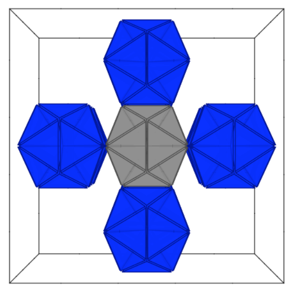
469.860333868828

$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5}+\frac{15350}{3}\sqrt{2}-\frac{6865}{3}\sqrt{10}$

.454035631348

$\mathbf{V}_{20}\ Q_2^{=}\ P_z^{\mathrm{VF}}$





$\mathbf{V}_{20}\ Q_3^{=}\ P_{xy}^{\mathrm{EE}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

2.448722667966

$b = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

2.448722667966

$c = \frac{20}{3}-\frac{4}{3}\sqrt{5}+\frac{1}{3}\sqrt{2}-\frac{1}{3}\sqrt{10}$

3.102554664069

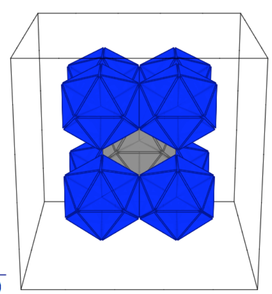
$V = \frac{6656}{27}+\frac{256}{27}\sqrt{5}+\frac{2432}{27}\sqrt{2}-\frac{832}{27}\sqrt{10}$

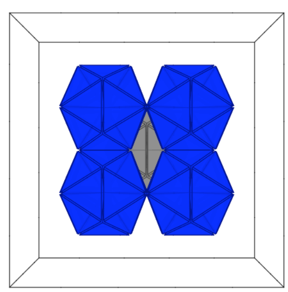
297.658732321156

$D = \frac{4280}{39}-\frac{1900}{39}\sqrt{5}-\frac{3010}{39}\sqrt{2}+\frac{1345}{39}\sqrt{10}$

.716704434201

$\mathbf{V}_{20}\ Q_2^{=}\ P_{xy}^{\mathrm{EE}}$





$\mathbf{V}_{20}\ Q_3^{=}\ P_{xy}^{\mathrm{VF}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets
80 tetrahedra

$a = \frac{8}{5}\sqrt{5}$
3.577708764000

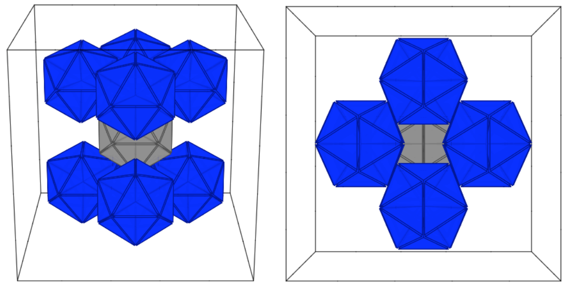
$b = 0$
.000000000000

$c = \frac{8}{5}\sqrt{5}$
3.577708764000

$V = \frac{4096}{25}\sqrt{5}$
366.357377433566

$D = \frac{25}{96}\sqrt{5}$
.582309369141

$\mathbf{V}_{20}\ Q_2^{=}\ P_{xy}^{\mathrm{VF}}$



$\mathbf{V}_{20}\ Q_3^{\neq}\ P_{z}^{\mathrm{EE}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets
80 tetrahedra

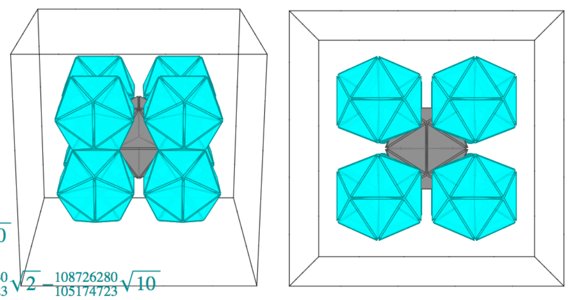
$a = \frac{11}{3}-\frac{1}{3}\sqrt{5}+\frac{1}{12}\sqrt{2}-\frac{1}{12}\sqrt{10}$
2.775638666017

$b = \frac{11}{3}-\frac{1}{3}\sqrt{5}+\frac{1}{12}\sqrt{2}-\frac{1}{12}\sqrt{10}$
2.775638666017

$c = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966

$V = -\frac{112}{27}+\frac{3568}{27}\sqrt{5}-\frac{2608}{27}\sqrt{2}+\frac{1256}{27}\sqrt{10}$
301.846011637841

$D = -\frac{323107760}{105174723}+\frac{178984240}{105174723}\sqrt{5}+\frac{241153840}{105174723}\sqrt{2}-\frac{108726280}{105174723}\sqrt{10}$
.706762140655



$V_A\langle r0,r1,r2\rangle = V_C\langle r0,r1,-r2\rangle + \langle 0,0,2c\rangle$
$E_A[\langle q0,q1,q2\rangle,\langle q2,q0,q1\rangle] = E_B[\langle -q0,-q2,-q1\rangle,\langle -q2,-q1,-q0\rangle] + \langle a,b,c\rangle$
$E_A[\langle q0,q1,q2\rangle,\langle q2,q0,q1\rangle] = E_B[\langle -q2,-q1,-q0\rangle,\langle -q1,-q0,-q2\rangle] + \langle a,b,c\rangle$
$E_A[\langle -q0,q1,q2\rangle,\langle -q2,q0,q1\rangle] = E_D[\langle q0,-q2,-q1\rangle,\langle q2,-q1,-q0\rangle] + \langle -b,a,c\rangle$
$E_A[\langle -q0,q1,q2\rangle,\langle -q2,q0,q1\rangle] = E_D[\langle q2,-q1,-q0\rangle,\langle q1,-q0,-q2\rangle] + \langle -b,a,c\rangle$
$b = a$ (minimum lattice volume)

$\mathbf{V}_{20}\ Q_3^{\neq}\ P_{z}^{\mathrm{VF}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets
80 tetrahedra

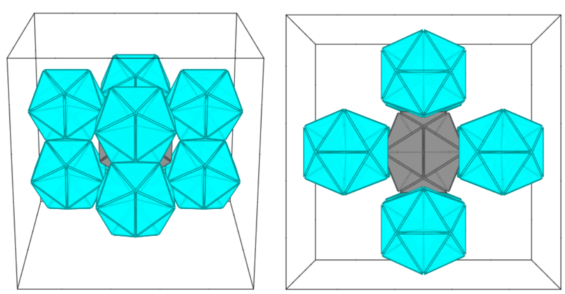
$a = -\frac{2}{3}+2\sqrt{5}+2\sqrt{2}-\frac{2}{3}\sqrt{10}$
4.525711306300

$b = 0$
.000000000000

$c = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966

$V = \frac{3328}{27}+\frac{1792}{27}\sqrt{5}+\frac{1472}{27}\sqrt{2}+\frac{448}{27}\sqrt{10}$
401.239132268454

$D = -\frac{310}{3}+\frac{140}{3}\sqrt{5}-\frac{865}{12}\sqrt{2}+\frac{385}{12}\sqrt{10}$
.531686259332



$V_A\langle r0,r1,r2\rangle = V_C\langle r0,r1,-r2\rangle + \langle 0,0,2c\rangle$
$V_A\langle r2,r0,r1\rangle = F_B[\langle -s1,0,-s2\rangle,\langle -r2,r1,-r0\rangle,\langle -r2,-r1,-r0\rangle] + \langle a,b,c\rangle$
$V_A\langle r2,r0,-r1\rangle = F_D[\langle -s1,0,s2\rangle,\langle -r2,r1,r0\rangle,\langle -r2,-r1,r0\rangle] + \langle a,b,-c\rangle$
$b = 0$ (minimum lattice volume)

$\mathbf{V}_{20}\ Q_3^{\neq}\ P_{xy}^{\mathrm{EE}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

2.448722667966

$b = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

2.448722667966

$c = \frac{20}{3}-\frac{4}{3}\sqrt{5}+\frac{1}{3}\sqrt{2}-\frac{1}{3}\sqrt{10}$

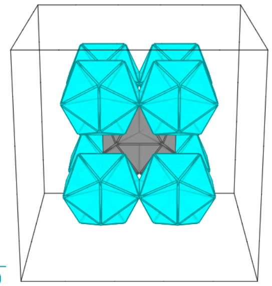
3.102554664069

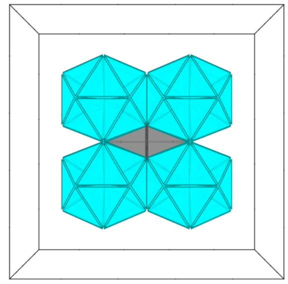
$V = \frac{6656}{27}+\frac{256}{27}\sqrt{5}+\frac{2432}{27}\sqrt{2}-\frac{832}{27}\sqrt{10}$

297.658732321156

$D = \frac{4280}{39}-\frac{1900}{39}\sqrt{5}-\frac{3010}{39}\sqrt{2}+\frac{1345}{39}\sqrt{10}$

.716704434201





$V_A\langle \mathrm{r1,r2,r0}\rangle = V_C\langle \mathrm{r1,-r2,r0}\rangle + \langle a-b,a+b,0\rangle$

$E_A[\langle \mathrm{q0,q1,q2}\rangle,\langle \mathrm{q2,q0,q1}\rangle] = E_B[\langle \mathrm{-q0,-q2,-q1}\rangle,\langle \mathrm{-q2,-q1,-q0}\rangle] + \langle a,b,c\rangle$

$E_A[\langle \mathrm{q0,q1,q2}\rangle,\langle \mathrm{q2,q0,q1}\rangle] = E_B[\langle \mathrm{-q2,-q1,-q0}\rangle,\langle \mathrm{-q1,-q0,-q2}\rangle] + \langle a,b,c\rangle$

$E_A[\langle \mathrm{-q0,q1,q2}\rangle,\langle \mathrm{-q2,q0,q1}\rangle] = E_D[\langle \mathrm{q0,-q2,-q1}\rangle,\langle \mathrm{q2,-q1,-q0}\rangle] + \langle -b,a,c\rangle$

$E_A[\langle \mathrm{-q0,q1,q2}\rangle,\langle \mathrm{-q2,q0,q1}\rangle] = E_D[\langle \mathrm{q2,-q1,-q0}\rangle,\langle \mathrm{q1,-q0,-q2}\rangle] + \langle -b,a,c\rangle$

$b = a$ (minimum lattice volume)

$\mathbf{V}_{20}\ Q_3^{\neq}\ P_{xy}^{\mathrm{VF}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{8}{5}\sqrt{5}$

3.577708764000

$b = 0$

.000000000000

$c = \frac{8}{3}+\frac{4}{15}\sqrt{5}+\frac{4}{3}\sqrt{2}-\frac{1}{3}\sqrt{10}$

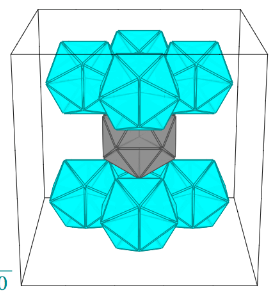
4.094476990441

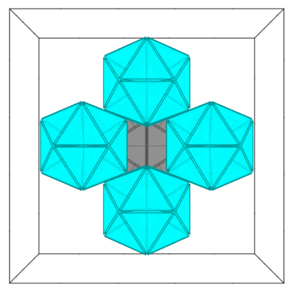
$V = \frac{4096}{15}+\frac{2048}{75}\sqrt{5}+\frac{2048}{15}\sqrt{2}-\frac{512}{15}\sqrt{10}$

419.274443821187

$D = -\frac{125}{813}+\frac{1825}{4878}\sqrt{5}+\frac{2125}{4878}\sqrt{2}-\frac{1625}{6504}\sqrt{10}$

.508815494188





$V_A\langle \mathrm{s1,s2,0}\} = F_C[\langle \mathrm{-s1,-s2,0}\},\langle \mathrm{-r2,-r0,r1}\},\langle \mathrm{-r2,-r0,-r1}\}] + \langle a-b,a+b,0\}$

$E_A[\langle \mathrm{s1,s2,0}\},\langle \mathrm{r2,r0,r1}\}] = E_C[\langle \mathrm{-s1,-s2,0}\},\langle \mathrm{-r2,-r0,r1}\}] + \langle a-b,a+b,0\}$

$V_A\langle \mathrm{s1,-s2,0}\} = F_C[\langle \mathrm{-s1,s2,0}\},\langle \mathrm{-r2,r0,r1}\},\langle \mathrm{-r2,r0,-r1}\}] + \langle a+b,-a+b,0\}$

$E_A[\langle \mathrm{s1,-s2,0}\},\langle \mathrm{r2,-r0,r1}\}] = E_C[\langle \mathrm{-s1,s2,0}\},\langle \mathrm{-r2,r0,r1}\}] + \langle a+b,-a+b,0\}$

$V_A\langle \mathrm{r0,r1,-r2}\} = F_B[\langle \mathrm{0,-s2,s1}\},\langle \mathrm{r1,-r0,r2}\},\langle \mathrm{-r1,-r0,r2}\}] + \langle -b,a,-c\}$

$V_A\langle \mathrm{r0,r1,r2}\} = F_D[\langle \mathrm{0,-s2,-s1}\},\langle \mathrm{r1,-r0,-r2}\},\langle \mathrm{-r1,-r0,-r2}\}] + \langle -b,a,c\}$

$\mathbf{V}_{20}\ Q_3^{+}\ P_{z}^{\mathrm{EE}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets

40 tetrahedra

$a = \frac{4}{3}+\frac{4}{3}\sqrt{5}-\frac{1}{3}\sqrt{2}+\frac{1}{3}\sqrt{10}$

4.897445335931

$b = 0$

.000000000000

$c = \frac{1}{3}+\frac{1}{3}\sqrt{5}-\frac{1}{12}\sqrt{2}+\frac{1}{12}\sqrt{10}$

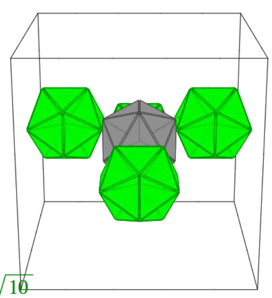
1.224361333983

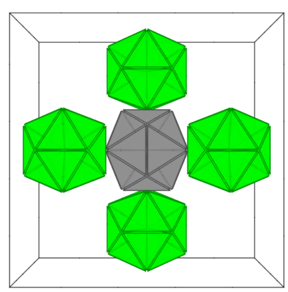
$V = \frac{1856}{27}+\frac{1216}{27}\sqrt{5}+\frac{320}{27}\sqrt{2}+\frac{416}{27}\sqrt{10}$

234.930166934414

$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5}+\frac{15350}{3}\sqrt{2}-\frac{6865}{3}\sqrt{10}$

.454035631348





$E_A[\langle \mathrm{r0,r1,r2}\rangle,\langle \mathrm{r0,-r1,r2}\rangle] = E_A[\langle \mathrm{r1,r0,-r2}\rangle,\langle \mathrm{-r1,r0,-r2}\rangle] + \langle 0,0,4c\rangle$

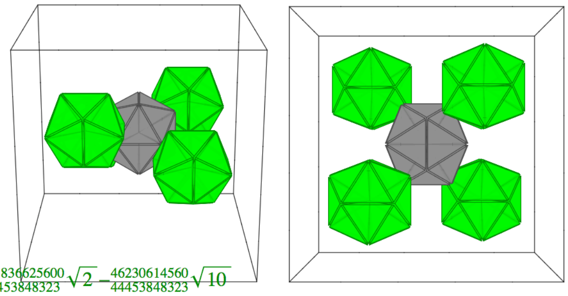
$E_A[\langle \mathrm{r2,r0,r1}\rangle,\langle \mathrm{r2,r0,-r1}\rangle] = E_B[\langle \mathrm{-r2,r1,r0}\rangle,\langle \mathrm{-r2,-r1,r0}\rangle] + \langle a,b,c\rangle$

$E_A[\langle \mathrm{r2,r0,r1}\rangle,\langle \mathrm{r2,r0,-r1}\rangle] = E_B[\langle \mathrm{-r2,r1,-r0}\rangle,\langle \mathrm{-r2,-r1,-r0}\rangle] + \langle a,b,c\rangle$

$b = 0$ (minimum lattice volume)

$\mathbf{V}_{20}\ Q_3^+\ P_z^{\mathrm{VF}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets
40 tetrahedra

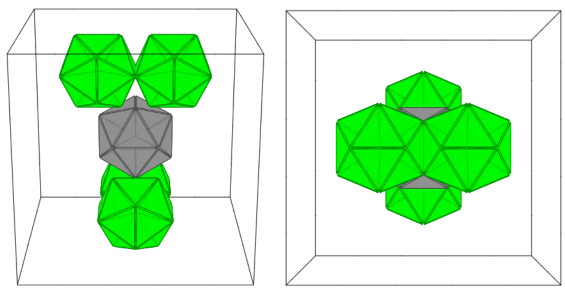


$a = \frac{23}{6}-\frac{1}{6}\sqrt{5}+\frac{1}{24}\sqrt{2}-\frac{1}{24}\sqrt{10}$
3.387819333009
$b = \frac{23}{6}-\frac{1}{6}\sqrt{5}+\frac{1}{24}\sqrt{2}-\frac{1}{24}\sqrt{10}$
3.387819333009
$c = \frac{1}{3}+\frac{1}{3}\sqrt{5}-\frac{1}{12}\sqrt{2}+\frac{1}{12}\sqrt{10}$
1.224361333983
$V = \frac{1066}{27}+\frac{2006}{27}\sqrt{5}-\frac{950}{27}\sqrt{2}+\frac{589}{27}\sqrt{10}$
224.838185942567
$D = -\frac{143135437760}{44453848323}+\frac{74416487360}{44453848323}\sqrt{5}+\frac{101836625600}{44453848323}\sqrt{2}-\frac{46230614560}{44453848323}\sqrt{10}$
.474415261000

$V_A\langle r0,r1,r2\rangle = V_C\langle r0,r1,-r2\rangle + \langle 0,0,4c\rangle$
$V_A\langle q1,q2,q0\rangle = F_B[\langle -q2,-q1,-q0\rangle,\langle -q1,-q0,-q2\rangle,\langle -q0,-q2,-q1\rangle] + \langle a,b,c\rangle$
$E_A[\langle q1,q2,q0\rangle,\langle q0,q1,q2\rangle] = E_B[\langle -q2,-q1,-q0\rangle,\langle -q1,-q0,-q2\rangle] + \langle a,b,c\rangle$
$E_A[\langle q1,q2,q0\rangle,\langle q2,q0,q1\rangle] = E_B[\langle -q2,-q1,-q0\rangle,\langle -q0,-q2,-q1\rangle] + \langle a,b,c\rangle$
$b = a$ (minimum lattice volume)

$\mathbf{V}_{20}\ Q_3^+\ P_{xy}^{\mathrm{EE}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

2 cosets
40 tetrahedra

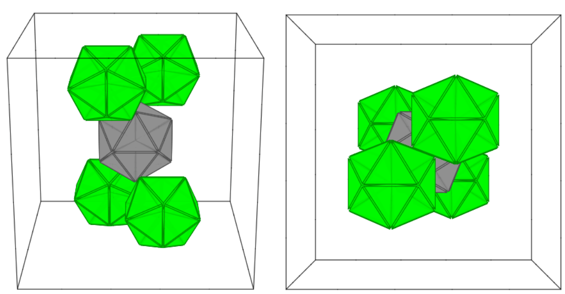


$a = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966
$b = 0$
.000000000000
$c = -\frac{2}{3}+2\sqrt{5}+2\sqrt{2}-\frac{2}{3}\sqrt{10}$
4.525711306300
$V = \frac{224}{3}+\frac{928}{27}\sqrt{5}+\frac{608}{27}\sqrt{2}+\frac{32}{3}\sqrt{10}$
217.098107228561
$D = -\frac{565}{6}+\frac{85}{2}\sqrt{5}+\frac{115}{2}\sqrt{2}-\frac{155}{6}\sqrt{10}$
.491329325844

$V_A\langle r2,r0,r1\rangle = V_A\langle -r2,r0,r1\rangle + \langle 2a,2b,0\rangle$
$V_A\langle r0,r1,-r2\rangle = F_B[\langle 0,-s2,s1\rangle,\langle r1,-r0,r2\rangle,\langle -r1,-r0,r2\rangle] + \langle -b,a,-c\rangle$

$\mathbf{V}_{20}\ Q_3^+\ P_{xy}^{\mathrm{VF}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,*,*\}$

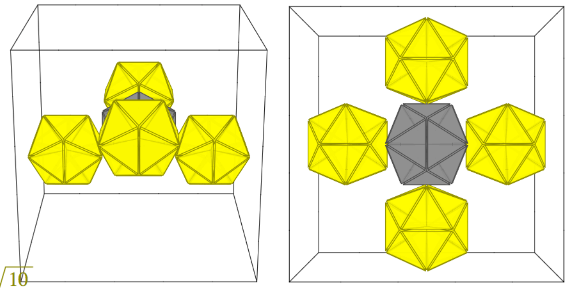
2 cosets
40 tetrahedra



$a = \frac{4}{5}\sqrt{5}$
1.788854382000
$b = \frac{4}{5}\sqrt{5}$
1.788854382000
$c = 8-\frac{8}{5}\sqrt{5}$
4.422291236000
$V = \frac{2048}{5}-\frac{2048}{25}\sqrt{5}$
226.421311283217
$D = \frac{125}{384}+\frac{25}{384}\sqrt{5}$
.471098175618

$V_A\langle s1,s2,0\rangle = F_A[\langle -s1,-s2,0\rangle,\langle -r2,-r0,r1\rangle,\langle -r2,-r0,-r1\rangle] + \langle 2a,2b,0\rangle$
$E_A[\langle s1,s2,0\rangle,\langle r2,r0,r1\rangle] = E_A[\langle -s1,-s2,0\rangle,\langle -r2,-r0,r1\rangle] + \langle 2a,2b,0\rangle$
$V_A\langle -s1,s2,0\rangle = F_A[\langle s1,-s2,0\rangle,\langle r2,-r0,r1\rangle,\langle r2,-r0,-r1\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle -s1,s2,0\rangle,\langle -r2,r0,r1\rangle] = E_A[\langle s1,-s2,0\rangle,\langle r2,-r0,r1\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle q0,q1,q2\rangle,\langle q2,q0,q1\rangle] = E_B[\langle -q1,-q0,-q2\rangle,\langle -q2,-q1,-q0\rangle] + \langle a,b,c\rangle$
$E_A[\langle q0,q1,q2\rangle,\langle q2,q0,q1\rangle] = E_B[\langle -q1,-q0,-q2\rangle,\langle -q0,-q2,-q1\rangle] + \langle a,b,c\rangle$

$\mathbf{V}_{20}\ Q_3^-\ P_z^{\mathrm{EE}}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$



2 cosets
40 tetrahedra

$a = \frac{4}{3}+\frac{4}{3}\sqrt{5}-\frac{1}{3}\sqrt{2}+\frac{1}{3}\sqrt{10}$
4.897445335931

$b = 0$
.000000000000

$c = \frac{1}{3}+\frac{1}{3}\sqrt{5}-\frac{1}{12}\sqrt{2}+\frac{1}{12}\sqrt{10}$
1.224361333983

$V = \frac{1856}{27}+\frac{1216}{27}\sqrt{5}+\frac{320}{27}\sqrt{2}+\frac{416}{27}\sqrt{10}$
234.930166934414

$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5}+\frac{15350}{3}\sqrt{2}-\frac{6865}{3}\sqrt{10}$
.454035631348

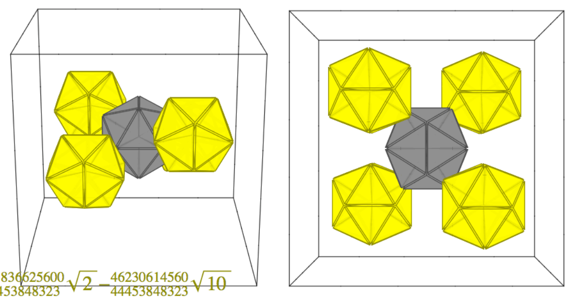
$E_A[\langle r0,r1,r2\rangle,\langle r0,-r1,r2\rangle] = E_A[\langle r1,r0,-r2\rangle,\langle -r1,r0,-r2\rangle] + \langle 0,0,4c\rangle$

$E_A[\langle r1,r2,r0\rangle,\langle -r1,r2,r0\rangle] = E_D[\langle r0,-r2,r1\rangle,\langle r0,-r2,-r1\rangle] + \langle -b,a,c\rangle$

$E_A[\langle r1,r2,-r0\rangle,\langle -r1,r2,-r0\rangle] = E_D[\langle r0,-r2,r1\rangle,\langle r0,-r2,-r1\rangle] + \langle -b,a,c\rangle$

$b = 0$ (minimum lattice volume)

$\mathbf{V}_{20}\ Q_3^-\ P_z^{\mathrm{VF}}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$



2 cosets
40 tetrahedra

$a = \frac{23}{6}-\frac{1}{6}\sqrt{5}+\frac{1}{24}\sqrt{2}-\frac{1}{24}\sqrt{10}$
3.387819333009

$b = \frac{23}{6}-\frac{1}{6}\sqrt{5}+\frac{1}{24}\sqrt{2}-\frac{1}{24}\sqrt{10}$
3.387819333009

$c = \frac{1}{3}+\frac{1}{3}\sqrt{5}-\frac{1}{12}\sqrt{2}+\frac{1}{12}\sqrt{10}$
1.224361333983

$V = \frac{1066}{27}+\frac{2006}{27}\sqrt{5}-\frac{950}{27}\sqrt{2}+\frac{589}{27}\sqrt{10}$
224.838185942567

$D = -\frac{143135437760}{44453848323}+\frac{74416487360}{44453848323}\sqrt{5}+\frac{101836625600}{44453848323}\sqrt{2}-\frac{46230614560}{44453848323}\sqrt{10}$
.474415261000

$V_A\langle r0,r1,r2\rangle = V_A\langle r0,r1,-r2\rangle + \langle 0,0,4C\rangle$

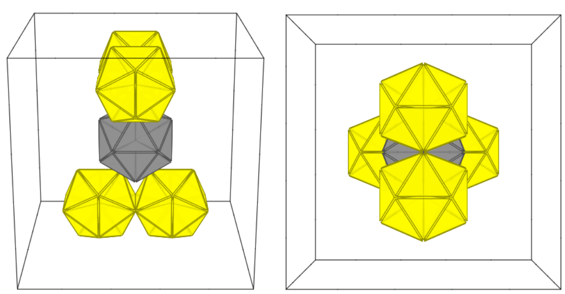
$V_A\langle -q1,q2,q0\rangle = F_D[\langle q2,-q1,-q0\rangle,\langle q1,-q0,-q2\rangle,\langle q0,-q2,-q1\rangle] + \langle -b,a,c\rangle$

$E_A[\langle -q1,q2,q0\rangle,\langle -q0,q1,q2\rangle] = E_D[\langle q2,-q1,-q0\rangle,\langle q1,-q0,-q2\rangle] + \langle -b,a,c\rangle$

$E_A[\langle -q1,q2,q0\rangle,\langle -q2,q0,q1\rangle] = E_D[\langle q2,-q1,-q0\rangle,\langle q0,-q2,-q1\rangle] + \langle -b,a,c\rangle$

$b = a$ (minimum lattice volume)

$\mathbf{V}_{20}\ Q_3^-\ P_{xy}^{\mathrm{EE}}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$



2 cosets
40 tetrahedra

$a = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$
2.448722667966

$b = 0$
.000000000000

$c = -\frac{2}{3}+2\sqrt{5}+2\sqrt{2}-\frac{2}{3}\sqrt{10}$
4.525711306300

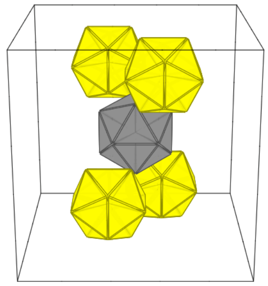
$V = \frac{224}{3}+\frac{928}{27}\sqrt{5}+\frac{608}{27}\sqrt{2}+\frac{32}{3}\sqrt{10}$
217.098107228561

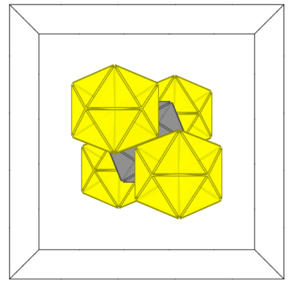
$D = -\frac{565}{6}+\frac{85}{2}\sqrt{5}+\frac{115}{2}\sqrt{2}-\frac{155}{6}\sqrt{10}$
.491329325844

$V_A\langle r2,r0,r1\rangle = V_A\langle -r2,r0,r1\rangle + \langle 2a,2b,0\rangle$

$V_A\langle r0,r1,r2\rangle = F_D[\langle 0,-s2,-s1\rangle,\langle r1,-r0,-r2\rangle,\langle -r1,-r0,-r2\rangle] + \langle -b,a,c\rangle$

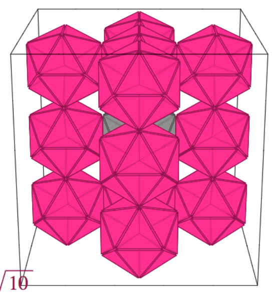
$\mathbf{V}_{20}\ Q_3^-\ P_{xy}^{\mathrm{VF}}\ \{+\mathbf{X}_A,*,*,-\mathbf{X}_D\}$
2 cosets
40 tetrahedra
$a = \frac{4}{5}\sqrt{5}$
1.788854382000
$b = \frac{4}{5}\sqrt{5}$
1.788854382000
$c = 8-\frac{8}{5}\sqrt{5}$
4.422291236000
$V = \frac{2048}{5}-\frac{2048}{25}\sqrt{5}$
226.421311283217
$D = \frac{125}{384}+\frac{25}{384}\sqrt{5}$
.471098175618

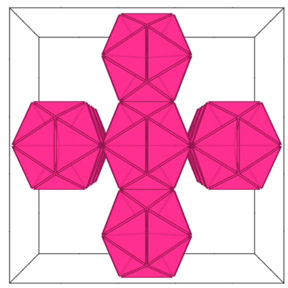




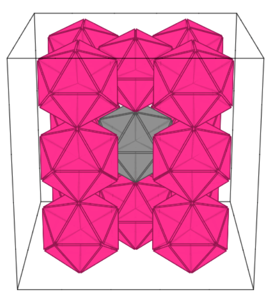
$V_A\langle s1,s2,0\rangle = F_A[\langle -s1,-s2,0\rangle,\langle -r2,-r0,r1\rangle,\langle -r2,-r0,-r1\rangle] + \langle 2a,2b,0\rangle$
$E_A[\langle s1,s2,0\rangle,\langle r2,r0,r1\rangle] = E_A[\langle -s1,-s2,0\rangle,\langle -r2,-r0,r1\rangle] + \langle 2a,2b,0\rangle$
$V_A\langle -s1,s2,0\rangle = F_A[\langle s1,-s2,0\rangle,\langle r2,-r0,r1\rangle,\langle r2,-r0,-r1\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle -s1,s2,0\rangle,\langle -r2,r0,r1\rangle] = E_A[\langle s1,-s2,0\rangle,\langle r2,-r0,r1\rangle] + \langle -2b,2a,0\rangle$
$E_A[\langle -q0,q1,q2\rangle,\langle -q2,q0,q1\rangle] = E_D[\langle q1,-q0,-q2\rangle,\langle q2,-q1,-q0\rangle] + \langle -b,a,c\rangle$
$E_A[\langle -q0,q1,q2\rangle,\langle -q2,q0,q1\rangle] = E_D[\langle q1,-q0,-q2\rangle,\langle q0,-q2,-q1\rangle] + \langle -b,a,c\rangle$

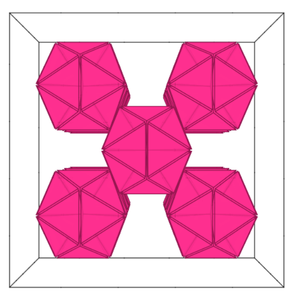
$\mathbf{V}_{20}\ Q_{\mathrm{F}}^{=}\ P^{\mathrm{EE}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$
4 cosets
80 tetrahedra
$a = \frac{4}{3}+\frac{4}{3}\sqrt{5} -\frac{1}{3}\sqrt{2} +\frac{1}{3}\sqrt{10}$
4.897445335931
$b = 0$
.000000000000
$c = \frac{2}{3}+\frac{2}{3}\sqrt{5} -\frac{1}{6}\sqrt{2} +\frac{1}{6}\sqrt{10}$
2.448722667966
$V = \frac{3712}{27}+\frac{2432}{27}\sqrt{5} +\frac{640}{27}\sqrt{2} +\frac{832}{27}\sqrt{10}$
469.860333868828
$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5} +\frac{15350}{3}\sqrt{2} -\frac{6865}{3}\sqrt{10}$
.454035631348
$\mathbf{V}_{20}\ Q_1^{=}\ P^{\mathrm{EE}}$





$\mathbf{V}_{20}\ Q_{\mathrm{F}}^{=}\ P^{\mathrm{VF}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$
4 cosets
80 tetrahedra
$a = \frac{8}{5}\sqrt{5}$
3.577708764000
$b = \frac{8}{5}\sqrt{5}$
3.577708764000
$c = \frac{2}{3}+\frac{2}{3}\sqrt{5} -\frac{1}{6}\sqrt{2} +\frac{1}{6}\sqrt{10}$
2.448722667966
$V = \frac{2048}{15}+\frac{2048}{15}\sqrt{5} -\frac{512}{15}\sqrt{2} +\frac{512}{15}\sqrt{10}$
501.498402399384
$D = -\frac{25}{6}+\frac{25}{12}\sqrt{5} +\frac{275}{96}\sqrt{2} -\frac{125}{96}\sqrt{10}$
.425391850328
$\mathbf{V}_{20}\ Q_1^{=}\ P^{\mathrm{VF}}$





$\mathbf{V}_{20}\ Q_{\mathrm{F}}^{\neq}\ P^{\mathrm{EE}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{4}{3}+\frac{4}{3}\sqrt{5}-\frac{1}{3}\sqrt{2}+\frac{1}{3}\sqrt{10}$

4.897445335931

$b = 0$

.000000000000

$c = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

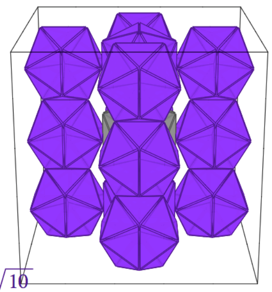
2.448722667966

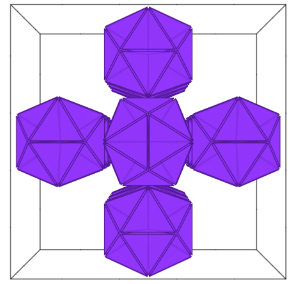
$V = \frac{3712}{27}+\frac{2432}{27}\sqrt{5}+\frac{640}{27}\sqrt{2}+\frac{832}{27}\sqrt{10}$

469.860333868828

$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5}+\frac{15350}{3}\sqrt{2}-\frac{6865}{3}\sqrt{10}$

.454035631348





$V_A\langle r0,r1,r2\rangle = V_C\langle r0,r1,-r2\rangle + \langle 0,0,2c\rangle$

$E_A[\langle r2,r0,r1\rangle,\langle r2,r0,-r1\rangle] = E_B[\langle -r2,r1,r0\rangle,\langle -r2,-r1,r0\rangle] + \langle a,b,0\rangle$

$E_A[\langle r1,r2,r0\rangle,\langle -r1,r2,r0\rangle] = E_D[\langle r0,-r2,r1\rangle,\langle r0,-r2,-r1\rangle] + \langle -b,a,0\rangle$

$b = 0$ (minimum lattice volume)

$\mathbf{V}_{20}\ Q_{\mathrm{F}}^{\neq}\ P^{\mathrm{VF}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{4}{3}\sqrt{5}+\frac{2}{3}\sqrt{2}$

3.924233011582

$b = \frac{4}{3}\sqrt{5}+\frac{2}{3}\sqrt{2}$

3.924233011582

$c = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

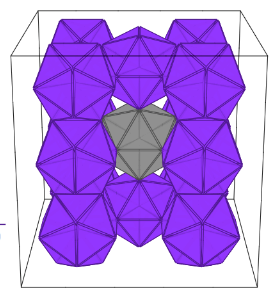
2.448722667966

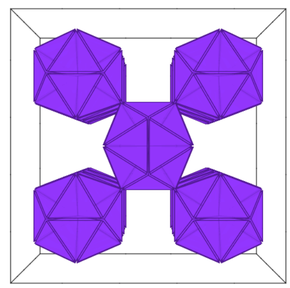
$V = \frac{4096}{27}+\frac{2560}{27}\sqrt{5}+\frac{1856}{27}\sqrt{2}+\frac{1216}{27}\sqrt{10}$

603.349778849210

$D = -\frac{95}{27}+\frac{55}{27}\sqrt{5}+\frac{205}{108}\sqrt{2}-\frac{115}{108}\sqrt{10}$

.353581522380





$V_A\langle r0,r1,r2\rangle = V_C\langle r0,r1,-r2\rangle + \langle 0,0,2c\rangle$

$V_A\langle s1,s2,0\rangle = V_B\langle -s2,-s1,0\rangle + \langle a,b,0\rangle$

$V_A\langle -s1,s2,0\rangle = V_D\langle s2,-s1,0\rangle + \langle -b,a,0\rangle$

$\mathbf{V}_{20}\ Q_{\mathrm{E}}^{=}\ P^{\mathrm{EE}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

2.448722667966

$b = \frac{2}{3}+\frac{2}{3}\sqrt{5}-\frac{1}{6}\sqrt{2}+\frac{1}{6}\sqrt{10}$

2.448722667966

$c = \frac{4}{3}+\frac{4}{3}\sqrt{5}-\frac{1}{3}\sqrt{2}+\frac{1}{3}\sqrt{10}$

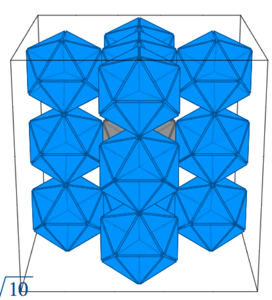
4.897445335931

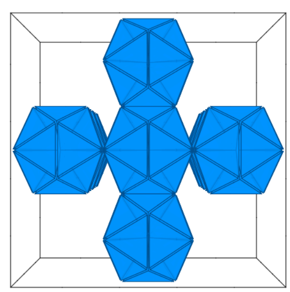
$V = \frac{3712}{27}+\frac{2432}{27}\sqrt{5}+\frac{640}{27}\sqrt{2}+\frac{832}{27}\sqrt{10}$

469.860333868828

$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5}+\frac{15350}{3}\sqrt{2}-\frac{6865}{3}\sqrt{10}$

.454035631348





$\mathbf{V}_{20}\ Q_{\mathrm{I}}^{=}\ P^{\mathrm{EE}}$

$\mathbf{V}_{20}\ Q^{=}_{\mathrm{E}}\ P^{\mathrm{VF}}\ \{+\mathbf{X}_A,+\mathbf{X}_B,+\mathbf{X}_C,+\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{8}{5}\sqrt{5}$

3.577708764000

$b = 0$

.000000000000

$c = \frac{4}{3}+\frac{4}{3}\sqrt{5} - \frac{1}{3}\sqrt{2} + \frac{1}{3}\sqrt{10}$

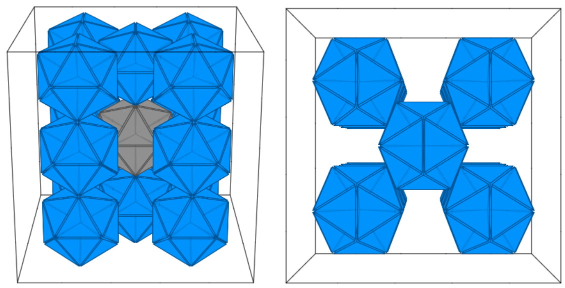
4.897445335931

$V = \frac{2048}{15}+\frac{2048}{15}\sqrt{5} - \frac{512}{15}\sqrt{2} + \frac{512}{15}\sqrt{10}$

501.498402399384

$D = -\frac{25}{6}+\frac{25}{12}\sqrt{5} + \frac{275}{96}\sqrt{2} - \frac{125}{96}\sqrt{10}$

.425391850328



$\mathbf{V}_{20}\ Q^{=}_{\mathrm{I}}\ P^{\mathrm{VF}}$

$\mathbf{V}_{20}\ Q^{\neq}_{\mathrm{E}}\ P^{\mathrm{EE}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{2}{3}+\frac{2}{3}\sqrt{5} - \frac{1}{6}\sqrt{2} + \frac{1}{6}\sqrt{10}$

2.448722667966

$b = \frac{2}{3}+\frac{2}{3}\sqrt{5} - \frac{1}{6}\sqrt{2} + \frac{1}{6}\sqrt{10}$

2.448722667966

$c = \frac{4}{3}+\frac{4}{3}\sqrt{5} - \frac{1}{3}\sqrt{2} + \frac{1}{3}\sqrt{10}$

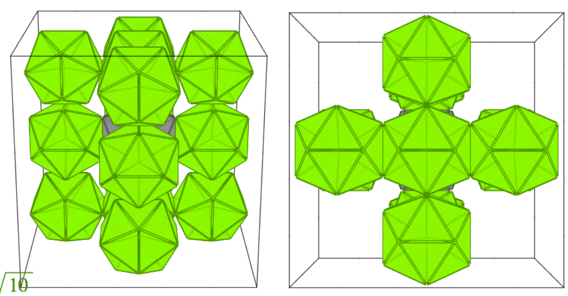
4.897445335931

$V = \frac{3712}{27}+\frac{2432}{27}\sqrt{5} + \frac{640}{27}\sqrt{2} + \frac{832}{27}\sqrt{10}$

469.860333868828

$D = -\frac{21710}{3}+\frac{9710}{3}\sqrt{5} + \frac{15350}{3}\sqrt{2} - \frac{6865}{3}\sqrt{10}$

.454035631348



$V_A\langle r1,r2,r0\rangle = V_C\langle r1,-r2,r0\rangle + \langle a-b,a+b,0\rangle$

$E_A[\langle r0,r1,r2\rangle,\langle r0,-r1,r2\rangle] = E_B[\langle r1,r0,-r2\rangle,\langle -r1,r0,-r2\rangle] + \langle 0,0,c\rangle$

$E_A[\langle r0,r1,-r2\rangle,\langle r0,-r1,-r2\rangle] = E_D[\langle r1,r0,r2\rangle,\langle -r1,r0,r2\rangle] + \langle 0,0,-c\rangle$

$\mathbf{V}_{20}\ Q^{\neq}_{\mathrm{E}}\ P^{\mathrm{VF}}\ \{+\mathbf{X}_A,-\mathbf{X}_B,+\mathbf{X}_C,-\mathbf{X}_D\}$

4 cosets

80 tetrahedra

$a = \frac{8}{5}\sqrt{5}$

3.577708764000

$b = 0$

.000000000000

$c = \frac{4}{3}+\frac{4}{3}\sqrt{5} - \frac{1}{3}\sqrt{2} + \frac{1}{3}\sqrt{10}$

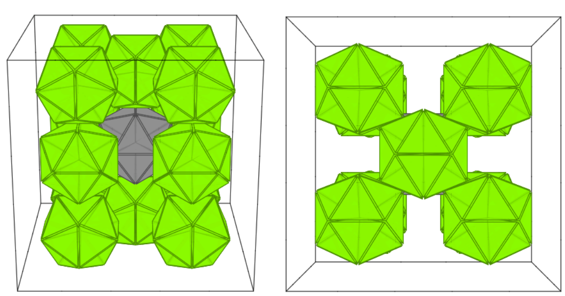
4.897445335931

$V = \frac{2048}{15}+\frac{2048}{15}\sqrt{5} - \frac{512}{15}\sqrt{2} + \frac{512}{15}\sqrt{10}$

501.498402399384

$D = -\frac{25}{6}+\frac{25}{12}\sqrt{5} + \frac{275}{96}\sqrt{2} - \frac{125}{96}\sqrt{10}$

.425391850328



$V_A\langle s1,s2,0\rangle = F_C[\langle -s1,-s2,0\rangle,\langle -r2,-r0,r1\rangle,\langle -r2,-r0,-r1\rangle] + \langle a-b,a+b,0\rangle$

$E_A[\langle s1,s2,0\rangle,\langle r2,r0,r1\rangle] = E_C[\langle -s1,-s2,0\rangle,\langle -r2,-r0,r1\rangle] + \langle a-b,a+b,0\rangle$

$V_A\langle s1,-s2,0\rangle = F_C[\langle -s1,s2,0\rangle,\langle -r2,r0,r1\rangle,\langle -r2,r0,-r1\rangle] + \langle a+b,-a+b,0\rangle$

$E_A[\langle s1,-s2,0\rangle,\langle r2,-r0,r1\rangle] = E_C[\langle -s1,s2,0\rangle,\langle -r2,r0,r1\rangle] + \langle a+b,-a+b,0\rangle$

$E_A[\langle r0,r1,r2\rangle,\langle r0,-r1,r2\rangle] = E_B[\langle r1,r0,-r2\rangle,\langle -r1,r0,-r2\rangle] + \langle 0,0,c\rangle$

$E_A[\langle r0,r1,-r2\rangle,\langle r0,-r1,-r2\rangle] = E_D[\langle r1,r0,r2\rangle,\langle -r1,r0,r2\rangle] + \langle 0,0,-c\rangle$