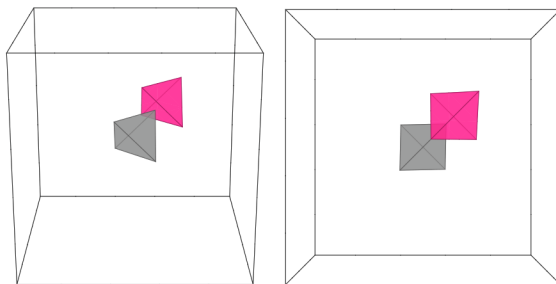


$\{+B_1,+B_1\}$

$a = \frac{4}{3}$

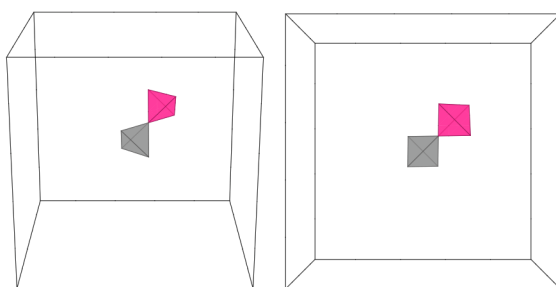
$\langle a,a,a \rangle$
1.333333333333



$\{+B_1,-B_1\}$

$a = 2$

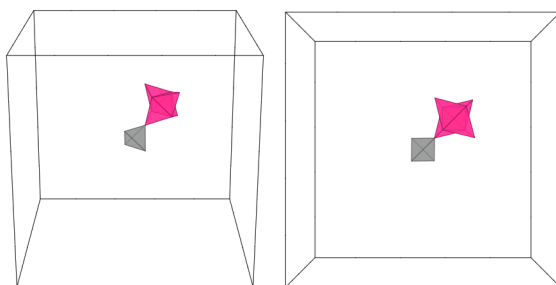
$\langle a,a,a \rangle$
2.000000000000



$\{+B_1,+B_5\}$

$a = \frac{8}{3}$

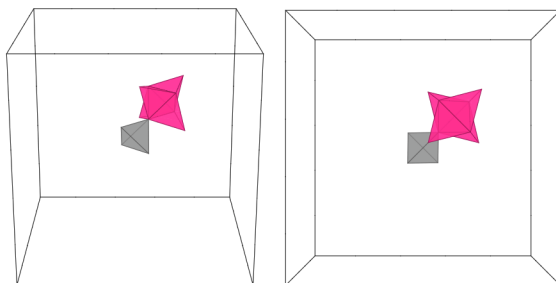
$\langle a,a,a \rangle$
2.666666666667



$\{+B_1,-B_5\}$

$a = 2$

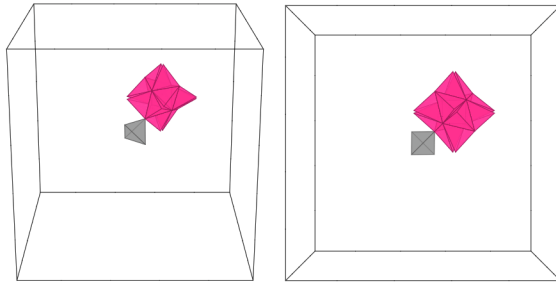
$\langle a,a,a \rangle$
2.000000000000



$\{+B_1,+B_{17}\}$

$a = \frac{8}{3}$

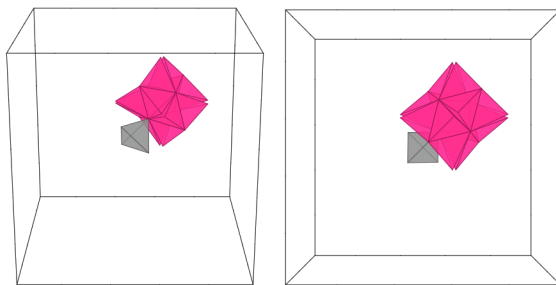
$\langle a,a,a \rangle$
2.666666666667



$\{+B_1,-B_{17}\}$

$a = 2$

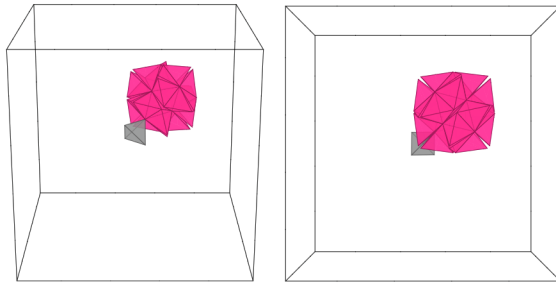
$\langle a,a,a \rangle$
2.000000000000



$\{+B_1,+B_{41}\}$

$a = \frac{8}{3}$

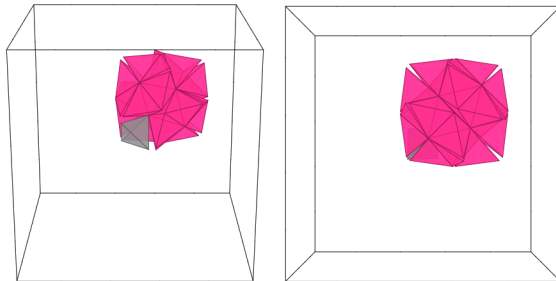
$\langle a,a,a \rangle$
2.666666666667



$\{+B_1,-B_{41}\}$

$a = 2$

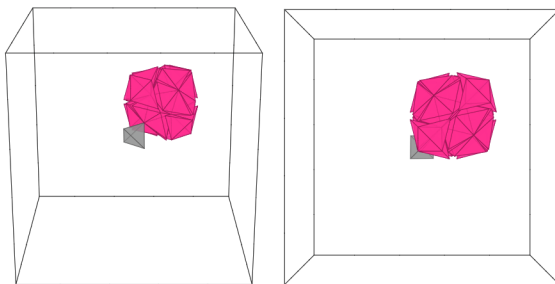
$\langle a,a,a \rangle$
2.000000000000



$\{+\mathbf{B}_1, +\mathbf{B}_{57}\}$

$$a = \frac{8}{3}$$

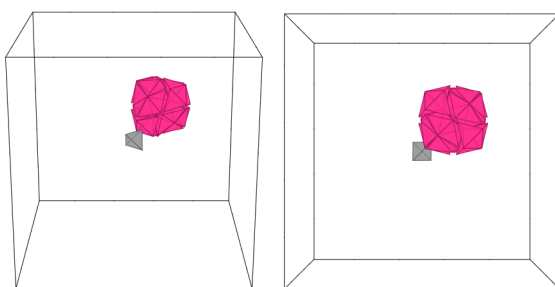
$\langle a, a, a \rangle$
2.666666666667



$\{+\mathbf{B}_1, -\mathbf{B}_{57}\}$

$$a = \frac{10}{3}$$

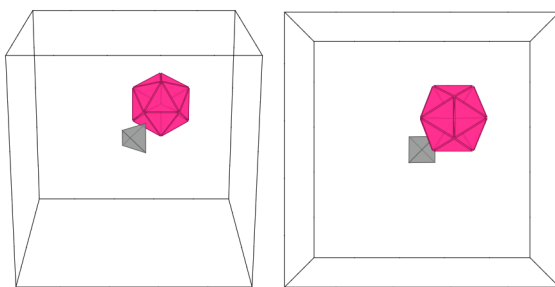
$\langle a, a, a \rangle$
3.333333333333



$\{+\mathbf{B}_1, +\mathbf{V}_{20}\}$

$$a = \frac{7}{3}$$

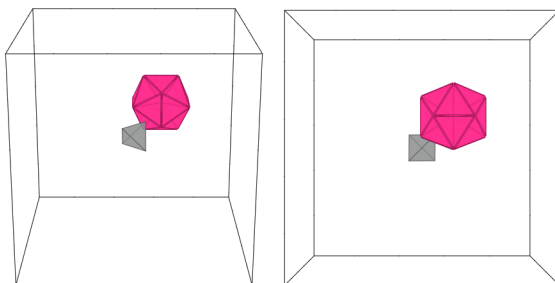
$\langle a, a, a \rangle$
2.333333333333



$\{+\mathbf{B}_1, -\mathbf{V}_{20}\}$

$$a = \frac{7}{3}$$

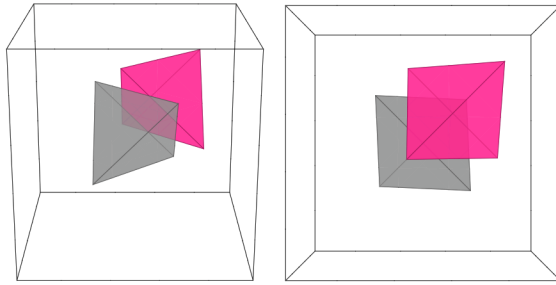
$\langle a, a, a \rangle$
2.333333333333



$\{-\mathbf{B}_1, +\mathbf{B}_1\}$

$a = \frac{2}{3}$

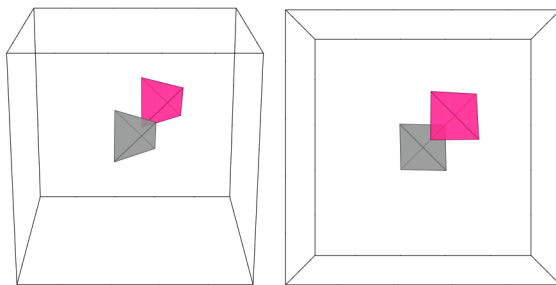
$\langle a, a, a \rangle$
.666666666667



$\{-\mathbf{B}_1, -\mathbf{B}_1\}$

$a = \frac{4}{3}$

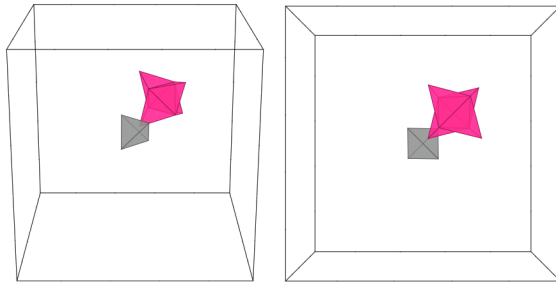
$\langle a, a, a \rangle$
1.333333333333



$\{-\mathbf{B}_1, +\mathbf{B}_5\}$

$a = 2$

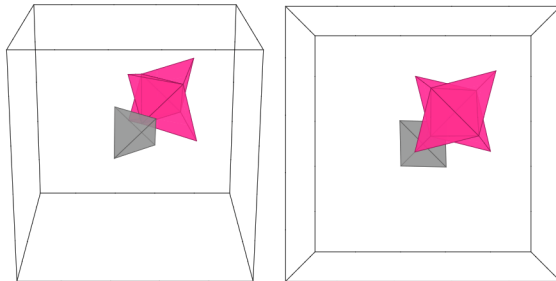
$\langle a, a, a \rangle$
2.000000000000



$\{-\mathbf{B}_1, -\mathbf{B}_5\}$

$a = \frac{4}{3}$

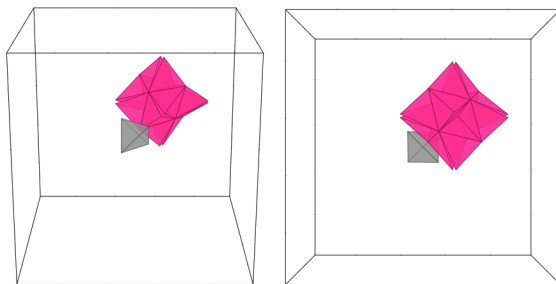
$\langle a, a, a \rangle$
1.333333333333



$\{-\mathbf{B}_1, +\mathbf{B}_{17}\}$

$a = 2$

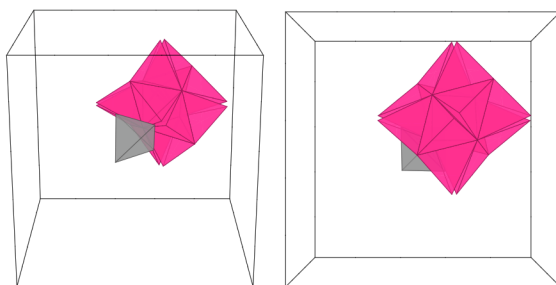
$\langle a, a, a \rangle$
2.000000000000



$\{-\mathbf{B}_1, -\mathbf{B}_{17}\}$

$a = \frac{24}{17}$

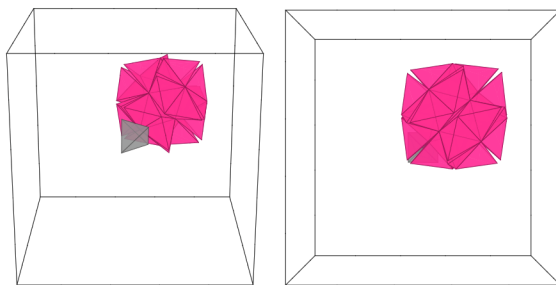
$\langle a, a, a \rangle$
1.411764705882



$\{-\mathbf{B}_1, +\mathbf{B}_{41}\}$

$a = \frac{190}{93}$

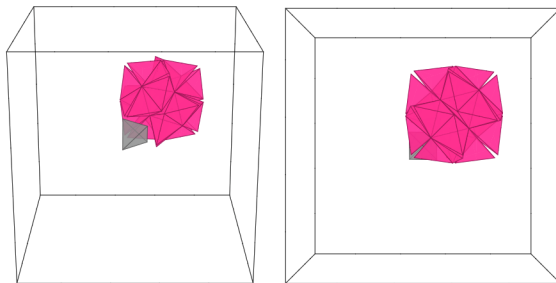
$\langle a, a, a \rangle$
2.043010752688



$\{-\mathbf{B}_1, -\mathbf{B}_{41}\}$

$a = \frac{120}{67}$

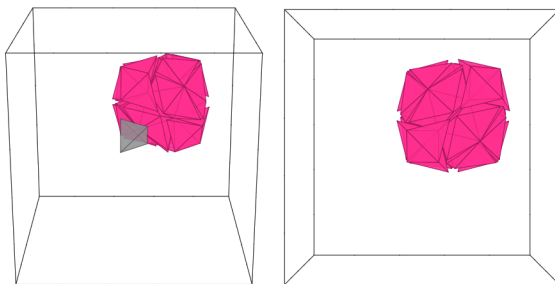
$\langle a, a, a \rangle$
1.791044776119



$\{-\mathbf{B}_1, +\mathbf{B}_{57}\}$

$$a = \frac{190}{93}$$

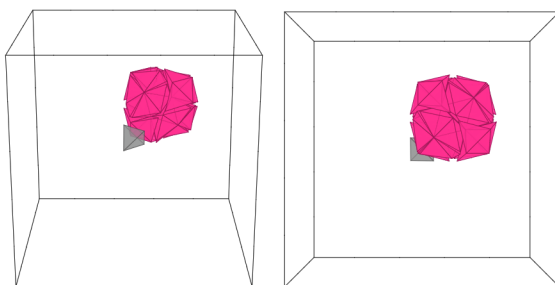
$\langle a, a, a \rangle$
2.043010752688



$\{-\mathbf{B}_1, -\mathbf{B}_{57}\}$

$$a = \frac{8}{3}$$

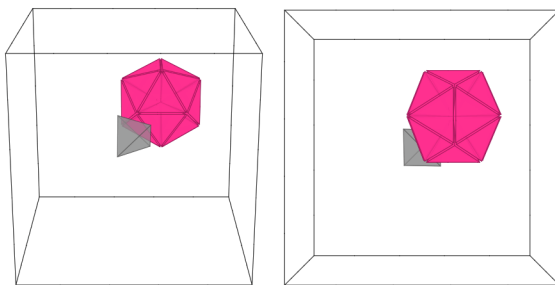
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_1, +\mathbf{V}_{20}\}$

$$a = \frac{5}{3}$$

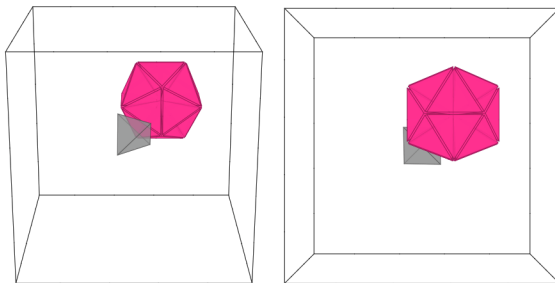
$\langle a, a, a \rangle$
1.666666666667



$\{-\mathbf{B}_1, -\mathbf{V}_{20}\}$

$$a = \frac{5}{3}$$

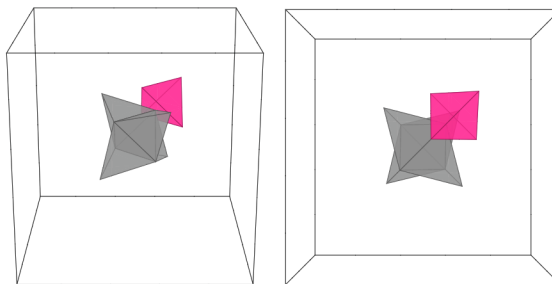
$\langle a, a, a \rangle$
1.666666666667



$\{+B_5,+B_1\}$

$a = \frac{4}{3}$

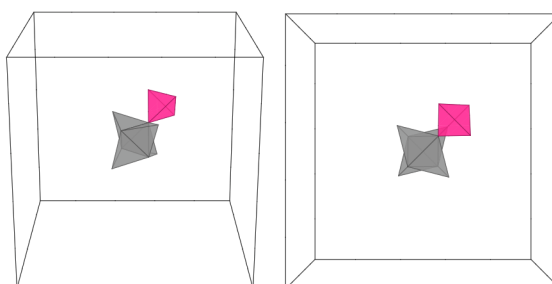
$\langle a,a,a \rangle$
1.333333333333



$\{+B_5,-B_1\}$

$a = 2$

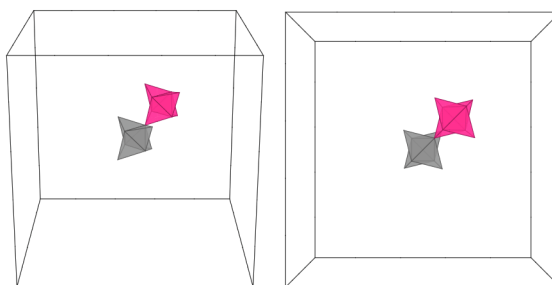
$\langle a,a,a \rangle$
2.000000000000



$\{+B_5,+B_5\}$

$a = \frac{8}{3}$

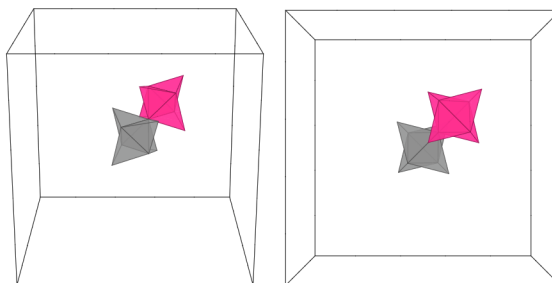
$\langle a,a,a \rangle$
2.666666666667



$\{+B_5,-B_5\}$

$a = 2$

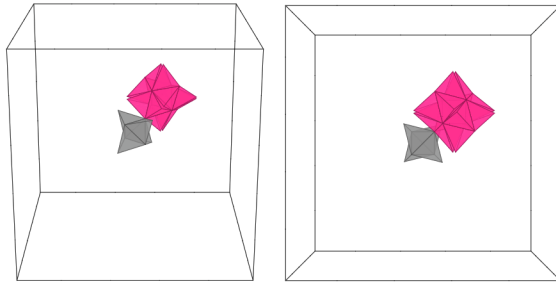
$\langle a,a,a \rangle$
2.000000000000



$\{+\mathbf{B}_5, +\mathbf{B}_{17}\}$

$a = \frac{8}{3}$

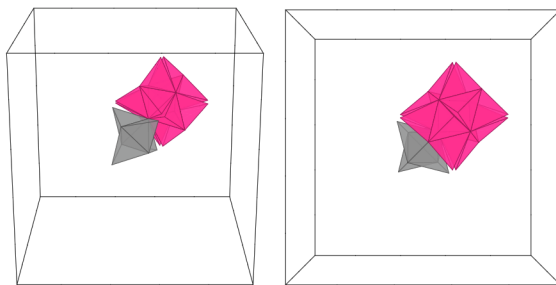
$\langle a, a, a \rangle$
2.666666666667



$\{+\mathbf{B}_5, -\mathbf{B}_{17}\}$

$a = 2$

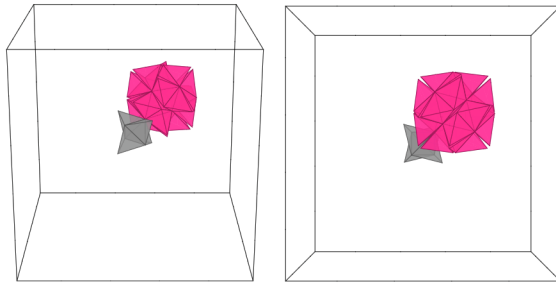
$\langle a, a, a \rangle$
2.000000000000



$\{+\mathbf{B}_5, +\mathbf{B}_{41}\}$

$a = \frac{8}{3}$

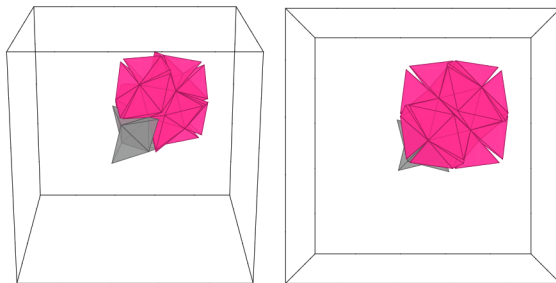
$\langle a, a, a \rangle$
2.666666666667



$\{+\mathbf{B}_5, -\mathbf{B}_{41}\}$

$a = 2$

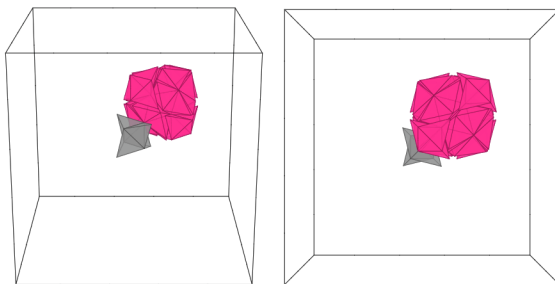
$\langle a, a, a \rangle$
2.000000000000



$\{+\mathbf{B}_5, +\mathbf{B}_{57}\}$

$$a = \frac{8}{3}$$

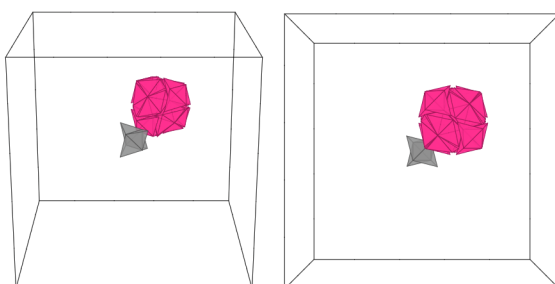
$\langle a, a, a \rangle$
2.666666666667



$\{+\mathbf{B}_5, -\mathbf{B}_{57}\}$

$$a = \frac{10}{3}$$

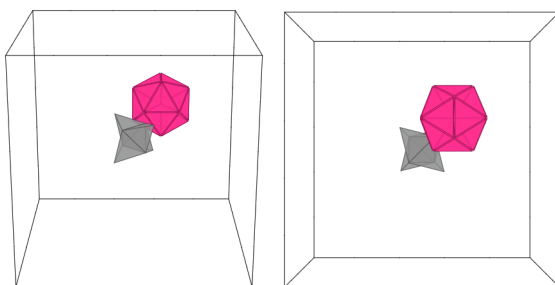
$\langle a, a, a \rangle$
3.333333333333



$\{+\mathbf{B}_5, +\mathbf{V}_{20}\}$

$$a = \frac{7}{3}$$

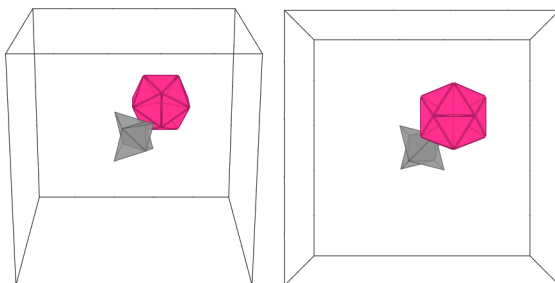
$\langle a, a, a \rangle$
2.333333333333



$\{+\mathbf{B}_5, -\mathbf{V}_{20}\}$

$$a = \frac{7}{3}$$

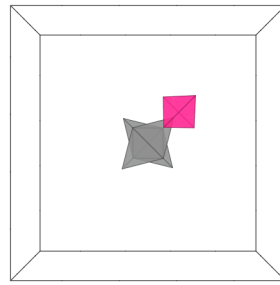
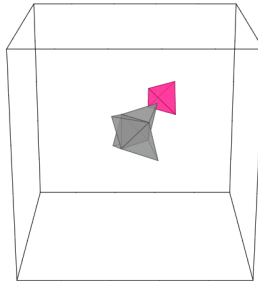
$\langle a, a, a \rangle$
2.333333333333



$\{-\mathbf{B}_5, +\mathbf{B}_1\}$

$a = 2$

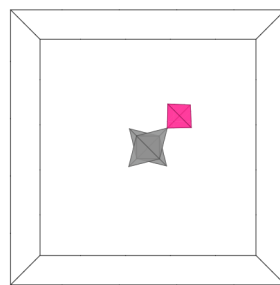
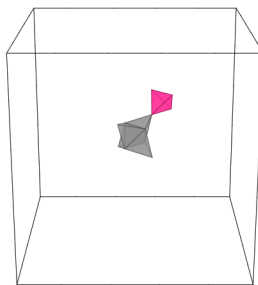
$\langle a, a, a \rangle$
2.000000000000



$\{-\mathbf{B}_5, -\mathbf{B}_1\}$

$a = \frac{8}{3}$

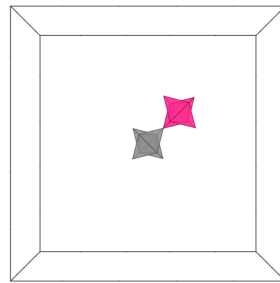
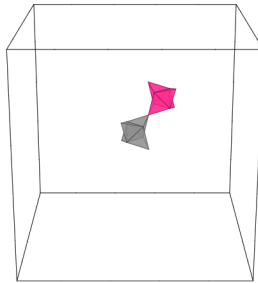
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_5, +\mathbf{B}_5\}$

$a = \frac{10}{3}$

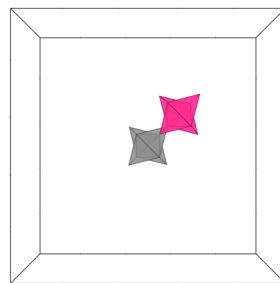
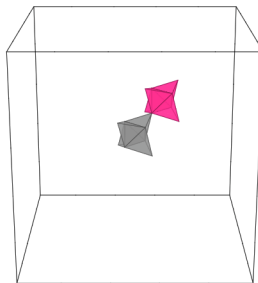
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_5, -\mathbf{B}_5\}$

$a = \frac{8}{3}$

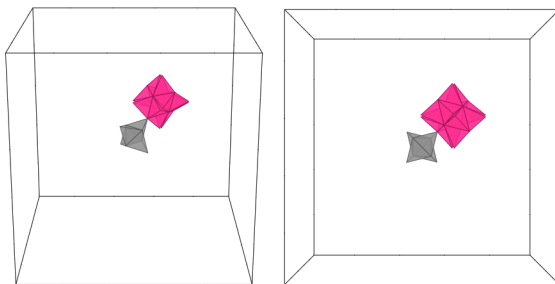
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_5, +\mathbf{B}_{17}\}$

$$a = \frac{10}{3}$$

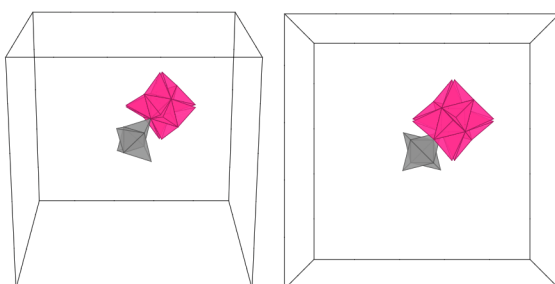
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_5, -\mathbf{B}_{17}\}$

$$a = \frac{8}{3}$$

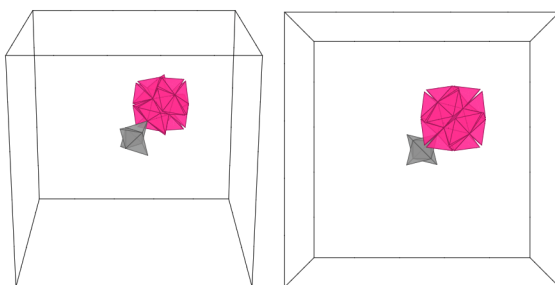
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_5, +\mathbf{B}_{41}\}$

$$a = \frac{10}{3}$$

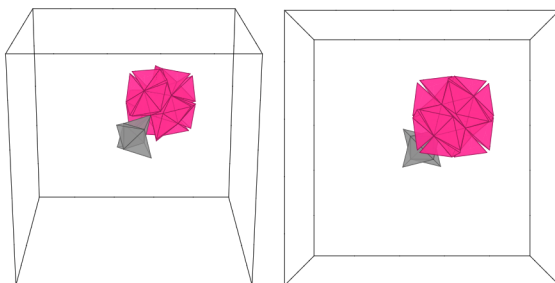
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_5, -\mathbf{B}_{41}\}$

$$a = \frac{8}{3}$$

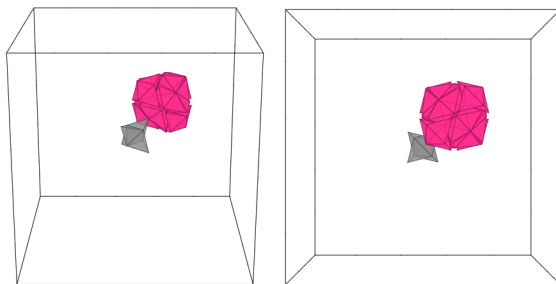
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_5, +\mathbf{B}_{57}\}$

$a = \frac{10}{3}$

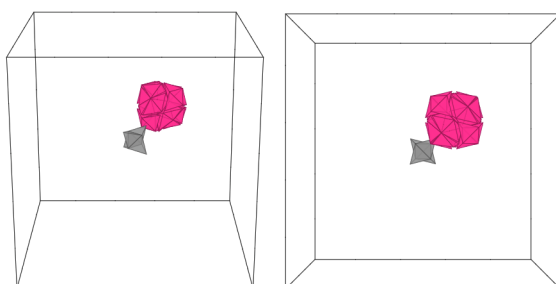
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_5, -\mathbf{B}_{57}\}$

$a = 4$

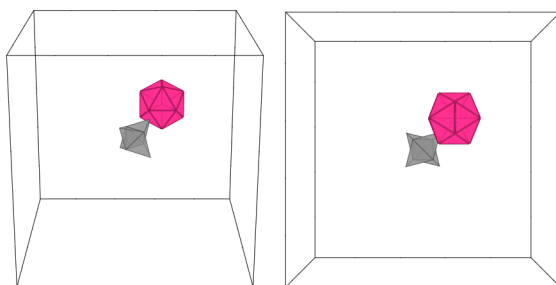
$\langle a, a, a \rangle$
4.000000000000



$\{-\mathbf{B}_5, +\mathbf{V}_{20}\}$

$a = 3$

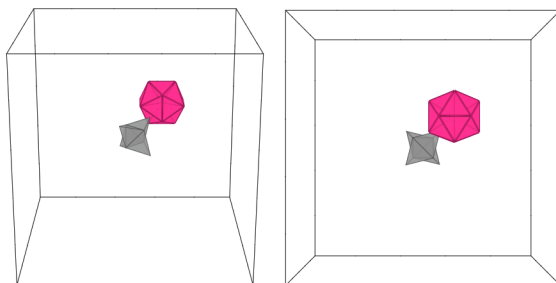
$\langle a, a, a \rangle$
3.000000000000



$\{-\mathbf{B}_5, -\mathbf{V}_{20}\}$

$a = 3$

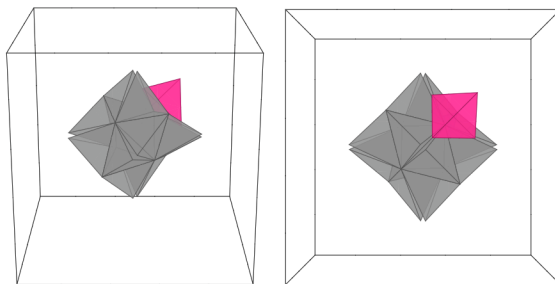
$\langle a, a, a \rangle$
3.000000000000



$\{+\mathbf{B}_{17},+\mathbf{B}_1\}$

$a = \frac{24}{17}$

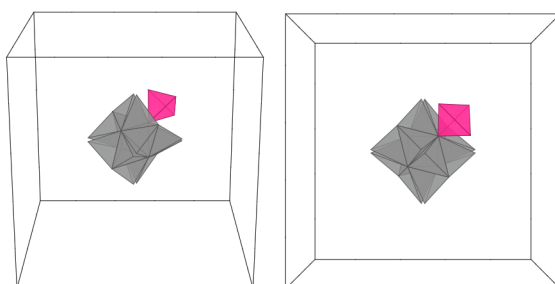
$\langle a,a,a \rangle$
1.411764705882



$\{+\mathbf{B}_{17},-\mathbf{B}_1\}$

$a = 2$

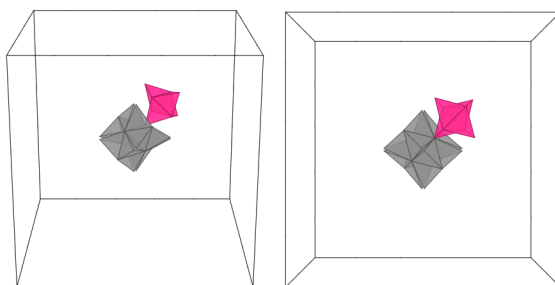
$\langle a,a,a \rangle$
2.000000000000



$\{+\mathbf{B}_{17},+\mathbf{B}_5\}$

$a = \frac{8}{3}$

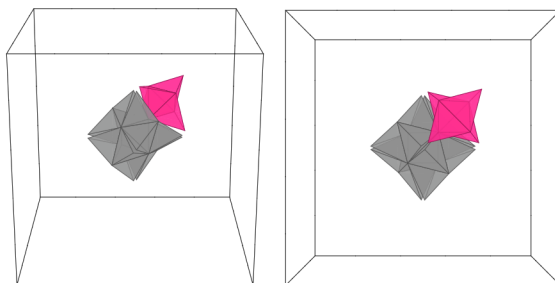
$\langle a,a,a \rangle$
2.666666666667



$\{+\mathbf{B}_{17},-\mathbf{B}_5\}$

$a = 2$

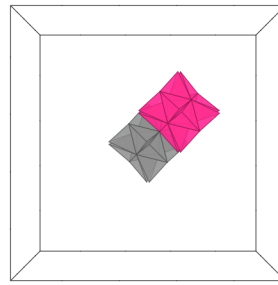
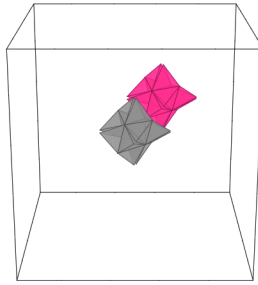
$\langle a,a,a \rangle$
2.000000000000



$\{+\mathbf{B}_{17},+\mathbf{B}_{17}\}$

$$a = \frac{8}{3}$$

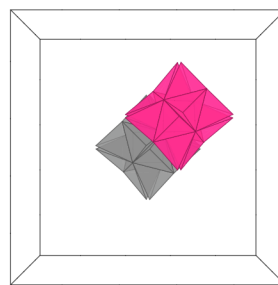
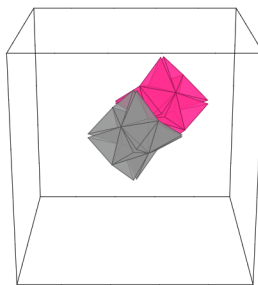
$\langle a,a,a \rangle$
2.666666666667



$\{+\mathbf{B}_{17},-\mathbf{B}_{17}\}$

$$a = 2$$

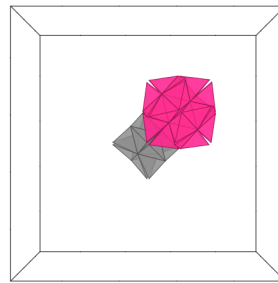
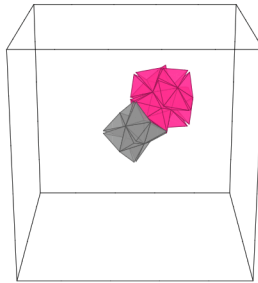
$\langle a,a,a \rangle$
2.000000000000



$\{+\mathbf{B}_{17},+\mathbf{B}_{41}\}$

$$a = \frac{80}{27}$$

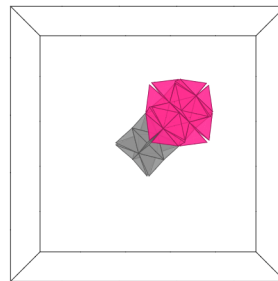
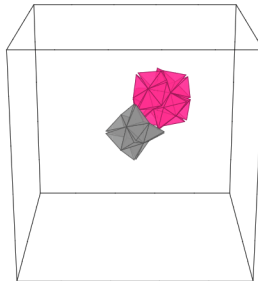
$\langle a,a,a \rangle$
2.962962962963



$\{+\mathbf{B}_{17},-\mathbf{B}_{41}\}$

$$a = \frac{2182}{675}$$

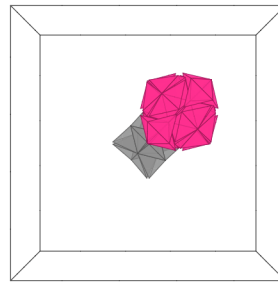
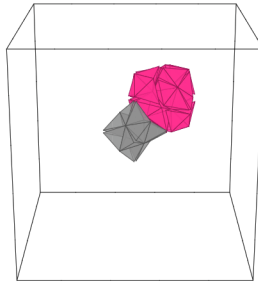
$\langle a,a,a \rangle$
3.232592592593



$\{+\mathbf{B}_{17},+\mathbf{B}_{57}\}$

$$a = \frac{80}{27}$$

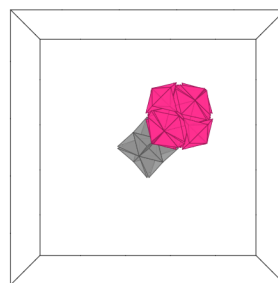
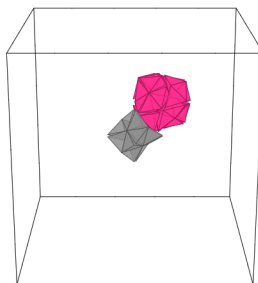
$\langle a,a,a \rangle$
2.962962962963



$\{+\mathbf{B}_{17},-\mathbf{B}_{57}\}$

$$a = \frac{58}{17}$$

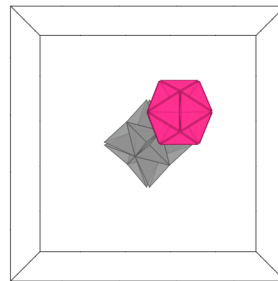
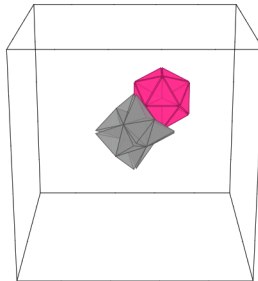
$\langle a,a,a \rangle$
3.411764705882



$\{+\mathbf{B}_{17},+\mathbf{V}_{20}\}$

$$a = \frac{7}{3} + \frac{2}{1199}\sqrt{2} + \frac{62}{3597}\sqrt{10}$$

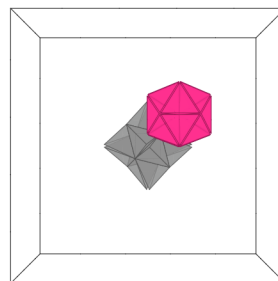
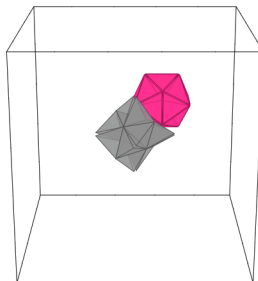
$\langle a,a,a \rangle$
2.390199192745



$\{+\mathbf{B}_{17},-\mathbf{V}_{20}\}$

$$a = \frac{7}{3} + \frac{2}{1199}\sqrt{2} + \frac{62}{3597}\sqrt{10}$$

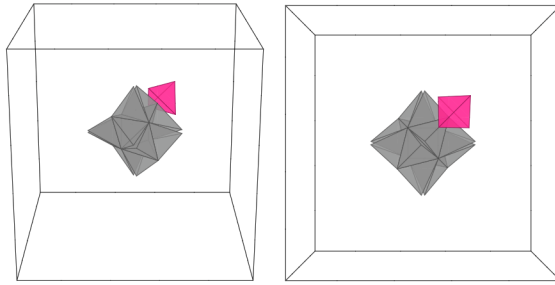
$\langle a,a,a \rangle$
2.390199192745



$\{-\mathbf{B}_{17}, +\mathbf{B}_1\}$

$a = 2$

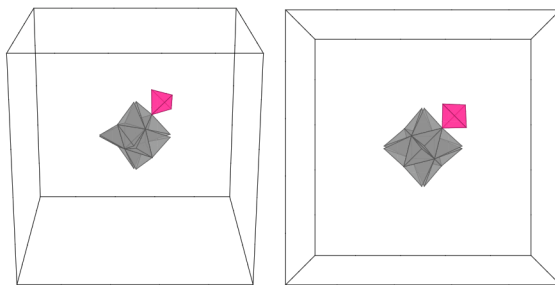
$\langle a, a, a \rangle$
2.000000000000



$\{-\mathbf{B}_{17}, -\mathbf{B}_1\}$

$a = \frac{8}{3}$

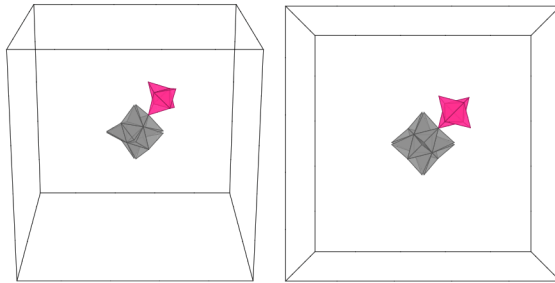
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_{17}, +\mathbf{B}_5\}$

$a = \frac{10}{3}$

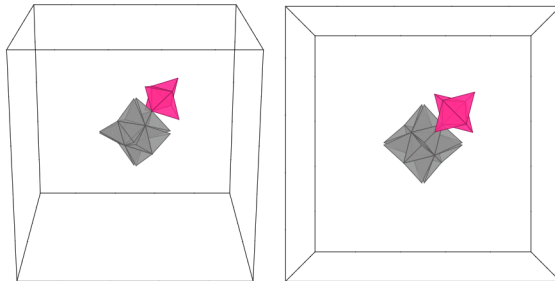
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_{17}, -\mathbf{B}_5\}$

$a = \frac{8}{3}$

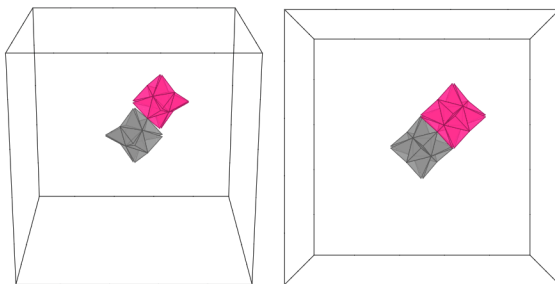
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_{17}, +\mathbf{B}_{17}\}$

$$a = \frac{10}{3}$$

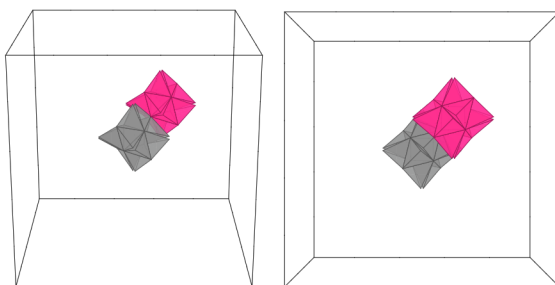
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_{17}, -\mathbf{B}_{17}\}$

$$a = \frac{8}{3}$$

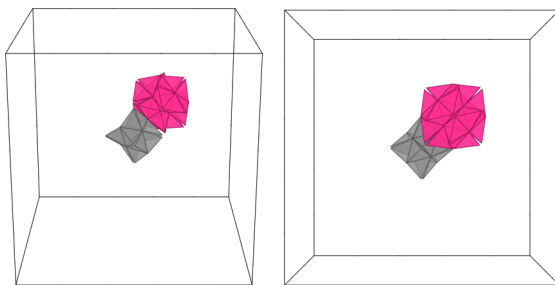
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_{17}, +\mathbf{B}_{41}\}$

$$a = \frac{10}{3}$$

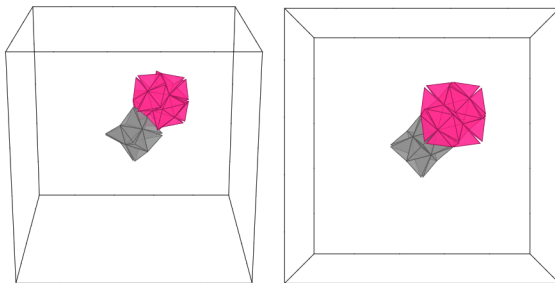
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_{17}, -\mathbf{B}_{41}\}$

$$a = \frac{1352}{405}$$

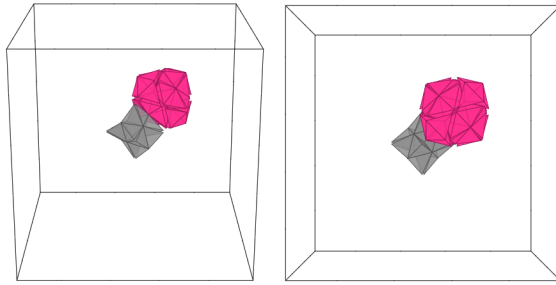
$\langle a, a, a \rangle$
3.338271604938



$\{-\mathbf{B}_{17}, +\mathbf{B}_{57}\}$

$a = \frac{10}{3}$

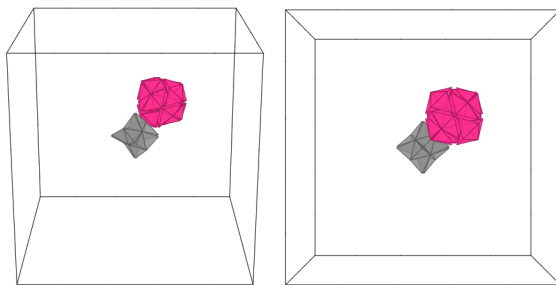
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_{17}, -\mathbf{B}_{57}\}$

$a = 4$

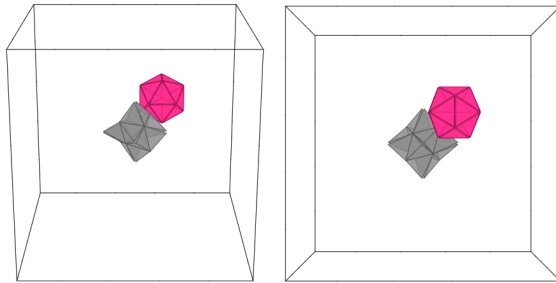
$\langle a, a, a \rangle$
4.000000000000



$\{-\mathbf{B}_{17}, +\mathbf{V}_{20}\}$

$a = 3$

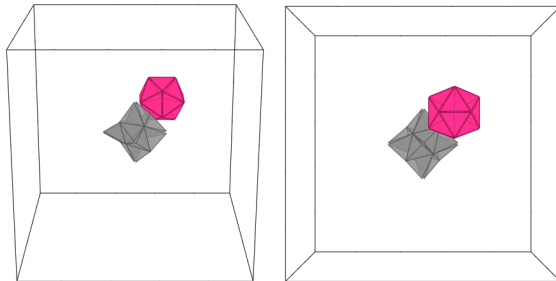
$\langle a, a, a \rangle$
3.000000000000



$\{-\mathbf{B}_{17}, -\mathbf{V}_{20}\}$

$a = 3$

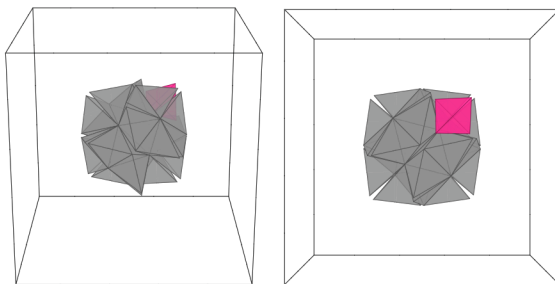
$\langle a, a, a \rangle$
3.000000000000



$\{+\mathbf{B}_{41},+\mathbf{B}_1\}$

$a = \frac{120}{67}$

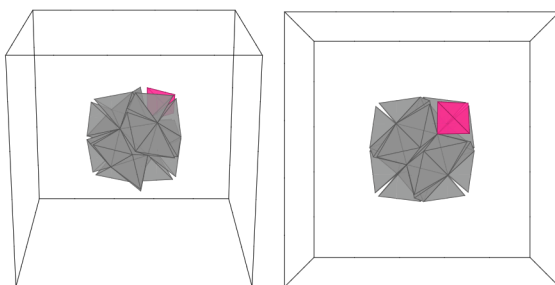
$\langle a,a,a \rangle$
1.791044776119



$\{+\mathbf{B}_{41},-\mathbf{B}_1\}$

$a = 2$

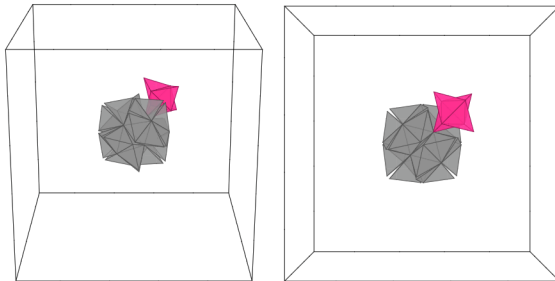
$\langle a,a,a \rangle$
2.000000000000



$\{+\mathbf{B}_{41},+\mathbf{B}_5\}$

$a = \frac{8}{3}$

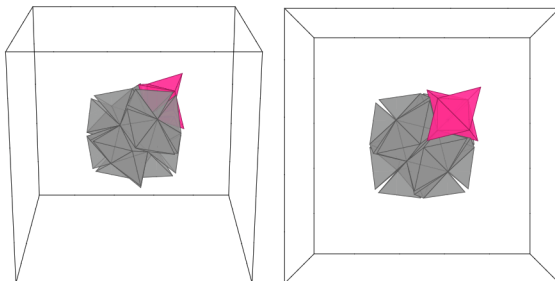
$\langle a,a,a \rangle$
2.666666666667



$\{+\mathbf{B}_{41},-\mathbf{B}_5\}$

$a = 2$

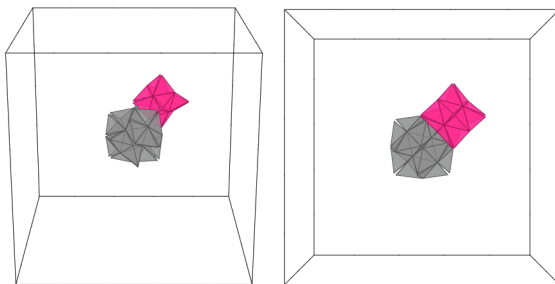
$\langle a,a,a \rangle$
2.000000000000



$\{+\mathbf{B}_{41}, +\mathbf{B}_{17}\}$

$$a = \frac{1352}{405}$$

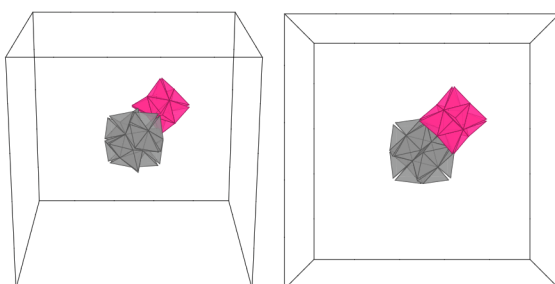
$\langle a, a, a \rangle$
3.338271604938



$\{+\mathbf{B}_{41}, -\mathbf{B}_{17}\}$

$$a = \frac{2182}{675}$$

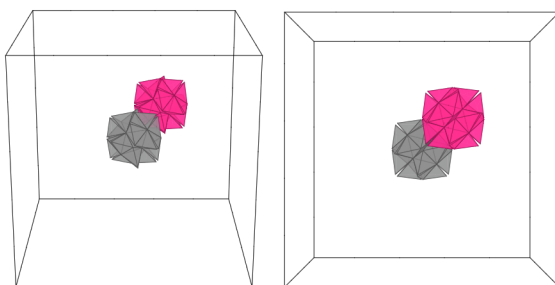
$\langle a, a, a \rangle$
3.232592592593



$\{+\mathbf{B}_{41}, +\mathbf{B}_{41}\}$

$$a = \frac{7084}{2025}$$

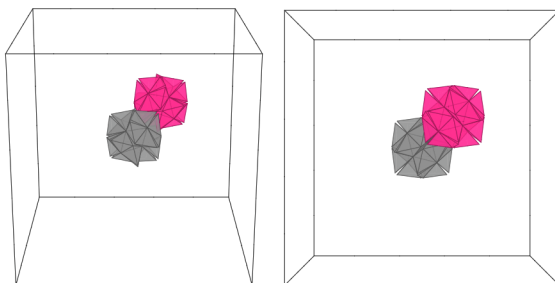
$\langle a, a, a \rangle$
3.498271604938



$\{+\mathbf{B}_{41}, -\mathbf{B}_{41}\}$

$$a = \frac{246}{71}$$

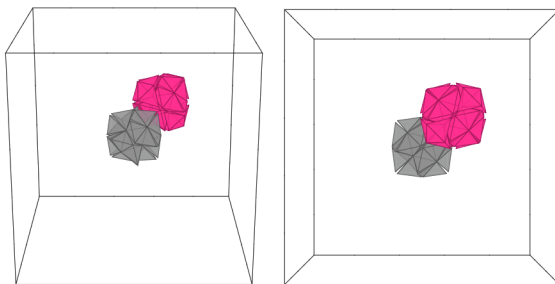
$\langle a, a, a \rangle$
3.464788732394



$\{+\mathbf{B}_{41}, +\mathbf{B}_{57}\}$

$$a = \frac{7084}{2025}$$

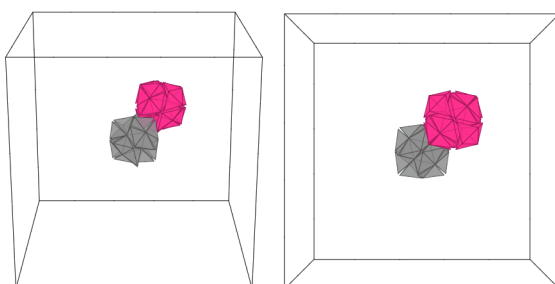
$\langle a, a, a \rangle$
3.498271604938



$\{+\mathbf{B}_{41}, -\mathbf{B}_{57}\}$

$$a = \frac{82}{21}$$

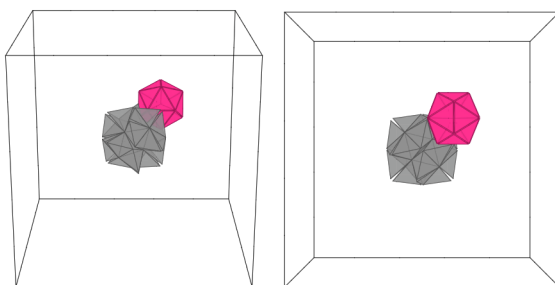
$\langle a, a, a \rangle$
3.904761904762



$\{+\mathbf{B}_{41}, +\mathbf{V}_{20}\}$

$$a = \frac{7}{5} + \frac{2}{3}\sqrt{5} - \frac{1}{6}\sqrt{2} + \frac{1}{10}\sqrt{10}$$

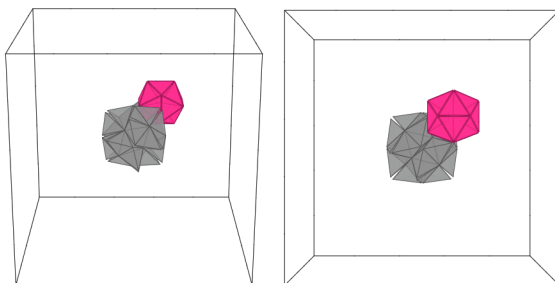
$\langle a, a, a \rangle$
2.971237490621



$\{+\mathbf{B}_{41}, -\mathbf{V}_{20}\}$

$$a = \frac{7}{5} + \frac{2}{3}\sqrt{5} - \frac{1}{6}\sqrt{2} + \frac{1}{10}\sqrt{10}$$

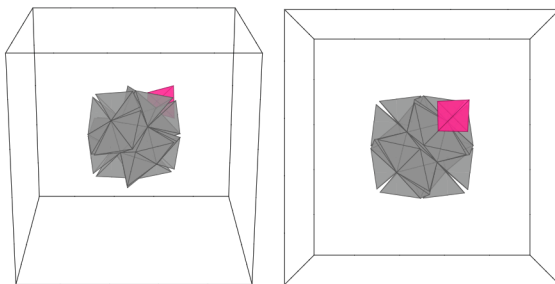
$\langle a, a, a \rangle$
2.971237490621



$\{-\mathbf{B}_{41}, +\mathbf{B}_1\}$

$$a = \frac{190}{93}$$

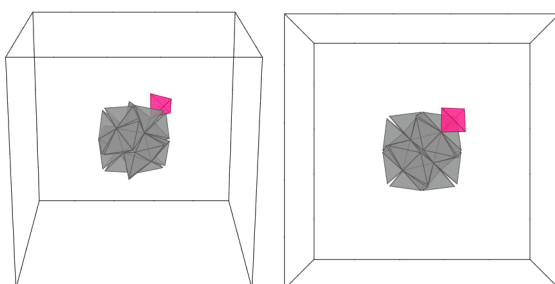
$\langle a, a, a \rangle$
2.043010752688



$\{-\mathbf{B}_{41}, -\mathbf{B}_1\}$

$$a = \frac{8}{3}$$

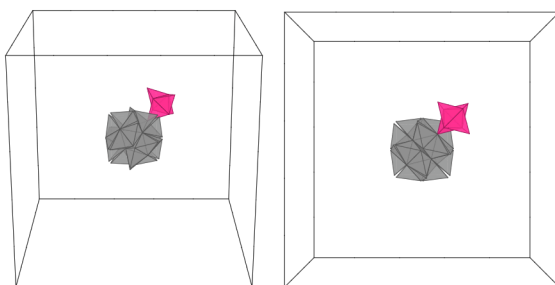
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_{41}, +\mathbf{B}_5\}$

$$a = \frac{10}{3}$$

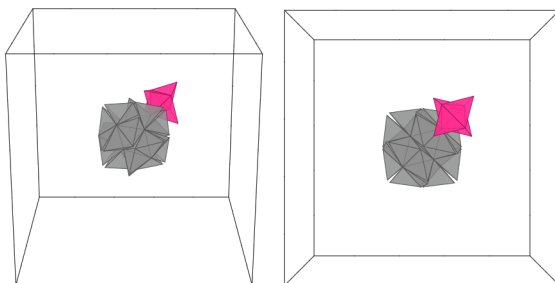
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_{41}, -\mathbf{B}_5\}$

$$a = \frac{8}{3}$$

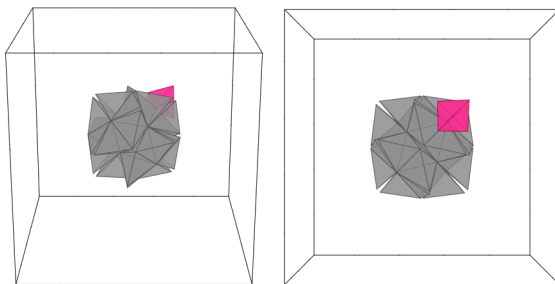
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_{41}, +\mathbf{B}_1\}$

$$a = \frac{190}{93}$$

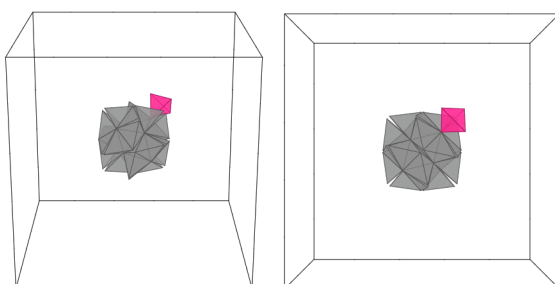
$\langle a, a, a \rangle$
2.043010752688



$\{-\mathbf{B}_{41}, -\mathbf{B}_1\}$

$$a = \frac{8}{3}$$

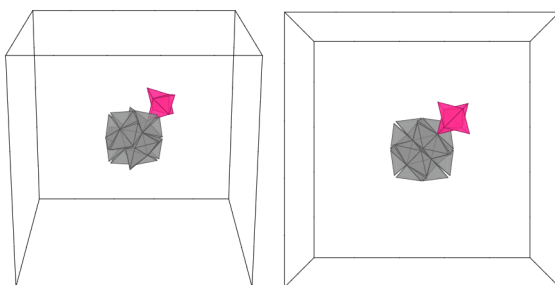
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_{41}, +\mathbf{B}_5\}$

$$a = \frac{10}{3}$$

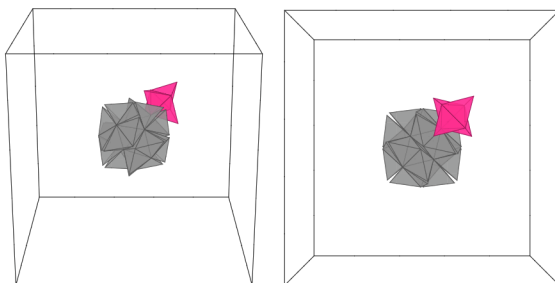
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_{41}, -\mathbf{B}_5\}$

$$a = \frac{8}{3}$$

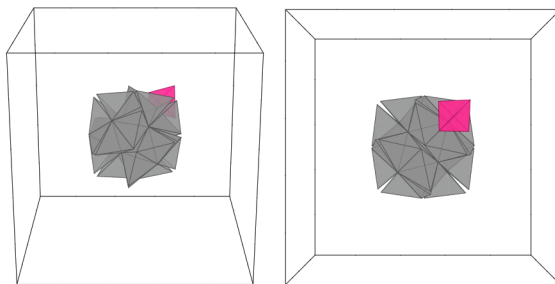
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_{41}, +\mathbf{B}_1\}$

$$a = \frac{190}{93}$$

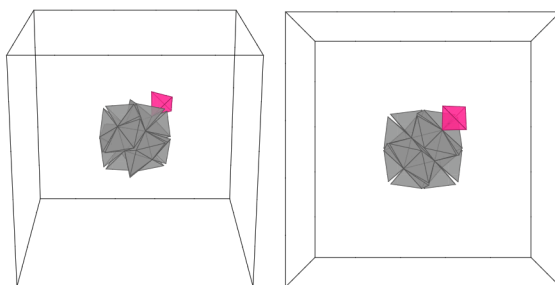
$\langle a, a, a \rangle$
2.043010752688



$\{-\mathbf{B}_{41}, -\mathbf{B}_1\}$

$$a = \frac{8}{3}$$

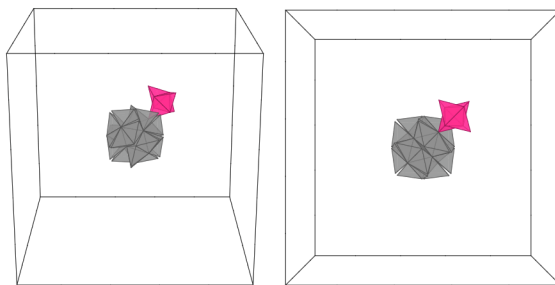
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_{41}, +\mathbf{B}_5\}$

$$a = \frac{10}{3}$$

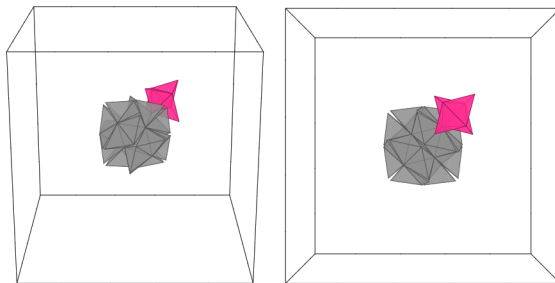
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_{41}, -\mathbf{B}_5\}$

$$a = \frac{8}{3}$$

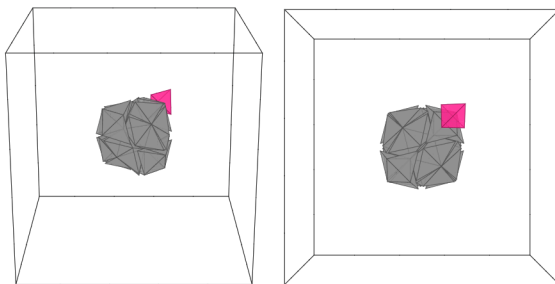
$\langle a, a, a \rangle$
2.666666666667



$\{+\mathbf{B}_{57},+\mathbf{B}_1\}$

$$a = \frac{8}{3}$$

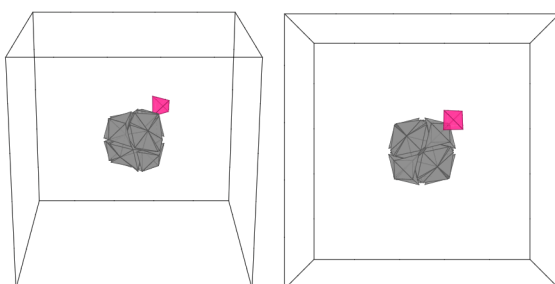
$\langle a,a,a \rangle$
2.666666666667



$\{+\mathbf{B}_{57},-\mathbf{B}_1\}$

$$a = \frac{10}{3}$$

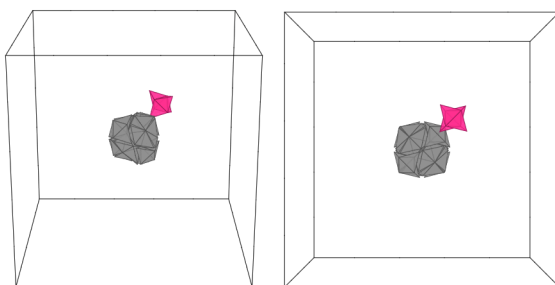
$\langle a,a,a \rangle$
3.333333333333



$\{+\mathbf{B}_{57},+\mathbf{B}_5\}$

$$a = 4$$

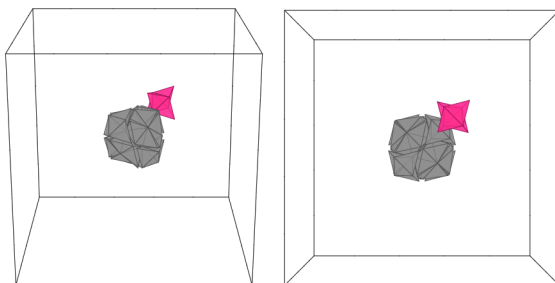
$\langle a,a,a \rangle$
4.000000000000



$\{+\mathbf{B}_{57},-\mathbf{B}_5\}$

$$a = \frac{10}{3}$$

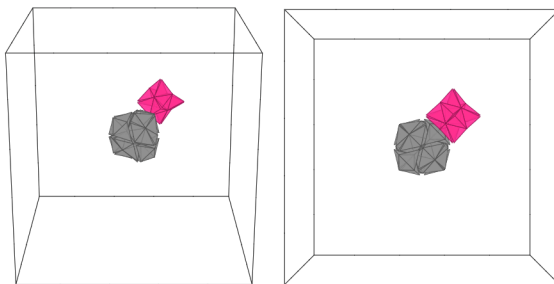
$\langle a,a,a \rangle$
3.333333333333



$\{+\mathbf{B}_{57},+\mathbf{B}_{17}\}$

$a = 4$

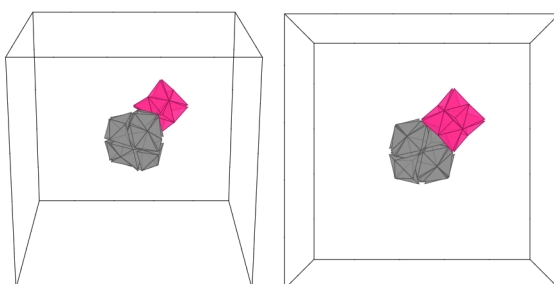
$\langle a,a,a \rangle$
4.000000000000



$\{+\mathbf{B}_{57},-\mathbf{B}_{17}\}$

$a = \frac{58}{17}$

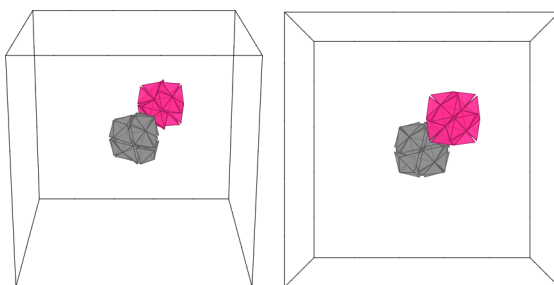
$\langle a,a,a \rangle$
3.411764705882



$\{+\mathbf{B}_{57},+\mathbf{B}_{41}\}$

$a = \frac{376}{93}$

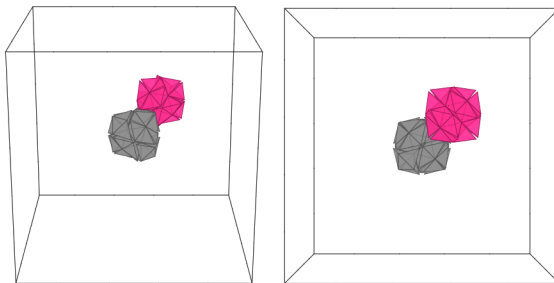
$\langle a,a,a \rangle$
4.043010752688



$\{+\mathbf{B}_{57},-\mathbf{B}_{41}\}$

$a = \frac{82}{21}$

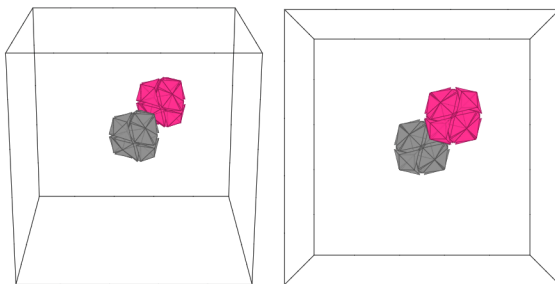
$\langle a,a,a \rangle$
3.904761904762



$\{+\mathbf{B}_{57},+\mathbf{B}_{57}\}$

$$a = \frac{376}{93}$$

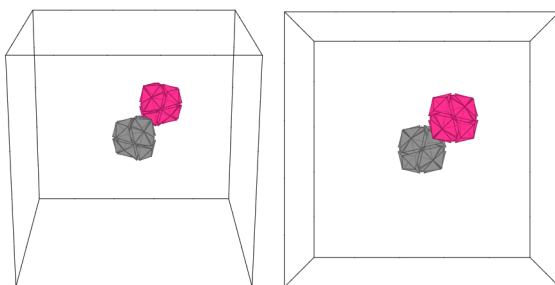
$\langle a,a,a \rangle$
4.043010752688



$\{+\mathbf{B}_{57},-\mathbf{B}_{57}\}$

$$a = \frac{14}{3}$$

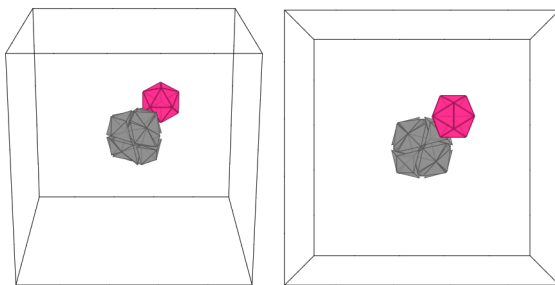
$\langle a,a,a \rangle$
4.666666666667



$\{+\mathbf{B}_{57},+\mathbf{V}_{20}\}$

$$a = \frac{11}{3}$$

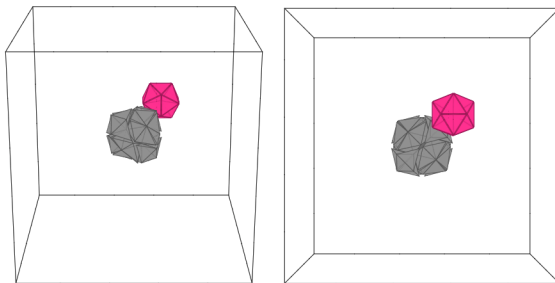
$\langle a,a,a \rangle$
3.666666666667



$\{+\mathbf{B}_{57},-\mathbf{V}_{20}\}$

$$a = \frac{11}{3}$$

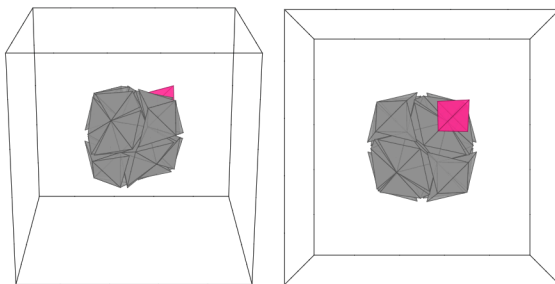
$\langle a,a,a \rangle$
3.666666666667



$\{-\mathbf{B}_{57}, +\mathbf{B}_1\}$

$$a = \frac{190}{93}$$

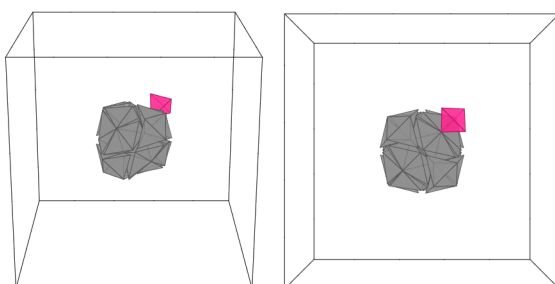
$\langle a, a, a \rangle$
2.043010752688



$\{-\mathbf{B}_{57}, -\mathbf{B}_1\}$

$$a = \frac{8}{3}$$

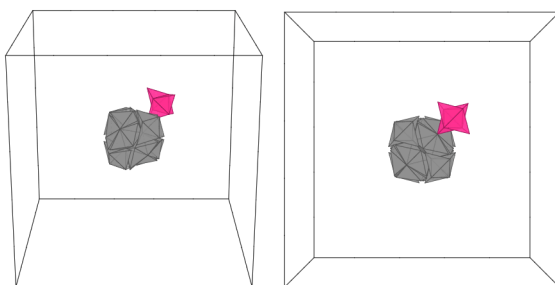
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_{57}, +\mathbf{B}_5\}$

$$a = \frac{10}{3}$$

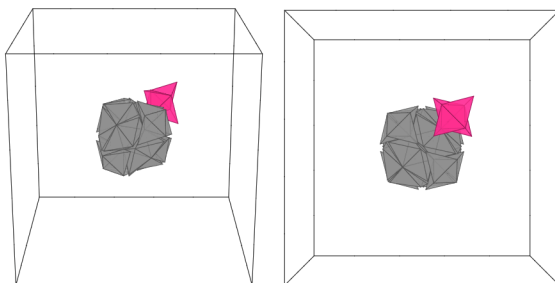
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_{57}, -\mathbf{B}_5\}$

$$a = \frac{8}{3}$$

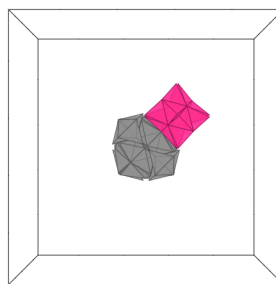
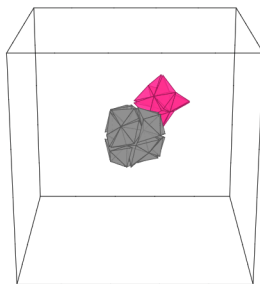
$\langle a, a, a \rangle$
2.666666666667



$\{-\mathbf{B}_{57}, +\mathbf{B}_{17}\}$

$$a = \frac{10}{3}$$

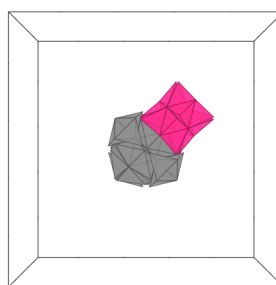
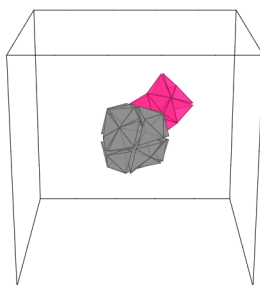
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_{57}, -\mathbf{B}_{17}\}$

$$a = \frac{80}{27}$$

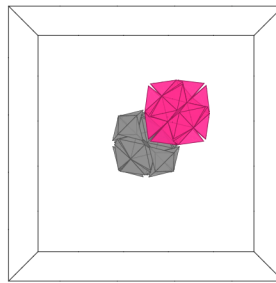
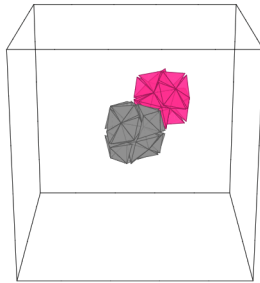
$\langle a, a, a \rangle$
2.962962962963



$\{-\mathbf{B}_{57}, +\mathbf{B}_{41}\}$

$$a = \frac{10}{3}$$

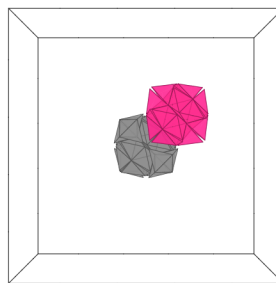
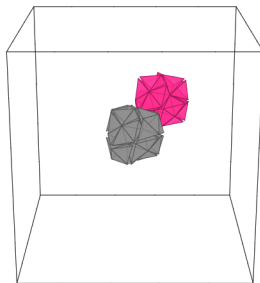
$\langle a, a, a \rangle$
3.333333333333



$\{-\mathbf{B}_{57}, -\mathbf{B}_{41}\}$

$$a = \frac{7084}{2025}$$

$\langle a, a, a \rangle$
3.498271604938

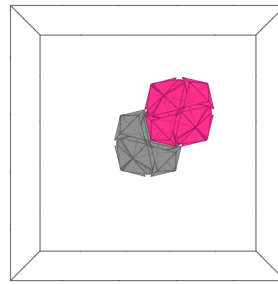
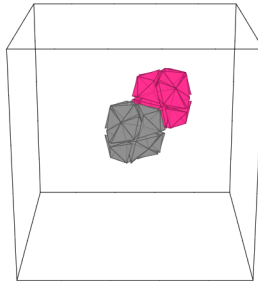


$$\{-\mathbf{B}_{57}, +\mathbf{B}_{57}\}$$

$$a = \frac{10}{3}$$

$$\langle a, a, a \rangle$$

$$3.333333333333$$

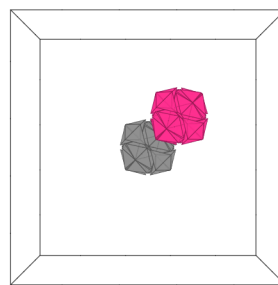
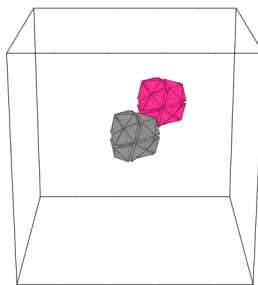


$$\{-\mathbf{B}_{57}, -\mathbf{B}_{57}\}$$

$$a = \frac{376}{93}$$

$$\langle a, a, a \rangle$$

$$4.043010752688$$

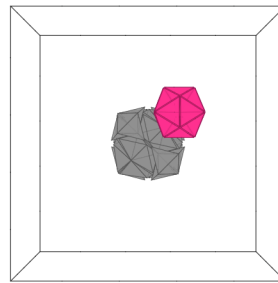
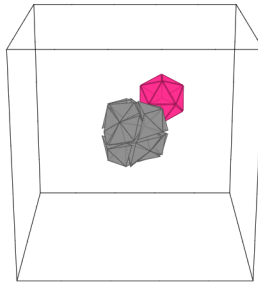


$$\{-\mathbf{B}_{57}, +\mathbf{V}_{20}\}$$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

$$\langle a, a, a \rangle$$

$$3.047553602722$$

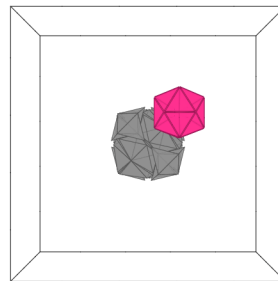
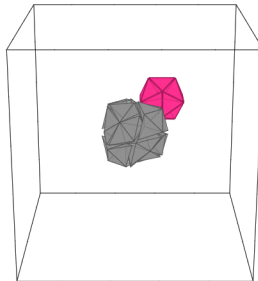


$$\{-\mathbf{B}_{57}, -\mathbf{V}_{20}\}$$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

$$\langle a, a, a \rangle$$

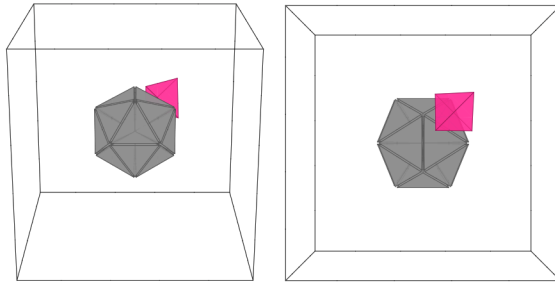
$$3.047553602722$$



$\{+V_{20}, +B_1\}$

$a = \frac{5}{3}$

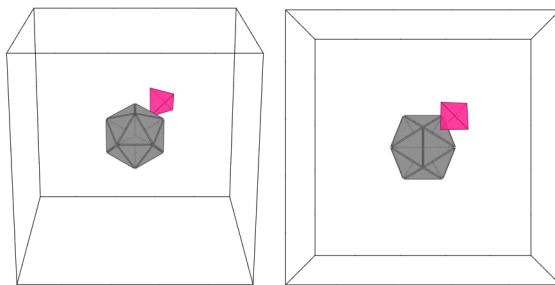
$\langle a, a, a \rangle$
1.666666666667



$\{+V_{20}, -B_1\}$

$a = \frac{7}{3}$

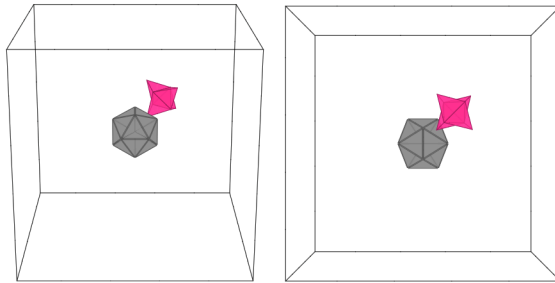
$\langle a, a, a \rangle$
2.333333333333



$\{+V_{20}, +B_5\}$

$a = 3$

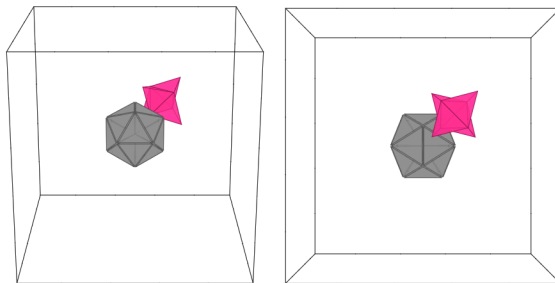
$\langle a, a, a \rangle$
3.000000000000



$\{+V_{20}, -B_5\}$

$a = \frac{7}{3}$

$\langle a, a, a \rangle$
2.333333333333

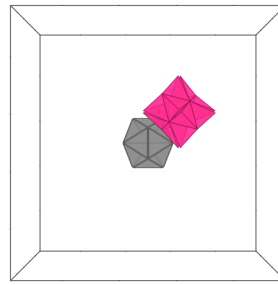
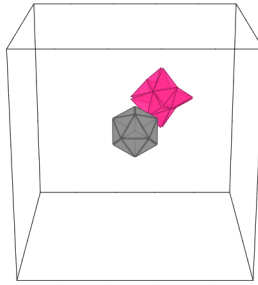


$$\{+\mathbf{V}_{20},+\mathbf{B}_{17}\}$$

$$a = 3$$

$$\langle a,a,a \rangle$$

$$3.000000000000$$

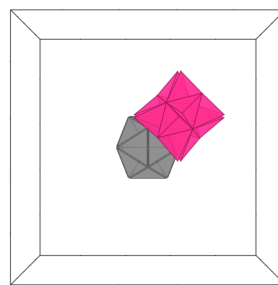
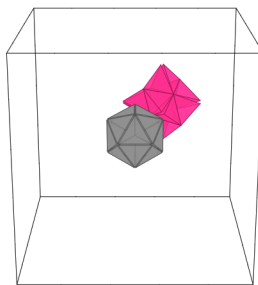


$$\{+\mathbf{V}_{20},-\mathbf{B}_{17}\}$$

$$a = \frac{7}{3} + \frac{2}{1199}\sqrt{2} + \frac{62}{3597}\sqrt{10}$$

$$\langle a,a,a \rangle$$

$$2.390199192745$$

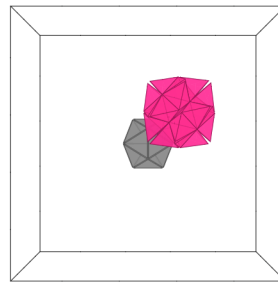
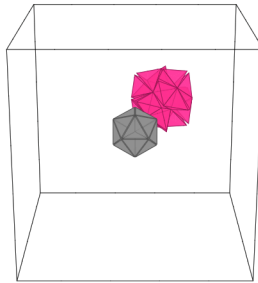


$$\{+\mathbf{V}_{20},+\mathbf{B}_{41}\}$$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

$$\langle a,a,a \rangle$$

$$3.047553602722$$

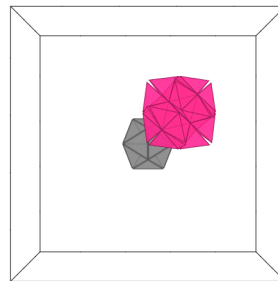
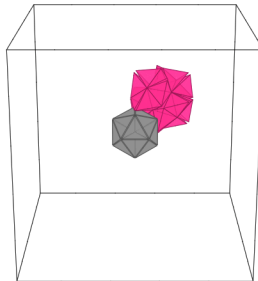


$$\{+\mathbf{V}_{20},-\mathbf{B}_{41}\}$$

$$a = \frac{7}{5} + \frac{2}{3}\sqrt{5} - \frac{1}{6}\sqrt{2} + \frac{1}{10}\sqrt{10}$$

$$\langle a,a,a \rangle$$

$$2.971237490621$$

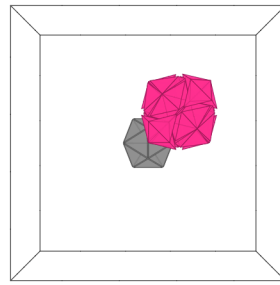
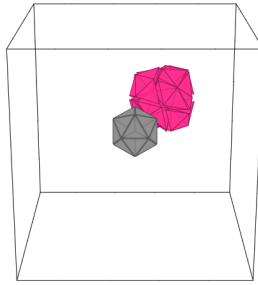


$$\{+\mathbf{V}_{20},+\mathbf{B}_{57}\}$$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

$$\langle a,a,a \rangle$$

$$3.047553602722$$

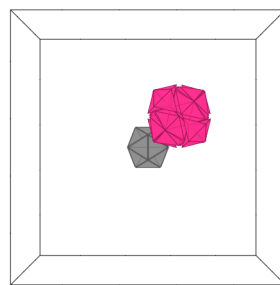
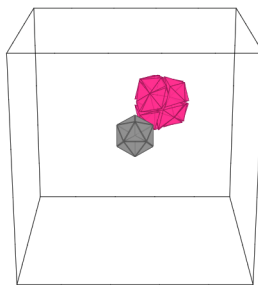


$$\{+\mathbf{V}_{20},-\mathbf{B}_{57}\}$$

$$a = \frac{11}{3}$$

$$\langle a,a,a \rangle$$

$$3.666666666667$$

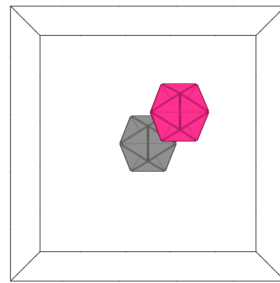
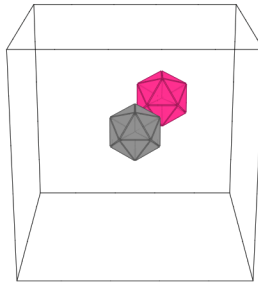


$$\{+\mathbf{V}_{20},+\mathbf{V}_{20}\}$$

$$a = \frac{8}{3}$$

$$\langle a,a,a \rangle$$

$$2.666666666667$$

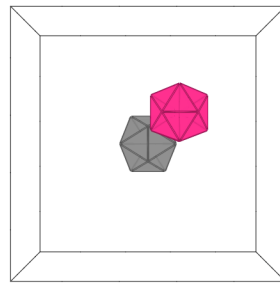
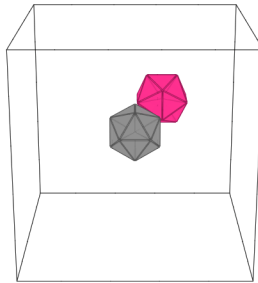


$$\{+\mathbf{V}_{20},-\mathbf{V}_{20}\}$$

$$a = \frac{8}{3}$$

$$\langle a,a,a \rangle$$

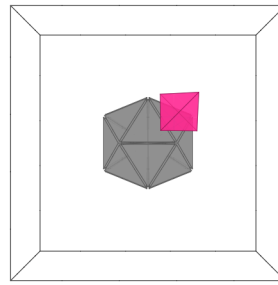
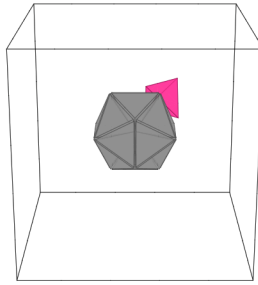
$$2.666666666667$$



$\{-V_{20}, +B_1\}$

$a = \frac{5}{3}$

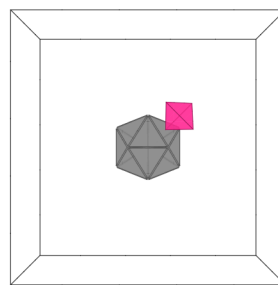
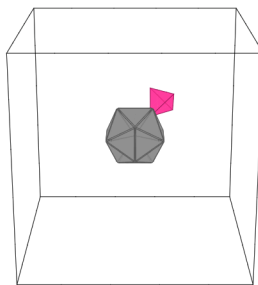
$\langle a, a, a \rangle$
1.666666666667



$\{-V_{20}, -B_1\}$

$a = \frac{7}{3}$

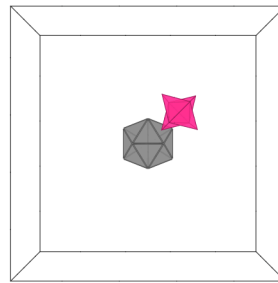
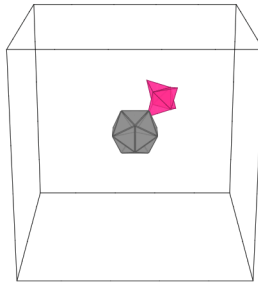
$\langle a, a, a \rangle$
2.333333333333



$\{-V_{20}, +B_5\}$

$a = 3$

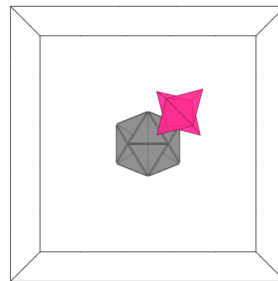
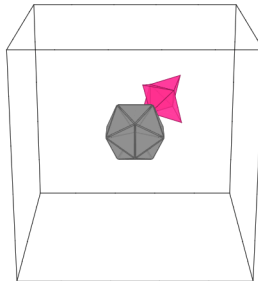
$\langle a, a, a \rangle$
3.000000000000



$\{-V_{20}, -B_5\}$

$a = \frac{7}{3}$

$\langle a, a, a \rangle$
2.333333333333

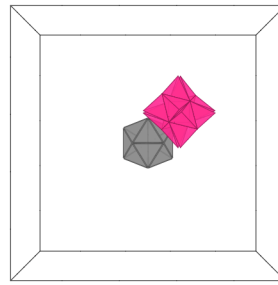
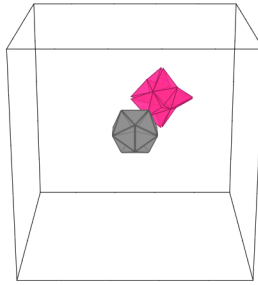


$$\{-\mathbf{V}_{20}, +\mathbf{B}_{17}\}$$

$$a = 3$$

$$\langle a, a, a \rangle$$

$$3.000000000000$$

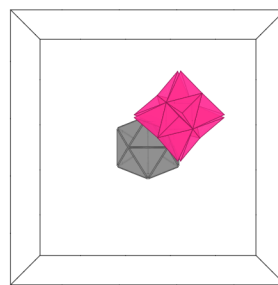
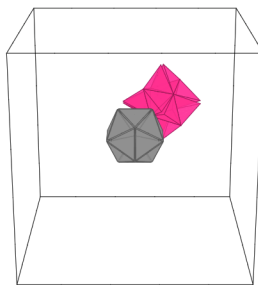


$$\{-\mathbf{V}_{20}, -\mathbf{B}_{17}\}$$

$$a = \frac{7}{3} + \frac{2}{1199}\sqrt{2} + \frac{62}{3597}\sqrt{10}$$

$$\langle a, a, a \rangle$$

$$2.390199192745$$

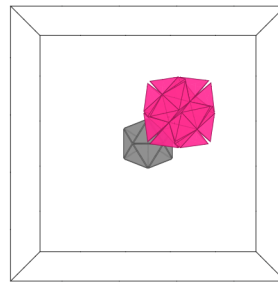
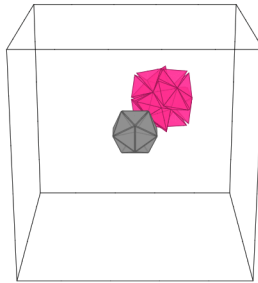


$$\{-\mathbf{V}_{20}, +\mathbf{B}_{41}\}$$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

$$\langle a, a, a \rangle$$

$$3.047553602722$$

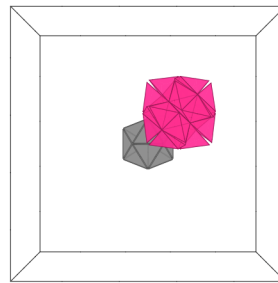
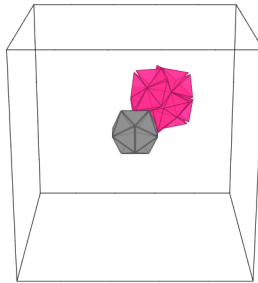


$$\{-\mathbf{V}_{20}, -\mathbf{B}_{41}\}$$

$$a = \frac{7}{5} + \frac{2}{3}\sqrt{5} - \frac{1}{6}\sqrt{2} + \frac{1}{10}\sqrt{10}$$

$$\langle a, a, a \rangle$$

$$2.971237490621$$

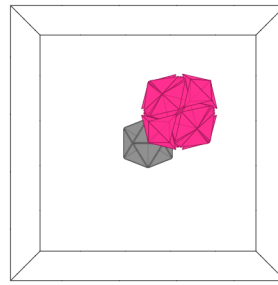
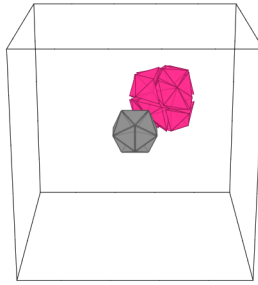


$$\{-\mathbf{V}_{20}, +\mathbf{B}_{57}\}$$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

$$\langle a, a, a \rangle$$

$$3.047553602722$$

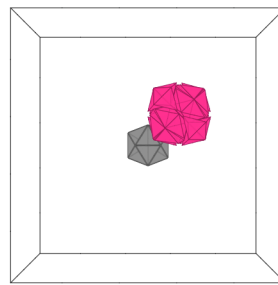
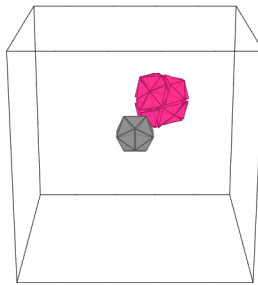


$$\{-\mathbf{V}_{20}, -\mathbf{B}_{57}\}$$

$$a = \frac{11}{3}$$

$$\langle a, a, a \rangle$$

$$3.666666666667$$

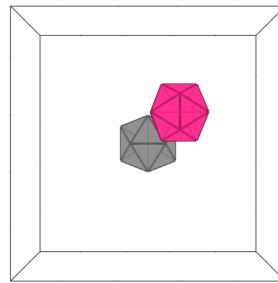
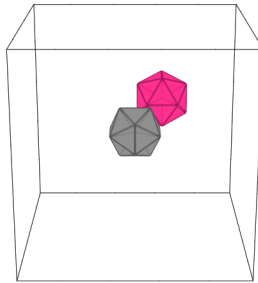


$$\{-\mathbf{V}_{20}, +\mathbf{V}_{20}\}$$

$$a = \frac{8}{3}$$

$$\langle a, a, a \rangle$$

$$2.666666666667$$

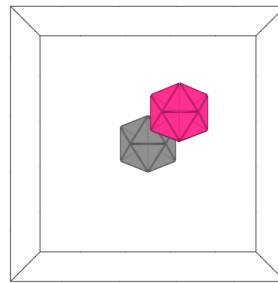
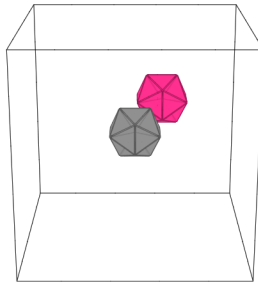


$$\{-\mathbf{V}_{20}, -\mathbf{V}_{20}\}$$

$$a = \frac{8}{3}$$

$$\langle a, a, a \rangle$$

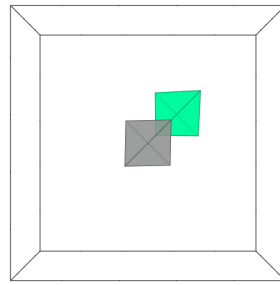
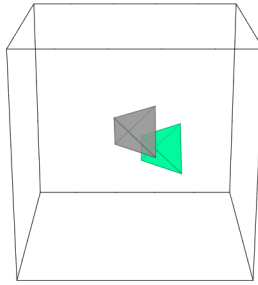
$$2.666666666667$$



$\{+B_1,+B_1\}$

$a = \frac{4}{3}$

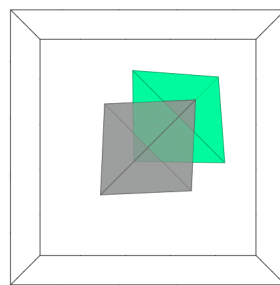
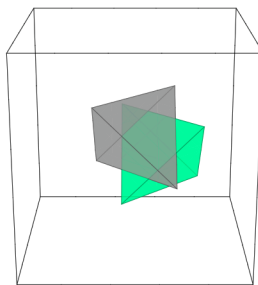
$\langle a,a,-a \rangle$
1.333333333333



$\{+B_1,-B_1\}$

$a = \frac{2}{3}$

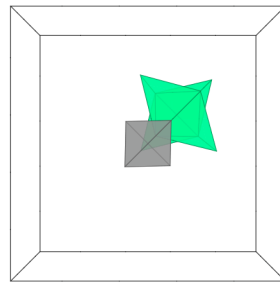
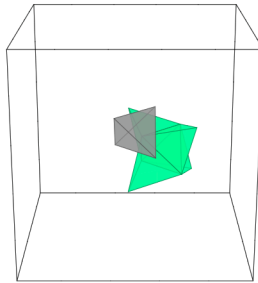
$\langle a,a,-a \rangle$
.666666666667



$\{+B_1,+B_5\}$

$a = \frac{4}{3}$

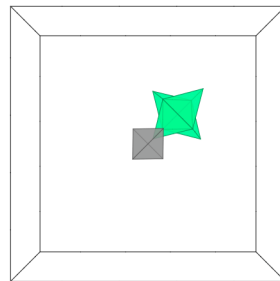
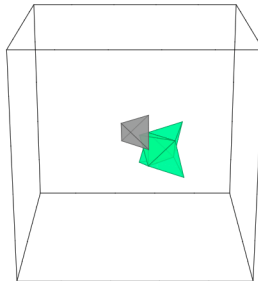
$\langle a,a,-a \rangle$
1.333333333333



$\{+B_1,-B_5\}$

$a = 2$

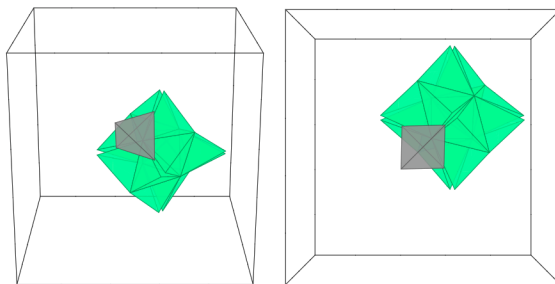
$\langle a,a,-a \rangle$
2.000000000000



$\{+\mathbf{B}_1, +\mathbf{B}_{17}\}$

$$a = \frac{24}{17}$$

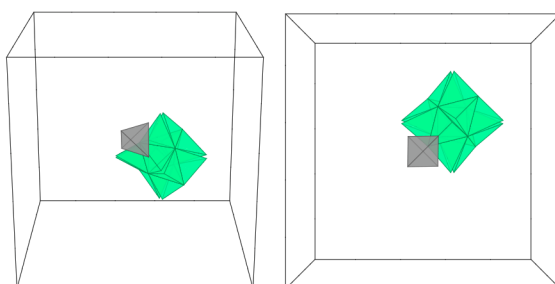
$\langle a, a, -a \rangle$
1.411764705882



$\{+\mathbf{B}_1, -\mathbf{B}_{17}\}$

$$a = 2$$

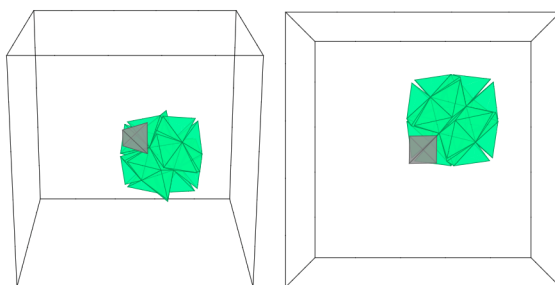
$\langle a, a, -a \rangle$
2.000000000000



$\{+\mathbf{B}_1, +\mathbf{B}_{41}\}$

$$a = \frac{120}{67}$$

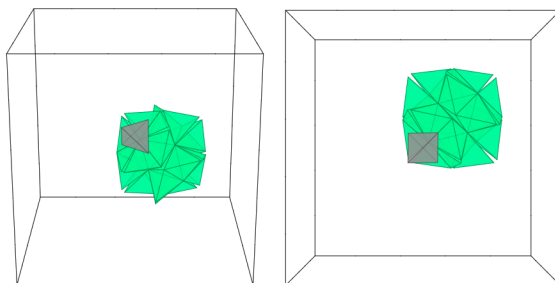
$\langle a, a, -a \rangle$
1.791044776119



$\{+\mathbf{B}_1, -\mathbf{B}_{41}\}$

$$a = \frac{190}{93}$$

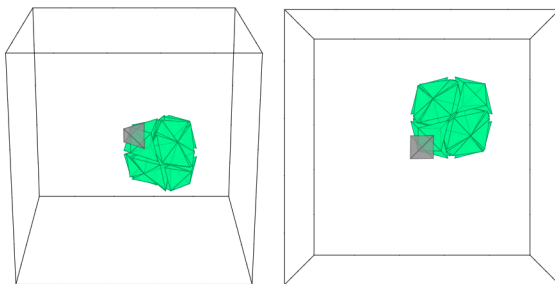
$\langle a, a, -a \rangle$
2.043010752688



$\{+\mathbf{B}_1, +\mathbf{B}_{57}\}$

$$a = \frac{8}{3}$$

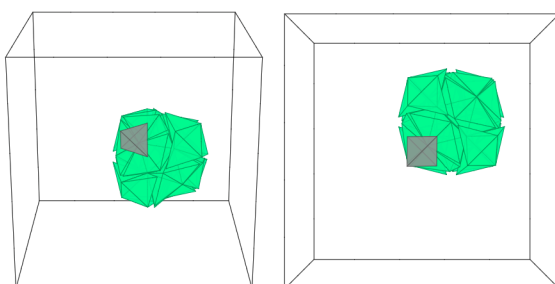
$\langle a, a, -a \rangle$
2.666666666667



$\{+\mathbf{B}_1, -\mathbf{B}_{57}\}$

$$a = \frac{190}{93}$$

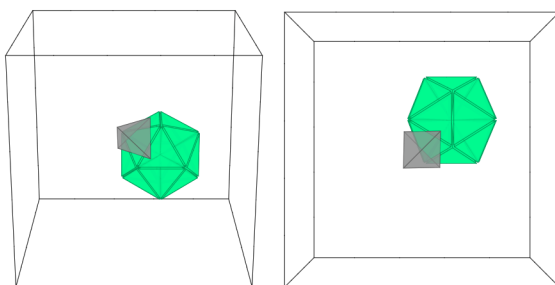
$\langle a, a, -a \rangle$
2.043010752688



$\{+\mathbf{B}_1, +\mathbf{V}_{20}\}$

$$a = \frac{5}{3}$$

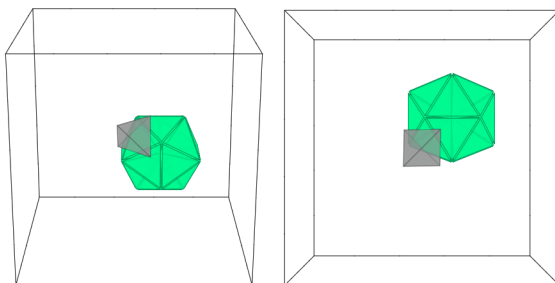
$\langle a, a, -a \rangle$
1.666666666667



$\{+\mathbf{B}_1, -\mathbf{V}_{20}\}$

$$a = \frac{5}{3}$$

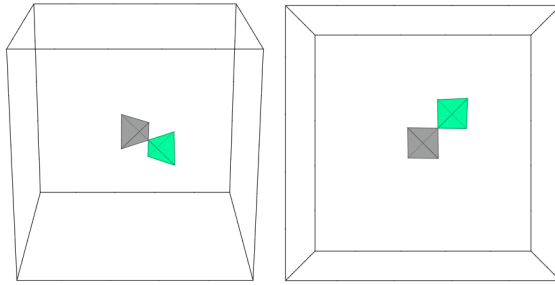
$\langle a, a, -a \rangle$
1.666666666667



$\{-\mathbf{B}_1, +\mathbf{B}_1\}$

$a = 2$

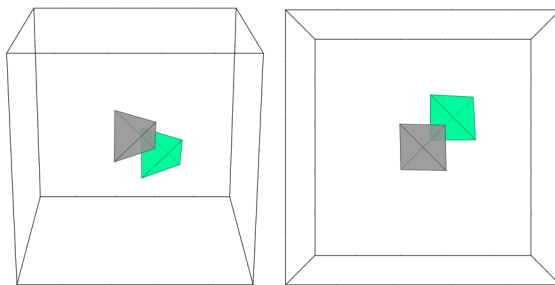
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_1, -\mathbf{B}_1\}$

$a = \frac{4}{3}$

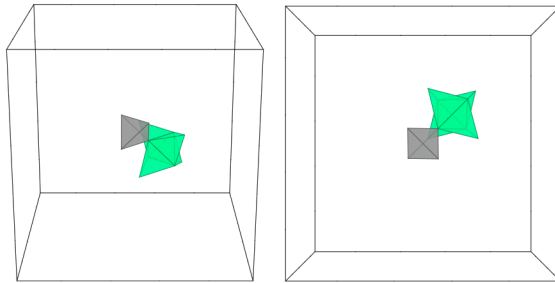
$\langle a, a, -a \rangle$
1.333333333333



$\{-\mathbf{B}_1, +\mathbf{B}_5\}$

$a = 2$

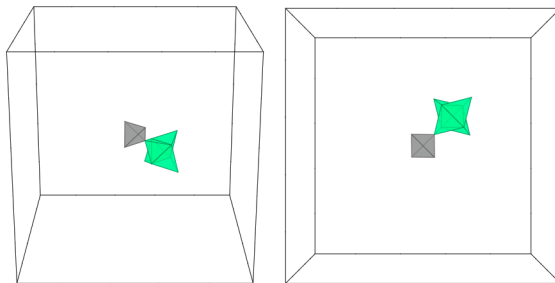
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_1, -\mathbf{B}_5\}$

$a = \frac{8}{3}$

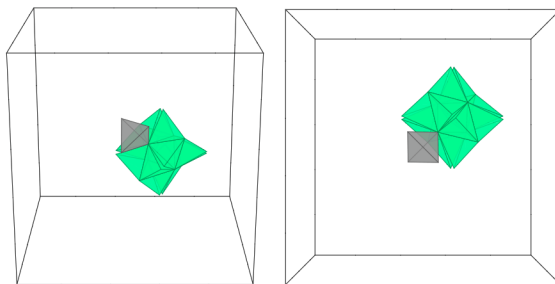
$\langle a, a, -a \rangle$
2.666666666667



$\{-\mathbf{B}_1, +\mathbf{B}_{17}\}$

$a = 2$

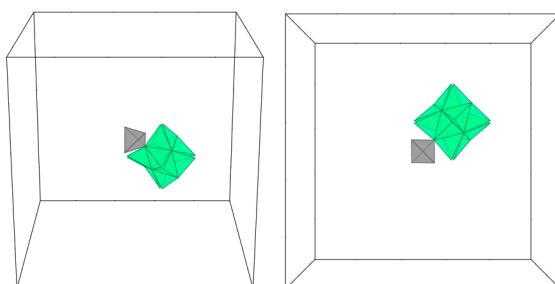
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_1, -\mathbf{B}_{17}\}$

$a = \frac{8}{3}$

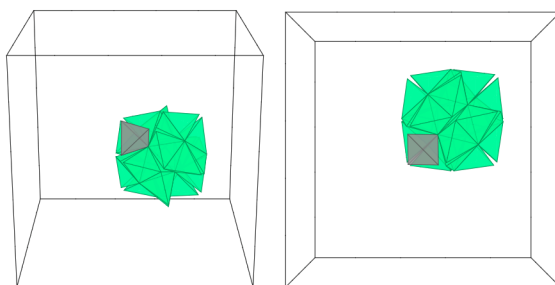
$\langle a, a, -a \rangle$
2.666666666667



$\{-\mathbf{B}_1, +\mathbf{B}_{41}\}$

$a = 2$

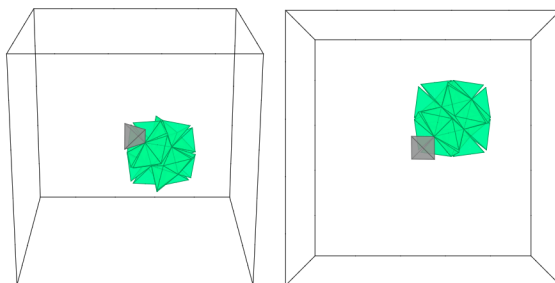
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_1, -\mathbf{B}_{41}\}$

$a = \frac{8}{3}$

$\langle a, a, -a \rangle$
2.666666666667

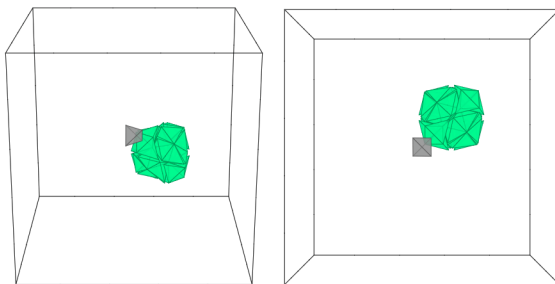


$\{-\mathbf{B}_1, +\mathbf{B}_{57}\}$

$a = \frac{10}{3}$

$\langle a, a, -a \rangle$

3.333333333333

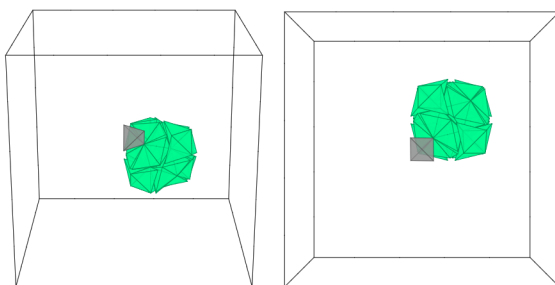


$\{-\mathbf{B}_1, -\mathbf{B}_{57}\}$

$a = \frac{8}{3}$

$\langle a, a, -a \rangle$

2.666666666667

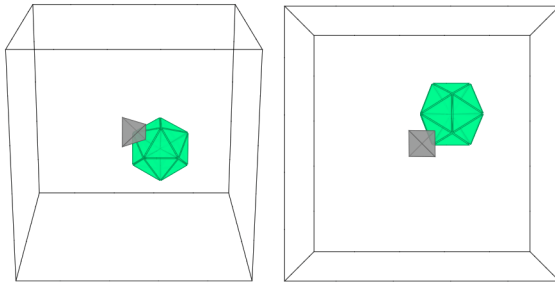


$\{-\mathbf{B}_1, +\mathbf{V}_{20}\}$

$a = \frac{7}{3}$

$\langle a, a, -a \rangle$

2.333333333333

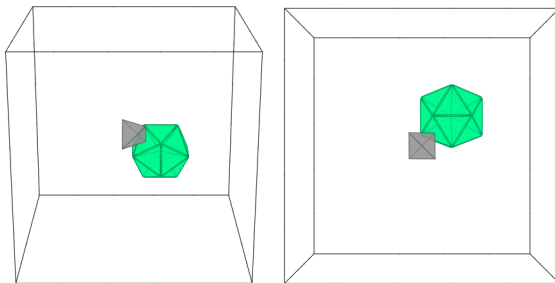


$\{-\mathbf{B}_1, -\mathbf{V}_{20}\}$

$a = \frac{7}{3}$

$\langle a, a, -a \rangle$

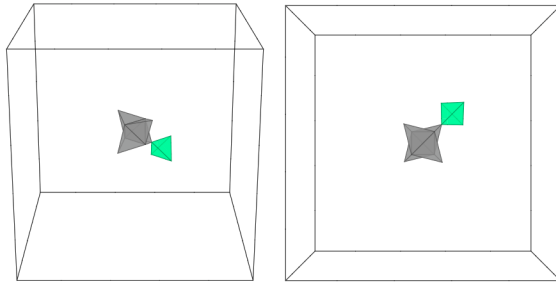
2.333333333333



$\{+B_5, +B_1\}$

$$a = \frac{8}{3}$$

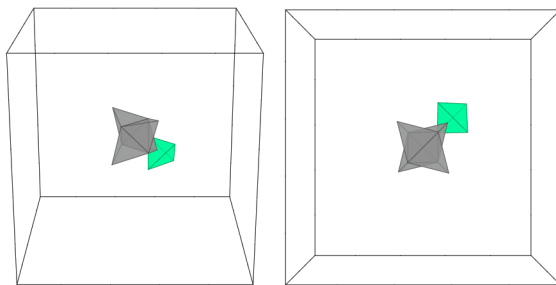
$\langle a, a, -a \rangle$
2.666666666667



$\{+B_5, -B_1\}$

$$a = 2$$

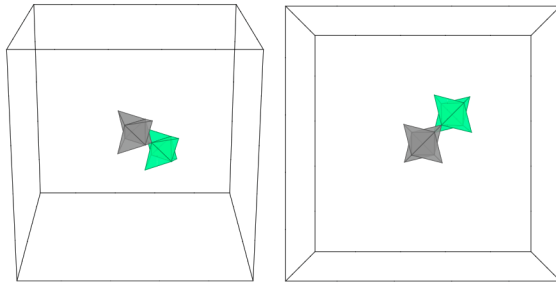
$\langle a, a, -a \rangle$
2.000000000000



$\{+B_5, +B_5\}$

$$a = \frac{8}{3}$$

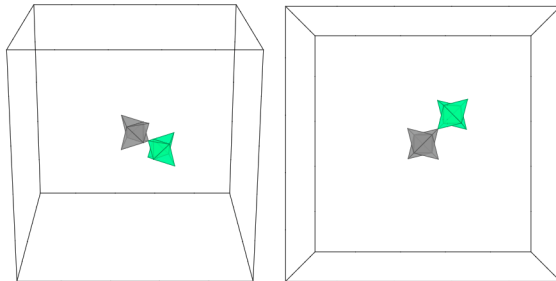
$\langle a, a, -a \rangle$
2.666666666667



$\{+B_5, -B_5\}$

$$a = \frac{10}{3}$$

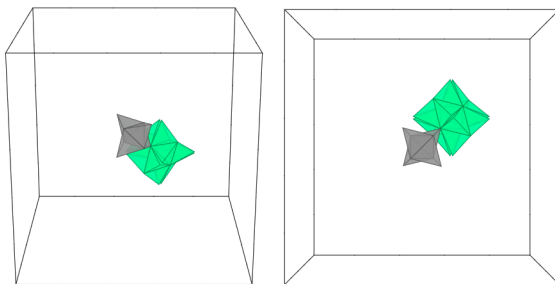
$\langle a, a, -a \rangle$
3.333333333333



$\{+B_5,+B_{17}\}$

$$a = \frac{8}{3}$$

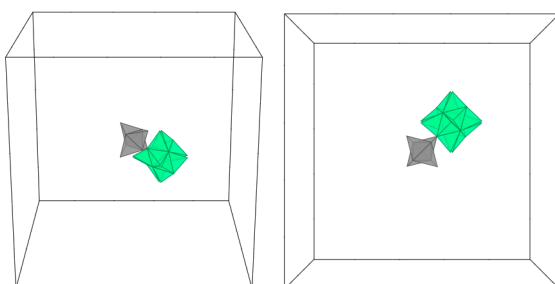
$\langle a,a,-a \rangle$
2.666666666667



$\{+B_5,-B_{17}\}$

$$a = \frac{10}{3}$$

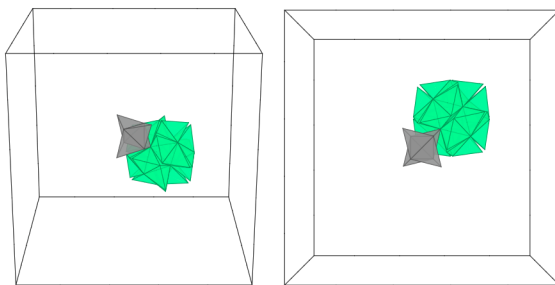
$\langle a,a,-a \rangle$
3.333333333333



$\{+B_5,+B_{41}\}$

$$a = \frac{8}{3}$$

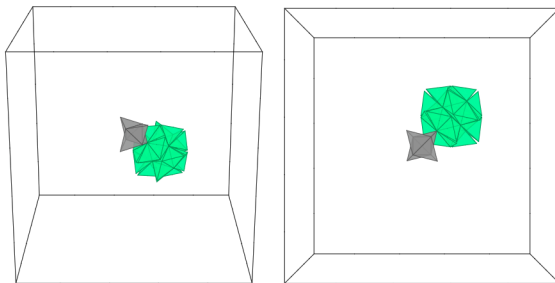
$\langle a,a,-a \rangle$
2.666666666667



$\{+B_5,-B_{41}\}$

$$a = \frac{10}{3}$$

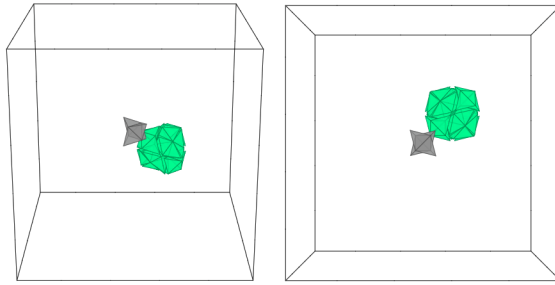
$\langle a,a,-a \rangle$
3.333333333333



$\{+\mathbf{B}_5, +\mathbf{B}_{57}\}$

$a = 4$

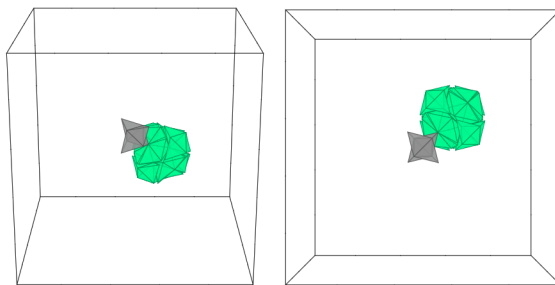
$\langle a, a, -a \rangle$
4.000000000000



$\{+\mathbf{B}_5, -\mathbf{B}_{57}\}$

$a = \frac{10}{3}$

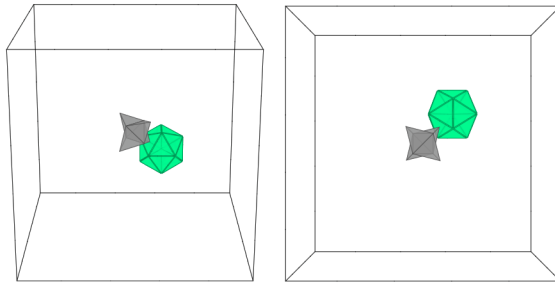
$\langle a, a, -a \rangle$
3.333333333333



$\{+\mathbf{B}_5, +\mathbf{V}_{20}\}$

$a = 3$

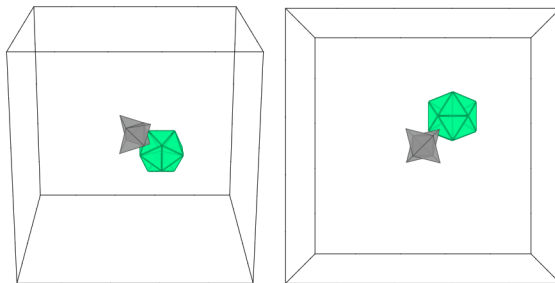
$\langle a, a, -a \rangle$
3.000000000000



$\{+\mathbf{B}_5, -\mathbf{V}_{20}\}$

$a = 3$

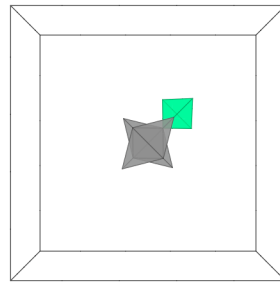
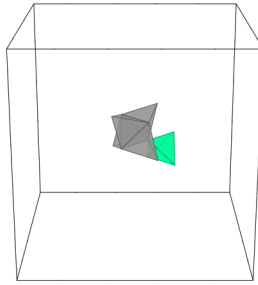
$\langle a, a, -a \rangle$
3.000000000000



$\{-\mathbf{B}_5, +\mathbf{B}_1\}$

$a = 2$

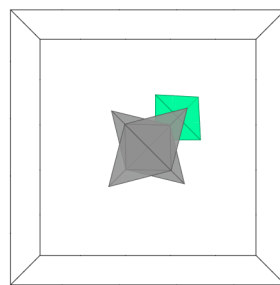
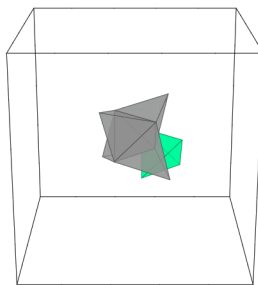
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_5, -\mathbf{B}_1\}$

$a = \frac{4}{3}$

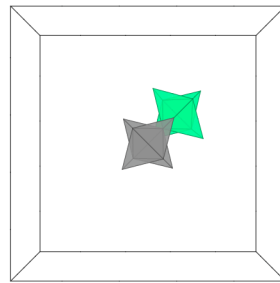
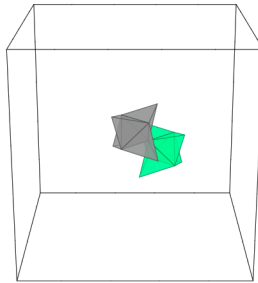
$\langle a, a, -a \rangle$
1.333333333333



$\{-\mathbf{B}_5, +\mathbf{B}_5\}$

$a = 2$

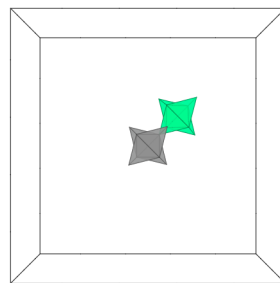
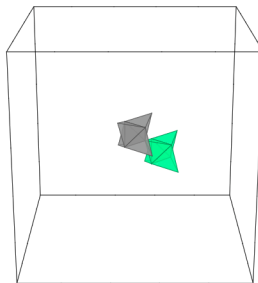
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_5, -\mathbf{B}_5\}$

$a = \frac{8}{3}$

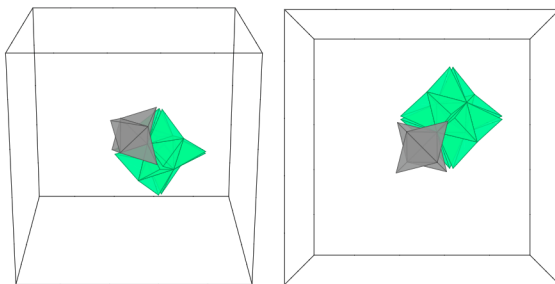
$\langle a, a, -a \rangle$
2.666666666667



$\{-\mathbf{B}_5, +\mathbf{B}_{17}\}$

$a = 2$

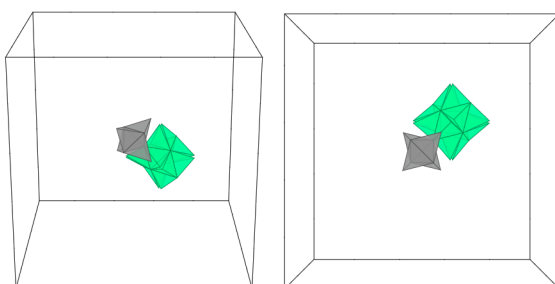
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_5, -\mathbf{B}_{17}\}$

$a = \frac{8}{3}$

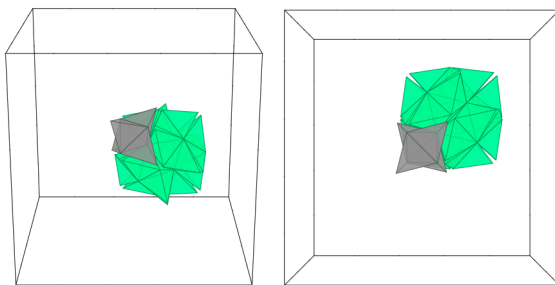
$\langle a, a, -a \rangle$
2.666666666667



$\{-\mathbf{B}_5, +\mathbf{B}_{41}\}$

$a = 2$

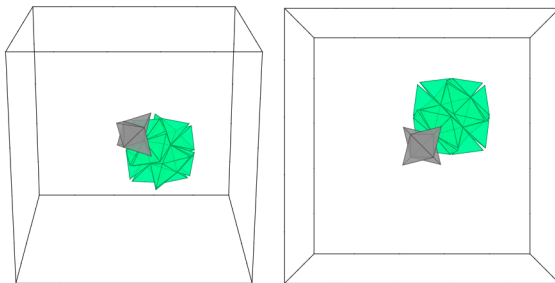
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_5, -\mathbf{B}_{41}\}$

$a = \frac{8}{3}$

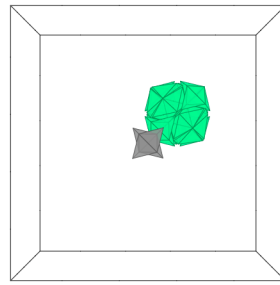
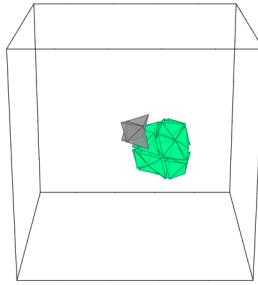
$\langle a, a, -a \rangle$
2.666666666667



$\{-\mathbf{B}_5, +\mathbf{B}_{57}\}$

$a = \frac{10}{3}$

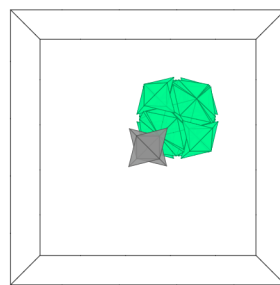
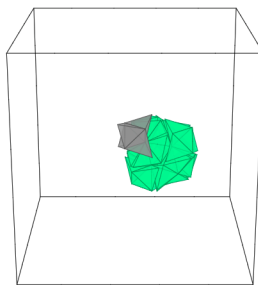
$\langle a, a, -a \rangle$
3.333333333333



$\{-\mathbf{B}_5, -\mathbf{B}_{57}\}$

$a = \frac{8}{3}$

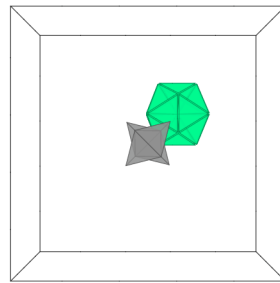
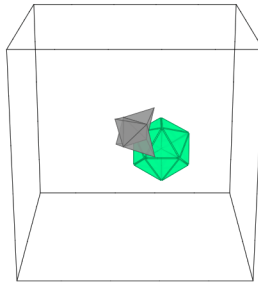
$\langle a, a, -a \rangle$
2.666666666667



$\{-\mathbf{B}_5, +\mathbf{V}_{20}\}$

$a = \frac{7}{3}$

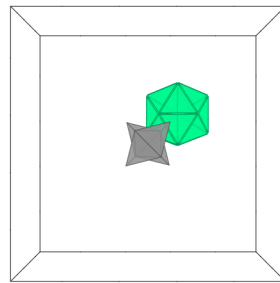
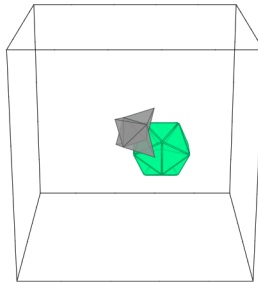
$\langle a, a, -a \rangle$
2.333333333333



$\{-\mathbf{B}_5, -\mathbf{V}_{20}\}$

$a = \frac{7}{3}$

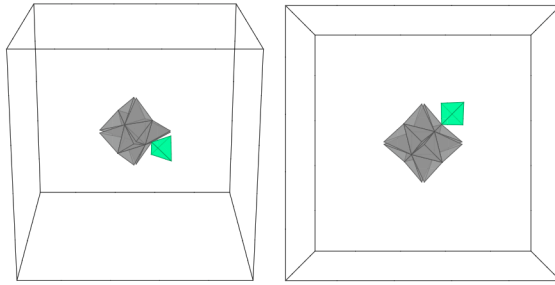
$\langle a, a, -a \rangle$
2.333333333333



$\{+\mathbf{B}_{17},+\mathbf{B}_1\}$

$$a = \frac{8}{3}$$

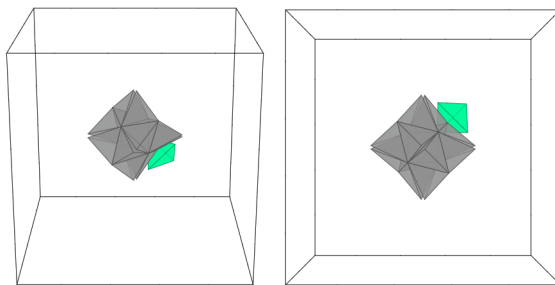
$\langle a,a,-a \rangle$
2.666666666667



$\{+\mathbf{B}_{17},-\mathbf{B}_1\}$

$$a = 2$$

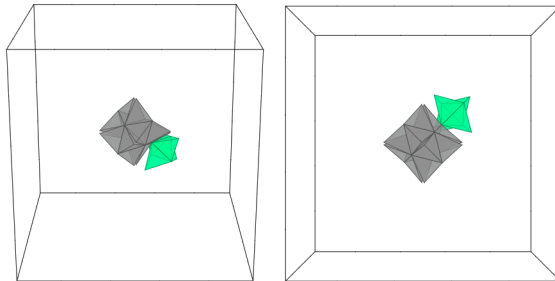
$\langle a,a,-a \rangle$
2.000000000000



$\{+\mathbf{B}_{17},+\mathbf{B}_5\}$

$$a = \frac{8}{3}$$

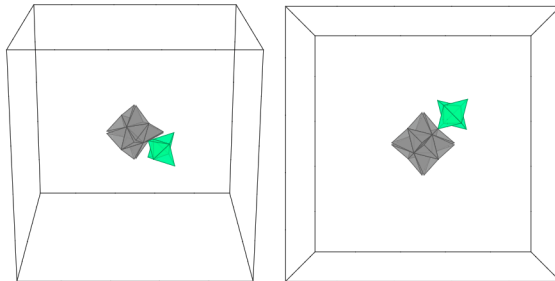
$\langle a,a,-a \rangle$
2.666666666667



$\{+\mathbf{B}_{17},-\mathbf{B}_5\}$

$$a = \frac{10}{3}$$

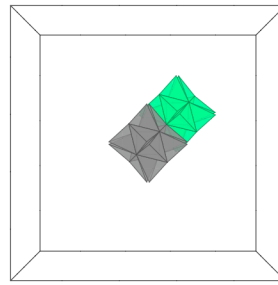
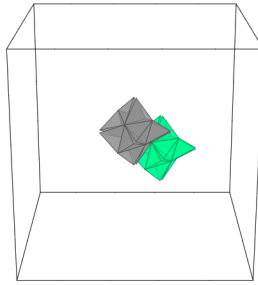
$\langle a,a,-a \rangle$
3.333333333333



$\{+\mathbf{B}_{17},+\mathbf{B}_{17}\}$

$$a = \frac{8}{3}$$

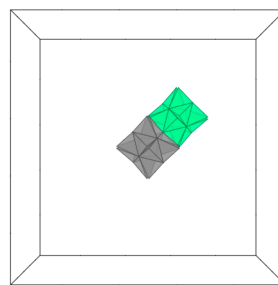
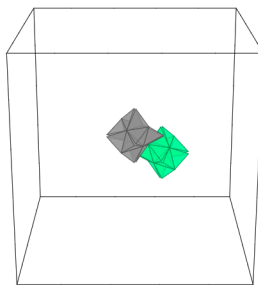
$\langle a,a,-a \rangle$
2.666666666667



$\{+\mathbf{B}_{17},-\mathbf{B}_{17}\}$

$$a = \frac{10}{3}$$

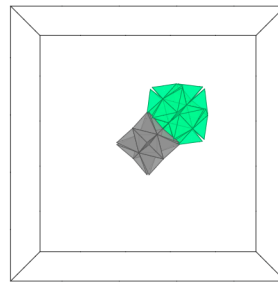
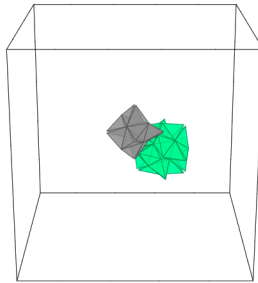
$\langle a,a,-a \rangle$
3.333333333333



$\{+\mathbf{B}_{17},+\mathbf{B}_{41}\}$

$$a = \frac{1352}{405}$$

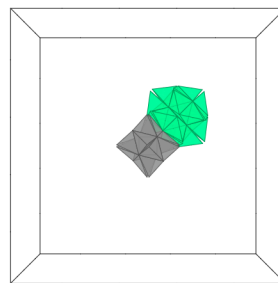
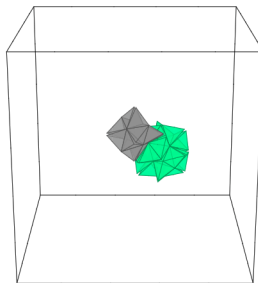
$\langle a,a,-a \rangle$
3.338271604938



$\{+\mathbf{B}_{17},-\mathbf{B}_{41}\}$

$$a = \frac{10}{3}$$

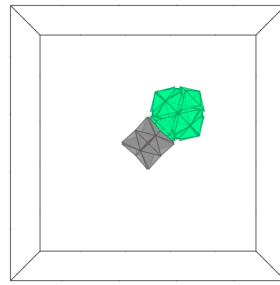
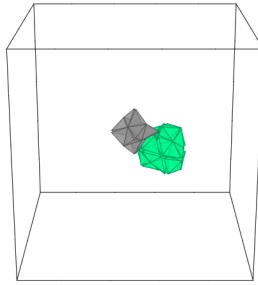
$\langle a,a,-a \rangle$
3.333333333333



$\{+\mathbf{B}_{17},+\mathbf{B}_{57}\}$

$a = 4$

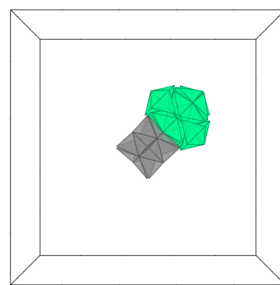
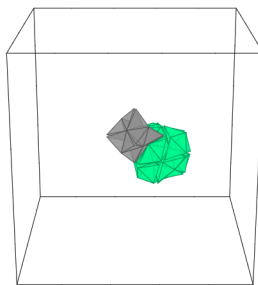
$\langle a,a,-a \rangle$
4.000000000000



$\{+\mathbf{B}_{17},-\mathbf{B}_{57}\}$

$a = \frac{10}{3}$

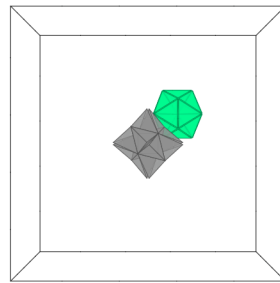
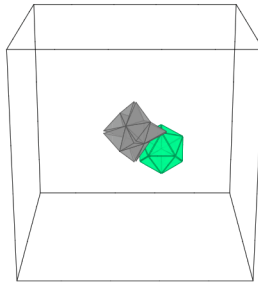
$\langle a,a,-a \rangle$
3.333333333333



$\{+\mathbf{B}_{17},+\mathbf{V}_{20}\}$

$a = 3$

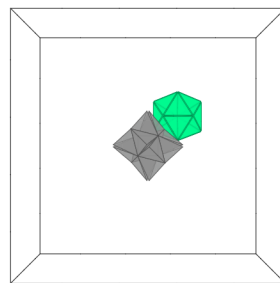
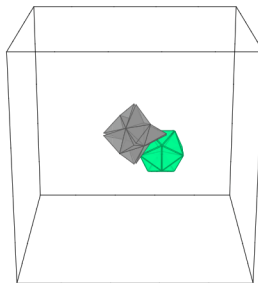
$\langle a,a,-a \rangle$
3.000000000000



$\{+\mathbf{B}_{17},-\mathbf{V}_{20}\}$

$a = 3$

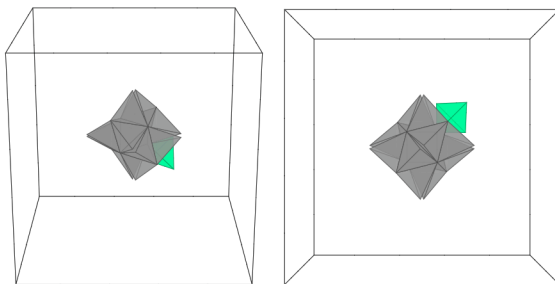
$\langle a,a,-a \rangle$
3.000000000000



$\{-\mathbf{B}_{17}, +\mathbf{B}_1\}$

$a = 2$

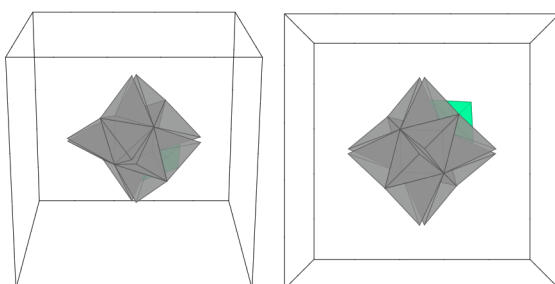
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_{17}, -\mathbf{B}_1\}$

$a = \frac{24}{17}$

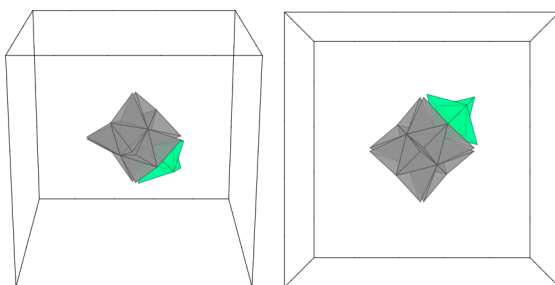
$\langle a, a, -a \rangle$
1.411764705882



$\{-\mathbf{B}_{17}, +\mathbf{B}_5\}$

$a = 2$

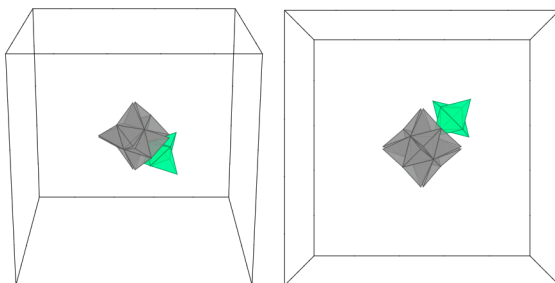
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_{17}, -\mathbf{B}_5\}$

$a = \frac{8}{3}$

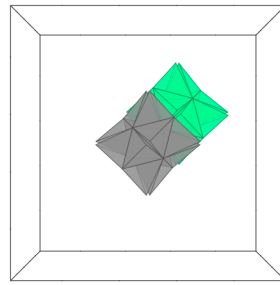
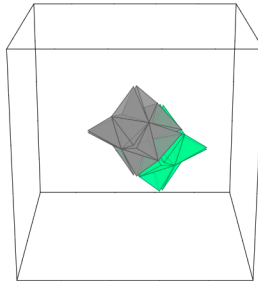
$\langle a, a, -a \rangle$
2.666666666667



$\{-\mathbf{B}_{17}, +\mathbf{B}_{17}\}$

$a = 2$

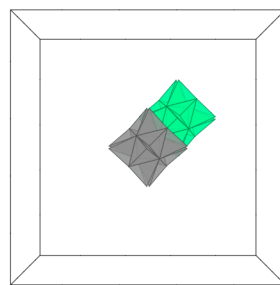
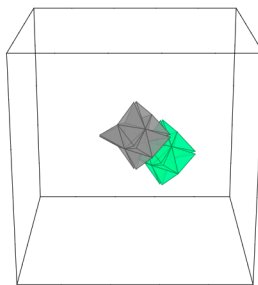
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_{17}, -\mathbf{B}_{17}\}$

$a = \frac{8}{3}$

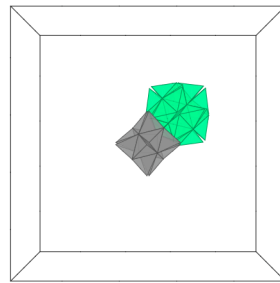
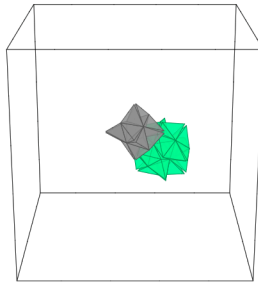
$\langle a, a, -a \rangle$
2.666666666667



$\{-\mathbf{B}_{17}, +\mathbf{B}_{41}\}$

$a = \frac{2182}{675}$

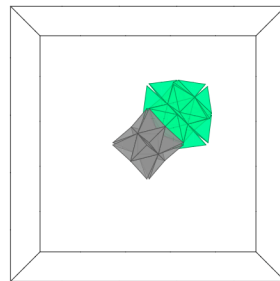
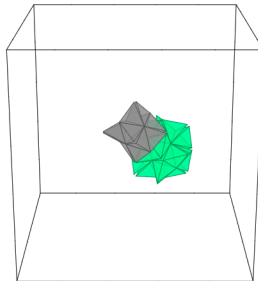
$\langle a, a, -a \rangle$
3.232592592593



$\{-\mathbf{B}_{17}, -\mathbf{B}_{41}\}$

$a = \frac{80}{27}$

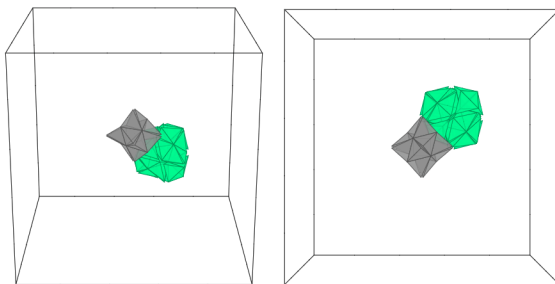
$\langle a, a, -a \rangle$
2.962962962963



$\{-\mathbf{B}_{17}, +\mathbf{B}_{57}\}$

$$a = \frac{58}{17}$$

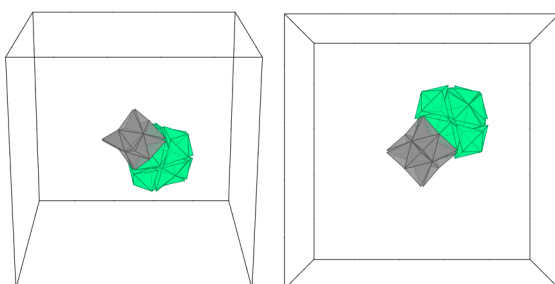
$\langle a, a, -a \rangle$
3.411764705882



$\{-\mathbf{B}_{17}, -\mathbf{B}_{57}\}$

$$a = \frac{80}{27}$$

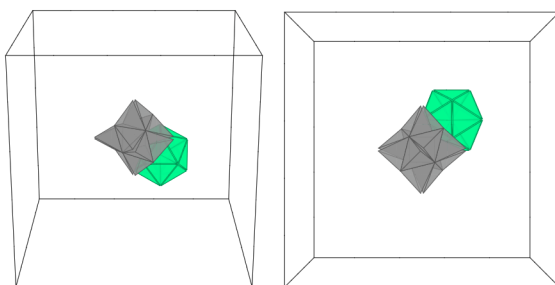
$\langle a, a, -a \rangle$
2.962962962963



$\{-\mathbf{B}_{17}, +\mathbf{V}_{20}\}$

$$a = \frac{7}{3} + \frac{2}{1199}\sqrt{2} + \frac{62}{3597}\sqrt{10}$$

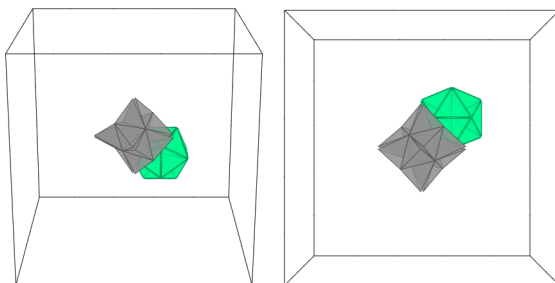
$\langle a, a, -a \rangle$
2.390199192745



$\{-\mathbf{B}_{17}, -\mathbf{V}_{20}\}$

$$a = \frac{7}{3} + \frac{2}{1199}\sqrt{2} + \frac{62}{3597}\sqrt{10}$$

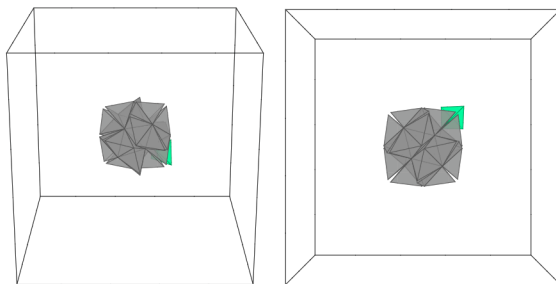
$\langle a, a, -a \rangle$
2.390199192745



$\{+\mathbf{B}_{41}, +\mathbf{B}_1\}$

$$a = \frac{8}{3}$$

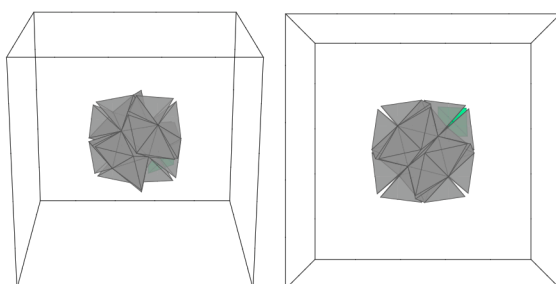
$\langle a, a, -a \rangle$
2.666666666667



$\{+\mathbf{B}_{41}, -\mathbf{B}_1\}$

$$a = \frac{190}{93}$$

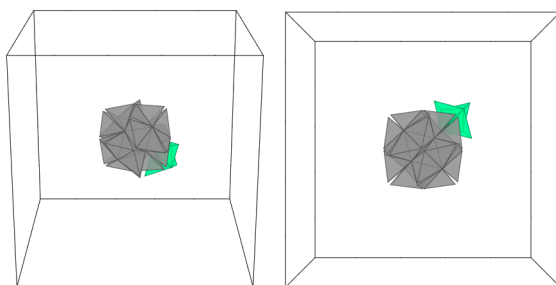
$\langle a, a, -a \rangle$
2.043010752688



$\{+\mathbf{B}_{41}, +\mathbf{B}_5\}$

$$a = \frac{8}{3}$$

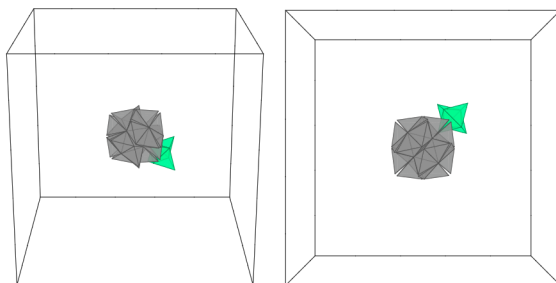
$\langle a, a, -a \rangle$
2.666666666667



$\{+\mathbf{B}_{41}, -\mathbf{B}_5\}$

$$a = \frac{10}{3}$$

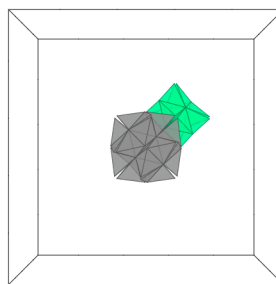
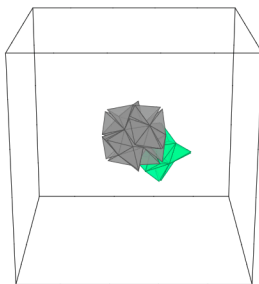
$\langle a, a, -a \rangle$
3.333333333333



$\{+\mathbf{B}_{41}, +\mathbf{B}_{17}\}$

$$a = \frac{80}{27}$$

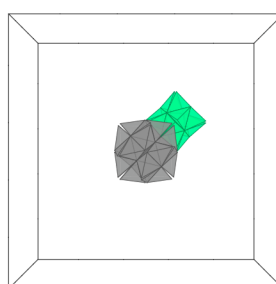
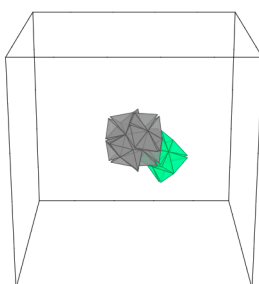
$\langle a, a, -a \rangle$
2.962962962963



$\{+\mathbf{B}_{41}, -\mathbf{B}_{17}\}$

$$a = \frac{10}{3}$$

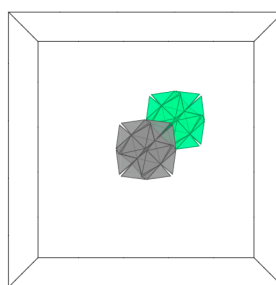
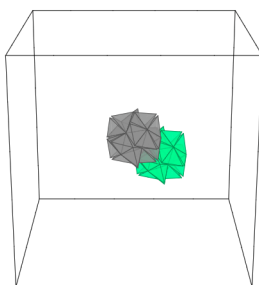
$\langle a, a, -a \rangle$
3.333333333333



$\{+\mathbf{B}_{41}, +\mathbf{B}_{41}\}$

$$a = \frac{7084}{2025}$$

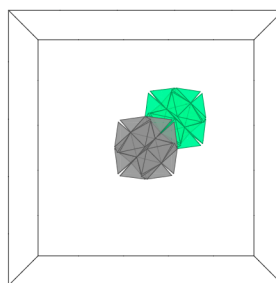
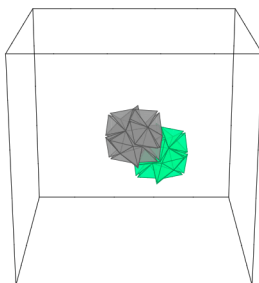
$\langle a, a, -a \rangle$
3.498271604938



$\{+\mathbf{B}_{41}, -\mathbf{B}_{41}\}$

$$a = \frac{10}{3}$$

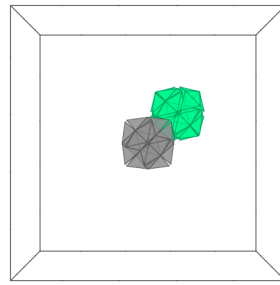
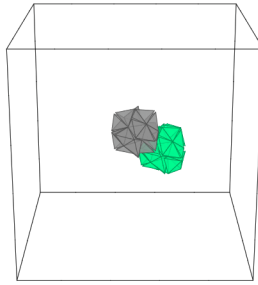
$\langle a, a, -a \rangle$
3.333333333333



$\{+\mathbf{B}_{41}, +\mathbf{B}_{57}\}$

$$a = \frac{376}{93}$$

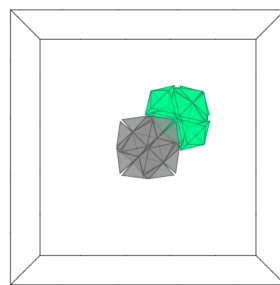
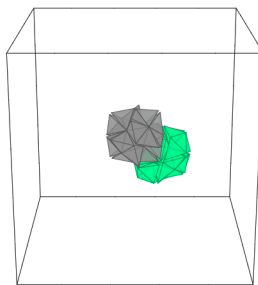
$\langle a, a, -a \rangle$
4.043010752688



$\{+\mathbf{B}_{41}, -\mathbf{B}_{57}\}$

$$a = \frac{10}{3}$$

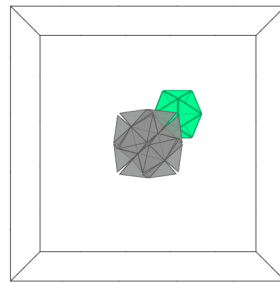
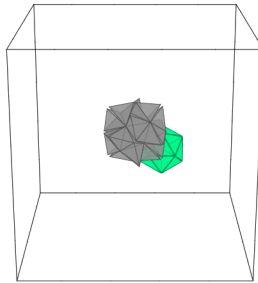
$\langle a, a, -a \rangle$
3.333333333333



$\{+\mathbf{B}_{41}, +\mathbf{V}_{20}\}$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

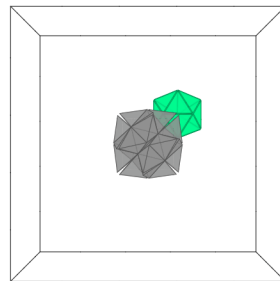
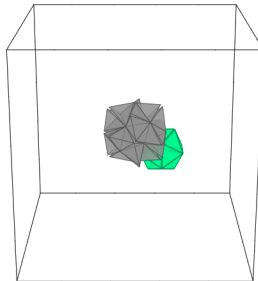
$\langle a, a, -a \rangle$
3.047553602722



$\{+\mathbf{B}_{41}, -\mathbf{V}_{20}\}$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

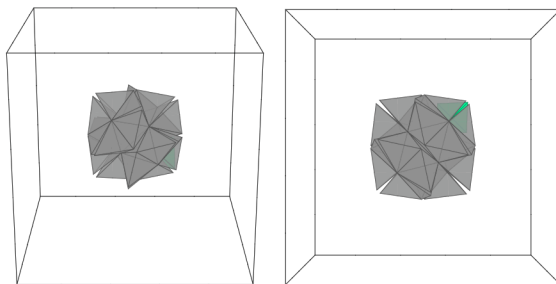
$\langle a, a, -a \rangle$
3.047553602722



$\{-\mathbf{B}_{41}, +\mathbf{B}_1\}$

$a = 2$

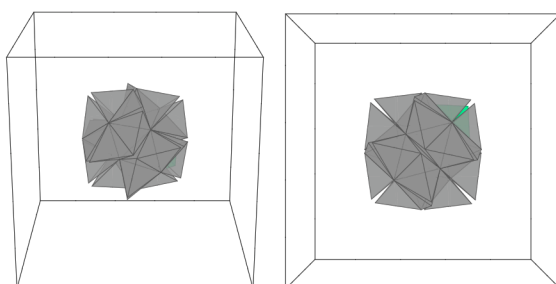
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_{41}, -\mathbf{B}_1\}$

$a = \frac{120}{67}$

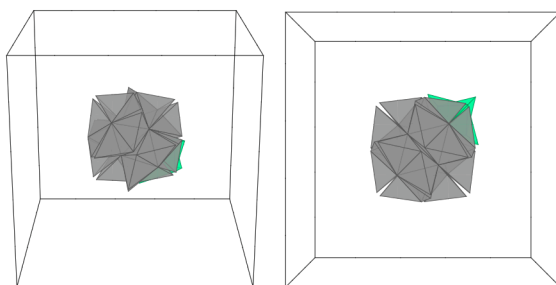
$\langle a, a, -a \rangle$
1.791044776119



$\{-\mathbf{B}_{41}, +\mathbf{B}_5\}$

$a = 2$

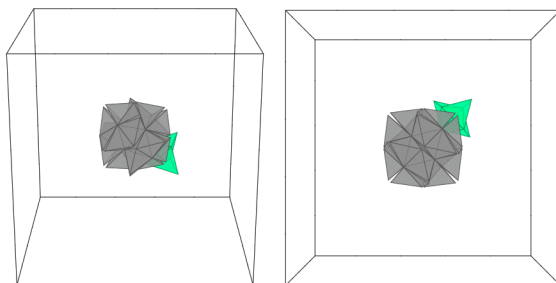
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_{41}, -\mathbf{B}_5\}$

$a = \frac{8}{3}$

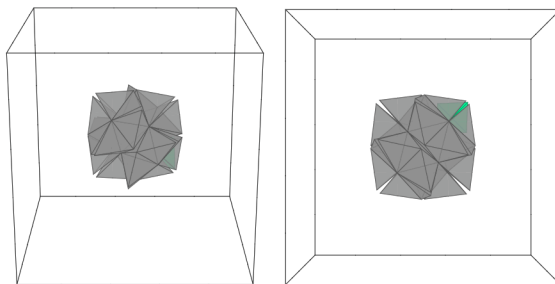
$\langle a, a, -a \rangle$
2.666666666667



$\{-\mathbf{B}_{41}, +\mathbf{B}_1\}$

$a = 2$

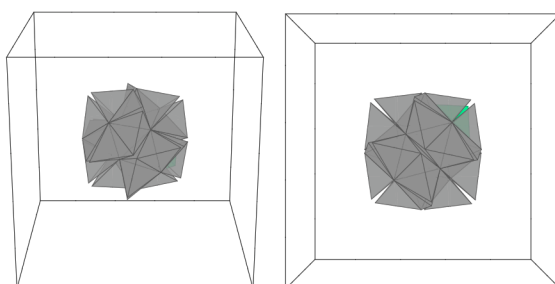
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_{41}, -\mathbf{B}_1\}$

$a = \frac{120}{67}$

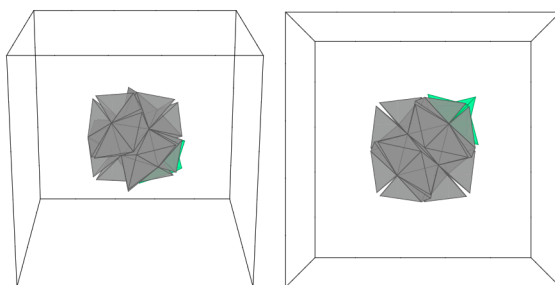
$\langle a, a, -a \rangle$
1.791044776119



$\{-\mathbf{B}_{41}, +\mathbf{B}_5\}$

$a = 2$

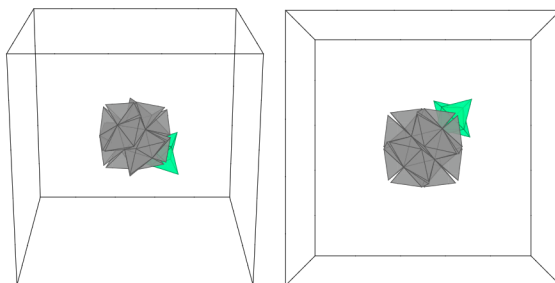
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_{41}, -\mathbf{B}_5\}$

$a = \frac{8}{3}$

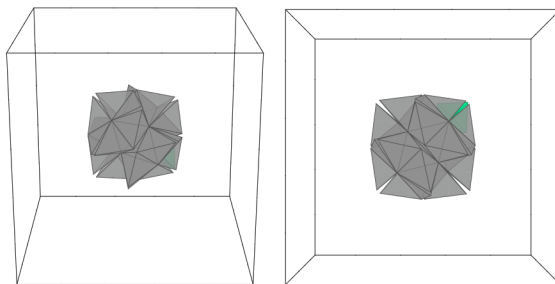
$\langle a, a, -a \rangle$
2.666666666667



$\{-\mathbf{B}_{41}, +\mathbf{B}_1\}$

$a = 2$

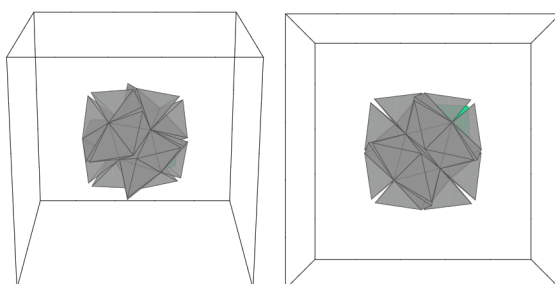
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_{41}, -\mathbf{B}_1\}$

$a = \frac{120}{67}$

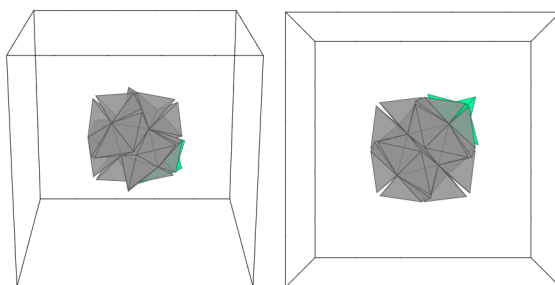
$\langle a, a, -a \rangle$
1.791044776119



$\{-\mathbf{B}_{41}, +\mathbf{B}_5\}$

$a = 2$

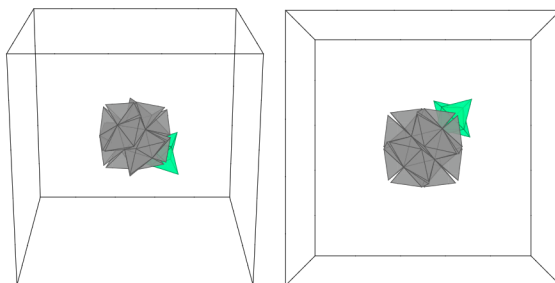
$\langle a, a, -a \rangle$
2.000000000000



$\{-\mathbf{B}_{41}, -\mathbf{B}_5\}$

$a = \frac{8}{3}$

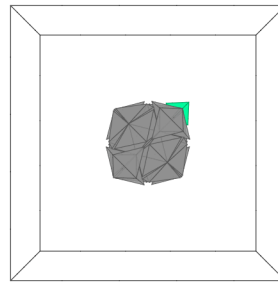
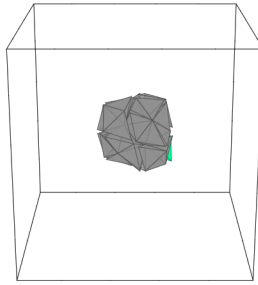
$\langle a, a, -a \rangle$
2.666666666667



$\{+\mathbf{B}_{57},+\mathbf{B}_1\}$

$$a = \frac{8}{3}$$

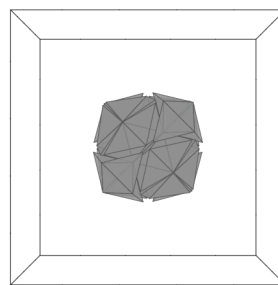
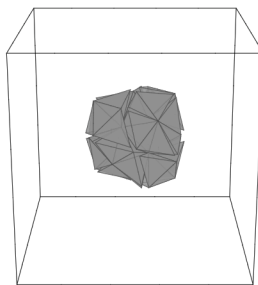
$\langle a,a,-a \rangle$
2.666666666667



$\{+\mathbf{B}_{57},-\mathbf{B}_1\}$

$$a = \frac{190}{93}$$

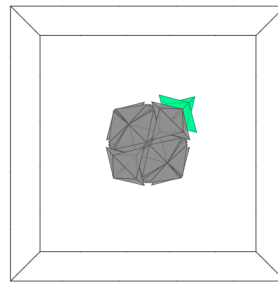
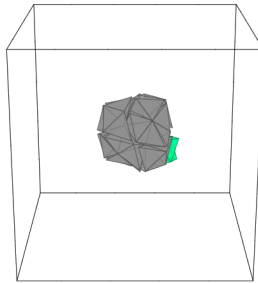
$\langle a,a,-a \rangle$
2.043010752688



$\{+\mathbf{B}_{57},+\mathbf{B}_5\}$

$$a = \frac{8}{3}$$

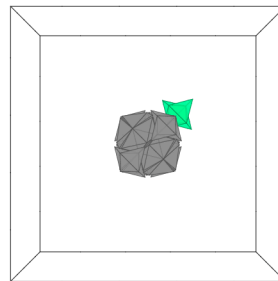
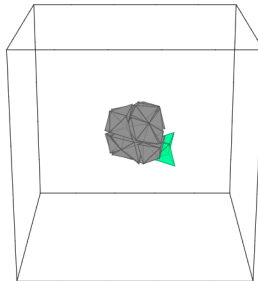
$\langle a,a,-a \rangle$
2.666666666667



$\{+\mathbf{B}_{57},-\mathbf{B}_5\}$

$$a = \frac{10}{3}$$

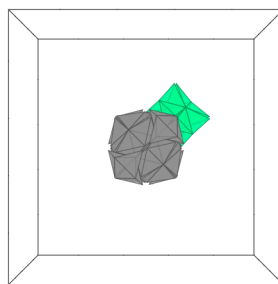
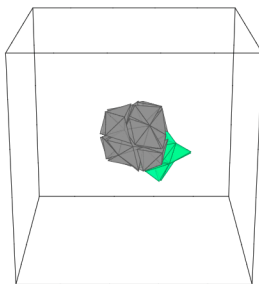
$\langle a,a,-a \rangle$
3.333333333333



$\{+\mathbf{B}_{57},+\mathbf{B}_{17}\}$

$$a = \frac{80}{27}$$

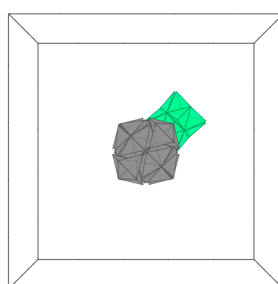
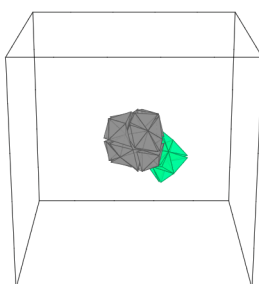
$\langle a,a,-a \rangle$
2.962962962963



$\{+\mathbf{B}_{57},-\mathbf{B}_{17}\}$

$$a = \frac{10}{3}$$

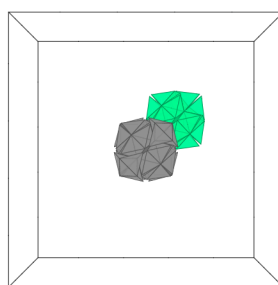
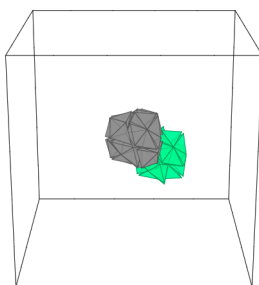
$\langle a,a,-a \rangle$
3.333333333333



$\{+\mathbf{B}_{57},+\mathbf{B}_{41}\}$

$$a = \frac{7084}{2025}$$

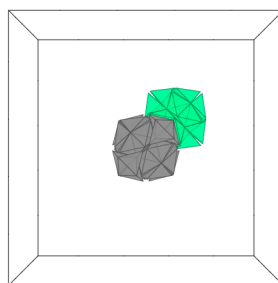
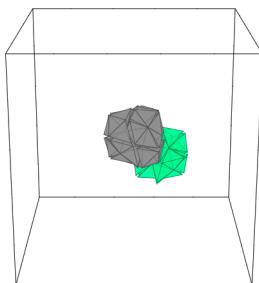
$\langle a,a,-a \rangle$
3.498271604938



$\{+\mathbf{B}_{57},-\mathbf{B}_{41}\}$

$$a = \frac{10}{3}$$

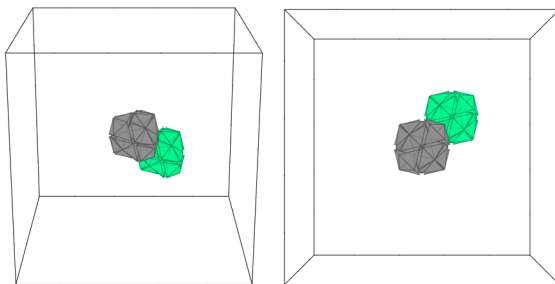
$\langle a,a,-a \rangle$
3.333333333333



$\{+\mathbf{B}_{57}, +\mathbf{B}_{57}\}$

$$a = \frac{376}{93}$$

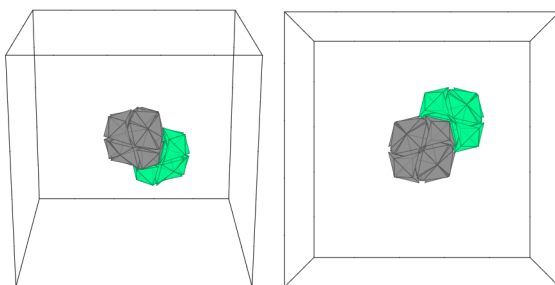
$\langle a, a, -a \rangle$
4.043010752688



$\{+\mathbf{B}_{57}, -\mathbf{B}_{57}\}$

$$a = \frac{10}{3}$$

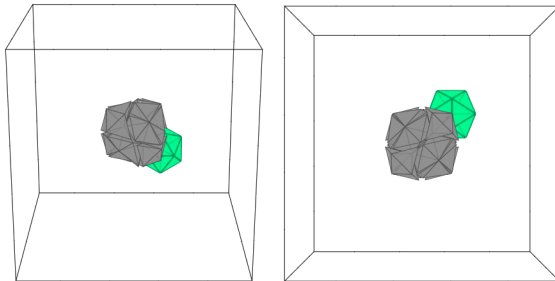
$\langle a, a, -a \rangle$
3.333333333333



$\{+\mathbf{B}_{57}, +\mathbf{V}_{20}\}$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

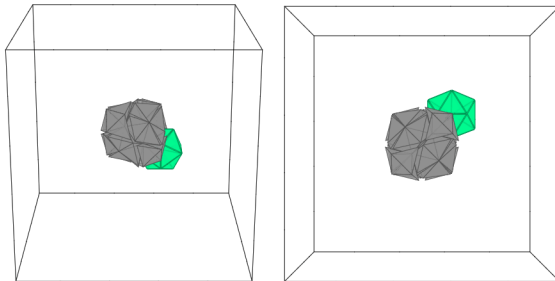
$\langle a, a, -a \rangle$
3.047553602722



$\{+\mathbf{B}_{57}, -\mathbf{V}_{20}\}$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

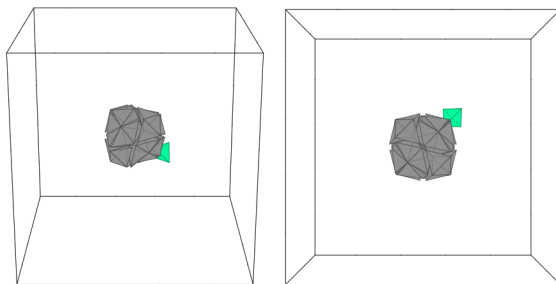
$\langle a, a, -a \rangle$
3.047553602722



$\{-\mathbf{B}_{57}, +\mathbf{B}_1\}$

$$a = \frac{10}{3}$$

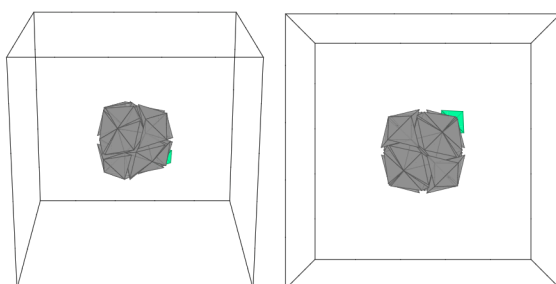
$\langle a, a, -a \rangle$
3.333333333333



$\{-\mathbf{B}_{57}, -\mathbf{B}_1\}$

$$a = \frac{8}{3}$$

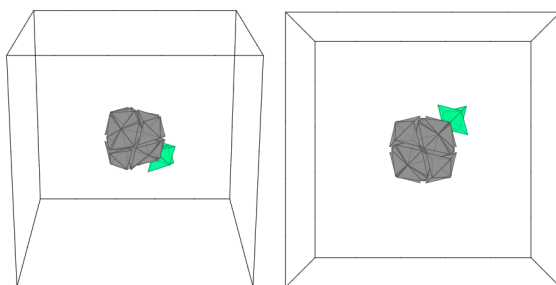
$\langle a, a, -a \rangle$
2.666666666667



$\{-\mathbf{B}_{57}, +\mathbf{B}_5\}$

$$a = \frac{10}{3}$$

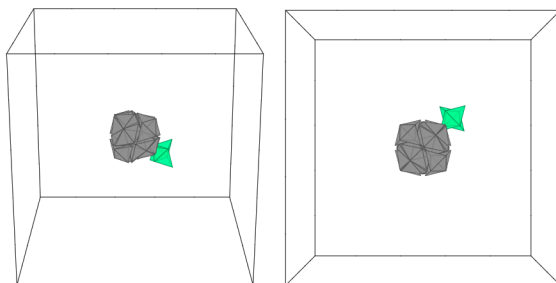
$\langle a, a, -a \rangle$
3.333333333333



$\{-\mathbf{B}_{57}, -\mathbf{B}_5\}$

$$a = 4$$

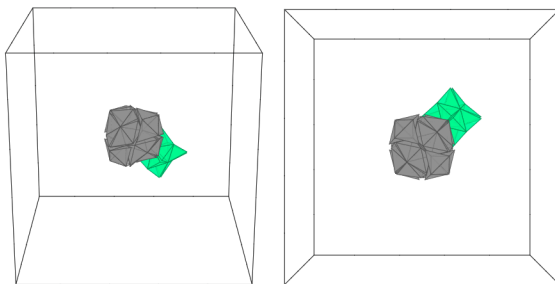
$\langle a, a, -a \rangle$
4.000000000000



$\{-\mathbf{B}_{57}, +\mathbf{B}_{17}\}$

$$a = \frac{58}{17}$$

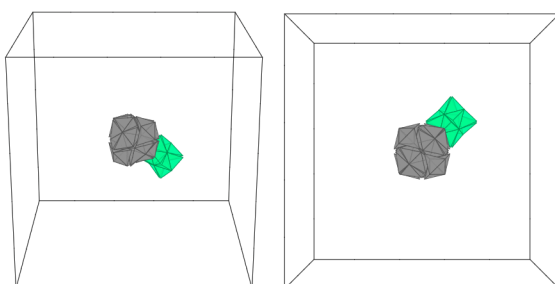
$\langle a, a, -a \rangle$
3.411764705882



$\{-\mathbf{B}_{57}, -\mathbf{B}_{17}\}$

$$a = 4$$

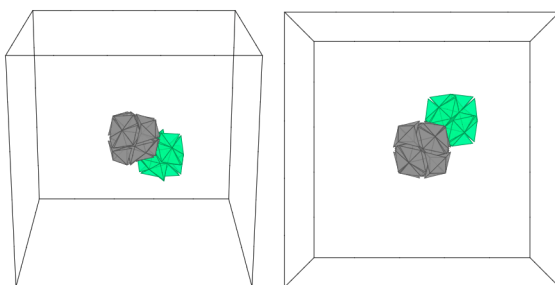
$\langle a, a, -a \rangle$
4.000000000000



$\{-\mathbf{B}_{57}, +\mathbf{B}_{41}\}$

$$a = \frac{82}{21}$$

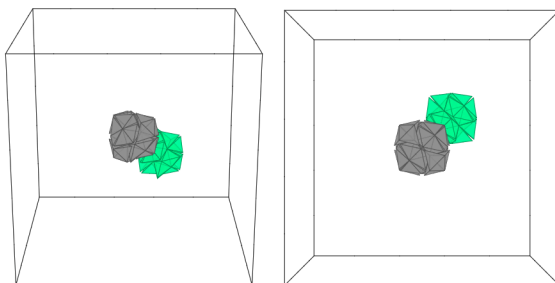
$\langle a, a, -a \rangle$
3.904761904762



$\{-\mathbf{B}_{57}, -\mathbf{B}_{41}\}$

$$a = \frac{376}{93}$$

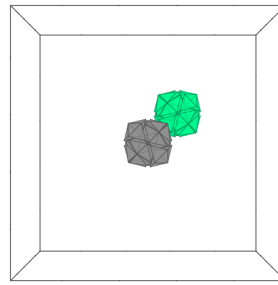
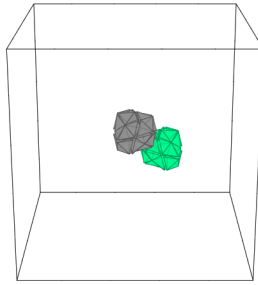
$\langle a, a, -a \rangle$
4.043010752688



$\{-\mathbf{B}_{57}, +\mathbf{B}_{57}\}$

$$a = \frac{14}{3}$$

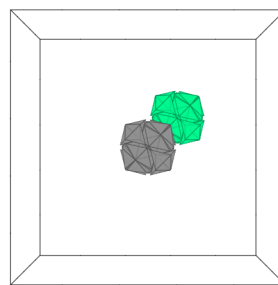
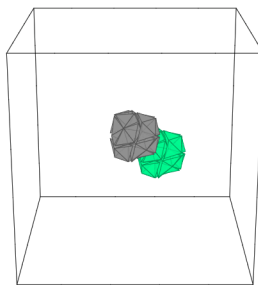
$\langle a, a, -a \rangle$
4.666666666667



$\{-\mathbf{B}_{57}, -\mathbf{B}_{57}\}$

$$a = \frac{376}{93}$$

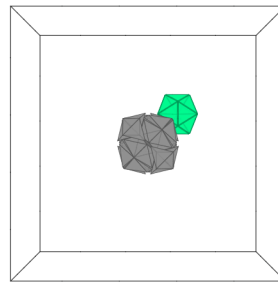
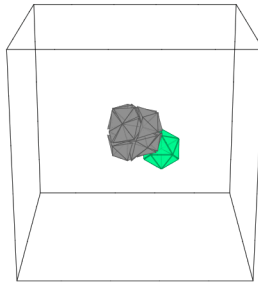
$\langle a, a, -a \rangle$
4.043010752688



$\{-\mathbf{B}_{57}, +\mathbf{V}_{20}\}$

$$a = \frac{11}{3}$$

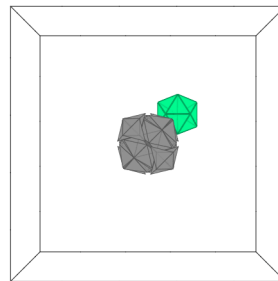
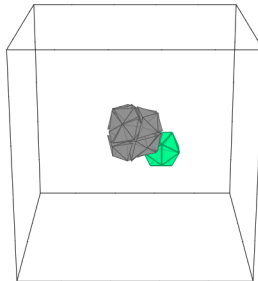
$\langle a, a, -a \rangle$
3.666666666667



$\{-\mathbf{B}_{57}, -\mathbf{V}_{20}\}$

$$a = \frac{11}{3}$$

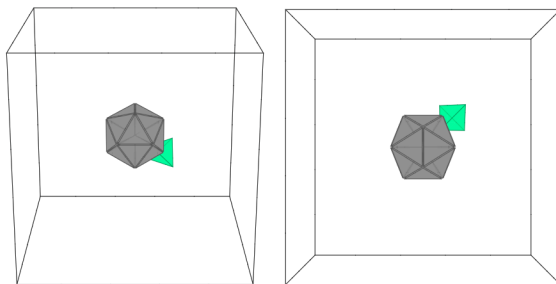
$\langle a, a, -a \rangle$
3.666666666667



$\{+V_{20}, +B_1\}$

$a = \frac{7}{3}$

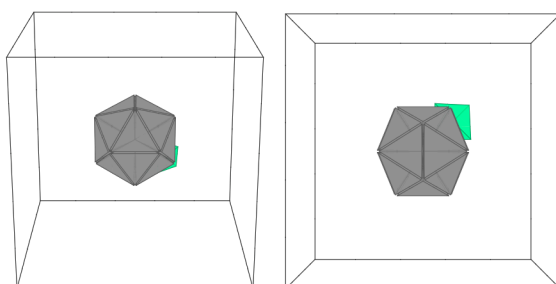
$\langle a, a, -a \rangle$
2.333333333333



$\{+V_{20}, -B_1\}$

$a = \frac{5}{3}$

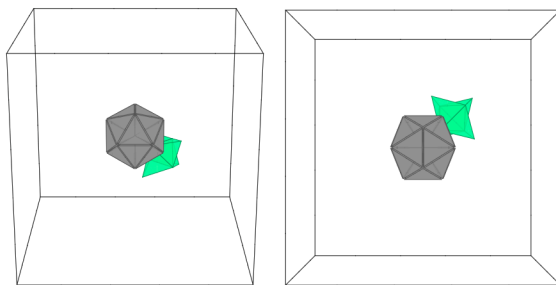
$\langle a, a, -a \rangle$
1.666666666667



$\{+V_{20}, +B_5\}$

$a = \frac{7}{3}$

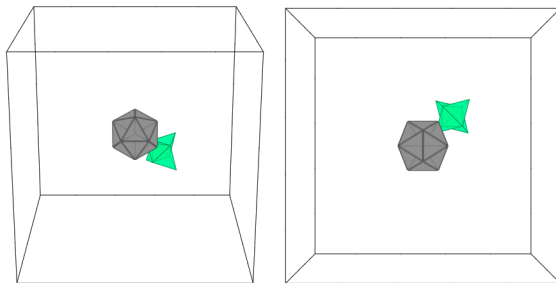
$\langle a, a, -a \rangle$
2.333333333333

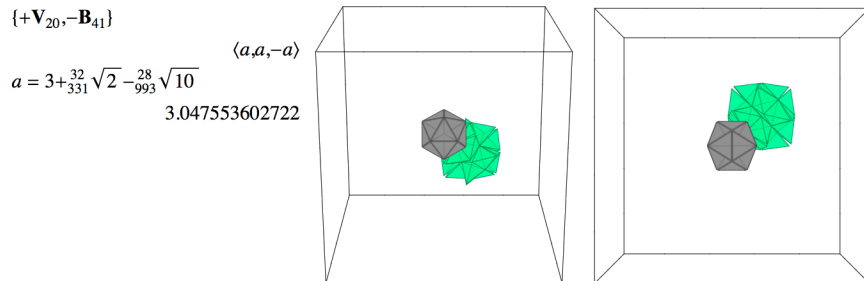
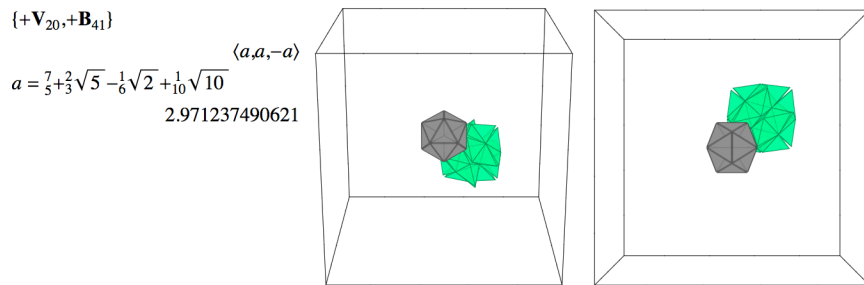
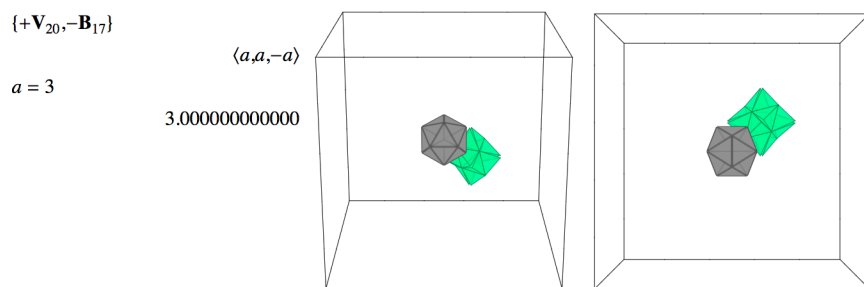
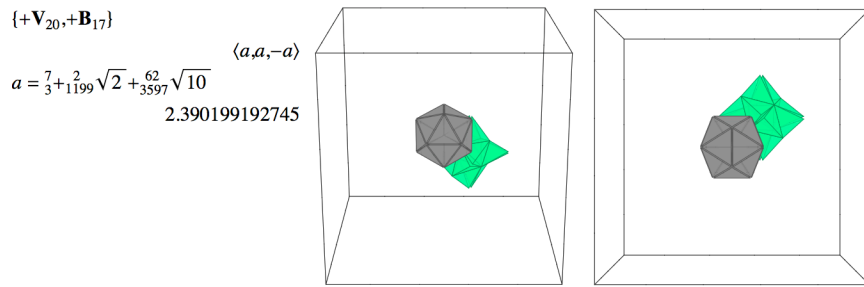


$\{+V_{20}, -B_5\}$

$a = 3$

$\langle a, a, -a \rangle$
3.000000000000

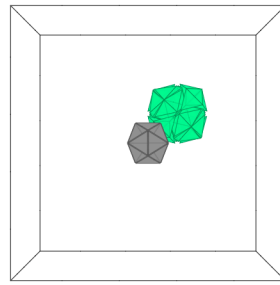
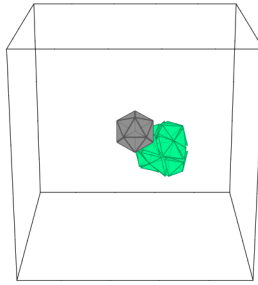




$\{+\mathbf{V}_{20},+\mathbf{B}_{57}\}$

$$a = \frac{11}{3}$$

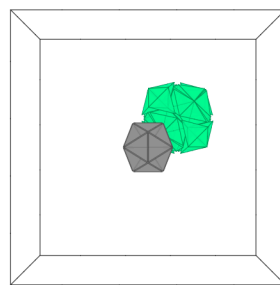
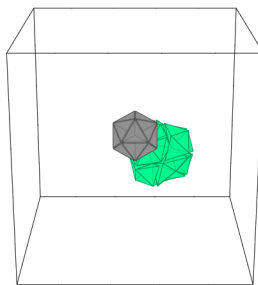
$\langle a,a,-a \rangle$
3.66666666667



$\{+\mathbf{V}_{20},-\mathbf{B}_{57}\}$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

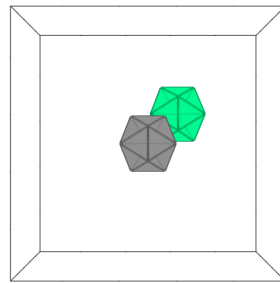
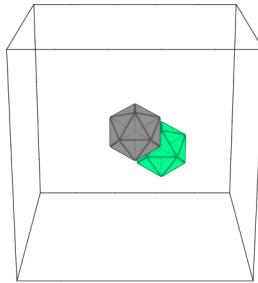
$\langle a,a,-a \rangle$
3.047553602722



$\{+\mathbf{V}_{20},+\mathbf{V}_{20}\}$

$$a = \frac{8}{3}$$

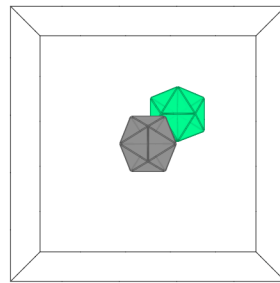
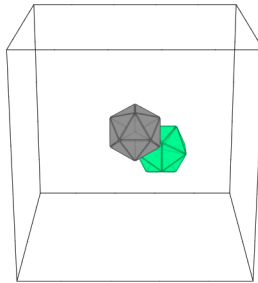
$\langle a,a,-a \rangle$
2.66666666667



$\{+\mathbf{V}_{20},-\mathbf{V}_{20}\}$

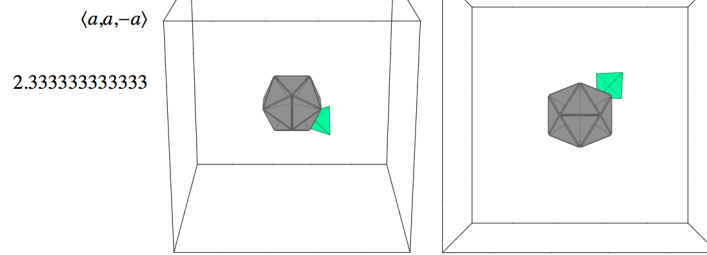
$$a = \frac{8}{3}$$

$\langle a,a,-a \rangle$
2.66666666667



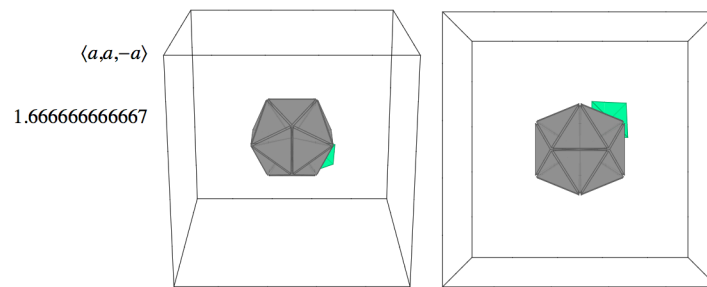
$\{-V_{20}, +B_1\}$

$a = \frac{7}{3}$



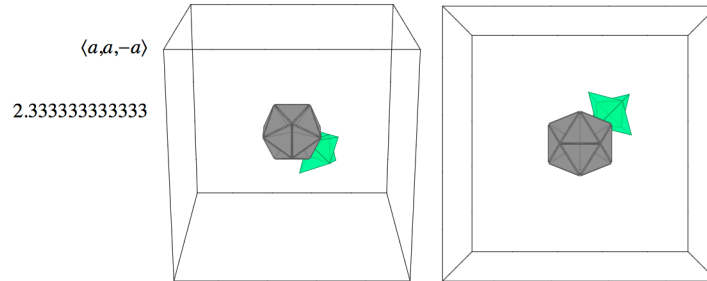
$\{-V_{20}, -B_1\}$

$a = \frac{5}{3}$



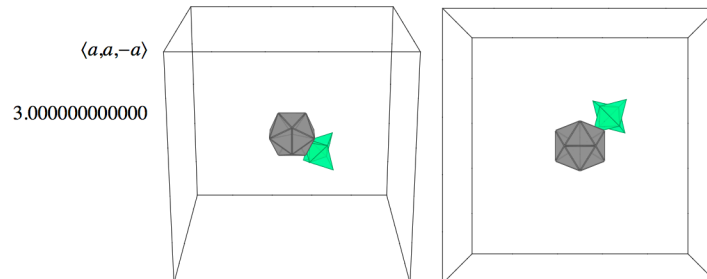
$\{-V_{20}, +B_5\}$

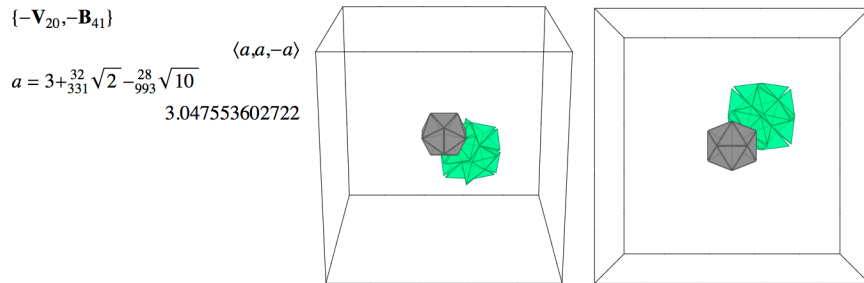
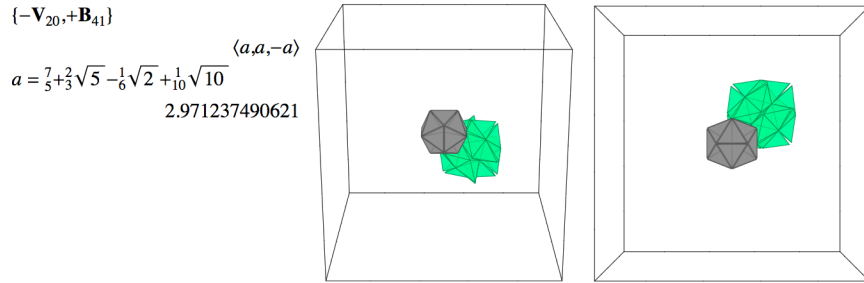
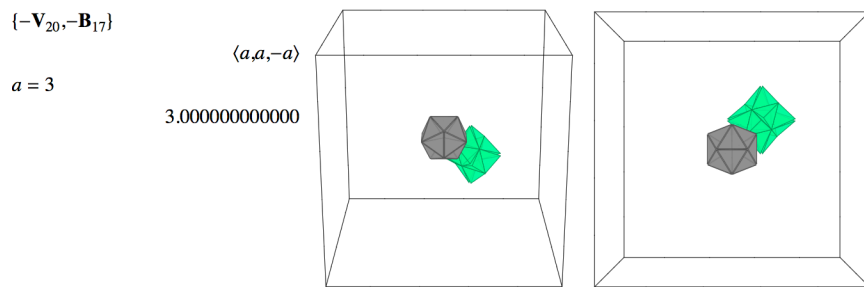
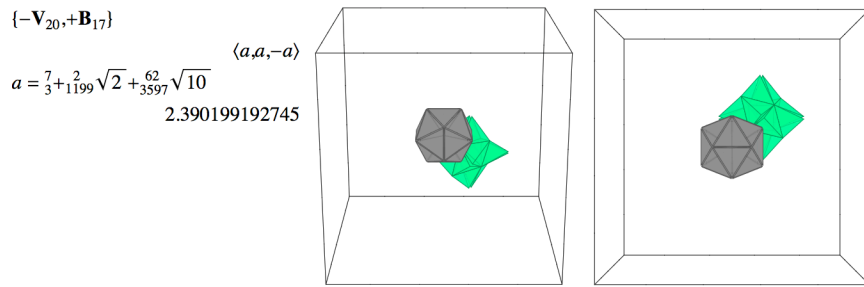
$a = \frac{7}{3}$



$\{-V_{20}, -B_5\}$

$a = 3$

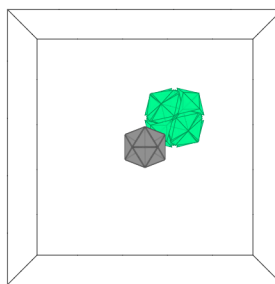
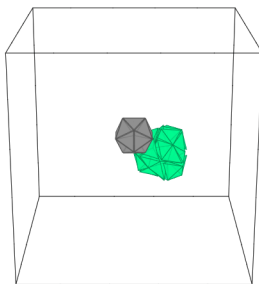




$\{-V_{20}, +B_{57}\}$

$$a = \frac{11}{3}$$

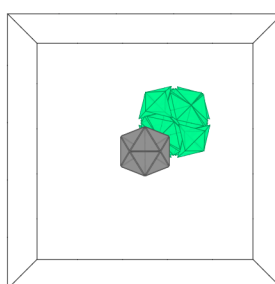
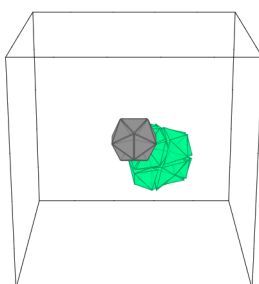
$\langle a, a, -a \rangle$
3.666666666667



$\{-V_{20}, -B_{57}\}$

$$a = 3 + \frac{32}{331}\sqrt{2} - \frac{28}{993}\sqrt{10}$$

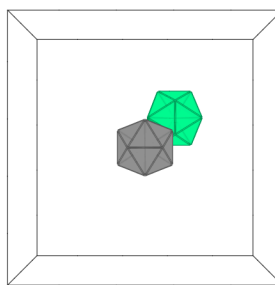
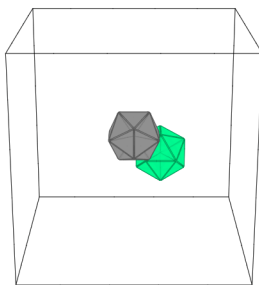
$\langle a, a, -a \rangle$
3.047553602722



$\{-V_{20}, +V_{20}\}$

$$a = \frac{8}{3}$$

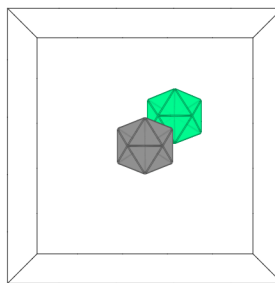
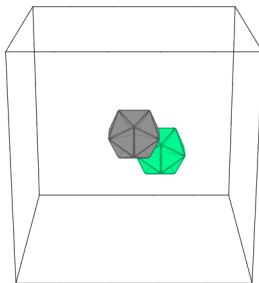
$\langle a, a, -a \rangle$
2.666666666667



$\{-V_{20}, -V_{20}\}$

$$a = \frac{8}{3}$$

$\langle a, a, -a \rangle$
2.666666666667



Chapter 12

Cubic family packings

Construct clusters with ‘cubic’ symmetry

$$\text{Let } A, B, C, D \in \{*, \pm\mathbf{B}_1, \pm\mathbf{B}_5, \pm\mathbf{B}_{17}, \pm\mathbf{B}_{41}, \pm\mathbf{B}_{57}, \pm\mathbf{V}_{20}\}$$

Construct the packing $\langle A, B, C, D \rangle$ with ‘cubic’ symmetry, for all values of A, B, C, D

Calculate the spacings between all pairs of clusters, in all directions

$$\text{in the 2-directions: } A \rightarrow A, B \rightarrow B, C \rightarrow C, D \rightarrow D$$

$$\text{in the } +3\text{-directions: } A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$$

$$\text{in the 1-directions: } A \rightarrow C, B \rightarrow D, C \rightarrow A, D \rightarrow B$$

$$\text{in the } -3\text{-directions: } A \rightarrow D, B \rightarrow A, C \rightarrow B, D \rightarrow C$$

Evaluate the lattice vector a , which is the maximum of all calculated spacings

Note: a is the minimum spacing between neighboring clusters

The lattice basis is $\{\langle 0, 2a, 2a \rangle, \langle 2a, 0, 2a \rangle, \langle 2a, 2a, 0 \rangle\}$

The fundamental domain $ABCD$ is the convex hull of $A = \langle 0, 0, 0 \rangle, B = \langle a, a, +a \rangle, C = \langle 2a, 0, 0 \rangle, D = \langle a, a, -a \rangle$

$$\text{Evaluate the lattice volume } V = \det \begin{bmatrix} 0 & 2a & 2a \\ 2a & 0 & 2a \\ 2a & 2a & 0 \end{bmatrix} = 16a^3$$

Evaluate the packing density $D = \frac{1}{6}n/a^3$, where n is the total number of tetrahedra from the 4 cosets $\{A, B, C, D\}$

Note: Each tetrahedron has volume $\frac{8}{3}$

Note: The following packings are congruent to $\{A, B, C, D\}$ (via the following isometries)

$$\begin{array}{ll} \{A, B, C, D\} & \langle x, y, z \rangle \rightarrow +\langle x, y, z \rangle + \langle 0, 0, 0 \rangle \\ \{D, A, B, C\} & \langle x, y, z \rangle \rightarrow +\langle x, y, z \rangle + \langle a, a, +a \rangle \\ \{C, D, A, B\} & \langle x, y, z \rangle \rightarrow +\langle x, y, z \rangle + \langle 2a, 0, 0 \rangle \\ \{B, C, D, A\} & \langle x, y, z \rangle \rightarrow +\langle x, y, z \rangle + \langle a, a, -a \rangle \\ \\ \{-A, -D, -C, -B\} & \langle x, y, z \rangle \rightarrow -\langle x, y, z \rangle + \langle 0, 0, 0 \rangle \\ \{-B, -A, -D, -C\} & \langle x, y, z \rangle \rightarrow -\langle x, y, z \rangle + \langle a, a, +a \rangle \\ \{-C, -B, -A, -D\} & \langle x, y, z \rangle \rightarrow -\langle x, y, z \rangle + \langle 2a, 0, 0 \rangle \\ \{-D, -C, -B, -A\} & \langle x, y, z \rangle \rightarrow -\langle x, y, z \rangle + \langle a, a, -a \rangle \end{array}$$

Note: The offsets are the vertices of the fundamental domain $ABCD$

We give all packings with packing density $D \geq \frac{4}{7}$

We also give the packing with minimum packing density

$(-\mathbf{B}_{17}, *, *, +\mathbf{B}_{17})$

2 cosets
34 tetrahedra

$$U = \frac{272}{3}$$

90.666666666667

$$a = 2$$

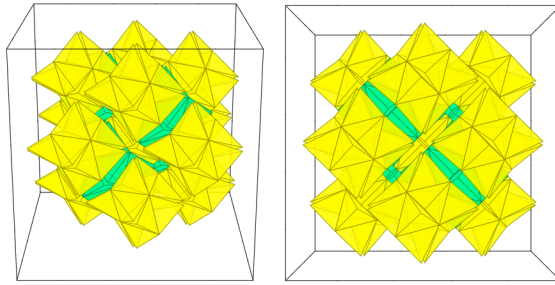
2.000000000000

$$V = 128$$

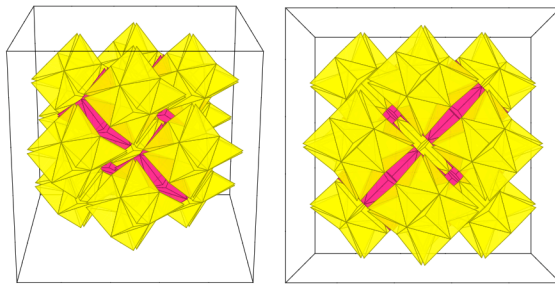
128.000000000000

$$D = \frac{17}{24}$$

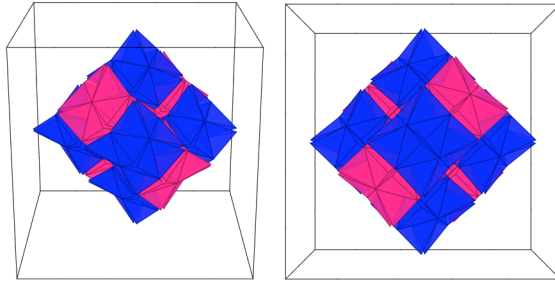
.708333333333



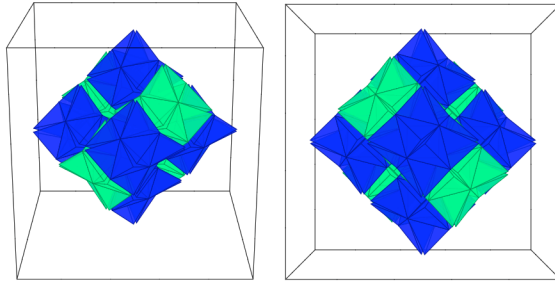
$(+\mathbf{B}_{17}, -\mathbf{B}_{17}, *, *)$



$(*, +\mathbf{B}_{17}, -\mathbf{B}_{17}, *)$



$(*, *, +\mathbf{B}_{17}, -\mathbf{B}_{17})$



$\{+V_{20}, +V_{20}, +V_{20}, +V_{20}\}$

4 cosets

80 tetrahedra

$$U = \frac{640}{3}$$

213.333333333333

$$a = \frac{8}{3}$$

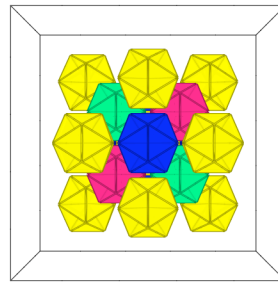
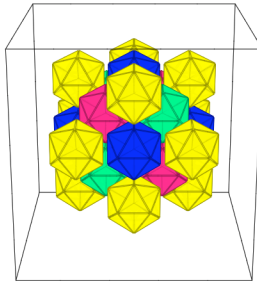
2.666666666667

$$V = \frac{8192}{27}$$

303.407407407407

$$D = \frac{45}{64}$$

.703125000000



$\{+\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{41}, -\mathbf{B}_{41}\}$

4 cosets

164 tetrahedra

$$U = \frac{1312}{3}$$

437.333333333333

$$a = \frac{246}{71}$$

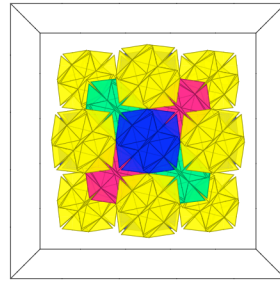
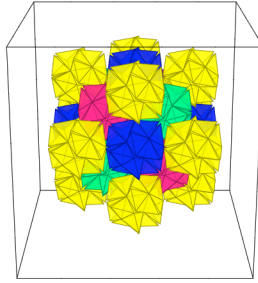
3.464788732394

$$V = \frac{238190976}{357911}$$

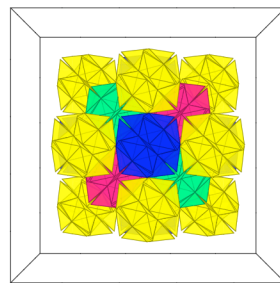
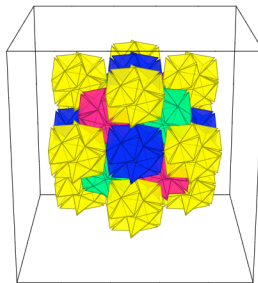
665.503368155771

$$D = \frac{357911}{544644}$$

.657146686643



$\{-\mathbf{B}_{41}, +\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{41}\}$



{+V₂₀,*,+B₅,*}

2 cosets
25 tetrahedra

$$U = \frac{200}{3}$$

66.66666666667

$$a = \frac{7}{8} + \frac{5}{12}\sqrt{5} - \frac{5}{48}\sqrt{2} + \frac{1}{16}\sqrt{10}$$

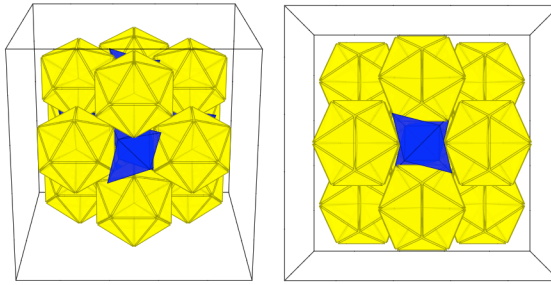
1.857023431638

$$V = \frac{377}{8} + \frac{18335}{864}\sqrt{5} + \frac{4385}{1728}\sqrt{2} + \frac{87}{64}\sqrt{10}$$

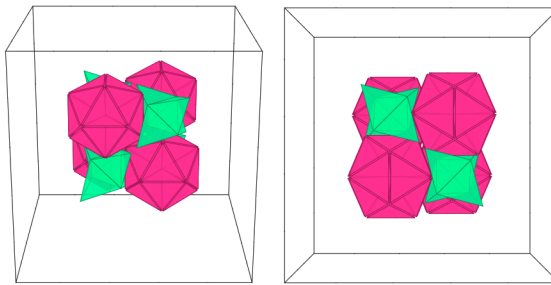
102.464195271793

$$D = \frac{224728886700800}{8093428886883} - \frac{97932582976000}{8093428886883}\sqrt{5} - \frac{341050262704000}{8093428886883}\sqrt{2} + \frac{152370804534400}{8093428886883}\sqrt{10}$$

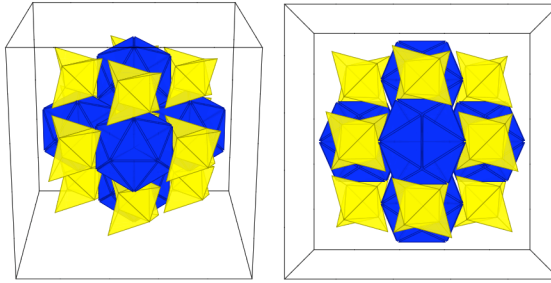
.650633779827



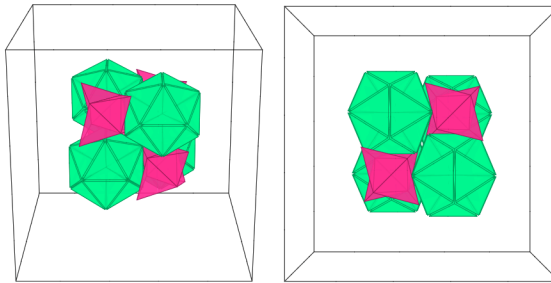
{*,+V₂₀,*,+B₅}



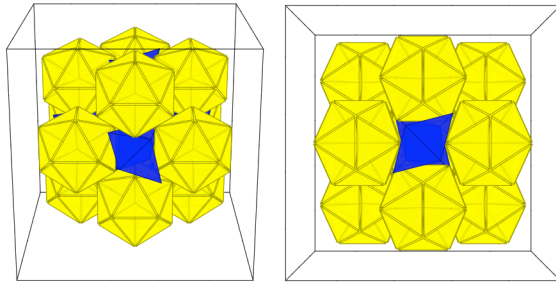
{+B₅,*,+V₂₀,*}



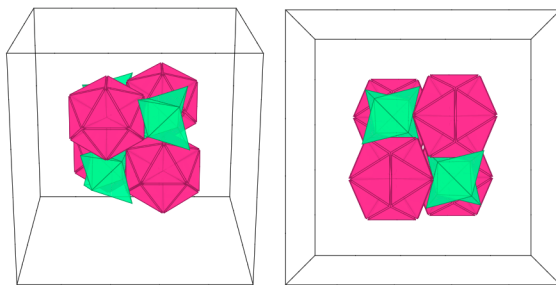
{*,+B₅,*,+V₂₀}



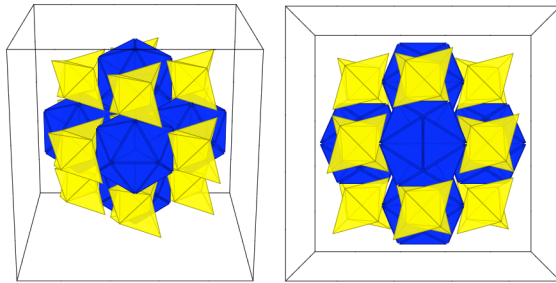
$\{+V_{20},*,-B_5,*\}$



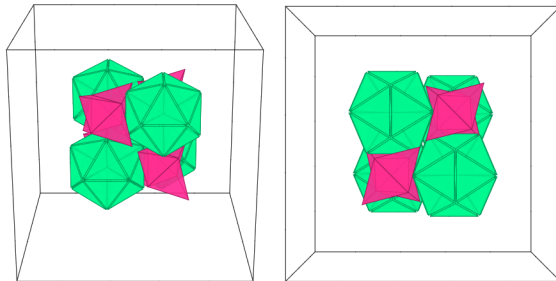
$\{*,+V_{20},*,-B_5\}$



$\{-B_5,*,+V_{20},*\}$



$\{*,-B_5,*,+V_{20}\}$



$\{+\mathbf{V}_{20},+\mathbf{B}_1,*,-\mathbf{B}_1\}$

3 cosets
22 tetrahedra

$$U = \frac{176}{3}$$

58.666666666667

$$a = \frac{4}{5}\sqrt{5}$$

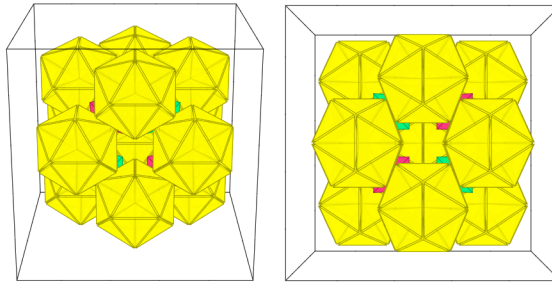
1.788854382000

$$V = \frac{1024}{25}\sqrt{5}$$

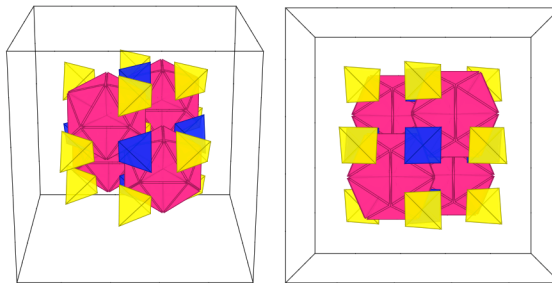
91.589344358391

$$D = \frac{55}{192}\sqrt{5}$$

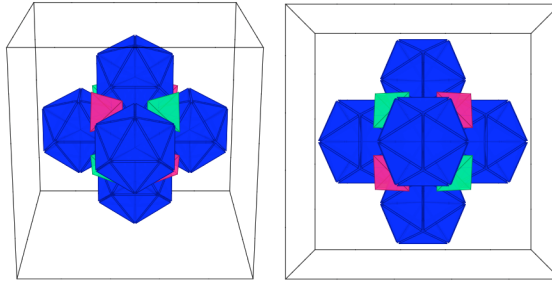
.640540306055



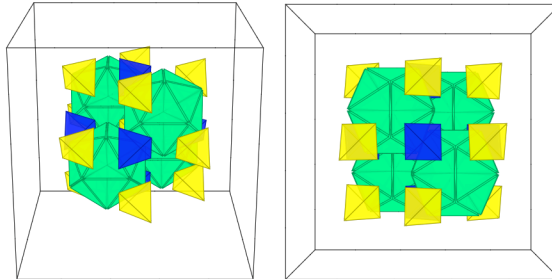
$\{-\mathbf{B}_1,+\mathbf{V}_{20},+\mathbf{B}_1,*\}$



$\{*,-\mathbf{B}_1,+\mathbf{V}_{20},+\mathbf{B}_1\}$



$\{+\mathbf{B}_1,*,-\mathbf{B}_1,+\mathbf{V}_{20}\}$



$\{+V_{20}, +B_1, +B_1, *\}$

3 cosets
22 tetrahedra

$$U = \frac{176}{3}$$

58.6666666666667

$$a = \frac{4}{5}\sqrt{5}$$

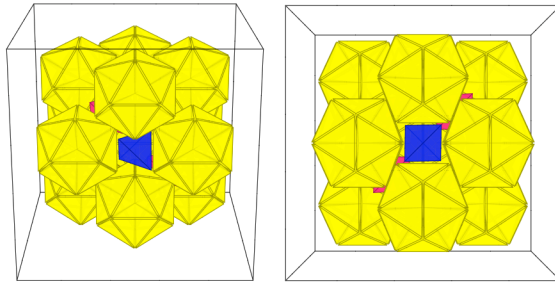
1.788854382000

$$V = \frac{1024}{25}\sqrt{5}$$

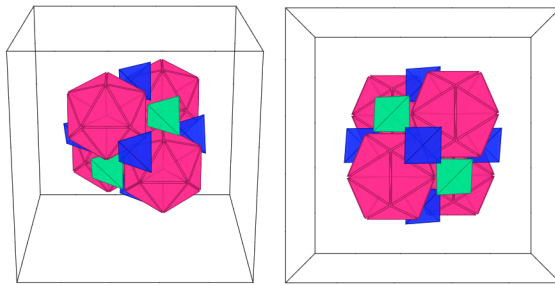
91.589344358391

$$D = \frac{55}{192}\sqrt{5}$$

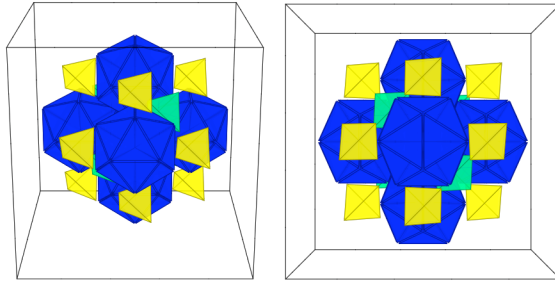
.640540306055



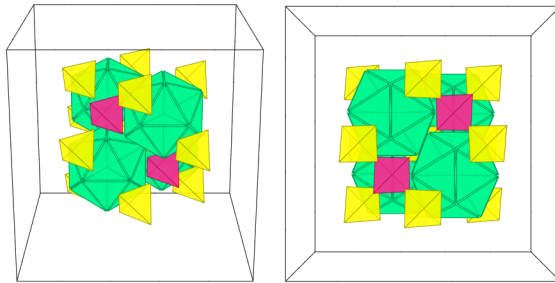
$\{*, +V_{20}, +B_1, +B_1\}$



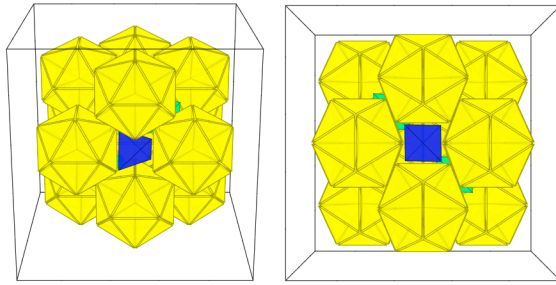
$\{+B_1, *, +V_{20}, +B_1\}$



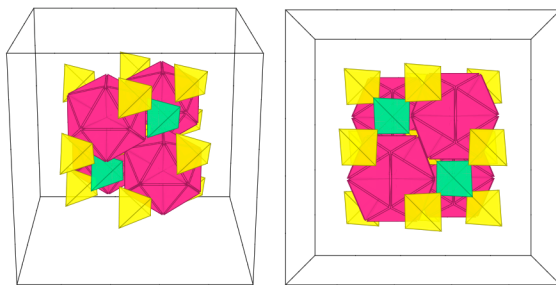
$\{+B_1, +B_1, *, +V_{20}\}$



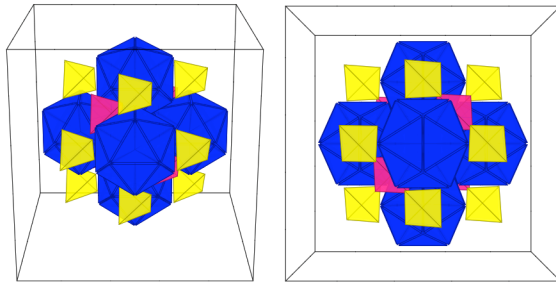
$\{+V_{20},*, -B_1, -B_1\}$



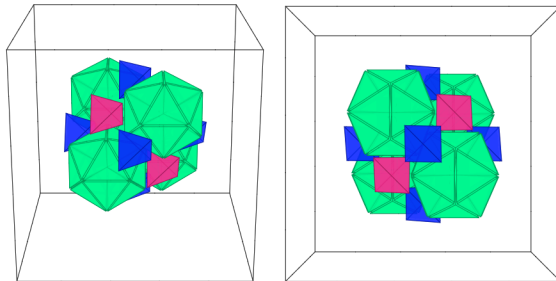
$\{-B_1,+V_{20},*, -B_1\}$



$\{-B_1,-B_1,+V_{20},*\}$



$\{*, -B_1, -B_1, +V_{20}\}$



$\{+B_{41}, +B_{41}, +B_{41}, +B_{41}\}$

4 cosets

164 tetrahedra

$$U = \frac{1312}{3}$$

437.333333333333

$$a = \frac{7084}{2025}$$

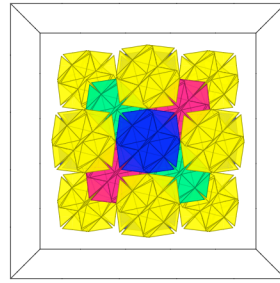
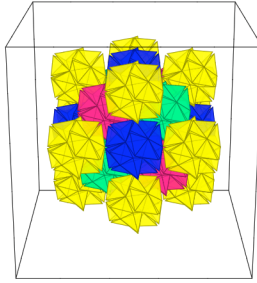
3.498271604938

$$V = \frac{5687948299264}{8303765625}$$

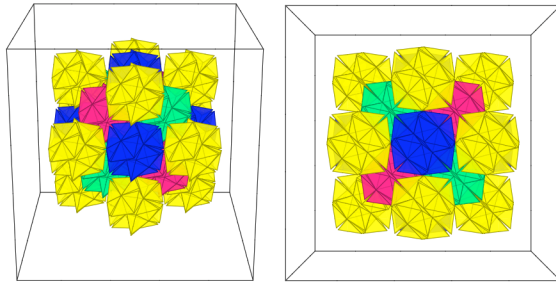
684.984205495805

$$D = \frac{113484796875}{177748384352}$$

.638457543728



$\{-\mathbf{B}_{41}, -\mathbf{B}_{41}, -\mathbf{B}_{41}, -\mathbf{B}_{41}\}$



$\{+B_{41}, +B_{41}, -B_{41}, -B_{41}\}$

4 cosets

164 tetrahedra

$$U = \frac{1312}{3}$$

437.333333333333

$$a = \frac{7084}{2025}$$

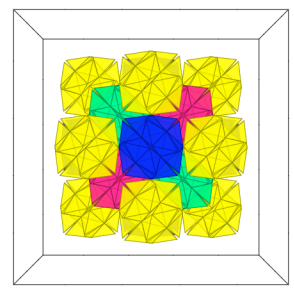
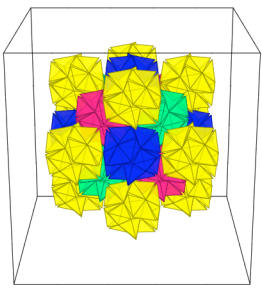
3.498271604938

$$V = \frac{5687948299264}{8303765625}$$

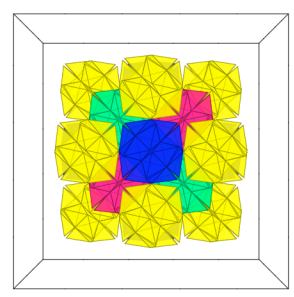
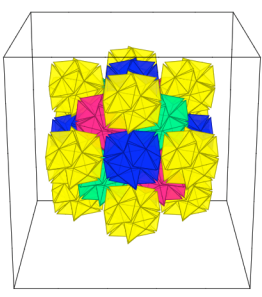
684.984205495805

$$D = \frac{113484796875}{177748384352}$$

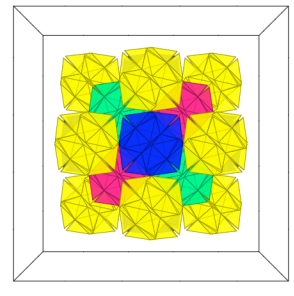
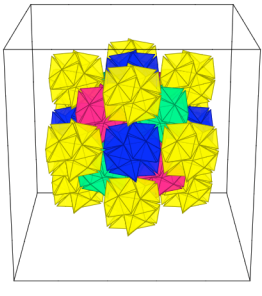
.638457543728



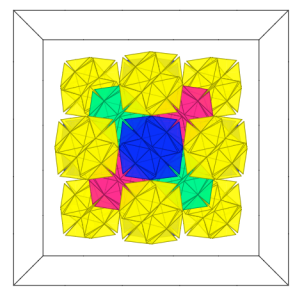
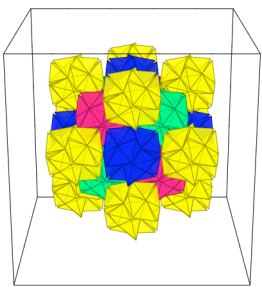
$\{-B_{41}, +B_{41}, +B_{41}, -B_{41}\}$



$\{-B_{41}, -B_{41}, +B_{41}, +B_{41}\}$



$\{+B_{41}, -B_{41}, -B_{41}, +B_{41}\}$



$$\{-\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{41}\}$$

4 cosets

164 tetrahedra

$$U = \frac{1312}{3}$$

437.333333333333

$$a = \frac{7084}{2025}$$

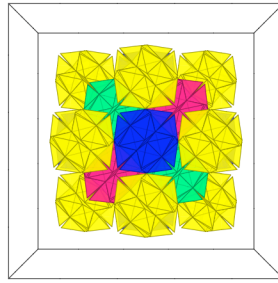
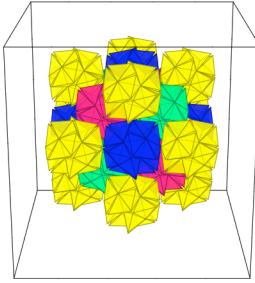
3.498271604938

$$V = \frac{5687948299264}{8303765625}$$

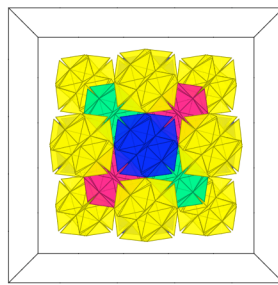
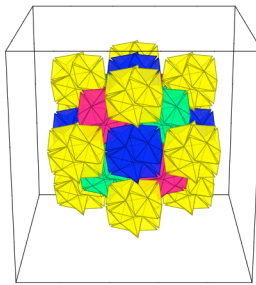
684.984205495805

$$D = \frac{113484796875}{177748384352}$$

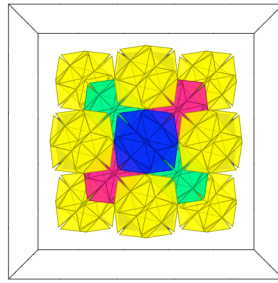
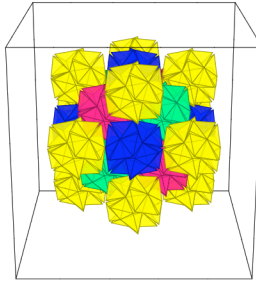
.638457543728



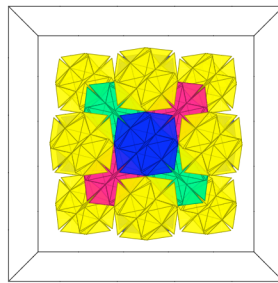
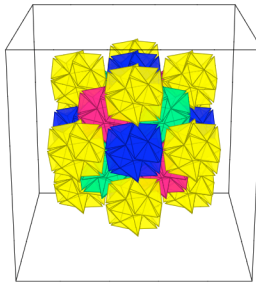
$$\{+\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{41}\}$$



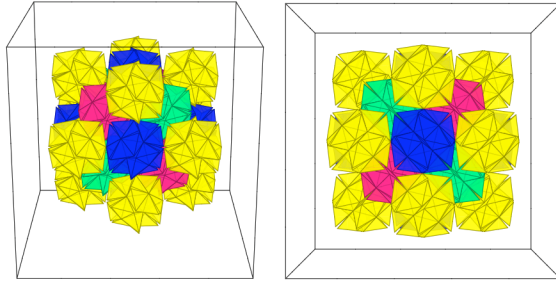
$$\{+\mathbf{B}_{41}, +\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{41}\}$$



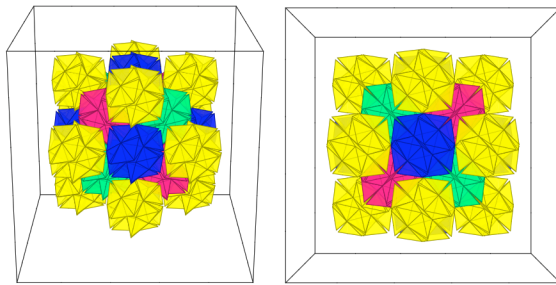
$$\{+\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{41}, -\mathbf{B}_{41}\}$$



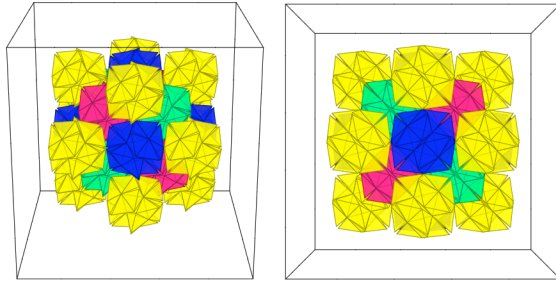
$\{+\mathbf{B}_{41}, -\mathbf{B}_{41}, -\mathbf{B}_{41}, -\mathbf{B}_{41}\}$



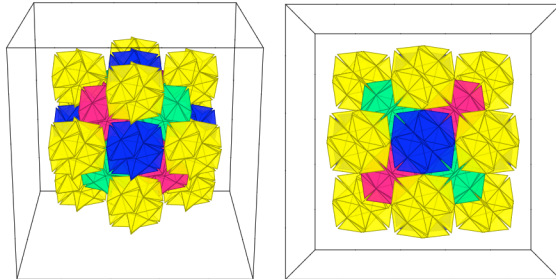
$\{-\mathbf{B}_{41}, +\mathbf{B}_{41}, -\mathbf{B}_{41}, -\mathbf{B}_{41}\}$



$\{-\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{41}, -\mathbf{B}_{41}\}$



$\{-\mathbf{B}_{41}, -\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{41}\}$



$$\{-\mathbf{B}_{57}, +\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{17}\}$$

4 cosets

156 tetrahedra

$$U = 416$$

416.000000000000

$$a = \frac{246}{71}$$

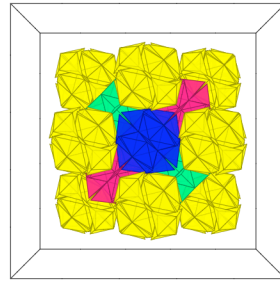
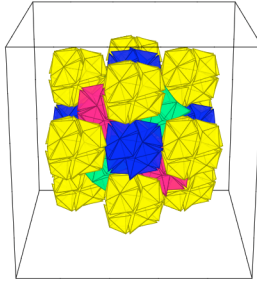
3.464788732394

$$V = \frac{238190976}{357911}$$

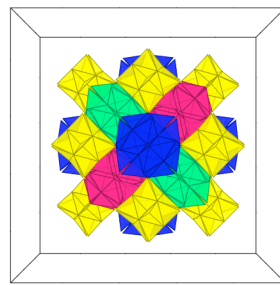
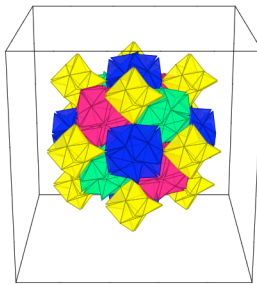
665.503368155771

$$D = \frac{4652843}{7443468}$$

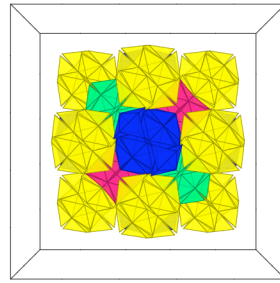
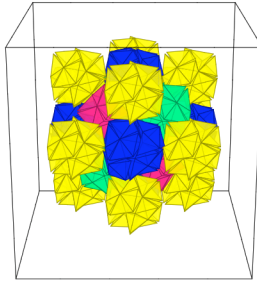
.625090750709



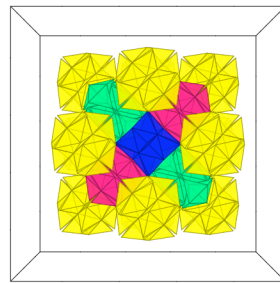
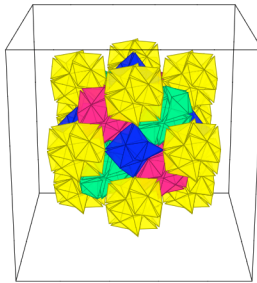
$$\{+\mathbf{B}_{17}, -\mathbf{B}_{57}, +\mathbf{B}_{41}, -\mathbf{B}_{41}\}$$



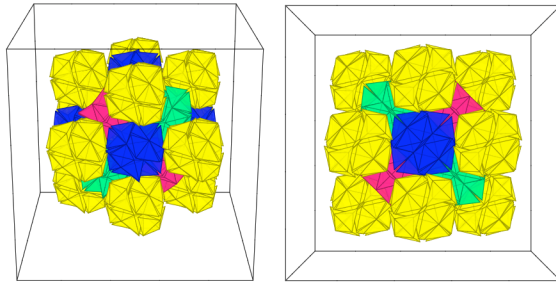
$$\{-\mathbf{B}_{41}, +\mathbf{B}_{17}, -\mathbf{B}_{57}, +\mathbf{B}_{41}\}$$



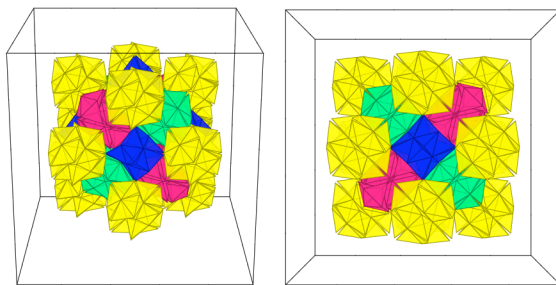
$$\{+\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{17}, -\mathbf{B}_{57}\}$$



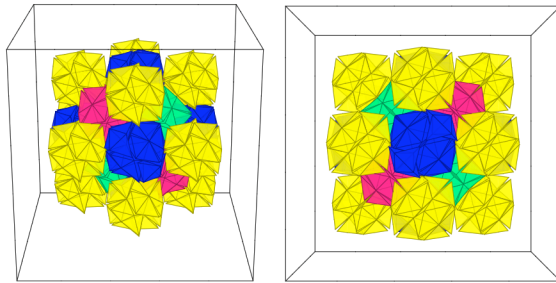
$\{+\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{B}_{41}, -\mathbf{B}_{41}\}$



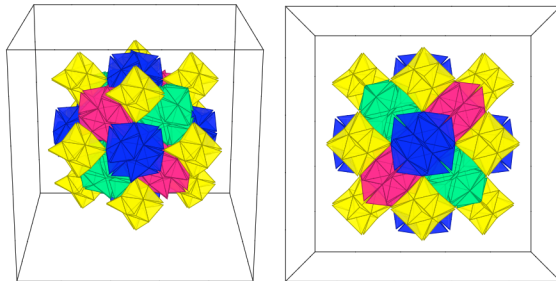
$\{-\mathbf{B}_{41}, +\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{B}_{41}\}$



$\{+\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{57}, -\mathbf{B}_{17}\}$



$\{-\mathbf{B}_{17}, +\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{57}\}$



$$\{+\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{B}_{17}, -\mathbf{B}_{57}\}$$

4 cosets

148 tetrahedra

$$U = \frac{1184}{3}$$

394.666666666667

$$a = \frac{24}{7}$$

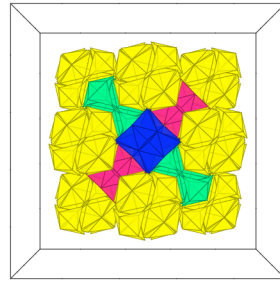
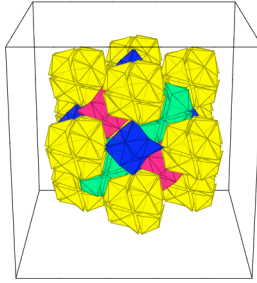
3.428571428571

$$V = \frac{221184}{343}$$

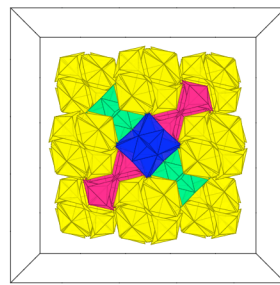
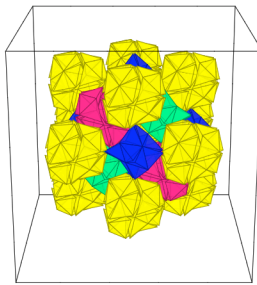
644.851311953353

$$D = \frac{12691}{20736}$$

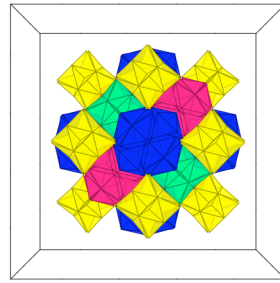
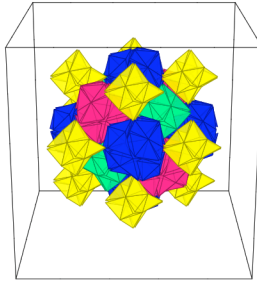
.612027391975



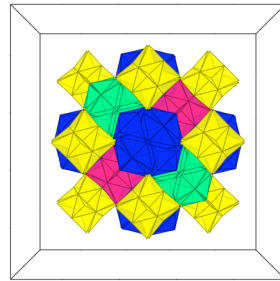
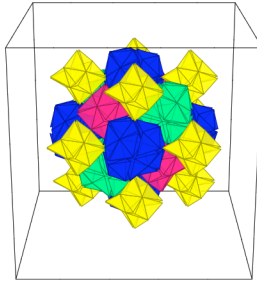
$$\{-\mathbf{B}_{57}, +\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{B}_{17}\}$$



$$\{+\mathbf{B}_{17}, -\mathbf{B}_{57}, +\mathbf{B}_{57}, -\mathbf{B}_{17}\}$$



$$\{-\mathbf{B}_{17}, +\mathbf{B}_{17}, -\mathbf{B}_{57}, +\mathbf{B}_{57}\}$$



$\{+\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{B}_{57}, -\mathbf{B}_{17}\}$

4 cosets

148 tetrahedra

$$U = \frac{1184}{3}$$

394.666666666667

$$a = \frac{24}{7}$$

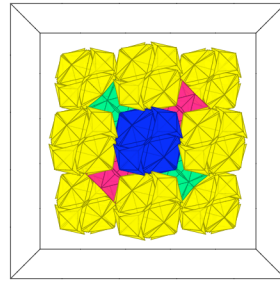
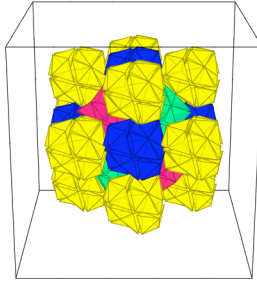
3.428571428571

$$V = \frac{221184}{343}$$

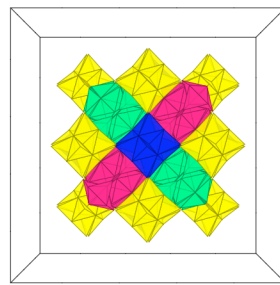
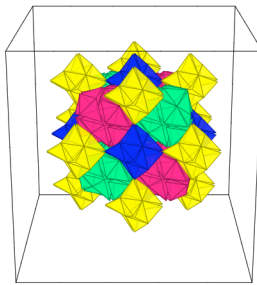
644.851311953353

$$D = \frac{12691}{20736}$$

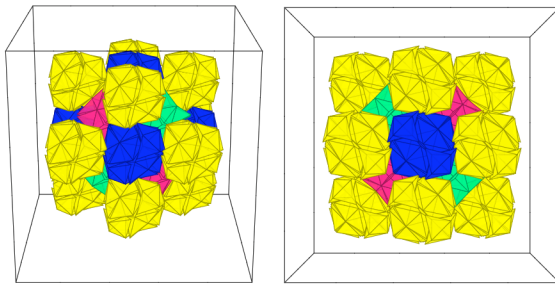
.612027391975



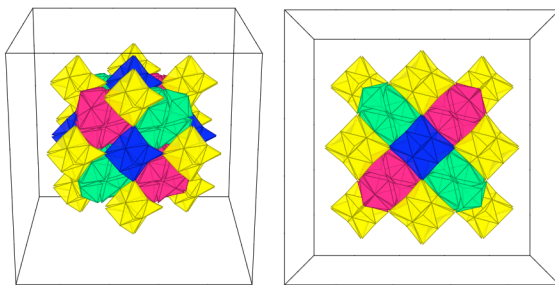
$\{-\mathbf{B}_{17}, +\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{B}_{57}\}$



$\{-\mathbf{B}_{57}, +\mathbf{B}_{17}, -\mathbf{B}_{57}, +\mathbf{B}_{17}\}$



$\{+\mathbf{B}_{17}, -\mathbf{B}_{57}, +\mathbf{B}_{17}, -\mathbf{B}_{57}\}$



$\{+\mathbf{V}_{20},*,+\mathbf{B}_1,*\}$

2 cosets
21 tetrahedra

$$U = 56$$

$$a = \frac{4}{5}\sqrt{5}$$

$$V = \frac{1024}{25}\sqrt{5}$$

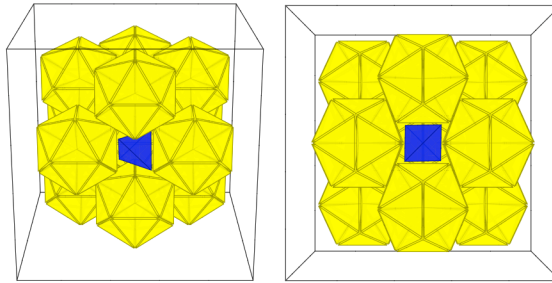
$$D = \frac{35}{128}\sqrt{5}$$

56.000000000000

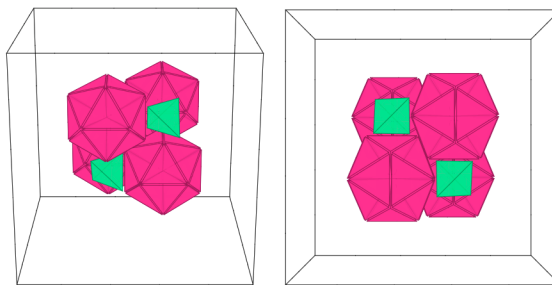
1.788854382000

91.589344358391

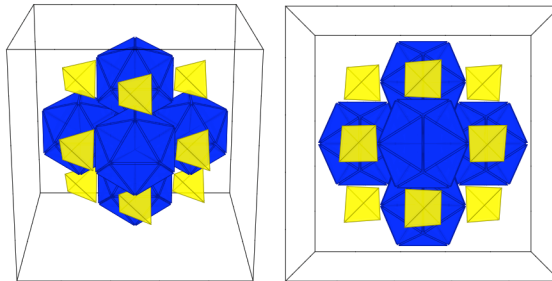
.611424837598



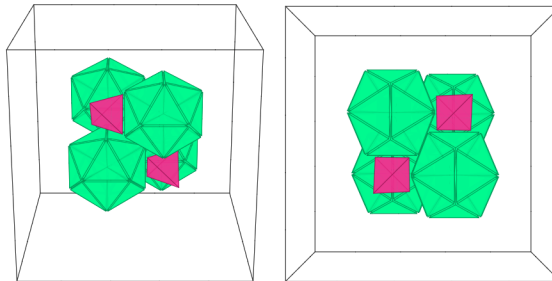
$\{*,+\mathbf{V}_{20},*,+\mathbf{B}_1\}$



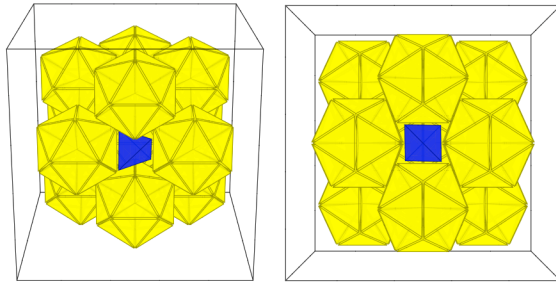
$\{+\mathbf{B}_1,*,+\mathbf{V}_{20},*\}$



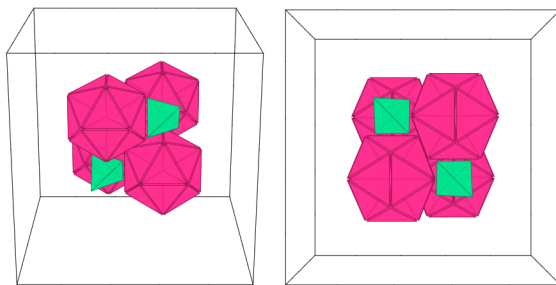
$\{*,+\mathbf{B}_1,*,+\mathbf{V}_{20}\}$



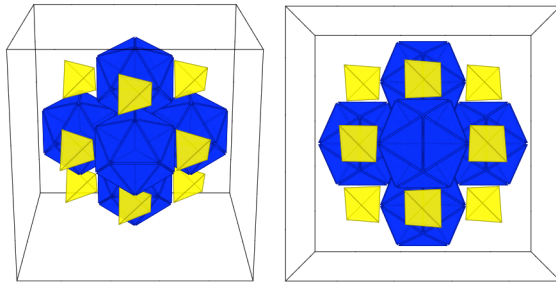
$\{+V_{20},*, -B_1,*\}$



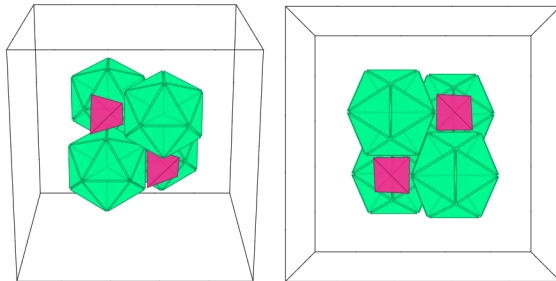
$\{*,+V_{20},*, -B_1\}$



$\{-B_1,*,+V_{20},*\}$



$\{*, -B_1,*,+V_{20}\}$



$\{+\mathbf{V}_{20},+\mathbf{B}_1,*,*\}$

2 cosets
21 tetrahedra

$$U = 56$$

$$a = \frac{4}{5}\sqrt{5}$$

$$V = \frac{1024}{25}\sqrt{5}$$

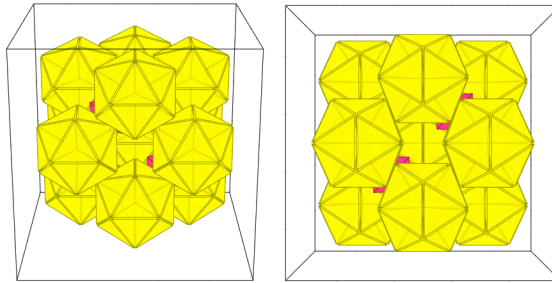
$$D = \frac{35}{128}\sqrt{5}$$

56.0000000000000

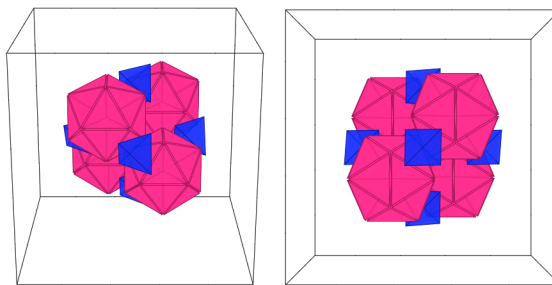
1.788854382000

91.589344358391

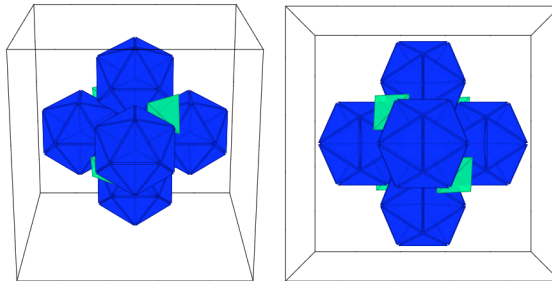
.611424837598



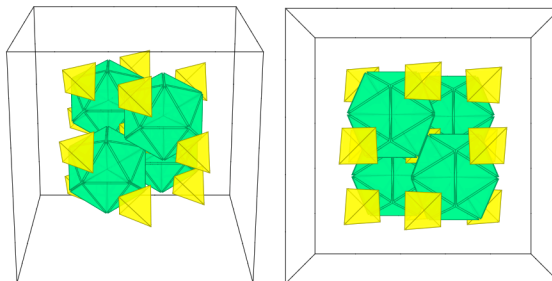
$\{*,+\mathbf{V}_{20},+\mathbf{B}_1,*\}$



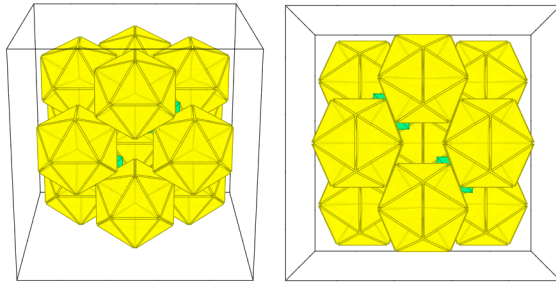
$\{*,*,+\mathbf{V}_{20},+\mathbf{B}_1\}$



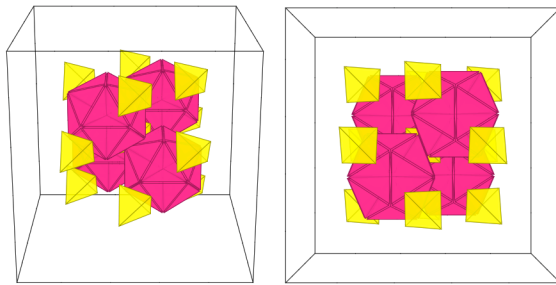
$\{+\mathbf{B}_1,*,*,+\mathbf{V}_{20}\}$



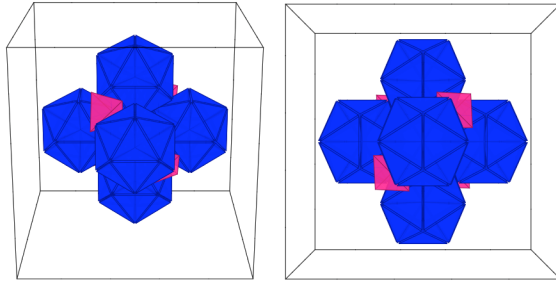
$\{+V_{20},*,*,-B_1\}$



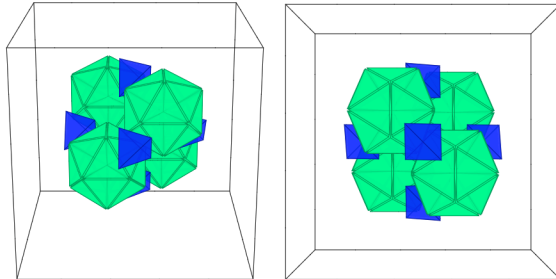
$\{-B_1,+V_{20},*,*\}$



$\{*,-B_1,+V_{20},*\}$



$\{*,*,-B_1,+V_{20}\}$



$\{-\mathbf{B}_{57}, -\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{17}\}$

4 cosets

156 tetrahedra

$U = 416$

416.000000000000

$a = \frac{7084}{2025}$

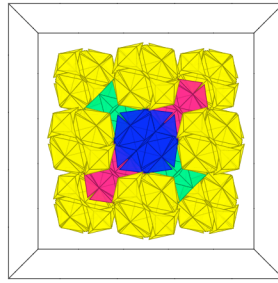
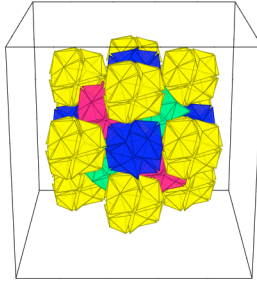
3.498271604938

$V = \frac{5687948299264}{8303765625}$

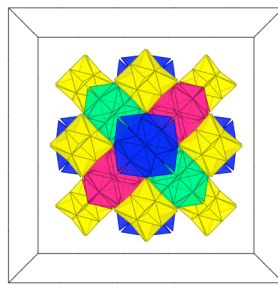
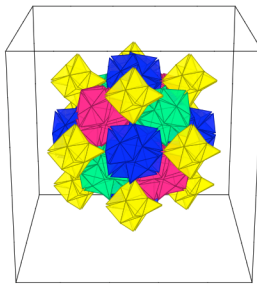
684.984205495805

$D = \frac{107948953125}{177748384352}$

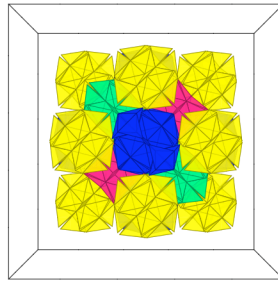
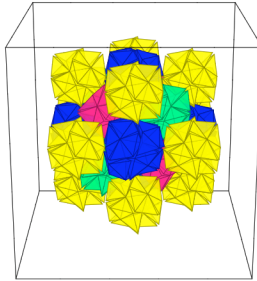
.607313273302



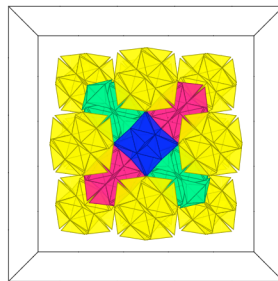
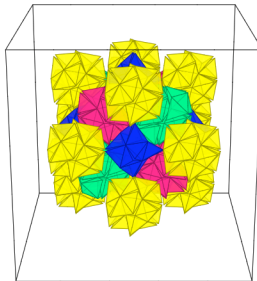
$\{+\mathbf{B}_{17}, -\mathbf{B}_{57}, -\mathbf{B}_{41}, +\mathbf{B}_{41}\}$



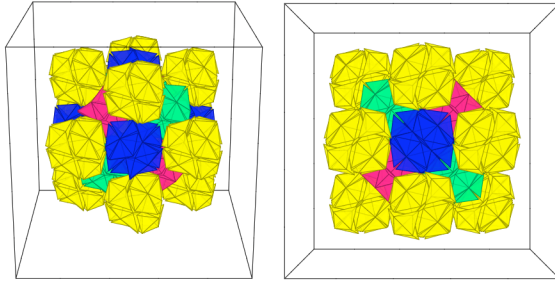
$\{+\mathbf{B}_{41}, +\mathbf{B}_{17}, -\mathbf{B}_{57}, -\mathbf{B}_{41}\}$



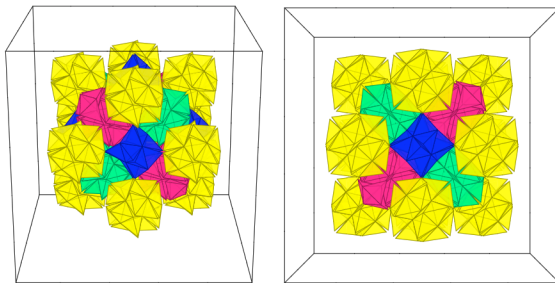
$\{-\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{17}, -\mathbf{B}_{57}\}$



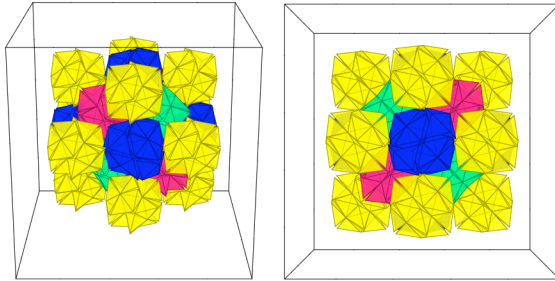
$\{+\mathbf{B}_{57}, -\mathbf{B}_{17}, -\mathbf{B}_{41}, +\mathbf{B}_{41}\}$



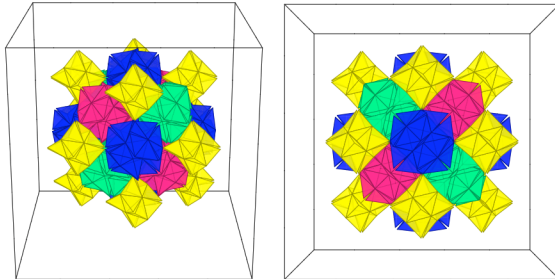
$\{+\mathbf{B}_{41}, +\mathbf{B}_{57}, -\mathbf{B}_{17}, -\mathbf{B}_{41}\}$



$\{-\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{57}, -\mathbf{B}_{17}\}$



$\{-\mathbf{B}_{17}, -\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{57}\}$



$\{-\mathbf{B}_{57}, +\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{17}\}$

4 cosets

156 tetrahedra

$U = 416$

416.000000000000

$a = \frac{7084}{2025}$

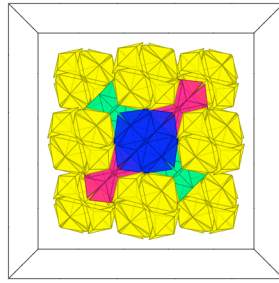
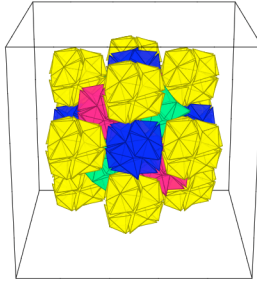
3.498271604938

$V = \frac{5687948299264}{8303765625}$

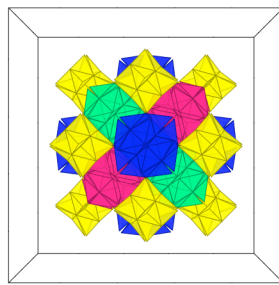
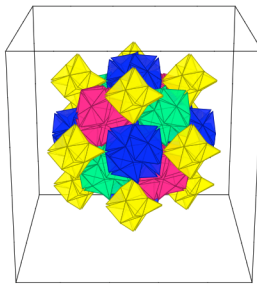
684.984205495805

$D = \frac{107948953125}{177748384352}$

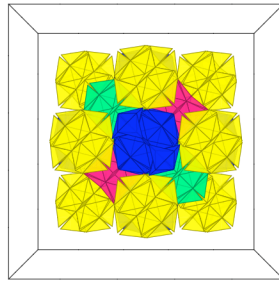
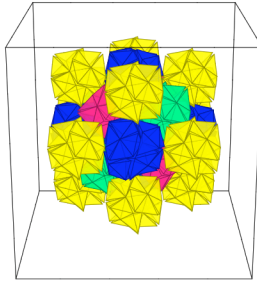
.607313273302



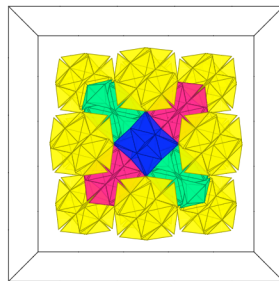
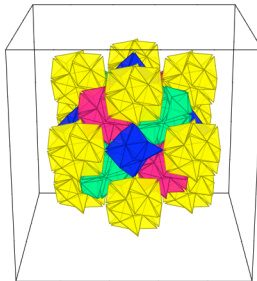
$\{+\mathbf{B}_{17}, -\mathbf{B}_{57}, +\mathbf{B}_{41}, +\mathbf{B}_{41}\}$



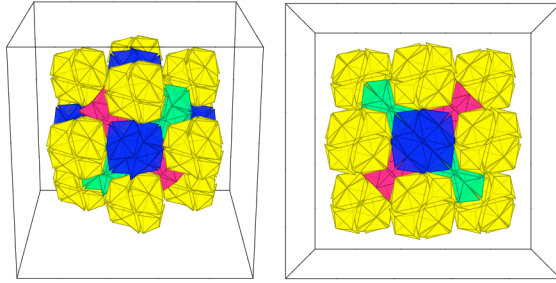
$\{+\mathbf{B}_{41}, +\mathbf{B}_{17}, -\mathbf{B}_{57}, +\mathbf{B}_{41}\}$



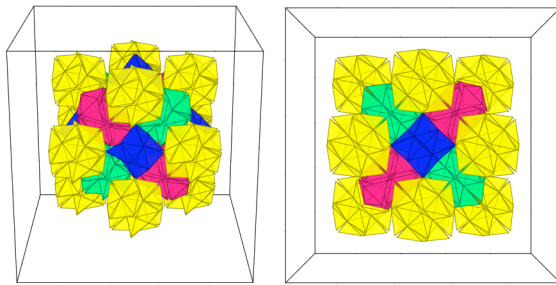
$\{+\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{17}, -\mathbf{B}_{57}\}$



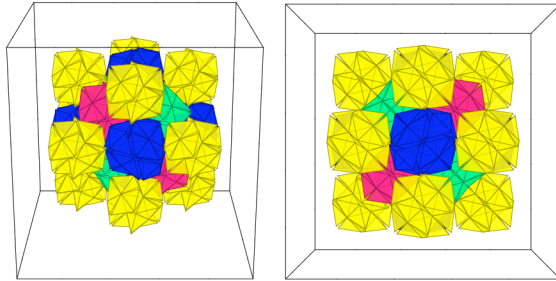
$\{+\mathbf{B}_{57}, -\mathbf{B}_{17}, -\mathbf{B}_{41}, -\mathbf{B}_{41}\}$



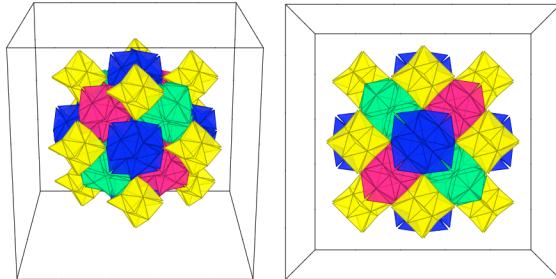
$\{-\mathbf{B}_{41}, +\mathbf{B}_{57}, -\mathbf{B}_{17}, -\mathbf{B}_{41}\}$



$\{-\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{57}, -\mathbf{B}_{17}\}$



$\{-\mathbf{B}_{17}, -\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{57}\}$



$\{-\mathbf{B}_{57}, -\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{17}\}$

4 cosets

156 tetrahedra

$U = 416$

416.000000000000

$a = \frac{7084}{2025}$

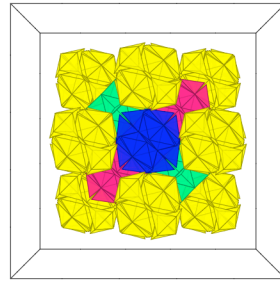
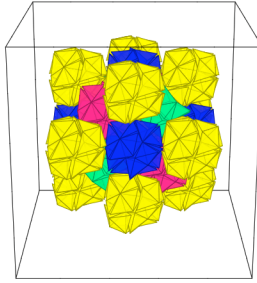
3.498271604938

$V = \frac{5687948299264}{8303765625}$

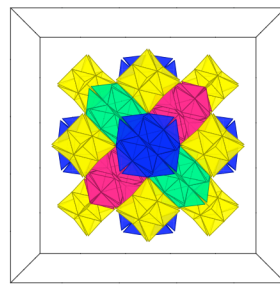
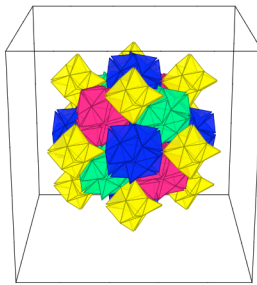
684.984205495805

$D = \frac{107948953125}{177748384352}$

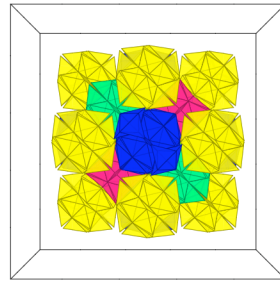
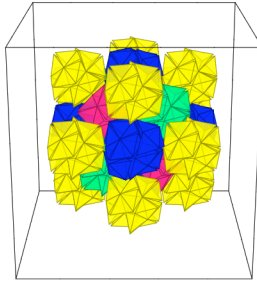
.607313273302



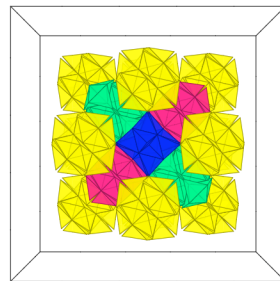
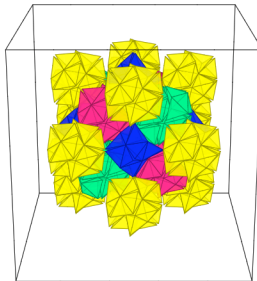
$\{+\mathbf{B}_{17}, -\mathbf{B}_{57}, -\mathbf{B}_{41}, -\mathbf{B}_{41}\}$



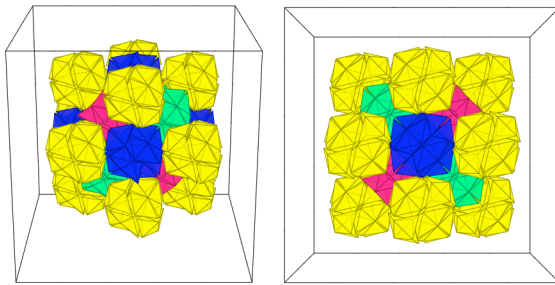
$\{-\mathbf{B}_{41}, +\mathbf{B}_{17}, -\mathbf{B}_{57}, -\mathbf{B}_{41}\}$



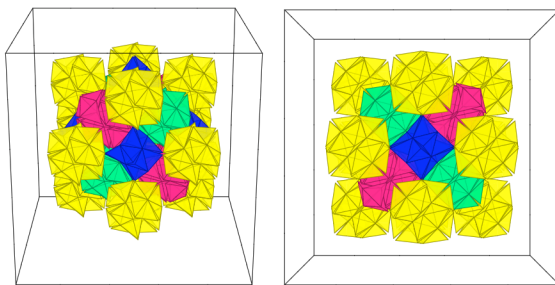
$\{-\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{17}, -\mathbf{B}_{57}\}$



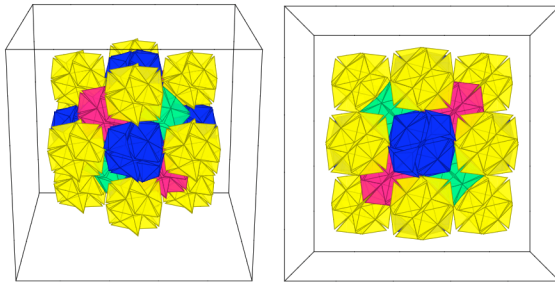
$\{+\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{B}_{41}, +\mathbf{B}_{41}\}$



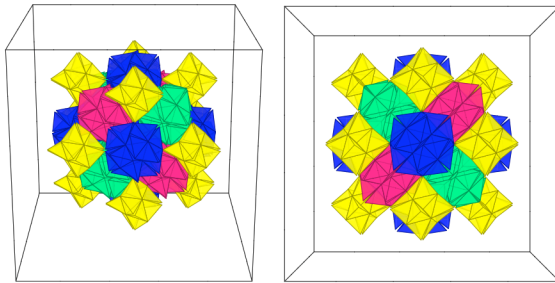
$\{+\mathbf{B}_{41}, +\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{B}_{41}\}$



$\{+\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{57}, -\mathbf{B}_{17}\}$



$\{-\mathbf{B}_{17}, +\mathbf{B}_{41}, +\mathbf{B}_{41}, +\mathbf{B}_{57}\}$



$\{+B_{41}, +V_{20}, +V_{20}, +B_{17}\}$

4 cosets

98 tetrahedra

$$U = \frac{784}{3}$$

261.333333333333

$$a = 3$$

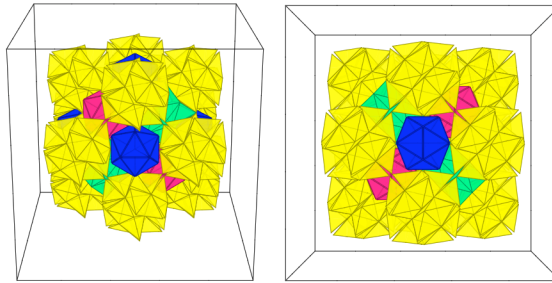
3.000000000000

$$V = 432$$

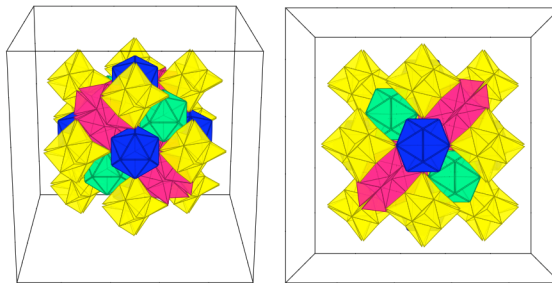
432.000000000000

$$D = \frac{49}{81}$$

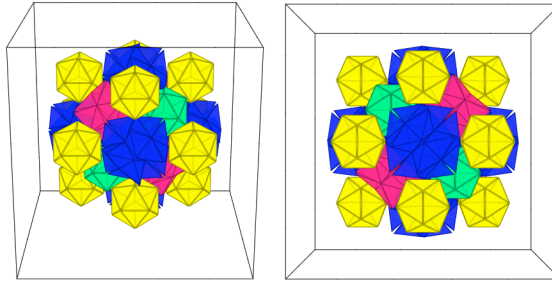
.604938271605



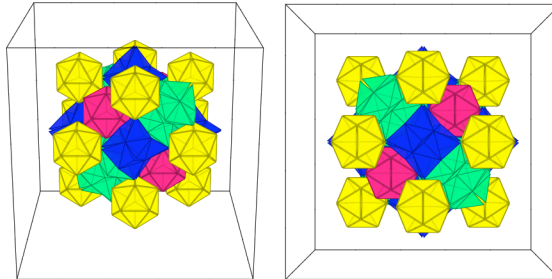
$\{+B_{17}, +B_{41}, +V_{20}, +V_{20}\}$



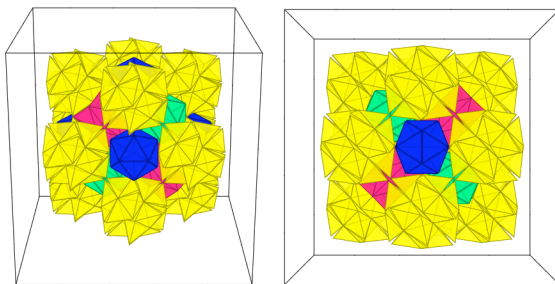
$\{+V_{20}, +B_{17}, +B_{41}, +V_{20}\}$



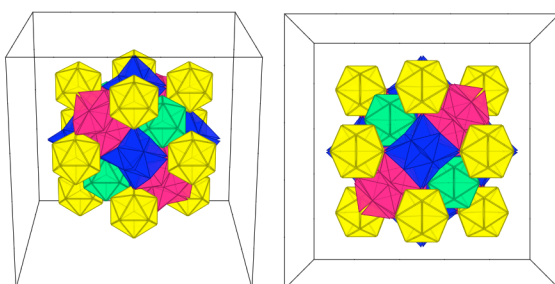
$\{+V_{20}, +V_{20}, +B_{17}, +B_{41}\}$



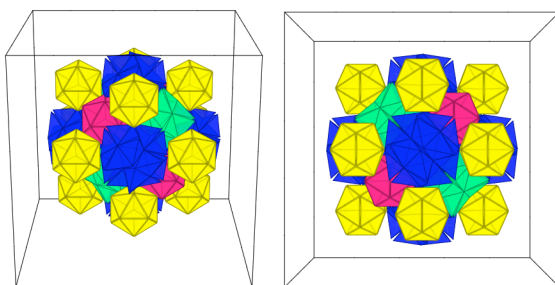
$\{-\mathbf{B}_{41}, -\mathbf{B}_{17}, +\mathbf{V}_{20}, +\mathbf{V}_{20}\}$



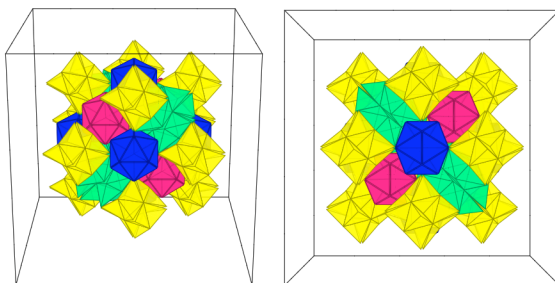
$\{+\mathbf{V}_{20}, -\mathbf{B}_{41}, -\mathbf{B}_{17}, +\mathbf{V}_{20}\}$



$\{+\mathbf{V}_{20}, +\mathbf{V}_{20}, -\mathbf{B}_{41}, -\mathbf{B}_{17}\}$



$\{-\mathbf{B}_{17}, +\mathbf{V}_{20}, +\mathbf{V}_{20}, -\mathbf{B}_{41}\}$



$$\{+B_{41}, +V_{20}, +V_{20}, +V_{20}\}$$

4 cosets

101 tetrahedra

$$U = \frac{808}{3}$$

269.333333333333

$$a = 3 + \frac{32}{331} \sqrt{2} - \frac{28}{993} \sqrt{10}$$

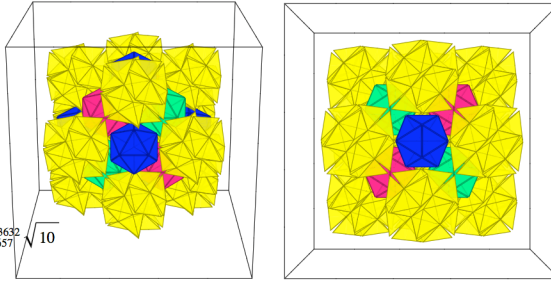
3.047553602722

$$V = \frac{47750704}{109561} \frac{172032}{109561} \sqrt{5} + \frac{4550873600}{108794073} \sqrt{2} - \frac{11955533632}{979146657} \sqrt{10}$$

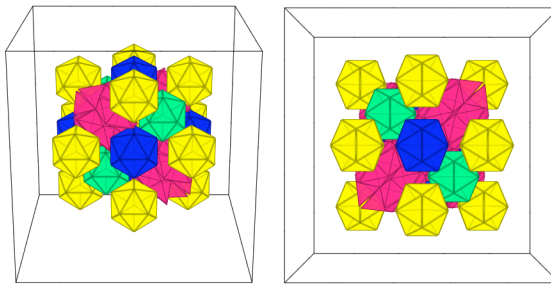
452.870510636688

$$D = \frac{463158693784537197520532733}{729747923656392828884526482} \frac{1678409195030769213347328}{364873961828196414442263241} \sqrt{5} - \frac{22343004837340359530263920}{364873961828196414442263241} \sqrt{2} + \frac{6568389768945839153419926}{364873961828196414442263241} \sqrt{10}$$

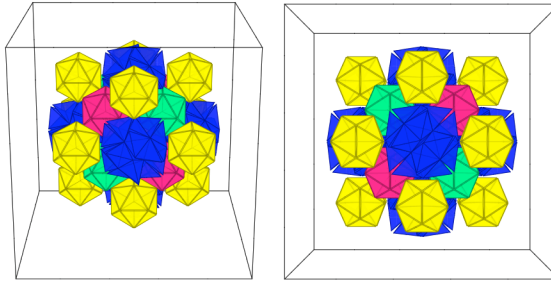
.594724820909



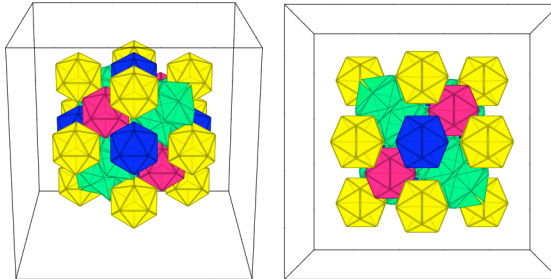
$$\{+V_{20}, +B_{41}, +V_{20}, +V_{20}\}$$



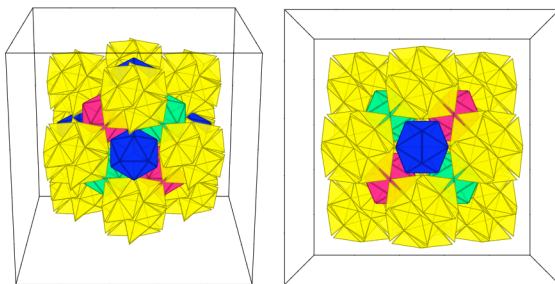
$$\{+V_{20}, +V_{20}, +B_{41}, +V_{20}\}$$



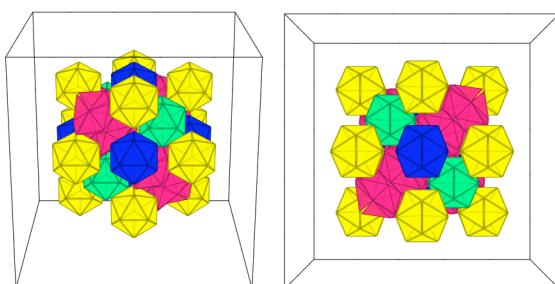
$$\{+V_{20}, +V_{20}, +V_{20}, +B_{41}\}$$



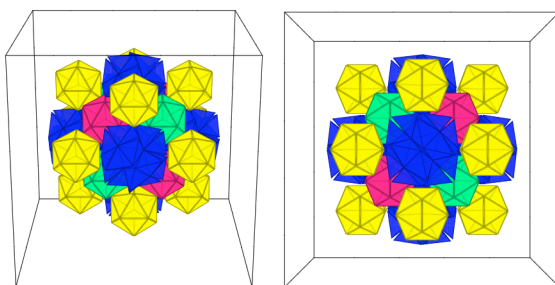
$\{-\mathbf{B}_{41}, +\mathbf{V}_{20}, +\mathbf{V}_{20}, +\mathbf{V}_{20}\}$



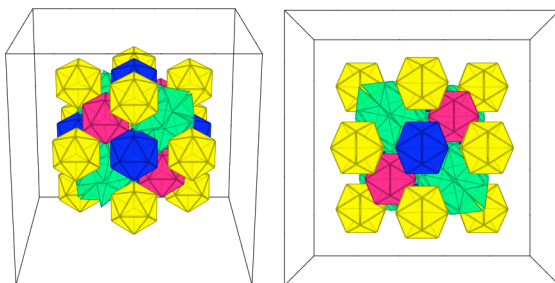
$\{+\mathbf{V}_{20}, -\mathbf{B}_{41}, +\mathbf{V}_{20}, +\mathbf{V}_{20}\}$



$\{+\mathbf{V}_{20}, +\mathbf{V}_{20}, -\mathbf{B}_{41}, +\mathbf{V}_{20}\}$



$\{+\mathbf{V}_{20}, +\mathbf{V}_{20}, +\mathbf{V}_{20}, -\mathbf{B}_{41}\}$



$$\{-\mathbf{B}_{57}, +\mathbf{B}_{41}, +\mathbf{B}_{57}, +\mathbf{V}_{20}\}$$

4 cosets

175 tetrahedra

$$U = \frac{1400}{3}$$

466.666666666667

$$a = \frac{11}{3}$$

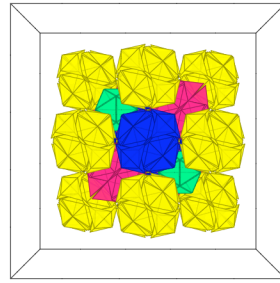
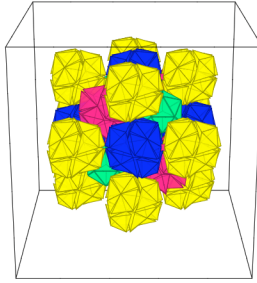
3.666666666667

$$V = \frac{21296}{27}$$

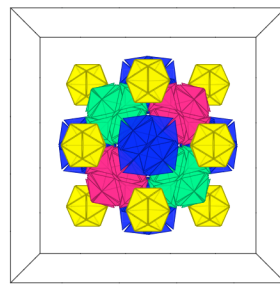
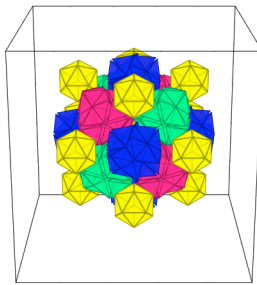
788.740740740741

$$D = \frac{1575}{2662}$$

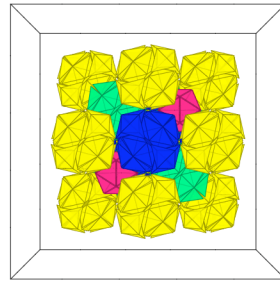
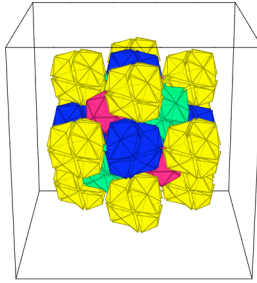
.591660405710



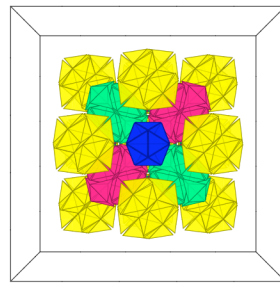
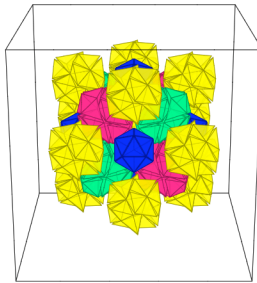
$$\{+\mathbf{V}_{20}, -\mathbf{B}_{57}, +\mathbf{B}_{41}, +\mathbf{B}_{57}\}$$



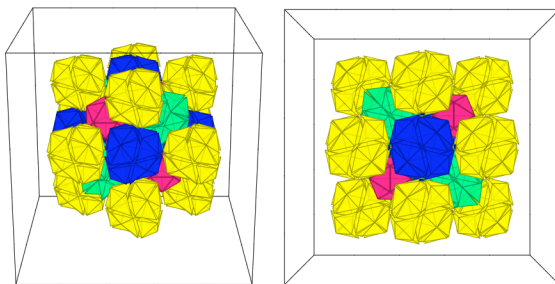
$$\{+\mathbf{B}_{57}, +\mathbf{V}_{20}, -\mathbf{B}_{57}, +\mathbf{B}_{41}\}$$



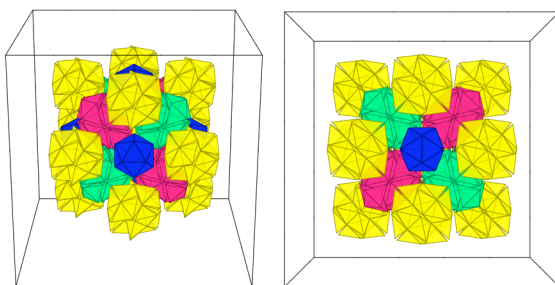
$$\{+\mathbf{B}_{41}, +\mathbf{B}_{57}, +\mathbf{V}_{20}, -\mathbf{B}_{57}\}$$



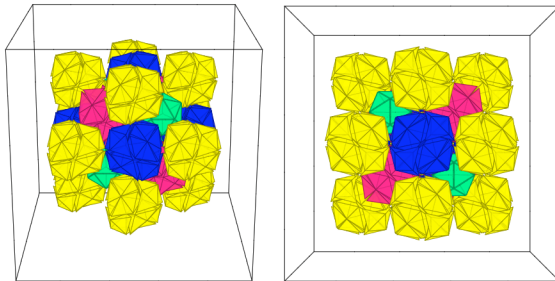
$$\{+\mathbf{B}_{57}, +\mathbf{V}_{20}, -\mathbf{B}_{57}, -\mathbf{B}_{41}\}$$



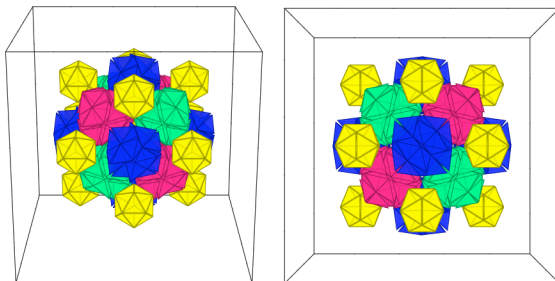
$$\{-\mathbf{B}_{41}, +\mathbf{B}_{57}, +\mathbf{V}_{20}, -\mathbf{B}_{57}\}$$



$$\{-\mathbf{B}_{57}, -\mathbf{B}_{41}, +\mathbf{B}_{57}, +\mathbf{V}_{20}\}$$



$$\{+\mathbf{V}_{20}, -\mathbf{B}_{57}, -\mathbf{B}_{41}, +\mathbf{B}_{57}\}$$



$$\{-\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{V}_{20}, -\mathbf{B}_1\}$$

4 cosets

95 tetrahedra

$$U = \frac{760}{3}$$

253.333333333333

$$a = \frac{7}{12} + \frac{13}{12}\sqrt{5}$$

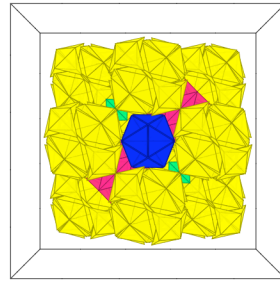
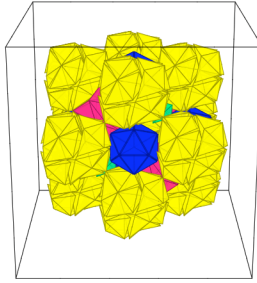
3.005740308958

$$V = \frac{4522}{27} + \frac{3224}{27}\sqrt{5}$$

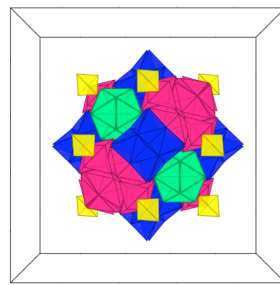
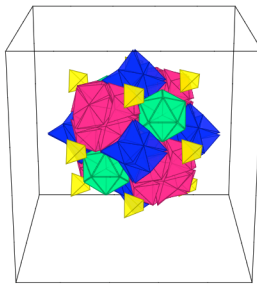
434.484561461456

$$D = \frac{-7732620 + 5513040\sqrt{5}}{7880599 + 7880599\sqrt{5}}$$

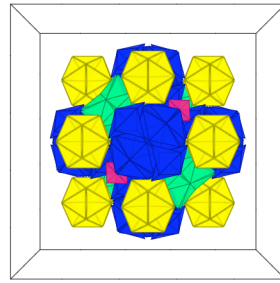
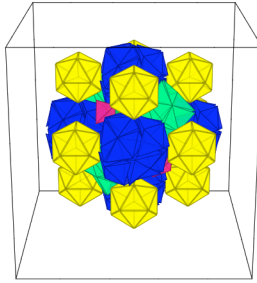
.583066363696



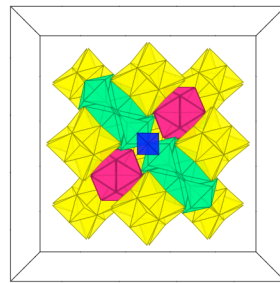
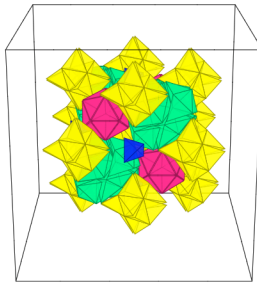
$$\{-\mathbf{B}_1, -\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{V}_{20}\}$$



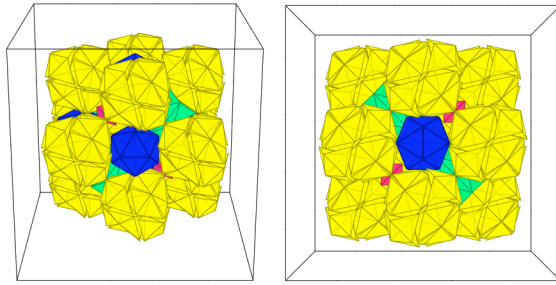
$$\{+\mathbf{V}_{20}, -\mathbf{B}_1, -\mathbf{B}_{57}, -\mathbf{B}_{17}\}$$



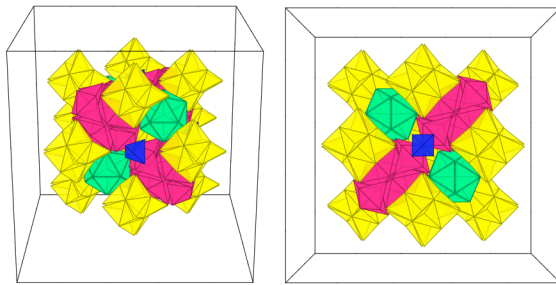
$$\{-\mathbf{B}_{17}, +\mathbf{V}_{20}, -\mathbf{B}_1, -\mathbf{B}_{57}\}$$



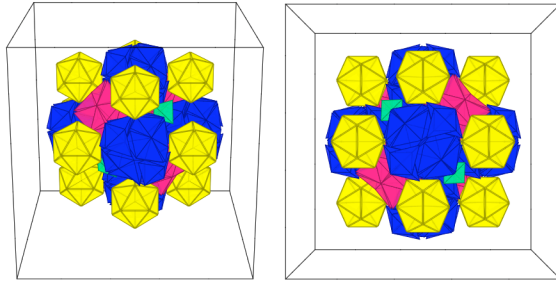
$\{+B_{57}, +B_1, +V_{20}, +B_{17}\}$



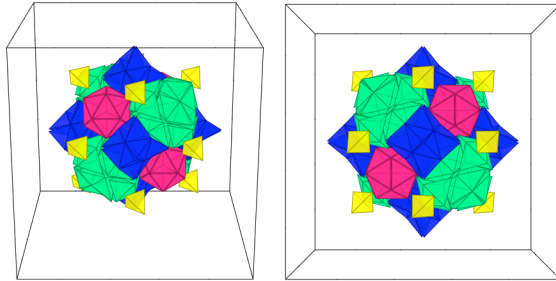
$\{+B_{17}, +B_{57}, +B_1, +V_{20}\}$



$\{+V_{20}, +B_{17}, +B_{57}, +B_1\}$



$\{+B_1, +V_{20}, +B_{17}, +B_{57}\}$



$\{+\mathbf{V}_{20},*,*,*\}$

1 coset
20 tetrahedra

$$U = \frac{160}{3}$$

53.333333333333

$$a = \frac{4}{5}\sqrt{5}$$

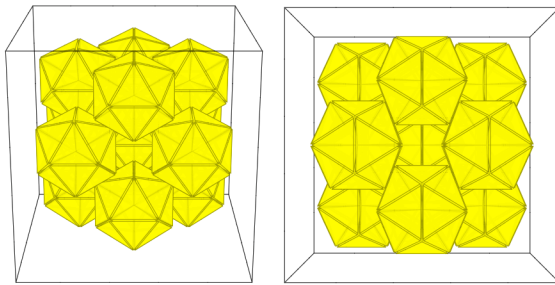
1.788854382000

$$V = \frac{1024}{25}\sqrt{5}$$

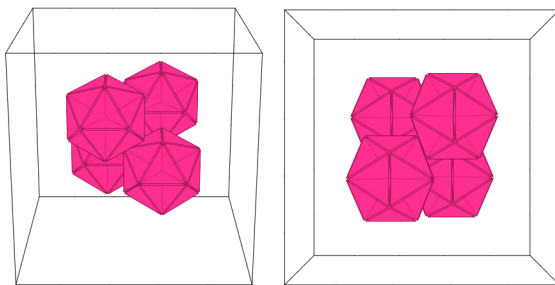
91.589344358391

$$D = \frac{25}{96}\sqrt{5}$$

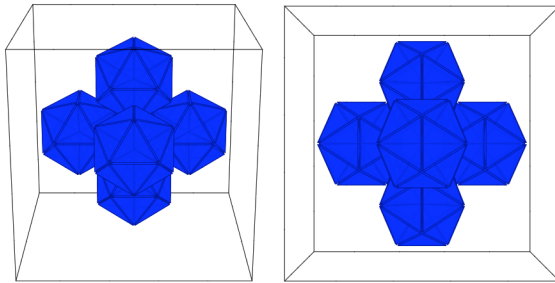
.582309369141



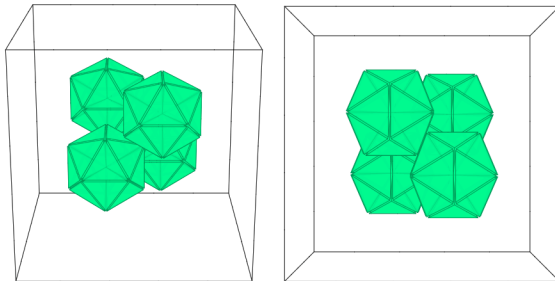
$\{*,+\mathbf{V}_{20},*,*\}$



$\{*,*,+\mathbf{V}_{20},*\}$



$\{*,*,*,+\mathbf{V}_{20}\}$



$$\{-\mathbf{B}_{57}, +\mathbf{V}_{20}, +\mathbf{V}_{20}, -\mathbf{B}_1\}$$

4 cosets

98 tetrahedra

$$U = \frac{784}{3}$$

261.333333333333

$$a = 3 + \frac{32}{331} \sqrt{2} - \frac{28}{993} \sqrt{10}$$

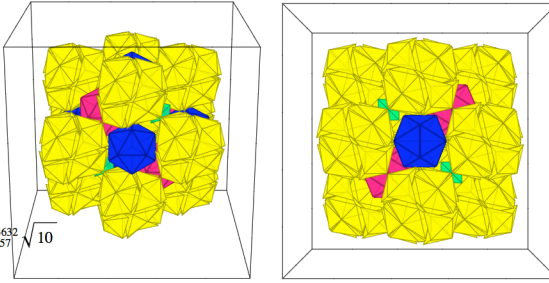
3.047553602722

$$V = \frac{47750704}{109561} - \frac{172032}{109561} \sqrt{5} + \frac{4550873600}{108794073} \sqrt{2} - \frac{11955533632}{979146657} \sqrt{10}$$

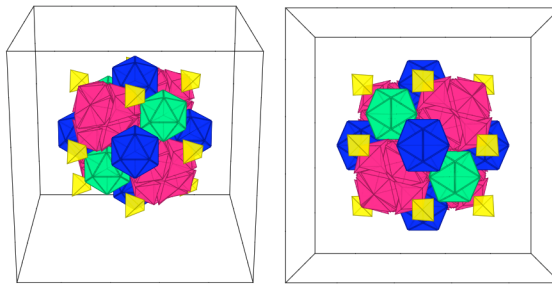
452.870510636688

$$D = \frac{224700752430122006717882217}{364873961828196414442263241} - \frac{1628555456564508741663744}{364873961828196414442263241} \sqrt{5} - \frac{21679351228310447861048160}{364873961828196414442263241} \sqrt{2} + \frac{6373289082739527099357948}{364873961828196414442263241} \sqrt{10}$$

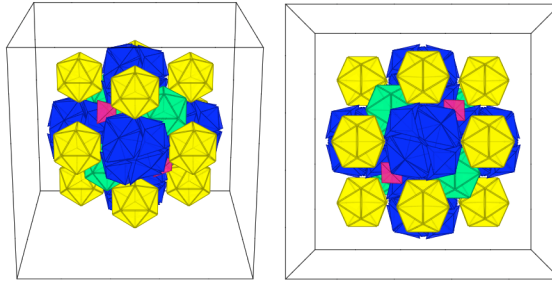
.577059727219



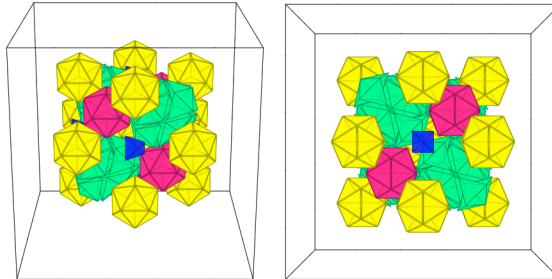
$$\{-\mathbf{B}_1, -\mathbf{B}_{57}, +\mathbf{V}_{20}, +\mathbf{V}_{20}\}$$



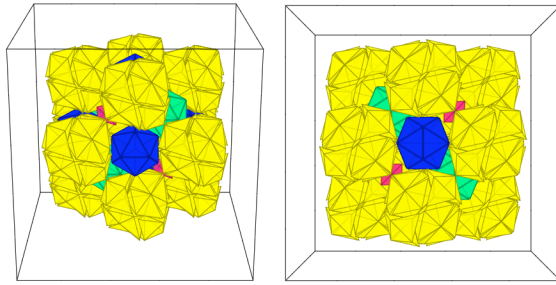
$$\{+\mathbf{V}_{20}, -\mathbf{B}_1, -\mathbf{B}_{57}, +\mathbf{V}_{20}\}$$



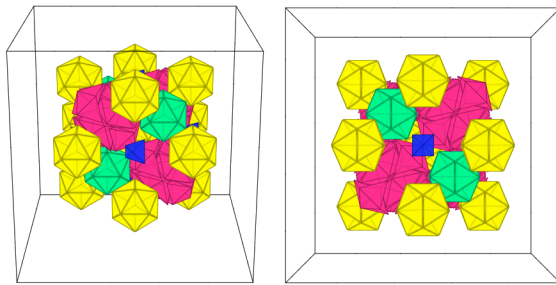
$$\{+\mathbf{V}_{20}, +\mathbf{V}_{20}, -\mathbf{B}_1, -\mathbf{B}_{57}\}$$



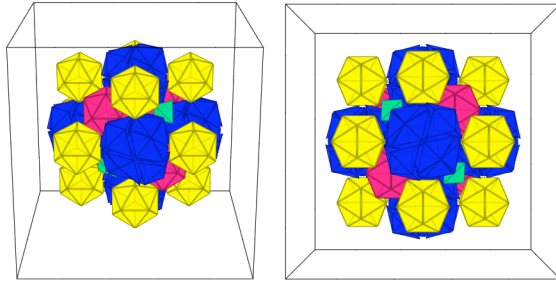
$\{+B_{57}, +B_1, +V_{20}, +V_{20}\}$



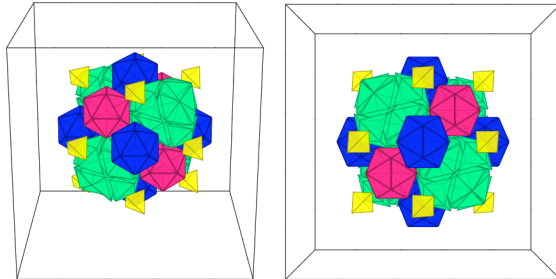
$\{+V_{20}, +B_{57}, +B_1, +V_{20}\}$



$\{+V_{20}, +V_{20}, +B_{57}, +B_1\}$



$\{+B_1, +V_{20}, +V_{20}, +B_{57}\}$



$\{-\mathbf{B}_{57}, +\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_5\}$

4 cosets

144 tetrahedra

$U = 384$

384.000000000000

$a = \frac{246}{71}$

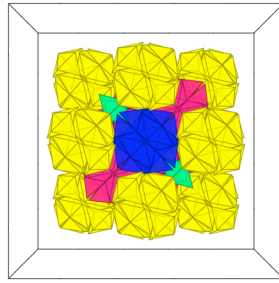
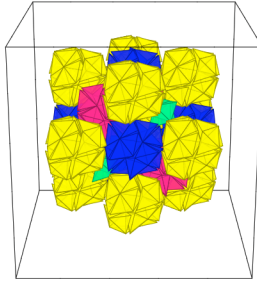
3.464788732394

$V = \frac{238190976}{357911}$

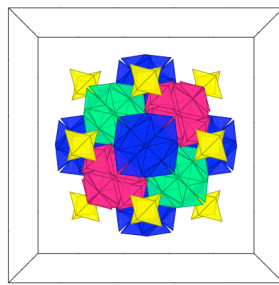
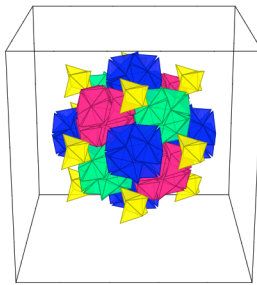
665.503368155771

$D = \frac{357911}{620289}$

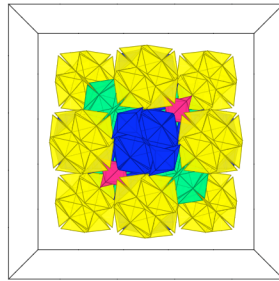
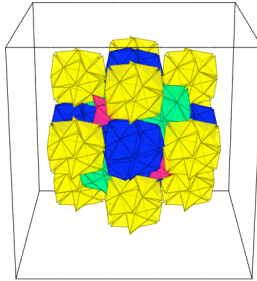
.577006846809



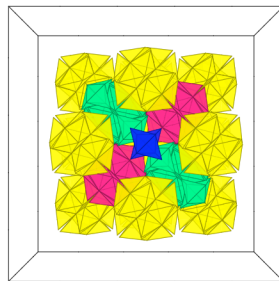
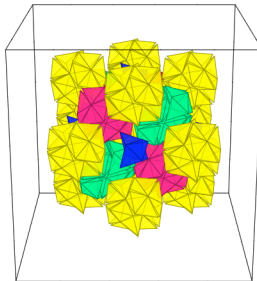
$\{+\mathbf{B}_5, -\mathbf{B}_{57}, +\mathbf{B}_{41}, -\mathbf{B}_{41}\}$



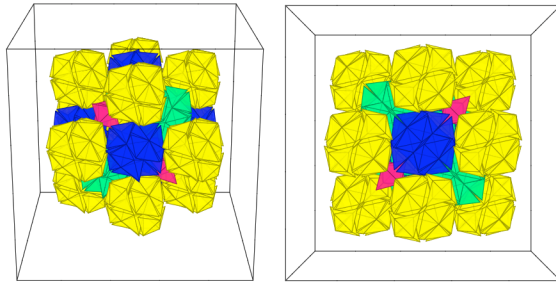
$\{-\mathbf{B}_{41}, +\mathbf{B}_5, -\mathbf{B}_{57}, +\mathbf{B}_{41}\}$



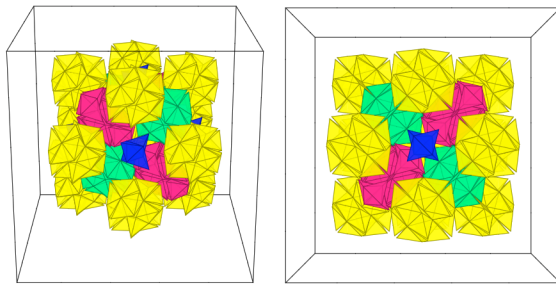
$\{+\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_5, -\mathbf{B}_{57}\}$



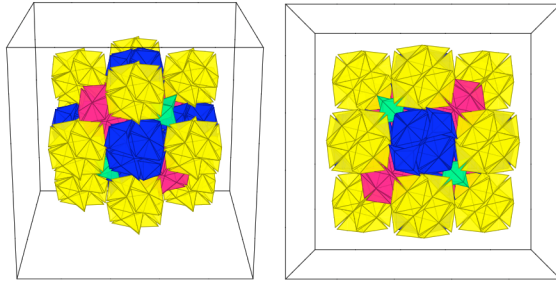
$\{+\mathbf{B}_{57}, -\mathbf{B}_5, +\mathbf{B}_{41}, -\mathbf{B}_{41}\}$



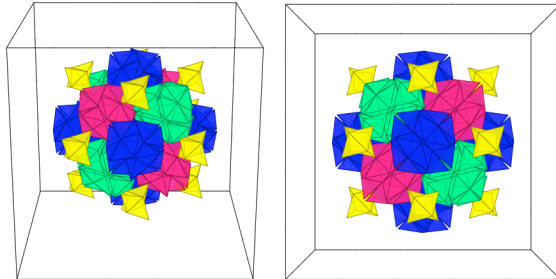
$\{-\mathbf{B}_{41}, +\mathbf{B}_{57}, -\mathbf{B}_5, +\mathbf{B}_{41}\}$



$\{+\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{57}, -\mathbf{B}_5\}$



$\{-\mathbf{B}_5, +\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{B}_{57}\}$



$$\{-\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{V}_{20}, *\}$$

4 cosets
94 tetrahedra

$$U = \frac{752}{3}$$

250.666666666667

$$a = \frac{7}{12} + \frac{13}{12}\sqrt{5}$$

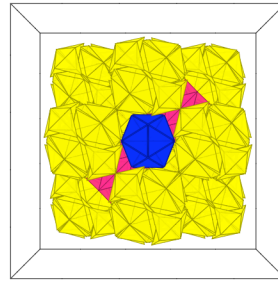
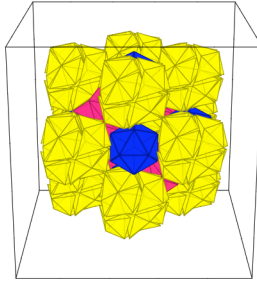
3.005740308958

$$V = \frac{4522}{27} + \frac{3224}{27}\sqrt{5}$$

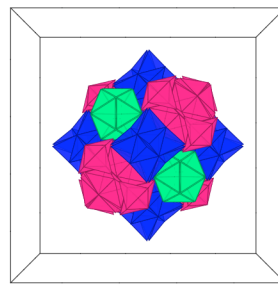
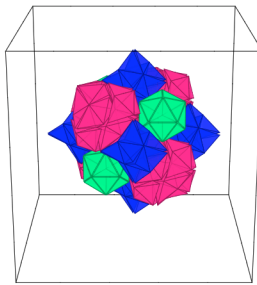
434.484561461456

$$D = \frac{-7651224}{7880599} + \frac{5455008}{7880599}\sqrt{5}$$

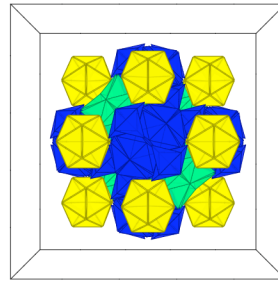
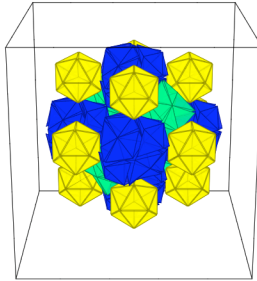
.576928823025



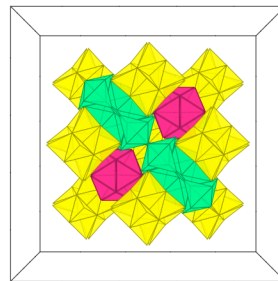
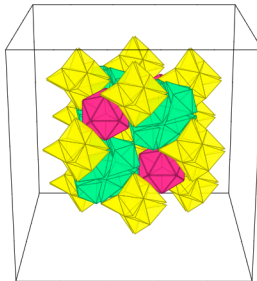
$$\{*, -\mathbf{B}_{57}, -\mathbf{B}_{17}, +\mathbf{V}_{20}\}$$



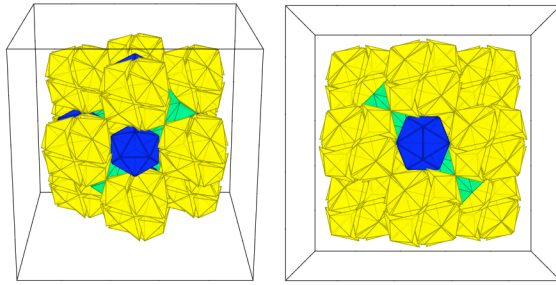
$$\{+\mathbf{V}_{20}, *, -\mathbf{B}_{57}, -\mathbf{B}_{17}\}$$



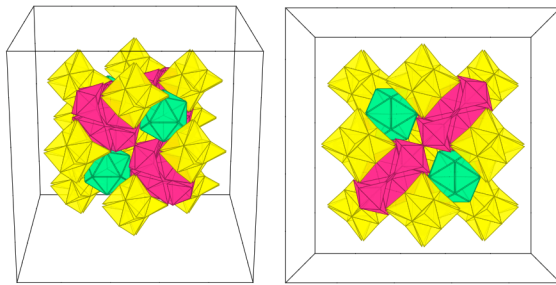
$$\{-\mathbf{B}_{17}, +\mathbf{V}_{20}, *, -\mathbf{B}_{57}\}$$



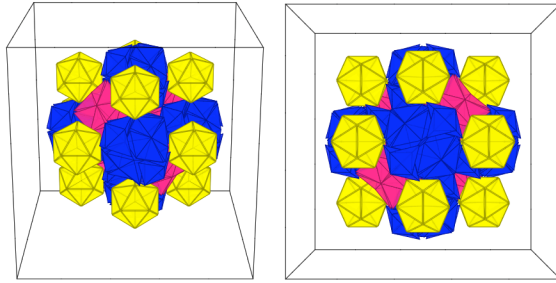
$\{+B_{57},*,+V_{20},+B_{17}\}$



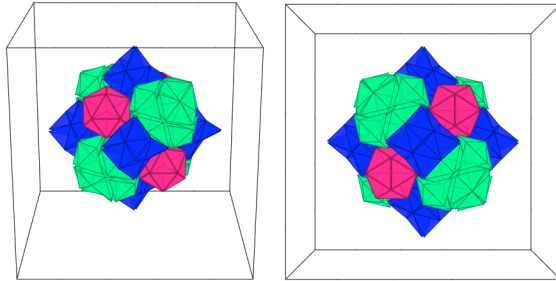
$\{+B_{17},+B_{57},*,+V_{20}\}$



$\{+V_{20},+B_{17},+B_{57},*\}$



$\{*,+V_{20},+B_{17},+B_{57}\}$



$\{+\mathbf{B}_{57}, +\mathbf{B}_{57}, +\mathbf{B}_{57}, +\mathbf{B}_{57}\}$

4 cosets

228 tetrahedra

$U = 608$

608.000000000000

$a = \frac{376}{93}$

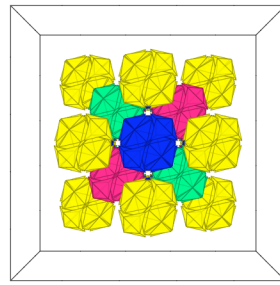
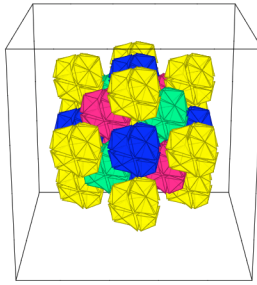
4.043010752688

$V = \frac{850518016}{804357}$

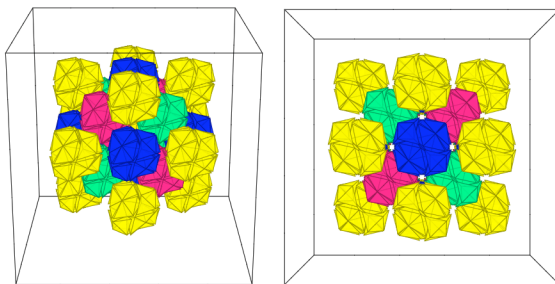
1057.388716701664

$D = \frac{15282783}{26578688}$

.575001407142



$(-\mathbf{B}_{57}, -\mathbf{B}_{57}, -\mathbf{B}_{57}, -\mathbf{B}_{57})$



$$\{+\mathbf{V}_{20},+\mathbf{B}_{41},-\mathbf{B}_{41},+\mathbf{B}_{41}\}$$

4 cosets

143 tetrahedra

$$U = \frac{1144}{3}$$

381.333333333333

$$a = \frac{246}{71}$$

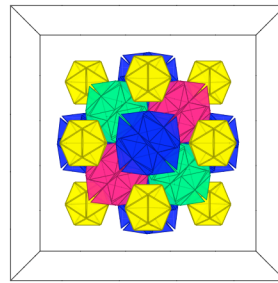
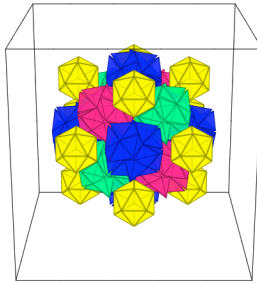
3.464788732394

$$V = \frac{238190976}{357911}$$

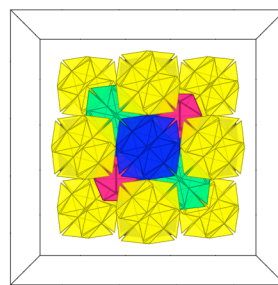
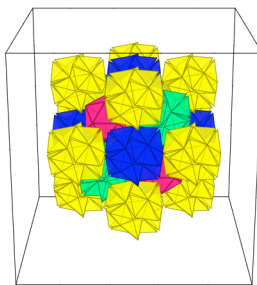
665.503368155771

$$D = \frac{51181273}{89321616}$$

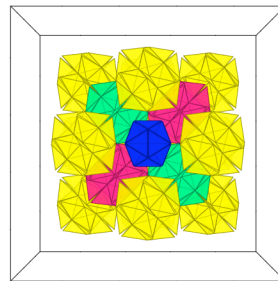
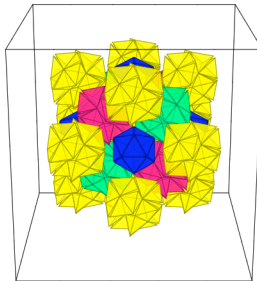
.572999854817



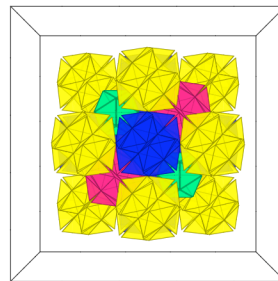
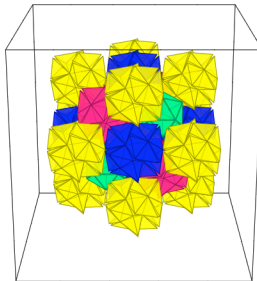
$$\{+\mathbf{B}_{41},+\mathbf{V}_{20},+\mathbf{B}_{41},-\mathbf{B}_{41}\}$$



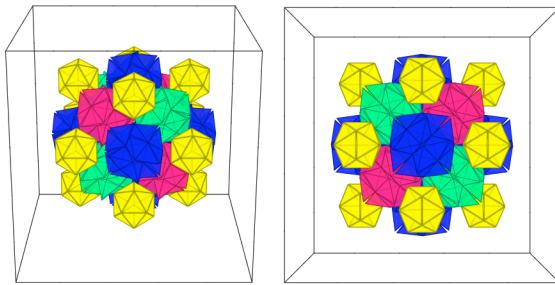
$$\{-\mathbf{B}_{41},+\mathbf{B}_{41},+\mathbf{V}_{20},+\mathbf{B}_{41}\}$$



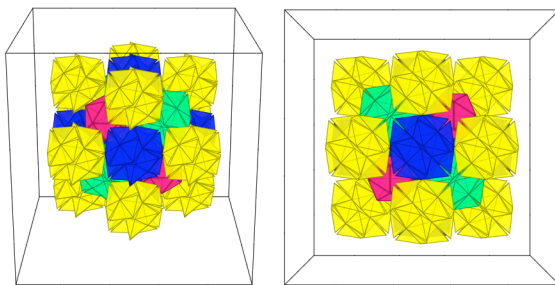
$$\{+\mathbf{B}_{41},-\mathbf{B}_{41},+\mathbf{B}_{41},+\mathbf{V}_{20}\}$$



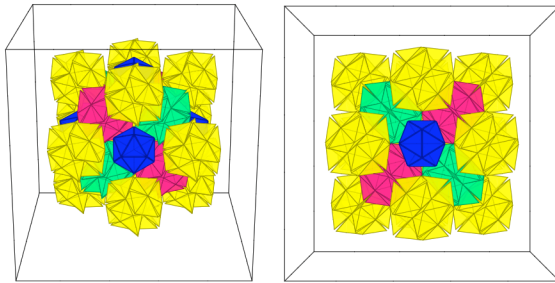
$$\{+\mathbf{V}_{20}, -\mathbf{B}_{41}, +\mathbf{B}_{41}, -\mathbf{B}_{41}\}$$



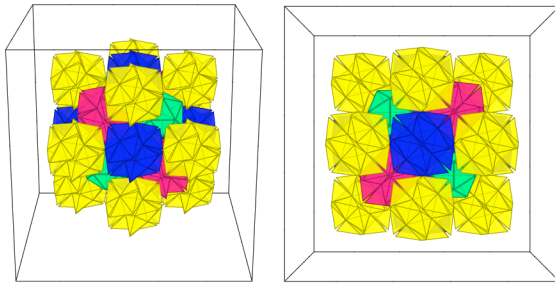
$$\{-\mathbf{B}_{41}, +\mathbf{V}_{20}, -\mathbf{B}_{41}, +\mathbf{B}_{41}\}$$



$$\{+\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{V}_{20}, -\mathbf{B}_{41}\}$$



$$\{-\mathbf{B}_{41}, +\mathbf{B}_{41}, -\mathbf{B}_{41}, +\mathbf{V}_{20}\}$$



$\{-\mathbf{B}_1, *, *, +\mathbf{B}_1\}$

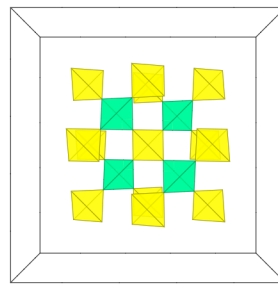
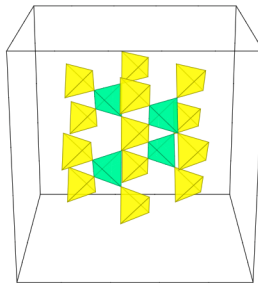
$U = \frac{16}{3}$

$a = 2$

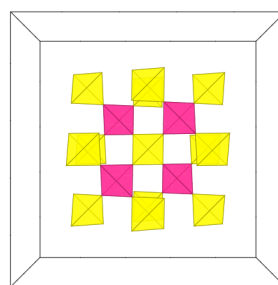
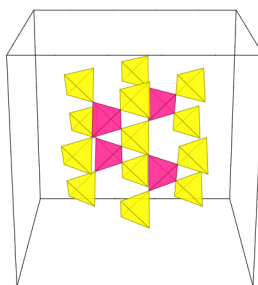
$V = 128$

$D = \frac{1}{24}$

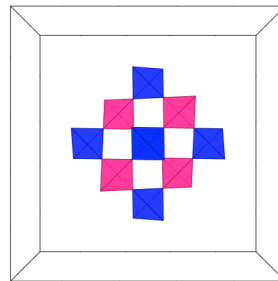
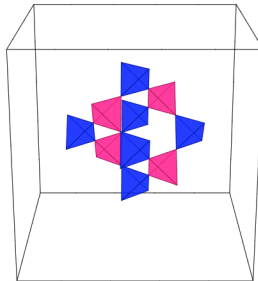
2 cosets
2 tetrahedra
5.33333333333333
2.00000000000000
128.000000000000
.041666666667



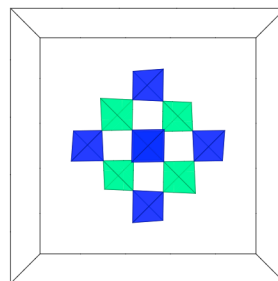
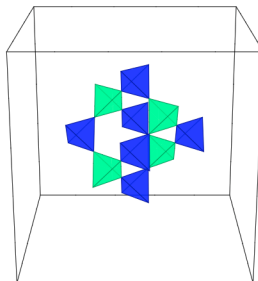
$\{+\mathbf{B}_1, -\mathbf{B}_1, *, *\}$



$\{*, +\mathbf{B}_1, -\mathbf{B}_1, *\}$



$\{*, *, +\mathbf{B}_1, -\mathbf{B}_1\}$



References

1. Ἀριστοτέλης: Περὶ οὐρανοῦ. Translation: Boethius: Aristotelēs. De Caelo. Translation: WKC Guthrie: Aristotle. On heavenly bodies. Leob Classical Library, vol. 338. Harvard University Press, vol. 6 (1986)
2. U Betke, M Henk: (FORTRAN computer program) lattice_packing.f (1999)
3. U Betke, M Henk: Densest lattice packings of 3-polytopes. *Comput. Geom.* **16**(3), 157-186 (2000)
4. ER Chen: A dense packing of regular tetrahedra. *Disc. Comp. Geom.* **40**, 214-240 (2008)
5. JH Conway, S Torquato: Packing, tiling, and covering with tetrahedra. *Proc. Natl. Acad. Sci. U.S.A.* **103**, 10612-10617 (2006)
6. H Grömer: Über die dichteste gitterförmige Lagerung kongruenter Tetraeder. *Mon. Math.* **66**, 12-15 (1962)
7. TC Hales: A proof of the Kepler conjecture. *Ann. Math.* **162**, 1065-1185 (2005)
8. TC Hales, SP Ferguson: Historical overview of the Kepler conjecture; A formulation of the Kepler conjecture; Sphere packings III: Extremal cases; Sphere packings IV: Detailed bounds; Sphere packings V: Pentahedral prisms; Sphere packings VI: Tame graphs and linear programs. *Discrete Comput. Geom.* **36**(1), 5-265 (2006)
9. DC Hilbert: Mathematische Probleme. *Nachr. Ges. Wiss. Gött., Math. Phys. Kl.* **3**, 253-297 (1900). Translation: Newson, M.W.: Mathematical problems. *Bull. Am. Math. Soc.* **8**, 437-479 (1902)
10. DJ Hoylman: The densest lattice packing of tetrahedra. *Bull. Am. Math. Soc.* **76**, 135-137 (1970)
11. AC Hurley: Some helical structures generated by reflexions. *Aust. J. Phys.* **38**(3), 299-310 (1985)
12. H Minkowski: Dichteste gitterförmige Lagerung kongruenter Körper. *Nachr. K. Ges. Wiss. Göttingen*, 311-355 (1904), in *Gesammelte Abhandlungen*. Vol. II, Teubner, Berlin, 3-42 (1911)
13. H Minkowski: *Geometrie der Zahlen*. Teubner, Leipzig (1896). Reprint: Minkowski, H.: *Geometrie der Zahlen*. Chelsea (1953)
14. M Senechal: Which tetrahedra fill space? *Math. Mag.* **54**(5), 227-243 (1981)
15. DJ Struik: De impletione loci. *Nieuw Arch. Wiskd.* **15**, 121-134 (1925)
16. S Torquato, Y Jiao: Dense packings of the Platonic and Archimedean solids. *Nature.* **460**, 876-879 (2009)