ENGINEERING RESEARCH INSTITUTE
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# MACH NUMBER AND YAW-ANGLE DETERMINATION FOR CONICAL-FLOW REGIMES USING TWO SURFACE-FLOW ANGLE INDICATORS

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This paper was delivered as part of the Programme of the Meeting of the Division of Fluid Dynamics American Physical Society on November 22, 1954, at the Hotel Chamberlin, Old Point Comfort, Fort Monroe, Virginia.

Project 2096

AIR FORCE CAMBRIDGE RESEARCH CENTER, U. S. AIR FORCE
CONTRACT NO. AF 19(604)-545

January, 1955

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## ABSTRACT

The research reported in this paper has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command.

In the search for principles applicable to measuring upper-air environments (i.e., temperature, density, wind vectors, etc.) from supersonic missiles in the regions above 30 kilometers altitude, it became evident that knowledge of the angle of surface flow on a right circular cone coupled with body-coordinate rotational parameters would permit calculation of the free-stream Mach number and the yaw angle. An instrumentation was developed for measuring the surface-flow angle utilizing a rectangular, metal plate, 3/4" x 1/8" x 0.003", driving a capacitive transducer. The electronic version of the instrument requires a minimum force of 60 dynes at the plate's center of pressure for accurate alignment with the stream. Correlation of the theory was provided in a series of wind-tunnel experiments with the subsequent realization that the vane angle as measured by an external optical system would provide a reliable Mach meter for wind-tunnel instrumentations, particularly low-density tunnels where conventional Pitot tubes become complicated by having to measure very low pressures. Such an optical transducer for the vane angle would require a lower actuating force than the electronic version of the instrument. The discussion will concern the governing relations, the solution of the resulting transcendental equations, experimental results, and the possible applications.

# INTRODUCTION

The use of supersonic missiles in upper-atmosphere research has increased the need for measurement techniques based on the properties of low-density supersonic flow. Fundamentally, the problem has been one of obtaining a measure of local conditions from a geophysical reference frame, i.e., atmospheric pressure, temperature, density, wind speed, and direction as a function of altitude. Instrumentations to date have been based on principles using densities or pressures existing on various geometries in a supersonic flow. These techniques have had to operate in an environment where the static pressure is decreasing at a very rapid rate (approximately one decade every ten seconds) beginning at approximately 100,000 feet with a static pressure of less than 5 mm hg at a static temperature of 240°K. Using conventional single-state rockets as available today, it is routine to reach a Mach 3 performance to altitudes where the limit of the gas continuum is exceeded.

This rapid pressure decrease presents the problem of degassing the measuring instruments and the missile without seriously contaminating the measurements. It also follows that there exist conventional problems of high-vacuum technique centering on making absolute calibration of pressure gages to pressure values less than 1 micron.

Other methods of approaching the solution to the problem have been tried utilizing knowledge of the shock-wave angle produced by miscellaneous geometries, the angle function being determined through use of pressure probes or of some flow-visualization technique. These shock-wave techniques have suffered poor results largely because of their use in low-density flow fields, while the pressure techniques are somewhat more successful. The use of pressure measurements to determine the direction of velocity has in the past been a conventional technique. However, it appears that velocity direction as a measure of the Mach number has not been considered seriously by other investigators. Such a technique should avoid many of the shortcomings of the pressure systems without introducing an equivalent number of its own aberrations.

A small, flat plate normal to the cone surface provides an element sensitive to the flow direction without seriously disturbing the flow regime.

There are some possible sources of practical difficulties arising from use of such a vane element for flow-direction indication leading to calculation of Mach number and yaw angles:

(1) The vane performs some sort of averaging process on the flow direction since its height above the cone surface exposes it to flow other than that precisely at the surface.

(2) Part or all of the vane is submerged in the boundary layer depending on its height.

(3) The vane as constructed is constrained in move in a plane tangent to the cone rather than parallel to the surface.

By a trial and error process, a vane design has been attained which seems to minimize the disadvantages so that the ensuing experimental data provides reasonable agreement with the theory.

## THEORETICAL

The surface-flow direction on a given right circular cone is dependent on the incident angle, the Mach number, and the body rotational position (Fig. 1). The figure combines in one drawing a side view of the cone and the vane equipment and also a top view with the vane rotated 90 degrees toward the viewer.

The notation in the figure has the conventional meaning: M is for the Mach number; $\epsilon$, for the incidence angle; $\psi$, for body rotation away from the plane of yaw; $\Theta_s$ is the apical half angle; $u_s$, the radial surface velocity; w, the tangential surface velocity; and $\beta$ is the surface-flow inclination.

The placement of the vane at a position eight tenths of the cone's axial length was determined experimentally as the position having the smallest deviation from the theories governing conical regimes. Figure 2 shows that the tangent of the flow angle can be represented as the ratio of the tangential surface velocity (w) to the radial surface velocity ($u_s$).

In terms of the second-order approximations in conical-flow theory, an expression involving the Mach number, incidence angle, and the body rotational position was approached. When this was written in functional form it could more easily be seen that the surface-flow angle reduced to dependence on the three parameters mentioned.

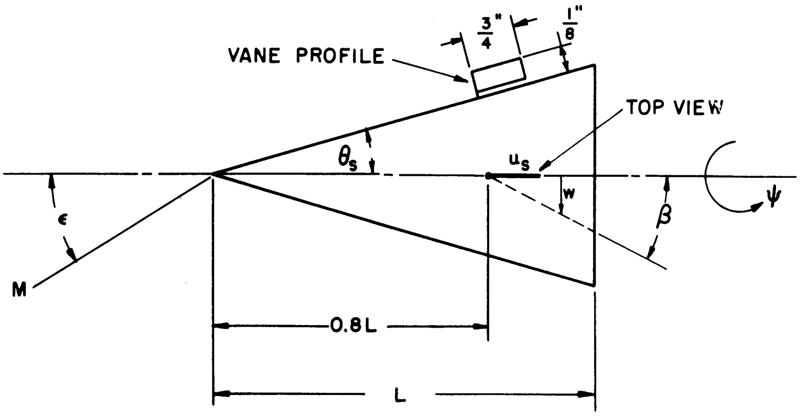

Fig. 1. Vane Physical Arrangement.

$$\beta = f\left(\epsilon, M, \phi(\epsilon,\psi),\ X_i(M,\theta_s)\right)$$

$$\mathrm{TAN}\ \beta = \frac{\epsilon Z\ \mathrm{SIN}\,\psi + \epsilon^2 (w_2 - u_s\ \mathrm{CSC}\,\theta_s - Z\ \mathrm{COT}\,\theta_s)\ \mathrm{SIN}\ 2\psi}{u_s + \epsilon\ X\ \mathrm{COS}\,\psi + \epsilon^2 (u_0 + \frac{1}{2}u_s + \frac{1}{2}X\ \mathrm{COT}\,\theta_s) + \epsilon^2 (u_2 + \frac{1}{2}u_s - \frac{1}{2}X\ \mathrm{COT}\,\theta_s)\ \mathrm{COS}\ 2\psi}$$

$\beta$ = SURFACE FLOW ANGLE

$\epsilon$ = YAW ANGLE

$\phi$ = WIND AXIS ROTATION COORDINATE

$\psi$ = BODY ROTATIONAL COORDINATE

$X_i$ = PERTURBATION COEFFICIENT

M = UPSTREAM MACH NUMBER

$\theta_s$ = CONE APICAL HALF ANGLE

Fig. 2. Functional Dependence of Surface-Flow Angle of Right Circular Cone.

Because of the transcendental nature of the governing expression, an explicit form for the Mach number was not derived. However, a graphical solution involving the Mach number and the flow angle was possible. This was seen in Fig. 3 where the flow angle is plotted versus the Mach number for families of rotational coordinates at a fixed angle of attack. For a cone apical half angle of 7.5 degrees and an angle of attack of 3 degrees the flow angle at a fixed rotational position should change approximately 1 degree for a Mach number change from 1.5 to 2.5. The maximum flow-angle change occurs in the vicinity of a 90-degree rotational position from the plane of yaw.

If the angle of attack is increased to 10 degrees then the flow-angle change becomes about 2 degrees for the same Mach number variation (see Fig. 4). It will be noted that the angle change as a function of Mach number diminishes as the $\psi$ values approach the plane of yaw. However, for wind-tunnel instrumentations the most desirable position for a vane instrumentation so that it could be read through an external optical system would be a $\psi$ value of 90 degrees where the maximum angle change occurs. The determination of the Mach number with such an instrumentation would require a right circular cone, the wind vane, and a cathetometer. The vane used by the authors was made from steel shim stock, 3/4" x 1/8" x 0.003"; it was mounted on a 1/32-inch-diameter shaft whose end face was slightly below the cone surface, so that only the metal vane was exposed to the flow. The space between the shaft and the hole through the cone was of the order of 0.002 inch (see Fig. 5).

Since the ultimate purpose of this instrumentation was for upper-air exploration, an electrical transducer was developed to permit remote metering of the experiments by means of a conventional telemeter system. The transducer (see Fig. 6) was designed with the thought of its use in flow fields where the restoring force is of the order of 50 dynes, i.e., equivalent to a static pressure of 10 microns. To achieve this sensitivity, jewel bearings were used for lateral support while the thrust constraint was obtained by holding the sharpened end of the vane shaft against a cap jewel through use of a small permanent magnet under the jeweled surface. The magnet prevented end play of the shaft without introducing large friction. Mounted perpendicular to the shaft was a small rectangular plate serving as one element of a variable capacitor. It was this variation in capacity that measured the flow angle. The overall experimental error of the system provided results with errors less than 0.03 degrees, which would permit Mach number determinations to be better than 0.02 units.

The wind-tunnel model shown in Fig. 7 was made with a motor drive so that the model could be rotated about its longitudinal axis reversing the direction of travel in 186 degrees and having rotational velocity of less than 0.5 foot per second. Since the tests were conducted in an intermittent-flow

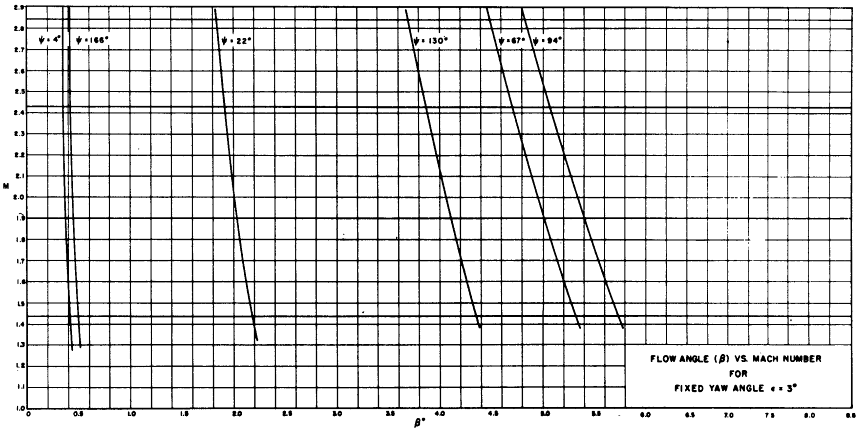
ψ = 4°
ψ = 166°
ψ = 22°
ψ = 130°
ψ = 67°
ψ = 94°
M
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
β°
FLOW ANGLE (β) VS. MACH NUMBER
FOR
FIXED YAW ANGLE ε = 3°

Fig. 3

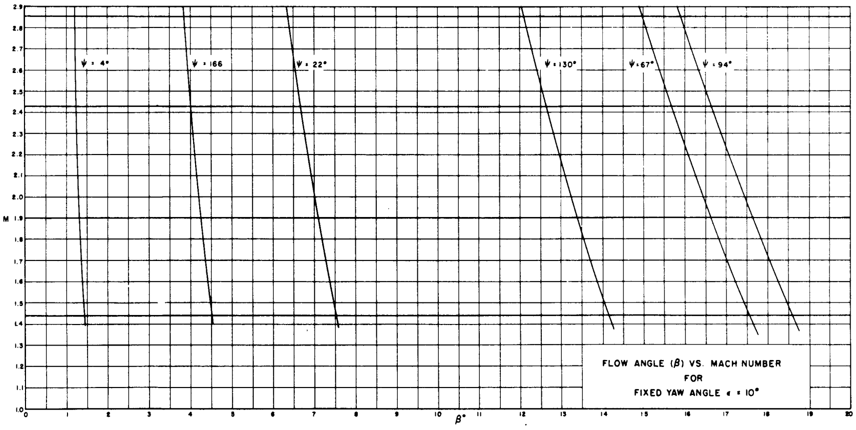
ψ = 4°
ψ = 166
ψ = 22°
ψ = 130°
ψ = 67°
ψ = 94°
M
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
β°
FLOW ANGLE (β) VS. MACH NUMBER
FOR
FIXED YAW ANGLE ε = 10°

Fig. 4

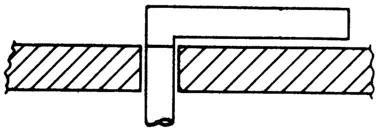

Fig. 5. Vane and Cone Surface Relationship.

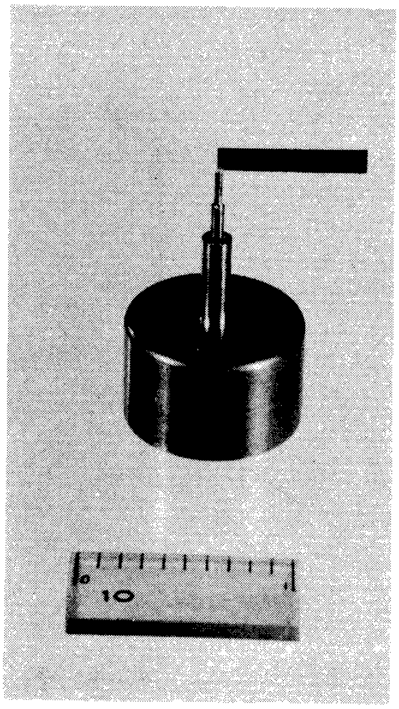

Fig. 6. Angle Transducer and Associate Flag.

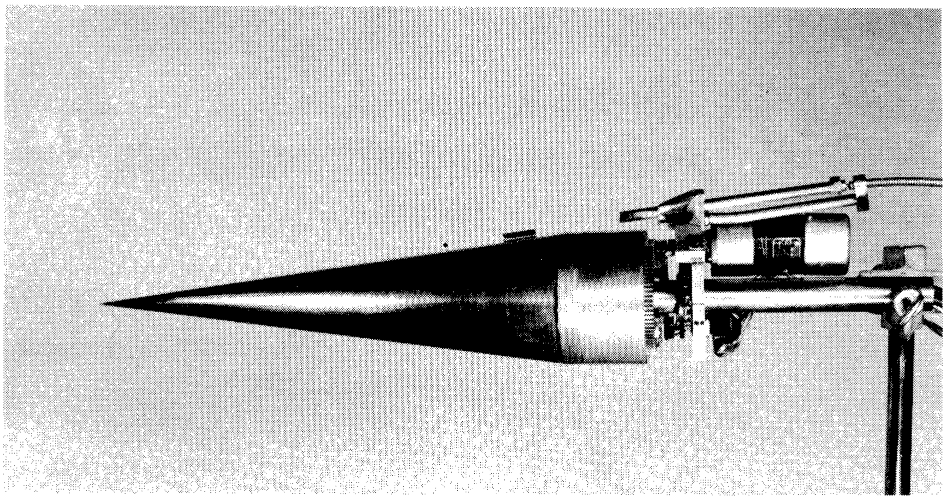

Fig. 7. Rotating Wind-Tunnel Model with Cover Retracted Exposing Flag.

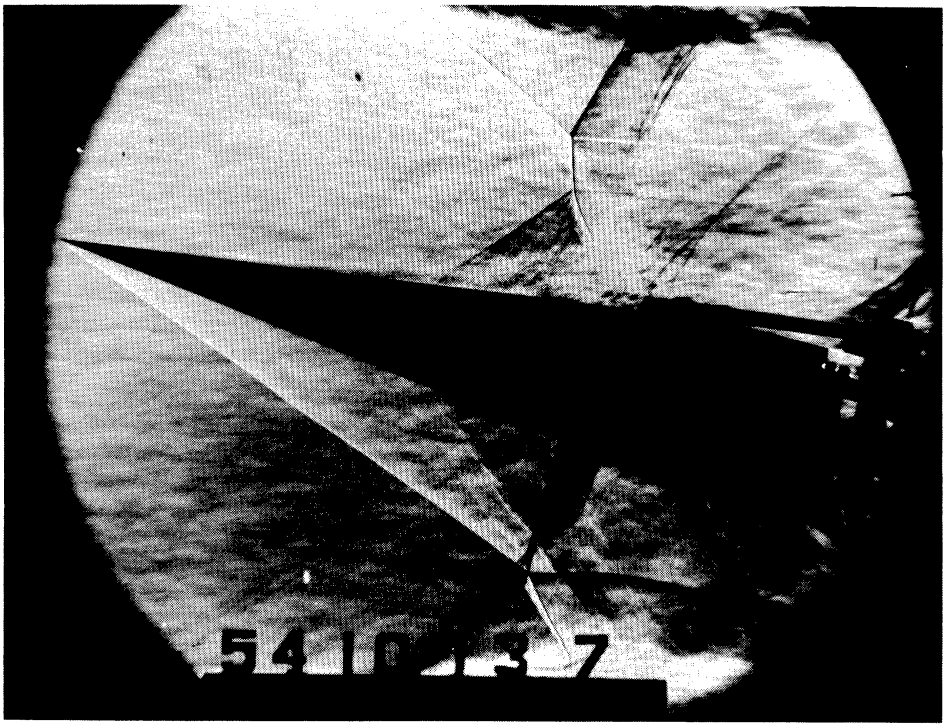

Fig. 8. Starting Shock Waves.

tunnel it was necessary to protect the metal vane during the starting and stopping of the flow. The starting shocks and flow reversals in the tunnel would cause the vane to spin very rapidly and oscillate as a plane surface which would destroy the vane. A covering arrangement was made so that the flag was protected during run starts by extending a slotted, metal rod from behind the model over the vane until the flow stabilized at which time the cover was retracted. Then the model would rotate through the desired range, returning to its starting position in time to be covered once again for protection from the flow breakup shocks. An example of these starting shocks is shown in Fig. 8, taken at Mach 1.90 and an incidence angle of 15 degrees. At the top of the model the cover in its extended position was seen protecting the vane.

In Fig. 9 the flow established just prior to retracting the cover and beginning the model rotation was seen. This happened to be at another incidence angle but the same Mach number. Finally the cover was retracted exposing the vane at the top of the model in Fig. 10. This is also in the plane of yaw so that use could be made of the fact that the flag is at zero flow angle in order that the cover could be advanced over the flag while the stream held it in position.

## EXPERIMENTAL DATA

The experimental data were taken in the University of Michigan supersonic wind tunnel, which is an intermittent vacuum type, its operating potential resulting from the pressure differential between air stored at atmospheric pressure and the low pressure of an evacuated tank. Through use of the rotating model described, each run taken at a given Mach number and angle of incidence provided forty-four points between $\psi$ equals zero and $\psi$ equals 186 degrees. The recording was done on a Miller oscillograph. The incidence angle was determined optically from schlieren photographs and observations through a theodolite in order to correct for the effects of aerodynamic loading. A correction was also made in the indicated flow angle to account for the effect due to gravitational attraction on the vane.

It is of interest to note that in the case where the wind vane is to be used on an upper-air missile it is not necessary to correct for the gravity effects since the missile is in a free-fall condition during the sampling period. The experimental data agreed extremely well with the theory for angles of incidence up to approximately the apical half angle of the cone. Figure 11 shows the results at Mach 1.94 for different incident angles. The theoretical and experimental results are shown as one line. The vertical

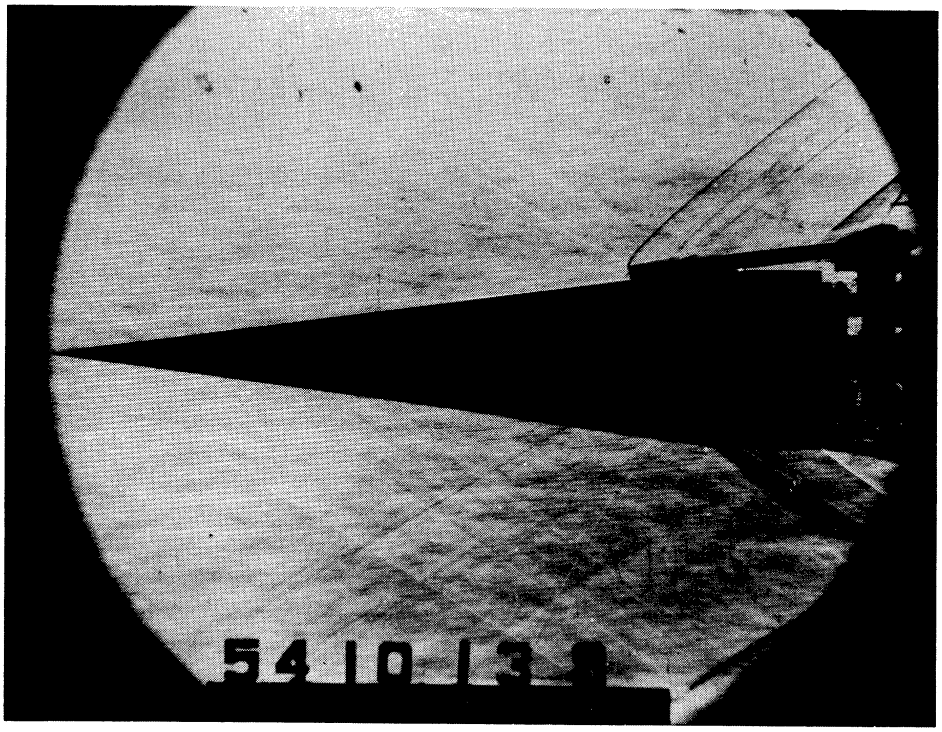

Fig. 9. Established Flow Prior to Flag Exposure.

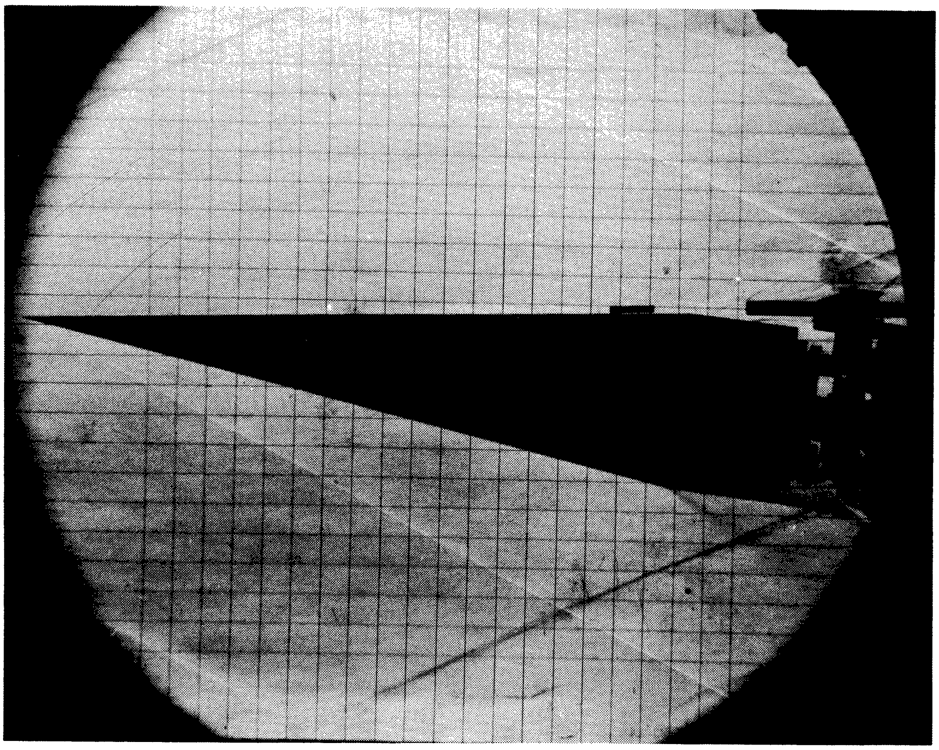

Fig. 10. Schlieren Photo of Model with Flag Exposed.

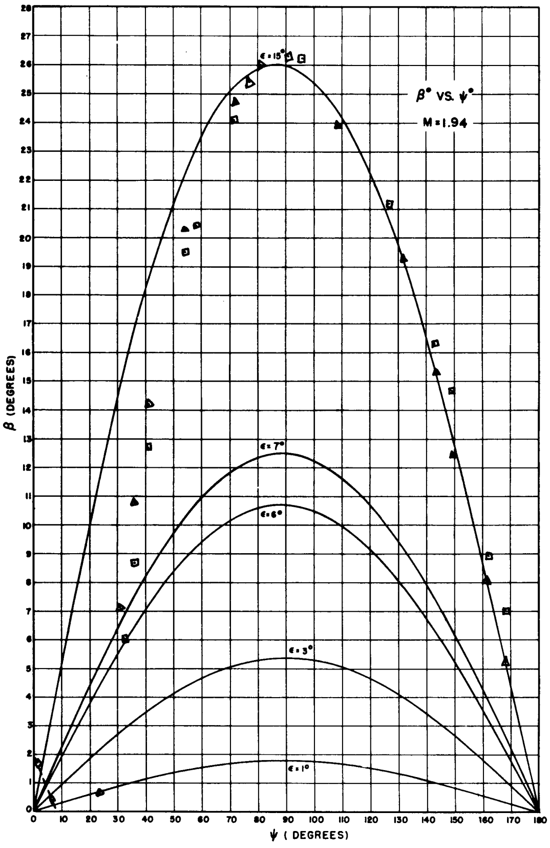
β° VS. ψ°
M = 1.94
ε = 15°
ε = 7°
ε = 6°
ε = 3°
ε = 1°
β (DEGREES)
ψ (DEGREES)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180

Fig. 11

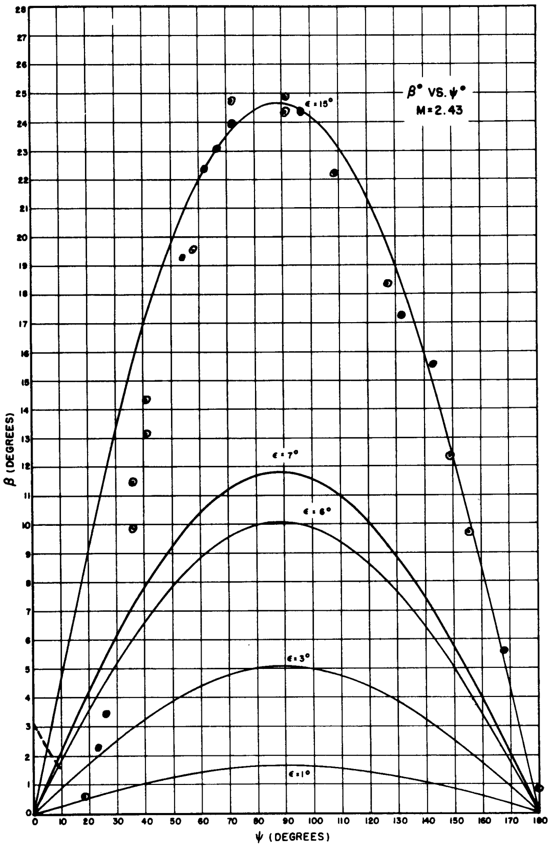
β° VS. ψ°
M = 2.43
ε = 15°
ε = 7°
ε = 6°
ε = 3°
ε = 1°
β (DEGREES)
ψ (DEGREES)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180

Fig. 12

coordinate is the flow angle and the horizontal coordinate is the rotational position. The results at epsilon 15 degrees do not show good agreement until approximately 70 degrees for $\psi$. The R deviation is probably due to flow separation on the top surface of the cone. The behavior is such that at $\psi$ equals zero the flow angle starts from 3 degrees and decreases as $\psi$ increases, instead starting from zero degrees and increasing thereafter. The inflection takes place at about 18 degrees. Similarly, in Fig. 12 the data for mach 2.43 agrees well when the incident angle is of the order of the cone half angle and again shows a similar divergence from the theoretical path at the high-incident angles.

## CONCLUSION

Solution for Mach numbers, yaw angles, or body rotational positions can be made graphically when two unknowns are sought or analytically when one is desired in the case of upper-air missile application. The range of variables is known so that the data gathering can be made on incremental basis. For example, the flow-angle data is rounded to the nearest 0.03 degree, the yaw angle to 0.3 degree, and so on. This allows the use of punched cards, such as the McBee keysort variety, for the reduction of data. Each card represents a solution of the flow field in terms of the four variables. Thus, the card system will permit the rapid solution of problems in terms of one or two unknowns. The expense of constructing such a card system would not be justified, except in the case of upper-air research where instrumentations are used repeatedly and data reduction must be carried out with nonscientific personnel. The cards can be searched for a solution at a rate of approximately 40,000 cards an hour.

The authors' interests have been directed toward the use of this instrument in upper-air sampling. However, it does appear that with some additional refinements the wind-vane principle could prove useful in low-density wind-tunnel instrumentations.