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Multidisciplinary Design Optimization of Elastomeric Mounting Systems in Automotive Vehicles

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Abstract

In this paper, a design optimization problem with multidisciplinary objectives is considered for a general purpose elastomeric mounting system (EMS). The multidisciplinary design objectives include quasi-static, dynamic and stability targets. Elastic stability of the EMS is investigated for the first time with the development of a general formulation that determines the critical buckling force and buckling mode of the system. Optimization is then performed to maximize the critical buckling force. The major contributions of this work include a systematic approach for the multidisciplinary design optimization of EMS and the stability analysis and optimization. The approach developed in this paper can be applied to a wide range of EMS design problems including body mounting system and powertrain mounting system. Reliability assessment of the optimum design is also conducted in order to consider uncertainties of the system parameters due to the manufacturing and assembling variations. Design optimization of a real vessel-mounting system in an innovative concept vehicle is used as an example to demonstrate the feasibility of the approach developed.

Keywords:

Multidisciplinary Design Optimization, Elastomeric Mounting System, Structural Optimization, Stability Analysis, Buckling, NVH, Durability, Crashworthiness.

Biographical Notes:

Dr. Zheng-Dong Ma is an *associate research scientist* in the Department of Mechanical Engineering at the University of Michigan. His research field is *computational mechanics* including structural optimization, flexible multibody systems, structural dynamics, and structural acoustics. He has published more than ninety technical papers and received five best paper awards from ASME, SAE, JSME, JSIAM, and JAAC, respectively.

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1. Introduction

Elastomeric mounting system (EMS) has extensive applications to control noise, vibration and harshness (NVH) in aerospace, automotive, marine, and other related fields. EMS can be made compact, they are cost-effective and easy to maintain. Therefore, EMS has been used to isolate vehicle structure from engine vibration since the 1930s (Lord, 1930). Extensive efforts have been made since then to improve the performance of the elastomeric mounts (Browne and Taylor, 1939, Coleman and Alstadt, 1959, Miller, Ahmadian, 1992). Another typical EMS in automotive vehicles is cabin-frame (as well as bed-frame) mounting system, which is used to isolate cabin (and bed) from the vibration of vehicle frame and to reduce the noise level so as to improve riding quality.

An EMS generally consists of three or more mounts. The behavior of the EMS not only depends on the performance of individual mounts but also on the complete system configuration. The design of an EMS involves the selection of materials for desired mechanical properties and determination of the locations and orientations of the individual mounts. The development of EMS has mostly concentrated on improvement of quasi-static (amplitude-dependent) and dynamics (frequency-dependent) properties.

The traditional "trial-and-error" methods in EMS design are highly dependent upon the engineer's experience and the allowable flexibility in modifying the system. Extensive experiments and analyses are required to meet the design criteria even in one aspect of the system performance, which turns out to be very time consuming. When multidisciplinary system performance objectives are considered, it becomes much more difficult to find a suitable design. A computerized automated design method such as optimization with reliable modeling techniques is highly desirable. Significant work has been done in this area since the late 1970s, but little work was found which consider multidisciplinary design objectives.

A simplest EMS model consists of a rigid body with a number of elastomeric mounts that support the rigid body. The rigid body can represent, for example, a powertrain unit, a cabin or a bed in a vehicle, which has six degrees of freedom (DOF). The rigid body can translate and rotate about the three-independent Cartesian axes. The mounts are usually modeled as springs and dampers with viscous elastic or viscoelastic properties in each of the three principal directions.

Different objectives of optimization have been considered in the literature. One objective of the optimization is to tune the natural frequencies of the mounting system to a desired range to avoid resonance and to improve the isolation of vibration and shock (Johnson and Subhaedar, 1979, Arai, Bernard and Starkey, 1983, Geck and Patton, 1984, Spiekermann, Radeliffe and Goodman, 1985, Kubozuka and Gray, 1993). Swanson, Wu and Ashrafiuno (1993) also showed that the transmitted forces through the mounts can be directly minimized in order to obtain a truly optimum design of the mounting system. Ashrafiuon (1993) further used these criteria to minimize the dynamic forces transmitted from the engine to the body. Other studies in the literature also used these two objectives (Wise and Reid, 1984, Suresh, Shankar and Bokil, 1993, Lee, Yin and Kim, 1995).

No work has been found in the literature related to the stability analysis of the general purpose EMS. In this paper, we derived an eigenvalue problem base on a second-order approximation of the original nonlinear dynamic equation of the EMS for the stability analysis. The eigenvalue problem can be solved to determine the buckling load and related buckling mode of the system. The stability related objective is first time introduced to the design optimization of EMS, which can be used to improve system behavior due to the nonlinear bifurcation. In addition to the stability objective, other design objectives, including quasi-static, dynamic, and durability targets, are also considered. Optimization with the multidisciplinary objectives leads to a much practical and reliable design in all aspects of the EMS.

In the practical EMS design, uncertainties of the system parameters have to be considered. For example, manufacturing variation will induce uncertainties in the stiffness of the individual mounts, and assembling errors may cause uncertainties in the locations and orientations of these mounts. In order to estimate the reliability and robustness of the optimum design, a reliability assessment is essential. Approximation techniques developed to assess a component or system reliability can be broadly categorized into two groups: a) random sampling methods, and b) analytical methods. The selection of methods depends on the problem involved. In this paper, a prevalent method- Monte Carlo simulation is used to assess the reliability of the optimum design.

This paper is organized as follows. In Section 2, the basic system equations for the quasi-static, frequency response, eigenvalue, and stability of the general purpose EMS are derived; the equations for the stability analysis of general EMS are first time considered. In Section 3, optimal design problems of the EMS are considered by defining different design objectives and assessing the reliability of the optimum design. As an example, in Section 4, design optimization of a real EMS in an innovative concept vehicle is considered with multidisciplinary objectives followed by the reliability assessment of the optimum design. Section 5 concludes the paper and further outlines possible future research.

2. Basic Equations of a General Elastomeric Mounting System

2.1 Quasi-static, frequency response and eigenvalue analyses

The EMS considered in this paper is modeled as a rigid body, which is supported by a number of elastomeric mounts. It is assumed that all mounts are seated on a rigid base. Note that this assumption can be easily extended to consider a flexible base. As shown in Fig. 1, the origin of the global coordinate system is at the center of gravity (C.G.) of the rigid-body, while X and Y-axes are parallel to the base, Z is normal to the base. The rigid body consists of six independent degrees of freedom (DOF), which include three translational and three rotational coordinates.

Assume that $\{\mathbf{r_c}\} = \{x_c, y_c, z_c\}^T$ is the translational displacement vector of the C.G. of the rigid body, and $\{\mathbf{\Theta}\} = \{\theta_x, \theta_y, \theta_z\}^T$ is the linear angle vector that represents a small rotation of the rigid body about its C.G., where $\theta_x, \theta_{\mathrm{y}}, \theta_{\mathrm{z}}$ are components of the rotation with respect to three axes of the global coordinate system, then a complete set of independent generalized coordinates for the EMS can be defined as

$$\{\mathbf{q}\} = \{\mathbf{r}_{\mathrm{c}}^T, \quad \mathbf{\Theta}^T\}^T \tag{1}$$

Under the assumption of "small" motion, the EMS equation can be linearized about its initial configuration and thus written as

$$[\mathbf{M}]\{\ddot{\mathbf{q}}\}+[\mathbf{C}]\{\dot{\mathbf{q}}\}+[\mathbf{K}]\{\mathbf{q}\}=\{\mathbf{p}\} \tag{2}$$

where $[\mathbf{M}]$ denotes the inertia matrix; $[\mathbf{C}]$ denotes the damping matrix; $[\mathbf{K}]$ denotes the stiffness matrix, and $\{\mathbf{p}\}$ is the force vector (including the torques) applied at the body C.G..

The stiffness and damping matrices are contributed from each mount, and in general we have

$$[\mathbf{K}]=\sum_{i=1}^{N}[\mathrm{K_i}], \quad [\mathbf{C}]=\sum_{i=1}^{N}[\mathbf{C_i}] \tag{3}$$

where N is the total number of the mounts that support the rigid body, and

$$[\mathbf{K}_\mathrm{i}]=\begin{bmatrix}[\mathbf{k}_\mathrm{i}] & -[\mathbf{k}_\mathrm{i}][\tilde{\mathbf{r}}_\mathrm{i}] \\ -[\tilde{\mathbf{r}}_\mathrm{i}]^T[\mathbf{k}_\mathrm{i}] & [\tilde{\mathbf{r}}_\mathrm{i}]^T[\mathbf{k}_\mathrm{i}][\tilde{\mathbf{r}}_\mathrm{i}]\end{bmatrix} \tag{4}$$

is the stiffness matrix from the i-th mount, here $[\tilde{\mathbf{r}}_\mathrm{i}]$ is a skew matrix of the position vector $\{\mathbf{r_i}\}$, while $\{\mathbf{r_i}\}=\{x_i, y_i, z_i\}^T$ is the position vector of the i-th mount, and we have

$$[\tilde{\mathbf{r}}_\mathbf{i}]=\begin{bmatrix}0 & -z_i & y_i \\ z_i & 0 & -x_i \\ -y_i & x_i & 0\end{bmatrix} \tag{5}$$

where x_i, y_i, z_i are the coordinates of the i-th mount measured at the body-fixed coordinate system (as shown in Fig. 1); $[\mathbf{k_i}]$ is the stiffness matrix of mount i measured in the global coordinate system.

Assuming a (linear) viscous elastic mount, $[\mathbf{k_i}]$ can be expressed as

$$[\mathbf{k}_\mathrm{i}]=[\mathbf{A}_\mathrm{i}][\mathbf{k}'_\mathrm{i}][\mathbf{A}_\mathrm{i}]^T \tag{6}$$

where $[\mathbf{k}'_\mathrm{i}]$ is the stiffness matrix of the i-th mount measured in the mount local coordinate system, and $[\mathbf{A}_\mathrm{i}]$ is the transposition matrix, which can be defined, for example, using Euler angles.

Assuming a viscous damping matrix for the i-th mount, namely,

$$[\mathbf{c}'_\mathrm{i}]=\eta_i[\mathbf{k}'_\mathrm{i}] \tag{7}$$

where η_i is the loss factor of the i-th mount, then the viscous damping matrix that contributes to the global damping matrix $[\mathbf{C}]$ in Eq. (3), of mount i, can be obtained as

$$[\mathbf{C}_i] = \eta_i [\mathbf{K}_i] \tag{8}$$

Based on Eq. (2), for a frequency response problem, we have

$$\left([\mathbf{K}] + j\omega[\mathbf{C}] - \omega^2[\mathbf{M}]\right)\{\mathbf{q}\} = \{\mathbf{p}\} \tag{9}$$

where $\{\mathbf{q}\}$ and $\{\mathbf{p}\}$ are the amplitudes of the body C.G. displacement and force vectors, respectively, ω is the excitation frequency.

For the quasi-static analysis, assuming

$$\{\mathbf{p}\} = -[\mathbf{M}]\{\mathbf{a}\} \tag{10}$$

is the inertia force applied on the body, where $\{\mathbf{a}\}$ is a given acceleration vector of the rigid body, then we have

$$[\mathbf{K}]\{\mathbf{q}\} = \{\mathbf{p}\} \tag{11}$$

Finally, for the modal analysis, we have

$$\left([\mathbf{K}] - \lambda_n[\mathbf{M}]\right)\boldsymbol{\varphi}_n = 0 \tag{12}$$

where λ_n donates the n-th eigenvalue of the EMS, and $\boldsymbol{\varphi}_n$ is the corresponding eigenvector.

Note that the displacement at each mount due to the rigid body motion $\{\mathbf{q}\}$ can be obtained as

$$\{\mathbf{U_i}\} = \{\mathbf{r_c}\} + [\tilde{\mathbf{r}}_\mathbf{i}]^T\{\boldsymbol{\Theta}\} \tag{13}$$

The force transmitted to the base through the i-th mount can be then obtained as

$$\{\mathbf{F_i}\} = -[\mathbf{k_i}]\{\mathbf{U_i}\} \tag{14}$$

2.2 Stability analysis

Consider a perturbation on the rigid body from its equilibrium position that results in a small displacement $\{\mathbf{q}\} = \{\mathbf{r}_c^T, \boldsymbol{\Theta}^T\}^T$. The potential energy due to the perturbation can be written as

$$V = \{\mathbf{r}_c\}^T\{\mathbf{F}\} + \{\boldsymbol{\Theta}\}^T\{\boldsymbol{\tau}\} \tag{15}$$

where $\{\mathbf{F}\} = \{F_x, F_y, F_z\}^T$ denotes the external force vector applied on the C.G., $\{\boldsymbol{\tau}\} = [\tilde{\mathbf{r}}_c]\{\mathbf{F}\}$ denotes the torque vector resulted from the perturbation of the C.G. and the force applied, and $[\tilde{\mathbf{r}}_c]$ is the skew matrix of the vector $\{\mathbf{r}_c\}$. Note that $\{\boldsymbol{\tau}\}$ defined in this paper is a higher-order non-linear effect, which is considered in here as it has a major contribution to the stability condition. Equation (15) can be then rewritten as

$$V = \{\mathbf{q}\}^T\{\mathbf{p}\} + \frac{1}{2}\lambda_b\{\mathbf{q}\}^T[\mathbf{K}_G]\{\mathbf{q}\} \tag{16}$$

where $\{\mathbf{p}\} = \{\mathbf{F}^T, \quad 0\}^T$, $\lambda_b = |\mathbf{F}|$ denotes the amplitude of $\{\mathbf{F}\}$, and $[\mathbf{K}_G]$ is so-called the geometry stiffness matrix,

$$[\mathbf{K}_G] = \begin{bmatrix} 0 & \mathbf{B} \\ \mathbf{B}^T & 0 \end{bmatrix} \tag{17}$$

where,

$$[\mathbf{B}] = \frac{1}{2}\begin{bmatrix} 0 & -\gamma & \beta \\ \gamma & 0 & -\alpha \\ -\beta & \alpha & 0 \end{bmatrix} \tag{18}$$

where α, β, γ are direction cosines of the force vector $\{\mathbf{F}\}$ measured at the global coordinate system.

The internal energy stored in the EMS due to the perturbation can be written as

$$U = \frac{1}{2}\{\mathbf{q}\}^T[\mathbf{K}]\{\mathbf{q}\} \tag{19}$$

The total energy stored in the EMS due to the perturbation then becomes

$$\Pi = U - V \tag{20}$$

The stability condition of the mechanical system requires the Hessian matrix of Π to be positive, which results in a critical condition:

$$\det\left[\frac{\partial^2 \Pi}{\partial q_i \partial q_j}\right] = 0 \tag{21}$$

or a corresponding eigenvalue problem

$$([\mathbf{K}] - \lambda_b[\mathbf{K}_G])\{\boldsymbol{\varphi}_b\} = 0 \tag{22}$$

where λ_b denotes the critical buckling force, and $\{\boldsymbol{\varphi}_b\}$ is the corresponding buckling mode.

3. Optimal Design Process

3.1 Optimization problems of EMS

Assuming $\{\mathbf{x}\} = \{x_1, x_2, \ldots, x_n\}^T$ stands for a vector of the design variables, an optimization problem of the general mounting system can be written as:

Find $\{\mathbf{x}\}$ such that

$$\begin{aligned} &\underset{\mathbf{x}}{\text{Minimize}} \quad f(\mathbf{x}) \text{ (or } -f(\mathbf{x})) \\ &\text{Subject to } \; h_j(\mathbf{x}) \le 0, \; (j = 1, 2, \ldots, m) \\ &\underline{x}_i \le x_i \le \overline{x}_i, \; (i = 1, 2, \ldots, n) \end{aligned} \tag{23}$$

where x_i denotes a design variable, which can be a location, orientation, stiffness or damping variable of an individual mount, $\underline{x}_i$ and $\overline{x}_i$ are the lower and upper bounds of x_i, ($i = 1, 2, \ldots, n$); $f(\mathbf{x})$ denotes the objective function, and $h_j(\mathbf{x})$ ($j = 1, 2, \ldots, m$) are the constraint functions. $f(\mathbf{x})$ can be defined as one of, or a combination of, the below:

1. Displacement and rotation of the body C.G., i.e

$$f_1 = (1-\alpha)\sqrt{\{\mathbf{r_c}\}^T \{\mathbf{r_c}\}} + \alpha\sqrt{\{\mathbf{\Theta}\}^T \{\mathbf{\Theta}\}} \tag{24}$$

where $\alpha \; (0 \le \alpha \le 1)$ is a given weighting parameter.

2. Mean eigenvalue of the system (Ma, Kikuchi and Cheng, 1995)

$$f_2 = \lambda_0 + \alpha\left(\sum_{i=1}^{m} \frac{w_i}{(\lambda_{n_i} - \lambda_{0_i})^n}\right)^{-1/n} \tag{25}$$

where $\lambda_{n_i} \; (i = 1,2,\ldots,m)$ are the eigenvalues to be optimized (defined in Eq. (12)), $n = 1, 2, -2$, or other, is a parameter used to define different design problems, $w_i \; (i = 1,2,\ldots,m)$ are given weighting coefficients, $\lambda_{0_i} \; (i = 1,2,\ldots,m)$ are given target eigenvalues, λ_0 and α are constants which are used only for adjusting the dimension of the objective function.

3. Critical buckling force of the system

$$f_3 = \lambda_b \tag{26}$$

where λ_b is defined in Eq. (22).

4. Maximum mounting force

$$f_4 = \max\{F_i, i = 1,2,\ldots,N\} \tag{27}$$

where $F_i = \sqrt{\{\mathbf{F_i}\}^T\{\mathbf{F_i}\}}$ is the magnitude of the transmitted force $\{\mathbf{F_i}\}$ at the i-th mount.

3.2 Multidisciplinary objectives reduction approach

In a multidisciplinary design optimization problem, it is critical to study the relationships (trading-offs) of all different objectives and to handle them in one single design process. In this paper, we introduce a general approach, which can be used to reduce the total number of the multidisciplinary design objectives in a practical structural optimization problem without debasing the optimality of the final design, so as to simplify the design problem. The approach can be explained as following: Firstly, a series of single objective optimization (SOO) are conducted for all individual objectives in the design problem. Secondly, the resultant SOO designs are evaluated for all the other multidisciplinary objectives. Based on the evaluation results, we can divide the objectives into different groups. In the same group, the objectives are consistent with each other, while in different groups, objectives are conflict. Finally, we can choose from each group a representative design objective. By considering the representative objective, the other objectives in the same group can be then reduced, which results in a much simpler design problem. This objective reduction approach is based on the natural characteristics of an engineering structure in responding to different physical processes, therefore, it can be generalized for the same class of structures. We will further demonstrate effectiveness of this approach through an example of real engineering design problem.

3.3 Reliability assessment for optimal design

For reliability analyses of the EMS, the probabilistic performance measure can be defined as

$$G(\mathbf{X}) = \frac{d_{\max}}{d} - 1 \quad \text{or} \quad G(\mathbf{X}) = \frac{d}{d_{\min}} - 1 \tag{28}$$

where $\{\mathbf{X}\}$ is a random vector representing the uncertainties of the design parameters, $d_{\max}$ ($d_{\min}$) is the maximum (minimum) value of the design target, d is the actual value of the design target. Here, a failure event is defined as $G(\mathbf{X}) \le 0$, and the probability of failure p_f is defined as

$$p_f = P\{G(\mathbf{X}) \le 0\} \tag{29}$$

which is generally calculated by the integral

$$p_f = \int \cdots \int_{G(\mathbf{X}) \le 0} f_{\mathbf{X}}(\mathbf{X}) \mathbf{dX} \tag{30}$$

where $f_{\mathbf{X}}(\mathbf{X})$ is the probability density function (PDF) of $\{\mathbf{X}\}$ and the probability is evaluated by the multidimensional integration over the failure region $G(\mathbf{X})$.

The reliability R is the probability that the EMS works properly, and it is given by

$$R = P\{G(\mathbf{X}) > 0\} = 1 - p_f \tag{31}$$

It is very difficult or even impossible to analytically compute the multidimensional integration in Eq. (30). Some approximation methods, such as the first order reliability method (FORM) (Madsen, Krenk and Lind, 1986) or the asymptotic second-order reliability method (SORM) (Hasofer and Lind, 1974) with a rotationally invariant reliability measure, have been developed to provide efficient solutions, while maintaining a reasonable level of accuracy. In this paper, instead, we simply use Monte Carlo simulation to investigate the robustness of the optimal design. The reason for this lies in the fact that calculating the response function of the general purpose EMS defined in this paper is not expensive, so the large number of function evaluations for an effective Monte Carlo simulation can be performed without much computational cost.

4. Example Design Problem

4.1 Design model

As an example design problem, we considered a vessel-mounting system which is employed in an innovative concept vehicle. The demonstration system has a vessel supported by four mounts made of elastomeric bushings; the mounts are connected to the frame of the vehicle.

In the current research, the vessel is assumed as a rigid body with a total mass of 256.7 Kg. The moment of inertia matrix is

$$\mathbf{I} = \begin{bmatrix} 7.7 & -1.1 & 3.7 \\ -1.1 & 52.0 & 1.0 \\ 3.7 & 1.0 & 55.2 \end{bmatrix} Kg \cdot m^2 \tag{32}$$

The bushings have axial stiffness coefficient $k'_x = 1.4e5$ N/m and radial stiffness $k'_y = k'_z = 1.4e6$ N/m. The three orthogonal local coordinate axes of each bushing are originally parallel to the axes of the global coordinate system. Damping effects of the bushings are neglected. The locations of the body C.G. and each bushing are listed in Table 1. The major load considered in the current design is the inertia forces of the vessel when the vehicle has accelerations or decelerations during braking or steering. The load is assumed as a worst case of 10g inertia force applied to the C.G. of the body in the X-Y plane with an angle θ counter clockwise from the positive X axis. Different loading conditions are considered by varying the angle θ with the constant amplitude of the load.

The design variables considered in the current design problem are the orientation angles of the bushings about the Z-axis. The design variable vector is therefore $\{\mathbf{x}\} = \{\alpha_1, \quad \alpha_2, \quad \alpha_3, \quad \alpha_4\}^T$, where α_i represents the orientation angle of the i-th bushing ($i = 1, 2, 3, 4$) with the lower and upper bounds $-\pi/2$ and $\pi/2$ for all orientation angles. Four design objectives are considered as following:

1. The maximum body C.G. displacement should not exceed 20 mm;
2. The fundamental eigenfrequency of the system should be greater than 10 Hz;
3. Critical buckling force should be greater than 100 KN;
4. The maximum bushing force should be less than 12 KN.

Note that the above objectives do not reflect the actual requirements of the vessel-mounting system design. The design objectives listed above are only for demonstration purpose.

4.2 Design results and discussion

The original design assumes all the four bushings are oriented with their axial direction parallel to the X-axis of the global coordinate system, i.e., $\{\mathbf{x}\} = \{0, \quad 0, \quad 0, \quad 0\}^{\mathrm{T}}$. This design provides a very weak support for the vessel along the forward-afterward direction, which results in a maximal 45mm forward-afterward movement of the vessel when the G-force is applied along the same direction.

To improve the EMS design, firstly, a design optimization is carried out to minimize the body C.G. displacement for the G-force along the X-direction, i.e.,

$$\underset{\mathbf{x}}{\text{Minimize}} \quad f(\mathbf{x}) = \left\{ f_1(\theta),\ \theta = 0 \right\} \tag{33}$$

The optimum angles obtained are listed in Table 2, which shows that all bushings should be oriented nearly 90 degrees. This new design is referred as Design 1. The objective function of the new design has been improved from original 45 mm to 5 mm in this case for the given load direction. However, the optimality of this design highly depends on the loading direction assumed.

Figure 2 illustrates the variation of the body C.G. displacement with different loading directions. It can be seen that both the original design and Design 1 are highly load-dependent, which produce small displacement only for certain loading directions. Both designs may fail to meet the design objective if the load is applied along a totally different direction.

Secondly, we allow the load varying its direction, and to minimize the maximal body displacement with respect to all possible load directions. The optimization problem is then given as

$$\underset{\mathbf{x}}{\text{Minimize}} \quad f(\mathbf{x}) = \max\left\{ f_1(\theta),\ \text{for all } \theta \right\} \tag{34}$$

It can be seen from Fig. 2 that the new design (which is referred as Design 2), will eliminate the load-dependency of the original design and Design 1. The maximum body C.G. displacement is 8.2 mm for all possible loading directions, which can satisfy the design objective.

In order to meet the eigenfrequency requirement, thirdly, two optimization processes are carried out to maximize the eigenfrequencies of the vessel-mounting system. Design 3 is to maximize the fundamental eigenfrequency with an optimization problem defined as

$$\underset{\mathbf{x}}{\text{Maximize}} \quad f(\mathbf{x}) = \lambda_1 \tag{35}$$

Design 4 is to maximize the mean-value of all the six eigenfrequencies of the system. The design problem is defined as

$$\underset{\mathbf{x}}{\text{Maximize}} \quad f(\mathbf{x}) = \left(\sum_{i=1}^{6} \frac{1}{\lambda_i} \right)^{-1} \tag{36}$$

The optimization results for Design 3 and Design 4 are listed in Table 3. It can be seen that the lowest eigenfrequency can be increased to more than twice of the original value. We noticed that the optimum value obtained in Design 4 are very close to that obtained from Design 3, this is because the lowest eigenfrequency λ_1 has large contribution to the mean-eigenvalue defined in Eq. (36). It will be seen later that Design 4 is slightly better than Design 3 in terms of all other objectives considered (refer to Table 3).

Design 4 will also eliminate the load-dependency of the original design for the quasi-static loading case as shown in Fig. 2. This can be interpreted by that the eignfrequencies of the system represent the system stiffness in a global sense and this characteristic is independent of the external loads. Moreover, Design 4 results in smaller body C.G. displacement than Design 3 (see Table 3) since it also increased the higher eigenfrequencies of the system.

Design 5 is to maximize the critical buckling force so as to obtain a most stable vessel-mounting system. The optimization problem is given as

$$\underset{\mathbf{x}}{\text{Maximize}} \quad f(\mathbf{x}) = \lambda_b \tag{37}$$

where λ_b is defined in Eq. (22) and the optimization is for all possible loading directions. Figure 3 shows the critical buckling force obtained from the design process, and compares it with the critical buckling force when the original design or Design 4 is used. It can be seen that the original design yields a low buckling force when θ is near 90° and 270°. This can be explained as follows: when force is applied along these two directions, the mounting system along the forcing direction is much stiffer than that of the perpendicular direction, but the system has only a very small resistance to the yaw motion of the vessel. This results in a worst stability condition for the system. It can also be seen from Fig. 3 that the optimum design can significantly improve the stability by a factor near to 3. It is important to note that Design 4 (from the eigenvalue optimization) is as good as the current design (Design 5). This indicates that maximizing system eigenvalues can also improve the stability of the system.

Figure 4 illustrates dependency of the critical buckling force on the axial stiffness of the bushings. It is seen that the stability of the original design strongly depends on the axial stiffness of the bushings. This indicates that when the axial stiffness becomes smaller and smaller, the stability of the system will become a critical issue, although the current system does not have a stability problem. It is also seen in Fig. 4 that Design 5 results in a buckling force which is linearly dependent of the axial stiffness, but with a much higher minimum value, while Design 4 has almost no dependency on the variation of the axial stiffness. In other word, Design 4 is a much better design in terms of absorbing the uncertainty of the bushing stiffness.

From the above study, bearing the objectives reduction approach in mind, we conclude that the first three design objectives are consistent and can be included in the same group, say, group 1, maximizing the mean eigenfrequency of the system (Design 4) is chosen as a representative objective of the group, whose resultant design can meet all the first three design objectives. With this, our design task becomes much simpler.

The last single design task is to minimize the forces transmitted through the bushings. The goal is to minimize the maximum bushing force carried by all bushings. This objective is set to reduce the failure of the bushing. For this purpose, the design problem is defined as

$$\underset{\mathbf{x}}{\text{Minmize}}\ f(\mathbf{x}) = \max\{\overline{F}_i(\theta), i = 1, 2, 3, 4 \text{ and for all } \theta\} \tag{38}$$

The resultant design (Design 6) has significantly reduced the maximum bushing forces, for example, from 14.7 KN in Design 5 to 7.4 KN in the current design (other comparisons are shown in Table 3). However, as can be seen in Table 3, the new design turns out to be a bad design with respect to the other three design objectives considered in this paper. It is also seen in Table 3 that the Design 4 is good for the first three design tasks, but it is the worst for the last objective (minimize the maximum bushing force). Bearing the objectives reduction approach in mind, we can conclude now that the last design objective is in a different group, say, group 2, which is conflict with group 1 as defined before. There exists some trading-offs between the two groups. In order to meet the requirements of multidisciplinary objectives by one design, we define an optimization problem, which constraints the maximum force transmitted through each bushing as 11 KN and to maximize the mean-eigenvalue defined in Design 4, namely,

$$\begin{aligned} &\underset{\mathbf{x}}{\text{Maximize}}\ \ f(\mathbf{x}) = \left(\sum_{i=1}^{6} \frac{1}{\lambda_i} \right)^{-1} \\ &\text{Subject to}\ \ \overline{F}_i(\theta) \le 11KN,\ (i = 1, 2, 3, 4 \text{ and for all } \theta) \end{aligned} \tag{39}$$

The final results are listed in Table 2 as Design 7, which has met all design objectives; it is, therefore, considered as the final design from the design process. Figure 5 compares the maximum bushing forces obtained for three different designs (Design 4, 6 and 7) in terms of the loading direction. Note that different design requirements may result in a different design decision. However, the process proposed in this paper is general enough to deal with various design requirements.

Table 2 summarizes the results for all design cases considered in this example. Table 3 summaries the objective values obtained for different designs.

4.3 Reliability assessment

It is crucial to provide a reliability assessment for the optimal design. In this study, only the reliabilities with respect to the four design variables $\alpha_i, (i = 1, 2, 3, 4)$ are considered. Assume that all four design variables are normally distributed with the same standard deviation of 5 degrees. The mean values of these design variables are the optimization results of Design 7. Figure 6 shows an example probability density distribution of the first design variable.

We then calculated the design reliabilities for all four objectives defined in this example, namely, body C.G. displacement, fundamental eigenfrequency, critical buckling force, and the maximum bushing force. Firstly, we assume that the example EMS will fail if the body C.G. goes a distance greater than 20 mm. The probabilistic performance measure is then defined as

$$G_1(\mathbf{X}) = \frac{20 \text{ mm}}{d} - 1 \tag{40}$$

The probability of failure p_f is next obtained by using the Monte Carlo simulation, which is 0.01 with the reliability $R = 1 - p_f = 0.99$. Figure 7 shows the probability density distribution of the body C.G. displacement. The result indicates that the system response in terms of the body C.G. displacement has 99% reliability if the optimum design is used.

By the same way, the reliability for the first eigenfrequency being greater than 10 Hz can be obtained as $R = 0.74$, and the reliabilities for the critical buckling force and maximum bushing force can be obtained as 1.0 and 0.57, respectively, for the given design targets.

5. Conclusions and Future Work

In this paper, a systematic approach is developed for the design optimization of a general elastomeric mounting system with multidisciplinary design objectives. The design objectives include quasi-static response, eigenfrequencies of the mounting system, critical buckling force, and the maximum force transmitted through the mounts. Elastic instability of the general mounting system is first time considered in the EMS design problems. A general formulation that determines the critical buckling force and associated buckling mode has been developed, which can be readily generalized to predict the instability of a multi-body system and other similar systems with elastomeric mounts and bushings.

An objective reduction approach is proposed in this paper as a general method for the multidisciplinary design optimization. This approach simplifies a multidisciplinary optimization problem by reducing the total number of the design objectives based on the natural characteristics of an engineering structure in responding to different physical processes. This approach and the results obtained through this approach can be generalized for other similar structural design problems with similar multidisciplinary objectives. For instance, the mean-eigenvalue defined in this paper represents a set of different objectives in a global sense, and this conclusion can be applied to other structures, such as a vehicle suspension system.

By conducting a real mounting system design problem for a concept vehicle, we conclude that maximizing the mean-eigenfrequency of the system yields a design that can meet all design objectives in the design problem except the maximum bushing force. A combined design optimization problem with the reduced objectives is thereafter considered in order to compromise the conflict design targets and achieve the overall design goals. Furthermore, reliability analyses are conducted to assess the reliability of the final design for considering uncertainties of the parameters in the EMS design problem.

Future work will further consider the flexibility of the body and the base, as well as to conduct reliability based design optimization for more general design problems.

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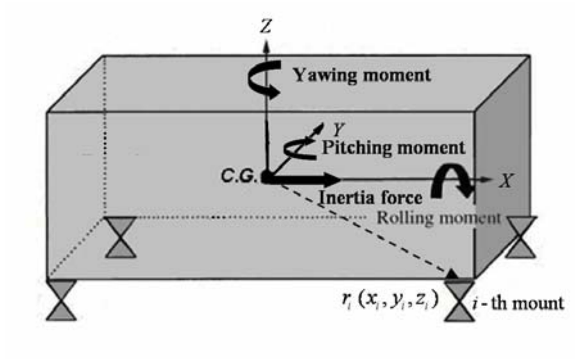


Figure 1. Rigid body on elastomeric mounts

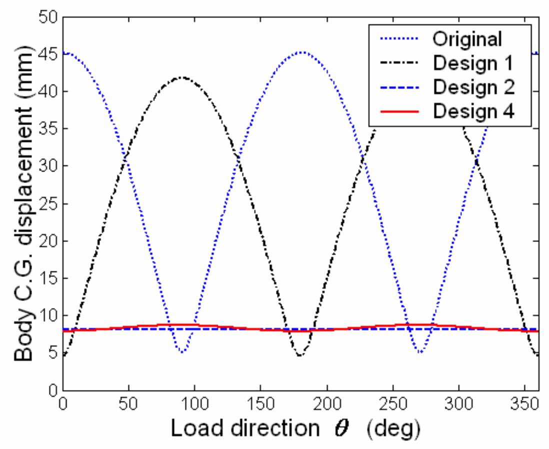


Figure 2. Load-dependent and Load-independent designs

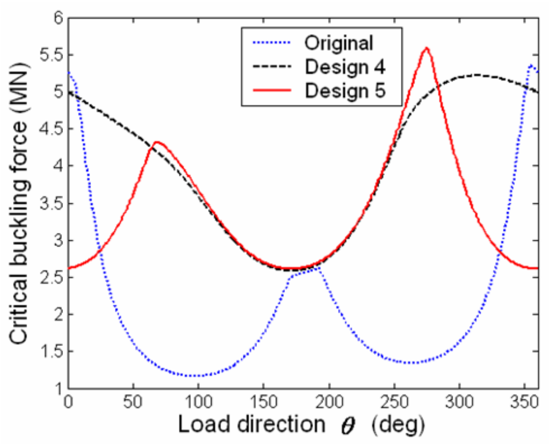


Figure 3. Comparison of critical buckling force in different designs

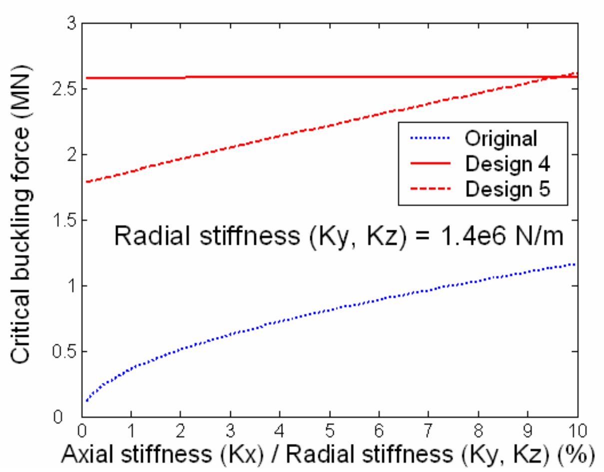


Figure 4. Comparison of critical buckling force in different designs by considering variation of bushing axial stiffness

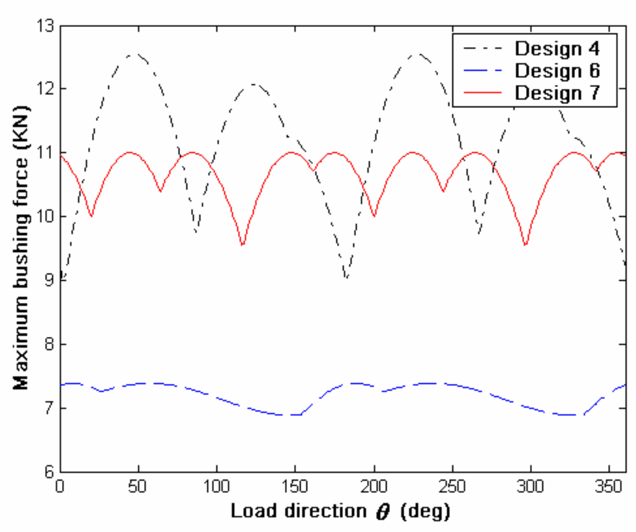
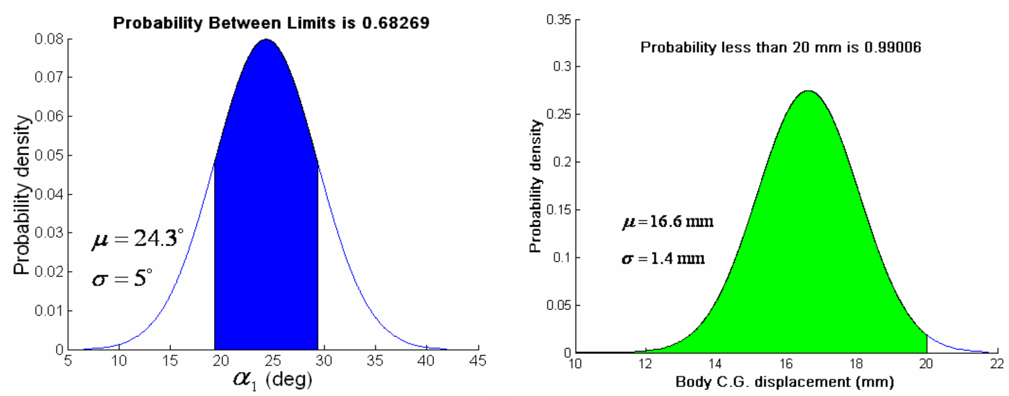


Figure 5. Comparison of maximum bushing force with different designs



a) Bushing angle probabilistic distribution b) C.G. displacement probabilistic distribution

Figure 6. Design variable and response probabilistic distributions

Table 1. Bushing locations

	X(mm)	Y(mm)	Z(mm)
Body C.G.	0	0	0
Bushing 1	-459.8	521.5	-77.2
Bushing 2	-457.0	-352.0	-76.6
Bushing 3	206.0	520.9	77.4
Bushing 4	206.2	-372.6	78.0

Table 2. Optimum value for different objectives

Design case	Design description / design objectives	Optimum angles (deg)
0	Original design	[0.0 0.0 0.0 0.0]
1	Minimizing the amplitude of the body C.G. displacement while $\theta = 0^{\circ}$	[83.0 86.8 -81.7 -87.2]
2	Minimizing the amplitude of the body C.G. displacement for arbitrary θ	[61.6 -28.4 -4.5 85.5]
3	Maximizing the fundamental eigenfrequency	[-53.8 43.7 40.9 -52.7]
4	Maximizing the mean value of all the six eigenfrequencies	[-47.7 51.1 41.3 -46.2]
5	Maximizing the critical buckling force	[-81.9 65.3 62.4 -72.3]
6	Minimizing the maximum bushing force	[85.9 69.1 -89.9 72.5]
7	Maximizing the mean value of all the six eigenfrequencies while constraining the maximum bushing force transmitted through each bushing	[24.3 82.3 15.4 -37.7]

Table 3. Comparison of design objectives with different designs

Objectives \ Design case		0	1	2	3	4	5	6	7
Maximum body C.G. displacement (mm)		45.1	41.8	8.2*	9.1	8.7	22.1	47.4	16.6
Natural frequencies (Hz)	**1**	7.4	7.7	14.9	16.4*	16.4*	10.6	7.1	10.4*
	2	16.2	16.6	16.7	16.4	16.8*	16.5	16.5	16.2*
	3	16.8	21.6	17.4	18.0	17.7*	21.8	22.2	19.1*
Critical buckling force (MN)		1.2	1.5	2.4	2.6	2.6	2.6*	1.0	1.0
Maximum bushing force (KN)		8.6	11.8	11.5	12.9	12.5	14.7	7.4*	11.0*

Note: numbers with (*) are objectives or constraints.