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MECHANICAL PROPERTIES OF CANCELLOUS BONE

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MECHANICAL PROPERTIES OF CANCELLOUS BONE†

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Abstract

Samples of human femora, vertebrae and cranial bone have been tested in compression. The results of these experiments have been analyzed, taking into account observed anisotropies and varying structures. Statistical correlations of properties have been made with density and a model proposed that summarizes these results. The cranial bones appear to be transversely isotropic and they are generally much stronger and stiffer in the transverse or tangent to the skull direction in comparison to the radial direction. The structure of the cancellous bone was found to be highly variable and this strongly influenced many of the mechanical responses. The model, however, explains much of the observed variation.

I. Introduction

Cancellous bone consists of a scaffolding of bony material arranged to form a spongelike porous structure. The pore cavities are quite variable in both size and shape and since most of the cells do communicate, the general arrangement may be approximated as an open cell two phase composite. One phase is bone, while the other phase consists of blood, red and yellow marrow, nerve tissue, miscellaneous cell tissue, interstitial fluid and blood vessels. These soft materials behave similarly to viscoelastic fluids (1) with a bulk modulus slightly larger than water (350,000 psi) and a shear modulus many orders of magnitude smaller.

Because of its calcified nature, bone can grow only by apposition. That is, bone can increase in size only by adding material to one of its surfaces. Bone remodeling and growth occurs with a combination of bone deposition and absorption occurring at the surfaces. New bone that is added to the sides of trabeculae (beams of bone) usually is deposited in the form of fairly even lamellae (plates). As new lamellae are added to the sides of a trabeculae in a cancellous network, the spaces are correspondingly narrowed and filled. Accordingly, the continued deposition of fresh bone lamellae on trabeculae that enclose spaces soon affects the character of the bone in that it changes from a structure consisting of large spaces with little bone (cancellous bone) to one of narrow spaces with much bone. When bone substance instead of spaces becomes the predominant features of the tissue, it is then called compact or dense bone(2). The femur, for example, consists of bone with a wide range of densities. The extremely porous bone of the condyles smoothly merges with the much less porous bone of the distal portion of the shaft.

The minimum pore size occurs in so called compact bone and is determined by the blood vessel (Haversian Canal) size required for proper nutrition of the bone cells (osteocytes), the size of the lacunae surrounding the bone cells and the canals (canaliculi) containing the interconnecting cytoplasmic processes of the bone cells. In thin plates of bone (as in the trabeculae) the Haversian canals are absent and the canaliculi open into the spaces of the cancellous tissue. Canaliculi in lamellar bone are about 0.35 microns in diameter and number 50-100 per lacuna. There are about 1,000,000 canaliculae per cubic millimeter of bone. The Haversian canals vary in size, some being as much as 0.12 mm in diameter; the average size in the human is about 0.05 mm. All bone is porous, therefore, and the important structural feature that distinguishes so called compact bone from cancellar cancellous is the quantity, size, shape and distribution of the cavities.

There have been many analysis of two phase composites ranging from the simple geometric models of Ishai (3) to the complex and relatively intractable elasticity approach typified by Biot(4). Unfortunately, the structural details of porous bone are so complex and variable that an analysis of only a single specimen requires an inordinate effort. The semi-empirical model described here relates the modulus or normalized stiffness of bone to its porosity. By matching the model responses to observed properties, much of the geometrical complexities can be relaxed and a useful correlation obtained.

II. Materials and Methods

Three types of bone were used in this study, femora, vertebral bodies and cranial bone. Fresh bovine femora and embalmed human femora were sectioned and tested. The vertebrae were excised from fresh cadavers at autopsy and were tested as soon as possible after removal, generally less than four days elapsed between the donors death and test.

The human skull bone was obtained from three sources: embalmed cadavers, craniotomies and autopsy. The primary source was embalmed cadavers from which entire calvariums were sectioned and tested. Fresh material from craniotomies and autopsy was also tested to verify the results of the tests on embalmed material. Previous work(5) indicates that the mechanical properties of embalmed bone are not significantly different from immediate postmortem properties. During the period after removal and prior to testing, the specimens were stored in isotonic saline buffered with calcium. All tests were made on bone in a moist condition.

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Cubiodal compression specimens were prepared by wet grinding on a metallographic polisher using specially designed fixtures with micrometer drives. This method produced high quality specimens with surfaces parallel (less than 1/1000) and smooth. For additional information on specimen size and test methods, see references 6 and 7.

Due to the sandwich construction of cranial bone, the skull test samples have a variable structural geometry. These specimens contain the full cross section except they are ground flat on the top and bottom corresponding to the inside and outside of the skull. This results in a specimen with an inner and outer table of compact bone and a core of cancellous bone (diploë). Due to the highly variable nature of the diploë region, a wide range of porosities can be obtained from these specimens.

The vertebral bodies were generally taken in the lumbar region at L_2, L_3, L_4 and L_5. A variety of specimen sizes were tested ranging from the whole *body with the processes removed down to 1/4 inch* cubes.

III. Density

The density of the specimens was determined by measuring the external dimensions of the specimen to determine the volume and dividing this into the dry weight. The specimens were dried by baking at 105°C until no further weight loss was observed. This was done after the mechanical properties tests were completed. This density is therefore the weight of dry bone per unit volume and provides a convenient method of estimating porosity.

IV. Compression

Compression tests were performed to determine the stiffness and strength of the various bone types. Many specimens were loaded first in one direction to about 1/3 the ultimate strength, while the deformation was measured in the direction of *the load and also perpendicular to it. This was* repeated in the other two directions yielding the stiffness and Poisson's ratio for three orthogonal axes. *For the femoral specimens, one axis was the* long axis of the bone, for the vertebral specimen, one axis was the central axis of the vertebrae and *for the cranial specimen, one axis was normal to* the skull. The load was taken to failure in the third of these tests.

The load was applied by a Tinius Olsen Electromatic Testing Machine at a constant velocity of 0.01 inches per minute. The load was monitored by a very stiff strain gage type load ring while the deformation in the direction of the load and perpendicular to the load were measured with very compliant strain gaged cantilever contact arms (Figure 1). These signals were continuously recorded on an x-y_1-y_2 recorder. The ultimate strength was taken as the stress corresponding to the maximum load the specimen can withstand generally characterized by a slope reversal. Stress and strain as used here, while computed in the usual fashion (load divided by the initial total cross sectional area and deformation divided by the initial length), are only normalized measures of load and deformation. That is, they are average values of complex stress and strain distributions.

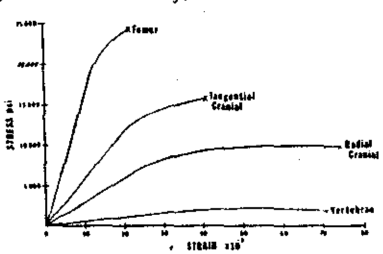
Figure 4 shows a photoelastic model of a typical cranial compression specimen indicating the effect of the gross geometry on the shear stress distribution. Local effects such as collagen and apatite crystal organization and osteocyte arrangement are not included and probably have a significant role in the reduction of stress concentrations.

V. Bone Properties

A summary of the results of these tests is presented in Table 1. The high values of the standard deviations are due to the naturally occurring variations of porosity in all bone types. For human skull bone, no significant difference was found for the modulus and ultimate strength due to loading in various tangential directions. Histological studies of tangent sections of the inner and outer tables and the diploë, revealed random patterns *without discernable geometrical organization.* The index of isotropy, which is a quite sensitive measure of isotropy(6) was low and the maximum and *minimum values of the modulus upon which it is* based occurred in random directions. Thus, it is concluded that skull bone is reasonably isotropic in directions *tangent to the skull surface. The* response in the radial or normal to the skull direction was of course significantly different for specimens that contained a full skull-section.

There was no significant difference in the mechanical properties of the human vertebral cancellous bone when loaded in different directions. Histological studies indicated differences in trabecular patterns when sectioned in various directions, however, and work is continuing to document this observation. For the purposes of this paper, the cancellous bone of the vertebral body will be considered homogeneous and isotropic in the large.

The results of the femoral tests corresponded well with Evans and Bang(8) and with Maj and Toajari(9). Compact bone from the shaft of the femur is very anisotropic. Histological studies indicate a high level of organization with Haversian System axes generally aligned with the *long axis of the bone. Specimen size effects were* studied in the vertebral body by testing whole bodies in an intact vertebral "unit" after Rockoff (10), *vertebral bodies with the processes removed* and the ends ground flat and cubes of cancellous bone only. No significant difference was observed *in the modulus or the ultimate compressive strength.* This is contrary to Rockoff's(10) results, but agrees well with Bartley(11).



Typical Stress-Strain Curves for Human Bone in Compression.

Figure 1

TABLE 1. PROPERTIES OF HUMAN BONE IN COMPRESSION

Density #/in³	No. of Donors	No. of Specimens	Age Range	Mean	Standard Deviation
Femur	32	160	28-86 yrs	0.067	0.027
Vertebral Body	72	288	1 mo-89 yrs	0.017	0.007
Cranium Full Section	14	240	56-73 yrs	0.051	0.019
Cranium Compacta	7	27	56-73 yrs	0.068	0.007
Modulus #/in²					
Femur Long Axis	32	160	28-86 yrs	1.84×10^6	0.4×10^6
Vertebral Body	72	288	1 mo-89 yrs	0.22×10^5	0.17×10^5
Cranium Full Section					
Radial	26	237	39-81 yrs	3.5×10^5	2.1×10^5
Tangential	14	219	56-73 yrs	8.1×10^5	4.4×10^5
Cranium Compacta	7	27	56-73 yrs	1.81×10^6	0.6×10^6
Ultimate Strength #/in²					
Femur Long Axis	32	160	28-86 yrs	2.6×10^4	6.1×10^3
Vertebral Body	72	288	1 mo-89 yrs	0.6×10^3	0.5×10^3
Cranium Full Section					
Radial	26	237	39-81 yrs	10.7×10^3	5.1×10^3
Tangential	14	210	56-73 yrs	14×10^3	5.2×10^3
Cranium Compacta	7	27	56-73 yrs	20.9×10^3	6.8×10^3
Poisson's Ratio					
Femur Long Axis	7	28	37-86 yrs	0.3	0.05
Vertebral Body	7	28	45-79 yrs	0.14	0.09
Cranium Full Section					
Radial	14	122	56-73 yrs	0.19	0.08
Tangential	18	327	56-81 yrs	0.22	0.11
Cranium Compacta	7	27	56-73 yrs	0.28	0.04

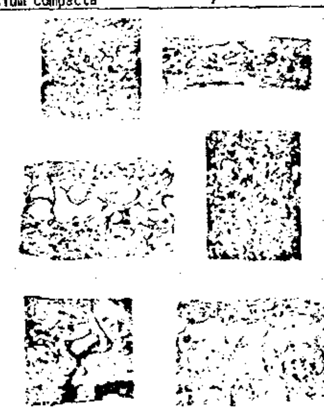


Figure 2. Sections Through Cranium

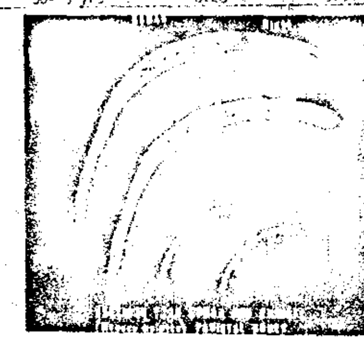


Figure 4.

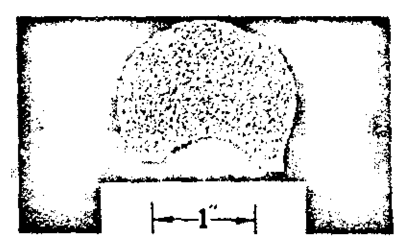


Figure 3. Cancellous Bone from the Vertebral Body

Figure 5. Photoelastic Study of Cranial Section

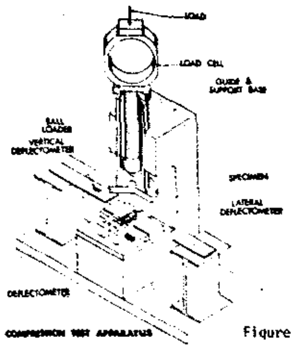


Figure 6.

Regression analyses were performed to correlate those properties that were anticipated to be functionally related. The following relationships were found for human cranial bone using data from those tests that produced classical stress-strain curves.

Radial Compression

$$E_R = (36\gamma - 1.3) \times 10^6;\ C_c = 0.62$$

$$E_R = (2.02\gamma^{5.13}) \times 10^{12};\ C_c = 0.86$$

$$\sigma = 2.9 \times 10^{-2} E_R;\ C_c = 0.78$$

$$\sigma = 1.2 \times 10^3 \gamma^{3.3};\ C_c = .91$$

Tangential Compression

$$E_t = (46\gamma - 1.52) \times 10^6;\ C_c = 0.65$$

$$\sigma = (770\gamma - 26) \times 10^3;\ C_c = 0.65$$

$$\sigma = 2.4 \times 10^{-2} E_t;\ C_c = 0.57$$

where:

E_R is the modulus of Elasticity loaded in the radial direction (psi)

E_t is the modulus of Elasticity loaded in the tangential direction (psi)

σ is the ultimate compressive strength (psi)

γ is the dry weight density (lbs/in³)

C_c is the correlation coefficient

The values of the correlation coefficients indicate that a significant part of the variation in the modulus and strength may be attributed to variations in the density. The relation between density and modulus and density and strength is non-linear as evidenced by the higher values of the correlation coefficients of the logarithmic regression analysis compared with the linear regression analysis.

The relation between the stress and modulus was approximately linear, however, and indicates that the maximum strain may be taken as a constant value independent of porosity. This provides a basis for a maximum strain theory of failure for skull bone with failure strains of 2.9×10^{-2} for radial

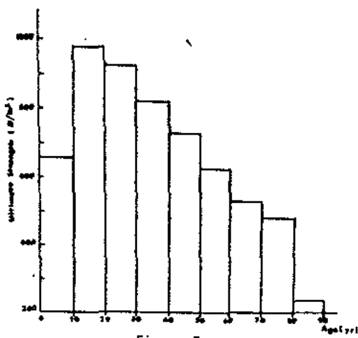


Figure 7.
Strength ~ Aging Characteristics of the Human Vertebral Body

compression and 2.4×10^{-2} for tangential compression indicated by the regression analysis. It should be recognized, however, that these regression analyses were performed only on the data from the classical stress-strain curves, and those specimens that failed progressively or by buckling had much higher maximum strain values. The regression equations given are approximately valid for dry weight densities between 0.061 and 0.042 #/in³ and should not be extrapolated beyond this range.

The vertebral body properties also correlated well with the dry weight density. The following linear regression equations were obtained. The symbols used are the same as above.

$$E = (3\gamma - 0.023) \times 10^6;\ C_c = 0.61$$

$$\sigma = (300\gamma - 5.9) \times 10^3;\ C_c = 0.67$$

$$\sigma = (3.2 \times 10^{-2})\ E;\ C_c = 0.71$$

The vertebral body data also shows a strong linear correlation between modulus and strength indicating that a maximum strain theory of failure applies also to cancellous bone from the vertebrae with a maximum failure strain of 3.2×10^{-2}. These regression equations give approximately valid results for dry weight densities between 0.022 and 0.010 #/in³.

While no significant correlation of the femoral or cranial bone was found with age, the strength of vertebral bone correlated at the 0.52 level. (Figure 7).

VII. Optical Density

Roentgenograms were prepared for the vertebral body sections mentioned above. The bodies were ground wet to a standard 1/2" thickness. Using a step wedge calibration techniques and an optical densitometer, a normalized film density was determined for each body. This number represented the average of ten 1/8 inch diameter spot readings over the body cross section. A linear regression analysis was performed relating the normalized optical density (y) with the ultimate compression strength (σ) and the measured weight density (γ). The following relationships describe the results of those analysis.

$$y = 1.6 + 9.7 \times 10^{-4}\sigma$$

coefficient of correlation 0.556
adjusted standard error 0.458

$$y = -6.5 + 210\gamma$$

coefficient of correlation 0.491
adjusted standard error 0.511

VIII. Porous Block Model

A semi-empirical model has been developed that allows the prediction of the modulus and strength of bone based on its density and internal geometry. The accuracy of this prediction is limited by the natural variation of the properties of compact bone, the base material of the model. The advantage of such a model is that it allows the description, in a relatively simple form, of the physical properties of all types of bone, ranging from the dense bone of the femoral shaft to the very porous bone of the vertebral body. A model of this type may provide some insight into failure patterns and hopefully treatment of multiple myeloma, osteoporosis and other diseases affecting the density of bone.

The primary assumption upon which this model is based is that all bone has the same physical properties in the small. That is, bone of different types has the same microscopic properties but varying macroscopic structure. In dealing with the varying structure, it is further assumed that local mechanical responses like the modulus of elasticity are proportional to the local density raised to some power (n) with n being determined empirically.

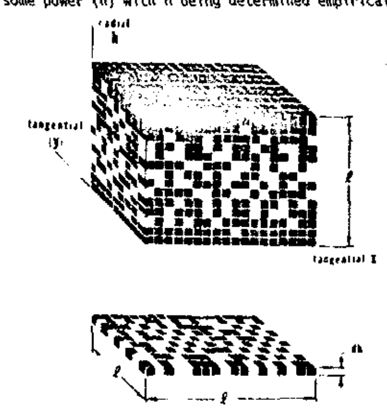


Figure 8.
POROUS BLOCK MODEL FOR CANCELLOUS BONE

Consider the porous block model shown in Figure 8 which consists of a number of small cubical aggregates arranged to form a larger transversely isotropic porous cube. In this model it is assumed that the absence of one or more of the cubical aggregates from the large block does not alter either the position or the behavior of the neighboring aggregates. The ratio of the void volume to the total volume is called the porosity C.

Thus $$C = 1 - \gamma/\gamma_o$$

where γ is the density of the sample and γ_o is the density of the base material.

The model indicates that the modulus of elasticity of transversely isotropic bone samples loaded axially may be simulated by juxtaposition of a large number of slabs or springs loaded in parallel while radial loading is simulated by a large number of layers or springs loaded in series. Therefore,

$$\frac{E_o}{E_r} = \int_0^h \frac{dh}{[1 - C(h)]^{1/n}} \text{ for radial compression,}$$

and

$$\frac{E_t}{E_o} = \int_0^h [1 - C(h)]^n dh \text{ for tangential compression,}$$

where E_o is the modulus of the base material and $C(h)$ is the porosity of a slab or layer.

For an essentially homogeneous distribution as occurs in vertebral bodies

$$C(h) = C_{ave} = \text{Constant}$$

and

$$E = E_o(1 - C)^n$$

Figure 9 shows this equation plotted for various values of n superimposed on the human vertebral body data.

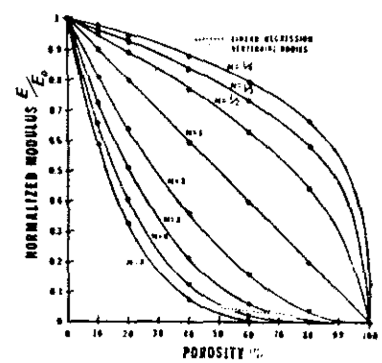


Figure 9 Homogeneous Model Responses

A study of serial sections of cranial bone samples conducted as part of this research program, has shown that the voids or marrow spaces are approximately homogeneously distributed in directions tangent to the inner and outer tables but in the radial direction there is a continuous increase in porosity as one moves either from the inner or outer table toward the center. Furthermore, the variation of porosity in the radial direction C(h) may be satisfactorily approximated by a Gaussian function of the form

$$C(h) = C_m e^{-18.42(h-1/2)^2}.$$

where $C_m e$ is the maximum porosity. With this form for $C_{(h)}$ the porosity of the tables is Cm/100.

The average porosity is then

$$C_a = \frac{1}{h}\int_0^h C_m e^{-18.42(h-1/2)^2}\,dh$$

It was found that for skewed distributions the modulus of the composite is independent of the location of $C_m e$ and no loss in generality is incurred by considering only symmetric distributions. These integrals do not possess closed form solutions but can be easily solved numerically on a digital computer.

Various values of the exponent n were examined by comparing the correlation coefficient relating the modulus data to the model predictions. The value n = 3 was found to yield a best fit with a correlation coefficient of 0.79 for cranial bone and 0.61 for the cancellous bone from the vertebral body.

In analyzing the cranial bone data several other porosity distributions were examined including parabolic and sinusoidal functions. The model responses for different distributions shown in Figure 10. The behavior of the model for a sandwich construction in radial compression, with a homogeneous porous center and n = 1 is shown in Figure 11.

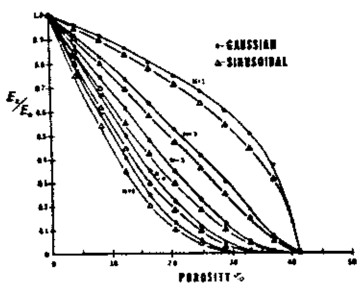


Figure 10. Model Responses for Nonuniform Diploë Distribution

Figures 12 and 13 show the relationship between modulus and porosity predicted by the final form of the model and compared with the regression analyses previously described.

The value of E_0 the modulus of the base material used to normalize the ordinate was 1.8×10^6 psi. This value was chosen from the data for the compact bone of the cranial tables which histologically compares favorably with the bony material of the traebeculae of cancellous bone. The porosity was based on a value of 0.068 #/in³ also based on the data from the cranial tables.

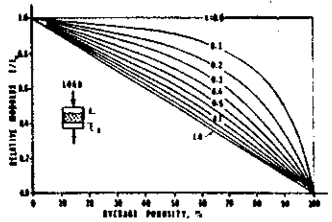


Figure 11. Homogeneous Sandwich Model Response

The model indicates that with a Gaussian distribution of the form used in Figures 12 and 13 for cranial bone, the average porosity can not exceed 41%. This is because the maximum porosity at the center of the diploë region is zero when the average porosity is 41% and of course the porosity can not be less than zero. The value of the modulus is therefore much more sensitive to porosity distribution in the high porosity material than in the low porosity material.

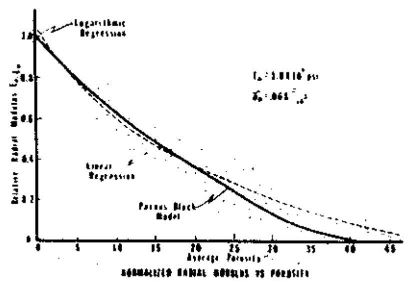


Figure 12. Human Cranial Bone in Radial Compression

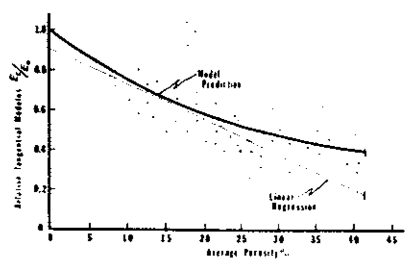


Figure 13. Human Cranial Bone in Tangential Compression

Figure 14 shows the model's prediction of the ratio of the radial modulus to the tangential modulus compared with measured values for cranial bone. The model explains quite well the observed increase of non-isotropic response with increasing porosity. The non-isotropic mechanical response of the model is based on the non-isotropic porosity distribution and is very sensitive to the form of the porosity distribution in the region approaching 41% average porosity. It is possible that much of the lack of fit in the low density region may be explained by the difference between the actual porosity distribution in the sample and the assumed Gaussian distribution.

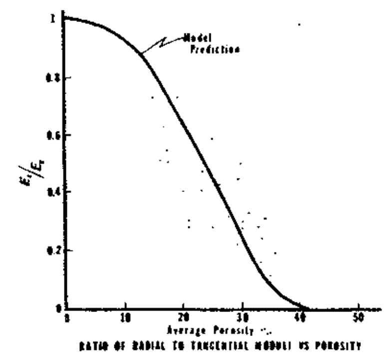


Figure 14.

IX. Discussion

As has been pointed out by Biot and Willis(12) a dry porous medium might not exhibit the same elastic properties as that of the saturated one. If the fluid filled pores significantly contribute to the stiffness, then one would expect both rate of loading and specimen size effects. As the porosity increased in the cancellous bone studied here, the effects of rate of loading and specimen size diminished until at approximately 30% porosity, they were inseparable from the background noise due to the inherent biological variations and therefore they become statistically insignificant.

Many of the mechanical responses of cancellous bone are strongly influenced by the structural arrangement of the diploë. Thus in these tests properties such as compressive strength and modulus are sturctural properties and the large values of the standard deviations observed for these properties are primarily due to variations in the porosity and internal arrangement of the traebeculae. The similarities of the properties and histology of compact bone indicates that a single material porous block model is justified as a first approximation in describing the relation between structure and mechanical response. Relating the value of a property to the density raised to some power n and then determining n empirically provides a means of incorporating in the model many of the structural elements that influence the response but are too complex to be included in detail. The model presented here is therefore complex enough to explain much of the property variation observed but much too simple to explain the mechanisms involved. The model shows that the modulus of bone is approximately proportional to the third power of the density. Thus, small porosity changes in bone of low relative density result in only small changes in strength and modulus while small porosity changes in bone of high relative density result in large changes in strength and modulus. The porosity distribution in a given sample of bone is much more significant in its effect on strength and modulus in bone of low relative density than in bone of high relative density. Of interest is the fact that the homogeneous version of this model fits much of Coble's(13) data on various porous ceramics indicating that bone is not unique in its response to porosity variations.

The material properties in the small i.e. hardness, density and local compression strength bone are not significantly different. The amount and distribution of the trabeculae however is quite variable and therefore the structural responses in particular the energy absorption, gross stiffness and damping characteristics which are strongly dependent on structure, will vary greatly.

The strong dependence of strength on physical density and the relatively weak dependence on optical density from roentgenographic analysis indicates that estimates of strength based on radiograph data should be made with great caution. While the conclusion could have been reached from purely theoretical arguments it is comforting that the empirical methods utilized here support it.

X. Acknowledgments

The results reported in this paper represent a compilation of work performed in the Biomaterials Laboratory, Highway Safety Research Institute, The University of Michigan and the Biomechanics Laboratory, Department of Theoretical and Applied Mechanics, West Virginia University. Particular acknowledgment is given to the contributions of Nabih Alem, Jack Wood and Iqbal Barodawala from the University of Michigan, and Bruce Durs, Frank Ammons, George Utt and Richard Stalnaker from West Virginia University who performed much of the experimental work presented here.

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