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[EMV-5]:

ANALYSIS OF THE TEHCNIQUES EMPLOYED IN EMV 2, EMV 3, EMV 4, AND ALSO A STUDY OF THE PRECISION OF THE PHOTOGRAPHIC TECHNIQUES EMPLOYED.

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In EMV-3, the author spoke about taking certain points from photographs. Since these reports might come into readers' hands who are not familiar with the great care taken in the photographic techniques employed, it becomes necessary to explain the photographic work of Mr. Cooper.

In the application of EMV 3 certain revisions in the original report seemed to add clarity in the application of this technique by the combustion engineer. These are discussed.

A discussion, giving the physical difficulties involved in the application of EMV 2, and an experimental method is suggested for making EMV 3 more general.

A conclusion is then given concerning the value of EMV 2, EMV 3 and EMV 4.

Throughout this report the writers have assumed that the reader is familiar with reports EMV 2, EMV 3, EMV 4.

PHOTOGRAPHIC TECHNIQUES

The procedures outlined below involved the problem of producing accurate photographs of thin titanium-tetrachloride smoke streams in a moving gas in addition to showing the inverted cone of flame produced and held in the gas stream.

The area photographed enclosed a vertically moving stream of gas, a V-flame of low visual intensity and blue color held in the stream, and two thin streamers of TT smoke produced in a vertical plane coincident with the tip of the flame cone. The pertinent portion of the photograph included the flame cone, the smoke, and the points of junction of the cone and smoke streamers. Since accurate measurements were to be taken from enlargements of the record negatives, it was necessary to keep the summation of photographic errors $<1\%$.

The photographic equipment was simple. The camera was a 4 X 5 Speed Graphic with a 6-3/8" Anastigmat lens in a Supermatic shutter. This was fastened on a sturdy stand made of 1" pipe. Placement of the camera was such that the lens and film planes were parallel to the plane of the titanium-tetrachloride smoke streams.

The source of light was a 500 W. Bardwell and McAllister spotlight, set to maximum brilliance. Its position was at 120° from the camera in the horizontal plane and 30° above horizontal in the vertical plane. This gave sufficient illumination so that ½ sec. exposure at F-5.6, through a Wratten A filter, was found to be adequate. The film used was Eastman Super-pancho-press film, Type B. The A filter was used, incidentally, because it decreased the haze around the flame which tended to obscure detail. The blue flame was of sufficient intensity to record itself in spite of the use of the filters.

Development of the film was extended to produce maximum contrast without chemical fog being produced. Six minutes in D-72 diluted 1 to 1 at 68° F. was found to give satisfactory results. Fixation was 5 minutes in fresh F-5, followed by a minimum satisfactory wash of 15 minutes in 68° F water. The film was then hung in a 75° F stream of clean air until dry. This procedure in handling insured minimum distortion of the film.

These negatives were found to have the necessary accuracy for the data desired.

It is axiomatic, however, that errors will occur in recording photographically.

Possible sources of error came from several points. It was necessary to investigate all in order to be sure that the summation of the maximum possible errors were less than 1%.

Both information from Eastman Kodak Co. and our tests showed that with our methods of handling the film that distortion in either the film base or emulsion was negligable. Differential distortion was < .1%.

It is possible to have a slight shift of the image relative to the base, but it was too slight, if present, to be measured.

Non-parallility of film, lens, and object was found to introduce the greatest error, but accuracy of alignment within 1° gave results still within limits when added to the other maximum errors in other stages.

Printing of these negatives required care to eliminate inaccuracies. The equipment and the paper both were called upon to work to tolerances much beyond the normal range of photographic accuracy.

The enlarger was an Omega D-2 4 x 5 enlarger with a Baush and Lomb 5½" enlarging Tessar lens. The regular dustless carrier was discarded and a glass sandwich type carrier substituted in order to keep the film flat.

Before any work was started, the enlarger was checked at the base - board,lens,and carrier with a machinists level to insure that they were parallel. An additional check was made fromthe easel surface to the carrier surface with a machinists height gage to both double check the original alignment and to check the easel for flatness and parallelism with the carrier.

The next step was to check the paper for differential distortion. It was single weight, Type XII, semi-matte bromide paper, Air Corps Specifications 75-157B, which is plastic impregnated.

The testing was accomplished by punching fine holes 18" apart on 20" x 24" paper both across and with the grain of the paper. Ten sheets were marked in this way with a beam compass and then processed. After drying, the compass was again placed over the holes and the shift from original placement was measured. In no case was differential distortion over .3%. This is in contrast to standard papers which have an inherent distortion which can reach 2.5%

Tests also showed that the paper was held sufficiently flat in the easel so that no error was introduced through buckling of the paper. As an added precaution, the paper was allowed to take a"set" in the easel before exposure to eliminate slight springing of the paper during the exposure. This may not have accomplished anything toward greater accuracy, but at least it was an added precaution.

The exposed paper was developed in D-72, diluted 2 parts water to one of developer, for $1\frac{1}{2}$ minutes. It was then rinsed in $1\frac{1}{2}\%$ Acetic acid. Fixation was carried out in fresh F-5 for 10 minutes.

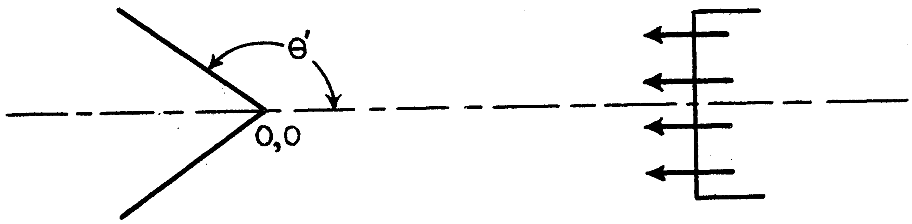
After fixation, the paper was washed for 20 minutes in a drum washer with water at 68° F. All times in liquids were kept at a minimum, compatible with proper processing. The prints were then blotted between photographic blotters and allowed to dry between fresh blotters laid on a table and weighted.

The use of this procedure and a paper designed for map mosiacs insured prints with accuracy of recording well within the limit of error of 1%.

In application of EMV 3 one presumes an understanding of the physical coordinate system as used by Durand in his "Aerodynamic Theory' on page 197, Vol. 1.

The fact that the sign changes on the velocity vector if flow is from left to right instead of right to left seems to play havoc with some readers. In order to change this system, all one has to do is to consider at all times that his flow is from right to left. Then the velocity vector at the jet is always in this direction. Refer back to the regular signs of polar coordinates and cartesian coordinates as far as signs of u and v, U, r, θ, x and y are concerned.

One needs to make the velocity at the jet - U in order to make it consistent with polar coordinates. That is, the velocity of the jet is the vector $\underline{U}$, but $\underline{U}$ looks like $\xleftarrow{U}$ which in polar coordinates is $\underline{U} = |U|\ e^{i\pi} = -|U|$; in other words the minus sign connotes direction of the vector $\underline{U}$ which has the magnitude $|U|$. In this system vectors will be signless until related to the coordinate system used.



The only changes this makes in EMV 3 are on Pages 3 and 4

Page 3

$\psi\ (r,\pi) = 0$	line 1
$u(r,\theta\ ) = \infty$ as $r \to \infty$	line 2
$v(r,\theta\ ) = \infty$ as $r \to \infty$	line 3
$u(r,\theta\ ) = \frac{\pi}{\theta'} U + C_1 U$	line 12

The last condition applies far from the origin and close to $\theta = \theta'$

$\pi - \theta_2 = \arctan \frac{.57}{.87}$	line 28
$\pi - \theta_2 = \pi - \theta'$ etc.	line 31

Page 4

Replace (-1.632), wherever it occurs, by (+.5705).

It might be helpful to point out that the infinite boundary conditions are employed to insist on conservation of energy, and are not to be used by the engineer as physical reality, because these forced boundary conditions are just to make it possible for the product of

$$\rho V A = \rho' V' A'$$

In our case we forced ρVA to be an indeterminate. ρ is a constant ($\rho = \rho'$) since the fluid is incompressible. Also in our case A was equal to zero at $r = \infty$ so V must be infinite in order that there be a chance that flow exist. It naturally is required that there be no violation of the equation of continuity or, as it's sometimes called, the law of continuity. (Ref.1). This is exactly what Durand said in his quotation, but perhaps a change of words might convey the thought more easily.

EMV 3 might be made completely general for wedge-like flames if one were to plot a family of curves C_1 versus $\frac{\rho \text{ unburned}}{\rho \text{ burned}}$ for different θ'. In this manner the geometry of the bead would determine the wedge and the angle of the wedge θ'. Given θ' one would then have C_1 determined for a particular gas. Given C_1 one then could determine U, knowing the given jet output velocity. Thus by knowing the geometry of the bead and knowing the output velocity of the jet and the type gas used, the physics of the effect of a wedge-like flame on a gas flow up to the flame front seems to be completely determined.

Perhaps it should be pointed out that even though this approximation appears almost perfect, in theory it cannot be exact except for a "flame" at the same temperature as that of the gas; otherwise the flow is not isentropic. However, we know that many irrotational flow assumptions seem to give good approximations for the non-isentropic cases*, except when actual measurement of direction of rotations becomes important. In our case we are only interested in the velocities at the flame front.

As a result this assumption is probably as valid as the rest of the assumptions especially to within the precision measure of the experiment.

EMV 2 was an attempted solution of the cone problem. Due to the lack of sufficient boundary conditions the solution had two arbitrary constants present.

*For example Sauer's approximations for Yawing Cones see Lotkin's article P 656 Nov.1948 Journal of Aeronautical Sciences.

$$\phi = C_6 \sin\theta' \, e^{+kx} J_o(kr) - Ux - C_6 \sin\theta'$$

$$\psi = - C_6 \sin\theta' \, r \, e^{+kx} J_o'(kr) - \frac{Ur^2}{2}$$

$$u = - C_6 \sin\theta' k \, e^{+kx} J_o(kr) + U$$

$$v = - C_6 \sin\theta' \, k \, e^{+kx} J_o'(kr)$$

where $J_o'(kr) = - J_1(kr)$

However in this problem we assumed that the flow was irrotational. This implies that there is no maximum or minimum within the flow. Thus, for our solution to be physically correct as it now stands, J_o and J_1 cannot have a maximum or a minimum within the flow.

Let us make a very rough estimate of kr by trying to set up a rough approximation on kr. For a typical example $.5 < r < 1.8$. (1)

But except for calculating the smallest zero of J_o for which n = 1

$$kr = (n\pi - \tfrac{1}{4}\pi) + \frac{1}{(n\pi - \frac{1}{4}\pi)} - \frac{31}{384(n\pi - \frac{1}{4}\pi)^3} + \frac{3779}{15360(n\pi - \frac{1}{4}\pi)^5} \quad (2)$$

This expansion (Ref.2) is adequate for computing all the zeros of J_o up to 5 decimal places.

Now let us examine the physics as expressed by the inequality (1)

Now assume we are between two zeros of J_o as we have to be, for irrotational flow.

Let the smaller zero be $(p = kr_n)$

Let the next zero be $(q = kr_{n+1})$

$$q \geq kr \geq p \qquad (3)$$

In other words p is equal to the sum of expansion (2) when n = a and q is equal to the sum of the expansion (2) when n = a + 1.

n = 1, 2, 3, 4, etc.

From (3)

$$q \geq k\,r \geq p$$

but this must hold for all possible physical values in the given experiment.

thus from (1) and (3) $k(.5) \geq p$ (4)

$k \geq 2p$ (5)

$q \geq 1.8\,(k)$ (6)

$.555\,q \geq k$ (7)

Thus from (5) and (7)

$.555\,q \geq 2p$

$q \geq 3.6\,p$ (8)

now supplying the one zero of J_o not given by (2) to within 5 decimal places

$J_{o,n}$ means n-th zero of J_o

$J_{o,1} = 2.40482$

$J_{o,2} = 5.52008$

$J_{o,3} = 8.05373$

etc.

Thus from examining expansion (2) we see that the greatest ratio of successive zeros of J_o occurs for the ratio of $\frac{J_{o,2}}{J_{o,1}} = \frac{5.52008}{2.40482} = 2.2954$ (9)

but if the physical assumptions are to be correct

$\frac{q}{p} \geq 3.6$ (8); thus there are no zeros of J_o which meet the physical requirements for the given experiment. In like manner it can be shown that the only possibility for an equation like (1) to be set up $c \leq r \leq d$ would be if c and d are very close to the same value. This can only occur physically when the cone angle is very large, ($\theta' \cong \frac{\pi}{2}$) or for the cone angle to be very small $\theta' \cong 0$. Perhaps one can see this better when one realizes that we have at most only $\frac{1}{2}$ the interval q-p available, since a maximum occurs in the middle between p and q. Even after this is set up there is a further shortening of the interval when the same requirements are put on J_1 that have just been put on J_o.

This analysis shows the futility of the use of Bessel functions for solving the conical flame problem. Possibly the physics is such that the flow is rotational before a conical flame front. EMV 4 further convinces the writer that this is the case. Thus EMV 3 and EMV 4 ask for experimental evidence of the irrotationality of subsonic flow before a conical flame; and they predict the flow will be rotational.

REFERENCES

1. Hemke, "Elementary Applied Aerodynamics" P. 21

2. "Theory of Bessel Functions" Watson P. 505