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Scientific Report No. RS-1

AN INTRODUCTION TO THE THEORY AND DATA REDUCTION METHOD FOR THE PITOT-STATIC TECHNIQUE OF UPPER ATMOSPHERE MEASUREMENT

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ORA Project 05776

under contract with:

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
CONTRACT NO. NAS5-3335
GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

administered through:

OFFICE OF RESEARCH ADMINISTRATION   ANN ARBOR

March 1964

PREFACE

The Department of Electrical Engineering, Space Physics Research Laboratory of The University of Michigan has been for many years actively engaged in various aspects of atmospheric research by rocket-borne techniques. As a result of this work much has been added to our present day knowledge of the earth's upper atmosphere. Because of this ever increasing knowledge and understanding of the upper atmosphere, more and more emphasis is being placed upon accuracy, basic understanding, and speed of processing of experimental data obtained from atmospheric measurements.

Thus, the purpose of this paper shall be twofold. First, it will serve to briefly introduce those persons involved in the data-reduction effort of the research program of this laboratory to the terminology and nature of the upper atmosphere as it is known today. Second, it shall develop in detail the theory, equations, conversions, and transformation required in the data-processing effort for this research program.

Thus, it is hoped that this paper will bridge the gap between the varying technical backgrounds of those persons involved in this most important work and the technical background required, thereby increasing the speed and accuracy of the data processing of any subsequent flights of rocket-borne upper-atmosphere probes.

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A	Avogadro's number 6.0247 x 10^{23} particles per mole
$\bar{A}$	a dimensional constant equal to 1435.064 $\left(\frac{ft^2\ kg}{sec^2 m^3\ [mm\ Hg]}\right)$
$\bar{B}$	a dimensional constant equal to 10.257 $\left(\frac{ft\ kg\ {}^\circ K^{1/2}}{sec\ m^3\ [mm\ Hg]}\right)$
c_m	most probable thermal particle speed given by $\sqrt{\frac{2kT}{m}}$
C	aerodynamic coefficient
d	mean molecular diameter
D	drag force
F(S)	a function of the parameter S given by $\left[e^{-S^2}+S\sqrt{\pi}\ (1+\text{erf } S)\right]$
g	acceleration due to gravity
h	distance
H	altitude
k	Boltzmann's constant 1.3804 x 10^{-23} joules/°K
K(M)	a function of Mach number given by $\frac{\gamma+1}{2\gamma}\left[\frac{(\gamma+1)^2M^2}{4\gamma M^2-2\gamma+2}\right]^{\frac{1}{\gamma-1}}$
ℓ	characteristic length
m	mass
M	Mach number
MW	molecular weight
N	number of particles

LIST OF SYMBOLS (Concluded)

P	pressure
R	gas constant per unit mass given as k/m
S	area element, velocity parameter given by $\frac{V}{c_{m_a}} \cos \alpha$
t	time
T	absolute temperature
v	particle thermal velocity
V	velocity
α	angle of attack
$\beta(T)$	one-half of the atmospheric scale parameter $\frac{m}{2kT}$
γ	ratio of specific heats $\frac{c_p}{c_v} = 1.4$ for air
$\bar{\lambda}$	particle mean free path
θ	spherical polar angle
ϕ	spherical azimuth angle
ρ	mass density
$\bar{\tau}$	particle mean free time

Subscripts

a	ambient conditions
D	drag
i	impact conditions
o	sea level conditions
s	condition directly behind a normal shock
x,y,z	direction along the coordinate axis
cal	information obtained directly from calibration data
gauge	actual conditions in a pressure sensor

CHAPTER I

THE UPPER ATMOSPHERE AS IT APPEARS TODAY

1.1 BASIS OF OUR KNOWLEDGE OF THE UPPER ATMOSPHERE

Although research on the upper atmosphere does antedate the Twentieth Century, current knowledge about the upper air comes primarily from work done during the past 50 years. Within this relatively short period of time the atmosphere has revealed itself as a very complex and stimulating object of study, and calls upon an impressive number and variety of techniques in its investigation. Modern high-altitude research is a major field of endeavor claiming the attention of many well-known scientists throughout the world.

One of the most challenging features of the upper atmosphere has been, and is, its inaccessability. This challenge has brought about two lines of attack. First, there have been devised a great many indirect* methods of studying the atmosphere and associated phenomena in terms of observations made at the earth's surface. Second, means have been sought to make the upper regions accessible, such as balloon, rocket, and satellite-borne probes. The latter two approaches, in which we are most interested, have met with a great deal of success in recent years.[3-5]

Before the advent of the rocket-borne probe, most upper atmosphere measurements were obtained by earth-bound observation of atmospheric-spectra, meteor trails, sound propagation, and by investigation of aurorae and the

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*Indirect shall refer to measurements made entirely by earth-bound observation.

earth's magnetic field. From the above and various theoretical studies, many inferences have been drawn about the structure, composition, and electrical properties of the high atmosphere. These early indirect attacks upon the problem have long been supplemented by the direct approach of making measurements with balloon-borne instruments, and most recently (within the past 20 years) rocket-borne instruments have extended the direct approach much beyond the balloon ceiling of about 30 km to over 1,000 km.

1.2 THE REGIONS OF THE ATMOSPHERE

The lowest portion of the atmosphere is referred to as the troposphere. It is in this region that the ordinary meteorological processes occur (rain, snow, hurricanes, tornadoes, monsoons, etc.). One might say it is the "weather sphere." Starting at the ground, the temperature and pressure decreases with height until a minimum temperature is reached at what is called the tropopause.[4,5] The height of the tropopause is, of course, variable and ranges from approximately 6 km above the poles of the earth to approximately 18 km above the equator. In the middle latitudes (Central United States) its height is roughly 12 km.

Above the troposphere and the tropopause lies the so-called stratosphere, so-called because in the early part of this century this region was thought to be isothermal (constant temperature) and quiet, and hence stratified (separated into distinct layers). In the higher latitudes (over Northern Canada) there does exist an almost isothermal region of 10 or more km in thickness, but it is found that over the equator there is a very sharp temperature inversion at the tropopause. In recent times, the stratosphere has come to be that portion of the atmosphere between 18 and 50 km with a stratopause occurring at approximately 50 km.

Continuing beyond the stratosphere there is the upper atmosphere. In the not too distant past this phrase has been used to denote the region of the atmosphere above 30 km (in general, the maximum attainable altitude of balloon-borne probes). It is most common today, however, to denote a middle atmosphere or mesosphere which extends above the stratopause from 50 to approximately 85 km with a mesopause occurring at about 85 km, and an exosphere or more recently named the thermosphere, at the very top where the most significant heating of the atmosphere occurs. Here the atmospheric density is so low that atmospheric particles rise and fall without collision in elliptic paths under the influence of gravity or following parabolic and hyperbolic trajectories, they will escape from the earth entirely. Thus, the thermosphere (exosphere) is accordingly the fringe region between the atmosphere and interplanetary space. The precise location of the upper edge of the thermosphere is of considerable interest and is at present not well-known by virtue of its very nature, but it is believed to extend well above 400 km.

1.3 SOME PROPERTIES OF THE ATMOSPHERE

1.3.1 Sea Level

At the surface of the earth, the following observations of the atmosphere have been made. The average composition of dry air at sea level is given in Table I.[7]

TABLE I

PERCENTAGE BY VOLUME OF ATMOSPHERIC GASES AT SEA LEVEL
(Dry Air)

Molecular Nitrogen	(N_2)	78.084%
Molecular Oxygen	(O_2)	20.9476%
Ozone	(O_3)	0-0.000007%
Argon	(Ar)	0.934%
Carbon Dioxide	(CO_2)	0.0314%
Molecular Hydrogen	(H_2)	0.00005%
Neon	(Ne)	0.001818%
Helium	(He)	0.000524%
Krypton	(Kr)	0.000114%
Nitrous Oxide	(N_2O)	0.0005%
Xenon	(Xe)	0.0000087%
Methane	(CH_4)	0.0002%

The actual composition at ground level is observed to vary slightly from the above with both position on the surface of the earth and time, and variable amounts of water vapor are always present. The sea level values of pressure, temperature, and density adopted in Ref. 7 are as follows:

$$P_o - 1.01325 \times 10^5 \text{ nt/m}^2$$

$$760 \text{ mm Hg}$$

$$\rho_o - 1.2250 \text{ kg/m}^3$$

$$T_o - 288.150^\circ\text{K} \quad .$$

The temperature, pressure, and density at the earth's surface are also variable with position and time but the extremes of variation about a mean value are quite limited on an absolute scale. The extreme variation in ground-level temperature is of the order of 15% of the mean in the middle latitudes and the deviation of atmospheric pressure and density are limited to a very small percent of the mean.

1.3.2 Temperature Variation With Altitude

Starting at sea level, temperature and pressure decrease almost, but not quite adiabatically with height through the troposphere until the temperature minimum of between approximately 200 and 230°K is reached at the tropopause, whose exact altitude and temperature value is variable with time and position over the earth.

The stratosphere exhibits a nearly constant temperature over an altitude region which is also variable with time and position.

Immediately above the stratosphere the atmospheric gases are heated by solar ultraviolet energy absorbed in the atmospheric ozone which is formed from atmospheric molecular oxygen by photochemical processes at the top of a layer sometimes called the ozonosphere which lies between 20 and 50 km. This heating causes a rise in temperature from the cold stratospheric value to a maximum of about 270°K at an altitude of approximately 50 km (the stratopause). Thereafter, the temperature falls again to a low value of the order of 200°K in the neighborhood of 80 km (mesopause). At still higher altitudes in the thermosphere the temperature rises again until it reaches an isothermal value of as low as 800°K at an altitude of approximately 200 km[8] during the quiet sun and as high as 1200-1300°K during the maximum solar activity. The exact nature of this heating process is not well known at present. However, several theoretical and experimental studies of the various physicochemical processes of this part of the atmosphere are being carried out with hopes that a proper explanation of the heating process will be determined. Figure I.1 graphically illustrates the various regions and temperature values of the atmosphere.

1.3.3 Variation of Atmospheric Pressure and Density With Altitude

Pressures and densities fall off roughly exponentially with altitude from sea level up to an altitude of approximately 110 km. This fact may by expressed mathematically by

$$P = P_0 e^{-2\beta(T)gH} \tag{I.1}$$

$$\rho = \rho_0 e^{-2\beta(T)gH} \quad . \tag{I.2}$$

Equations (I.1) and (I.2) are strictly correct if $\beta(T)$ is considered constant for a small range in altitude where the atmospheric temperature may be considered constant. An approximate value at sea level of $2\beta(T)g_0$ is 1/9.6 km^{-1} for which Eqs. (I.1) and (I.2) shows that all but approximately 35% of the total mass of the atmosphere lies below 9.6 km. Above 110 km the decrease of pressure and density is still exponential; however, the rates of decrease of these parameters is observed to be much less than at the lower altitudes.

A good mnemonic is that up to 110 km both pressure and density are divided by 10 for every 10-mile increase in altitude. Also, if pressure is expressed in millimeters of mercury (mm Hg) and density in grams per cubic meter, the numerical values of the two at a given altitude are about the same.

1.3.4 Electrical Properties of the Upper Atmosphere

The atmosphere may also be subdivided according to its electrical proper-

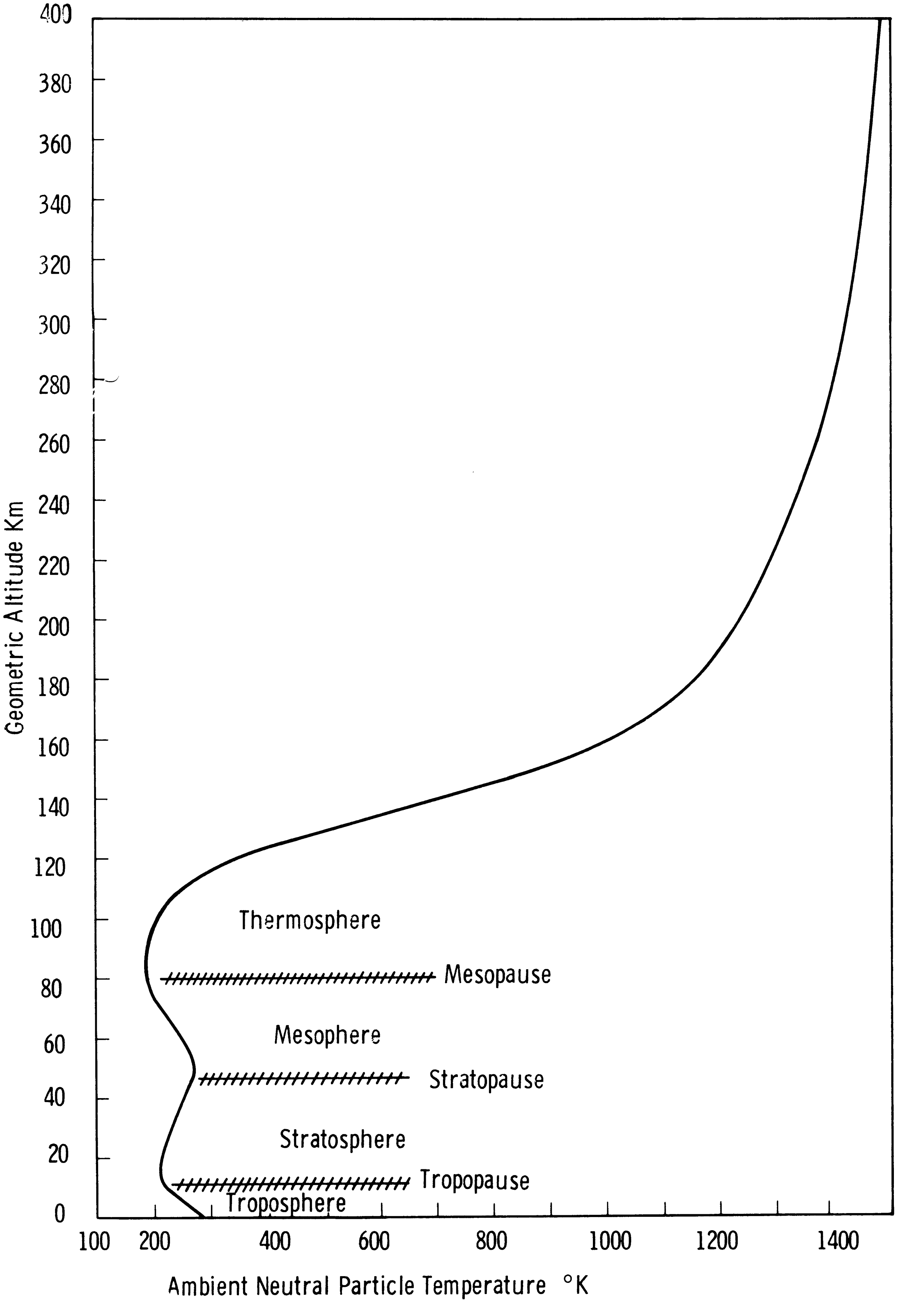

Fig. I.1. Ambient temperature vs. altitude U.S. standard atmosphere, 1962.

ties. From this point of view, the ionosphere is that portion of the atmosphere in which significant numbers of the constituent particles are charged. The total ionization in the ionosphere varies from altitude to altitude and several well-pronounced maxima are found. The first, or lowest altitude, of these layers which is sometimes found in the neighborhood of 60 km is the D-region. This region is often associated with strong radio fadeouts. Next in altitude are the E- and F-regions which are the most familiar and most important parts of the ionosphere.[9] During the day the F-region contains two layers designated by F_1 and F_2, which at night coalesce. The E-region diminishes at night and during the day may sometimes exhibit two layers, E_1 and E_2. From ground-based radio propagation studies and ejectable electrostatic-probe experiments the approximate height of the maximum ionization in the E-region is placed somewhere between 90 and 120 km, that of F_1 appears at about 175 km, with that of F_2 at approximately 300 km. Sudden intense clouds of ionization often appear within the E-region and are referred to as sporadic E. Their cause is not well-known, but it is believed that meteors may be one cause of the ionization of the sporadic E. The direct measurement of the properties of the ionosphere is relatively young and is the subject of an intense research effort by many laboratories including the Space Physics Research Laboratory.

1.3.5 Winds in the Upper Atmosphere

Many observations by various techniques (long-enduring meteor trains, radar-chaff, sodium vapor trails, etc.) indicate the existence of winds in the upper atmosphere. These observations show that adjacent layers in the high atmosphere often have winds of quite different speeds and directions. Vertical components are also observed in some of the winds. These high-altitude winds sometimes exhibit velocities of several hundred kilometers per hour with average values between 100 and 200 km/hr. Radar studies of meteor trails confirm and supplement visual observations and extend the results to much higher altitudes.[10] Rocket-borne techniques for measuring wind are presently being worked out but results are still fragmentary. One notable characteristic of high-altitude winds is their seeming correspondence to and dependence on the atmospheric temperature profile.[11] It is found that at points of temperature inversion wind shears between adjacent layers are usually present and of a very violent nature.

1.4 CONCLUDING REMARKS

It is realized that the foregoing descriptive treatment of the atmosphere has been very general and has offered little in the way of explanation as to the cause of the observed phenomena. It is hoped, however, that it has served to stimulate the interest of the reader in the subject and give some basis for further study. To pursue this study, the interested reader is referred to the references at the end of this chapter.

References for Chapter I

1. Sicinski, H. S., Spencer, N. W., and Dow, W. G., "Rocket Measurement of Upper Atmosphere Ambient Temperature and Pressure in the 30 to 70 Kilometer Region," *Journal of Applied Physics*, *25*, 2, 161-168, Feb., 1954.

2. Spencer, N. W., Brace, L. H., and Carignan, G. R., "Electron Temperature Evidence for Non Thermal Equil. in the Ionosphere," *J.G.R.*, *67*, p. 157, 1962.

3. Horvath, J. J., Simmons, R. W., and Brace, L. H., "Theory and Implementation of the Pitot-Static Technique for Upper Atmospheric Measurements," Office of Research Administration, The University of Michigan, *Scientific Report No. NS1*, March 1962.

4. Newell, H. E., *High Altitude Rocket Research*, Academic Press, New York, 1953.

5. Ratner, B., *Temperature, Pressure, and Relative Humidity Over the United States and Alaska*, U.S. Department of Commerce, Washington, D.C., 1945.

6. Humphreys, W. J., Composition of the Atmosphere. *International Critical Tables*, Vol. I, McGraw-Hill Book Company, New York, 1926.

7. Dubin, M., Sessenwine, N., and Wexler, H., *U.S. Standard Atmosphere, 1962*, U.S. Government Printing Office, Washington, D.C., Dec., 1962.

8. Results of NASA 6.06 Univ. of Mich., and NASA GSFC.

9. Mitra, S. K., *The Upper Atmosphere*, 2nd ed. The Asiatic Society, Calcutta, 1952.

10. Malone, T. F., ed., *Compendium of Meteorology*. American Meteorological Society, Boston, 1951.

11. Murgatroyd, R. J., "Winds and Temperatures Between 20 Km and 100 Km—A Review," *Quarterly Journal of the Royal Meteorological Society*, *83*, No. 358, pp. 417-454, Oct., 1957.

CHAPTER II

A COMPARISON AND BRIEF DESCRIPTION OF UPPER ATMOSPHERE NEUTRAL PARTICLE MEASUREMENTS

2.1 CLASSIFICATION OF NEUTRAL UPPER ATMOSPHERE MEASUREMENTS

In the investigation of the neutral upper atmosphere the parameters of interest to the experimenter are ambient pressure, temperature, density, and mass motion or wind in the atmosphere. Thus, the study of the upper atmosphere by rocket-borne techniques has brought about many different approaches to the design and implementation of neutral particle measurements. Among the more common theoretical approaches taken in the problem of experiment design are those involving the theory of sound propagation, the theory of gaseous diffusion and dispersion, and basic aerodynamic theory. Thus, experiments which yield measurements of the parameters of the neutral upper atmosphere may be classified according to the theoretical concepts upon which they are based. As will be seen, these experiments involve straightforward applications of well-known theoretical concepts or simple extensions of these well-known ideas and will fall into the categories mentioned above.

2.2 COMMON NEUTRAL PARTICLE EXPERIMENTS

As indicated, several rocket-borne techniques have proved very useful in the direct measurement of the upper atmosphere above balloon altitudes. Among the more successful of these is the rocket-grenade experiment which is based on the theory of sound propagation in a gaseous media such as the atmosphere and is well suited to determining an average temperature profile in the upper atmosphere.

A second technique, which has been highly successful and is well adapted to the measurement of winds, is the sodium release experiment which is based on gaseous diffusion and dispersion of a sodium vapor trail left by an ascending rocket under the influence of atmospheric wind.

A third experiment which is based on aerodynamic theory involves the measurement of the atmospheric drag on a sphere which is ejected from an ascending rocket and allowed to fall freely back to earth. The measurement of the drag encountered by the falling sphere leads to the determination of atmospheric density.

A fourth technique, the pitot-static probe, also based on aerodynamic theory, uses pressure measurements made at suitable points on a rocket surface to obtain the ambient pressure, temperature, and density in the upper stratosphere, mesosphere, and lower thermosphere.

A fifth experimental approach based on aerodynamic theory and particularly well suited to determination of ambient pressure, temperature, and density at very high altitudes is the thermosphere probe. This device makes use of a cyclic pressure measurement at a suitable point on a tumbling cylindrical probe ejected from an ascending rocket vehicle.

2.3 DESCRIPTION OF COMMON NEUTRAL PARTICLE EXPERIMENT

2.3.1 The Rocket Grenade Experiment

In this experiment,[1] which is based on the speed of propagation of sound through the atmosphere, it is assumed that the ratio of specific heat at constant pressure to that at constant volume, and the average molecular mass or composition of the atmosphere remains essentially unchanged with altitude up to approximately 90 km. Under these conditions it is well-known that the speed of a sound wave traveling through the atmosphere is a function only of the atmospheric temperature.[2] This functional relationship forms the theoretical basis of the rocket-grenade experiment which leads to a determination of an average upper atmosphere temperature profile.

The implementation of this experiment requires that grenades be ejected from a flying rocket and exploded at high altitude. Both the times and locations in space of the explosions are accurately measured. The time of arrival of the sound from each grenade at the earth's surface is also measured accurately at each of several listening stations arranged on the ground directly below the rocket.

In the ideal case, all grenades would be exploded directly above a listening station, the sound traveling vertically downward to the recording point or listening station. Thus, if A is a listening station on the ground, B a point of explosion vertically above A, and C the position of a second explosion vertically above B, then the sound path from C to A would overlap that of B to A along BA. The transit time of the sound wave from the second explosion minus that of the wave from the first, would then give the transit time from B to C. This transit time could then be used along with the known distance between B and C to calculate the average velocity of sound on the path BC, from which, through the theoretical relation between speed of sound and temperature, and average temperature between B and C can be derived.

The practical implementation of the experiment, however, departs from the above idealized situation. The explosion points may be displaced by several degrees away from the vertical over the listening stations. In this real case, the sound wave undergoes continuous refraction (turning) on its way downward toward the recording point, and accordingly follows a curved rather than straight path from the point of explosion to the listening point. In such a case, the various sound paths do not overlap thus making the use of other more limiting assumptions necessary in the derivation of the average temperature profile from this technique.

There is, in addition to the above practical limitation, still another effect which must be considered. It is known that appreciable atmospheric winds exist at various altitudes along the sound path. Wind components along the path change the apparent velocity of sound, whereas components of wind perpendicular to the path alter its curvature. Thus, if not corrected for, winds can introduce errors into the determination of air temperatures.

Since winds have a directional effect, their influence on the sound path can be separated from that of temperature by measuring not only the times but also angles of arrival of the sound at the various listening stations. With such data and balloon determination of wind and temperature to as high an altitude as possible, the still higher altitude winds can be estimated and taken into account as a correction factor in the determination of temperature by the rocket grenade technique.

This experiment has been flown many times by various experimenters with great success in determining average atmospheric temperature profiles to an altitude of approximately 90 km where it becomes very difficult to receive the sound from the grenade burst because of atmospheric attenuation of the sound wave. Also, the experiment has, with some degree of success, given an estimate of atmospheric winds in the upper regions to approximately 90 km.

2.3.2 The Sodium Release Experiment

As mentioned earlier, this experiment is used in determining upper atmosphere wind profiles.

The implementation of this technique requires that a rocket nose cone be equipped with a canister containing sodium metal and a remotely controlled release valve. As the ascending rocket reaches the altitude at which measurement is to commence, the valve in the canister is opened, thus allowing the sodium to escape into the rare atmosphere where the sodium vaporizes producing a continuous trail of sodium vapor behind the rocket vehicle. This trail is then very accurately photographed from the ground by several precision motion picture cameras from several different locations. Thus, by comparison of the various films a triangulation scheme is used to determine the rate and direction of the dispersion of the sodium vapor trail which yields a direct determination of the upper atmosphere wind which caused the dispersion.

A great number of these experiments have been used and have added much to our present-day knowledge of the upper atmosphere winds to altitudes well in excess of 200 km.

This technique, because of its dependence upon ground based photography for data acquisition is severely limited for a number of reasons. First, a very accurately surveyed launch area is required for the triangulation procedure used in data analysis. Second, in order to assure data acquisition, very severe limits are placed upon allowable cloud cover. Finally, to assure adequate and proper illumination of the vapor trail, launchings must be scheduled for either sunset or sunrise.

2.3.3 The Falling Sphere Experiment

The theoretical basis for this experiment is found in basic aerodynamics; for it is well known that the drag force encountered by a body moving through a fluid media is a direct function of the fluid density, the square of the relative velocity, and the shape of the body. Mathematically this expression would be

$$D = 1/2 \, \rho \, V^2 \, C_D S \qquad \text{(II.1)}$$

where

D is the drag force

ρ is the fluid density

V is the relative velocity

C_D is a dimensionless drag coefficient (a function primarily of the body shape)

S is the cross-sectional area of the body.

In addition to this, Newton's second law of motion states that the sum of the forces acting on a body is equal to the product of the mass of the body and its net acceleration, or:

$$D - mg = m\frac{dV}{dt} \qquad \text{(II.2)}$$

where

D is the drag force

m is the mass of the body

g is the acceleration due to gravity

$\frac{dV}{dt}$ is the rate of change of velocity with respect to time or acceleration.

Now combining Eqs. (II.1) and (II.2) and solving

$$\rho = \frac{2m\left(\frac{dV}{dt} + g\right)}{V^2 C_D S} \quad . \qquad \text{(II.3)}$$

Thus, Eq. (II.3) gives a precise mathematical statement of atmospheric density in terms of known quantities m, g, C_D, and S, and quantities to be measured, V, and dV/dt.

The implementation of the experiment is carried out by ejecting a spherical body, whose drag coefficients, cross-sectional area, and mass is known, from an ascending rocket and allowed to fall freely back to earth. During the period of free fall the velocity altitude and acceleration of the sphere are carefully determined. The velocity and altitude are determined by ground-based radar tracking and the acceleration is determined by an on-board accelerometer in the case of the active falling sphere, or by differentiation of the velocity time profile from radar in the case of the passive falling sphere.

Through the use of Eq. (II.3) and the velocity-acceleration profile, the atmospheric density as a function of altitude may be deduced.

It is easily seen that the falling sphere experiment is very modest in its requirement of ground support equipment and thus highly portable.

This experiment has been used extensively with good results and is presently limited to a maximum altitude of approximately 100 km where, owing to the insensitivity of the device to extremely small drag forces, and inadequate understanding of the very low density drag coefficients, the derived densities are apparently meaningless.

2.4 PURELY AERODYNAMIC UPPER ATMOSPHERIC MEASUREMENTS

As with the development of any physical science the need for ever increasing range of measurement and basic accuracy imposes itself on those involved in the science. The pitot-static probe and the thermosphere probe have been developed,[3,4] to this end, through the pioneering efforts of the Naval Research Laboratories, NASA, and The University of Michigan.

The pitot-static probe and thermosphere probe techniques are based upon a purely aerodynamic approach to the measurement of pressure, temperature, and density in the upper atmosphere, and is the subject of subsequent chapters of this report. In these following chapters a detailed development of the theory, implementation, and subsequent data analysis for the pitot-static probe will be given.

References for Chapter II

1. Newell, H. E., High Altitude Rocket Research, Academic Press, New York, 1953.

2. Liepmann, H. W., and Puckett, A. E., Aerodynamics of a Compressible Fluid, John Wiley and Sons, Inc., New York, 1947.

3. Horvath, J. J., Simmons, R. W., and Brace, L. H., "Theory and Implementation of the Pitot-Static Technique for Upper Atmospheric Measurements," Scientific Report No. NS1, ORA, The University of Michigan, March, 1962.

4. Ainsworth, J. E., Fox, D. F., and LaGow, H. E., "Upper Atmosphere Structure Measurement Made With the Pitot-Static Tube," J.G.R., 66(10), 3191-3211, 1961.

CHAPTER III

THEORY OF AERODYNAMIC UPPER ATMOSPHERE NEUTRAL PARTICLE MEASUREMENT

3.1 INTRODUCTION

In this chapter the theoretical basis for the interpretation of aerodynamic pressure measurements made at suitable points on a rocket-borne probe surface in terms of the ambient properties of the upper atmosphere will be discussed and the governing equations developed. These equations will be developed in detail starting with well-known aerodynamic and gas dynamical concepts, with this development leading to general equations which are readily specialized to the interpretation of a measured quantity in terms of one or more of the desired properties of the atmosphere.

In light of the above, certain models of the gaseous media which is known as the upper atmosphere must be presented, and a mathematical analysis of this model made. The results of such analysis leads then to the desired concepts and governing equations.

To facilitate the readers' understanding of the next few sections, the following definitions of important terms are presented.

1. Density.—The total mass of the molecules in a unit volume is defined as the density of the gas. Mass is defined as weight divided by the local acceleration of gravity.

2. Pressure.—When molecules strike a surface, they rebound; and, by Newton's law, the surface experiences a force equal and opposite to the time

rate of change of momentum of the rebounding molecules. Pressure is defined as the force per unit area exerted on a surface immersed in the fluid. Because the molecular bombardment of the surface is considered perfectly elastic we have a first important result: fluid pressure acts normal to a surface. As will be shown in a later section a second important result concerning fluid pressure is: the pressure is proportional to the kinetic energy of the random motion per unit volume.

3. Temperature.—According to the kinetic theory of gases, the absolute temperature is defined to be proportional to the mean kinetic energy of the molecules. It will also be shown to be interpreted in terms of the Equation of State for an ideal gas as

$$P = \rho RT$$

where P and ρ are the pressure and density, R is the gas constant per unit mass, and T is the absolute temperature. Thus, for systems in which the density remains constant, such as the addition of heat, that increase in kinetic energy of random motion will increase the temperature and pressure by proportional amounts.

3.2 THE EQUATION OF STATE AND MEAN FREE PATH

3.2.1 The Maxwell-Boltzmann Velocity Distribution Law

As has been shown the earth's atmosphere consists of a gaseous mixture of several constituents and exhibits certain measurable properties such as pressure, temperature, density, etc. In order to further understand these properties, a theoretical model has been postulated and several of these known properties have been linked together on the basis of this theoretical model. In order to set down a concrete theory for aerodynamic atmospheric measurements, these concepts must be called upon. The first of these shall be the concept of a molecular velocity distribution function.

As a good approximation to the actual gaseous atmosphere let us consider a finite volume of an ideal gas. A finite volume of an ideal gas consists of a huge number of individual molecules of negligible size. These molecules are in random motion and collide with each other and the walls of the containing vessel all the time and at a very high rate. In order to represent mathematically the behavior of this gas we must adopt a molecular point of view where the individual molecule is the important physical quantity. A method of representing this molecular motion is to use a velocity phase space rather than the physical space and use a velocity distribution function which gives the density of the gas molecular velocities through the spectrum of all possible velocities.

Consider the velocity space of Fig. III.1a where

$$|\vec{v}| = \sqrt{v_x^2+v_y^2+v_z^2} \quad .$$

If at a given time the velocities of all of the molecules could be found, then the behavior of the gas would be known. This leads to the determination of a velocity distribution function $f(v_x,v_y,v_z)$ which gives the density of the molecular velocities through the spectrum. Again let us consider the above velocity space with several velocities of individual molecules shown, as in Fig. III.1b, and further we see that

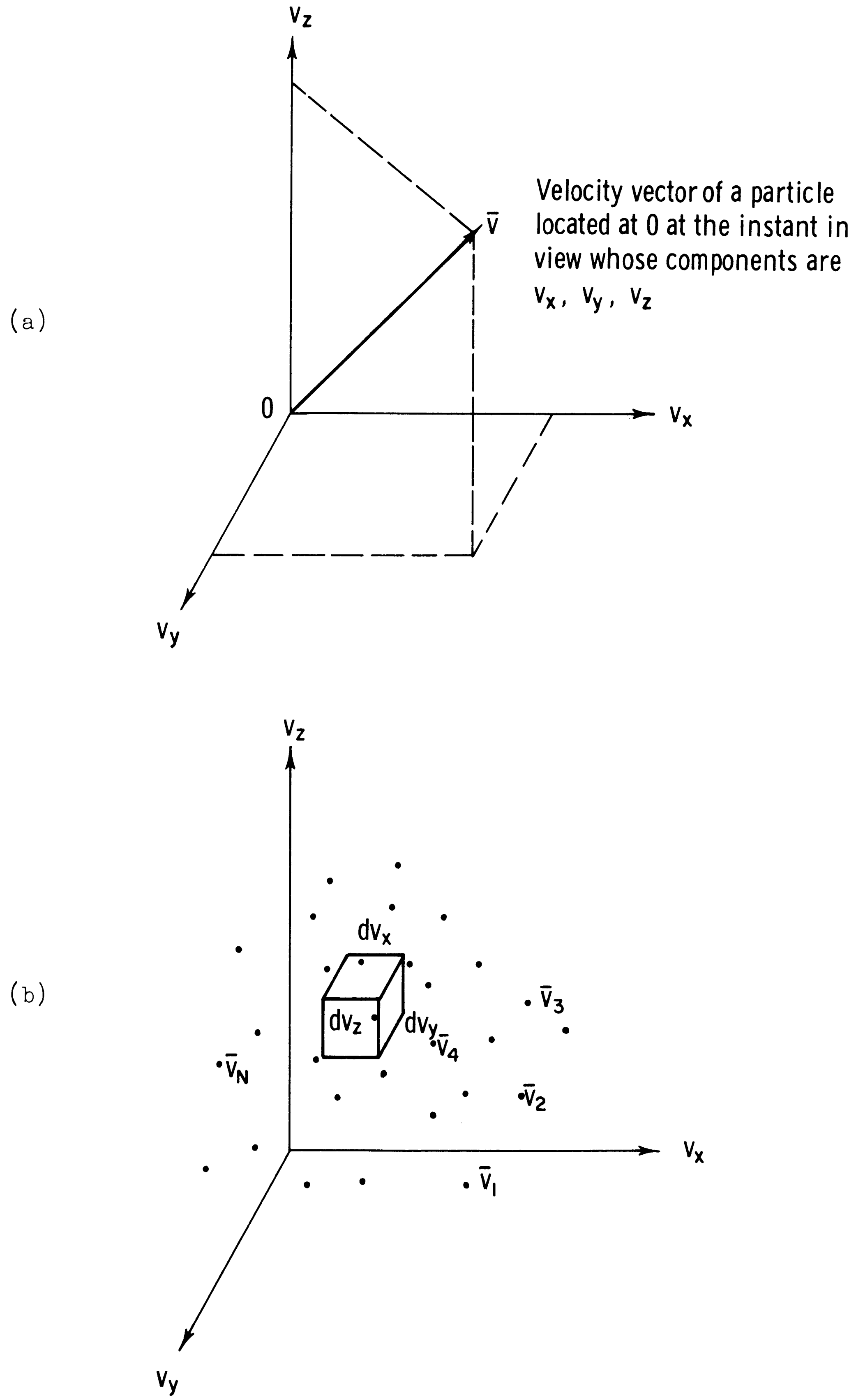

Fig. III.1. Velocity space retangular coordinates.

$$dv = dv_x dv_y dv_z \quad .$$

Now $f(v_x,v_y,v_z)dv_x dv_y dv_z$ equals the fraction of the number of molecules per unit volume whose velocity lies in the range

$$v \text{ and } v + dv$$

or

$$\begin{aligned} &v_x, v_x + dv_x \\ &v_y, v_y + dv_y \\ &v_z, v_z + dv_z \quad . \end{aligned}$$

Further, if we let n be the number density of the molecules or the number of molecules per unit volume then $nf(v_x,v_y,v_z)dv_x dv_y dv_z$ equals the number of molecules per unit volume whose velocity lies in the range v and v+dv. It is easily seen that f(v), the distribution function, represents the probability of the velocity of a molecule lying between v and v+dv.

To determine f(v) we will follow the approach of Maxwell,[1,2] which employs the following postulate:

$$f(v) \equiv f(v_x,v_y,v_z) = f_1(v_x)f_1(v_y)f_1(v_z) \qquad \text{(III.1)}$$

where $f_1(v_x)$ represents the distribution function of the x component of molecular velocity. Physically this postulate implies; (1) the velocity components, v_x, v_y, and v_z, are statistically independent; and (2) the distribution function, f(v), is isotropic (spherically symmetric).

Working now with Eq. (III.1) after taking the logarithm of both sides we see

$$\ln f(v) = \ln f_1(v_x) + \ln f_1(v_y) + \ln f_1(v_z) \quad .$$

Taking the derivative of the above equation with respect to v_x where

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

gives

$$\frac{f'(v)}{f(v)} \frac{\partial v}{\partial v_x} = \frac{f_1'(v_x)}{f_1(v_x)}$$

but

$$\frac{\partial v}{\partial v_x} = \frac{v_x}{v}$$

thus

$$\frac{1}{v} \frac{f'(v)}{f(v)} = \frac{1}{v_x} \frac{f_1'(v_x)}{f_1(v_x)} \quad .$$

Introducing the notation

$$\Phi(v) = \frac{1}{v} \frac{f'(v)}{f(v)}$$

$$\Phi_1(v) = \frac{1}{v_x} \frac{f_1'(v_x)}{f_1'(v_x)}$$

we see that

$$\Phi(v) = \Phi_1(v) \quad . \qquad \text{(III.2)}$$

Now taking the derivative of Eq. (III.2) with respect to v_y or v_z leads to

$$\Phi'(v) = 0 \quad .$$

Integrating

$$\Phi(v) = -2\beta$$

where β is a constant. From Eq. (III.2)

$$\Phi_1(v_x) = \frac{1}{v_x}\frac{f_1'(v_x)}{f_1(v_x)} = -2\beta \quad . \qquad \text{(III.3)}$$

Integrating Eq. (III.3)

$$\ln f_1(v_x) = \alpha - \beta v_x^2$$

where α and β are constants. Finally

$$f_1(v_x) = e^{\alpha-\beta v_x^2} = e^{\alpha} e^{-\beta v_x^2}$$

or

$$f_1(v_x) = A\, e^{-\beta v_x^2} \qquad \text{(III.4)}$$

where

$$A = e^{\alpha} \quad .$$

Now from Eq. (III.1) the form of the distribution function f(v) becomes

$$f(v) = A^3 e^{-\beta(v_x^2+v_y^2+v_z^2)} \quad . \qquad \text{(III.5)}$$

To normalize Eq. (III.5) we must determine $A = e^{\alpha}$. We know that

$$nf(v)dv = n\,A^3 e^{-\beta(v_x^2+v_y^2+v_z^2)} dv_x dv_y dv_z = dn$$

where dn is the fraction of the number of molecules per unit volume whose velocity lies between v and v+dv. Now integrating over the entire range of values of v_x, v_y, and v_z,

$$\iiint_{-\infty}^{\infty} nf(v)dv = \iiint_{-\infty}^{\infty} n\,A^3 e^{-\beta(v_x^2+v_y^2+v_z^2)} dv_x dv_y dv_z = n$$

we find that

$$A = \left(\frac{\beta}{\pi}\right)^{1/2} .$$

Thus,

$$f(v) = \left(\frac{\beta}{\pi}\right)^{3/2} e^{-\beta v^2} . \qquad \text{(III.6)}$$

Equation (III.6) represents the basic form of the Maxwellian velocity distribution function, but because the use of this function often involves rather complicated integrals, it is more convenient to express it in a slightly different form which simplifies the usual integration problem. To facilitate this change we shall define the spherical polar coordinate system shown in Fig. III.2. The volume element in this velocity space is then given by

$$dv = v^2 \sin\Theta \, d\Theta \, d\phi \, dv$$

and

$$f(v)dv = \left(\frac{\beta}{\pi}\right)^{3/2} e^{-\beta v^2} v^2 \sin\Theta \, d\Theta \, d\phi \, dv$$

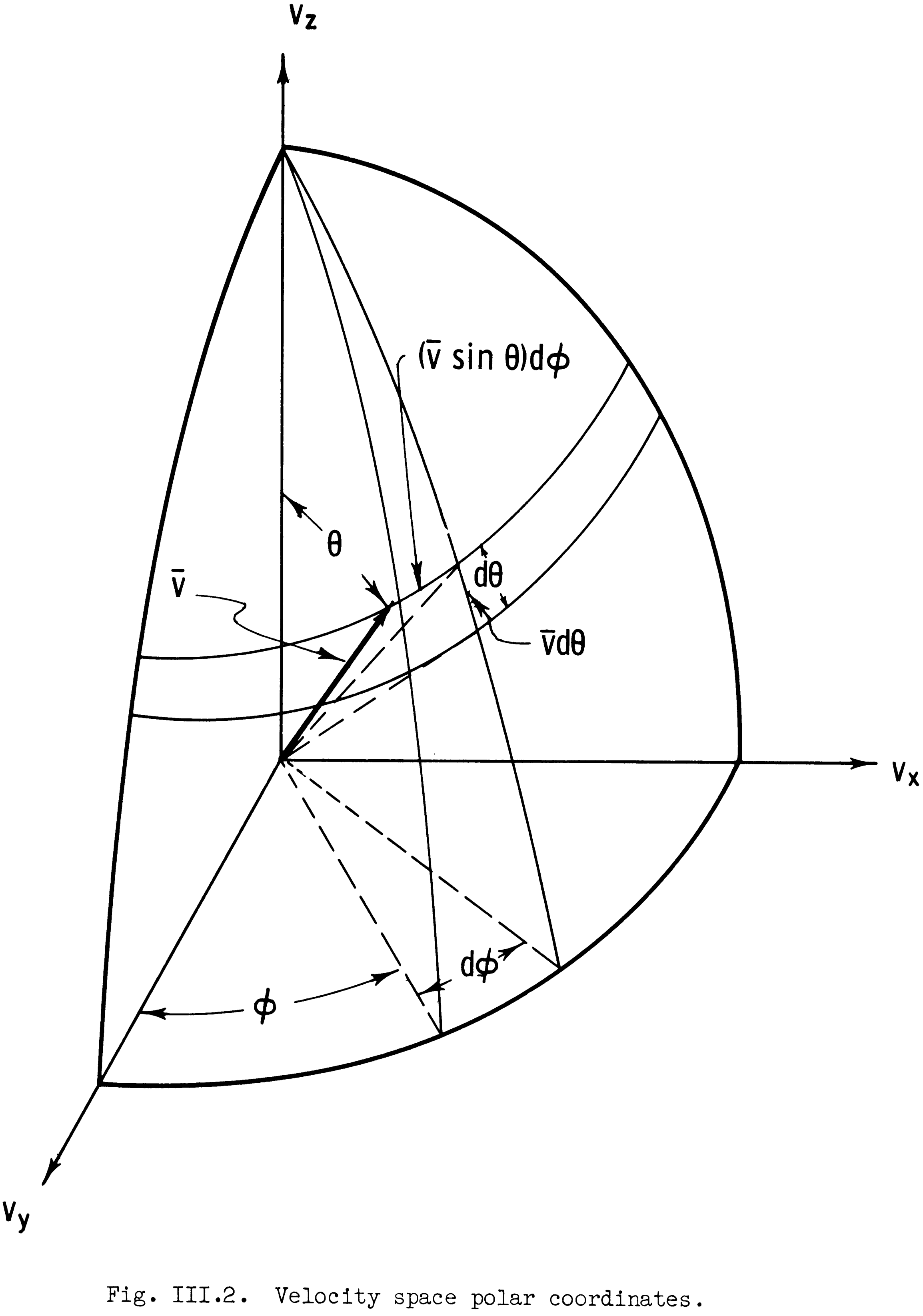

Fig. III.2. Velocity space polar coordinates.

define

$$F(v)dv = \int_0^{\pi} \sin \theta \, d\theta \int_0^{2\pi} d\phi \left(\frac{\beta}{\pi}\right)^{3/2} e^{-\beta v^2} v^2 dv$$

carrying out the integration

$$f(v)dv = 4\pi \left(\frac{\beta}{\pi}\right)^{3/2} e^{-\beta v^2} v^2 dv \tag{III.7}$$

Equation (III.7) now represents the fraction of the molecules per unit volume whose speed lies between v and v+dv.

Equations (III.6) or (III.7) may now be used to calculate some average quantities of the gas we are dealing with such as the mean kinetic energy of the molecules:

$$\overline{1/2 \, mv^2} = 1/2 \, m\overline{v^2}$$

which is the mathematical expression of mean kinetic energy, where m is the molecular mass. Thus, we see that the problem reduces to that of obtaining an expression for $\overline{v^2}$ (the mean square velocity) in terms of the distribution functions.

Recalling that

$$\overline{v^2} = \int_0^{\infty} v^2 F(v) dv$$

which upon substitution of Eq. (III.7) becomes

$$\overline{v^2} = \int_0^{\infty} v^4 4\pi \left(\frac{\beta}{\pi}\right)^{3/2} e^{-\beta v^2} dv \quad .$$

Integrating we obtain

$$\overline{v^2} = \frac{3}{2}\frac{1}{\beta} \quad .$$

Thus, the mean kinetic energy of a molecule is given by

$$\frac{1}{2}m\overline{v^2} = \frac{1}{2}m\frac{3}{2}\frac{1}{\beta} . \tag{III.8}$$

At this time it is convenient to introduce the concept of absolute temperature and relate this quantity to the mean kinetic energy. This relationship stated mathematically is:

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

where

m is the molecular mass

k is Boltzmann's constant or, the gas constant per molecule

T is the absolute temperature.

This theoretical relationship along with Eq. (III.8) leads to the determination of the constant β.

$$\frac{1}{2}m\overline{v^2} = \frac{1}{2}m\frac{3}{2}\frac{1}{\beta} = \frac{3}{2}kT .$$

Thus,

$$\beta = \frac{m}{2kT} . \tag{III.9}$$

Substituting Eq. (III.9) into Eqs. (III.6) and (III.7) we obtain the Maxwell-Boltzmann velocity and speed distribution function in terms of the absolute temperature, T.

$$f(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} \tag{III.10}$$

and

$$F(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} v^2 \quad . \qquad \text{(III.11)}$$

These equations represent the most frequently encountered forms of the Maxwellian distribution law, and have been verified experimentally to very accurately represent the actual velocity distribution for a real gas.

3.2.2 Equation of State

The results of the previous derivation may now be utilized to develop an equation of state for an ideal gas. To facilitate this development let the physical model, illustrated by Fig. III.3, be considered.

To calculate the pressure exerted on the walls of the container we must calculate the force applied to the wall through momentum exchange from the colliding molecules. That is, an expression must be obtained for the momentum transfer to area, S, per unit time.

Now, the condition on the molecule at time, $t = o$, to insure collision with the wall at S in time increment, δt, is that it have velocity component, v_x, such that

$$h \leq v_x \, \delta t$$

or another way of saying this is that at time, $t = o$, the molecule with velocity component, v_x, must be with a volume

$$hS \leq v_x \, \delta t \, S \quad .$$

Further, the number of molecules, N_{v_x}, in the volume V, of the container having a component of velocity, v_x, between v_x and v_x+dv_x is given by

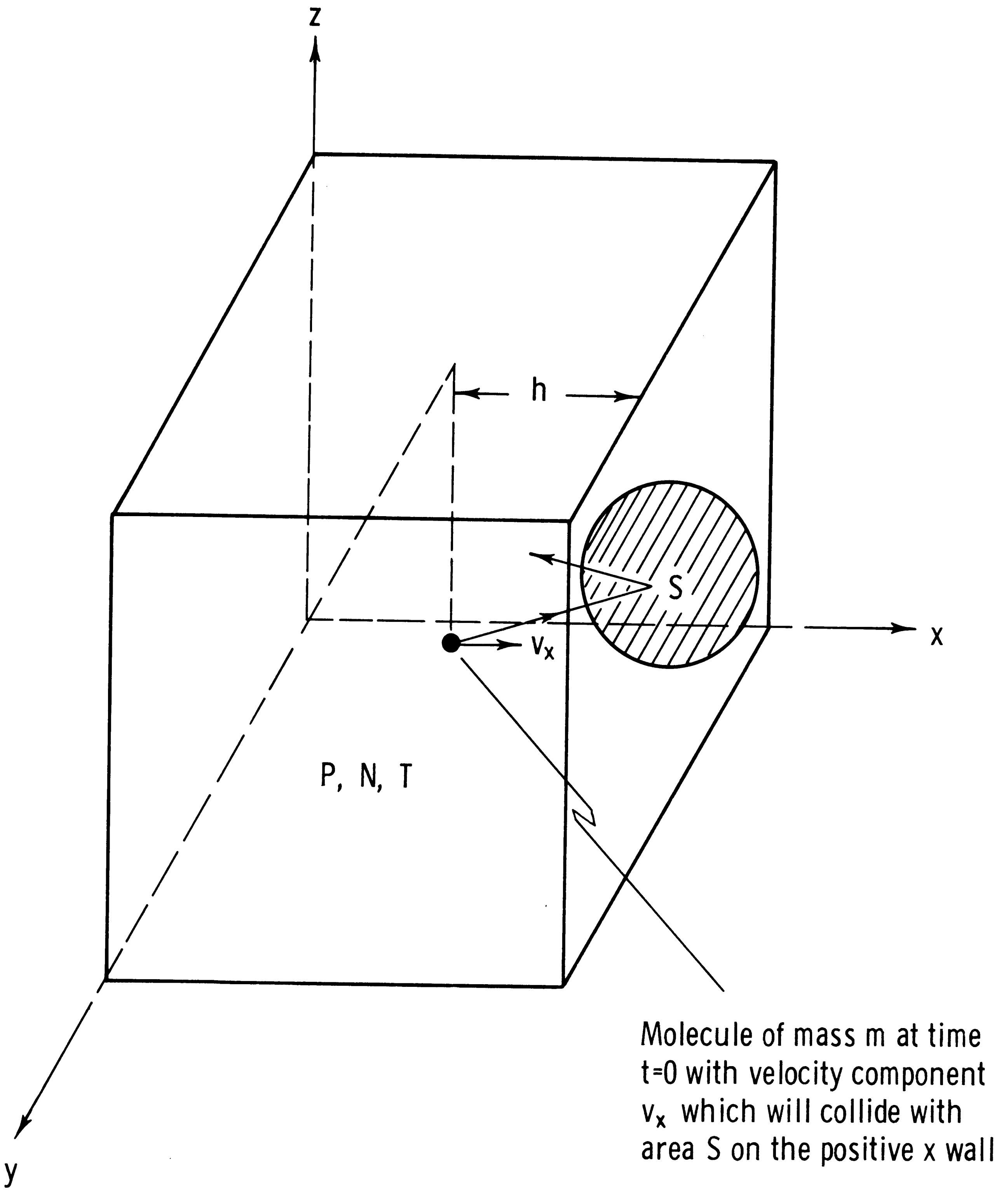

Fig. III.3 The ideal gas law model.

$$N_{v_x} = nf_1(v_x)dv_x \quad .$$

And the number of molecules hitting surface, S, in time, δt, N_{v_x}, S is given by

$$N_{v_x,S} = v_x\ \delta t\ S\ nf_1(v_x)dv_x \quad .$$

The momentum transferred to S by the individual molecules specified by $N_{v_x,S}$ is

$$dM_{v_x,S} = 2m\ v_x\ v_x\ S\ \delta t\ nf_1(v_x)dv_x \quad .$$

The total momentum transferred to S in δt is found by integrating the above over the positive range of values of v_x.

$$M_{v_x,S} = \int_0^\infty 2mv_x^2\ S\ \delta t\ nf_1(v_x)dv_x$$

$$M_{v_x,S} = 2nm\ S\ \delta t \int_0^\infty v_x^2\ f_1(f_x)dv_x$$

but

$$\int_0^\infty v_x^2\ f_1(v_x)dv_x = \frac{1}{2}\ \overline{v_x^2} \quad .$$

Therefore,

$$M_{v_x,S} = 2n\ S\ \delta t\ \frac{1}{2}\ m\overline{v_x^2} \quad .$$

Recalling that pressure is given by force per unit area and force is given by total exchange of momentum per unit time, we see that the pressure, P, exerted on the walls of the container can be expressed by

$$P = \frac{M_{v_x},S}{S\ \delta t}$$

or

$$P = 2n \frac{1}{2} m\overline{v_x^2} \quad .$$

From the previous expression of the absolute temperature and the theory of equipartition of energy we also have that if

$$\frac{1}{2} m\overline{v^2} = \frac{3}{2} kT$$

then

$$\frac{1}{2} m\overline{v_x^2} = \frac{1}{2} kT \quad .$$

Substituting this in the above expression for P

$$P = nkT \quad .$$

Now if we define

$nm = \rho$ the mass density

and

$\frac{k}{m} = R$ the gas constant per unit mass.

We have as the final form of the equation of state for an ideal gas:

$$P = \rho RT \qquad \text{(III.12)}$$

where

P is the pressure (force per unit area)

ρ is the mass density of the gas

R is the constant per unit mass

T is the absolute temperature.

Thus, we have a very simple equation which links the three properties of a gas to each other. Experiments with real gases show that this equation accurately predicts the behavior of the gas to within a very minute percent error over a large range of conditions of temperature, pressure, and density.

3.2.3 Mean Free Path

Up to now, it has been apparent that molecular collisions must take place, but the details of such collisions have been neglected. Let us now consider the molecular model in which the following assumptions are made: (1) all molecules are spherically symmetric and undeformable; and (2) the force of interaction is in the direction of the line of centers of the two molecules, and its magnitude depends on the distance between the two (i.e., a central force system).

Under the conditions of the above assumptions and following the further development of the Kinetic Theory of Gases,[1,2] one is lead to the following results:

$$\bar{\tau} = \frac{1}{\sqrt{2}\, n(\pi d^2)\bar{v}} \qquad \text{(III.13)}$$

where

τ is the mean free time between intermolecular collisions in an ideal gas consisting of spherical elastic molecules

n is the number density of the gas

d is the mean molecular diameter

$\bar{v}$ is the mean velocity of the molecules given by $\sqrt{\frac{8kT}{\pi m}}$

and

$$\bar{\lambda} = \frac{\bar{v}}{\sqrt{2}\, n(\pi d^2)\bar{v}} = \frac{1}{\sqrt{2}\, n\pi d^2}$$

or

$$\bar{\lambda} = \frac{m}{\sqrt{2}\, \rho\pi d^2} \qquad \text{(III.14)}$$

where

$\bar{\lambda}$ is the mean free path length between intermolecular collisions in an ideal gas consisting of spherical elastic molecules

ρ is the mass density of the gas

m is the molecular mass

d is the mean molecular diameter

$\bar{v}$ is mean molecular velocity.

The above results are obtained under the influence of some rather restrictive assumptions, but are found to be very good first-order approximations to the phenomena observed in real gases.

3.3 THE HYDROSTATIC LAW

Considering the atmosphere surrounding the earth and the effects of the gravity field of the earth on the atmospheric gases, a very important physical result is obtained. Namely, that known as the hydrostatic law.

Now, except for mass motions, such as winds and convection, the atmospheric pressure, as defined previously, is in equilibrium with the forces of gravity. That is, in an atmosphere at rest the pressure at a specific point is equal to the weight of the gas in a vertical column of unit cross-sectional area above that point. Thus, the difference between two altitude levels is due to the weight of the air contained between those levels.

To mathematically show the above, consider Fig. III.4 in which a unit cross-sectional cylinder of atmospheric gas between the geometric altitudes H and H+dH above the surface of the earth where the acceleration due to gravity, g, is shown. The weight of gas contained in the incremental volume 1 dH between H_1 and H_2 is 1 ρg dH, acting at the center of the elementary volume. Equating these forces, which is required by equilibrium, we obtain

$$P = \left(P + \frac{\partial P}{\partial H} dH \right) + \rho g \, dH \quad .$$

Cancelling common terms and rearranging

$$\frac{\partial P}{\partial H} = - \rho g$$

and since we will assume

$$P = P(H)$$

then

$$\frac{\partial P}{\partial H} = \frac{dP}{dH}$$

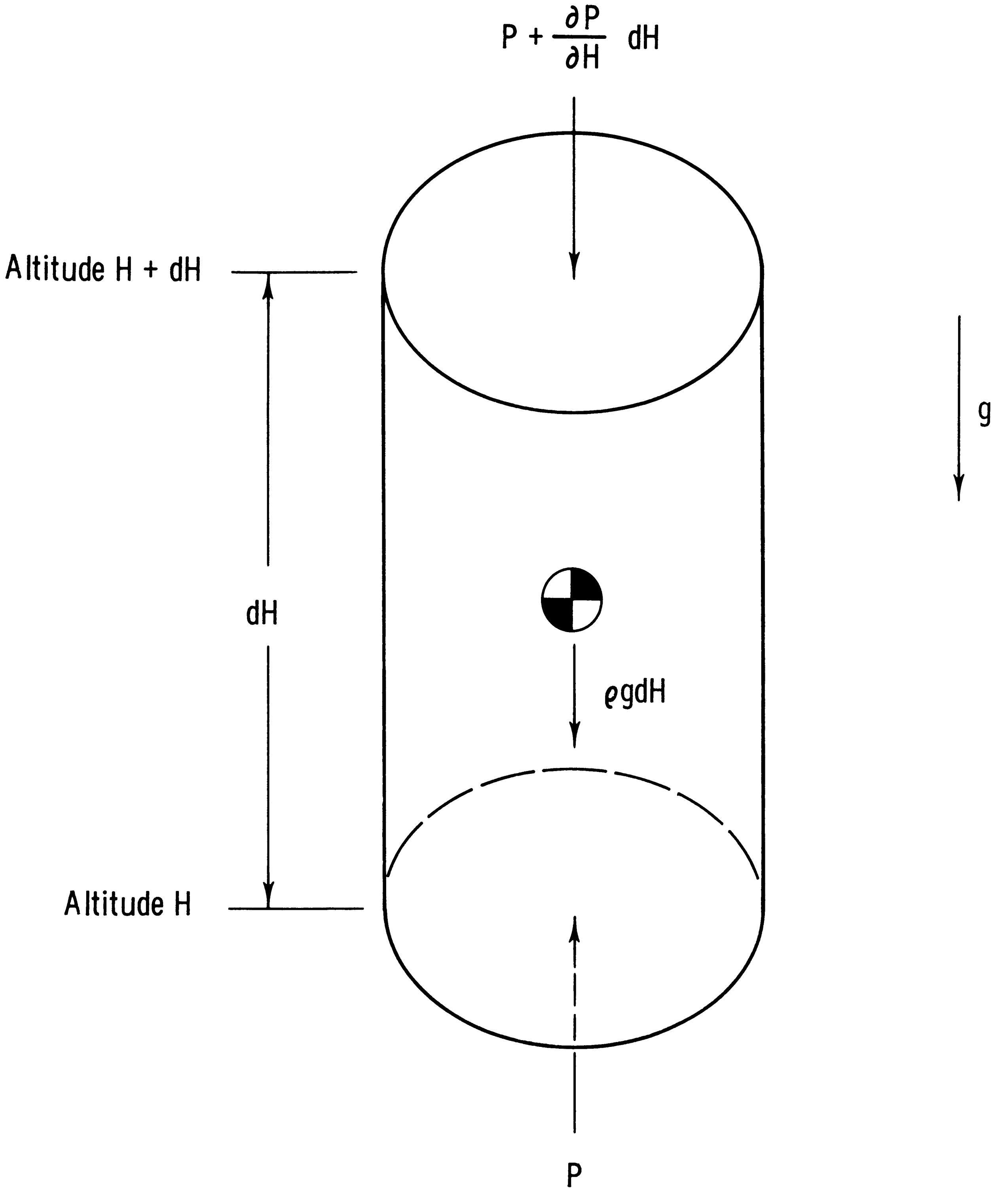

Fig. III.4. The hydrostatic law model.

and knowing that

$$g = g(H)$$

thus, we have

$$\frac{dP}{dH} = - \rho g(H) \qquad \text{(III.15)}$$

which is the elementary form of the hydrostatic law, and is valid for all real gases which are under the influence of a central gravity field.

3.4 AERODYNAMIC FLOW REGIONS

In the previous sections we have devoted the discussion to the general equation resulting from the consideration of a gas from the molecular point of view. We must now make use of a previously obtained result which becomes important when one considers the measurement of atmospheric parameters by means of a supersonic rocket-borne probe. That is, we must now make use of the mean-free path concept in the determination of aerodynamic flow regions encountered by a supersonic rocket vehicle on a high-altitude mission.

In this section, three aerodynamic flow regions will be defined in terms of the mean-free path of the atmospheric particles and pertinent characteristics of each stated.

3.4.1 The Continuum Region

The continuum region of the atmosphere is defined as that portion of the atmosphere where the mean-free path of the molecules is much less than a characteristic dimension defined by the dimensions of the instrument used in making physical measurements of the atmospheric parameters. For example, the flow region is considered continuous if the mean-free path is less than the orifice diameter of a pressure gauge used to make aerodynamic measurements. It is found that when this condition exists, with respect to the pressure sensor, there are many more intermolecular collisions than collisions with the walls of the sensor, and thus, to the sensor the gas appears to be a continuous media.

Mathematically (III.14) may be used as a first approximation to determine the density for the flow-region to be classified as continuous.

$$\bar{\lambda} = \frac{m}{\sqrt{2}\ \rho\pi d^2} < \ell$$

where

m is the molecular mass considered constant

ρ is the mass density

d is the mean molecular diameter considered constant

ℓ is a characteristic dimension of the measuring instrument

The above may be written in the following form:

$$\frac{m}{\sqrt{2}\ \ell\pi d^2} < \rho \quad . \qquad \text{(III.16)}$$

Equation (III.16) thus gives a lower limit of the magnitude of the density for which continuum flow exists, and further shows that the continuum flow region exists in the portion of the atmosphere directly adjacent to the surface of the earth where the density is the highest.

3.4.1.1 The Rayleigh Pitot Tube Equation

In the continuum region the gas dynamics of a compressible fluid dictates the theoretical interpretation of aerodynamic pressure measurements made at points on a supersonic vehicle.

Let us now consider a rocket-borne probe flying at supersonic speed at zero angle of attack in the continuum region. Further, let this probe assume a configuration similar to that shown in Fig. III.5. The probe is moving forward with x directed velocity V_x which is greater than the local speed of sound and thus is accompanied by the shock front shown in the figure. This implies that the incoming flow Mach M is greater than one. Further, let the undisturbed atmospheric parameters in front of the shock wave be P_a, ρ_a, T_a (ambient pressure, ambient density, and ambient temperature). Also, let the pressure directly behind the shock front be P_s and the pressure appearing at the orifice of the impact pressure sensor be P_i.

Now, following the development of Ref. 3 we see that

$$\frac{P_i}{P_a} = \left(\frac{P_i}{P_s}\right)\left(\frac{P_s}{P_a}\right) .$$

Now

$$\frac{P_i}{P_s} = \left(1 + \frac{\gamma-1}{2} M_s^2\right)^{\gamma/\gamma-1}$$

where

γ is ratio of the specific heat of air at constant pressure to that at constant volume

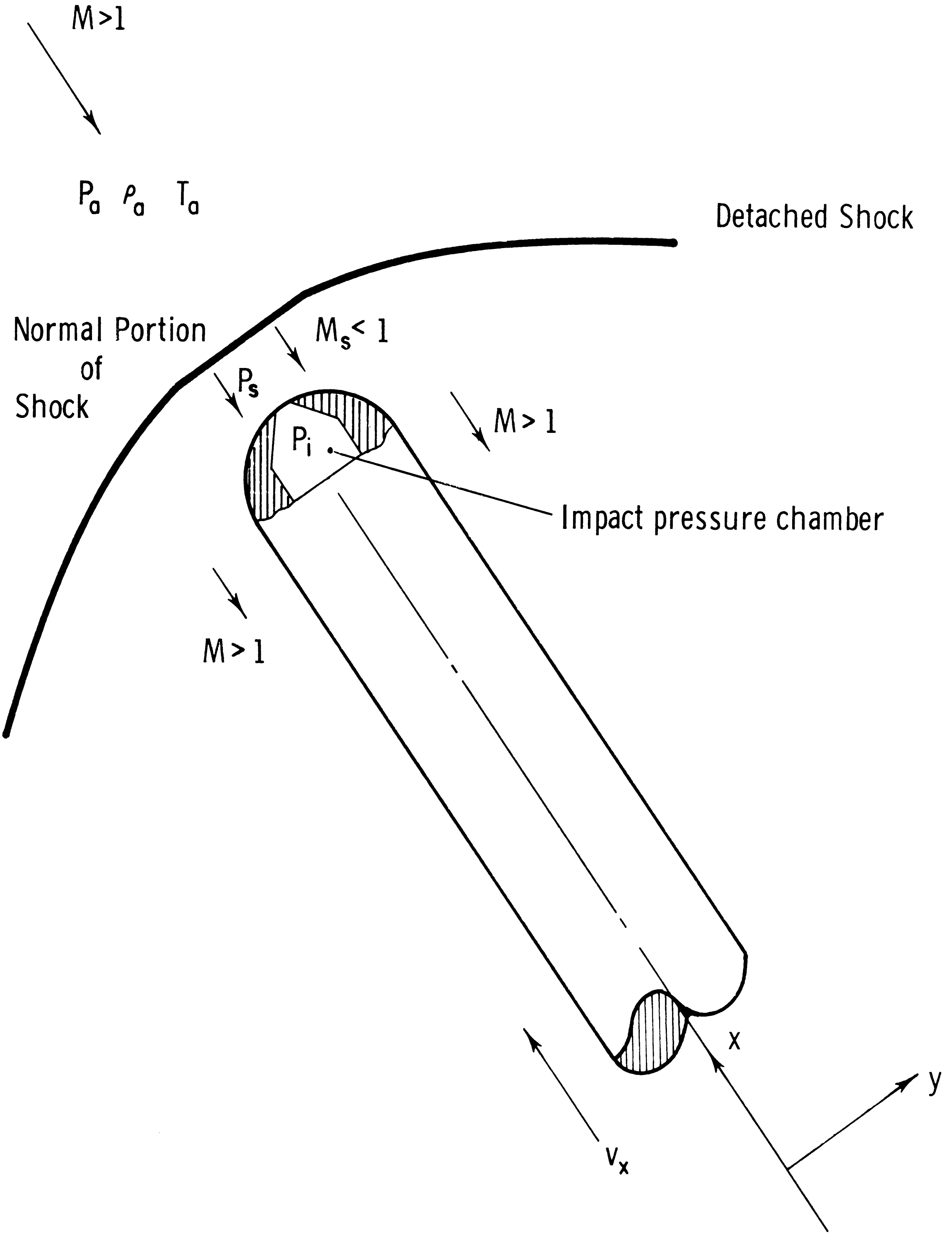

Fig. III.5. The Rayleigh pitot tube equation model.

M_s is the Mach number directly behind the normal shock position of the shock front

and

$$M_s^2 = \frac{M^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1} M^2 - 1}$$

where M is the incoming flow Mach number. Thus, substituting

$$\frac{P_i}{P_s} = \left(1 + \frac{\gamma-1}{2}\left[\frac{M^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1} M^2 - 1}\right]\right)^{\gamma/\gamma-1}$$

which after some manipulation becomes

$$\frac{P_i}{P_s} = \left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2 - 2\gamma + 2}\right]^{\gamma/\gamma-1} .$$

We now need

$$\frac{P_s}{P_a} = \frac{2\gamma}{\gamma+1} M^2 - \left(\frac{\gamma-1}{\gamma+1}\right)$$

which may be written

$$\frac{P_s}{P_a} = \frac{2\gamma M^2 - \gamma + 1}{\gamma+1} .$$

Combining these results

$$\frac{P_i}{P_a} = \left(\frac{P_i}{P_s}\right)\left(\frac{P_s}{P_a}\right) = \left[\frac{2\gamma M^2 - \gamma + 1}{\gamma+1}\right]\left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2 - 2\gamma + 2}\right]^{\gamma/\gamma-1} .$$

Now, by multiplying and dividing the above by the term

$$2\left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2 - 2\gamma + 2}\right]$$

we achieve the final desired form

$$\frac{P_i}{P_a} = \left(\frac{\gamma+1}{2}\right) M^2 \left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2-2\gamma+2}\right]^{1/\gamma-1} . \qquad \text{(III.17)}$$

Where

P_i is the measured stagnation pressure or impact pressure

P_a is the undisturbed ambient atmospheric pressure

M is the incoming flow Mach number

γ is the ratio of specific heats.

Equation (III.17) represents a most useful form of the Rayleigh supersonic pitot-tube equation. In a later chapter it will be shown that this equation allows one to interpret the measured impact pressure in terms of the ambient atmospheric density provided the condition of continuum flow is met.

3.4.2 Free Molecule Flow

The free molecule flow region, or FMF as we shall refer to it, is defined as that region of the atmosphere where the ambient density is sufficiently low so that the fluid media ceases to act as a continuum. Unlike continuum flow, where the important physical phenomena is intermolecular collision, in FMF, the importance is placed upon molecular collision with the walls of an intruding measuring device. In FMF the mean-free path, because of the low density, is sufficiently long compared to the dimensions of the measuring device, that with respect to the sensor, the fluid appears to be a collection of a large number of individual molecules in random motion. In other words, the sensor sees the molecules of the gas as essentially independent entities obeying the laws of physics as individuals rather than as individual groups of molecules as is the case with continuum flow.

In terms of our mean-free path analysis, the condition for the existence of FMF is:

$$\bar{\lambda} = \frac{m}{\sqrt{2}\ \rho \pi d^2} > \ell$$

which may be written

$$\rho < \frac{m}{\sqrt{2}\ \ell \pi d^2} \qquad \text{(III.18)}$$

where

m is the mean molecular mass

ρ is the mass density

d is the mean molecular diameter

ℓ is a characteristic dimension of the measuring instrument.

Equation (III.18) thus gives an upper limit on the magnitude of the ambient density for which FMF exists, and also shows that FMF can exist only at altitudes above a certain critical altitude where Eq. (III.18) is satisfied.

Because of the nature of FMF, it is noted that the aerodynamics of the rocket-borne probes are considerably different than that found in continuum flow. In FMF, the familiar shock wave associated with continuum flow disappears and the term "speed of sound" becomes physically meaningless. Therefore, a theoretical analysis based upon the Kinetic Theory of Gases is again called for. The next section will present such an analysis which will lead to a result analogous to the Rayleigh supersonic pitot-tube equation which will, in a later chapter, be specialized to the interpretation of measured pressures in terms of the desired ambient properties of the atmosphere.

3.4.2.1 The Equations for Gas Flow Equilibrium at the Entrance to a Measuring Chamber in Free Molecule Flow

Following the analysis given in Refs. 1,2,6, and 12, we shall direct our attention to a chamber with an opening in one face. The situation is illustrated in Fig. III.6 where

P_a is the ambient pressure outside the chamber

T_a is the ambient absolute temperature outside the chamber

P_i is the measured impact pressure inside the chamber

T_i is the measured absolute temperature inside the chamber

V_x is the x component of relative velocity between the chamber and the gas

v_x is the x component of random particle velocity of the ambient gas

δt is the time increment under consideration

n_a is the number density of the ambient gas

n_i is the number density of the gas within the chamber.

The chamber shown has a velocity relative to the ambient atmosphere, which velocity is assumed to have component V_x normal to the face of the chamber opening. The gas molecules external to the chamber are assumed to have instantaneous thermal velocities v which vary from particle to particle and are a function of time and temperature for a single molecule. The x directed component of v is v_x. The absolute temperature and pressure within the chamber

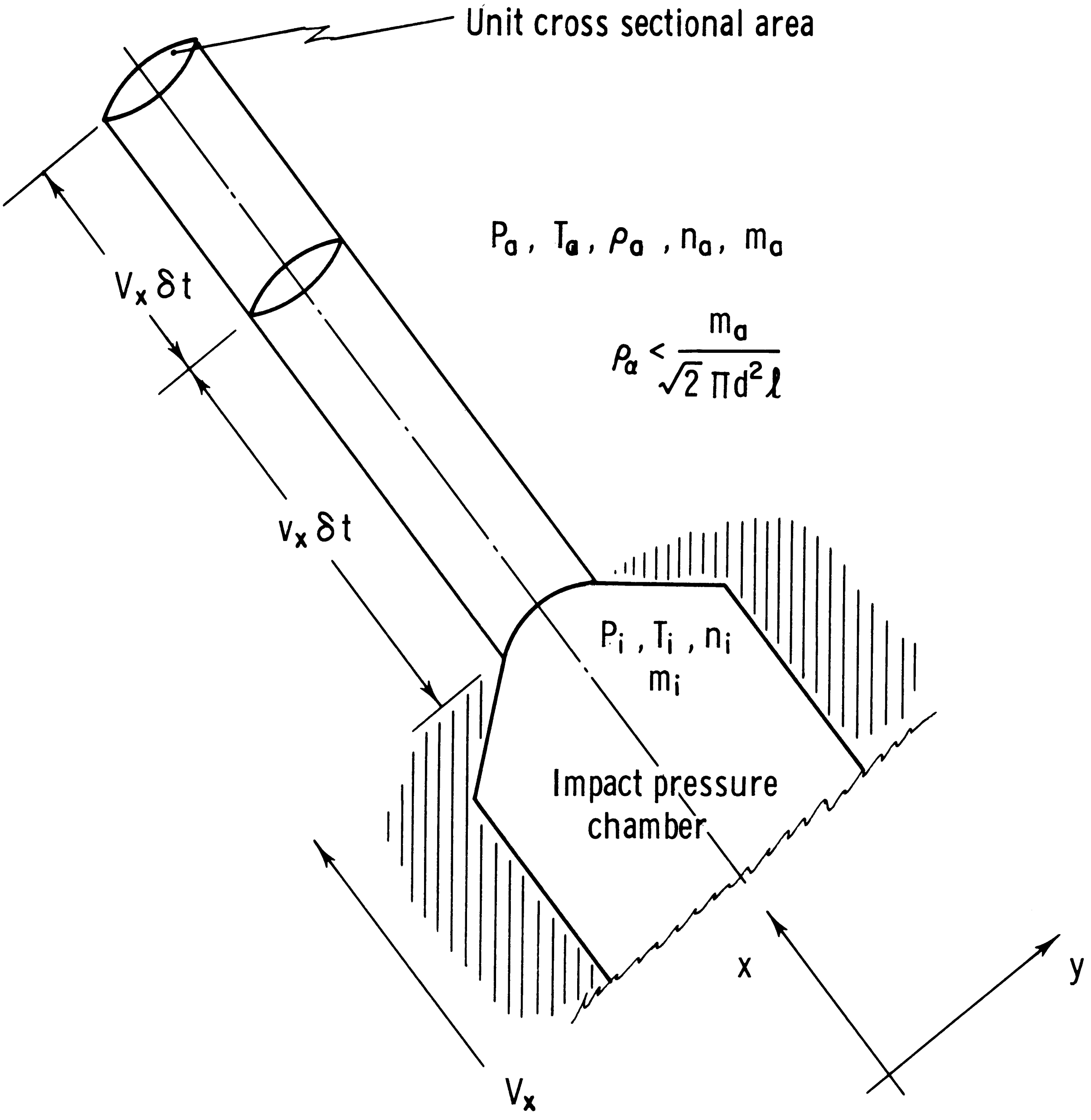

Fig. III.6. Free molecule flow across an orifice.

will be T_i and P_i, respectively, and those of the ambient atmosphere will be T_a and P_a.

Further assumptions required by this analysis are that the molecules both inside and outside the chamber obey the previously derived Maxwell-Boltzmann velocity distribution law and that the particles in the chamber are in thermodynamic equilibrium with the walls of the chamber. Also, P_a, T_a, P_i, T_i, and V_x must be assumed independent of time for a period time δt where δt is an increment of time long enough to assure that an equilibrium state exists and short enough so that P_a, T_a, P_i, T_i, and V_x may be considered time independent.

Now, for equilibrium the number of particles entering the chamber through the orifice must be equal to the number leaving. We shall first consider striking the surface area of the orifice per unit time, assuming that there is no relative velocity between the chamber and the atmosphere. In this case V_x is zero.

Consider the imaginary cylinder $v_x\delta t$ long of unit cross-sectional area shown in the previous figure. Any particle within this cylinder that has a velocity with an x component of at least v_x will reach the orifice in the time increment δt, provided it does not suffer an intermolecular collision before entering the chamber. Because we are considering FMF we will assume that the number of such collisions is negligible.

Now, let us lift the restriction on V_x and assume that a relative drift velocity V does exist whose x directed component is V_x. Thus, it is necessary to change the length of the imaginary cylinder from $v_x\delta t$ to $(v_x+V_x)\delta t$

as shown in the previous figure in order to insure that a molecule whose x directed component of thermal velocity v_x will enter the chamber. Either v_x or V_x may be negative, as long as their sum is positive, otherwise no molecules will enter the chamber volume.

Now, let dN_a be defined as the number of external molecules with x component of thermal velocity in the range between v_x and v_x+dv_x that strike the surface area of the orifice in time increment δt. The number dN_a may be determined by multiplying the volume of the imaginary cylinder by dn_a, which is the number of external molecules per unit volume whose x directed thermal velocity is between v_x and v_x+dv_x. This may be stated mathematically:

$$dN_a = (v_x+V_x)1\ \delta t\ dn_a \quad .$$

Now, v_x may vary from large negative values to large positive values so, in order to find the total number N_a of particles entering the chamber is δt, it is necessary to integrate the above expression over all possible values of (v_x+V_x), recalling that dn_a is a function only of v_x. Thus:

$$N_a = \int_{(v_x+V_x) = 0}^{(v_x+V_x) = \infty} (v_x+V_x)\delta t\ dn_a$$

recalling that

$$dn_a = n_a f_1(v_x) dv_x$$

which may be written in terms of the most probable x directed thermal velocity which we shall call c_{m_a} which is given by

$$c_{m_a} = \sqrt{\frac{2kT_a}{m}}$$

where

c_{m_a} is the most probable thermal velocity of the ambient atmospheric molecules

k is Boltzmann's constant

T_a is the absolute ambient temperature

m is the mean molecular mass.

Introducing the expression for c_{m_a} into Eq. (III.4) with appropriate value of A inserted, we find

$$f_1\left(\frac{v_x}{c_{m_a}}\right) = \left(\frac{m}{2\pi kT_a}\right)^{1/2} e^{-\left(\frac{v_x}{c_{m_a}}\right)^2} .$$

Similarly

$$dv_x = \left(\frac{2kT_a}{m}\right)^{1/2} d\left(\frac{v_x}{c_{m_a}}\right) .$$

Thus

$$dn_a = \frac{n_a}{\sqrt{\pi}} e^{-\left(\frac{v_x}{c_{m_a}}\right)^2} d\left(\frac{v_x}{c_{m_a}}\right) .$$

Introducing the above equation into the expression for N_a with the appropriate change of limits, one finds

$$N_a = \int_{\frac{v_x}{c_{m_a}} = -\frac{V_x}{c_{m_a}}}^{\frac{v_x}{c_{m_a}} = \infty} (v_x+V_x)\,\delta t\,\frac{n_a}{\sqrt{\pi}} e^{-\left(\frac{v_x}{c_{m_a}}\right)^2} d\left(\frac{v_x}{c_{m_a}}\right)$$

which can also be written as

$$N_a = \frac{\delta t \; n_a \; c_{m_a}}{\sqrt{\pi}} \int_{-\frac{v_x}{c_{m_a}}}^{\infty} \frac{v_x}{c_{m_a}} \, e^{-\left(\frac{v_x}{c_{m_a}}\right)^2} d\left(\frac{v_x}{c_{m_a}}\right)$$

$$+ \frac{\delta t \; n_a \; V_x}{\sqrt{\pi}} \int_{-\frac{V_x}{c_{m_a}}}^{\infty} e^{-\left(\frac{v_x}{c_{m_a}}\right)} d\left(\frac{v_x}{c_{m_a}}\right) \quad .$$

Let us introduce the notation

$$S = \frac{V_x}{c_{m_a}} \quad \text{and} \quad s = \frac{v_x}{c_{m_a}} \quad .$$

Then

$$N_a = \frac{\delta t \; n_a \; c_{m_a}}{\sqrt{\pi}} \int_{-S}^{\infty} s \, e^{-s^2} ds + \frac{\delta t \; n_a \; V_x}{\sqrt{\pi}} \int_{-S}^{\infty} e^{-s^2} ds$$

which may be separated as follows

$$N_a = \frac{\delta t \; n_a \; c_{m_a}}{\sqrt{\pi}} \int_{-S}^{\infty} s \, e^{-s^2} ds + \frac{\delta t \; n_a \; V_x}{\sqrt{\pi}} \int_{-S}^{0} e^{-s^2} ds$$

$$+ \frac{\delta t \; n_a \; V_x}{2} \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-s^2} ds \quad .$$

Carrying out the integration with the aid of Refs. 9 and 10 we see that

$$\frac{\delta t \; n_a \; c_{m_a}}{\sqrt{\pi}} \int_{-S}^{\infty} s \, e^{-s^2} ds = \left[\frac{\delta t \; n_a \; c_{m_a}}{2\sqrt{\pi}} \, e^{-S^2} \right] \quad .$$

And

$$\frac{\delta t \; n_a \; V_x}{\sqrt{\pi}} \int_{-S}^{0} e^{-s^2} ds = \frac{\delta t \; n_a \; V_x}{2} \left[\frac{2}{\sqrt{\pi}} \int_{0}^{S} e^{-s^2} ds \right] \quad .$$

But

$$\frac{2}{\sqrt{\pi}} \int_{0}^{S} e^{-s^2} ds = \text{erf } S \quad .$$

Thus, the above integral becomes

$$\frac{\delta t \; n_a \; V_x}{\sqrt{\pi}} \int_{-S}^{0} e^{-s^2} ds = \frac{\delta t \; n_a \; V_x}{\sqrt{\pi}} \text{erf } S \quad .$$

Now, finally

$$\frac{\delta t \; n_a \; V_x}{2} \left[\frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-s^2} ds \right] = \frac{\delta t \; n_a \; V_x}{2} \quad .$$

Therefore, our expression for N_a becomes after some manipulation

$$N_a = \frac{\delta t \; n_a \; c_{m_a}}{2\sqrt{\pi}} \left[e^{-S^2} + \frac{\sqrt{\pi} V_x}{c_{m_a}} \text{erf } S + \frac{\sqrt{\pi} V_x}{c_{m_a}} \right] \quad .$$

Recall that

$$S = \frac{V_x}{c_{m_a}} \quad .$$

So

$$N_a = \frac{\delta t \; n_a \; c_{m_a}}{2\sqrt{\pi}} \left[e^{-S^2} + S\sqrt{\pi} \; (1+\text{erf } S) \right] \quad . \qquad \text{(III.19)}$$

Equation (III.19) represents the number of ambient atmospheric molecules entering the chamber orifice in time increment δt per unit area of orifice opening.

Now, we must consider the number of particles leaving the chamber through the orifice in time δt per unit area of orifice. Let this number be N_i. This number may be calculated in a manner similar to that used to calculate N_a, remembering that V_x is equal to zero and that we must use values of n, dn, and c_m for the interior of the chamber.

Thus,

$$N_i = \int_{v_x = o}^{v_x = \infty} \delta t \; v_x \; dn_i$$

where

$$dn_i = \frac{n_i}{\sqrt{\pi}} e^{-\left(\frac{v_x}{c_{m_i}}\right)^2} d\left(\frac{v_x}{c_{m_i}}\right) .$$

Substituting into the above integral gives

$$N_i = \frac{\delta t \; n_i \; c_{m_i}}{\sqrt{\pi}} \int_{\frac{v_x}{c_{m_i}} = o}^{\frac{v_x}{c_{m_i}} = \infty} \frac{V_x}{c_{m_i}} e^{-\left(\frac{v_x}{c_{m_i}}\right)^2} d\left(\frac{v_x}{c_{m_i}}\right) .$$

After integration we find

$$N_i = \frac{\delta t \; n_i \; c_{m_i}}{2\sqrt{\pi}} . \qquad \text{(III.20)}$$

Recalling now that equilibrium exists when the number of particles entering the chamber is equal to the number leaving. That is to say

$$N_i = N_a$$

or

$$\frac{\delta t \; n_a \; c_{m_a}}{2\sqrt{\pi}} \left[e^{-S^2} + S\sqrt{\pi} \; (1+\text{erf } S)\right] = \frac{\delta t \; n_i \; c_{m_i}}{2\sqrt{\pi}} .$$

Rearranging

$$\frac{n_i c_{m_i}}{n_a c_{m_a}} = \left[e^{-S^2} + S\sqrt{\pi} \; (1+\text{erf } S)\right] . \qquad \text{(III.21)}$$

In terms of pressure, we know from the equation of state that

$$P_i = n_i \, k \, T_i$$

and

$$P_a = n_a \, k \, T_a \quad .$$

Now

$$c_m^2 = \frac{2kT}{m} \quad .$$

Therefore,

$$T = \frac{m \, c_m^2}{2k} \quad .$$

Using this expression in the equation of state

$$P_i = \frac{1}{2} n_i m_i c_{m_i}^{\ 2}$$

and

$$P_a = \frac{1}{2} n_a m_a c_{m_a}^{\ 2}$$

we will now assume that

$$m_i = m_a \quad .$$

With this assumption the ratio

$$\frac{P_i}{P_a} = \frac{n_i c_{m_i}^{\ 2}}{n_a c_{m_a}^{\ 2}}$$

or

$$\frac{n_i c_{m_i}}{n_a c_{m_a}} = \frac{c_{m_a} P_i}{c_{m_i} P_a} \quad .$$

Substituting this expression into Eq. (III.21) and rearranging, we find

$$\frac{P_i}{P_a} = \frac{c_{m_i}}{c_{m_a}} \left[e^{-S^2} + S\sqrt{\pi}\,(1+\mathrm{erf}\ S) \right] \quad .$$

But

$$\frac{c_{m_i}}{c_{m_a}} = \sqrt{\frac{T_i}{T_a}} \quad .$$

So

$$\frac{P_i}{P_a} = \sqrt{\frac{T_i}{T_a}} \left[e^{-S^2} + S\sqrt{\pi}\,(1+\mathrm{erf}\ S) \right] \quad .$$

Let us now define

$$F(S) = \left[e^{-S^2} + S\sqrt{\pi}\,(1+\mathrm{erf}\ S) \right] \quad .$$

Finally

$$\frac{P_i}{P_a} = \sqrt{\frac{T_i}{T_a}}\, F(S) \qquad \text{(III.22)}$$

where

P_i is a measured impact pressure

P_a is the ambient atmospheric pressure

T_i is the measured absolute temperature interior to the chamber

T_a is the absolute ambient atmospheric temperature

$$F(S) = \left[e^{-S^2} + S\sqrt{\pi}\,(1+\text{erf } S)\right]$$

$$S = \frac{V_x}{c_{m_a}} = \frac{V_x}{\sqrt{\frac{2kT_a}{m}}} \quad .$$

Equation (III.22) now gives the desired relationship between the measured impact pressure and the ambient atmospheric pressure. This expression will be specialized in a later chapter to allow the interpretation of measured impact pressure in terms of the ambient atmospheric density.

It is interesting to note the behavior of F(S), (see Appendix) and the fact that for S equal to zero, (corresponding to V_x equal to zero) the value of F(S) is unity, that is

$$\frac{P_i}{P_a} = \sqrt{\frac{T_i}{T_a}} \qquad \text{(III.23)}$$

when there is no relative drift velocity between the chamber orifice and the gas ($V_x = 0$).

3.4.3 The Transition Region

As a logical conclusion, in light of the previous two sections, there must exist a flow region intermediate to continuum and free molecule flow regions. This is the region known as transition. In this region, the flow exhibits properties of both continuum and free molecule flow, and because of the rather wide separation between the theoretical approaches used in analyzing the two flow regions, it does not lend itself readily to theoretical analysis. Considerable effort along the line has been given by several individuals. One very promising effort is given in Ref. 11.

Again using the mean-free path approach, the transition region is that region where

$$\bar{\lambda} = \frac{m}{\sqrt{2}\,\rho\pi d^2} = \ell$$

or

$$\rho \simeq \frac{m}{\sqrt{2}\,\pi d^2 \ell} \quad . \qquad \text{(III.24)}$$

Here we see that the mean-free path is approximately equal to the characteristic dimension, so that both intermolecular and molecular-wall collision become important.

Let us then conclude that, to date, no satisfactory theory exists by which one may interpret measured aerodynamics pressures in terms of ambient atmospheric properties in the transition region.

3.5 SUMMARY OF THEORETICAL RESULTS

By way of summary, let us simply restate the important final equations developed in the previous sections of this chapter and the appropriate assumptions and restriction on the use of each.

Maxwell-Boltzmann Velocity Distribution Law

$$f(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\left(\frac{mv^2}{2kT}\right)} \qquad \text{(III.10)}$$

where

m is the mean molecular mass

k is the Boltzmann constant or gas constant per molecule

v is the random thermal velocity

Assumptions:

(1) The velocity components of v_x, v_y and v_z, of v are statistically independent.

(2) The distribution function f(v) is isotropic.

Validity:

Several direct experimental verifications of the law for several common gases and vapors have been made and may be found in the literature. On the basis of this experiment we shall consider the law valid for the atmospheric gas and thus the atmosphere.

The Equation of State for an Ideal Gas

$$P = \rho RT \qquad \text{(III.12)}$$

where

P is the gas pressure exerted normal to a surface immersed in the fluid

ρ is the gas mass density

R is the gas constant per unit mass

T is the absolute temperature of the gas.

Assumptions:

(1) Ideal gas

Validity:

Even though the law applies strictly to an ideal gas, it has been shown experimentally to accurately predict the properties of a real gas at the value pressures and densities encountered in the atmosphere. It does not hold for extremely high densities and pressure approaching those of the critical values for the gas. This is because the extremely strong intermolecular forces are not accounted for in the ideal gas analysis.

Thus, because a real gas approaches the behavior of an ideal gas at and below the values of pressure and density on the surface of the earth, we shall consider the law valid for the properties of the earth's atmosphere.

The Hydrostatic Law

$$\frac{dP_a}{dH} = -\rho_a g \qquad \text{(III.15)}$$

where

$\frac{dP_a}{dH}$ is the rate of change of ambient atmospheric pressure with altitude

H is the altitude in the atmosphere above the surface of the earth

P_a is the ambient atmospheric pressure

ρ_a is the ambient atmosphere gas mass density

g is the local acceleration due to gravity

Assumptions:

(1) No mass motion

(2) Equilibrium between atmospheric pressure and the forces of gravity.

Validity:

It is known that over a large altitude interval, very violent mass motion of the atmosphere occurs. However, if the altitude increment dH is chosen arbitrarily small, then the mass motion occurring within the increment may be neglected and again only equilibrium considered.

The Rayleigh Supersonic Pitot-Tube Equation

$$\frac{P_i}{P_a} = \left(\frac{\gamma+1}{2}\right) M^2 \left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2 - 2\gamma + 2}\right]^{\frac{1}{\gamma-1}} \qquad \text{(III.17)}$$

where

P_i is the measured impact pressure

P_a is the ambient atmospheric pressure

M is the incoming flow Mach number

γ is the ratio of specific heats.

Assumptions:

(1) $M > 1$

(2) Continuum flow

(3) Normal shock exists in front of measuring sensor orifice

(4) Zero angle of attack.

Validity:

In all upper atmosphere rocket-borne probes the first three conditions above are met for at least the low-altitude portion of the flight. Condition four, however, is rarely satisfied. However, wind-tunnel tests of devices similar to the one to be considered[4] show that negligible error is introduced by angles of attack up to at least 10°.

Gas Flow Equilibrium at an Orifice in Free Molecule Flow

$$\frac{P_i}{P_a} = \sqrt{\frac{T_i}{T_a}}\, F(S) \qquad \text{(III.22)}$$

where

P_i is a measured impact pressure

P_a is the ambient atmospheric pressure

T_i is the measured absolute temperature interior to the measuring chamber

$$F(S) = \left[e^{-S^2} + \sqrt{\pi}\, S(1+\text{erf } S)\right]$$

S is the ratio of the x directed component of relative drift velocity to the most probable thermal velocity.

Assumptions:

(1) Free molecule flow

(2) Thermodynamic equilibrium between particles within the measuring chamber and the chamber walls.

Validity:

Provided that a high enough altitude is reached by the rocket-borne probe, the conditions for FMF will be met. Thermodynamics equilibrium can be assured if at least three collisions with the interior of the wall take place before the processes of measurement occur. For the third assumption, it is realized that there is some disassociation of atmospheric oxygen molecules above the altitude where transition to FMF occurs for the probe to be considered. It is therefore, entirely possible that there may be some recombination within the chamber, where the gas is not subject to the strong radiation fields which originally caused disassociation, and because the probability of recombination within the chamber is much greater than that outside. The magnitude of this effect can be a source of error in any measurement made; however, from past experience it is believed that this error is negligible when compared to other errors which may be present.

In view of this then, we shall consider Eq. (III.22) valid for all measurements made by the device in question in the free molecule flow region.

References for Chapter III

1. Patterson, G. N., *Molecular Flow of Gases*, John Wiley and Sons, 1956.
2. Liu, V. C., "Class Notes for Topics in Gas Dynamics," The University of Michigan, College of Engineering, Dept. of Aero. and Astro. Eng., Ann Arbor, Spring, 1962.
3. Shapiro, A. H., *Dynamics and Thermodynamics of Compressible Fluid Flow*, *1*, The Ronald Press, New York, 1953.
4. Laurmann, J. A., "Low Density Characteristics of an Aerobee-Hi Pitot-Static Probe," Univ. of Calif., Institute of Engineering Research, *Tech. Report*, *HE-150-156*, May, 1950.
5. Kuethe, A. M., and Schetzer, J. D., *Foundations of Aerodynamics*, John Wiley and Sons, New York, 1950.
6. Schultz, F. V., Spencer, N. W., and Reifman, A., "Atmospheric Pressure and Temperature Measurements Between the Altitudes of 40 and 110 Kilometers." *Upper Atmosphere Report No. 2*, Univ. of Mich. Research Institute, Ann Arbor, July, 1948.
7. Perkins, E. W., and Jorgensen, L. H., "Comparison of Experimental and Theoretical Normal-Force Distribution (Including Reynolds Number Effects) on an Ogive-Cylinder Body at Mach Number 1.98," NACA Technical Note 3716, Washington, D.C., May, 1956.
8. Ainsworth, J. E., Fox, D. F., and LaGow, H. E., "Upper Atmosphere Structure Measurements Made With the Pitot-Static Tube," *J.G.R.* *66*(10), 3191-3211, 1961.

References (Concluded)

9. Hodgman, C. D., Ed., Standard Mathematical Tables, Tenth Edition, Chemical Rubber Publishing Co., Cleveland, Ohio, 1955.

10. Churchill, R. V., Operational Mathematics, Second Edition, McGraw-Hill Publishing Co., New York, 1958.

11. Lui, V. C., "On Pitot Pressure in an Almost-Free-Molecule Flow—A Physical Theory for Rarefied-Gas Flows," Journal of the Aero/Space Sciences, 25, No. 1, December, 1958.

12. Hsu, Y. S., "The Thermal Transpiration Equation; Tables of F(S) vs. S," Scientific Report No. MS-2, ORA, Univ. of Mich., June, 1963.

CHAPTER IV

THE PITOT-STATIC PROBE

4.1 INTRODUCTION

Following Refs. 1 and 2, the pitot-static technique is an aerodynamic approach to the measurement of pressure, temperature, and density in the upper atmosphere through a simple implementation of the basic theories presented in Chapter III. "Pitot-static" implies the two primary measurements which are inherent to the method: (1) the pitot or impact pressure; and (2) the static or ambient pressure. The pitot-static probe design presented here is extended to include a spin pressure measurement which, after further study and development, may lead to a determination of upper atmosphere winds.

The pitot-static technique has been extensively studied and developed by the scientists and engineers of the Space Physics Research Laboratory (SPRL) of The University of Michigan. It is this design and instrumentation which will be discussed in this chapter.

4.2 THE PITOT-STATIC PROBE CONFIGURATION

The configuration of the SPRL pitot-static probe is shown in Fig. IV.1.

As can be seen, the probe is a cylindrical tube, 3.5 in. in diameter, approximately 37 in. long, with a hemispherical nose tip and a cylinder-cone-cylinder afterbody.

The figure shows the single orifice of the impact and spin pressure chambers and the many orifices of the manifold ambient pressure chamber.

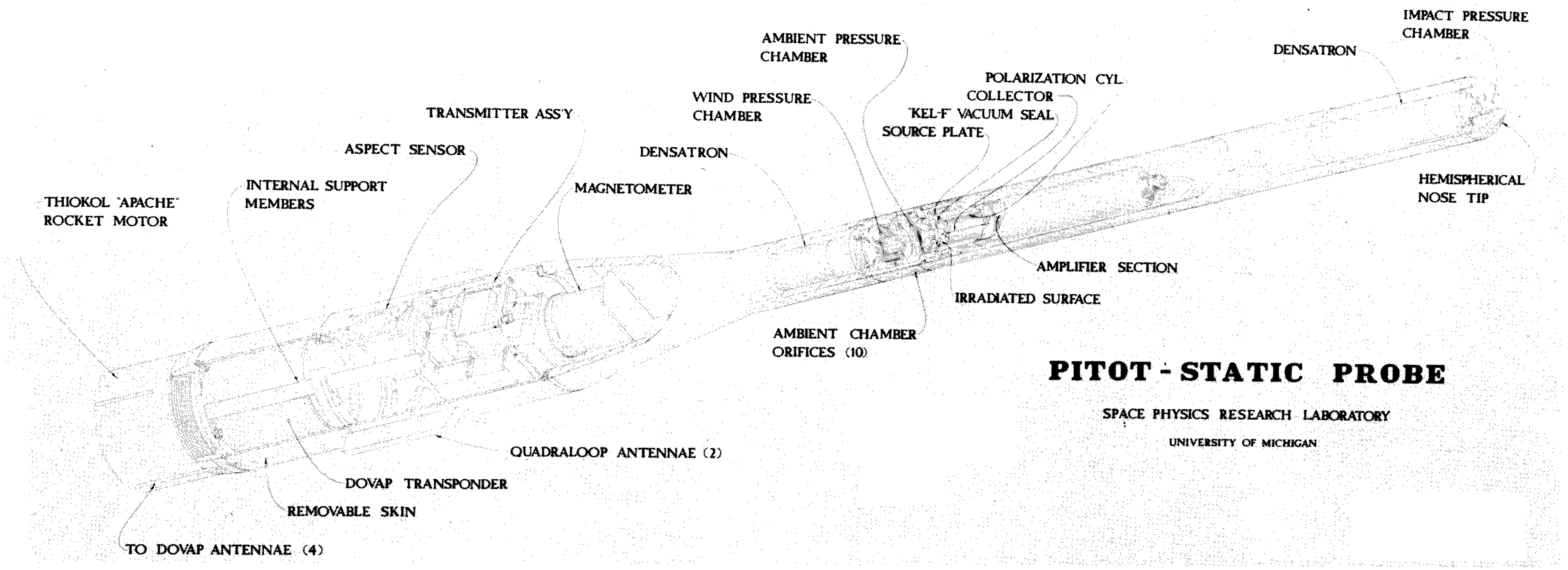

Fig. IV.1. The pitot-static probe.

This particular configuration was chosen for two reasons: first, it meets the requirements of good supersonic aerodynamic design of a rocket-borne probe; and second, this configuration readily lends itself to theoretical analysis and, in particular, to the theories presented in Chapter III.

4.3 INSTRUMENTATION

The raw data from the experiment consists of the continuous measurement of pressure in the three chambers by electronic pressure sensors which are connected directly to the three chambers. These pressure sensors convert the pressure information received from the chambers into their electrical analog. This analog or electric voltage, as is the exact case, is then transferred by internal cable to the instrumentation section whereit is further acted upon and combined with other analog information and finally transmitted via UHF FM radio to receivers and data recorders located on the ground.

In this section we will discuss in detail the pressure sensors and associated rocket-borne support equipment.

4.3.1 The Radioactive Ionization-Gauge Densatron

The pressure sensors called densatrons utilized with the SPRL pitot-static probe combine a radioactive ionization chamber with a multirange electrometer amplifier to form an instrument capable of detecting with good resolution the pressures found in the region of the earth's atmosphere from 30 to 120 km.

As with other ionization gauges, neutral gas ionization results in a current which may be measured with a suitable ammeter, and through laboratory calibration, interpreted in terms of gauge pressure. In view of this, it is to be noted that in an ionization gauge the important factor in deriving a current to be measured is the number of gas molecules present in the gauge, or the gas density, rather than gas pressure. However, if the temperature is known, the gas density is proportional to the pressure. So, in effect, all ionization gauges are density sensing units which are calibrated in terms of pressure.

The actual configuration of the densatron ionization chamber or gauge is shown in Fig. IV.2. The figure shows the important features of this ionization chamber. In operation, the neutral gas enters the chamber through the gas admittance port in the top seal plate. Upon entering the chamber it is bombarded by primary beta particles emitted by the radioactive source material (tritium) which is contained on the inside surface of the top seal plate. This process causes the neutral gas to become ionized. The ions and electrons are then acted upon by the electric field existing in the chamber between the polarization cylinder which is maintained at a positive potential with respect to ground and the central collector wire which is negative. Under the influence of this field the positive ions move to the collector where they are collected to form the ion current which is then measured and converted to an output voltage from the electrometer amplifier.

Now, because the densatron is a density sensor which is calibrated in terms of pressure at a constant gauge temperature, we must determine what effect a change in gauge temperature will have on the pressure measurement. This is easily done through use of the equation of state. Here we assume

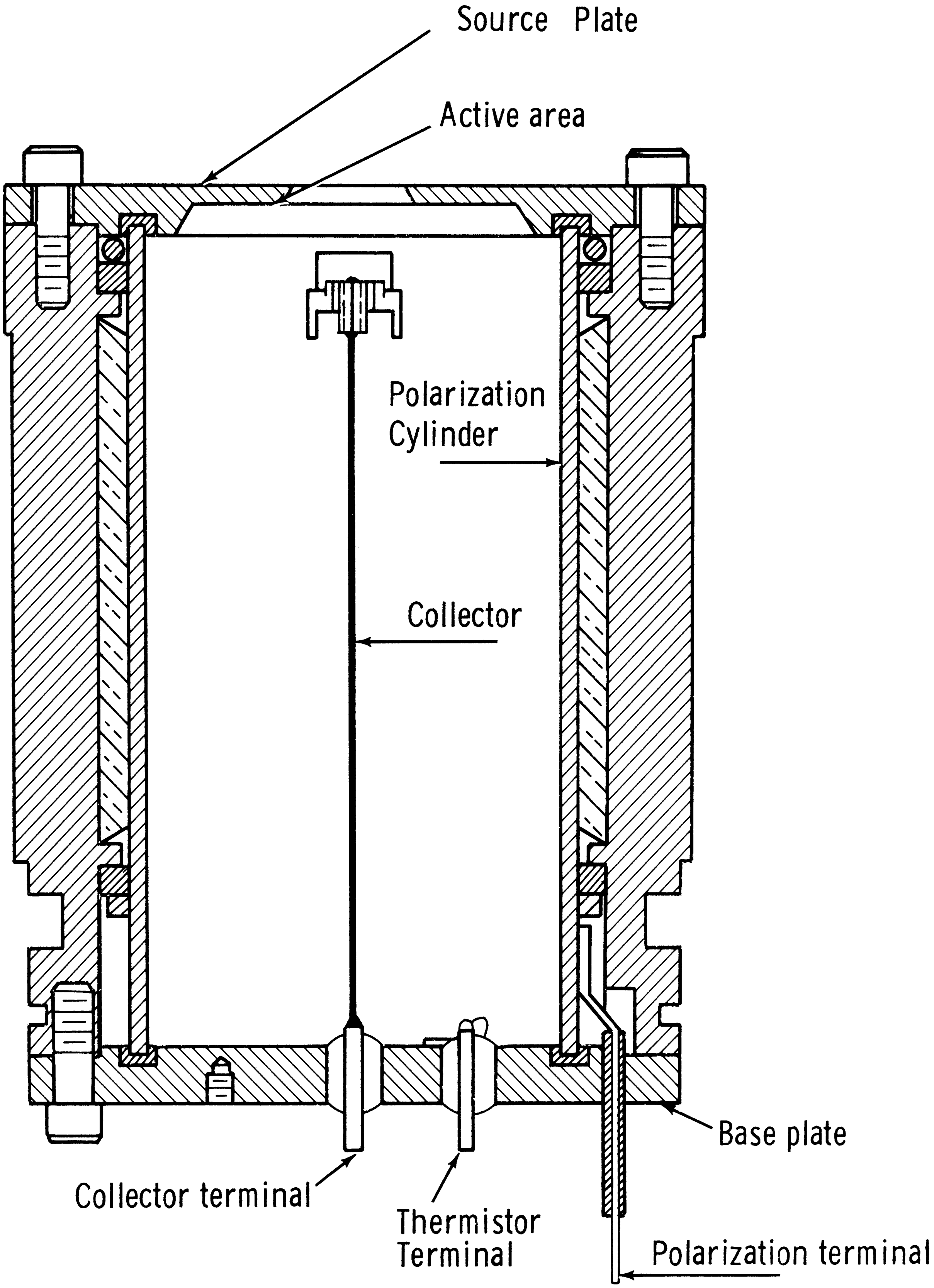

Fig. IV.2. The densatron gauge.

that during the calibration the gauge temperature and gas temperature are identical and constant at a value T_{cal}. In this case

$$P_{cal} = \rho R\, T_{cal}$$

where

P_{cal} is the pressure corresponding to ρ and T_{cal}

ρ is the gas density

T_{cal} is the absolute gauge temperature during calibration

R is the gas constant per unit mass.

Now, let us maintain the density ρ in the gauge constant (which will give the same densatron output voltage as in the previous case) but vary the gauge temperature, and thus the gas temperature, to a value T_{gauge}. We now have

$$P_{gauge} = \rho R\, T_{gauge}$$

where

P_{gauge} is the pressure corresponding to ρ and T_{gauge}

ρ is the gas density

T_{gauge} is the new absolute gauge temperature

R is the gas constant per unit mass.

Now, since ρ remained constant, we have

$$\frac{P_{gauge}}{R\, T_{gauge}} = \frac{P_{cal}}{R\, T_{cal}}$$

which gives

$$P_{gauge} = P_{cal} \frac{T_{gauge}}{T_{cal}} \qquad \text{(IV.1)}$$

where

P_{gauge} is the real gas pressure for a given densatron output voltage at any gauge temperature T_{cal}.

T_{gauge} is the absolute gauge temperature during actual operation as measured by the glass bead thermistor in the densatron gauge unit.

P_{cal} is the pressure obtained during calibration at gauge temperature T_{cal}, and is the pressure indicated by the gauge calibration for a given output voltage.

T_{cal} is the absolute gauge temperature during calibration.

Equation (IV.1) must be used whenever the gauge temperature during operation is different from that at which gauge calibration took place.

4.3.2 Supporting Rocket-Borne Equipment

In addition to the various pressure measurements, the pitot-static payload includes a four-channel FM/FM telemetry system, a DOVAP (Doppler-Velocity-And-Position) transponder for velocity and altitude (trajectory) determination, and an optical aspect system for angle of attack determination. This equipment is housed along with the battery power supply in the lower cylindrical portion of the probe called the instrumentation section.

The FM/FM telemetry system consists of a UHF FM radio transmitter and associated electronics which serve as the data transmission link for the analog data from the probe during flight to the ground.

The DOVAP transponder enables a supporting agency to determine the precise altitude and velocity versus time history of the probe during flight.

The data from the optical aspect sensor allows the determination of the probe in-flight angle of attack. This information, as will be seen later, is of prime importance in subsequent data reduction of primary probe measurements.

References for Chapter IV

1. Ainsworth, J. E., Fox, D. F., and LaGow, H. E., "Upper Atmosphere Structure Measurement Made With the Pitot-Static Tube," J.G.R. 66(10), 3191-3211, 1961.

2. Horvath, J. J., Simmons, R. W., and Brace, L. H., "Theory and Implementation of the Pitot-Static Technique for Upper Atmospheric Measurements," Scientific Report No. NS1, ORA, The University of Michigan, March, 1962.

CHAPTER V

TYPICAL DATA OBTAINED FROM THE SPRL PITOT-STATIC PROBE AND PRELIMINARY PROCESSING

5.1 INTRODUCTION

As stated earlier, all primary (pressure) and secondary (gauge temperature, etc.) data from the pitot-static probe are transmitted via radio from the probe in flight to ground based receivers. The analog data is then transferred from the receiver on to magnetic tape for storage and to a "real time" oscillograph paper record for immediate use. For the present, we shall be primarily concerned with the "real time" oscillograph record.

This chapter will explain in detail, the process by which the pressure and gauge temperature information is obtained from the "real time" record and combined with the appropriate trajectory information to give final raw data, which may then be interpreted in terms of the three desired atmospheric parameters.

5.2 THE ANALOG DATA

The analog data from a typical SPRL pitot-static probe flight is received at SPRL in the form of a "real time" paper oscillograph record. This record is nothing more than a graph of analog voltages versus time, where the voltage is given as the vertical axis and time as the horizontal. The time axis increments are spaced 0.001 sec apart with the actual time given by a binary code in days, hours, minutes, and seconds with respect to the solar observatory at Greenwich, England, or Greenwich Mean Time (GMT). The voltages presented on the vertical axis in general lie somewhere on an arbitrary linear scale whose end points are zero volts and five volts respectively. A portion of the flight record, Fig. V.1 of NASA 14.21 SPRL pitot-static probe, is given for reference. On this sample of record is shown the 0-5 volt vertical scale, and the horizontal binary time coded scale, in addition, this sample shows 30 actual commutated data segments which are repeated cyclically throughout the entire flight. Each data segment represents one analog data point. That is, each data segment gives one voltage output which occurred at the time given on the horizontal axis. An example of this is shown for segment no. 28.

With the aid of the inflight voltage calibrate points, which are usually located in the first few segments of each cycle, and the various oscillograph reading devices available in the laboratory, each data point voltage is read and recorded along with time (GMT) at which each occurred. This procedure usually involves the reading of approximately 7,500 data point voltages, 7,500 segment numbers, and 250 reference times for each pitot-static probe flight. From the 250 reference times, the time of each individual data segment is computed and recorded. Thus, for each pitot-static flight approximately 22,500 bits of information are obtained, 7,500 voltage points, 7,500 times, and 7,500 commutator segment numbers. For proper handling of this information, it must be recorded as ordered triples of numbers, a segment number, an output voltage, and the time of the data segment.

Having completed the reading and recording of the flight data as described above, the flight information is subjected to a sorting procedure.

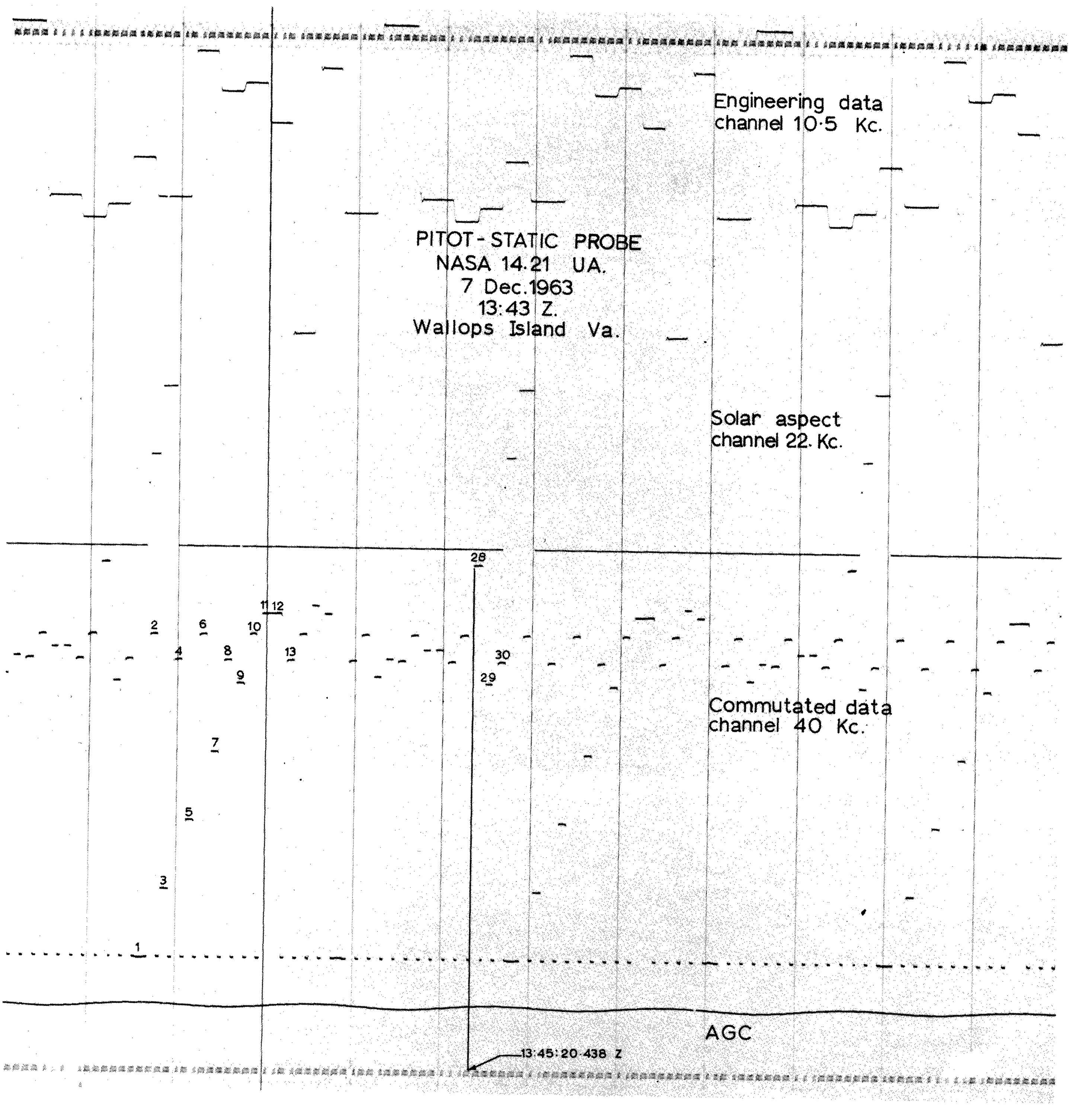

Fig. V.1. Real time oscillograph data record.

The sorting is carried out according to segment number and time sequence. That is, the flight information is sorted according to experiment by segment number in increasing time sequence. This sorting process has now eliminated the segment or bit number from the triplet and replaced it with a title or experiment name. The primary experiment titles are Impact Densatron Output, Ambient Densatron Output, Impact Densatron Range, Ambient Densatron Range, etc. Some secondary experiment titles are Impact Densatron Gauge Thermistor Output, Impact Densatron Amplifier Thermistor Output, etc.

Having completed the sorting process, the preliminary data "conditioning" is complete and the actual data reduction process may proceed.

5.3 CONVERSION OF THE ANALOG DATA TO "GAUGE PRESSURE"

In this section, we shall determine the process by which the previously obtained analog voltages are converted into actual "gauge pressures" through the use of the densatron pressure calibration curves obtained in the laboratory preceding each flight.

5.3.1 Densatron Calibration Curves

Each densatron to be flown must, prior to flight, be placed on a laboratory vacuum system where a pressure-voltage characteristic is determined. This procedure is known as "calibration." During the calibration, the pressure in each densatron unit is varied over a range that best simulates the pressure range that will be experienced by the unit during flight. While this procedure is being carried out a particular pressure level in the pressure range is set in each densatron unit and is carefully measured by means of a precision pressure standard. This pressure is then recorded along with the individual densatron output voltages and densatron gauge temperatures. Upon completing the measurement of this pressure-voltage-temperature point, the pressure in the densatron units is changed to a new value and the above measurements are again made and recorded. This procedure is continued unitl the entire pressure range of the densatron units has been covered.

In order to assure proper pressure sensitivity throughout its entire pressure measuring region, the densatron is so designed that this region is divided into five separate ranges with each successive range six times as sensitive as the preceding. To facilitate the interpretation of the voltage data in terms of pressure, the above recorded pressure-voltage information is presented in the form of five separate graphs of Calibration Pressure versus Densatron Output Voltage at constant calibration temperature. Each graph then represents the voltage-pressure characteristic of a particular densatron over one of its five ranges. In addition to the actual graph of Calibration Pressure versus Densatron Output Voltage, there will also appear on each curve a constant calibration temperature and Densatron Amplifier Calibration Voltage. The use of these numbers will be demonstrated later.

It must be remembered that each densatron has one set of five pressure calibration curves. Thus, for each pitot-static probe in which there are three densatrons, there must be 15 such separate calibration curves, one for each range of each densatron.

It is now a simple matter to use the calibration curves to determine the measured pressure corresponding to each analog point tabulated in the manner described in the last section.

5.3.2 Thermistor Calibration Curves

A thermistor is simply a semiconductor resistance element that exhibits a negative thermal coefficient of resistance. The thermistor may, therefore, when employed in a proper circuit, be used to determine an analog temperature. This analog is again electrical voltage. Similarly, a temperature-voltage calibration of the thermistor circuit must be made. Such a calibration results in a graph of temperature versus thermistor voltage output.

In the SPRL pitot-static porbe, all thermistors are uniform and therefore require only one such calibration curve.

With the thermistor calibration curve, the inflight thermistor output voltages may easily be converted to temperature.

5.3.3 Measured Pressures and Temperatures

Assuming that all of the necessary calibration curves are available along with the recorded densatron flight data mentioned in Section 5.2 we may now proceed with the preliminary data processing. Our goal is to convert the tabulated data to pressures and temperatures.

To accomplish this, we must first make use of the Amplifier Calibration Voltage. This voltage gives a direct measure of the performance of the Densatron Amplifier during flight, and allows us to correct any error in the Densatron Output Voltage caused by amplifier drift. This correction will be illustrated below.

Assume for example that

$$\left(\begin{array}{l}\text{Impact}\\ \text{Densatron Output}\\ \text{Voltage}\end{array}\right) = 1.22 \text{ volts}$$

and

$$\left(\begin{array}{l}\text{Impact}\\ \text{Densatron Amplifier}\\ \text{Calibration}\\ \text{Voltage}\end{array}\right) = 2.54 \text{ volts} .$$

Now we shall assume for the sake of illustration that Impact Densatron Amplifier Calibration Voltage given on the Range 1 Impact Densatron Pressure Calibration curve is equal to 2.56 volts. Comparing this with flight data we see a difference of 0.020 volts. This means that a correction of 0.020 volts must be made to the output voltage given above. Or simply

$$\left(\begin{array}{l}\text{Amplifier Drift}\\ \text{Densatron Impact}\\ \text{Densatron Output}\\ \text{Voltage}\end{array}\right) = 1.22 \text{ volts} + 0.020 \text{ volts}$$

This correction must be made to all densatron output voltages as long as the inflight Densatron Amplifier Calibration Voltage remains different from that given on the individual densatron calibration curves.

As a general rule for making this correction to the analog data let us state the following:

Let

$DACV_c$ be a Densatron Amplifier Calibration Voltage obtained during pressure calibration

and

$DACV_f$ be the Densatron Amplifier Calibration Voltage obtained from flight data.

Further, take the absolute magnitude of the difference of $DACV_c$ and $DACV_f$ which is

$$|DACV_c - DACV_f| = X \quad .$$

Now is $X = 0$ then a correction to the Densatron Output Voltage is required, and may be applied as follows:

If

$$DACV_c > DACV_f$$

then add X to all Densatron Output Voltages to obtain the corrected values, that is

$$\begin{pmatrix}\text{Amplifier Drift}\\ \text{Corrected Densatron}\\ \text{Output Voltage}\end{pmatrix} = \begin{pmatrix}\text{Densatron}\\ \text{Output Voltage}\end{pmatrix} + X \quad .$$

If

$$DACV_c < DACV_f$$

then subtract X from Densatron Output Voltages to obtain the correct values, that is

$$\begin{pmatrix}\text{Amplifier Drift}\\ \text{Corrected Densatron}\\ \text{Output Voltage}\end{pmatrix} = \begin{pmatrix}\text{Densatron}\\ \text{Output Voltage}\end{pmatrix} - X \; .$$

At this time, the analog data for each pressure experiment has been recorded in time sequence along with the appropriate thermistor output data. The amplifier drift correction may now be applied to the Densatron Output Voltage as previously described.

Having completed the above correction for each of the densatrons we may now make use of the densatron and thermistor calibration curves to find the pressure and temperature data corresponding to each output voltage. This is easily done by simply looking up the pressure or temperature corresponding to the given voltage. In the case of the densatron, be sure to use the Amplifier Drift Corrected Value in obtaining the pressure.

This procedure must be carried out through the entire measurement range of each densatron pressure experiment. This data now represents the measured pressures and temperatures with respect to time.

5.3.4 Gauge Pressure

To complete the preliminary data processing, it is now necessary to utilize Eq. (IV.1) and the measured gauge temperature along with the constant calibration temperature to compute the actual pressures (P_{gauge}) experienced by the densatrons during flight.

Recalling Eq. (IV.1) we see,

$$P_{guage} = P_{cal} \frac{T_{guage}}{T_{cal}} \qquad \text{(IV.1)}$$

where

P_{gauge} is the actual densatron pressure

P_{cal} is the pressure obtained for a given densatron output from the densatron calibration curve

T_{gauge} is the absolute temperature existing in the densatron gauge during flight as measured by the densatron thermistor

T_{cal} is the absolute temperature of the densatron gauge and gas temperature during calibration.

Equation (IV.1) may be used directly to compute the actual gauge pressures from the previously obtained measured pressure as follows:

For each measured densatron pressure and densatron thermistor temperature, compute the ratio T_{guage}/T_{cal} using the value of T_{cal} given on the particular densatron calibration curve, multiply the measured pressure by this ratio, thus obtaining the desired value of actual densatron gauge pressure. Such a procedure must be completed for each of the flight densatron experiments (Impact, Ambient, and Spin).

This completes all but one final step in processing and organizing the raw flight data.

5.4 THE TRAJECTORY

The trajectory, which is given as the position and velocity of the pitot-static probe in space versus time, is received by SPRL from an outside agency. It is received in the form of punched IBM cards and a set of tabulated information printed from the IBM cards. The information contained on the cards is a time history of total probe velocity, Cartesian total velocity components, probe range, and the Cartesian range components.

A computer program has been written which uses the information on the punched IBM cards as input data whose output consists of a tabulation of probe total velocity and altitude at the exact time from launch of each of the flight data points.

As a final step in the organization of the raw data, the above trajectory information (total velocity, V, and altitude, H) is included in the tabulation given below.

5.5 EXAMPLE OF RAW FLIGHT DATA IN FINAL FORM

The data presented in this section is actual flight data from SPRL NASA 14.21 flown from Wallops Island, Virginia on 7 Dec. 1963. It is given as illustration of the final form of raw flight data.

FLIGHT DATA
GAUGE PRESSURE

FLIGHT NO. NASA 14.21 UA
DATE OF FLIGHT 7 Dec. 1963
G.M.T. OF LAUNCH 13:43 Z
LAUNCH SITE Wallops Is., Va.

DATE 12/27/63
RECORDER
SHEET ___ OF ___

DENSATRON Impact
SERIAL NO. Unit No. 18 Source No. 7
DENSATRON SEGMENTS 4, 8, 13, 18, 22, 26, 30
RANGE SEGMENTS 15
THERMISTOR SEGMENTS (GAGE) 24
(AMPLIFIER) 20

TIME FROM LAUNCH (SEC.)	DENSATRON OUTPUT VOLTS	RANGE OUTPUT VOLTS	DENSATRON THERMISTOR OUTPUT VOLTS	DENSATRON AMPLIFIER CALIBRATION VOLTAGE	AMPLIFIER CORRECTION FACTOR VOLTS	AMPLIFIER DRIFT CORRECT DENSATRON OUTPUT	MEASURED PRESSURE P_{CAL} (MM HG)		DENSATRON GAUGE TEMP. T_{GAUGE} (°K)	$\frac{T_{GAUGE}}{T_{CAL}}$	GAUGE PRESSURE P_{GAUGE} (MMHG)		TOTAL VELOCITY V (FT/SEC)	ALTITUDE H (KM)
57.009	2.00	1.04	4.56	0.490	0.03	2.03	4.08	0	285	.973	3.97	0	4.367 3	57.62
57.065	2.02					2.05	4.05				3.94		4.365	57.69
57.135	2.05					2.08	4.01				3.90		4.362	57.78
57.205	2.08					2.11	3.97				3.86		4.360	57.86
57.261	2.10					2.13	3.94				3.83		4.355	57.93
57.317	2.14					2.17	3.89				3.78		4.357	58.00
57.373	2.18					2.21	3.83				3.73		4.355	58.07
57.428	2.18	1.05	4.57	0.490		2.21	3.83		285	.973	3.73		4.353	58.14
57.484	2.21					2.24	3.79				3.69		4.351	58.21
57.552	2.24					2.27	3.75				3.65		4.349	58.30
57.622	2.28					2.31	3.69				3.59		4.347	58.38
57.678	2.30					2.33	3.67				3.57		4.346	58.45
57.734	2.34					2.37	3.61				3.51		4.344	58.52
57.789	2.36					2.39	3.58				3.48		4.343	58.59
57.844	2.38	1.04	4.56	0.490	0.03	2.41	3.55		285	.973	3.45		4.341	58.66
57.900	2.42					2.45	3.50				3.41		4.340	58.73
57.970	2.46					2.49	3.44				3.35		4.338	58.82
58.040	2.49					2.52	3.40				3.31		4.336	58.90
58.096	2.52					2.55	3.37				3.28		4.334	58.97
58.152	2.54					2.57	3.34				3.25		4.333	59.04
58.208	2.56					2.59	3.31				3.22		4.331	59.11
58.264	2.58	1.04	4.54	0.490	0.03	2.61	3.29		285	.973	3.20		4.329	59.18
58.320	2.60					2.63	3.26				3.17		4.327	59.25
58.388	2.62					2.65	3.24				3.15		4.325	59.34
58.458	2.64					2.67	3.21				3.12		4.322	59.42
58.514	2.68					2.71	3.16				3.07		4.321	59.49
58.570	2.68					2.71	3.16				3.07		4.319	59.56
58.627	2.72					2.75	3.11				3.03		4.318	59.63
58.682	2.74	1.04	4.56	0.490	0.03	2.77	3.08		285	.973	3.00		4.316	59.70
58.736	2.76					2.79	3.06				2.98		4.315	59.77
58.805	2.79					2.82	3.02				2.94		4.313	59.85
58.874	2.83					2.86	2.97				2.89		4.311	59.94
58.930	2.84					2.87	2.95				2.87		4.310	60.00
58.984	2.88					2.91	2.90				2.82		4.308	60.07
59.040	2.90	1.04	4.56	0.490	0.03	2.93	2.88	0	2.85	.973	2.80	0	4.307	60.14
												/		

CHAPTER VI

DETERMINATION OF AMBIENT ATMOSPHERIC DENSITY FROM IMPACT GAUGE PRESSURE

6.1 INTRODUCTION

In this chapter, the technique by which the impact gauge pressure is interpreted in terms of the ambient density of the atmosphere will be developed. The technique will be based on the theory presented in Chapter III.

As mentioned earlier, the aerodynamics of the probe vehicle depend on the flow region. The flow regions being defined by the dimensions of the probe relative to the mean-free path of the atmospheric particles. Because the mean-free path is a function of the density, and the density a function of altitude, the flow regions may be specified as ranges in altitude for any given probe.

In light of the above, the derivation of ambient density will be carried out using continuum theory at altitudes below approximately 85 km (continuum flow region for the present SPRL pitot-static probe), and free molecular flow theory at altitudes above approximately 90 km. The transition region for the present probe exists between 85 and 90 km.

The system of units to be used in this, and following chapters will be a modified MKS system.

6.2 AMBIENT DENSITY DETERMINATION BELOW 85 KM

In the continuum region, the equations employed to deduce the ambient density from impact gauge pressure are the Rayleigh Supersonic Pitot-Tube Equation, Eq. (III.17) and the equation of state, Eq. (III.12) which are

$$\frac{P_i}{P_a} = \left(\frac{\gamma+1}{2}\right) M^2 \left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2 - 2\gamma + 2}\right]^{\frac{1}{(\gamma-1)}} \qquad \text{(III.17)}$$

and

$$P_a = \rho_a R T_a \qquad \text{(III.12)}$$

where the symbols have the meanings assigned in Chapter III. Equation (III.12) can be rearranged into the form

$$\frac{P_a}{\rho_a} = RT_a \quad .$$

To obtain an expression for the ambient density, the above equation is multiplied with Eq. (III.17) with the following result

$$\frac{P_i}{\rho_a} = \left(\frac{\gamma+1}{2}\right) M^2 R T_a \left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2 - 2\gamma + 2}\right]^{\frac{1}{(\gamma-1)}} \quad .$$

Recalling from gas dynamics that

$$a^2 = \gamma R T_a$$

where

a is the speed of sound in the atmosphere at temperature T_a

γ is the ratio of specific heats for air

R is the gas constant per unit mass

T_a is the ambient atmospheric temperature and that the definition of Mach number is

$$M^2 = \frac{V^2}{a^2} = \frac{V^2}{\gamma R T_a}$$

where

V is the total velocity of the vehicle to which the Mach number M refers

M is the Mach number

a is the local speed of sound.

Substituting these equations into the expression for P_i/ρ_a

$$\frac{P_i}{\rho_a} = \left(\frac{\gamma+1}{2}\right)\frac{V^2}{\gamma R T_a} R T_a \left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2-2\gamma+2}\right]^{\frac{1}{(\gamma-1)}}$$

which after simplification becomes

$$\frac{P_i}{\rho_a} = V^2\left(\frac{\gamma+1}{2\gamma}\right)\left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2-2\gamma+2}\right]^{\frac{1}{(\gamma-1)}} .$$

Solving for the ambient density, ρ_a we have

$$\rho_a = \frac{P_i}{V^2\left(\frac{\gamma+1}{2\gamma}\right)\left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2-2\gamma+2}\right]^{\frac{1}{(\gamma-1)}}} .$$

Let us define the function

$$K(M) = \left(\frac{\gamma+1}{2\gamma}\right)\left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2-2\gamma+2}\right]^{\frac{1}{(\gamma-1)}} .$$

K(M) is found to be a very weak function of Mach number (see Appendix) which converges very rapidly to its asymptotic value of

$$K(\infty) = \frac{\gamma+1}{2\gamma}\left[\frac{(\gamma+1)^2}{4\gamma}\right]^{\frac{1}{\gamma-1}} .$$

Because of this weak dependence upon M and its rapid convergence, it can be approximated to within less than 1% error over a small range of Mach number by a known constant. Let this constant be $K(M_1)$ where M_1 is the approximate Mach number of the probe based on its known velocity and altitude and the standard speed of sound at that altitude taken from a standard reference atmosphere.

Using the value of $K(M_1)$ and the proper units conversion factor for our modified MKS system we see that

$$\rho_a \cong \frac{\bar{A}\ P_i}{K(M_1)V^2} \tag{VI.1}$$

where

ρ_a is the ambient atmospheric density (kg/m^3) for altitudes less than 85 km

$\bar{A}$* is a units conversion factor equal to $1435.064 \left(\frac{ft^2\ kg}{m^3\,[mmHg]\,sec^2}\right)$

P_i is the impact gauge pressure (mmHg)

V is the probe total velocity (ft/sec)

$K(M_1)$** is the approximate probe Mach number.

Equation (VI.1) represents a very simple and very close approximation to

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*For derivation of the constant A, see the Appendix.

**For tabulated values of K(M) versus M, see the Appendix.

the ambient atmospheric density in terms of the impact gauge pressure for the continuum flow region. The degree of approximation in Eq. (VI.1) may be made closer through an itterative technique. This, however, is usually not necessary. The results using the first approximation are to within 1% when M_1 is properly chosen and at the angles of attack experienced in flight.

6.3 AMBIENT DENSITY DETERMINATION ABOVE 90 KM

In the free molecule flow region, the interpretation of impact gauge pressure is modified due to the change in flow conditions, related to the change in mean-free path of the ambient atmospheric particles relative to the probe dimensions. In this region the governing equation is Eq. (III.22) which is

$$\frac{P_i}{P_a} = \sqrt{\frac{T_i}{T_a}}\, F(S) \tag{III.22}$$

where

$$F(S) = e^{-S^2} + S\sqrt{\pi}\,[1+\mathrm{erf}\ S] \quad .$$

A careful study of the behavior of F(S) (see Appendix for plot of F(S) versus S) shows that F(S) becomes linear in S for S sufficiently large. That is with negligible error

$$F(S) = 2\sqrt{\pi}\, S \qquad S > 1.5 \quad .$$

It is easily shown that the condition that S be greater than one implies that the Mach number M of the probe must satisfy the following condition

$$M > S \sqrt{\frac{2}{\gamma}} \cong 1.80$$

for

$$\gamma = 1.4 \quad .$$

This condition is easily satisfied by the SPRL pitot-static probe throughout its entire measurement region from 20 to 120 km.

Under this minor restriction, then, Eq. (III.22) may be rewritten

$$\frac{P_i}{P_a} \cong 2\sqrt{\pi}\, S \sqrt{\frac{T_i}{P_a}} \quad .$$

Recalling that

$$S = \frac{V_x}{\sqrt{2RT_a}} \quad .$$

The above statement becomes

$$\frac{P_i}{P_a} \simeq \frac{2\sqrt{\pi}\, V_x}{T_a} \sqrt{\frac{T_i}{2R}} \quad .$$

Making use of the equation of state, Eq. (III.12) in the above, we have that

$$P_a = \rho_a R T_a$$

and

$$\frac{P_i}{\rho_a} \cong 2\sqrt{\pi}\, RV_x \sqrt{\frac{T_i}{2R}} \quad .$$

Now, solving for the ambient atmospheric density, ρ_a, and simplifying

$$\rho_a \simeq \frac{P_i}{\sqrt{2\pi RT_i}\, V_x} \quad .$$

We also know that

$$V_x = V \cos \alpha$$

where

V is the total velocity of the probe

α is the probe angle of attack.

Making this substitution, the expression for ρ_a becomes

$$\rho_a \simeq \frac{P_i}{\sqrt{2\pi R}\,\sqrt{T_i}\; V \cos \alpha} \quad .$$

Let us now define

$$\overline{B} = \frac{\overline{A}}{\sqrt{2\pi R}}$$

where

$\overline{A}$ is the same constant given in Eq. (VI.2)

with this, we have finally

$$\rho_a \cong \frac{\overline{B}\, P_i}{\sqrt{T_i}\; V \cos \alpha}$$

where

ρ_a is the ambient atmosphere density (kg/m^3) for altitudes greater than 90 km

$\overline{B}$* is a dimensional constant equal to 10.257 $\frac{\text{ft kg}\sqrt{K^\circ}}{\text{sec m}^3\,[\text{mm Hg}]}$ based on the assumed value of molecular weight for air of 28.72 which is valid for the atmosphere between 90 and 120 km

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*The constant $\overline{B}$ is derived in the Appendix.

P_i is the impact gauge pressure (mm Hg)

T_i is the measured impact densatron gauge absolute temperature (°K)

V is the probe total velocity (ft/sec)

α is the probe angle of attack (degrees).

Through the use of Eq. (VI.2) we are now in the position to interpret the impact gauge pressure in terms of the ambient atmospheric density at altitudes above 90 km for the present SPRL pitot-static probe. This equation, it must be remembered, is an approximation dependent not only upon the simplifying assumptions made in its derivation, but on the mean molecular mass of the atmospheric particle. The error involved in the assumption that $F(S)$ is linear in S for $M > 1.5$ introduces negligible error in the final density expression. The error introduced by assuming a constant molecular weight for the entire FMF measurement (90-120 km) may be further minimized by assuming a variable composition. This, however, based on our present knowledge of the composition of the Upper Atmosphere, is not necessary, for the error introduced by the assumption is of the order of 0.3% at the end points of the region.

6.4 SUMMARY OF DENSITY FORMULAE

To summarize the results of the two previous sections, we will simply restate the final expressions and the applicable flow region.

Continuum Flow

$$\rho_a \cong \frac{1435.064\ P_i}{K(M_1)V^2} \qquad \text{(VI.1)}$$

where

ρ_a is the ambient atmospheric density (kg/m^3)

P_i is the impact gauge pressure (mm Hg)

V is the probe total velocity (ft/sec)

$K(M_1)$ is the value of $K(M)$ at M_1 (see the Appendix)

M_1 is the approximate probe Mach number.

Equation (VI.1) is applicable for altitudes below 85 km.

Free Molecule Flow

$$\rho_a \cong \frac{10.257\ P_i}{\sqrt{T_i}\ V \cos \alpha} \qquad \text{(VI.2)}$$

where

ρ_a is the ambient atmospheric density (kg/m^3)

P_i is the impact gauge pressure (mm Hg)

T_i is the impact densatron gauge temperature (°K)

V is the probe total velocity (ft/sec)

a is the probe angle of attack (degrees)

Equation (VI.2) is applicable for altitudes between 90 and 120 km.

CHAPTER VII

DETERMINATION OF AMBIENT ATMOSPHERIC PRESSURE

7.1 INTRODUCTION

The determination of the ambient atmospheric pressure from the raw data of the pitot-static probe is considerably easier than either of the other two desired parameters. The reason for this is, in accordance with Refs. 1 and 2, that the SPRL pitot-static probe is designed such that the ambient densatron gauge pressure is very close to the ambient atmospheric pressure in the continuum flow region (below approximately 85 km). In addition to this direct measurement of the ambient atmospheric pressure, the Hydrostatic Law may be used in a straightforward manner to deduce the ambient atmospheric pressure from the previously determined ambient density.

7.2 DIRECTLY MEASURED AMBIENT PRESSURE

In the continuum-flow region, the ambient pressure is measured directly at the manifold ambient chamber located as shown in Fig. IV.1. Wind tunnel investigations by Laurmann[1] show that the surface pressure at this location on the cylindrical body of the probe is within less than 1% of ambient, and that the manifold chamber minimizes deviations from the ambient pressure caused by bow-shock perturbations, boundary layer interference due to the expansion at the flare, and nonaxial flow about the probe. Recent flight test-data from two SPRL pitot-static probes indicates, contrary to wind tunnel information, that surface pressure at the ambient chamber is approximately 4%

lower than actual ambient pressure.

Expressed mathematically, the above statement becomes

$$\frac{P_a}{P_{gauge}} \cong 1.04 \qquad \text{(VII.1)}$$

where

P_{gauge} is the ambient densatron gauge pressure

P_a is the ambient atmospheric pressure.

Equation (VII.1) is valid in the continuum flow region only.

As discussed in Chapter III the pressure measured in the ambient chamber in the free molecule flow region will differ in magnitude from the actual ambient pressure due to the thermal transpiration effect as given by Eq. (III.23)

$$\frac{P_{gauge}}{P_a} = \sqrt{\frac{T_{gauge}}{T_a}} \qquad \text{(III.23)}$$

where

P_{gauge} is the ambient densatron gauge pressure

P_a is the ambient atmospheric pressure

T_{gauge} is the ambient densatron gauge temperature

T_a is the ambient atmospheric temperature.

Thus, according to Eq. (III.23), the ambient pressure cannot be found by using the manifold chamber pressure until the ambient atmospheric temperature is known. This problem is circumvented by the consideration of the next section.

7.3 AMBIENT PRESSURE DETERMINATION THROUGH THE USE OF THE HYDROSTATIC LAW

An alternate approach, which is independent of the flow region, to the determination of the ambient atmospheric pressure utilizes the previously determined ambient density and the Hydrostatic Law as given by Eq. (III.15)

$$\frac{dP_a}{dH} = -\rho_a g(H) \quad . \tag{III.15}$$

Equation (III.15) may be integrated with respect to H as follows

$$P_a]_{H_1} = \int_{H_{ref}}^{H_1} \rho_a g \, dH + P_a]_{H_{ref}} \tag{VII.2}$$

where

H_1 is an arbitrary altitude at which the value of P_a is desired

H_{ref} is an arbitrary reference altitude greater than H_1.

The integral indicated by Eq. (VII.2) must be evaluated numerically. To illustrate the use of this technique in determining the ambient pressure profile, let us rewrite Eq. (VII.2) in a slightly different but equivalent form.

$$P_{a,H_j} = P_{a,H_{ref}} + \Delta P_{a,1} + \Delta P_{a,2} + \dots + \Delta P_{a,j}$$

or

$$P_{a,H_j} = P_{a,H_{ref}} + \sum_{n=1}^{j} \Delta P_{a,n} \tag{VII.3}$$

where

P_{a,H_j} is the ambient atmospheric pressure at an altitude H_j

$P_{a,H_{ref}}$ is the ambient atmospheric pressure at an arbitrary high reference altitude.

$$\Delta P_{a,n} = \int_{H(n-1)}^{Hn} \rho g \, dH$$

$$H(n-1) > Hn, H_o = H_{ref} \quad .$$

To evaluate the integral in Eq. (VII.3), we first plot, using linear scales, the ambient atmospheric density in kilograms per cubic meter versus altitude in kilometers. On this plot subdivide the altitude range into one-half kilometer increments and integrate graphically, using the value of g corresponding to the altitude in question. The value of $\Delta P_{a,n}$ would then be given as

$$\Delta P_{a,n} = S_n(SF) \frac{g}{g_o} D \qquad \text{(VII.4)}$$

where

$$\Delta P_{a,n} = \int_{H(n-1)}^{Hn} \rho g \, dH \text{ (mm Hg)}$$

S_n is the total area under the density profile as determined by graphical integration between H(n-1) and Hn

SF is the scale factor of the plot of ambient density versus altitude (kg/m^2 $in.^2$)

g is the acceleration due to gravity at an altitude midway between H(n-1) and H(n). (m/sec^2)

g_o is the acceleration due to gravity at the surface of the earth. (m/sec^2)

D is a units conversion factor equal to $0.07356 \left(\frac{[\text{mm Hg}]m^2}{kg}\right)$.

The value of $P_{a,H_{ref}}$ or reference pressure may be determined in either of two ways, the latter being preferred. First, $P_{a,H_{ref}}$ may be assumed to be the value of P_a obtained from a Standard Reference Atmosphere corresponding to the altitude H_{ref}. Second, $P_{a,H_{ref}}$ may be computed by substituting the previously determined density at H_{ref} and an assumed value of atmospheric temperature obtained from the Standard Atmosphere corresponding to the altitude H_{ref} into the equation of state.

Since the actual integration process described here takes place starting at a high altitude (H_{ref}) and proceeds to a low altitude, the error incurred because of an improper choice of $P_{a,H_{ref}}$ is minimized. It can be shown that the choice of $P_{a,H_{ref}}$ is relatively unimportant and Eq. (VII.3) will rapidly converge to within less than 1% of the true ambient pressure when the difference between the altitude in question and the altitude H_{ref} is equal to approximately 15 km. A poor choice of $P_{a,H_{ref}}$ is readily apparent by its complete incompatability with the resulting pressure profile, and another value whice gives more rapid convergence should be selected.

7.4 CONCLUSION

As is readily seen, the ambient atmospheric pressure is measured directly at the manifold chamber in the continuum flow region, and is found through the use of the Hydrostatic Law in both the free molecule and continuum flow regions. This redundency of measurement is inherent in the multiple sensing system employed by the pitot-static probe. It is important to maintain a rather high degree of measurement redundency in the design of upper atmosphere experiments for several reasons. Two of which are: (1)

if self-consistency exists among the data resulting from the various methods of determining an atmospheric parameter, increased confidence can be had in the measured values of these parameters; and (2) redundency also helps maintain minimum data loss in the event of failure or improper operation of independent parts of the overall system.

References for Chapter VII

1. Laurmann, J. A., "Low Density Characteristics of an Aerobee-Hi Pitot-Static Probe," Univ. of Calif., Institute of Engineering Research, Tech. Report, HE-150-156, May, 1950.

2. Perkins, E. W., and L. H. Jorgensen, "Comparison of Experimental and Theoretical Normal-Force Distribution (Including Reynolds Number Effects) on an Ogive-Cylinder Body at Mach Number 1.98," NACA Technical Note 3716, Washington, D.C., May, 1956.

CHAPTER VIII

DETERMINATION OF AMBIENT ATMOSPHERIC TEMPERATURE

8.1 INTRODUCTION

Ambient temperature in the upper atmosphere is perhaps the most significant of the three parameters yet discussed. It is this parameter which is most closely related to the important physiochemical reactions which result in the heating and cooling of the upper atmosphere. These heating and cooling processes then determine the basic meteorology of the upper atmosphere. Thus, a need for a knowledge of the ambient temperature is readily apparent.

In addition to being the more basic and important of the parameters of the atmosphere it is also the most difficult to obtain. At present, there is no simple method by which the ambient temperature may be measured directly with the degree of accuracy required. Thus, the experimenter must rely on techniques for interpreting directly measured quantities in terms of the ambient temperature.

8.2 AMBIENT TEMPERATURE THROUGH THE USE OF THE STATE EQUATION

A first method for deducing the ambient atmospheric temperature in terms of the ambient density and pressure is a straightforward substituting of the known quantities into the equation of state which may be stated

$$T_a = \frac{P_a}{\rho_a R} \tag{VIII.1}$$

where

T_a is the ambient atmospheric temperature at an altitude H(°K)

P_a is the ambient atmospheric pressure at the altitude H

ρ_a is the ambient [mm Hg] atmospheric density at the altitude H. (kg/m^3)

R is the gas constant per unit mass whose value is $2.153 \frac{[\text{mm Hg}]\text{m}^3}{\text{kg }^\circ\text{K}}$ based on a molecular weight for air of 28.9644.

This equation may easily be set up in tabular form and the required calculation carried out for each altitude at which a value of ambient density and pressure exists.

Equation (VIII.1) represents an extremely simple method for determing the ambient temperature from the primary data obtained from the pitot-static probe. Because the primary data from the pitot-static is continuous with respect to altitude, a continuous temperature profile is readily obtained.

8.3 AMBIENT TEMPERATURE THROUGH USE OF THE HYDROSTATIC LAW IN COMBINATION WITH THE EQUATION OF STATE

Still a second technique which is based on the so called pressure scale height may be derived from the Hydrostatic Law as given by Eq. (III.15)

$$\frac{dP_a}{dH} = -\rho_a g(H) \qquad \text{(III.15)}$$

and Eq. (III.12)

$$P_a = \rho_a R T_a \quad . \qquad \text{(III.12)}$$

Equations (III.15) and (III.12) may be combined as follows

$$\frac{dP_a}{P_a} = \frac{g}{RT_a}\,dH$$

which, upon integration between the limits of P_{a1}, and P_{a2}, and H_1 and H_2 respectively,

$$\ln P_{a2} - \ln P_{a1} = \frac{g}{RT_a}(H_2 - H_1) \quad .$$

Here it should be noted that g, R, and T_a have been considered constant with respect to the integrand dH on the right hand side of the equation. This assumption is valid as long as the altitude increment H_2-H_1 is kept very small.

The above expression may now be solved for T_a, and after some manipulation the expression for T_a becomes

$$T_a = \frac{g_o}{R}\,\frac{g}{g_o}\,\frac{\Delta H}{\ln \frac{P_1}{P_2}} \qquad \text{(VIII.2)}$$

where

T_a is the ambient atmospheric temperature (°K)

g is the acceleration due to gravity at altitude $H(m/sec^2)$

g_o is the acceleration due to gravity at the surface of the earth (m/sec^2)

g_o/R is the ratio of the acceleration due to gravity at the earth's surface to the gas constant per unit mass whose value is 34.154 (°K/km) based on a molecular weight of 28.9644 for air

ΔH is the small altitude increment (H_2-H_1) under consideration (km)

P_1 is the ambient atmospheric pressure at altitude H_1 (mm Hg)

P_2 is the ambient atmospheric pressure at altitude H_2 (mm Hg).

Equation (VIII.2) is readily adapted to tabular computation using the values of g/g_0 given in the Appendix. The value of ΔH should be the order-of one kilometer to avoid the possibility of "too much averaging" in the final resulting temperature. This calculation should be carried out using all ambient pressure data obtained, thus yielding a continuous temperature profile for the entire measurements region of the pitot-static probe.

8.4 AMBIENT TEMPERATURE IN THE CONTINUUM FLOW REGION FROM THE RAYLEIGH-PITOT-TUBE EQUATION

Still another technique for deducing the ambient atmospheric temperature in the continuum flow region from the primary pressure data of the pitot probe may be derived from the Rayleigh Supersonic Pitot-Tube Equation, Eq. (III.17)

$$\frac{P_i}{P} = \left(\frac{\gamma+1}{2}\right) M^2 \left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2 - 2 + 2}\right]^{\frac{1}{\gamma-1}}$$

which may be put in the form of Eq. (VI.1)

$$\rho_a \cong \frac{\bar{A}\ P_i}{K(M_1)V^2} \quad . \qquad \text{(VI.1)}$$

Recall the equation of state

$$P_a = \rho_a R T_a \qquad \text{(III.12)}$$

or

$$\rho_a = \frac{P_a}{RT_a} \quad .$$

This expression may be substituted into Eq. (VI.1) to give

$$\frac{P}{RT_a} = \frac{\bar{A}\,P_i}{K(M_1)V^2}$$

which may now be solved for the ambient temperature as

$$T_a = \frac{K(M_1)V^2}{\bar{A}\,R}\,\frac{P_a}{P_i} \tag{VIII.3}$$

where

T_a is the ambient atmospheric temperature (°K)

$K(M_1)$ is the value of $K(M)$ at M_1 dimensionless

M_1 is the approximate probe Mach number

V is the probe total velocity

P_i is the impact densatron gauge pressure (mm Hg)

P_a is the ambient atmospheric pressure (mm Hg)

$\bar{A}$ is a unit conversion factor equal to $1435.064 \left(\frac{\text{ft}^2\text{kg}}{\text{m}^3[\text{mm Hg}]\text{sec}^2}\right)$

R is the gas constant per unit mass equal to $2.153 \left(\frac{[\text{mm Hg}]\text{m}^3}{\text{kg }^\circ\text{K}}\right)$.

Equation (VIII.3) may be used on a point by point basis in the continuum flow region to obtain a continuous ambient temperature profile over the entire flow region.

It is important to remember that Eq. (VIII.3) is not valid in the free molecule flow region.

8.5 AMBIENT TEMPERATURE IN THE FREE MOLECULE FLOW REGION

An expression for the ambient atmospheric temperature in the free molecule flow region may be developed in a manner similar to that for Eq. (VIII.3). For this development we make use of Eq. (VI.2)

$$\rho_a \cong \frac{\bar{B} \, P_i}{\sqrt{T_i} \, V \cos \alpha} \qquad \text{(VI.2)}$$

and the equation of state

$$P_a = \rho_a R T_a \quad . \qquad \text{(III.12)}$$

Combining Eq. (VI.2) and (III.12) and solving for T_a we have

$$T_a = \frac{\sqrt{T_i} \, V \cos \alpha}{\bar{B} R} \, \frac{P_a}{P_i} \qquad \text{(VIII.4)}$$

where

T_a is the ambient atmospheric temperature (°K)

T_i is the measured impact densatron gauge temperature (°K)

V is the probe total velocity (ft/sec)

α is the probe angle of attack (degrees)

P_a is the ambient atmospheric pressure (mm Hg)

P_i is the impact densatron gauge pressure (mm Hg)

$\bar{B}$ is a units conversion factor equal to 10.257 $\frac{\text{ft kg K}^\circ}{\text{sec m}^3 \, [\text{mm Hg}]}$ based on the assumed value of molecular weight for air of 28.72 for altitudes between 90 and 120 km

R is the gas constant per unit mass equal to 2.172 $\frac{[\text{mm Hg}]\text{m}^3}{\text{kg } ^\circ\text{K}}$ based on molecular weight of 28.72.

Equation (VIII.4) thus gives the ambient temperature in terms of the primary pressure measurements made by the pitot-static probe in the free molecule flow region.

Another interesting approach to the determination of ambient temperature in the free molecule flow region which has not yet been employed with the pitot-static probe data is based on the previously mentioned thermal transpiration effect. This technique requires an accurate determination of ambient gauge pressure and gauge temperature. To illustrate, recall Eq. (III.23)

$$\frac{P_{gauge}}{P_a} = \sqrt{\frac{T_{gauge}}{T_a}} \qquad \text{(III.23)}$$

where

P_{gauge} is the ambient gauge pressure in free molecule flow

P_a is the ambient atmospheric pressure

T_{gauge} is the ambient gauge temperature

T_a is the ambient atmospheric temperature.

Writing Eq. (III.23) in terms of ambient temperature, we have

$$T_a = T_{gauge}\left(\frac{P_a}{P_{gauge}}\right)^2 \quad . \qquad \text{(VIII.5)}$$

Equation (VIII.5) shows that if the three parameters, T_{gauge}, P_a, and P_{gauge} can be accurately determined in the free molecule flow region, then the ambient temperature may easily be deduced.

This method shows much promise for the near future, for it is now becoming possible, with increased sensitivity of the densatron, to obtain an accurate measure of the ambient gauge pressure in free molecule flow. The gauge temperature is directly measured, and the ambient atmospheric pressure is easily obtained from the impact pressure measurement.

8.6 AMBIENT TEMPERATURE FROM DIRECT INTEGRATION OF AMBIENT DENSITY

In addition to the method of determining developed above, one may develop an expression for atmospheric temperature at altitude "H" in terms of the temperature at a reference altitude "H_{ref}" and the ambient density profile.

This development again makes use of the theory presented in Chapter III.

To begin, we make use of Eq. (III.15) which is the Hydrostatic Law,

$$\frac{dP_a}{dH} = - \rho_a g(H) \tag{III.15}$$

and the equation of state

$$P_a = \rho_a RT \quad . \tag{III.12}$$

Equation (III.12) may be differentiated to yield

$$dP_a = Rd(\rho_a T)$$

which may be substituted into Eq. (III.15) and written as

$$d(\rho T_a) = - \frac{\rho_a g \; dH}{R} \quad . \tag{VIII.6}$$

Equation (VIII.6) may be integrated between the limits H and H_{ref} giving the desired result

$$T_{a,H} = \frac{T_{a,H_{ref}} \; \rho_{a,H_{ref}}}{\rho_{a,H}} \quad \frac{1}{\rho_{a,H} R} \int_{H_{ref}}^{H} \rho_a g \; dH \tag{VIII.7}$$

$$H_{ref} > H$$

where

$T_{a,H}$	is the ambient atmospheric temperature at altitude H (°K)
T_a, H_{ref}	is the ambient atmospheric temperature at altitude H_{ref}
ρ_a, H_{ref}	is the ambient atmospheric density at altitude H_{ref}
$\rho_{a,H}$	is the ambient atmospheric density at altitude H
R	is the gas constant per unit mass
H_{ref}	is an arbitrary high reference altitude
H	is a general altitude
g	is the acceleration due to gravity.

In the above, Eq. (VIII.7), H_{ref} is specified to be greater than H for the following reason: If the computation of temperature is begun at a high altitude and continued downward, the error incurred in the integration becomes progressively more independent of the error in T_a, H_{ref} and of the error in the density profile in the vicinity of ρ_a, H_{ref}, but becomes almost entirely dependent upon the error in the density profile in the region just prior to $\rho_{a,H}$.

8.7 CONCLUSION

An important conclusion to be drawn at this time is based on the fact that even though we have present four formulae for determining the ambient atmospheric temperature, there is in reality only two basic approaches used. Careful study of Eqs. (VIII.1), (VIII.3) and (VIII.4) show that these are actually the equation of state written in a form which utilizes the pitot-static probe data directly. Equations (VIII.2) and (VIII.7) on the other hand are simply forms of the Hydrostatic Law. There are of course several other expressions of ambient temperature in terms of the primary pitot-static data but they must all be based either on the equation of state or the Hydrostatic Law.

APPENDIX

PHYSICAL CONSTANTS AND CONVERSION FACTORS

A.1 INTRODUCTION

To facilitate the data reduction procedures presented in the body of this report, the following sections containing necessary constants, conversion factors, and tabulated data are presented.

In these sections, the following information will be found: (1) Often used physical constants and units conversion factors; (2) Derivation of the constants $\bar{A}$ and $\bar{B}$ found in Chapter VI; (3) Tabulation and plot of the function K(M) found in Chapter VI; (4) Tabulation and plot of the function F(S) also found in Chapter VI; Tabulation of the values of the ratio g/g_0 for an altitude range of 0 to 100 km.

A.2 PHYSICAL CONSTANTS AND CONVERSION FACTORS

A.2.1 Physical Constants and Formulae

The following is a list of the most often used physical constants:

(1) Avogadro's Number (number of molecules in one mole)

$$A = 6.0247 \times 10^{-23} \frac{\text{molecules}}{\text{mole}}$$

(2) Boltzmann's Constant

$$k = 1.3804 \times 10^{-23} \text{ joules}/{}^\circ K$$

(3) Density of Mercury at 273.15°K

$$\rho_{Hg} = 13{,}595.04 \ \frac{kg}{m^3}$$

(4) Acceleration due to Gravity at Sea Level

$$9.8066 \ \frac{m}{sec^2}$$

(5) Molecular Weight of Air at Sea Level

$$MW_o = 28.9644 \times 10^{-3} \ \frac{kg}{mole}$$

(6) Gas Constant per Unit Mass of Air at Sea Level

$$R = 287.128 \ \frac{joule}{kg \ {}^\circ K}$$

$$R = 2.153 \ \frac{m^3 [mm \ Hg]}{kg \ {}^\circ K}$$

(7) The Ratio g_o/R for Air at Sea Level

$$g_o/R = 34.154 \times 10^{-3} \ \frac{{}^\circ K}{m} \quad .$$

A.2.2 Conversion Factors

The following important, often used, conversions are presented for reference.

$$1 \ nt = \frac{1 \ kg \ m}{sec^2}$$

$$1 \ joule = 1 \ nt \ m = 1 \ \frac{kg \ m^2}{sec^2}$$

$$1 \ kg_{FORCE} = 9.8066 \ \frac{kg \ m}{sec^2}$$

$$1 \text{ mm Hg} = 13.595 \frac{\text{kg}_{\text{FORCE}}}{\text{m}^2}$$

$$1 \frac{\text{kg}_{\text{FORCE}}}{\text{m}^2} = 0.07356 \text{ [mm Hg]} \quad .$$

A.2.3 Conversion Formulae

In general, the physical constants presented in A.2.1 are valid for use in the data reduction formulae in the altitude range of 0 to 90 km. In the event, that it is necessary to reduce data for altitudes above 90 km, the following formulae may be used to recompute the values of the necessary constants taking into account the change in molecular weight of air which occurs above this altitude.

$$R = \frac{\text{kA}}{\text{MW}} \frac{\text{joule}}{\text{kg } ^\circ\text{K}} = \frac{8.3165}{(\text{MW})} \frac{\text{joule}}{\text{kg } ^\circ\text{K}}$$

$$R = \frac{\text{kA}}{\text{MW}} \times 0.00750 \frac{\text{m}^3 \text{[mm Hg]}}{\text{kg } ^\circ\text{K}} = \frac{0.06237}{(\text{MW})} \frac{\text{m}^3 \text{[mm Hg]}}{\text{kg } ^\circ\text{K}}$$

$$g_0/R = \frac{g_0 (\text{MW})}{\text{kA}} \frac{^\circ\text{K}}{\text{m}} = 1.1792 \, (\text{MW}) \frac{^\circ\text{K}}{\text{m}} \quad .$$

A.3 DERIVATION OF THE CONSTANTS $\overline{A}$ AND $\overline{B}$

As stated in Chapter VI, the constants $\overline{A}$ and $\overline{B}$ are units conversion factors. To determine the value for each of these constants, it is necessary to make a dimensional analysis of the formulae in which they are found.

The Constant $\overline{A}$

This constant is found in Eq. (VI.1)

$$\rho_a = \frac{\overline{A} \, P_i}{K(M_1)V^2} \frac{\text{kg}}{\text{m}^3} \quad . \tag{VI.1}$$

If P_i is given in [mm Hg] and V^2 in $(\text{ft/sec})^2$ then:

$$\frac{\text{kg}}{\text{m}^3} = \frac{\bar{A}\,[\text{mm Hg}]\,\text{sec}^2}{\text{ft}^2} \quad .$$

Thus, the units of $\bar{A}$ are:

$$\frac{\text{kg}}{\text{m}^3}\,\frac{\text{ft}^2}{\text{sec}^2}\,\frac{1}{[\text{mm Hg}]}$$

or

$$\bar{A} = 10.764\,\frac{\text{ft}^2}{\text{m}^2} \times 13.595\,\frac{\text{kg}_{\text{FORCE}}}{\text{m}^2\,[\text{mm Hg}]} \times 9.8066\,\frac{\text{m}}{\text{sec}^2}\,\frac{\text{kg}}{\text{kg}_{\text{FORCE}}} \quad .$$

Carrying out the operations

$$\bar{A} = 1435.064\,\frac{\text{kg}}{\text{m}^3}\,\frac{\text{ft}^2}{\text{sec}^2}\,\frac{1}{[\text{mm Hg}]} \quad .$$

The Constant B

Similarly, this constant is found in Eq. (VI.2)

$$\rho_a = \frac{\bar{B}\,P_i}{\sqrt{T_i}\,V\cos\alpha}\,\frac{\text{kg}}{\text{m}^3} \tag{VI.2}$$

where $\bar{B}$ is given by

$$\bar{B} = \frac{\bar{A}}{\sqrt{2\pi R}}\,\frac{\text{ft}}{\text{sec}}\,\frac{\text{kg}}{\text{m}^3}\,\frac{\sqrt{{}^\circ\text{K}}}{[\text{mm Hg}]}$$

or

$$\bar{B} = \frac{\bar{A}\sqrt{MW}}{\sqrt{2\pi}\sqrt{kA}\;3.2808}$$

$$\bar{B} = \frac{1435.064\sqrt{MW}}{7.2283 \times 3.2808}$$

$$\bar{B} = 60.514 \sqrt{MW} \frac{\text{ft}}{\text{sec}} \frac{\text{kg}}{\text{m}^3} \frac{\sqrt{°K}}{[\text{mm Hg}]}$$

which is the general equation for $\bar{B}$.

If the molecular weight of air in the altitude range 90 to 120 km is taken as

$$MW = 28.72 \times 10^{-3} \frac{\text{kg}}{\text{mole}} \quad .$$

Then $\bar{B}$ becomes

$$\bar{B} = 10.257 \frac{\text{ft}}{\text{sec}} \frac{\text{kg}}{\text{m}^3} \frac{\sqrt{°K}}{[\text{mm Hg}]}$$

A.4 TABLE AND GRAPH OF K(M) VS. M FOR AIR (pps. 123-126)

$\gamma = 1.4$

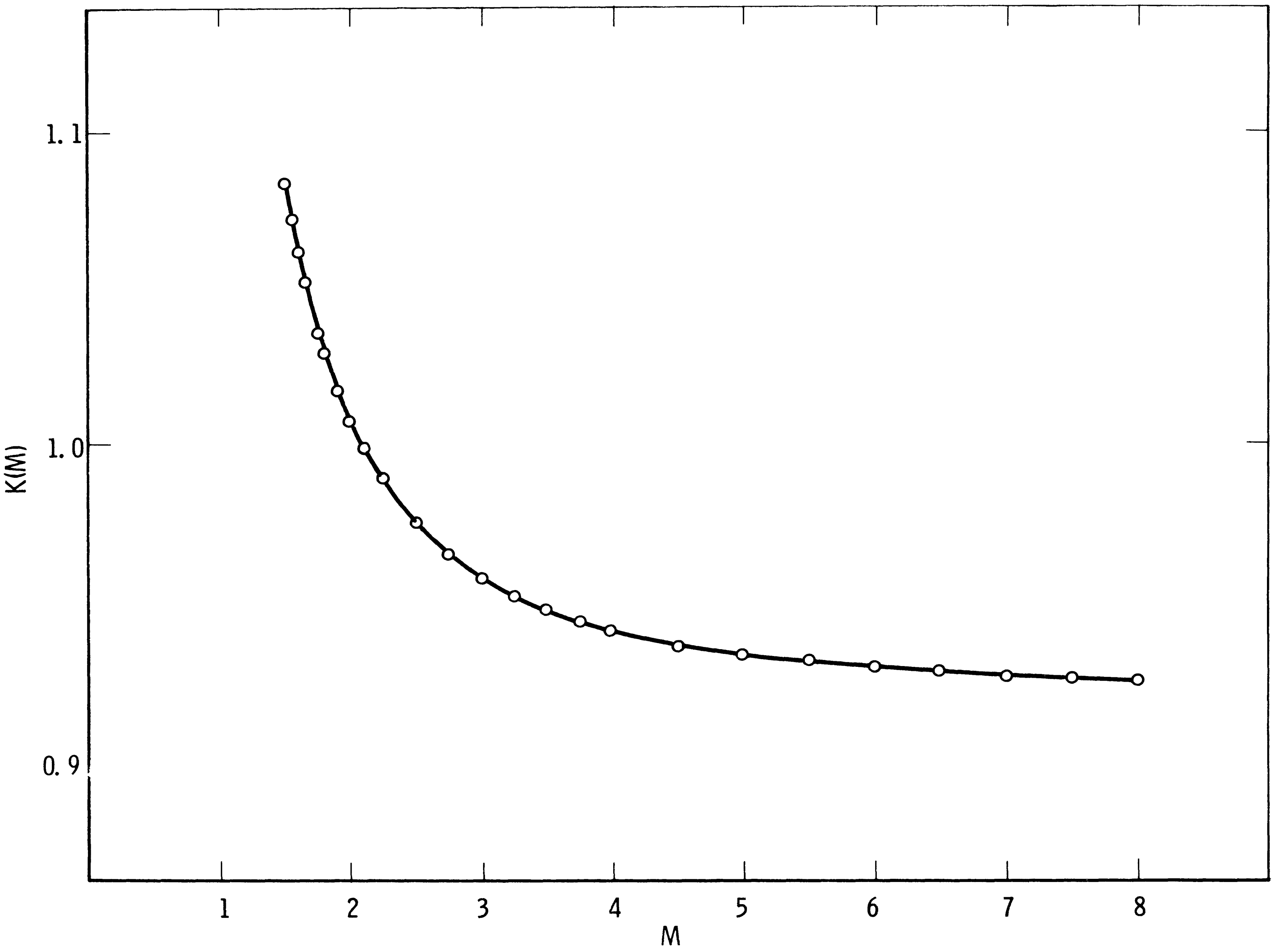
GRAPH: K(M) vs. M

$\gamma = 1.4$

M	K(M)	M	K(M)	M	K(M)	M	K(M)	M	K(M)
1.50	1.08358	2.05	1.00274	2.60	.97013	3.15	.95364	3.70	.94412
1.51	1.08116	2.06	1.00189	2.61	.96973	3.16	.95342	3.71	.94399
1.52	1.07879	2.07	1.00105	2.62	.96933	3.17	.95320	3.72	.94386
1.53	1.07648	2.08	1.00022	2.63	.96894	3.18	.95299	3.73	.94372
1.54	1.07422	2.09	.99941	2.64	.96856	3.19	.95277	3.74	.94359
1.55	1.07201	2.10	.99860	2.65	.96818	3.20	.95256	3.75	.94346
1.56	1.06985	2.11	.99781	2.66	.96780	3.21	.95235	3.76	.94334
1.57	1.06773	2.12	.99704	2.67	.96743	3.22	.95214	3.77	.94321
1.58	1.06566	2.13	.99627	2.68	.96706	3.23	.95194	3.78	.94308
1.59	1.06364	2.14	.99551	2.69	.96669	3.24	.95174	3.79	.94296
1.60	1.06166	2.15	.99477	2.70	.96633	3.25	.95153	3.80	.94283
1.61	1.05972	2.16	.99404	2.71	.96598	3.26	.95133	3.81	.94271
1.62	1.05782	2.17	.99332	2.72	.96563	3.27	.95114	3.82	.94259
1.63	1.05596	2.18	.99261	2.73	.96528	3.28	.95094	3.83	.94246
1.64	1.05413	2.19	.99191	2.74	.96494	3.29	.95075	3.84	.94234
1.65	1.05235	2.20	.99122	2.75	.96460	3.30	.95055	3.85	.94222
1.66	1.05060	2.21	.99054	2.76	.96426	3.31	.95036	3.86	.94211
1.67	1.04889	2.22	.98987	2.77	.96393	3.32	.95017	3.87	.94199
1.68	1.04721	2.23	.98920	2.78	.96360	3.33	.94999	3.88	.94187
1.69	1.04557	2.24	.98855	2.79	.96328	3.34	.94980	3.89	.94176
1.70	1.04395	2.25	.98791	2.80	.96295	3.35	.94962	3.90	.94164
1.71	1.04237	2.26	.98728	2.81	.96264	3.36	.94944	3.91	.94153
1.72	1.04082	2.27	.98665	2.82	.96232	3.37	.94926	3.92	.94141
1.73	1.03930	2.28	.98604	2.83	.96201	3.38	.94908	3.93	.94130
1.74	1.03781	2.29	.98543	2.84	.96171	3.39	.94890	3.94	.94119
1.75	1.03635	2.30	.98483	2.85	.96140	3.40	.94872	3.95	.94108
1.76	1.03491	2.31	.98424	2.86	.96110	3.41	.94855	3.96	.94097
1.77	1.03350	2.32	.98366	2.87	.96080	3.42	.94838	3.97	.94086
1.78	1.03212	2.33	.98309	2.88	.96051	3.43	.94821	3.98	.94075
1.79	1.03077	2.34	.98252	2.89	.96022	3.44	.94804	3.99	.94065
1.80	1.02943	2.35	.98196	2.90	.95993	3.45	.94787	4.00	.94054
1.81	1.02813	2.36	.98141	2.91	.95965	3.46	.94771	4.01	.94043
1.82	1.02684	2.37	.98087	2.92	.95937	3.47	.94754	4.02	.94033
1.83	1.02558	2.38	.98033	2.93	.95909	3.48	.94738	4.03	.94023
1.84	1.02435	2.39	.97980	2.94	.95881	3.49	.94722	4.04	.94012
1.85	1.02313	2.40	.97928	2.95	.95854	3.50	.94706	4.05	.94002
1.86	1.02194	2.41	.97876	2.96	.95827	3.51	.94690	4.06	.93992
1.87	1.02077	2.42	.97826	2.97	.95800	3.52	.94674	4.07	.93982
1.88	1.01961	2.43	.97775	2.98	.95774	3.53	.94658	4.08	.93972
1.89	1.01848	2.44	.97726	2.99	.95748	3.54	.94643	4.09	.93962
1.90	1.01737	2.45	.97677	3.00	.95722	3.55	.94627	4.10	.93952
1.91	1.01627	2.46	.97629	3.01	.95696	3.56	.94612	4.11	.93942
1.92	1.01520	2.47	.97581	3.02	.95671	3.57	.94597	4.12	.93932
1.93	1.01414	2.48	.97534	3.03	.95646	3.58	.94582	4.13	.93923
1.94	1.01310	2.49	.97487	3.04	.95621	3.59	.94567	4.14	.93913
1.95	1.01208	2.50	.97442	3.05	.95597	3.60	.94553	4.15	.93904
1.96	1.01108	2.51	.97396	3.06	.95572	3.61	.94538	4.16	.93894
1.97	1.01009	2.52	.97352	3.07	.95548	3.62	.94524	4.17	.93885
1.98	1.00912	2.53	.97307	3.08	.95524	3.63	.94509	4.18	.93876
1.99	1.00816	2.54	.97264	3.09	.95501	3.64	.94495	4.19	.93866
2.00	1.00722	2.55	.97221	3.10	.95477	3.65	.94481	4.20	.93857
2.01	1.00630	2.56	.97178	3.11	.95454	3.66	.94467	4.21	.93848
2.02	1.00539	2.57	.97136	3.12	.95431	3.67	.94453	4.22	.93839
2.03	1.00449	2.58	.97095	3.13	.95409	3.68	.94439	4.23	.93830
2.04	1.00361	2.59	.97053	3.14	.95386	3.69	.94426	4.24	.93821

M	K(M)	M	K(M)	M	K(M)	M	K(M)	M	K(M)
4.25	.93812	4.80	.93410	5.35	.93126	5.90	.92919	6.45	.92763
4.26	.93804	4.81	.93404	5.36	.93122	5.91	.92916	6.46	.92760
4.27	.93795	4.82	.93398	5.37	.93118	5.92	.92912	6.47	.92758
4.28	.93786	4.83	.93392	5.38	.93113	5.93	.92909	6.48	.92755
4.29	.93778	4.84	.93386	5.39	.93109	5.94	.92906	6.49	.92753
4.30	.93769	4.85	.93380	5.40	.93105	5.95	.92903	6.50	.92751
4.31	.93761	4.86	.93374	5.41	.93100	5.96	.92900	6.51	.92748
4.32	.93752	4.87	.93368	5.42	.93096	5.97	.92897	6.52	.92746
4.33	.93744	4.88	.93362	5.43	.93092	5.98	.92894	6.53	.92743
4.34	.93736	4.89	.93357	5.44	.93088	5.99	.92890	6.54	.92741
4.35	.93728	4.90	.93351	5.45	.93084	6.00	.92887	6.55	.92739
4.36	.93719	4.91	.93345	5.46	.93080	6.01	.92884	6.56	.92736
4.37	.93711	4.92	.93340	5.47	.93076	6.02	.92881	6.57	.92734
4.38	.93703	4.93	.93334	5.48	.93071	6.03	.92878	6.58	.92732
4.39	.93695	4.94	.93328	5.49	.93067	6.04	.92875	6.59	.92729
4.40	.93687	4.95	.93323	5.50	.93063	6.05	.92872	6.60	.92727
4.41	.93679	4.96	.93317	5.51	.93059	6.06	.92869	6.61	.92725
4.42	.93672	4.97	.93312	5.52	.93055	6.07	.92866	6.62	.92722
4.43	.93664	4.98	.93306	5.53	.93051	6.08	.92863	6.63	.92720
4.44	.93656	4.99	.93301	5.54	.93048	6.09	.92860	6.64	.92718
4.45	.93648	5.00	.93296	5.55	.93044	6.10	.92857	6.65	.92715
4.46	.93641	5.01	.93290	5.56	.93040	6.11	.92854	6.66	.92713
4.47	.93633	5.02	.93285	5.57	.93036	6.12	.92851	6.67	.92711
4.48	.93626	5.03	.93280	5.58	.93032	6.13	.92848	6.68	.92709
4.49	.93618	5.04	.93274	5.59	.93028	6.14	.92846	6.69	.92707
4.50	.93611	5.05	.93269	5.60	.93024	6.15	.92843	6.70	.92704
4.51	.93603	5.06	.93264	5.61	.93021	6.16	.92840	6.71	.92702
4.52	.93596	5.07	.93259	5.62	.93017	6.17	.92837	6.72	.92700
4.53	.93589	5.08	.93254	5.63	.93013	6.18	.92834	6.73	.92698
4.54	.93582	5.09	.93249	5.64	.93009	6.19	.92831	6.74	.92696
4.55	.93574	5.10	.93244	5.65	.93006	6.20	.92829	6.75	.92693
4.56	.93567	5.11	.93239	5.66	.93002	6.21	.92826	6.76	.92691
4.57	.93560	5.12	.93234	5.67	.92998	6.22	.92823	6.77	.92689
4.58	.93553	5.13	.93229	5.68	.92995	6.23	.92820	6.78	.92687
4.59	.93546	5.14	.93224	5.69	.92991	6.24	.92818	6.79	.92685
4.60	.93539	5.15	.93219	5.70	.92987	6.25	.92815	6.80	.92683
4.61	.93532	5.16	.93214	5.71	.92984	6.26	.92812	6.81	.92681
4.62	.93526	5.17	.93209	5.72	.92980	6.27	.92809	6.82	.92679
4.63	.93519	5.18	.93204	5.73	.92977	6.28	.92807	6.83	.92676
4.64	.93512	5.19	.93199	5.74	.92973	6.29	.92804	6.84	.92674
4.65	.93505	5.20	.93195	5.75	.92970	6.30	.92801	6.85	.92672
4.66	.93499	5.21	.93190	5.76	.92966	6.31	.92799	6.86	.92670
4.67	.93492	5.22	.93185	5.77	.92963	6.32	.92796	6.87	.92668
4.68	.93485	5.23	.93180	5.78	.92959	6.33	.92793	6.88	.92666
4.69	.93479	5.24	.93176	5.79	.92956	6.34	.92791	6.89	.92664
4.70	.93472	5.25	.93171	5.80	.92952	6.35	.92788	6.90	.92662
4.71	.93466	5.26	.93167	5.81	.92949	6.36	.92786	6.91	.92660
4.72	.93460	5.27	.93162	5.82	.92945	6.37	.92783	6.92	.92658
4.73	.93453	5.28	.93157	5.83	.92942	6.38	.92780	6.93	.92656
4.74	.93447	5.29	.93153	5.84	.92939	6.39	.92778	6.94	.92654
4.75	.93441	5.30	.93148	5.85	.92935	6.40	.92775	6.95	.92652
4.76	.93434	5.31	.93144	5.86	.92932	6.41	.92773	6.96	.92650
4.77	.93428	5.32	.93139	5.87	.92929	6.42	.92770	6.97	.92648
4.78	.93422	5.33	.93135	5.88	.92925	6.43	.92768	6.98	.92646
4.79	.93416	5.34	.93131	5.89	.92922	6.44	.92765	6.99	.92644

M	K(M)	M	K(M)	M	K(M)	M	K(M)	M	K(M)
7.00	.92642	7.21	.92603	7.42	.92568	7.62	.92537	7.82	.92508
7.01	.92640	7.22	.92602	7.43	.92566	7.63	.92535	7.83	.92506
7.02	.92638	7.23	.92600	7.44	.92565	7.64	.92534	7.84	.92505
7.03	.92637	7.24	.92598	7.45	.92563	7.65	.92532	7.85	.92504
7.04	.92635	7.25	.92596	7.46	.92561	7.66	.92531	7.86	.92502
7.05	.92633	7.26	.92595	7.47	.92560	7.67	.92529	7.87	.92501
7.06	.92631	7.27	.92593	7.48	.92558	7.68	.92528	7.88	.92500
7.07	.92629	7.28	.92591	7.49	.92557	7.69	.92526	7.89	.92498
7.08	.92627	7.29	.92590	7.50	.92555	7.70	.92525	7.90	.92497
7.09	.92625	7.30	.92588	7.51	.92554	7.71	.92523	7.91	.92496
7.10	.92623	7.31	.92586	7.52	.92552	7.72	.92522	7.92	.92494
7.11	.92622	7.32	.92584	7.53	.92550	7.73	.92521	7.93	.92493
7.12	.92620	7.33	.92583	7.54	.92549	7.74	.92519	7.94	.92492
7.13	.92618	7.34	.92581	7.55	.92547	7.75	.92518	7.95	.92490
7.14	.92616	7.35	.92579	7.56	.92546	7.76	.92516	7.96	.92489
7.15	.92614	7.36	.92578	7.57	.92544	7.77	.92515	7.97	.92488
7.16	.92612	7.37	.92576	7.58	.92543	7.78	.92513	7.98	.92486
7.17	.92611	7.38	.92574	7.59	.92541	7.79	.92512	7.99	.92485
7.18	.92609	7.39	.92573	7.60	.92540	7.80	.92511	8.00	.92484
7.19	.92607	7.40	.92571	7.61	.92538	7.81	.92509	8.01	.92482
7.20	.92605	7.41	.92569						



K(M) vs. M
$\gamma = 1.4$

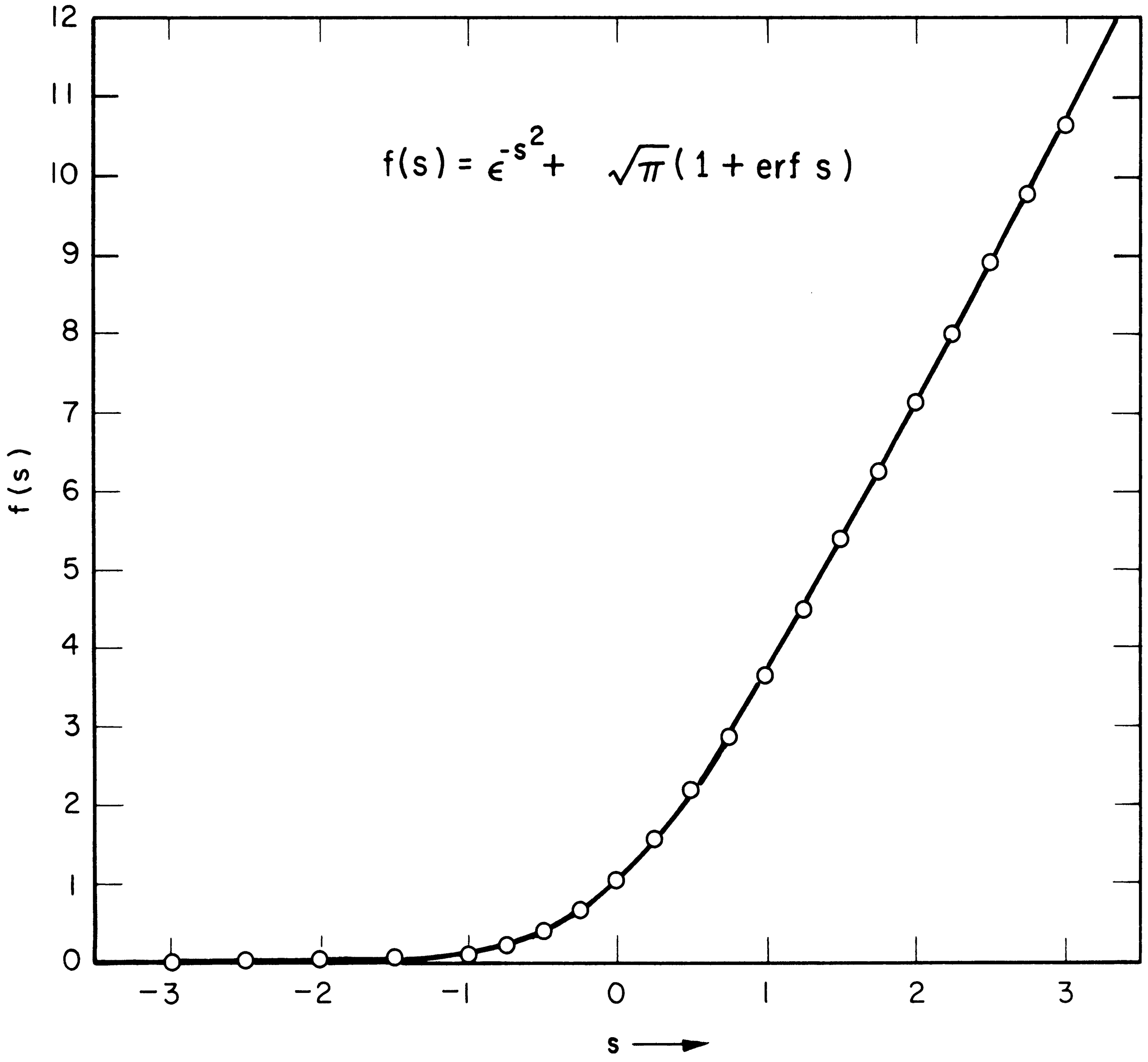
A.5 TABLE AND GRAPH OF F(S) VS. S (pps. 128-132)

GRAPH: F(S) vs. S
F(S) vs. -S

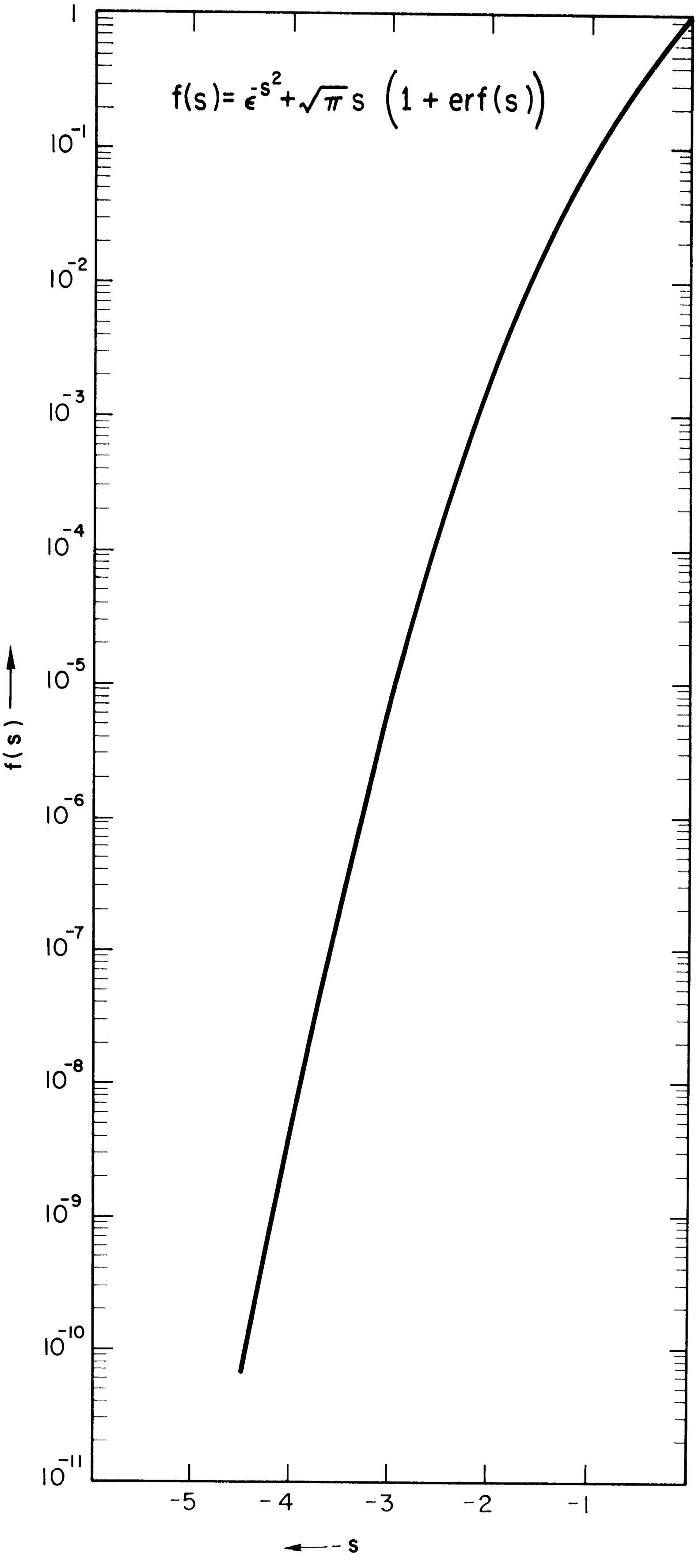
S	F(S)
-3.00	.00001
-2.90	.00001
-2.80	.00002
-2.70	.00004
-2.60	.00007
-2.50	.00013
-2.40	.00022
-2.30	.00038
-2.20	.00064
-2.10	.00107
-2.00	.00173
-1.90	.00277
-1.80	.00436
-1.70	.00674
-1.60	.01023
-1.50	.01529
-1.40	.02246
-1.30	.03247
-1.20	.04618
-1.10	.06464
-1.00	.08908
-.90	.12089
-.80	.16161
-.70	.21288
-.60	.27640
-.50	.35387
-.40	.44690
-.30	.55695
-.20	.68525
-.10	.83274
.00	1.00000
.10	1.18722
.20	1.39421
.30	1.62039
.40	1.86482
.50	2.12627
.60	2.40328
.70	2.69424
.80	2.99745
.90	3.31121
1.00	3.63388
1.10	3.96392
1.20	4.29994
1.30	4.64071
1.40	4.98518
1.50	5.33249
1.60	5.68191
1.70	6.03290
1.80	6.38500
1.90	6.73789
2.00	7.09133
2.10	7.44515
2.20	7.79920
2.30	8.15342
2.40	8.50774

S	F(S)
2.50	8.86213
2.60	9.21655
2.70	9.57100
2.80	9.92546
2.90	10.27993
3.00	10.63441
3.10	10.98888
3.20	11.34336
3.30	11.69784
3.40	12.05232
3.50	12.40680
3.60	12.76128
3.70	13.11576
3.80	13.47024
3.90	13.82472
4.00	14.17920
4.10	14.53368
4.20	14.88816
4.30	15.24264
4.40	15.59712
4.50	15.95160
4.60	16.30608
4.70	16.66056
4.80	17.01504
4.90	17.36952
5.00	17.72400
5.10	18.07848
5.20	18.43296
5.30	18.78744
5.40	19.14192
5.50	19.49640
5.60	19.85088
5.70	20.20536
5.80	20.55984
5.90	20.91432
6.00	21.26880
6.10	21.62328
6.20	21.97776
6.30	22.33224
6.40	22.68672
6.50	23.04120
6.60	23.39568
6.70	23.75016
6.80	24.10464
6.90	24.45912
7.00	24.81360
7.10	25.16808
7.20	25.52256
7.30	25.87704
7.40	26.23152
7.50	26.58600
7.60	26.94048
7.70	27.29496
7.80	27.64944
7.90	28.00392

```
                 S                        F(S)

                8.00                    28.35840
                8.10                    28.71288
                8.20                    29.06736
                8.30                    29.42184
                8.40                    29.77632
                8.50                    30.13080
                8.60                    30.48528
                8.70                    30.83976
                8.80                    31.19424
                8.90                    31.54872
                9.00                    31.90320




**** ALL INPUT DATA HAVE BEEN PROCESSED.
     AT LOC 26650
```


$f(s) = \epsilon^{-s^2} + \sqrt{\pi}(1 + \mathrm{erf}\, s)$
f(s)
s
12
11
10
9
8
7
6
5
4
3
2
1
0
−3
−2
−1
0
1
2
3

F(S) vs. S



$f(s) = \epsilon^{-s^2} + \sqrt{\pi}\, s \left(1 + \mathrm{erf}(s)\right)$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
f(s) ⟶
-5
-4
-3
-2
-1
⟵ -s

F(S) vs. -S

A.6 TABLE OF g, g/g_0, AND $(g/g_0)(g_0/R)$ VS. ALTITUDE FOR AIR FROM 0 TO 100 km (pp. 134-135)

ALT	G	G/GO	(G/GO)(GO/R)
0	9.8066	1.0000	34.1540
1	9.8035	.9997	34.1434
2	9.8005	.9994	34.1328
3	9.7974	.9991	34.1220
4	9.7943	.9987	34.1112
5	9.7912	.9984	34.1005
6	9.7882	.9981	34.0899
7	9.7851	.9978	34.0791
8	9.7820	.9975	34.0683
9	9.7789	.9972	34.0577
10	9.7759	.9969	34.0471
11	9.7728	.9966	34.0363
12	9.7697	.9962	34.0255
13	9.7666	.9959	34.0149
14	9.7636	.9956	34.0042
15	9.7605	.9953	33.9936
16	9.7575	.9950	33.9830
17	9.7544	.9947	33.9722
18	9.7513	.9944	33.9614
19	9.7482	.9940	33.9508
20	9.7452	.9937	33.9402
21	9.7421	.9934	33.9295
22	9.7391	.9931	33.9189
23	9.7360	.9928	33.9083
24	9.7330	.9925	33.8977
25	9.7299	.9922	33.8870
26	9.7269	.9919	33.8764
27	9.7238	.9916	33.8658
28	9.7208	.9913	33.8552
29	9.7177	.9909	33.8446
30	9.7147	.9906	33.8339
31	9.7116	.9903	33.8233
32	9.7086	.9900	33.8127
33	9.7056	.9897	33.8022
34	9.7026	.9894	33.7918
35	9.6995	.9891	33.7812
36	9.6965	.9888	33.7705
37	9.6934	.9885	33.7599
38	9.6904	.9882	33.7493
39	9.6874	.9878	33.7389
40	9.6844	.9875	33.7284
41	9.6813	.9872	33.7178
42	9.6783	.9869	33.7072
43	9.6753	.9866	33.6967
44	9.6723	.9863	33.6863
45	9.6693	.9860	33.6758
46	9.6663	.9857	33.6654
47	9.6632	.9854	33.6547
48	9.6602	.9851	33.6441
49	9.6557	.9846	33.6285
50	9.6512	.9842	33.6128
51	9.6497	.9840	33.6076
52	9.6482	.9838	33.6023
53	9.6451	.9835	33.5917
54	9.6421	.9832	33.5811

```
ALT              G                  G/GO            (G/GO)(GO/R)

 55            9.6391               .9829              33.5706
 56            9.6361               .9826              33.5602
 57            9.6331               .9823              33.5497
 58            9.6301               .9820              33.5393
 59            9.6271               .9817              33.5288
 60            9.6241               .9814              33.5184
 61            9.6211               .9811              33.5079
 62            9.6181               .9808              33.4975
 63            9.6151               .9805              33.4871
 64            9.6121               .9802              33.4766
 65            9.6091               .9799              33.4662
 66            9.6061               .9796              33.4557
 67            9.6031               .9793              33.4454
 68            9.6002               .9790              33.4352
 69            9.5972               .9786              33.4247
 70            9.5942               .9783              33.4143
 71            9.5912               .9780              33.4038
 72            9.5882               .9777              33.3934
 73            9.5852               .9774              33.3831
 74            9.5823               .9771              33.3728
 75            9.5793               .9768              33.3624
 76            9.5763               .9765              33.3519
 77            9.5733               .9762              33.3415
 78            9.5703               .9759              33.3310
 79            9.5673               .9756              33.3207
 80            9.5644               .9753              33.3105
 81            9.5614               .9750              33.3002
 82            9.5585               .9747              33.2899
 83            9.5555               .9744              33.2795
 84            9.5525               .9741              33.2690
 85            9.5495               .9738              33.2588
 86            9.5466               .9735              33.2485
 87            9.5436               .9732              33.2382
 88            9.5407               .9729              33.2279
 89            9.5377               .9726              33.2177
 90            9.5348               .9723              33.2074
 91            9.5318               .9720              33.1970
 92            9.5288               .9717              33.1867
 93            9.5259               .9714              33.1763
 94            9.5229               .9711              33.1659
 95            9.5199               .9708              33.1557
 96            9.5170               .9705              33.1454
 97            9.5140               .9702              33.1351
 98            9.5111               .9699              33.1248
 99            9.5081               .9696              33.1146
100            9.5052               .9693              33.1043


**** ERROR RETURN TO SYSTEM
```