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Aircraft Family Design Using Decomposition-based Methods

James Allison[(1)*], Brian Roth[(2)†], Michael Kokkolaras[(1)‡],
Ilan Kroo[(2)§] and Panos Y. Papalambros[(1)¶]

(1) *University of Michigan, Ann Arbor, Michigan, 48109, USA*

(2) *Stanford University, Stanford, California, 94305, USA*

This paper explores the use of decomposition-based methods for aircraft family design. The traditional approach in multidisciplinary design optimization is to decompose a problem along disciplinary lines. For aircraft family design problems, a more natural approach is decomposition by individual aircraft. This decomposition facilitates the concurrent development of several aircraft variants, providing substantial autonomy to individual aircraft development programs. Two decomposition-based methods are applied to the aircraft family problem: collaborative optimization and analytical target cascading. This paper marks the beginning of a collaborative effort to clarify the distinctions between these two methods, and to identify how these differences impact the relative performance and applicability of these methods. Initial product family results illustrate how decomposition-based methods can be applied to the aircraft family problem.

I. Introduction

A product family is a set of individual products that share common components or subsystems and address a related set of market applications.[1] In an aerospace context, a product family is usually comprised of a baseline aircraft and its derivatives or variants. Aircraft design often takes place with an eye towards derivative development. This is evidenced in the selection of power plants with growth capabilities and in wing design, with respect to the structural implications of tip extensions and winglets. An aerospace product family is not limited to a baseline aircraft and derivatives. It can involve two or more aircraft with dissimilar missions that share only a few key parts or systems.

Strong motivation exists for aerospace design based on product families.[2,3] As the aerospace industry has matured, emphasis has shifted from "higher, faster, farther" to "better, faster, cheaper." One opportunity for cost savings is through improved efficiency in manufacturing. When multiple aircraft share major structural components, costs can be saved in tooling and assembly. Product families also enable aircraft manufacturers to cater to the needs of potential customers by offering a wider selection of aircraft. From an airline's perspective, commonality is also an advantage. For example, avionics commonality speeds pilot cross-training among member aircraft in a product family. Additional advantages of commonality include simplification of maintenance procedures, flexibility in scheduling, and reduced spare-parts inventory. Thus, product families add value for the manufacturer and the customer.

Although a product family approach can reduce costs, shared components may lead to a performance penalty.[4–7] Common components may no longer be optimal for any one aircraft in a family, since they are designed to optimize some collective measure of merit. Multidisciplinary design optimization (MDO) provides a natural context in which to consider tradeoffs in design of product families. Just as it has been used for trades between aerodynamics and structures, it can be employed to consider trades between performance and cost.

*Doctoral Candidate, Department of Mechanical Engineering, AIAA Student Member
†Doctoral Candidate, Department of Aeronautics and Astronautics, AIAA Student Member
‡Associate Research Scientist, Department of Mechanical Engineering, AIAA Senior Member
§Professor, Department of Aeronautics and Astronautics, AIAA Fellow
¶Professor, Department of Mechanical Engineering, AIAA Senior Member

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This paper explores the use of decomposition-based methods for aircraft family design. Recent work in aerospace family design considered two approaches to the problem: sequential and simultaneous design.[8] Previous work in decomposition-based methods extended the analytical target cascading (ATC) formulation to design of product families.[9] Here, we illustrate the use of decomposition-based optimization to perform the simultaneous design of an aircraft family. As illustrated in Willcox & Wakayama,[8] decomposition is not necessary at the conceptual design level. However, during subsequent design steps (preliminary design), a certain level of autonomy may be desired between aircraft development programs, particularly if only a few parts are shared. Methods such as collaborative optimization (CO) and ATC enable decision making at the individual aircraft level consistent with overall product family goals.

The paper is organized as follows. Section II provides a brief overview of the two decomposition methods, comparing similarities and differences in their formulations. Section III introduces the considered aircraft family design problem and presents the formulations of the two approaches. Section IV discusses the obtained results.

II. Optimal Design of Decomposed Systems

Both CO[10] and ATC[11] are decomposition-based methods for solving complex system optimization problems. They were developed independently in response to different needs for design and product development. Both deal with interactions between elements of a partitioned system optimization problem. Their basic mathematical formulations exhibit similarities, yet each approach retains important distinctions, such as solution process. This paper builds on recent work[12] to establish CO and ATC as distinctly different methods with complementary characteristics using an aircraft family design example.

When a design problem is partitioned into smaller subproblems, additional terminology is necessary. The terminology presented here is common to many MDO formulations. The design vector $\mathbf{x}$ can be partitioned into local variables $\mathbf{x}_{\ell i}$ that are pertinent only to subproblem i, and shared variables $\mathbf{x}_{si}$ that are inputs to subproblem i and at least one other subproblem. The vector $\mathbf{x}_i$ contains local and shared variables required for subproblem i. In addition, subproblems are connected through interactions, i.e., analysis outputs of one subproblem may be required as analysis inputs for another. The vector of coupling variables $\mathbf{y}_{ij}$ is the set of values computed by subproblem j required as inputs to subproblem i. The collection of all coupling variables $\mathbf{y}$ has no common components with $\mathbf{x}$.

CO coordinates the solution of disciplinary subspaces using a system optimizer. It has been applied successfully to aerospace problems.[13,14] Subspaces can be executed in parallel, and the subspaces are consistent at convergence.[15] ATC was originally conceived for product development[16] and has been successfully implemented in automotive,[17,18] architectural,[19] product design,[20] and multiple-regime aircraft design.[21] ATC convergence properties have been proven for a specific class of coordination strategies under standard convexity and smoothness assumptions. ATC was developed as a tool for setting performance targets for a product at system, subsystem, and component levels such that top-level targets are met and the resulting system is consistent. Like CO, ATC also provides each specialist or team with substantial design freedom while accounting for critical interactions between system elements. While early papers[16] acknowledged the similarity between CO and ATC, only recently have formal comparisons been made between the two techniques. These comparisons were based on single product design.[12,22] This paper explores CO and ATC in the context of an aircraft family design problem.

A. Analytical Target Cascading

Analytical target cascading was developed based on needs in the automotive industry to translate top-level product targets into detailed design specifications. It is applicable to systems that possess hierarchical relationships. An example of a hierarchical system is shown in Figure 1. Each element in the hierarchy computes its own local analysis responses, and may require as inputs analysis responses (coupling variables) from lower level elements, in addition to local and shared variables.

The objective of the ATC process is to determine design specifications for each element in the hierarchy that account for interaction so that design teams can proceed with detail design independently. An optimization problem is formulated for each element. The formulation allows for a local objective and observes local design constraints. ATC allows the optimization algorithm to choose coupling variable values, and uses penalty functions (instead of equality constraints) to ensure system consistency. Recent ATC formula-

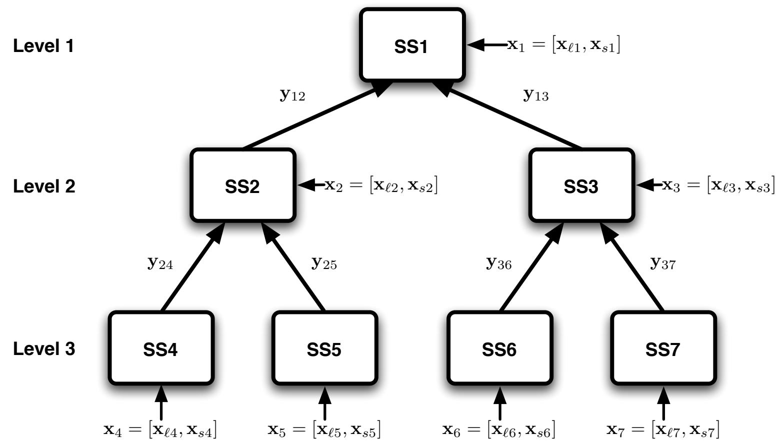

Figure 1. Hierarchical system decomposition

tions[12,23] allow multidirectional coupling, and coupling between same-level elements. The ATC formulation for subproblem P_i is

$$\begin{aligned} \min_{\mathbf{x}_i, \mathbf{y}_{ij}, \mathbf{x}_{s\mathcal{C}_i}, \mathbf{y}_{\mathcal{C}_i}} \quad & f(\mathbf{x}_i, \mathbf{y}_{ij}) + \pi(\mathbf{c}) \\ \text{subject to} \quad & \mathbf{g}_i(\mathbf{x}_i, \mathbf{y}_{ij}) \leq \mathbf{0} \\ & \mathbf{h}_i(\mathbf{x}_i, \mathbf{y}_{ij}) = \mathbf{0} \\ \text{where} \quad & \mathbf{c} = \mathbf{z}_i - \hat{\mathbf{z}}_i \end{aligned}$$

The objective is to minimize the local objective (if it exists) and a function π that penalizes nonzero values in the deviation vector $\mathbf{c}$, subject to local design constraints. The deviation vector quantifies the difference between shared quantities computed locally, $\mathbf{z}_i$, and the corresponding shared quantities computed by other subproblems, $\hat{\mathbf{z}}_i$. Shared quantities for element i consist of shared variables ($\mathbf{x}_{si}$), input and output coupling variables ($\mathbf{y}_{ij}$ and $\mathbf{y}_{ji}$), and shared and coupling variables that link elements that are children of element i ($\mathbf{x}_{s\mathcal{C}_i}$ and $\mathbf{y}_{\mathcal{C}_i}$). The components of $\hat{\mathbf{z}}_i$ are fixed parameters during the optimization of subsystem i.

Since each optimization problem is decoupled, we can solve all of the subproblems at a particular level in parallel. A popular ATC coordination strategy is to solve the top level problem (with initial guesses for top-level targets), use the results to update the target values for the next level down, solve the problems in the second level, and so on until the bottom level is reached. This large outer loop is repeated until all of the deviation vector values stop changing. Efficient penalty function methods can speed convergence, and have been shown to produce convergence in as few as 3 outer loop iterations.[21,23]

B. Collaborative Optimization

The CO method is designed to promote disciplinary autonomy while achieving interdisciplinary compatibility in non-hierarchical problems (Figure 2). Problem decomposition typically is made along analysis-convenient boundaries. A subspace optimizer is integrated with each analysis-block, and a system optimizer coordinates subspace solution. This approach decouples the subspace, while guiding the process toward a consistent solution. Each subspace has control over local design variables, and is charged with satisfying its own domain-specific constraints. As with ATC, discipline-specific optimization algorithms may be used.

Although a direct parallel between the top-level problem in an ATC formulation and the CO system optimizer may seem to exist, these elements fill different roles. The top ATC subproblem is similar to other subproblems, except that its targets are fixed. It seeks to bring the design of its subsystem into agreement with the rest of the system; it does not act to coordinate the solution of the entire system design problem. A separate coordination algorithm determines when each subproblem should be executed, and guides the system toward consistency. The CO system optimizer is not associated with an analysis block from the analysis structure (Figure 2), as is an ATC element and lower-level CO subspaces. Its role is similar to that of the ATC coordination algorithm; it guides the entire system toward consistency.

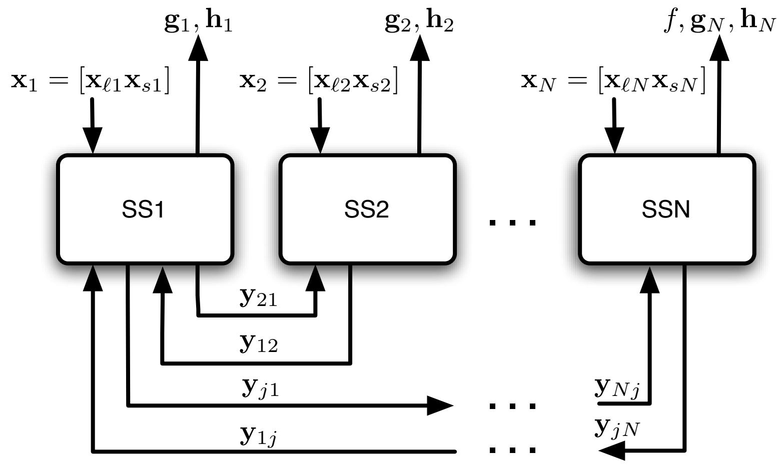

Figure 2. Non-hierarchical system decomposition

The system-level optimizer guides the system toward an optimal and consistent solution by minimizing a system objective function f, while enforcing system consistency via auxiliary constraints ($\mathbf{J}^* = [J_1^*, J_2^* \dots J_N^*]^{\mathrm{T}} = \mathbf{0}$). Requiring all subspace objectives to be zero at convergence results in consistency between all shared and coupling variables. The auxiliary constraints decouple the subspaces, facilitating parallel execution. System optimization is performed with respect to the system targets $\hat{\mathbf{z}}$. The system level sends subspace i the targets $\hat{\mathbf{z}}_i$, a subset of the system targets pertinent to subspace i. Subspace i returns its best response, $\mathbf{z}_i$, to meet these system targets. The system level constraints are then defined by the (square of the) difference between target values, $\hat{\mathbf{z}}_i$, and returned values, $\mathbf{z}_i$. The CO formulation is

System Level Formulation

$$\begin{aligned} \min_{\hat{\mathbf{z}}} \quad & f(\hat{\mathbf{z}}) \\ \text{subject to} \quad & \mathbf{J}^*(\hat{\mathbf{z}}) = \mathbf{0} \end{aligned}$$

Subspace Formulation

$$\begin{aligned} \min_{\mathbf{x}_{si},\mathbf{x}_{\ell i},\mathbf{y}_{ij}} \quad & J_i(\mathbf{x}_{si}, \mathbf{x}_{\ell i}, \mathbf{y}_{ij}) = \|\mathbf{z}_i - \hat{\mathbf{z}}_i\|_2^2 \\ \text{subject to} \quad & \mathbf{g}_i(\mathbf{x}_{si}, \mathbf{x}_{\ell i}, \mathbf{y}_{ij}) \leq \mathbf{0} \\ & \mathbf{h}_i(\mathbf{x}_{si}, \mathbf{x}_{\ell i}, \mathbf{y}_{ij}) = \mathbf{0} \end{aligned}$$

A target in the vector $\hat{\mathbf{z}}_i$ exists for every shared variable $\mathbf{x}_{si}$ used in subspace i and for every input $\mathbf{y}_{ij}$ and output $\mathbf{y}_{ji}$ coupling variable. Subspaces determine the value for local design variables $\mathbf{x}_{\ell i}$. The entire variable set in subproblem i is $\mathbf{x}_i = [\mathbf{z}_i, \mathbf{x}_{\ell i}] = [\mathbf{x}_{si}, \mathbf{y}_{ij}, \mathbf{y}_{ji}, \mathbf{x}_{\ell i}]$. The subspace objective J_i measures the discrepancy between subproblem targets and the corresponding responses: $J_i(\mathbf{x}_{si}, \mathbf{x}_{\ell i}, \mathbf{y}_{ij}) = \|\mathbf{z}_i - \hat{\mathbf{z}}_i\|_2^2$. Local targets $\hat{\mathbf{z}}_i$ are fixed parameters set by the system optimizer, and the subspace optimizer seeks to match these targets by varying the local and shared design variables, and the input coupling variables, subject to local design constraints $\mathbf{g}_i(\mathbf{x}_i)$ and $\mathbf{h}_i(\mathbf{x}_i)$. The output coupling variables $\mathbf{y}_{ij}$ are computed based on these decision variables, and are incorporated into J_i. At every system level iteration, the optimal value of the subspace objective function J_i^* is passed to the system optimizer and used as a system-level auxiliary constraint. Thus, CO is implemented as a nested optimization process.

C. Discussion of CO and ATC Formulations

An initial comparison of CO and ATC based on a single-product design problem[12] cited differences along four important dimensions: solution process, targets and communication patterns, intended structure of corresponding design problems, and paradigm. This section provides an updated perspective.

The fundamental difference between CO and ATC exists in the optimization process. CO utilizes nested optimization, while ATC solves a sequence of optimization subproblems at each level. In CO, the system-level optimization problem is solved only once, while the subspaces are solved many times (once during every system-level iteration). In ATC, a coordination strategy initializes the top-level optimization problem (with initial guesses for top-level targets), uses the resulting solution to update the target values for the next level down, initializes the problems in the second level, and so on until the bottom level is reached. This process is repeated until convergence.

The fundamental process difference between CO and ATC leads to a number of algorithmic distinctions. Firstly, since each element in ATC is solved repeatedly, inexact penalty relaxations may be used instead of equality constraints to ensure system consistency. Relaxation also helps the ATC process move more efficiently toward the solution. Secondly, analysis in ATC is conducted at all levels (including the system level), while analysis in CO is typically confined to the subproblems. This makes ATC well suited to object-based decomposition, where each element in a hierarchical multilevel system involves analysis. In fact, hierarchical analysis structures motivate the solution process of ATC. Thirdly, each element in ATC may pursue a local objective in addition to striving for compatibility. In CO, the subspaces' sole objective is to match targets provided by the system level. In addition to their algorithmic distinctions, CO and ATC also use different techniques to improve efficiency. For example, an augmented Lagrangian ATC formulation was recently proposed,[23] while CO often employs response surface models in the subproblems.[13,24]

III. Design of an Aircraft Family

The purpose of a product family is to reduce cost by sharing common components or systems to address a related set of market applications. In specific, the objective function should be able to differentiate between unique aircraft and product family solutions. Maximum takeoff weight, often used as an estimate for cost, will not capture the advantages of commonality. Life cycle cost is a rigorous approach, but is more complex than necessary. The primary goal is not to accurately predict total cost, but rather to quantify the benefits of a product family and define the preliminary design of its members. The ideal objective should include cost measures that distinguish between unique aircraft and families of aircraft, namely, a detailed model of acquisition cost and a reasonable estimate of fuel cost.

The acquisition cost model used in this paper is based on recent work by Markish.[25] Acquisition cost is split into manufacturing and development costs. A manufacturing learning curve is applied such that cost decreases with the number of units produced. For example, the 100th unit costs less to manufacture than the 1st unit. Development cost is non-recurring and is averaged over the total number of aircraft produced. For every part of a new aircraft design that has already been developed for another aircraft (i.e., for another aircraft in the family), the non-recurring cost is significantly lower. Thus, the effects of commonality are captured by the acquisition cost model.

Many airline labor costs, such as pension plans, are relatively unaffected by an airline's choice of aircraft fleet. Other labor costs, however, such as crew scheduling, training, and maintenance, are significantly impacted by choice of aircraft fleet.[26] These costs are difficult to model and have not been included in the present cost model. Though not specifically addressed in this paper, product families provide a potentially significant benefit in this area.

Although fuel cost prediction is a worthy challenge in its own right, this study uses a fixed fuel price per gallon as a simplification. Fuel cost is then computed based on the Breguet range equation.[27] In summary, the objective function is a carefully constructed cost measure that captures the key differences between unique aircraft and product family solutions, including acquisition and fuel costs.

Aircraft performance is evaluated using the Program for Aircraft Synthesis Studies (PASS), an aircraft conceptual design tool based on a collection of McDonnell-Douglas methods, DATCOM correlations, and new analyses developed specifically for conceptual design and performance. PASS has evolved over more than 15 years.[28] A detailed description of these methods may be found on the website of an aircraft design course at Stanford University.[29]

While existing conceptual design tools such as PASS are well-suited for the design of individual aircraft, a more detailed structural model is required for aircraft family design. For example, wing weight is computed using the following semi-empirical equation

$$W_{wing} = 4.22 S_{wg} + 1.642 * 10^{-6} \frac{N_{ult} b^3 \sqrt{W_{TO} W_{ZFW}}(1+2\lambda)}{(t/c)_{avg} cos^2(\Lambda_{ea}) S_{wg} (1+\lambda)}. \tag{1}$$

Note that wing weight is a function of wing geometry (S_{wg}, b, λ, etc.) as well as aircraft weight (W_{TO}). Thus, sharing a common wing geometry is not sufficient to ensure wing commonality. An additional issue is the need to compute the weight of individual wing sections such as root and tip extensions. These issues associated with wing commonality suggest the need for a more detailed wing weight model. While a finite element model was an option, the goal was a low-fidelity model consistent with existing conceptual design tools that captured the desired effects and was computationally efficient. The solution was a simple wing-box model in which the wing skin carried the bending load. An analysis estimated the load distribution on the wing and computed the material necessary to resist the resulting bending moment. Since high-lift systems, control surfaces, and minimum gauge material add to the final wing weight, a new equation was developed based on "bending material" and correlated to existing aircraft. This equation is listed below, where W_{str} is the weight of material needed to resist bending, W_{min} is the weight of minimum gauge material, and S_{wing} is the wing area

$$W_{wing} = 1.35(W_{str} - W_{min}) + 4.9S_{wing}. \tag{2}$$

Given a wing weight equation appropriate for modeling commonality between family members, the next step was to identify an appropriate means of parameterizing the wing for use in a decomposed optimization problem. The goal was to minimize the dimensionality while ensuring commonality. It was noted that an approximately quadratic relationship exists between skin thickness and spanwise location in the simple wing model. This enabled a three-term parameterization, where the skin thickness was defined at the following spanwise locations: wing root (T_1), 33% span (T_2), and 67% span (T_3) (of the main wing section). The wing tip was intentionally avoided in this parameterization since it is often sized by minimum gauge requirements rather than stress constraints. This yielded the following set of eight variables that uniquely define the main wing section: S_{wing}, AR_{wing}, λ, Λ, (t/c), T_1, T_2, and T_3. (Note that the current investigation focuses on commonality of the main wing section, with each aircraft allowed to have a unique wing tip extension. Future work will include the capability for wing root and wing tip extensions.)

A. Problem Statement

The considered product family includes two aircraft types, A and B, designed to fulfill missions 1 and 2, respectively. Mission 1 requires a range of 3400 nautical miles (nmi) and an aircraft capacity of 296 passengers. Mission 2 requires a range of 8200 nmi and an aircraft capacity of 259 passengers. Forecasts suggest a market need for 800 type A aircraft, and a need for 400 type B aircraft. In addition to mission requirements, constraints such as balanced field length and second segment climb are included.

To facilitate comparison between the CO and ATC formulations, the same bi-level decomposition is used for both. The system (product family) level seeks to minimize a cost measure, subject to compatibility of common parts. The subproblem (individual aircraft) level seeks to satisfy compatibility while meeting individual aircraft performance requirements. Local design variables specify all portions of the aircraft not shared in common with other aircraft in the family. Component commonality in the present study is limited to the main wing. Each family member has the freedom to specify its own wing tip extension area.

The aircraft family design problem requires the specification of 16 design variables for each of the two aircraft types. The design variables for each aircraft ($x_{1i} \dots x_{16i},\ i \in \{A, B\}$) are described in Table 1. The product family design problem imposes the constraint that the variables $x_{10i} \dots x_{16i}$ are equal for each aircraft, since these pertain to the common component—the main wing. The vector of shared variables is

$$\mathbf{x}_s = [x_{10A} \dots x_{16A}]^{\mathrm{T}} = [x_{10B} \dots x_{16B}]^{\mathrm{T}}.$$

The local variables for aircraft A and B are

$$\mathbf{x}_{\ell A} = [x_{1A} \dots x_{9A}]^{\mathrm{T}} \quad \text{and} \quad \mathbf{x}_{\ell B} = [x_{1B} \dots x_{9B}]^{\mathrm{T}}.$$

The complete set of design variables for the product family design problem is

$$\mathbf{x} = [\mathbf{x}_{\ell A}^{\mathrm{T}}\ \mathbf{x}_{\ell B}^{\mathrm{T}}\ \mathbf{x}_s^{\mathrm{T}}]^{\mathrm{T}}.$$

Each aircraft must comply with a set of five performance constraints, whose numeric values are specific to the mission each aircraft is designed to fly (see Table 2).

The objective of the aircraft family design problem is to minimize a composite cost metric for the family, where the cost metric for each mission is normalized by the number of aircraft that fly each mission. The cost metric model is based on an estimate of direct and indirect operating costs[29] with specific attention given to acquisition cost.[25] The system objective function is given in Equation (3), where n_A and n_B are the number of aircraft A and B in the family, respectively, and p_A and p_B are the cost metrics for each aircraft

$$f(\mathbf{x}) = \frac{n_A}{n_A + n_B} p_A(\mathbf{x}_{\ell A}, \mathbf{x}_s) + \frac{n_B}{n_A + n_B} p_B(\mathbf{x}_{\ell B}, \mathbf{x}_s). \tag{3}$$

Table 1. Design variables for the aircraft family design problem

Variable	Name	Description	Aircraft A Variable Bounds	Aircraft B Variable Bounds
x_{1i}	W_{TO}	takeoff weight	300,000 - 450,000 lbs	450,000 - 600,000 lbs
x_{2i}	$thrust$	sea level static thrust	50,000 - 70,000 lbs	75,000 - 105,000 lbs
x_{3i}	X_{wing}	location of wing root leading edge	0.20 - 0.40	0.20 - 0.40
x_{4i}	S_h/S_{ref}	nondimensional horizontal tail area	0.20 - 0.35	0.20 - 0.35
x_{5i}	Alt_I	initial cruise altitude	32,000 - 45,000 ft	32,000 - 45,000 ft
x_{6i}	Alt_F	final cruise altitude	32,000 - 45,000 ft	32,000 - 45,000 ft
x_{7i}	$Mach$	Mach number at start of cruise	0.75 - 0.92	0.75 - 0.92
x_{8i}	$flap_{TO}$	takeoff flap deflection	0.0 - 15.0	0.0 - 15.0
x_{9i}	S_{wt}	wing tip extension area	0 - 125 ft^2	0 - 125 ft^2
x_{10i}	S_{wm}	main wing area	2000 - 4000 ft^2	2000 - 4000 ft^2
x_{11i}	AR_{wm}	main wing aspect ratio	7.0 - 12.0	7.0 - 12.0
x_{12i}	(t/c)	thickness to chord ratio	0.80 - 0.14	0.80 - 0.14
x_{13i}	Λ	wing sweep	20.0 - 35.0	20.0 - 35.0
x_{14i}	T_1	skin thickness at root of main wing	0.06 - 2.5	0.06 - 2.5
x_{15i}	T_2	skin thickness at 33% span of main wing	0.06 - 2.0	0.06 - 2.0
x_{16i}	T_3	skin thickness at 67% span of main wing	0.06 - 1.5	0.06 - 1.5

Table 2. Design constraints for the aircraft family design problem

Constraint	Name	Description	Aircraft A	Aircraft B
g_1	$Range$	min range	3,400 nmi	8,200 nmi
g_2	$TOFL$	max takeoff field length	7,000 ft	10,000 ft
g_3	LFL	max landing field length	5,200 ft	6,000 ft
g_4	γ_2	min 2$^{\text{nd}}$ seg. climb grad	0.024	0.024
g_5	$stab$	stability requirement	≥ 0	≥ 0
g_6	$\hat{\sigma}_1$	normalized stress at wing root	≤ 0	≤ 0
g_7	$\hat{\sigma}_2$	normalized stress at 33% span	≤ 0	≤ 0
g_8	$\hat{\sigma}_3$	normalized stress at 67% span	≤ 0	≤ 0

B. ATC Formulation

The aircraft family design problem is decomposed into a bi-level ATC formulation with three elements. The top level problem P_1 seeks to attain agreement between the lower-level subproblems with respect to shared variables, while minimizing the problem objective f. The two lower-level problems, P_2 and P_3, seek to match targets set by P_1, while meeting local design performance constraints. P_2 corresponds to the design of aircraft A, and P_3 corresponds to the design of aircraft B. Alternative ATC decompositions exist, but a comparison of these is left for future work. For clarity in the ATC formulations, a superscript in parentheses indicates the subproblem in which a value is computed. Problem P_1 is formulated as

$$\begin{aligned}
\min_{\bar{\mathbf{x}}_1=\left[\mathbf{x}_s^{(1)\mathrm{T}}\ p_A^{(1)}\ p_B^{(1)}\right]^{\mathrm{T}}} \quad & f\left(p_A^{(1)},\ p_B^{(1)}\right) + \pi(\mathbf{c}_1) \\
\text{where:} \quad & \pi(\mathbf{c}_1) = \mathbf{v}_1^{\mathrm{T}}\mathbf{c}_1 + \|\mathbf{w}_1 \circ \mathbf{c}_1\|_2^2 \\
& \mathbf{c}_1 = \left[\mathbf{x}_s^{(1)\mathrm{T}}\ \mathbf{x}_s^{(1)\mathrm{T}}\ p_A^{(1)}\ p_B^{(1)}\right]^{\mathrm{T}} - \left[\mathbf{x}_s^{(2)\mathrm{T}}\ \mathbf{x}_s^{(3)\mathrm{T}}\ p_A^{(2)}\ p_B^{(3)}\right]^{\mathrm{T}}
\end{aligned}$$

The deviation vector $\mathbf{c}_1$ quantifies the difference between the targets set by P_1 and the achievable responses of P_2 and P_3. The responses are fixed parameters with respect to P_1. Note that f is a function only of target cost metrics, since these are independent decision variables in P_1. The penalty function $\pi(\mathbf{c}_1)$ guides the ATC process toward consistency. The linear and quadratic penalty weights, $\mathbf{v}_1$ and $\mathbf{w}_1$, are updated with every execution of P_1 using the formulas:[23]

$$\begin{aligned}
\mathbf{w}_1^{k+1} &= \beta \mathbf{w}_1^k \\
\mathbf{v}_1^{k+1} &= \mathbf{v}_1^k + 2\mathbf{w}_1^k \circ \mathbf{w}_1^k \circ \mathbf{c}_1^k
\end{aligned}$$

Typically $1 < \beta < 3$ and $\mathbf{v}_1^0 = \mathbf{0}$. In this case P_1 has no associated analysis, and the objective is a quadratic function, enabling direct solution without the use of an optimization algorithm. P_1 can be solved by finding $\bar{\mathbf{x}}_1$ such that $\nabla_{\bar{\mathbf{x}}_1} f_1 = \mathbf{0}$, where $f_1 = f + \pi$. Problem P_2 is formulated as

$$\begin{aligned}
\min_{\bar{\mathbf{x}}_2 = \left[\mathbf{x}_s^{(2)\mathrm{T}}\ \mathbf{x}_{\ell A}^{(2)\mathrm{T}}\right]^{\mathrm{T}}} \quad & \pi(\mathbf{c}_2) \\
\text{subject to} \quad & \mathbf{g}_A(\bar{\mathbf{x}}_2) \leq \mathbf{0} \\
\text{where:} \quad & \pi(\mathbf{c}_2) = \mathbf{v}_2^{\mathrm{T}} \mathbf{c}_2 + \|\mathbf{w}_2 \circ \mathbf{c}_2\|_2^2 \\
& \mathbf{c}_2 = \left[\mathbf{x}_s^{(2)\mathrm{T}}\ p_A^{(2)}\right]^{\mathrm{T}} - \left[\mathbf{x}_s^{(1)\mathrm{T}}\ p_A^{(1)}\right]^{\mathrm{T}}
\end{aligned}$$

The penalty weight vectors are updated using the same algorithm described above. The formulation of Problem P_3 is similar (one has simply to replace subscript or superscript 2 with 3 and subscript A with B).

C. CO Formulation

The CO formulation uses the same problem partition as ATC. Each subspace is tasked with designing one member of the family. The subspaces seek to match targets set by the system level, while satisfying local design performance constraints. The system level seeks to minimize a family cost measure, while satisfying compatibility of the subspaces. Since a non-gradient-based optimizer was used at the system level (as detailed in the next section), this allowed an L^1-norm constraint formulation, as shown below. The system problem formulation is

$$\begin{aligned}
\min_{\hat{\mathbf{z}} = [\mathbf{x}_s^{\mathrm{T}}\ p_A\ p_B]^{\mathrm{T}}} \quad & f\left(p_A,\ p_B\right) \\
\text{subject to} \quad & \mathbf{J}^*(\hat{\mathbf{z}}) = \mathbf{0}
\end{aligned}$$

The shared variables and cost metrics at each system-level iteration are passed to the appropriate subspace as fixed targets. The formulation for each subspace i is

$$\begin{aligned}
\min_{\mathbf{x}_s, \mathbf{x}_{\ell i}} \quad & J_i(\mathbf{x}_s, \mathbf{x}_{\ell i}) = \|\mathbf{z}_i - \hat{\mathbf{z}}_i\|_1 \\
\text{subject to} \quad & \mathbf{g}_i(\mathbf{x}_s, \mathbf{x}_{\ell i}) \leq \mathbf{0} \\
\text{where:} \quad & i \in \{A, B\} \\
& \mathbf{z}_i = [\mathbf{x}_s\ p_i(\mathbf{x}_s, \mathbf{x}_{\ell i})]^{\mathrm{T}} \\
& \hat{\mathbf{z}}_i = [\mathbf{x}_s\ p_i]^{\mathrm{T}} \ \text{(values set by system optimizer)}
\end{aligned}$$

IV. Implementation and Results

This section describes how the ATC and CO formulations were implemented to obtain solutions to the aircraft family design problem; and it presents the corresponding results. A challenge common to both implementations was the presence of gradient discontinuities in the responses of the PASS analysis software. For example, one source of gradient discontinuities in PASS is the calculation of worst-case aerodynamic loads on the wing, which are a function of load factor. Load factor is based on the larger of two quantities:

gust load and maneuver load. A change in critical load criteria can trigger a significant gradient discontinuity. This (and other) discontinuities caused slow convergence of gradient-based algorithms to suboptimal points, motivating the use of gradient-free algorithms for each implementation.

A. ATC Implementation

The ATC subproblems P_2 and P_3 were solved using NOMADm,[30] an implementation of mesh adaptive direct search.[31,32] This algorithm effectively handled the non-smooth responses of the PASS analysis software. The mesh tolerance used in determining convergence was 0.001, and subproblem optimizations typically required between 400 and 600 function evaluations. ATC required between 8 and 18 NOMADm optimizations to obtain a solution, depending on the value chosen for β in the penalty updates.

The P_1 subproblem objective function is quadratic, and required very little computational effort to solve. Two approaches were used to solve P_1: solving for $\nabla_{\bar{\mathbf{x}}_1} f_1 = \mathbf{0}$ (where $f_1 = f + \pi(\mathbf{c})$), and using a gradient-based algorithm to minimize f_1. The former was extremely efficient, but the latter proved more robust.

System consistency was quantified using the root mean square of the combined deviation vector

$$RMS(\mathbf{c}) = \sqrt{\frac{1}{|\mathbf{c}|}\mathbf{c}^{\mathrm{T}}\mathbf{c}}\ ,$$

where

$$\mathbf{c} = \left[\mathbf{c}_1^{\mathrm{T}}\ \mathbf{c}_2^{\mathrm{T}}\ \mathbf{c}_3^{\mathrm{T}}\right]^{\mathrm{T}},\ |\mathbf{c}| = \text{cardinality of } \mathbf{c}.$$

The convergence of ATC is strongly influenced by the choice of β when the penalty update algorithm described in the previous section is used. A larger β value can help force the system into tighter consistency, but can result in a stiff system that requires more iterations to converge. The problem was solved using a range of different β values to illustrate this influence. Figure 3 illustrates how larger values of β require more iterations of the ATC process. It was also observed that larger β values led to slightly larger objective function values, even when system consistency was approximately equal. This indicates that a stiff solution process can impede the identification of better designs.

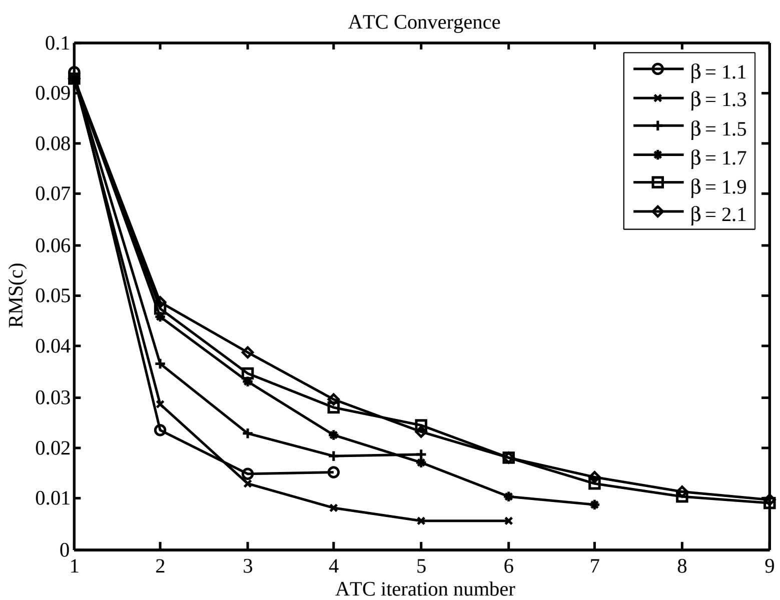

Figure 3. Influence of β on $RMS(\mathbf{c})$ (system consistency)

B. CO Implementation

The original goal was to use SNOPT[33] as the subproblem- and system-level optimizer in CO. However, SNOPT yielded subspace solutions that were only loosely converged. Gradient accuracy was not sufficient to enable the use of SNOPT for system-level optimization, but did permit the use of a gradient-free system-level optimizer. A robust (but computationally expensive) option was a genetic algorithm. While not ideal, this approach was robust to "noise" from the subspace level and yielded results that effectively illustrate the use of decomposition-based methods for aircraft family design. Future work will focus on implementing a more efficient alternative for handling non-smooth functions, such as response surface models of subspace responses. Response surfaces have been successfully employed with CO to resolve several common issues with non-smooth responses and slow system-level convergence.[13]

C. Results

Both CO and ATC provide reasonable designs, as detailed in Table 3.

Table 3. Aircraft family design results

			Aircraft A		Aircraft B	
	Name	Shared	CO Results	ATC Results	CO Results	ATC Results
p	$Cost$		\$377	\$390	\$1016	\$1072
g_1	$Range$ (nmi)		3400	3405	8200	8157
g_2	$TOFL$ (ft)		6394	6802	10000	10058
g_3	LFL (ft)		3197	3893	3576	4399
g_4	γ_2		0.027	0.039	0.033	0.031
g_5	$stab$		0.000	0.029	0.000	0.102
g_6	$\hat{\sigma}_1$		-0.560	-0.510	-0.121	-0.006
g_7	$\hat{\sigma}_2$		-0.718	-0.500	-0.251	-0.008
g_8	$\hat{\sigma}_3$		-0.302	-1.08	0.000	-0.296
x_1	W_{TO} (lbs)		$3.82 \cdot 10^5$	$3.88 \cdot 10^5$	$5.63 \cdot 10^5$	$6.02 \cdot 10^5$
x_2	$thrust$ (lbs)		$5.50 \cdot 10^4$	$6.61 \cdot 10^4$	$0.88 \cdot 10^5$	$1.03 \cdot 10^5$
x_3	X_{wing}		0.23	0.25	0.25	0.28
x_4	S_h/S_{ref}		0.20	0.24	0.20	0.26
x_5	Alt_I (ft)		$3.32 \cdot 10^4$	$3.80 \cdot 10^4$	$3.20 \cdot 10^4$	$3.27 \cdot 10^4$
x_6	Alt_F (ft)		$4.20 \cdot 10^4$	$3.32 \cdot 10^4$	$3.90 \cdot 10^4$	$3.64 \cdot 10^4$
x_7	$Mach$		0.838	0.789	0.824	0.791
x_8	$flap_{TO}$ (deg)		7.1	9.2	1.5	14.9
x_9	$S_{wt}(ft^2)$		124.9	12.1	104.8	20.7
x_{10}	$S_{wm}(ft^2)$	✓	$4.00 \cdot 10^3$	$3.20 \cdot 10^3$	$4.00 \cdot 10^3$	$3.30 \cdot 10^3$
x_{11}	AR_{wm}	✓	7.6	9.4	7.6	9.3
x_{12}	(t/c)	✓	0.123	0.114	0.123	0.110
x_{13}	Λ (deg)	✓	33.0	28.5	33.0	28.7
x_{14}	T_1 (in)	✓	1.10	1.50	1.10	1.50
x_{15}	T_2 (in)	✓	1.00	1.10	1.00	1.13
x_{16}	T_3 (in)	✓	0.50	0.73	0.50	0.72

The two approaches yield similar, but not identical, results. The differences can be attributed to the different optimization algorithms used for each method, and do not necessarily reflect the capabilities of the CO and ATC methods. We emphasize that the results are reported to demonstrate the applicability of the two approaches for solving the family design problem. They are not presented as the suggested designs. Additional work is required both in modeling the problem (e.g., accounting for additional part commonality) and in fine-tuning implementations and optimization algorithms.

V. Closing Remarks

The fundamental difference between collaborative optimization (CO) and analytical target cascading (ATC) is related to the optimization process. CO uses nested optimization, where each iteration of the system-level problem requires complete subspace optimizations. ATC solves a sequence of optimization problems, each of which is associated with an element of a multilevel hierarchy. This difference leads to several algorithmic distinctions and has an impact on the types of problems for which each strategy is best suited.

Aircraft family design using decomposition methods offers the same benefits afforded by disciplinary decomposition. While decomposition is only moderately beneficial for the simple aircraft family problem investigated in this paper, it would be advantageous (if not essential) for higher fidelity analysis where subproblem-specific optimization techniques can be exploited, where it is impossible to integrate existing codes, or where organizational structure may require decomposition.

This paper highlights needs and opportunities for future research work. While comparative work has sought to identify the key differences between CO and ATC, additional study is needed to explore the implications of these differences. Design space discontinuities present a challenge for MDO techniques, emphasizing the need for better approaches to subspace optimization. The example problem detailed here is just a first step towards aircraft family design. Higher levels of commonality should be considered to enable comparison of product family design solutions to individual aircraft design solutions.

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