A GENERALIZED GUIDANCE LAW FOR COLLISION COURSES

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Abstract

A new generalized guidance law for collision courses is presented. When the missile and target axial accelerations or decelerations are constant, there exists a rectilinear collision course. The guidance law presented, which is called the true guidance law, gives theoretical lateral acceleration commands to guide the missile on a collision course. Since it is very difficult, however, to realize the true guidance law on most existing tactical missiles, this paper shows a method for simply implementing the guidance law: this is called the simplified guidance law. The small perturbation equation of the true guidance law shows thal the definition of an effective navigation constant is the same expression as that in the case of conventional proportional navigation. The performance of the two guidance laws presented is compared with that of proportional navigation using simulation studies of a simple model of a short range air-lo-air missile. The simulation results show that the guidance laws presented can intercept the targel, using far smaller lateral acceleration commands than prepared for proportional navigation. The inner launch envelope shows that the guidance laws presented provide an overall performance improvement over proportional navigntion

Introduction

It is well known that a conventional proportional navigation (PN) is an adequate missile guidance law when the missile and target velocities remain conslant.[1-3] In practice, however, the missile and target velocities may change significantly. For instance, a short range air-to-air missile (AAM) has nearly constanl axial acceleration during the boost phase. Also. the axial deceleration of a surface-to-air missile due to air drag after thrusl. cutoff can he assumed nearly constant. On the other hand, a target may have constant axial acceleration or slowdown. These kinds of axial acceleration or deceleration may seriously influence the performance of the missile guided by PN. Chadwick developed the approximate analytical solution for the miss distance of a proportional navigation missile with axial slowdown after sustainer motor cutoff, but he mentioned nothing about a guidance law.[4]

When the missile and target accelerations or decelerations are constant, there exists the rectilinear collision course. Then, if we know the magnitudes of these accelerations or decelerations and the initial time-to-go, the missile can be guided to a collision course. A missile flying on this collision course does not require further acceleration commands to hit the target. For example, Pig. 1 compares the rectilinear collision course with the trajectory achived with PN. Since PN does not take into account changes in velocity. the trajectory with PN is curved as shown in Pig. 1. In other words, the missile guided by PN requires more acceleration commands as it gets close to the target.

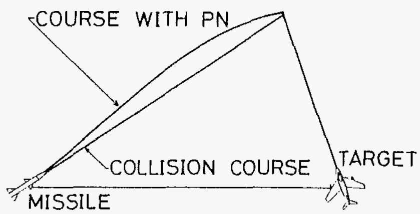

Fig. 1 The rectilinear collision course and the course achieved with PN

First, this paper presents the generalized guidance law for a missile with constant axial acceleralion against a target with constant acceleration. Next. the small perturbation equation of the guidance law is derived. This shows that lhe definition of an effective navigation constant reduces to the same expression as thal in the case of PN. An appropriate effective navigation constant can then be determined by integrating the small perturbation equalion. Though the generalizd guidance law gives the

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theoretical acceleration Lo guide a missile on a collision course. it is very difficult to implement this guidance law on the most existing tacical missiles. Therfore. this paper shows a simplified method for implementing the guidance law. Finally, the performance of the guidance laws presented is compared with thal of PN using simulation studies of the simple AAM models.

Nomenclature

$\vec{r}_m$	:	missile position vector
r_{mc}	:	correct missile position vector
AT,,,	:	$\vec{r}_m - \vec{r}_{mc}$
x, y	:	elements of $\Delta\vec{r}_m$
$\vec{e}_1$, $\vec{E}_2$:		unit vectors
$\vec{r}_t$	:	target position vector
$\vec{R}$	:	relative distance vector $(=\vec{r}_t - \vec{r}_m)$, $R=\|\vec{R}\|$
$\vec{V}_m$	:	missile velocity vector, $V_m=\|\vec{V}_m\|$
$\vec{V}_{mc}$	:	correct missile velocity vector. $V_m=\|\vec{V}_{mc}\|$
$\Delta\vec{V}_m$	:	$\vec{V}_m - \vec{V}_{mc}$
$\vec{V}_t$	:	target velocity vector. $V_t=\|\vec{V}_t\|$
$\vec{F}$	:	desired acceleration command vectcr
$\vec{a}_m$	:	missile axial acceleration vector, $a_m=\|\vec{a}_m\|$
$\vec{a}_t$	:	target axial acceleration vector, $a_t=\|\vec{a}_t\|$
σ	:	LOS angle
$\dot{\vec{\sigma}}$		LOS rate vector
θ		missile flight path angle
$\dot{\vec{\theta}}$		missile flight path rate vector
φ_m		missile flight path angle to LOS
φ_t		target flight path angle to LOS
μ		$\varphi_t - \varphi_m$
N		navigation constant
N_e		effective navigation constant
t_f		total flight time
t_{go}		time-lo-go
s		differential operator
T_m		missile time constant
n_{max}		maximum lateral load factor
MD		miss distance

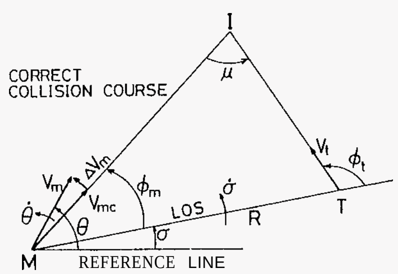


Fig. 2 Intercept geometry

Derivation of A Generalized Guidance Law

Fig. 2 shows the intercept geometry of a missile intercepting a target M nnd T represent the actual positions of a missile and a target. respectively. at Lime t From Pig. 2, the line-of-sight (LOS) rate is given by the following vector equation:

$$\dot{\vec{\sigma}} = \frac{\vec{R} \times (\vec{V}_t - \vec{V}_m)}{R^2} \tag{1}$$

Assuming that the missile is flying at constanb acceleration a, and the target is flying at coristanl. acceleration a_t, let the triangle ITM be a collision triangle. That is. I indicates the point of impact. $\vec{V}_{mc}$ is the correct missile velocity vector to obtain a collision at I and $\Delta\vec{V}_m$ is the deviation of missile velocity vector from $\vec{V}_{mc}$. Then we have

$$\vec{V}_m = \vec{V}_{mc} + \Delta\vec{V}_M \tag{2}$$

Substituting Eq. (2) into Eq. (1), we obtain

$$\dot{\vec{\sigma}} = \frac{\vec{R} \times (\vec{V}_t - \vec{V}_{mc})}{R^2} + \frac{\vec{R} \times (-\Delta\vec{V}_m)}{R^2} \tag{3}$$

The first term of the right side of Eq. (3) represents the correcl. LOS rate when the missile flies along the collision course The second term is the deviation of the LOS rate from the correct one I€ a missile is guided with a flight-path rate in proportion to the deviation of the LOS rate, assuming no missile dynamic lags, the missile flight-path rate becomes

$$\dot{\vec{\theta}} = N\frac{a \times (-\Delta\vec{V}_m)}{R^2} \tag{4}$$

where N is the navigebion constant. Then. the required lateral acceleration command €or a missile to fly along the correct collision course is given by

$$\begin{aligned}\vec{F} &= \dot{\theta} \times \vec{V}_m \\ &= -N\frac{((\vec{V}_{mc} - \vec{V}_m) \times \vec{R}) \times \vec{V}_m}{R^2}\end{aligned} \tag{5}$$

From Fig. 2, $\vec{V}_{mc}$ can be written as

$$\vec{V}_{mc} = V_m\left(\frac{\sin\varphi_m}{\sin\varphi_t}\frac{\vec{V}_t}{V_t} + \frac{\sin\mu}{\sin\varphi_t}\frac{\vec{R}}{R}\right) \tag{6}$$

Substituting Eq. (6) into Eq. (5), we obtain

$$\vec{F} = \frac{N}{R^2}((\vec{V}_m - \frac{V_m\sin\varphi_m}{V_t\sin\varphi_t}\vec{V}_t) \times \vec{R}) \times \vec{V}_m \tag{7}$$

If we let t_{go} be the time-to-go, from Pig. 2, we have

$$\vec{V}_{mc}t_{go} + \frac{a_m}{2}t_{go}^2 = \vec{R} + \vec{V}_t t_{go} + \frac{\vec{a}_t}{2}t_{go}^2 \tag{8}$$

From Fig. 2. the component of Eq. (8) perpendicular to $\vec{R}$ is given by

$$(V_m t_{go} + \frac{a_m}{2}t_{go}^2)\sin\varphi_m = (V_t t_{go} + \frac{a_t}{2}t_{go}^2)\sin\varphi_t \tag{9}$$

Dividing Eq. (9) by $t_{go}\sin\varphi_t$, we obtain

$$V_m(1 + \frac{a_m t_{go}}{2V_m})\frac{\sin\varphi_m}{\sin\varphi_t} = V_t(1 + \frac{a_t t_{go}}{2V_t}) \tag{10}$$

Eq. (10) can be rewritten as

$$\frac{V_m \sin\varphi_m}{V_t \sin\varphi_t} = \frac{1+\varepsilon_t}{1+\varepsilon_m} \tag{11}$$

where

$$\varepsilon_m = \frac{a_m t_{go}}{2V_m} \tag{12}$$

$$\varepsilon_t = \frac{a_t t_{go}}{2V_t} \tag{13}$$

Substituting Eq. (11) into Eq. (7), we obtain

$$\vec{F} = \frac{N}{R^2}\{(\vec{V}_m - k\vec{V}_t) \times \vec{R}\} \times \vec{V}_m \tag{14a}$$

where

$$k = \frac{1+\varepsilon_t}{1+\varepsilon_m} \tag{14b}$$

This is the generalized guidance law for a missile with constant axial acceleration to intercept a target with constant axial acceleration. If both a missile and a target have no acceleraton, that is, constant velocity, ε_m and ε_t are zero and Eq. (14a) reduces to a conventional proportional navigation guidance law. Since the value of t_{go} is required in order to compute ε_m and ε_t, we derive the equation for t_{go}. From Pig 2, we have

$$(V_m + \frac{a_m t_{go}}{2})\, t_{go}\cos\mu = R\cos\varphi_t + V_t t_{go} + \frac{a_t}{2} t_{go}^2 \tag{15}$$

$$(V_m + \frac{a_m t_{go}}{2})\, t_{go}\sin\mu = R\sin\varphi_t \tag{16}$$

Squaring both sides of Eqs. (15) and (16) and adding them, we obtain

$$(V_m t_{go} + \frac{a_m}{2} t_{go}^2)^2 = R^2 + (V_t t_{go} + \frac{a_t}{2} t_{go}^2)^2 + 2R(V_t t_{go} + \frac{a_t}{2} t_{go}^2)\cos\varphi_t \tag{17}$$

Rearranging Eq. (17), we have

$$(\frac{a_t^2 - a_m^2}{4})\, t_{go}^4 + (V_t a_t - V_m a_m)\, t_{go}^3 + (V_t^2 - V_m^2)\, t_{go}^2 + 2R(V_t t_{go} + \frac{a_t}{2} t_{go}^2)\cos\varphi_t + R^2 = 0 \tag{18}$$

Assuming that V_m, V_t, a_m, a_t, R and φ_t are known, t_{go} can be computed from Eq. (18), ε_m and ε_t can be obtained from Eqs. (12) and (13) and the required guidance acceleration commands are then computed from Eq. (14). We call this guidance law the true guidance law.

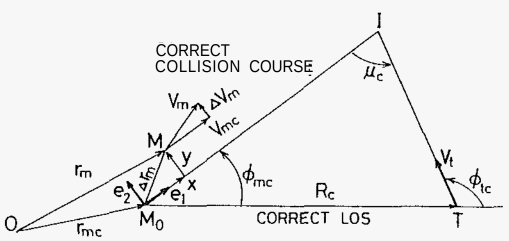


Fig. 3 Engagement model for linearization

Small Perturbation Equation

Eq. (14) gives the generalized guidance law for collision courses, where time-to-go is computed Prom Eq. (18). Since this equation is complicated, however, it is difficult to select the value of the navigation constant N. In order to choose N, it is useful to linearize Eq. (14a) about a correct collision course. For simplicity, we will consider the problem in two dimensions. Pig. 3 shows the engagement model for linearization, where the target is flying along the reference trajectory. In Pig. 3, O is the origin ol the inertial frame, M_0 indicates the position that a missile would have had at time t in the correct trajectory case and M displays the actual position of a missile at Lime t. φ_{mc}, φ_{tc} and μ_c are the correct angles such that $M_0 I$ and TI become the correct, collision courses for missile and a target, respectively. Prom Pig. 3, we have

$$\vec{r}_m = \vec{r}_{mc} + \Delta\vec{r}_m \tag{19}$$

$$\vec{V}_m = \vec{V}_{mc} + \Delta\vec{V}_m \tag{20}$$

$$\Delta\vec{r}_m = x\vec{e}_1 + y\vec{e}_2 \tag{21}$$

$$\Delta\vec{V}_m = \dot{x}\vec{e}_1 + \dot{y}\vec{e}_2 \tag{22}$$

$$\vec{R} = \vec{r}_t - \vec{r}_{mc} - \Delta\vec{r}_m \tag{23}$$

$$\vec{R}_c = \vec{r}_t - \vec{r}_{mc} \tag{24}$$

Substituting Eqs. (19), (20), (23), (24) into the numerator of Eq. (14a) and neglecting terms higher than second orders in Δr_m and ΔV_m, we obtain

$$\begin{aligned}(\vec{V}_m - k\vec{V}_t) \times \vec{R} &= \{(\vec{V}_{mc} + \Delta\vec{V}_m) - k\vec{V}_t\} \times (\vec{r}_t - \vec{r}_{mc} - \Delta\vec{r}_m) \\ &= \{(\vec{V}_{mc} - k\vec{V}_t) + \Delta\vec{V}_m\} \times (\vec{R}_c - \Delta\vec{r}_m) \\ &\fallingdotseq (k\vec{V}_t - \vec{V}_{mc}) \times \Delta\vec{r}_m + \Delta\vec{V}_m \times \vec{R}_c \end{aligned} \tag{25}$$

Then, we have

$$\begin{aligned}\{(\vec{V}_m - k\vec{V}_t) \times \vec{R}\} \times \vec{V}_m &= \{(\vec{V}_m - k\vec{V}_t) \times \vec{R}\} \times (\vec{V}_{mc} + \Delta\vec{V}_m) \\ &\fallingdotseq -\vec{V}_{mc} \times \{(k\vec{V}_t - \vec{V}_{mc}) \times \Delta\vec{r}_m + \Delta\vec{V}_m \times \vec{R}_c\}\end{aligned} \tag{26}$$

Substituting Eqs. (21), (22) into Eq. (26), we obtain

$$\begin{aligned}\{(\vec{V}_m - k\vec{V}_t) \times \vec{R}\} \times \vec{V}_m \fallingdotseq -V_m (\dot{x}R\sin\varphi_{mc} + kxV_t\sin\mu_c \\ + \dot{y}R\cos\varphi_{mc} - kyV_t\cos\mu_c + V_m y)\vec{e}_2\end{aligned} \tag{27}$$

Substituting Eq. (27) into Eq. (14a), we have

$$\begin{aligned}\vec{F} = \frac{NV_m}{R^2}(-\dot{x}R\sin\varphi_{mc} - xkV_t\sin\mu_c \\ - \dot{y}R\cos\varphi_{mc} + ykV_t\cos\mu_c - V_m y)\vec{e}_2\end{aligned} \tag{28}$$

Since the external force on the missile is not applied in the direction of $\vec{e}_1$, except lor the constant. acceleration a_m, we have

$$\ddot{x}(t) = 0 \tag{29}$$

Assuming $x(0)=\dot{x}(0)=0$, we obtain

$$x(t) = 0 \tag{30}$$

On the other hand, we have

$$\vec{F} = \ddot{y}\vec{e}_2 \tag{31}$$

Substituting Eqs. (30), (31) into Eq. (28), we have

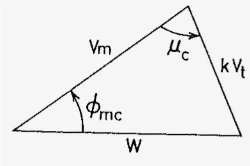
$$\ddot{y} = -\frac{NV_m\cos\varphi_{mc}}{R}\dot{y} + \frac{NV_m}{R^2}(-V_m + kV_t\cos\mu_o)y \quad (32)$$

Let us define V_o and W as follows:

$$V_o = V_m\cos\varphi_{mo} - V_t\cos\varphi_{to} \quad (33)$$

$$W = (k^2V_t^2 \ t \ V_m^2 - 2kV_tV_m\cos\mu_o)^{0.5} \quad (34)$$

Pig. 4 Definition of W



V_o becomes the closing velocity and W is the length of the side of the triangle, as shown in Fig. 4. From Pig. 4, we obtain

$$V_m - kV_t\cos\mu_o = W\cos\varphi_{mo} \quad (35)$$

From Eqs. (33) and (35), Eq. (32) can be written as

$$\ddot{y} = -\frac{NV_m\cos\varphi_{mo}}{R}(\dot{y} + \frac{W}{R}y)$$

$$= -N_e\frac{V_o}{R}(\dot{y} + \frac{W}{R}y) \quad (36)$$

where the parameter N_e is defined by

$$N_e = \frac{NV_m\cos\varphi_{mo}}{V_o} \quad (37)$$

As is obvious from Eq. (37), N_e is the same expression as Lhe effeclive navigation constant in the case ol PN. Thus we also call N_e an effective navigation constant in this paper. Eq. (36) is the small perturbation equation €or Eq. (14a). The initial conditions for a launch error are given by

$$y(0) = 0, \quad \dot{y}(0) = V_m(0)r_0 \quad (38)$$

and the miss distance is given by

$$MD = y(t_f) \quad (39)$$

As mentioned earlier, we have assumed until now that lhere are no missile dynamic lags. However. if we assume that the missile dynamics can be represented as a linear system with transfer operator $Y(s)$, where s is the operator d/dt and $Y(s)$ is the ratio of two polynomials in s, then the perturbation equation for the generalized guidance law becomes. instead of Eq. (36),

$$\ddot{y} = -N_eY(s)\frac{V_o}{R}(\dot{y} + \frac{W}{R}y) \quad (40)$$

As a specific example, consider the case of a missile with a simple time lag T_m . Thus let

$$Y(s) = \frac{1}{1 + T_ms} \quad (41)$$

Substituting Eq. (41) into **Eq.** (40), we obtain

$$T_m\dddot{y} \ t \ \ddot{y} = -N_e\frac{V_o}{R}(\dot{y} + \frac{W}{R}y) \quad (42)$$

Let us consider solving Eq. (42) numerically for a launch error r_0. From Eq. (38), the initial values become

$$y(0) = 0, \quad \dot{y}(0) = V_m(0)r_0, \quad \ddot{y}(0) = 0 \quad (43)$$

In order to solve Eq. (42), we assume I,, is given by

$$t_{go} = 1, - t \quad (44)$$

where t_f is the total flight lime. V_m and V_t are given by

$$V_m = V_{,,} \ \text{ı} \ a_mt \quad (45)$$

$$V_t = V_{to} + a_tt \quad (46)$$

where V_{m0} and V_{t0} are the initial values of V_m and V_t. Letting V_{ma} and V_{ta} be the average velocities ol the missile and target, respectively, we have

$$V_{ma} = V_{m0} \ t \ \frac{a_mt_f}{2} \quad (47)$$

$$V_{ta} = V_{t0} \ \text{ı} \ \frac{a_tt_f}{2} \quad \textbf{(48)}$$

Then, the average closing velocity V_{oa} becomes

$$V_{oa} = V_{ma}\cos\varphi_{mo} - V_{ta}\cos\varphi_{to} \quad (49)$$

The relative distance is given by

$$R = V_{oa}t_f - \{(V_{m0} + \frac{a_m}{2}t)\cos\varphi_{mo}$$

$$- (V_{t0} \ t \ \frac{a_t}{2}t)\cos\varphi_{to}\}t \quad (50)$$

If V_{m0}, V_{t0}, a_m, a,. μ_o and 1, are given, φ_{mo} is computed froin the lollowing equation:

$$\varphi_{mo} = \sin^{-1}\left\{\frac{\sin\mu_o(V_{t0}\text{I,} \ t \ a_t t_f^2/2)}{h}\right\} \quad (51)$$

where

$$h = \{(V_{t0}t_f \ t \ \frac{a_t}{2}t_f^2)^2 + (V_{m0}t_f + \frac{a_m}{2}t_f^2)^2$$

$$- 2(V_{t0}t_f + \frac{a_t}{2}t_f^2)(V_{m0}t_f \ t \ \frac{a_m}{2}t_f^2)\cos\mu_o)^{0.5} \quad (52)$$

Figs. 5, 6 and 7 show the results obtained by integrating Eq. (42) numerically lor the case where V_{m0}=288m/s, V_{t0}=288m/s, a_m=154m/s^2, a_t=0, T_m=0.4sec and μ_o=90deg. Pig. 5 illustrates the dimensionless miss distance $y(t_f)/V_{ma}r_0T_m$ vs. dimensionless time of flight t_f/T_m lor N_e=3, **4,** 5 and 6. Pigs. 6 and 7 show lhe deviation histories lrom the correct position and the lateral acceleration command histories in normalized form lor N_e=3, **4.** 5 and 6, where t_f is set equal to 4sec. Though Lhese three ligures were obtained for μ_o=90deg, other computation results have shown that. the lorm of Lhe curves does not depend significantly on the values of μ_o. In other words. Figs. 5, 6 and 7 show the typical miss distance. deviation hislories and acceleration command histories. respectively, and a reasonable N_e can be determined from Lhese ligures. In this exmple, 4 to 5 is considered the appropriate range in value €or N_e .

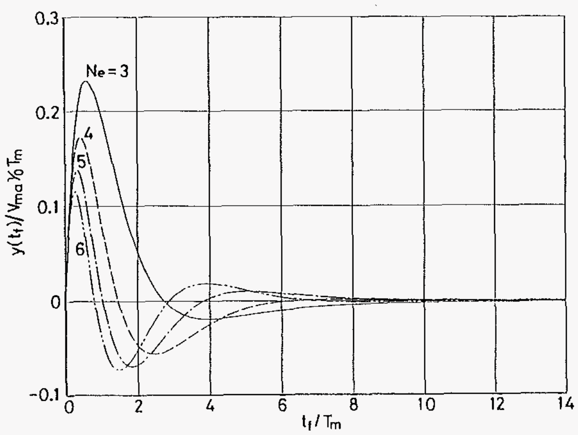


Fig. 5 Normalized miss due to launch error

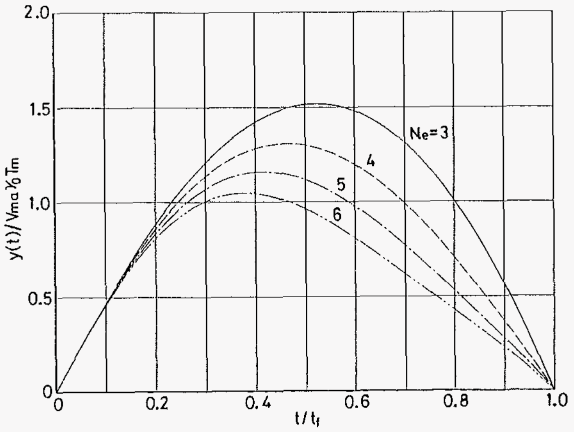


Fig. 6 Normalized deviation histories

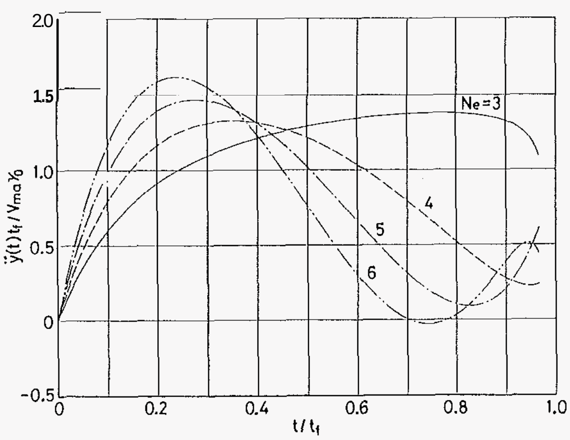


Fig. 7 Normalized acceleration commands histories

Simplified Construction of the Guidance Law

In order to construct the true guidance law given by Eq. (14), we must measure or estimate V_m, V_t, a_m, a_t, R and φ_t and compute t_{go} from Eq. (18) in real time. In general, however, most present tactical missiles do not carry the kind of instruments needed to measure all of these variables. Thus it becomes very difficult to realize Eq. (14). In this section, let us cosider the real mechanization of Eq. (14). The term $\vec{V}_m - k\vec{V}_t$ in Eq. (14a) can be rewritten as

$$\vec{V}_m - k\vec{V}_t = \vec{V}_m - \frac{1+\varepsilon_t}{1+\varepsilon_m}\vec{V}_t$$

$$= \left(\frac{1+\varepsilon_t}{1+\varepsilon_m}\right)(\vec{V}_m - \vec{V}_t) + \left(\frac{\varepsilon_m-\varepsilon_t}{1+\varepsilon_m}\right)\vec{V}_m \qquad (53)$$

Substituting Eq (53) into Eq (14a), we obtain

$$\vec{F} = \frac{N(1+\varepsilon_t)}{1+\varepsilon_m}\frac{((\vec{V}_m - \vec{V}_t)\times\vec{R})\times\vec{V}_m}{R^2} + \frac{N(\varepsilon_m-\varepsilon_t)}{1+\varepsilon_m}\frac{(\vec{V}_m\times\vec{R})\times\vec{V}_m}{R^2} \qquad (54)$$

The first term represents proportional navigation with the navigation constant $N(1+\varepsilon_t)/(1+\varepsilon_m)$ and the second term represnts pure pursuit navigation with the navigation constant $N(\varepsilon_m-\varepsilon_t)/(1+\varepsilon_m)$. We need the values of ε_m and ε_t to determine the two navigation constans. Though V_m, V_t, a_m, a_t, R and φ_t must be measured in order to compute ε_m and ε_t precisely, we do not necessarily need the correct values of ε_m and ε_t. This is because even if the navigation constants change somewhat from the optimal values, the performances of the guidance law will not be affected significantly. Therefore, if t_f, a_m, a_t, V_{m0} and V_{t0} are given from the launcher or the parent aircraft before launch, we can estimate the values of t_{go}, V_m and V_t as follows:

$$t_{go} = t_f - t \qquad (55)$$

$$V_m = V_{m0} + a_m t \qquad (56)$$

$$V_t = V_{t0} + a_t t \qquad (57)$$

Using these values, we can compute ε_m and ε_t from Eqs. (12) and (13) and simply mechanize Eq. (54). The block diagram representation of Eq. (54) is depicted in Fig. 8. Here, the effective navigation constant N_e defined by Eq. (37) is used instead of N. We call this implementation the simplified guidance law. In Fig. 8, K_p represents the gain for pure pursuit guidance and is chosen so that the trajectory achieved with the simplified guidance law approachs that with the true guidance law as nearly as possible. In other words, if $d(t)$ represents the relative distance between the two trajectories, K_p is chosen to minimize the following PI:

$$PI = \int_0^{t_f} d(t)\,dt \qquad (58)$$

Application to a Short Range AAM

Let us apply the true guidance law and the simplified guidance low to the simple model of a short range air-to-air missile (SRAAM) and compare the results with that achieved with PN. Since most SRAAM have solid rocket motors, their

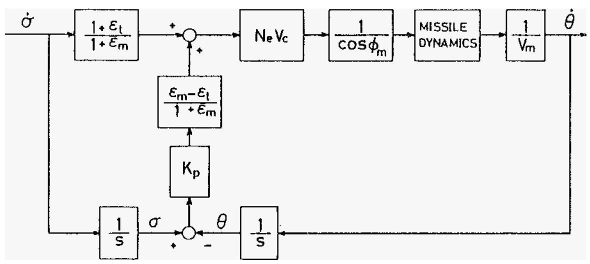


Fig. 8 Block diagram for the simplified guidance law

axial accelerations can he assumed nearly constant during boost phase. On the other hand, the target velocity can be considered nearly constant because it is very difficult to change significantly the target velocity during the short intercept lime. Let us assume that both the missile and the target are particles. their trajectories are limited to two dimensions and the total dynamics of the guidance system including the missile dynamics. a noise filter and so on is given by a first order lag with time constant 0.4*sec*. The velocity of the target is constant at 288*m/s*. The initial velocity **of** the missile is 288*m/s* and the acceleration is 154m/s^2. These are the same data that were used to obtain Pigs. 5, 6 and 7. Then, based on our previous discussion, we set N_e=4.5. Pigs. 9 and 10 are the graphs used to decide K_p. Fig. 9 shows *PI* vs. K_p, where μ=90*deg* and t_f=4*sec*. From this figure, the optimal K_p is 0.2 for μ=90*deg*. Fig. 10 displays the optimal K_p vs. μ. Though the optimal value of K_p depends on μ as shown in Fig. 10, we set K_p=0.2 because a broadside attack is the most severe case lor a missile. K_p is also a function of t_f However, simulation results have shown that the optimal value of K_p does not depend very much on t_f Pig. 11 displays one of the simula-Lion results, where the target is flying straight and the missile is launched along the collision course The figure shows that the missile guided by the true guidance law flies straight without acceleration commands. The trajectory achieved with PN is curved and requires large acceleration commands The effective navigation constant For PN was set equal to 4 5 The trajetory achieved with the simplified guidance law is also curved, hut the deviation from the ideal trajectory is much smaller than the deviation with PN Also, the required acceleration commands are smaller Figs 12, 13 arid 14 show the launch boundaries lor the miss distance and the maximum lateral load Factor specified as 3*m* and 30*g*, respectively Pig 12 is based on the true guidance law. Fig. 13 is with the simplified guidance law and Pig 14 is with PN Here it is assumed that the target. is flying straight and the missile is launced against the target without lead angle, other condilions are the same as those described before The inner launch envelope lor SRAAM must simultaneously satisfy al least these two boundaries Pig 15 compares the three inner launch envelopes achieved with the different guidance laws Pigs 14 and 15 show that the envelope witti PN mainly depends on the boundary of the maximum load factor

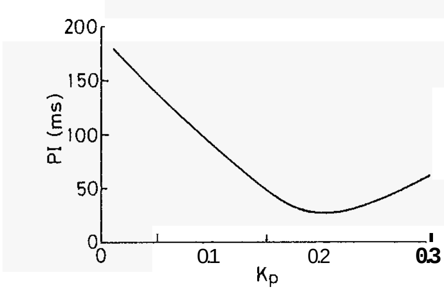


Fig. 9 *PI* vs. K_p

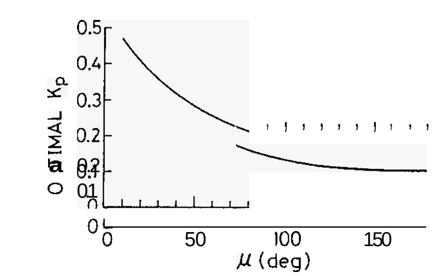
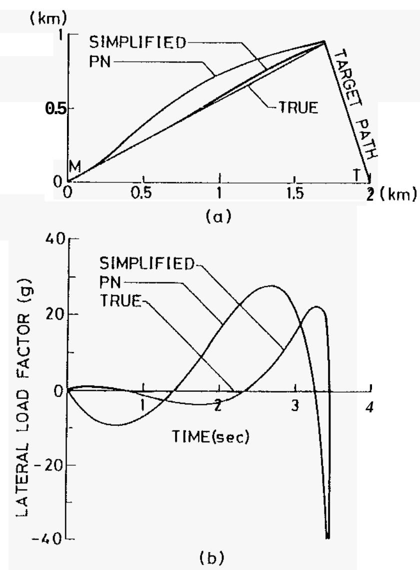


Fig. 10 Optimal K_p as a function of μ



Pig 11 Missile and target trajectories and lateral load factor histories

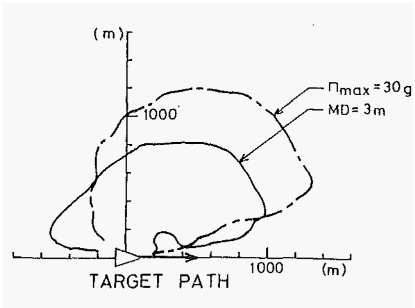


Fig. 12 The launch boundaries for miss and maximum lateral load factor to be $3m$ and $30g$, respectively (True guidance law)

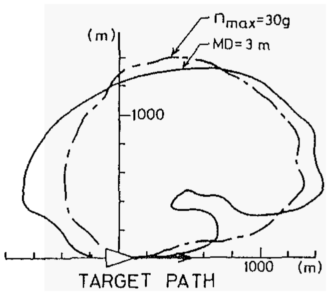
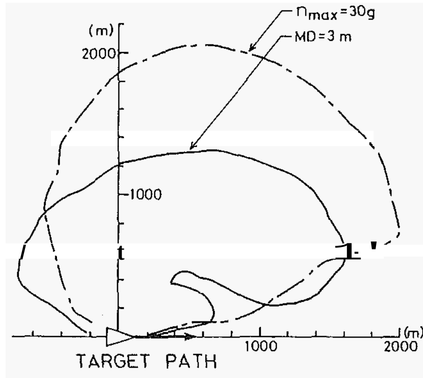
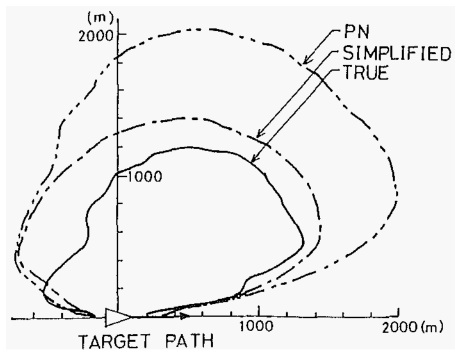


Fig. 13 The launch boundaries for miss and maximum lateral load factor to be $3m$ and $30g$, respectively (Simplified guidance law)



Pig. 14 'The launch boundaries for miss and maximum lateral load factor to be $3m$ and $30g$, respectively (Proportional navigation)



Pig. 15 Inner launch envelopes **of AAM**

Conclusion

A new generalized guidance law for collision courses has been presented. When the missile and target axial accelerations or decelerations are constant, there exists a rectilinear collision course. The guidance law presented, which is called the true guidance law, gives the theoretical guidance acceleration commands to guide a missile on the collison course. The implementation of the true guidance law requires values of missile velocity, missile axial acceleration, target velocity. target axial acceleration. the relative distance between the missile and the target and the target flight-path angle to LOS. In general. however, most present tactical missiles do not carry instruments Lo measure all these variables. Thus. it is very difficult to realize the true guidance law. Therefore. this paper has presented a method for approximately implementing the true guidance law by use of only Lhe target and missile initial velocities and accelerations, and the initial values of time-to-go. which are given from a launcher or a parent aircraft at launch. This implementation is called the simplified guidance law. The small perturbation equation of the lrue guidance law has shown that, the definition of an effective navigation conslant is the same as that in the case of PN. Also. the appropriate value of the navigation constant can be defined by integrating the small perturbation equation. The guidance laws presented as well as PN were applied to a simple model of a short range air-to-air missile with constant acceleration. From simulations, Lhe inner launch envelopes were generoted and the following results were oblained.

1) The missile guided by Lhe true guidance law flies straight and hits the target without further acceleration commands, provided there is no initial heading error.

2) The Lrajectory achieved with PN is quite curved. and large acceleration commands are required near the trajectory end even if the missile is launched along a collision course.

3) The trajectory with the simplified guidance law is also curved. but the deviation from the true collision course is far smaller than that with PN. Also, the required acceleration commands are smaller.

4) The inner launch envelope has shown that the guidance laws presented provide an overall performance improvement over PN.

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