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NONLINEAR INCREMENTAL INVERSE PERTURBATION METHOD FOR STRUCTURAL REDESIGN

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Abstract

A procedure is described for the redesign of undamped structural systems to meet natural frequency and/or mode shape objectives. The procedure can be applied to large or small modal changes and is based on a single finite element analysis of the baseline system. Perturbation of the baseline system is used to develop a set of equations which characterize the redesign process. Depending on the number of modal objectives S and design variables σ the problem is formulated as: (1) an underconstrained problem if $S < \sigma$, (2) a properly constrained problem if $S = \sigma$ or (3) an overconstrained problem if $S > \sigma$. All three problems are solved using an incremental predictor-corrector technique within the feasibility domain defined by the practical constraints imposed on the design variables. The procedure is illustrated by two examples: (1) a redesign of a cantilever beam to achieve frequency and mode shape objectives and (2) a redesign of a 1254 degree of freedom casting for a frequency objective.

Nomenclature

c_{ij}	Participation of the j^{th} mode to changes in the i^{th} mode.
i	Index associated with the i^{th} mode.
j	Index associated with the j^{th} mode.
$\lceil K \rfloor$	Generalized stiffness matrix of the baseline structure.
$\lceil K' \rfloor$	Generalized stiffness matrix of the objective structure.
[k]	Stiffness matrix of the baseline structure.
[k']	Stiffness matrix of the objective structure.
$[k_e]$	Stiffness matrix of element e .
$\lceil M \rfloor$	Generalized mass matrix of the baseline structure.
$\lceil M' \rfloor$	Generalized mass matrix of the objective structure.
[m]	Mass matrix of the baseline structure.
[m']	Mass matrix of the objective structure.
$[m_e]$	Mass matrix of element e .
N	Total number of increments in predictor-corrector solution.
n	Number of degrees of freedom in the structural model.
S	Number of modal objectives.
s	Number of modes involved in the redesign process.
$[\ ]^T$, $\{\ \}^T$	Denotes transpose of a matrix and vector respectively.

Greek Symbols

α_e	Fractional change to element e .
Δ	Prefix denoting change.
$[\Delta k]$	Change to stiffness matrix.
$[\Delta m]$	Change to mass matrix.
$[\Delta\phi]$	Matrix of mode shape vector changes.
$\Delta(\omega_i^2)$	Change to the i^{th} baseline structure eigenvalue, ω_i^2.
σ	Number of structural changes.
$[\phi]$	Matrix of mode shape vectors of the baseline system.
$[\phi']$	Matrix of mode shape vectors of the objective system.
$\{\psi\}_i$	i^{th} mode shape vector of $[\phi]$.
ψ_{ki}	k^{th} degree of freedom of the i^{th} baseline mode shape.
$\{\psi'\}_i$	i^{th} mode shape vector of $[\phi']$.
ω_i	i^{th} baseline structure natural frequency.
ω_i'	i^{th} objective structure natural frequency.

I. Introduction

In many structural problems the criteria for an acceptable design involves constraints on the free vibration characteristics of a structural system. The constraints may be on one or more of the natural frequencies and/or mode shapes. Several degrees of freedom may be constrained on any one mode. In most cases the first design does not satisfy all the free vibration objectives and/or practical constraints. Therefore reanalysis of the structural system which requires expensive finite element formulation and analysis is necessary. Alternatively, the redesign procedure of the baseline system (first design), based on the perturbation technique proposed in this work can be used. This procedure does not require additional finite element analyses and can be applied to large or small modal changes. The procedure is hereafter called Inverse Perturbation Redesign (IPR) procedure, because it gives the designer the capability of determining structural system particulars based on system response properties without using iterative methods.

Let σ be the number of design variables in the IPR procedure, that is, the number of structural system particulars allowed to change during the redesign, and S be the number of modal objectives. If s is the number of modes involved in the redesign procedure and the practical constraints, the problem can be reduced to one of the following:

Problem P1: Underconstrained problem if $S < \sigma$. In this case the design is not unique and the problem can be formulated as an optimization problem. For this purpose an optimality criterion

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is needed such as minimum structural weight or minimum change from the baseline system. The optimization variables in this problem are the σ design variables in the IPR, plus s·n where n is the number of structural degrees of freedom. The S redesign objectives and the s·n free vibration equations become equality constraints in the optimization problem. Further, practical constraints in the form of inequalities may be imposed.

Problem P2: Properly constrained problem if $S = \sigma$. In this case there are S + s·n equations which can be simultaneously solved for the σ + s·n unknowns yielding either a finite number of structural designs or no solution. From the point of view of optimization this is a constraint bound problem which can produce a finite number of feasible designs or prove the problem to be infeasible. In the latter case the problem should be treated as in case P3 below.

Problem P3: Overconstrained problem if $S > \sigma$. In this case the S + s·n equations cannot be satisfied by the σ + s·n unknowns. Consequently a minimum error criterion is needed in order to get a finite number of acceptable designs which will satisfy the equations and practical constraints within a minimum error.

II. Literature Review

Historically procedures involving natural frequency objectives were first developed. Since the first procedure developed by Rayleigh in 1873, many methods have been proposed.[1] Only recently have methods aiming at solving the combined frequency and mode shape objective problems, such as the one presented in this paper, been developed. Some of the methods falling in both categories are briefly described in this section.

Frequency objective procedures usually are of type P1 described in the introduction, that is, their goal is to minimize the mass of a structure which has the specified frequency values or maximize the frequency for a given total mass. Practical constraints are sometimes placed on design variables such as thickness of plates, cross sectional area of axial bars or moment of inertia of beams. Turner proposed one of the first methods to solve this problem.[2] The free vibration equations were considered as equality constraints and handled using the Lagrange multiplier method. Taylor solved the problem for an axially vibrating bar by minimizing the total energy of system using Hamilton's principle.[3] In an extension of his work, Taylor introduced inequality constraints on the cross-sectional area of the bar in addition to the total mass constraint.[4] This new constraint was included in the problem through a continuous Lagrange multiplier. Sheu extended the work of Turner and Taylor to situations where the number of constant stiffness segments was specified, but the boundries and specific stiffness values of the segments were design variables in the minimum bar weight problem.[5] Sippel considered similiar problems using a variational method to derive the minimum mass optimality criterion.[6] Structural systems composed of N-element sandwich-type structures supporting nonstructural mass were considered. McCart used an iterative process to solve the minimum mass problem applied to portal frames.[7] The boundary value nature of the free vibration equations was used in conjunction with a steepest descent method. Rubin used a two step process in which he assumed the optimal design laid on a frequency constraint.[8] The first step was a frequency modification mode where separate gradient equations were developed to achieve the natural frequency goal. In the second step he used the method of steepest descent to find the minimum weight structure for the specified natural frequency. Armand developed the problem as an optimal control problem with distributed parameters.[9] The method is powerful for simple structures and was demonstrated on a plate-like structure. For a more detailed review of many of these earlier methods the reader is referred to the survey by Pierson.[10]

In more recent work Taylor investigated the frequency only constrained problem in terms of model correlation.[11] A procedure was developed to scale an existing structural model to meet experimentally measured natural frequencies. The modification scheme is based on the first order terms of a Taylor series expansion about the baseline model. Bellagamba employed an exterior penalty function technique based on the first derivatives of the violated constraints.[12] Additional constraints were imposed on static displacements and element stresses.

The combined natural frequency and mode shape constrained problem has lately received considerable attention in terms of perturbation based solution techniques. Stetson proposed a first order perturbation method based on the assumption that the new mode shapes could be expressed as admixtures of the baseline mode shapes.[13] In subsequent works, the technique was cast in terms of finite elements and was applied to several problems.[14-16] Stetson's procedure, however, used a method of specifying mode shape constraints based on admixture coefficients which had no obvious physical interpretation. Sandström developed first order equations which are similar to Stetson's, but provided a method for specifying mode shape constraints based on physical quantities.[17,18] Kim formulated the problem using the complete nonlinear perturbation equations.[19] He employed a penalty function method where the objective function was a minimum weight condition and the penalty term was a set of residual force errors.

III. Mathematical Formulation

The undamped free vibration of a baseline discrete structural system is governed by the matrix equation

$$[m]_{nxn}\{\ddot{\psi}\}_{nx1} + [k]_{nxn}\{\psi\}_{nx1} = \{0\}_{nx1} \qquad (1)$$

An eigenvalue analysis produces the eigenvectors

$$[\phi] = [\{\psi\}_1, \{\psi\}_2, \ldots, \{\psi\}_n] \qquad (2)$$

and the natural frequencies

$$\lceil \omega^2 \rfloor = \begin{bmatrix} \omega_1{}^2 & & & 0 \\ & \omega_2{}^2 & & \\ & & \ddots & \\ 0 & & & \omega_n{}^2 \end{bmatrix} \qquad (3)$$

Using the calculated eigenvectors the governing equations can be uncoupled and written as

$$\lceil K \rfloor = \lceil M \rfloor \lceil \omega^2 \rfloor \quad (4)$$

where $\lceil K \rfloor$ is the generalized stiffness matrix

$$\lceil K \rfloor = [\phi]^T[k][\phi] \quad (5)$$

and $\lceil M \rfloor$ is the generalized mass matrix

$$\lceil M \rfloor = [\phi]^T[m][\phi] \quad (6)$$

Similarily, the uncoupled equations of an objective system are

$$\lceil K' \rfloor = \lceil M' \rfloor \lceil \omega'^2 \rfloor \quad (7)$$

where $\lceil K' \rfloor$ is the objective system generalized stiffness matrix

$$\lceil K' \rfloor = [\phi']^T[k'][\phi'] \quad (8)$$

and $\lceil M' \rfloor$ is the corresponding generalized mass matrix

$$\lceil M' \rfloor = [\phi']^T[m'][\phi'] \quad (9)$$

Relationships between the two systems can be defined in terms of perturbations of the baseline system, that is,

$$\begin{aligned} [m'] &= [m] + [\Delta m] && (10) \\ [k'] &= [k] + [\Delta m] && (11) \\ \lceil \omega'^2 \rfloor &= \lceil \omega^2 \rfloor + \lceil \Delta(\omega^2) \rfloor && (12) \\ [\phi'] &= [\phi] + [\Delta\phi] && (13) \end{aligned}$$

Through these definitions, the uncoupled equations (7) of the objective system can be rewritten in terms of the baseline system as

$$[\phi']^T[\Delta k][\phi'] - [\phi']^T[\Delta m][\phi'] \lceil \omega'^2 \rfloor = [\phi']^T[m][\phi'] \lceil \omega'^2 \rfloor - [\phi']^T[k][\phi'] \quad (14)$$

Equation (14) is called the general perturbation equation. It is nonlinear in terms of the modal quantities, $[\phi']$ and $[\omega'^2]$, but is linear with respect to the desired structural changes, $[\Delta k]$ and $[\Delta m]$. To facilitate the solution of these equations we note that (14) is composed of n^2 scalar equations of the following form

$$\{\psi'\}_j^T[\Delta k]\{\psi'\}_i - \{\psi'\}_j^T[\Delta m]\{\psi'\}_i \omega_i'^2 = \{\psi'\}_j^T[m]\{\psi'\}_i \omega_i'^2 - \{\psi'\}_j^T[k]\{\psi'\}_i \quad (15)$$

for $i, j = 1,2,\ldots, n$. Equation (14) or (15) must be satisfied by the objective system. Further it should satisfy the S modal objectives.

The structural change can be decomposed into σ element change properties, α_e. In sheet metal or die cast systems, many elements are required to change together for manufacturability. In this case, σ is the number of groups of perturbed elements.

$$[\Delta k]_{system} = \sum_{e=1}^{\sigma} [\Delta k_e] \quad (16)$$

$$[\Delta m]_{system} = \sum_{e=1}^{\sigma} [\Delta m_e] \quad (17)$$

Furthermore, each element change can be expressed as a fractional change from the baseline system (or a sum of terms as needed to separate bending, stretching, and torsion) as

$$[\Delta k_e] = [k_e]\ \alpha_e \quad (18)$$
$$[\Delta m_e] = [m_e]\ \alpha_e \quad (19)$$

These linear equations of α_e are used in the IPR procedure developed in this paper. Nonlinear relations can readily be implemented in the algorithm for other applications. For example, the effect of plate thickness on stretching stiffness is linear, while its effect on bending is of third order. The σ change properties α_e are the design variables in the IPR. Finally, several of the design variables may be subject to practical constraints. These may be maximum or minimum size constraints or relative size constraints between the elements.

In problem P1, defined in the introduction, equations (14) or (15) and the S modal objectives constitute equality constraints and the practical constraints constitute the inequality constraints. In problem P2, equations (14) or (15) and the S modal objectives can be simultaneously solved for the σ α_e's which must fall in the feasibility domain defined by the practical constraints. In problem P3, equations (14) or (15) and the S modal objectives can be satisfied along with the practical constraints within a specified error to yield the σ α_e's.

IV. Solution Technique

Solution of equations (14) or (15) will provide the required structural changes to meet the modal objectives. Direct solution, however, is usually not possible because of the implicit nature and complexity of these equations. The solution of the general perturbation equations (15) developed in this work is based on a predictor-corrector technique. The predictor phase uses the solution procedure for the first order equation to provide a first approximation to the elemental changes $[\Delta k]$ and $[\Delta m]$. Using these approximate elemental changes, approximate objective eigenvectors are calculated. In the corrector phase these eigenvectors are used in the general perturbation equations (15) to correct the elemental changes. The solution algorithm is diagrammatically shown in Figure 1. In the case of large modal characteristic changes, the algorithm is applied in increments.

Predictor Phase

In the predictor phase of the solution algorithm a first order solution to equations (15) is required. The first order equations developed here were originally proposed by Stetson and extended by Sandström.[13-18] In the first order perturbation development of Stetson, the mode shape changes were represented as linear combinations of the mode shapes obtained in the analysis of the baseline system

$$[\Delta\phi] = [\phi][c]^T \quad (20)$$

where the admixture coefficients c_{ij}, $i, j = 1,2, \ldots, n$ are small and $c_{ii} = 0$. This representation transforms the space spanned by the eigenvectors $\{\psi\}_i$, $i = 1,2, \ldots, n$ into one which is numerically more convenient. Any method which uses such a transformation is an indirect search method since it searches for the admixture coefficient values instead of the eigenvector changes.

Applying this relationship for the eigenvector changes and neglecting terms of order Δ^2, Δ^3 and Δ^4 we get the first order perturbation equations in terms of Δ

$$\{\psi\}_j^T[\Delta k]\{\psi\}_i^T - \{\psi\}_j^T[\Delta m]\{\psi\}_i\omega_i^2 = \begin{cases} M_i\ \Delta(\omega_i^2) & \text{for } i = j \\ M_j\ c_{ij}(\omega_i^2 - \omega_j^2) & \text{for } i \neq j \end{cases} \quad (21)$$

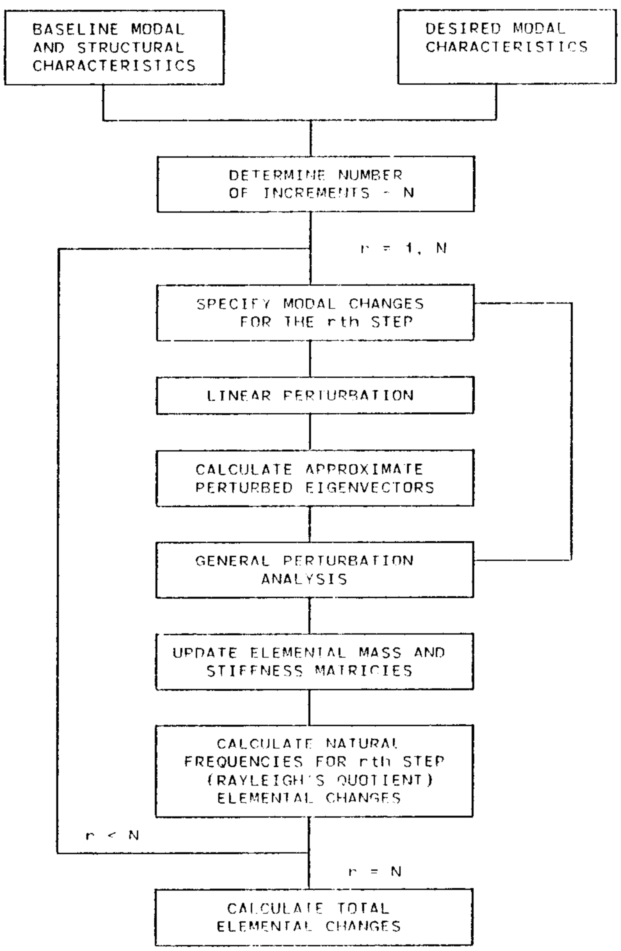

Figure 1 Predictor-Corrector Solution Technique

Solution of the first order equations require the specification of the frequency changes, $\Delta(\omega_i^2)$, and the mode shape changes, $\Delta\psi_{ki}$, in terms of the admixture coefficients, c_{ij}, where $\Delta\psi_{ki}$ is the change in the k^{th} degree of freedom of the i^{th} mode.

In order to eliminate the admixture coefficients, whose physical interpretation is difficult, the following algebraic manipulations are performed. Note that the change to the k^{th} degree of freedom of the i^{th} mode in terms of the admixture coefficients is

$$\Delta\psi_{ki} = c_{i1}\psi_{k1} + c_{i2}\psi_{k2} + \cdots + c_{i,i-1}\psi_{k,i-1} + c_{i,i+1}\psi_{k,i+1} + \cdots + c_{in}\psi_{kn} = \sum_{\substack{j=1 \\ j\neq i}}^{n} c_{ij}\psi_{kj} \quad (22)$$

Also note that by rearranging equation (21) when $i \neq j$ the admixture coefficient c_{ij} can be expressed as

$$c_{ij} = \frac{1}{M_j(\omega_i^2 - \omega_j^2)} \left(\{\psi\}_j^T[\Delta k]\{\psi\}_i - \omega_i^2\{\psi\}_j^T[\Delta m]\{\psi\}_i\right) \quad (23)$$

Applying equation (23) to (22) we develop an expression which directly relates the physical mode shape changes to the structural changes

$$\Delta\psi_{ki} = \sum_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{\psi_{kj}}{M_j(\omega_i^2 - \omega_j^2)} \left(\{\psi\}_j^T[\Delta k]\{\psi\}_i - \omega_i^2\{\psi\}_j^T[\Delta m]\{\psi\}_i\right)\right) \quad (24)$$

Using the relationships for the structural changes defined by equations (18) and (19), the first order perturbation equations for a natural frequency change to the i^{th} mode can be written as

$$\Delta(\omega_i^2) = \sum_{e=1}^{\sigma} \left(\{\psi\}_i^T[k_e]\{\psi\}_i - \omega_i^2\{\psi\}_i^T[m_e]\{\psi\}_i\right)\alpha_e \quad (25)$$

and for changes to the k^{th} degree of freedom on the i^{th} mode as

$$\Delta\psi_{ki} = \sum_{e=1}^{\sigma} \left(\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{\psi_{kj}}{M_j(\omega_i^2 - \omega_j^2)} \left(\{\psi\}_j^T[\Delta k]\{\psi\}_i - \omega_i^2\{\psi\}_j^T[\Delta m]\{\psi\}_i\right)\right)\alpha_e \quad (26)$$

Equations (25) and (26) are a set of linear equations with respect to the α_e's which, when solved, will provide a first order approximation to the structural changes. Should nonlinear expressions be used instead of equations (18) and (19) then equations (25) and (26) would be nonlinear.

The predictor phase problem can be of P1, P2 or P3 type depending on the relation between σ and S. In any case the problem solution will yield first order approximate values to the σ element change properties α_e.

Corrector Phase

In the corrector phase, the first order approximations to the structural changes are used to calculate a first order approximation to the objective eigenvectors. These approximate eigenvectors are used in the general perturbation equations to correct the elemental changes.

The approximate eigenvectors are calculated using the linear mapping provided by equation (20), where the admixture coefficients, c_{ij}'s, are calculated from equation (23). With the approximate eigenvectors developed, the general perturbation equations (15) combined with the definitions for structural changes (18) and (19) are used to develop equation (27)

$$\sum_{e=1}^{\sigma}(\{\psi'\}_j^T[k_e]\{\psi'\}_i - \omega_i'^2\{\psi'\}_j^T[m_e]\{\psi'\}_i)\alpha_e = \{\psi'\}_j^T[m]\{\psi'\}_i\omega_i^2 - \{\psi'\}_j^T[k]\{\psi'\}_i \qquad (27)$$

which provides the corrected structural changes. This equation is used for all modes where frequency constraint is specified. The corrector phase problem can be of type P1, P2 or P3 depending on the relation between σ and the number of specified natural frequencies. At the end of the predictor phase, however, all the unconstrained modal degrees of freedom have been computed to first order and are known in the corrector phase problem. Thus the only unknowns are the σ structural changes, α_e's, and in practice the corrector phase problem is usually type P3. When i=j equation (27) enforces the frequency constraint on the i^{th} mode. When $i \neq j$ equation (27) is interpreted as enforcing orthogonality between the i^{th} and j^{th} mode shapes.

V. Cantilever Beam Example

Vibration of the 2-element cantilever beam shown in Figure 2 is used to illustrate the procedure. Only planar motion is considered where shear deformation and axial displacement are excluded. The structural characteristics of the baseline system are shown in Table 1.

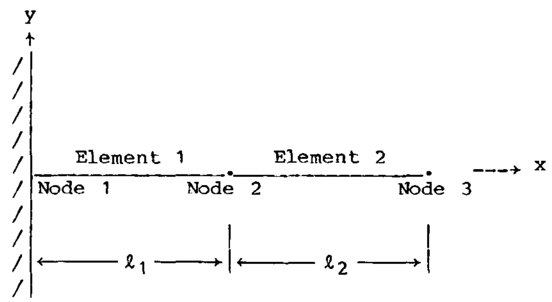

Figure 2 Cantilever Beam Model

An eigensolution of the baseline structure using the inverse power method option in MSC/NASTRAN provided the modal characteristics shown in Table 2. The natural modes were normalized by setting the generalized mass of each mode to unity.

In order to demonstate the potential of the method, both a mode shape objective and a frequency objective are imposed on the structure. The objective structure is a system where the natural frequency of the first mode is increased by 12.6% from 3.551 rad/sec to 4.000 rad/sec. Further, let the objective value for the node 3 translation of the first mode ($\{\psi_5\}_1$) be increased by 6.1% to 2.100 from a baseline value of 1.979.

Two structural properties are allowed to change (σ = 2), that is the second area moment of cross section of elements 1 and 2 respectively.

The procedure is applied to the structure with a single increment which results in the prediction of the following baseline system modifications,

I_1 - Increased by 40.8% to 1.408 in^4
I_2 - Decreased by 67.9% to 0.514 in^4

Analysis of the modified system using MSC/NASTRAN yields a first mode natural frequency of 4.001 rad/sec and the node 3 translation of the first mode of 2.097. These values represent a 0.02% error in the desired frequency value and a 0.14% error in the desired mode shape value. It should be noted that in this example the constraints imposed on the objective system provide a unique system of equations in the predictor phase (problem P2) and an overconstrained system in the corrector phase (problem P3).

Comparison of the results with the first order method demonstrates the advantages of this procedure. The first order solution predicts a 36.8% increase of I_1 and a 276.7% decrease of I_2. A reduction of I_2 by more than 100.0% is not physically possible. Therefore the linear method failed in this example.

VI. Disk Drive Aluminum Casting Redesign

The purpose of this example is to redesign the aluminum casting of the Irwin-Olivetti Winchester disk drive to raise the first natural frequency by 30%. Solution of the problem is considered using the design objective of a least change from the baseline structure.

	Element Number	
	1	2
I - Second Area Moment of Cross Section (in^4)	1.0	1.6
A - Area (in^2)	1.0	1.0
ℓ - Length (in)	0.5	0.5
E - Young's Modulus (lbs/in^2)	1.0	1.0
ν - Poisson's Ratio	0.3	0.3
ρ - Density ($lbs\text{-}sec^2/in^4$)	1.0	1.0

Table 1 Baseline Cantilever Beam Element Properties

		MODE			
	DOF	1	2	3	4
Frequency, ω_i (rad/sec)	-	3.551	24.369	86.637	273.500
Node 1 Translation	1	0.0	0.0	0.0	0.0
Node 1 Rotation	2	0.0	0.0	0.0	0.0
$\{\psi\}_i$ = Node 2 Translation	3	0.693	1.457	0.112	0.967
Node 2 Rotation	4	-2.372	2.181	17.479	-19.255
Node 3 Translation	5	1.979	-1.908	2.304	3.807
Node 3 Rotation	6	-2.643	8.762	-20.751	-73.304

Table 2 Baseline Cantilever Beam Modal Characteristics

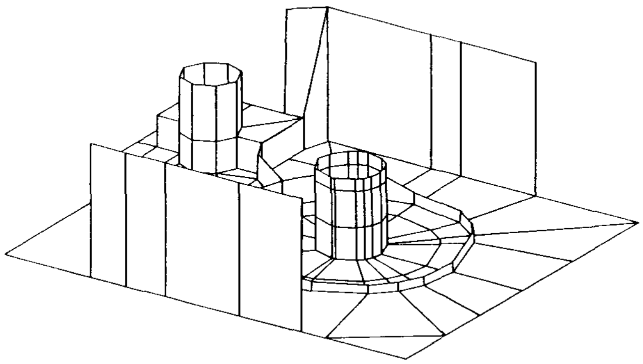

Figure 3 Disk Drive Aluminum Casting

Finite Element Model

It is desired to consider the vibration of the structure as a free body in space. In order to remove the rigid body motions the structure is supported by a soft foundation.

A total of 312 structural elements were used, involving 209 nodes and 1254 degrees of freedom (Figure 3). There were 144 beam elements, 8 spring elements, 159 quadrilateral plate elements and 1 triangular plate element. The quadrilateral and triangular plate elements were used to model the basic casting geometry while the beam elements were used for the various stiffeners. The spring elements were used for the soft foundation.

The eigenvalue analysis was performed using the inverse power method option in MSC/NASTRAN. The natural modes were normalized by setting the maximum value of each mode to unity. The first natural frequency of the baseline structure occurred at 351 Hz. The motion of the structure corresponds to a twisting of the main floor (Figure 4).

Design Variables

The design variables for the analysis are the plate thicknesses of the structural elements. Beam and elastic spring elements are held at their baseline values. Variation of plate thickness only is not considered a significant limitation on the analysis since the plate elements contain over 75% of the strain energy in the 1^{st} mode. Design variable linking is employed to link the thickness of several elements to one design parameter. A total of 16 design parameters remain after linking the 160 plate elements. Linking is performed on the basis of similar geometric and structural properties. Manufacturability and perturbation analysis cost considerations are primary reasons for variable linking.

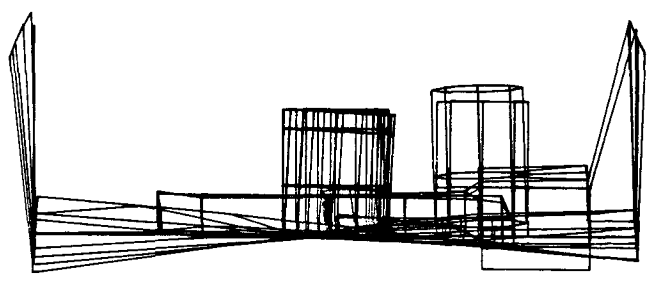

Figure 4 1^{st} Mode - Floor Twist

To further reduce the number of significant variables the strain energy of each one of the 16 parameter set was calculated. Only set numbers 1 and 4 through 8 have significant strain energy values for the 1^{st} mode. Therefore only these sets are included in the analysis. Due to constraints described below, however, set numbers 2 and 3 are also included in the analysis (Figure 5).

Constraints on Design Variables

Several constraints are imposed on the design variables to ensure a practical design. The thickness of any plate is limited to a minimum of 0.070 inches while the maximum thickness is unlimited. Furthermore, the thicknesses of the plates composing the lower surface are constrained to be within ±25% of other plate thicknesses on the lower surface. This placed the following inequality constraints on sets 2, 3, 4 and 6,

$$0.75\ t_i \leq t_j \qquad i,\ j = 2,3,4,6$$

where t_i and t_j are the thicknesses corresponding to the i^{th} and j^{th} element sets.

Results

The analysis was performed using 3 increments of the predictor-corrector technique. In the first increment the objective for the first mode natural frequency was 375.0 Hz., in the second 410.0 Hz. and in the third the objective was the final value of 456.0 Hz. These represent 6.8%, 16.8% and 30.0% increases over the baseline value of 351 Hz.

In the predictor phase the problem was formulated as a type P1 problem with

(1) A minimum change objective: $\min. \sum_{e=1}^{\sigma} \alpha_e^2$.

(2) 8 optimization variables ($\sigma = 8$).

(3) 1 equality constraint, that is, equation (25) for the first natural frequency.

(4) 8 inequality constraints defining the lower bounds for plate thicknesses and 8 more defining the relative thickness bounds.

This optimization problem was solved using the Nelder and Mead simplex method.[20]

In the corrector phase the problem was formulated as a type P2 problem with

(1) 8 unknowns ($\sigma = 8$).

(2) 8 equations of type (27) for the first 8 modes.

The simultaneous solution of the 8 equations yielded one solution which fell outside the feasibility domain. The problem was then reformulated as a type P3 problem which produced a solution that minimized the error in the equality and inequality constraints.

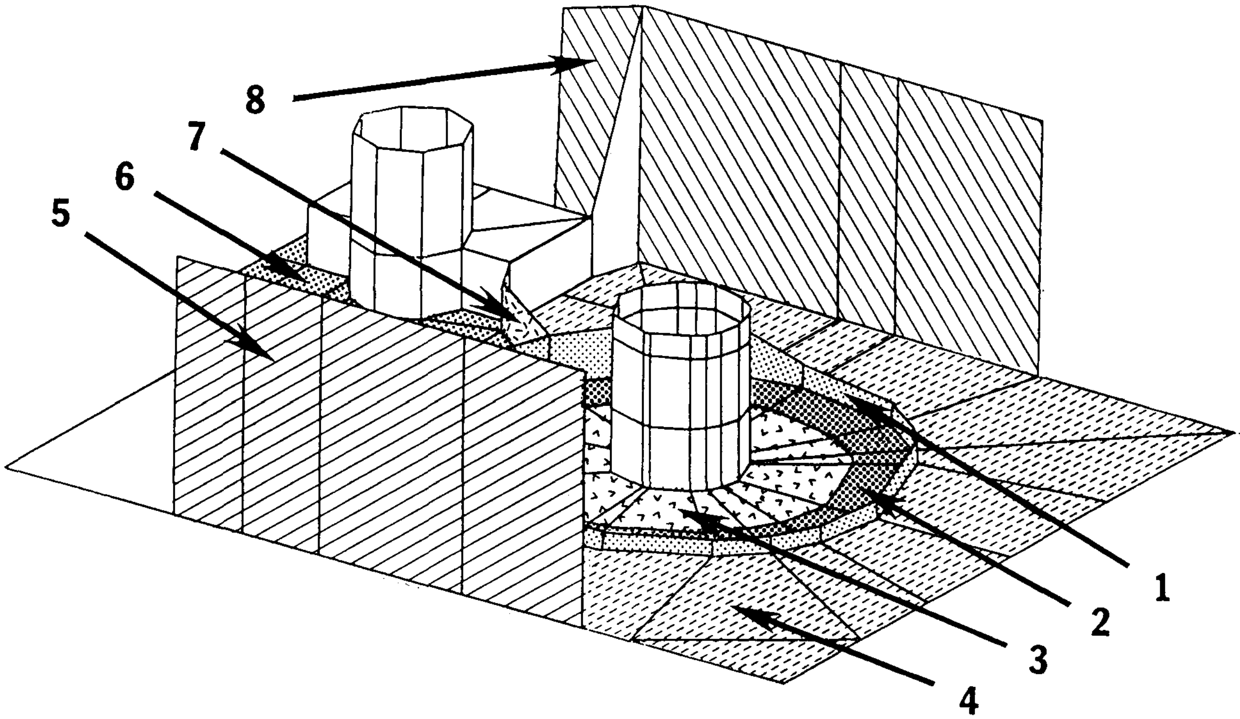

Figure 5 Aluminum Casting Design Parameter Sets

Results for the predictor and corrector phases of each increment, as well as the total changes, corresponding to each parameter set are shown in Table 3. In each increment the difference between the predicted changes and the corrected changes is significant. This is interpreted as the adjustment of the predictor phase changes to account for higher order effects and enforcement of the orthogonality conditions between modes.

Analysis of the structure using the changes determined above resulted in a first mode natural frequency of 434.0 Hz. This represents a 24% increase over the baseline structure versus the desired increase of 30%. This difference in the results is due to the combined effect of the reduction of the feasibility domain by the constraints imposed on the design variables and error accumulation caused by the large number of operations which are required for systems of high number of degrees of freedom. Significantly better results were obtained for the corresponding unconstrained problem.

Summary

A nonlinear incremental inverse perturbation method for structural redesign has been developed. The method uses a single finite element analysis of an undamped baseline structural system, and can be applied to large or small natural frequency and/or mode shape changes. The redesign problem is solved using an incremental predictor-corrector technique. A cantilever beam has been redesigned to achieve frequency and mode shape objectives and a 1254 degree of freedom aluminum casting has been redesigned for a frequency objective.

Acknowledgements

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	Increment Number						
	1		2		3		
Set Number	Predictor	Corrector	Predictor	Corrector	Predictor	Corrector	TOTAL
1	9.7	9.7	15.3	6.8	15.0	6.8	25.2
2	8.0	13.4	12.0	20.2	13.2	9.2	48.9
3	11.7	13.4	15.9	21.5	14.3	14.0	57.1
4	9.9	6.7	9.4	14.4	13.8	9.4	33.5
5	7.4	7.2	13.9	5.6	9.7	-3.0	9.8
6	8.9	9.4	13.3	20.3	15.8	12.4	48.0
7	9.6	11.6	9.9	11.9	10.4	25.1	56.3
8	2.2	5.3	15.7	-14.3	7.9	4.9	-5.3

Table 3 Element Set Percentage Thickness Changes

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