

**A STRUCTURAL OPTIMIZATION METHOD FOR
MULTI-DISCIPLINARY AND WEAKLY COUPLED PROBLEMS**

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Abstract

We shall describe a method to construct a simple but sufficiently flexible shape and sizing optimization system for structural design by using existing modules of modeling, finite element analysis, and optimization algorithm after a critical review of recent development of structural design optimization systems and theories. We shall solve several example problems of shape and sizing design optimization of structures to demonstrate the capability of the optimization system developed here. It will be also shown that the present approach can deal with not only linear problems but also nonlinear, multi-disciplinary, and weakly coupled problems.

Introduction

Reduction of the duration of design and manufacturing processes becomes much more important than reducing the cost of raw material of a product while the required functionality is fulfilled, to reduce the overall cost and to lead success of a new product. In order to reflect this situation to research and development in design methodology, we must reconstruct the notion of structural optimization in mechanical design. In past most structural design optimization tried to minimize the cost of raw material under certain constraints which are implied from mechanics and manufacturing requirement. But now, the most important matter is how easily certain design can be improved with minimal effort by design engineers rather than just considering minimization of the cost (i.e., weight) of raw material. In other words it becomes much important to examine how the optimal design can be achieved.

Although design optimization capability is additionally implemented into some finite element analysis programs in 1980s¹⁻²², limitations are found in these systems. One is the closedness of these systems and the other is limited in shape design capability.

More precisely, state equations we can deal with in these systems are very limited to the ones which are originally designed in finite element analysis codes. Majority of existing design optimization is for stress analysis for linear elasticity, while other analysis areas such as heat conduction, fluid flow, metal forming, and others are at this moment excluded from existing design optimization systems. However, these wide range state equations must be involved in structural optimization for mechanical design, and they might be simultaneous. That is, thus it is required in a design system that we can deal with any kind of state equations using different analysis software; i.e., a design system must be open. If other discrete methods such as boundary element and finite difference methods are applied to solve state equations, we should still be able to incorporate with these different type analysis capabilities in a design optimization system.

It is noted that structural optimization had studied mainly by structural engineers in aeronautical and aerospace industry. They are based on the theory of linear elasticity and linear thermoelasticity. On the other hand, mechanical design requires optimal design not only the sizes of a structure but also its shape, and it mostly deals with design of components or parts of a structure rather than a large scale whole body of complex structure. The number of design variables is much less than that of aerospace type structures, but the shape design becomes far more important in mechanical design. Thus, analysis model developing with shape geometry is the most important in mechanical design. No matter how capable sensitivity analysis and optimization algorithms are developed in a design system, if an appropriate finite element model cannot be generated automatically at every design step during execution of the system, the optimum cannot be easily obtained. This implied that a design system must involve flexible modeling capability that can reflect any geometrical design change by an optimization algorithm. In other words, a design system may be constructed based on a modeling software rather than finite element or sensitivity/optimization software by adding other necessary modules for design optimization.

It is thus required to study and to develop a simple

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methodology that can deal with both sizing and shape optimization as well as multi-disciplinary problems.

In this article we shall describe a method to construct a rather small scale but sufficiently flexible optimization system by integration of existing mesh generator, finite element analysis, and optimization algorithm capability using the concept of open-ended software modules in UNIX operating system. More precisely, we shall develop an optimization system using C-Shell scripts that can control the system flow and execute application programs in UNIX OS environment so that all function can be integrated in a system. It is noteworthy that the concept that we shall introduce to develop an optimization system should be general in the sense that the present approach is applicable to any software that has control flow commands to execute variety of modules and communication commands to the operating system. These are used as high level computer language to write a program of design optimization. The present system allows to deal with sizing and shape optimization in the same manner, while the nature of static/dynamic, or linear/nonlinear problems for analysis does not affect to the system itself since analysis is assumed to be independent of the system. In this sense, this can provide much flexible and powerful capability than existing one in commercially available codes for design optimization, and yet the size of the system is far smaller than any of existing programs.

Structural Optimization Procedure

Standard procedure of structural optimization as a class of mechanical design consists of the following four major modules:

1. Development of a Model for Analysis
2. Finite Element Analysis
3. Design Sensitivity Analysis
4. Application of Optimization Algorithms.

Development of a finite element analysis model is nothing but usual preprocessing for finite element analysis in sizing optimization. However, it must be a module inside of the design system for shape optimization, since geometric modeling itself is subject to design change during optimization. In general, finite element methods are applied to solve the state equation, but other analysis methods such as boundary element and finite difference methods should be also applicable. In any case a discrete model must be developed, and is directly related to shape design. It is noteworthy that design sensitivity calculation depends on analysis method applied. An appropriate method of sensitivity analysis may be determined by the characteristics of the state equation and a choice of method in the analysis module. The last part of the design optimization is the set of

optimization algorithms to solve the optimal design problem that is formulated as a constrained nonlinear minimization problem.

There are many well-developed sophisticated software for finite element analysis and optimization at present, and they are used daily in design practice. Thus it becomes rather ineffective to start writing all the program of structural optimization from scratch. It is better to develop a design system that can utilize these as intact existing modules so that it does not require additional effort to be familiar with analysis and optimization modules. Furthermore, since each organization has its own preference and rather extensive experience in use of a particular finite element code, the design optimization system should be able to integrate this specific one as well as other choices for other users. This means that the optimization system should be able to choose any of them according to the nature of the state equation and to the preference of a user. Since most of commercially available design optimization codes are developed as an enhancement of their original finite element analysis programs, they do not possess this flexibility and openness. As mentioned earlier, state equations in mechanical design are usually multi-disciplinary settings. Thus a design system should not be restricted by particular finite element analysis capability. It should be designed with new concept based on the nature of mechanical design that has much larger scope than analysis of the state equation.

Structural Shape Optimization

Since shape design is an important issue to be studied as structural optimization in mechanical design, an optimization system should easily be able to deal with the shape optimization of a structure as well as sizing optimization. The characteristic of shape design is that the geometric finite element model must be modified during optimization procedure, while sizing problems can be solved with a fixed geometrical finite element model. This nature of adaptation of the finite element mesh has been the major difficulty in shape optimization. Many researchers have been challenging to solve this difficulty and it is not completely solved yet especially for three-dimensional structures. The notion of shape optimization is first introduced by Zienkiewicz and Campbell²³. In early days, the direction of mesh movement is specified according to the shape change of a structure, while the topology of finite element connectivity is fixed as the initial one^{19,24}. This approach failed to yield the optimum in many problems by crashing elements or destroying convexity of finite elements unless we can specify appropriate direction of mesh movement by predicting the final shape in advance. Thus, we have reached to the notion that finite element meshes must be regenerated completely only by using

boundary information that defines the shape of a structure for fully automated shape optimization methods, see references²⁵⁻²⁹. This means that a design optimization system must contain the capability of finite element mesh generation. In the sense this becomes most important module for a design system involving shape change of a structure. We shall now briefly review development of shape optimization of an elastic structure. Extensive literature on this subject may be found, for example, in references^{30,26}.

There are two approaches to describe the shape of the design boundary. Majority of early days works on shape optimization define the design boundary by piece wise linear segments connecting nodes of a finite element model on the design boundary. Then the design variables are the coordinates of such nodes. In this case the number of design variables becomes significantly large if a refined finite element model is used to assure accuracy of approximation.

Another disadvantage of this approach is that mere application of resizing algorithm of the nodal location may yield a physically unrealistic oscillatory optimum shape, because of "instability" of stress and strain on the boundary³¹. The total number of degrees of freedom of displacement should be larger than the ones for stress and strain in the displacement finite element approximation. However, if the least squares method³² is applied to compute stress and strain on the boundary, those degrees of freedom exceed the one for the displacement. This leads instability in approximation. Stress and strains computed by the least squares method possess slightly oscillatory distribution, and then this slight oscillation is magnified during the iterations for shape optimization. Thus some smoothing algorithms of resizing or adaptive schemes to have accurate finite element approximation of stress and strain must be introduced to obtain a smooth optimum shape, see reference³³.

To overcome the defect due to the definition of the boundary by nodal points, Braibant and Fleury introduced the method of B-spline to describe the shape of design boundaries³¹. After decomposing the design boundary into a set of design segments consisting of lines, arcs, and curves, each boundary segment is represented by an appropriate spline function. Then a finite element model is constructed independently of the number of control points of the splines. In most of shape optimization problems, design boundary segments are defined by few numbers of control points. If a cubic curve is expected, four control points are required. If an arc is desired, the location of the center and the radius are regarded as the control points. Despite that no matters how refined finite element models are introduced, the number of control points stays the same, that is, the total number of the discrete design variables can be fixed and not so many. Furthermore, as reference³¹ showed, the smooth optimum shape can be obtained without

introducing special techniques of smoothing and adaptive finite element methods. Therefore, the majority of researchers and structural engineers are now using this second approach to define the design boundary.

Sensitivity analysis for shape optimization has been studied by Choi, Haug, Hou, and Yoo³⁴⁻³⁶, and others. Since extensive treatment of sensitivity analysis can be found in reference^{37,38}, we shall only give brief description of shape sensitivity. If the design boundary segment is described by the spline expression:

$$\mathbf{x} = \sum_{i=1}^{N_c} \mathbf{x}_{ci} \phi_i(s) \quad 0 \leq s \leq 1 \quad (1)$$

where N_c is the number of control points, \mathbf{x}_{ci} , $i = 1, \dots, N_c$, are the coordinates of the control points, s is the parametric coordinate, and ϕ_i are the basis function for spline expression. If either analytical or semi-analytical method is applied to compute the design sensitivity, we must calculate the sensitivity

$$\begin{aligned} \frac{Dg}{D\mathbf{x}_{cj}}(\mathbf{u}) &= \frac{\partial g}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial \mathbf{x}_{cj}} = \frac{\partial g}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial \mathbf{x}(s)} \frac{\partial \mathbf{x}(s)}{\partial \mathbf{x}_{cj}} \\ &= \frac{\partial g}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial \mathbf{x}(s)} \frac{\partial}{\partial \mathbf{x}_{cj}} \sum_{i=1}^{N_c} \mathbf{x}_{ci} \phi_i(s) = \frac{\partial g}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial \mathbf{x}(s)} \phi_j(s) \end{aligned} \quad (2)$$

for a performance function g that is a function of only the displacement \mathbf{u} . If N_{smax} number of nodes are placed on this spline, sensitivity of g is thus computed by

$$\frac{Dg}{D\mathbf{x}_{cj}}(\mathbf{u}) = \frac{\partial g}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial \mathbf{x}_{cj}} = \frac{\partial g}{\partial \mathbf{u}} \sum_{i=1}^{N_{smax}} \frac{\partial \mathbf{u}}{\partial \mathbf{x}_i} \phi_j(s_i) \quad (3)$$

where \mathbf{x}_i , $i = 1, \dots, N_{smax}$, are the nodal coordinates of the finite element model on the design boundary segment, and s_i are the parametric coordinates corresponding to \mathbf{x}_i . Therefore, we must compute the sensitivity of the displacement \mathbf{u} with respect to all of the nodal coordinates on the design boundary segment of a finite element model. If a refined finite element model is used for stress analysis, computing sensitivity might become fairly large despite a few numbers of control points for the spline expression in the case that analytical or semi-analytical method is applied. It thus follows from this fact that design sensitivity for shape optimization should be effectively computed by finite difference methods by taking advantage of a few control points of spline expression.

Sensitivity Analysis by the Finite Difference Method

In order to utilize an optimization algorithm, it is at least required to compute the value and its first derivative

of a performance function (that is the objective and constraint functions) g with respect to a design variable d , i.e., $g(\mathbf{u}, d)$ and $\frac{Dg}{Dd}(\mathbf{u}, d)$, where \mathbf{u} is the state variable to define the state equation (the equilibrium equation for stress analysis, the equation of motion in dynamics, the heat conduction equation for thermal analysis, ...) of a physical system to be optimized.

If the state equation is static and linear, and if a discrete model of the state equation is expressed by

$$\mathbf{K}\mathbf{u} = \mathbf{f} \quad (4)$$

then differentiation of this in a design variable d yields the sensitivity of the state variable

$$\frac{\partial \mathbf{K}}{\partial d} \mathbf{u} + \mathbf{K} \frac{\partial \mathbf{u}}{\partial d} = \frac{\partial \mathbf{f}}{\partial d} \Rightarrow \frac{\partial \mathbf{u}}{\partial d} = \mathbf{K}^{-1} \left(\frac{\partial \mathbf{f}}{\partial d} - \frac{\partial \mathbf{K}}{\partial d} \mathbf{u} \right) \quad (5)$$

where \mathbf{K}^{-1} is the inverse of a linear operator (i.e., the global stiffness matrix in stress analysis by the finite element method). Thus the sensitivity of the performance function g , i.e., the first derivative of g with respect to a design variable d is given by

$$\frac{Dg}{Dd} = \frac{\partial g}{\partial d} + \frac{\partial g}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial d} = \frac{\partial g}{\partial d} + \frac{\partial g}{\partial \mathbf{u}} \left\{ \mathbf{K}^{-1} \left(\frac{\partial \mathbf{f}}{\partial d} - \frac{\partial \mathbf{K}}{\partial d} \mathbf{u} \right) \right\} \quad (6)$$

This form of sensitivity requires explicit representation of a performance function in terms of the state variable \mathbf{u} and a design variable d . And it has to be assured that state variable \mathbf{u} is differentiable with respect to design variable d .

Thus application of the above analytically derived sensitivity might not be useful for performance functions which are implicit functions of the state variable \mathbf{u} such as the principal stresses of a three-dimensional solid structure. Even in the case that performance functions are explicitly expressed in the state and design variables, utilization of the above form of sensitivity might not be practical since we have to compute the first derivative of the linear operator (stiffness matrix in stress analysis) \mathbf{K} of the state equation. For example, in a three-dimensional shell structure, the analytical partial derivative of the stiffness matrix \mathbf{K} with respect to a nodal coordinate of a node may not be obtained in a simple form. For shell formulation in the finite element method we use a local coordinate system attached into each finite element that is defined by the nodal coordinates of the four corner nodes of the element. The element stiffness matrix \mathbf{K}_{local}^e is transformed to the one \mathbf{K}_{global}^e in the global coordinate system and it is assembled to the global stiffness matrix:

$$\mathbf{K}_{global}^e = \mathbf{R}^T \mathbf{K}_{local}^e \mathbf{R} \quad (7)$$

where \mathbf{R} is the transformation matrix from the local to the global coordinate system. The first derivative of the stiffness matrix is then computed by

$$\frac{\partial \mathbf{K}_{global}^e}{\partial d} = \left(\frac{\partial \mathbf{R}}{\partial d} \right)^T \mathbf{K}_{local}^e \mathbf{R} + \mathbf{R}^T \frac{\partial \mathbf{K}_{local}^e}{\partial d} \mathbf{R} + \mathbf{R}^T \mathbf{K}_{local}^e \frac{\partial \mathbf{R}}{\partial d} \quad (8)$$

that is, the first derivatives of both coordinate transformation and element stiffness matrices must be analytically obtained. Since the transformation matrix \mathbf{R} is not a straightforward function of the nodal coordinates, it is not practical to compute the analytical first derivative of the stiffness matrix. Furthermore, unless the exact formulation of the stiffness matrix is known, this form of sensitivity is not applicable. In many commercially available finite element codes, this formulation is not opened to public in general. This leads a finite difference scheme to compute the first derivative of \mathbf{K} of the state equation:

$$\frac{\partial \mathbf{K}}{\partial d} \approx \frac{\mathbf{K}|_{d+\Delta d} - \mathbf{K}|_{d-\Delta d}}{2\Delta d} \quad (9)$$

by evaluating \mathbf{K} at the two perturbed designs $d+\Delta d$ and $d-\Delta d$, where Δd is a sufficiently small design change at the current design d . This approximation yields the so-called semi-analytical method to compute the design sensitivity

$$\frac{Dg}{Dd} = \frac{\partial g}{\partial d} + \frac{\partial g}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial d} \approx \frac{\partial g}{\partial d} + \frac{\partial g}{\partial \mathbf{u}} \left\{ \mathbf{K}^{-1} \left(\frac{\mathbf{f}|_{d+\Delta d} - \mathbf{f}|_{d-\Delta d}}{2\Delta d} - \frac{\mathbf{K}|_{d+\Delta d} - \mathbf{K}|_{d-\Delta d}}{2\Delta d} \mathbf{u} \right) \right\} \quad (10)$$

The semi-analytical method does not require to know the exact explicit form of function in the state variable \mathbf{u} . If the operator can be evaluated at two different designs in analysis procedure (that is, if we can output the stiffness matrix and the load vector at two different design stages), we can compute the design sensitivity by using the semi-analytical method. Then, we may not need direct access to the source code of finite element analysis to develop a sensitivity analysis program using the semi-analytical method. However, it would be very inefficient.

Therefore, for multi-disciplinary problems, it might be much simpler to apply the finite difference approximation to compute the sensitivity of a performance function g :

$$\frac{Dg}{Dd} \approx \frac{g|_{d+\Delta d} - g|_{d-\Delta d}}{2\Delta d} \quad (11)$$

There are four reasons it is concluded that the finite difference method is the most appropriate method to compute sensitivity for most of practical optimization problems in mechanical design.

Differentiability. In the finite difference scheme, the explicit function form of g in d and u is not required, and even the first derivative of the linear operator of the state equation need not be calculated. Even the differentiability of the state variable with respect to design variables does not have to be considered.

Flexibility. The finite difference approximation is applicable both for linear and for nonlinear state equations, while time dependency of the state equation does not affect at all in computation of the design sensitivity. Mechanical design requires consideration of multi-disciplinary settings. This nature of mechanical design yields consequence that analytical, even semi-analytical methods are not practical to compute design sensitivity. Flexibility of the finite difference method for sensitivity is thus truly great.

Development Cost. We cannot expect a single software to integrate the capability to calculate sensitivity in the different disciplines (e.g., stress and fluid flow) based on the analytical or semi-analytical methods at least at present. The finite difference method is, however, applicable together with already existing analysis programs for different kind of state equations to compute sensitivity without any modification or enhancement of these at all.

Calculation Cost. The disadvantage of this method is requirement of two analyses per design variable to compute the sensitivity. Thus, if m number of performance functions and n number of design variables are involved in a design problem, $(2n+1)m$ analyses are required to compute the sensitivity and the value of performance functions. If the number of design variables is large as in sizing problems for aerospace structures, the finite difference method is not practical at all. However, if the number of design variables is rather small, it becomes powerful. As mentioned earlier, most of mechanical design problems, the number of design variables is very small, because of the requirement from cost effective manufacturing.

Here the central finite difference approximation is applied instead of forward or backward finite difference scheme, although it requires one more additional analysis. It is noted that the central difference method is a second order accurate approximation. Indeed, if a performance function g is sufficiently smooth, the approximation errors are given as follows:

$$\frac{g|_{d+\Delta d} - g|_{d-\Delta d}}{2\Delta d} - \frac{Dg}{Dd}\bigg|_d \approx \frac{1}{3}(\Delta d)^2 \frac{D^3g}{Dd^3}\bigg|_d \quad (12)$$

It is not necessary to assume too small design perturbation Δd to calculate the sensitivity. If the central difference scheme is applied, errors in the forward and backward difference schemes can be estimated by

$$\frac{1}{2}(\Delta d) \frac{D^2g}{Dd^2}\bigg|_d \approx \frac{1}{2} \frac{g|_{d+\Delta d} - 2g|_d + g|_{d-\Delta d}}{\Delta d} \quad (13)$$

This may be applicable to modify the size of design perturbation Δd by specifying the allowable tolerance ϵ for the approximation error in the forward or backward difference scheme. Indeed, we first estimate the second derivative of the performance function using a trial design perturbation Δd_{trial} , and then we check whether the estimated error

$$E_{estimated} = \left| \frac{1}{2}(\Delta d_{trial}) \frac{g|_{d+\Delta d} - 2g|_d + g|_{d-\Delta d}}{\Delta d_{trial}^2} \right| \quad (14)$$

is smaller or larger than a given tolerance ϵ . If this estimated error is larger than ϵ , then we define a new perturbation $\Delta d_{desired}$ by

$$\Delta d_{desired} \leq \sqrt{\frac{2\epsilon \Delta d_{trial}^2}{|g|_{d+\Delta d} - 2g|_d + g|_{d-\Delta d}}} \quad (15)$$

For the sensitivity, however, we shall apply the central difference scheme to assure at least one order higher accuracy than this estimation based on the forward or backward difference approximation. Since it is possible to estimate the upper bound of the amount of the approximation error, the finite difference method is now applicable with more confidence to compute design sensitivity.

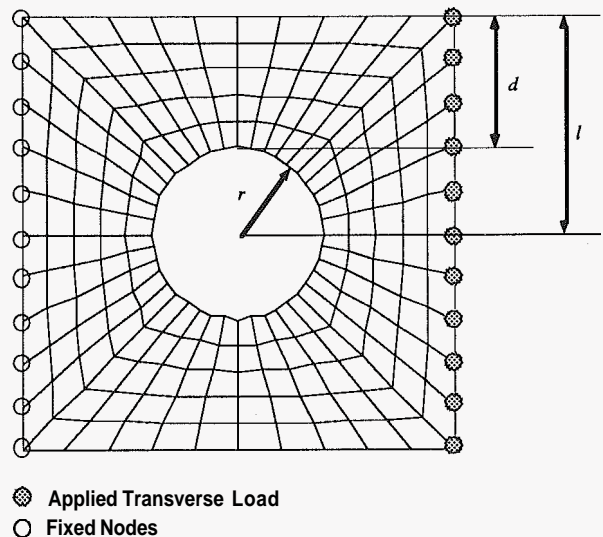


Fig. 1. Finite Element Mesh and Definition of Design Variable

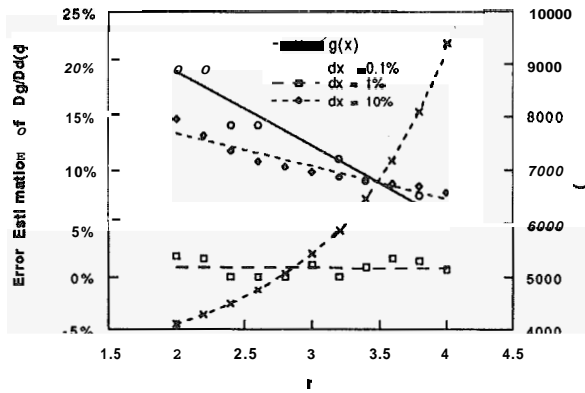


Fig. 2. Estimated Error in the Modified Design Variable
 $(dx = \frac{\Delta d}{d} \times 100\%)$

We shall provide an example of error estimation of the forward or backward finite difference method for sensitivity analysis related to the shape of the body. To do this, the bending problem of a plate with a circular hole is considered shown in Fig. 1. The nodes on the left edge are fixed, and transverse force is applied uniformly on the right edge. If a performance function g is the maximum principal stress $\bar{\sigma}$ of the structure, and design variable is $d = (1 - r)$ the finite difference approximation error is estimated by eqn. (14).

The size of perturbation Δd is determined by $\Delta d = 10^a d$ ($a = -1, -2, -3$). Figure 2 shows the estimated error of the sensitivity of the maximum principal stress with respect to the radius of the circular hole. The estimated error ratio is plotted with the radius r changed from initial value 2.0 to the upper limit 4.0. It follows from the plotting of the estimated error in Fig. 2 that the lowest error is obtained at $a = -2$ while much worse results are obtained for $a = -1$ and -3 . In the case that $a = -1$, we may say too large Δd is assumed. However, for $a = -3$, the round off error becomes much bigger than approximation error. In the finite difference approximation, the round off error occur in the calculation of $g|_{d+\Delta d} - g|_{d-\Delta d}$, thus the error is quantified by

$$E_{\text{RoundOff}} = \frac{g|_{d+\Delta d} - g|_{d-\Delta d}}{g|_d} \quad (16)$$

In the present example, the round off error is $O(\Delta d) = 10^{-3}$ at $a = -3$. This implies we would lose 3 digits in the calculation. This is the reason the error declines when the value of performance function and the perturbation size Δd become large.

From this error analysis, the finite difference approximation is practically accurate enough to perform optimization. In other words, the error is not too large

(less than 2% at $a = -2$) even in the forward or backward finite difference approximation that is one order less accurate approximation than the central difference scheme applied in our optimization study.

Optimization Algorithms

Many algorithms have been developed to solve the optimization problems modeled as following 38,39;

$$\begin{aligned} \text{Minimize} \quad & f(x) \\ \text{Subject to:} \quad & g_i(x) \leq 0 \quad i = 1, 2, \dots, m \\ & h_j(x) = 0 \quad j = 1, 2, \dots, l \\ & \underline{x}_k \leq x_k \leq \bar{x}_k \quad k = 1, 2, \dots, n \end{aligned} \quad (17)$$

If application range of a design system is limited in a single disciplinary problem for linear elastic or eigenvalue analysis of a structure, it is much more efficient to develop an optimization system using the optimality criteria method to find the optimum. However, the optimality criteria method is highly depend upon the nature of state equations in the design problem as well as a "single" design constraint, and further it is very difficult to extend to solve multi-disciplinary design optimization problems. The method of Mathematical Programming, especially Non Linear Programming is most suitable for the system because the method is independent from the state equation and most cases of problems have highly non linear objective functions or constraints.

There are many available commercial and public domain packages of optimization algorithms based on the theory of mathematical programming methods both for linear and nonlinear programming problems 39-47.

Implementation of Structural Optimization

On the basis of the open-ended concept, a prototype of optimization system has been implemented by integrating modules. The system consists of 1) MAZE 49 as a geometric modeler and finite element mesh generator, 2) the system driver written in UNIX C shell script 3) TOPAZ2D 50 as FEM analysis code for heat conduction and magnetic field problem, 4) NIKE2D 51 as FEM analysis code for thermal stress problem, 5) DOT 42 for a sequential linear programming optimizer.

Because these modules are all complete package software and independent from the other modules, the communication between the modules has to be taken by using input and output data files, and these modules have to be controlled by high level (OS level) language. This is the reason the system driver 2) has to be written in UNIX C-Shell Language.

Fig. 3 shows the basic flow of the optimization for the case of temperature-thermal stress coupled case. First

of all the model of initial design is created. After design variables, an objective function and constraints are defined, optimizer DOT is initialized. In the main loop, the system calculates the value or the sensitivities of performance functions, which are required by DOT. If the value is required, the geometric model is updated according to the current design variables, then finite element model and boundary conditions are created using MAZE. Temperature is analyzed by TOPAZ2D and thermal stress is analyzed by NIKE2D using the results from TOPAZ2D. The values of performance functions are calculated from the output file of the analysis and they are given to the optimizer. If the sensitivities are required, perturbation sizes of the design variables are calculated and the model is passed to the sensitivity analysis sub system.

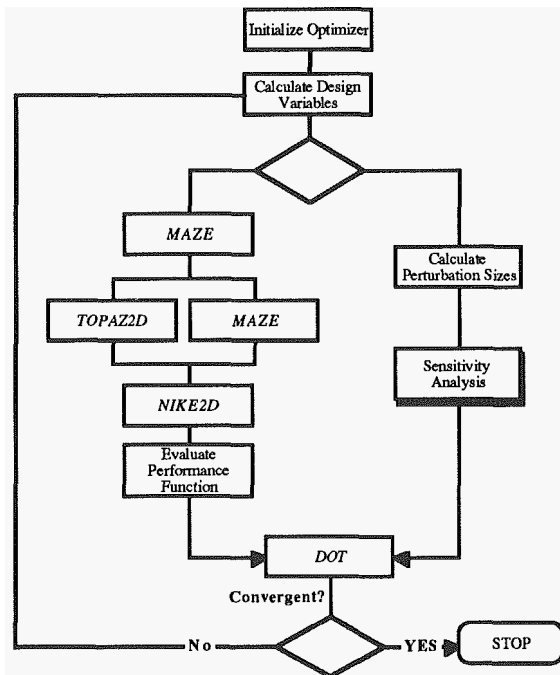


Fig. 3. Optimization Procedure

Using the values and sensitivities of the performance functions optimizer will calculate the values of the design variables of the next trial design. After the optimization is converged, the program exits from the main loop and outputs the optimal design.

Fig. 4 shows the basic flow of the sensitivity analysis sub system. According to the perturbation size, two kinds of perturbed model, one has slightly large value of the design variable and the other has small value, are created and analyzed. From the values of performance functions of perturbed models, the sensitivities are calculated by the central finite difference scheme. The sensitivities are given to the optimizer, and those are used to calculate the values of the variables of the next trial design.

Because of the choice of sensitivity calculation

using the central finite difference scheme, the main routine becomes very simple. If different geometric modules, analysis codes, and optimizers are intended to use in the system of optimization, we simply call desired modules. Most of the rest of the program of the optimization is unchanged.

It is clear that if the total numbers of design variables are reasonably small, and if computing speed is not so critical, this simple and small design optimization system written in UNIX C-Shell scripts can solve multi-disciplinary design optimization problems both in size and shape design.

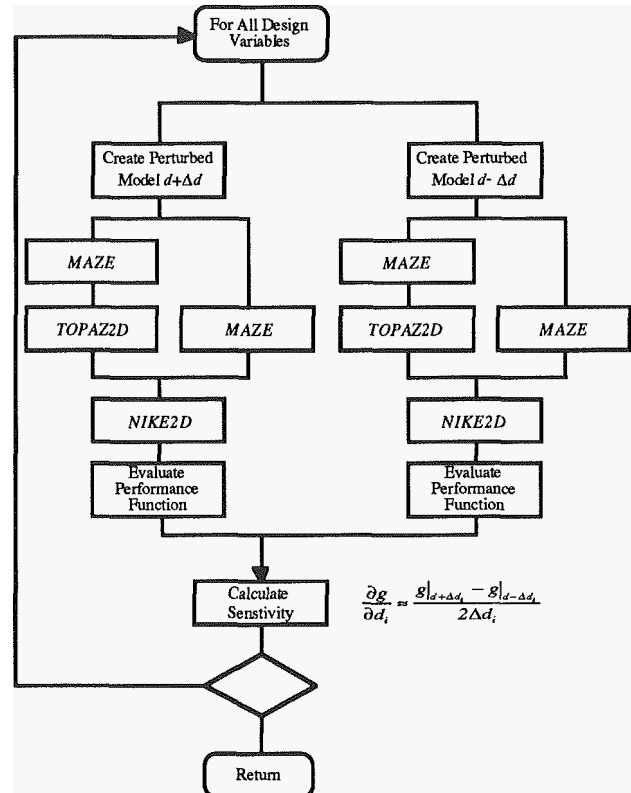


Fig. 4. Sensitivity Analysis by the Central Finite Difference Scheme

Examples at Present System

Some examples in mechanical design using the present design system will be presented for multi disciplinary and weakly coupled problems. TOPAZ2D is used for heat conduction and magnetic field analysis, while NIKE2D is used for thermal stress analysis.

Thermal and Magnetic Field Problem

The first example is a shape design problem of a plate that is placed in a field (Fig. 5). The plate is subject to two kinds of field, one is magnetic field (Fig. 6) and

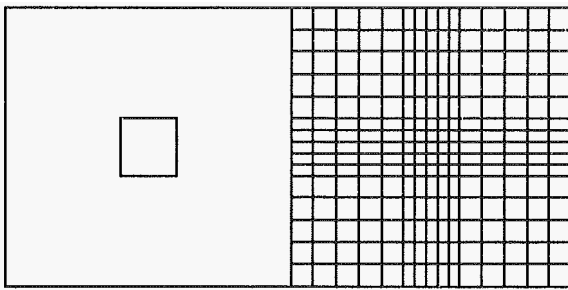


Fig. 3 Geometric and Finite Element Model for the Initial Design

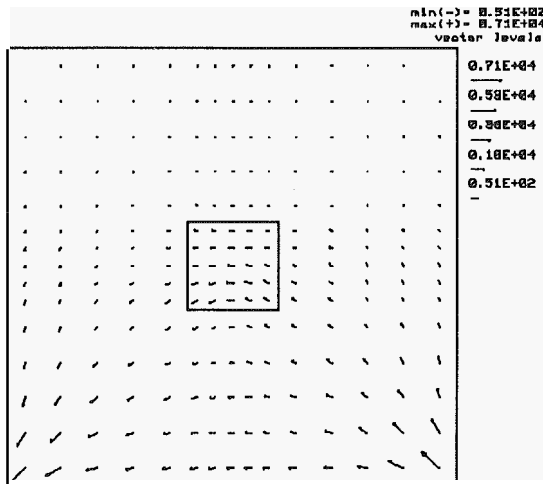


Fig. 4. Magnetic Flux of the Initial Design

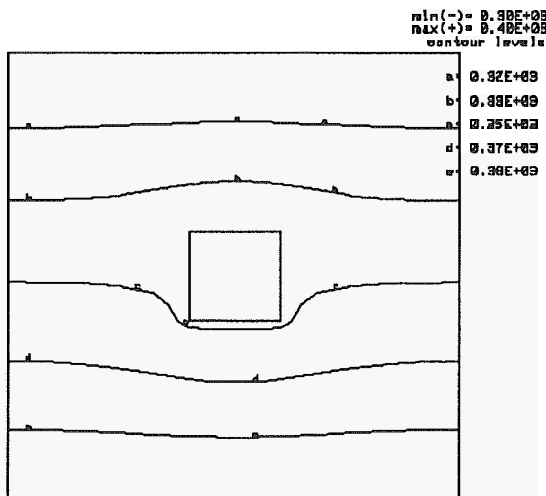


Fig. 5. Temperature Field of the Initial Design

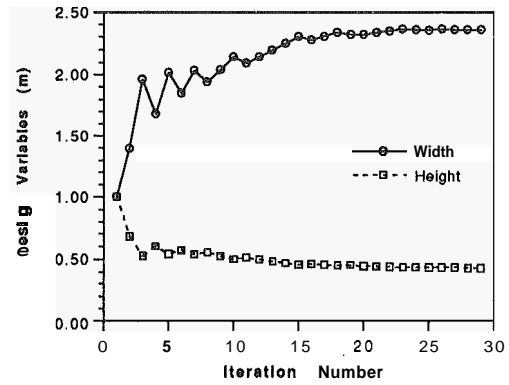


Fig. 6. Iteration History of Design Variables

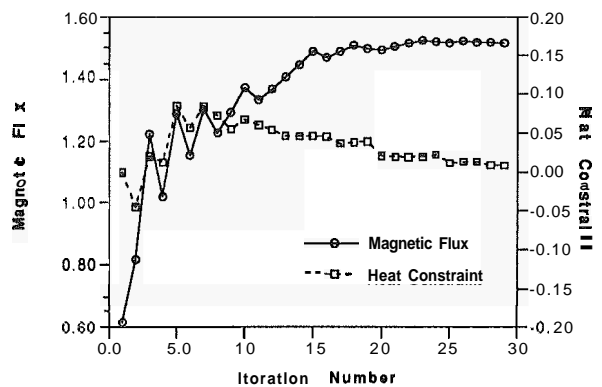


Fig. 7. Iteration History of Performance Functions

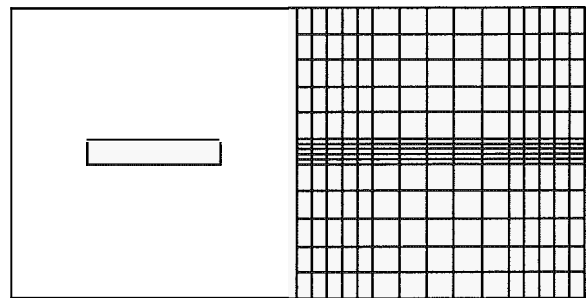


Fig. 8. Geometric and Finite Element Model of the Optimal Design

The finite element model of the optimum design is shown in Fig. 10. Figure 11 shows the vector plot of magnetic flux at the optimum.

Weakly Coupled Thermal Stress Problem

The other problem is a shape optimization of an L-type structure in the temperature field (Fig. 12). Figure 13 shows the deformed shape with a finite element mesh at the initial design. This is modeled using plane strain finite elements. The material of the structure is expanded

the other is temperature field (Fig. 7). The magnetic flux passed through the plate is maximized without increasing the heat given to the plate and its weight by varying the dimensions (sizes) of the width and height. Figures 8 and 9 show the iteration history of the optimization process.

After optimizing this design problem, the width of the plate is increase while its height is reduced, and the magnetic flux is increased 146% within the constraint.

by changing the temperature (Fig. 14), and thermal stress is generated inside the structure (Fig. 15). The objective of this problem is to minimize the maximum stress in the structure by modifying the shape of the structure. The total volume of the structure is bounded to the one at the initial design. The design variables here are the sizes of x_i , $i=1,2$.

The iteration history of the design variables is shown in Fig. 16, while the history of the performance functions

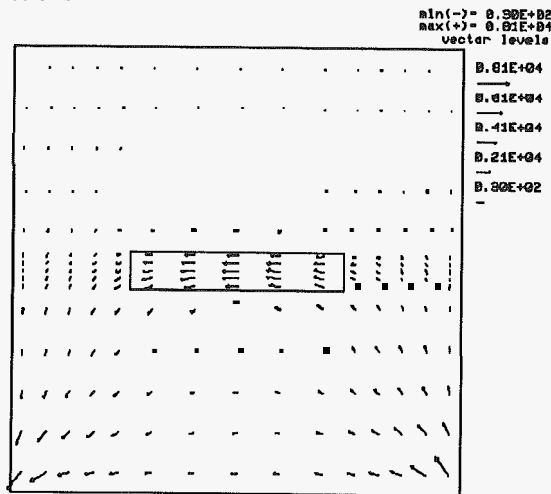


Fig. 9. Magnetic Flux of the Optimal Design

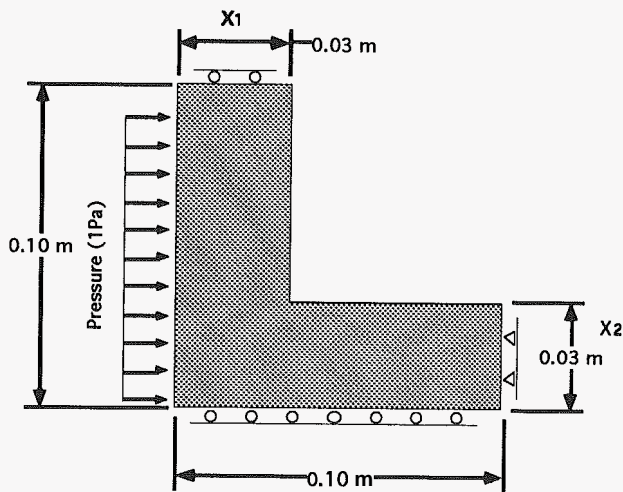


Fig. 10. Initial Design and Boundary Conditions

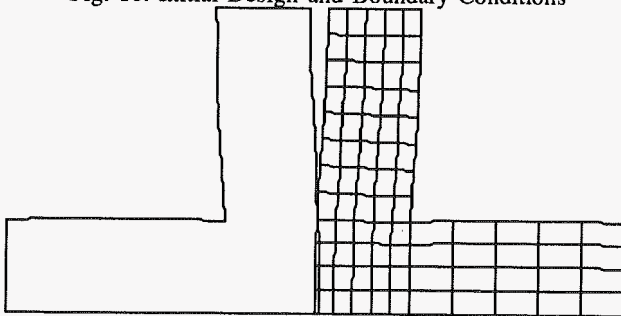


Fig. 11. Deformed Shape and FE Mesh of Initial Design

is shown in Fig. 17.

As the result of the optimization process, the shape of the structure is modified, and the maximum stress in the structure is reduced 25.4% from 9.15 to 6.83. Figure 16 shows the deformed shape and finite element model of the optimal design.

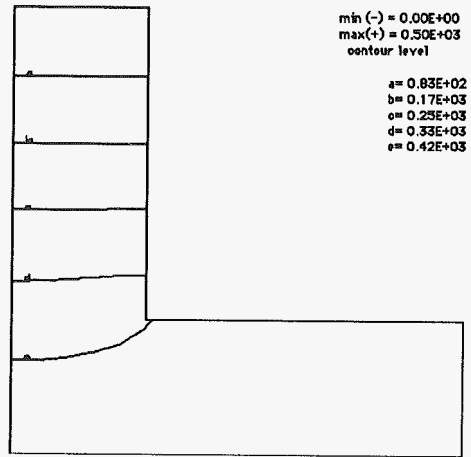


Fig. 12. Temperature of Initial Design

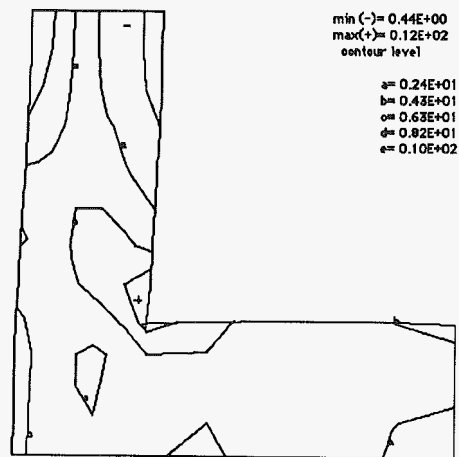


Fig. 13. Thermal Stress of Initial Design

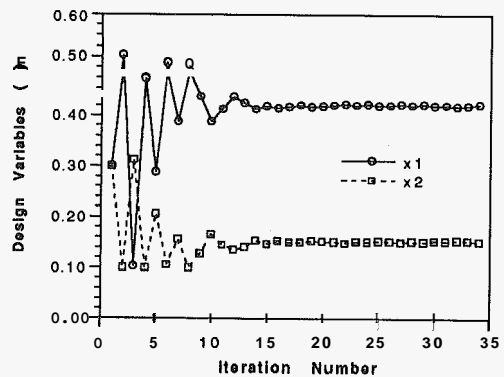


Fig. 14. Iteration History of Design Variables

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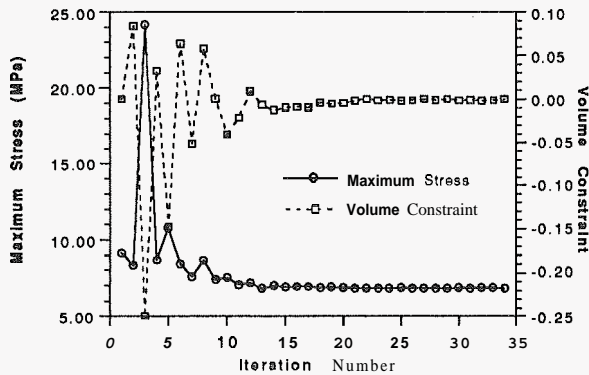


Fig. 15. Iteration History of Performance Functions

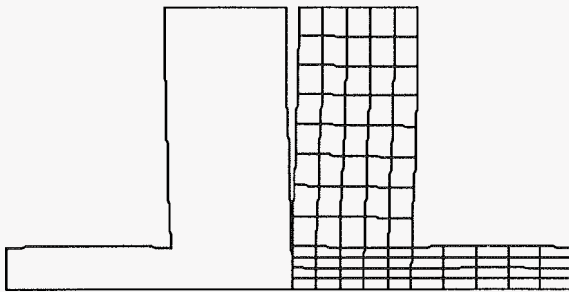


Fig. 16. Deformed Shape and FE Mesh of Optimal Design

Conclusion

As shown above, a system based on the open-ended concept can solve both multi disciplinary and weakly coupled structural optimization problems by calling different type of finite element analysis programs, an optimization code, and a mesh generation program from the system. If the speed of the process is not too concerned, the present system can handle most of optimization problems in mechanical design.

The key of the system is the capability of parametric mesh generation of the analysis model, since shape optimization is a much more important in mechanical design problems. Although only 2D problems are presented here, the present concept can be easily extended to the 3D problem using 3D mesh generating program and solid geometric modeler. The disadvantage of the present method is the necessity of large amount of calculation. It is certain that we have used more than one workstation to make finite element analysis to reduce the overall processing time. But still **this** may be insufficient if the number of design variables becomes large. At this moment, we must restrict the number of design variables into at most 20 ~ 30.

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