The lifting kernel function is of central importance in the determination of aerodynamic forces due to airfoil oscillations, wing gust responses, and engine inlet distortions. The nonlifting kernel function is of primary interest when 1) flow separations are involved for both cascade and isolated wing oscillations,[4] and 2) the panel flutter problem is considered for the external type of flow.[5-7]

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Modeling the Plasma Near-Wakes

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I. Introduction

WHEN a flat-based projectile moves rapidly in a gaseous medium, a cavity is momentarily created behind the moving body. The ambient gas tends to fill the cavity at the speed of its thermal motion. If the ambient gas is ionized as in the case of an ionosphere, the ambient electrons, having a greater mobility, implode into the empty space with the ions left behind, thus creating an electric field of charge separation. The field, so induced, will decelerate the advancing electrons very rapidly and accelerate the heavy ions slightly until a quasiequilibrium state is established, whereby they implode into the central core with a common speed of flow which is often designated as the speed of the wavefront. In addition to the fact that the plasma near-wake phenomenon is of intrinsic physical interest by itself, the importance of its relevancy to the in situ plasma measurements onboard the spacecraft is noteworthy.[1-4]

In order to sharpen our focus on the essential physics of the near-wakes in a tenuous plasma, consider an ambient, unmagnetized plasma in which a flat-based projectile of radius R

and zero surface potential moves along its axis of symmetry at a constant mesothermal speed; to wit, the body speed V_b lies intermediate between the thermal speeds of the undisturbed ambient ions and electrons such that the ion speed $(\sqrt{2kT_i/m_i}) \ll V_b \ll$ the electron speed $(\sqrt{2kT_e/m_e})$. It is also assumed that the Debye length λ_D of the ambient plasma be small compared with the characteristic size of the body R. Both of the above-assumed conditions represent well a typical space plasma wake. The contemporary theories of plasma near-wakes[1] usually proceed to iterate for an approximate self-consistent solution of the particle and field distributions in the wake using the Vlasov equation for the particles and the Poisson equation for the field. The computations in such a numerical iteration are unusually heavy and often plagued with computational instabilities if, for example, the grid sizes used in the computations are not appropriately chosen. A satisfactory self-consistent theory of plasma near-wakes is still not in sight. The experimental results of plasma wakes in the plasma wind tunnels under the strict conditions of mesothermal speeds and large bodies $(\gg \lambda_D)$ are still scanty.[5,6] In view of the inherently nonstationary nature of the near-wake in free flight at a constant mesothermal body speed, special discretion is advised of its simulation in a conventional plasma wind tunnel. In essence, the status of our present understanding of the physics of plasma near-wakes can be considered rudimentary only. Furthermore, the above-mentioned kinetic theories of near-wakes have not taken into account the likelihood of plasma instabilities in the wake flow. The prospect of extending them to the turbulent near-wake following the kinetic approach is very dim indeed considering the present state-of-the-art.

II. Analogy to Near-Wakes

The purpose of this Note is to consider an alternative conceptual view of the near-wake phenomenon: the plasma cavity-filling process in developing a near-wake, under the mesothermal flow conditions, can be likened to the transient flow associated with a radially imploding cylindrical shock seen immediately after the rupture of a cylindrical diaphragm of radius R, which separates a empty cylindrical chamber from the ambient plasma, by an observer moving with the velocity of the projectile (of radius R). It is noted that, under the mesothermal flow condition, the plasma expansion into the near-wake develops on a time scale that is long compared with the electron plasma relaxation time; consequently, the ions move in a self-consistent electric field with which the electrons are already in Boltzmann equilibrium. In view of the fact that plasma is continuously being replenished from the ambient, it is assumed that the electron temperature T_e remains constant and is much higher than the ion temperature T_i in the plasma. The cold ion approximation $(T_i \ll T_e)$, which justifies the domination of induced field effect over the pressure gradient effect on ion motion, is used in the following simplifed fluid approach. This bithermal nature of the upper ionosphere was previously discussed.[1]

It is important to the present modeling of the plasma near-wakes to note that the time constant of the cavity formation, which is inversely proportional to the projectile speed V_b, is much smaller than that of the implosive plasma flow; the latter is of the same order as the ion thermal motion. Hence, it is stipulated that the ambient plasma at the leading edge of the initial cavity is primarily responsible for populating the near-wake; the plasma ions at the trailing edge of the cavity, on the other hand, do not contribute significantly because of the following condition: $\sqrt{2kT_i/m_i} \ll V_b$. According to the present modeling, in the near-wake behind a projectile moving at constant speed V_b along the axial z axis, the radial and axial distributions of ion density $n_i(r,z)$ and field $\phi(r,z)$ can be determined from their respective counterparts; namely, the radial and time-dependent $n_i(r,t)$ and $\phi(r,t)$ of a cylindrical imploding plasma flow, which starts at $t=0$ from

an ambient $(n_i = n_e = n_0)$ at $r \geq R$ (the radius of the flat-based projectile) to $n_i = 0$ at $r < R$, by a coordinate transformation $t = z/|V_b|$ with z measuring downstream. The mathematical similitude of a steady near-wake and a cylindrical implosion will be presented later. The present modeling is expected to be valid in the time domain prior to the reflection of the implosive wavefront at the center of the tube, in a region excluding a thin layer adjacent to the projectile base within which the adjustment of the field potential difference from the projectile to the wake takes place. The objectives of the present modeling of the plasma near-wakes at mesothermal speeds are twofold: 1) it helps to circumvent the experimental difficulties of the mesothermal plasma wind tunnel testings by the use of a relatively simple implosion experiment; and 2) it makes the studies of turbulent plasma near-wakes more accessible.

III. Fluid Theory of Cylindrical Implosion

In the following, the development of the initial stages of a cylindrical freely imploding collisionless plasma is formulated as an illustration of the present modeling scheme. As noted in the above discussion, the plasma implosion, from the ambient $(n_i = n_e = n_0)$ into the empty core of radius R at the rupture of the cylindrical diaphragm at $t = 0$, has a characteristic flow time much larger than the electron plasma relaxation time. It is assumed that the plasma electrons reach the Boltzmann equilibrium with the local electrostatic potential $\phi(r)$ in the whole wake instantaneously at a constant electron temperature T_e. The plasma ions thus flow into the cavity as a result of the free expansion and the newly developed electric field-of-charge separation. Following Widner et al.,[7] we describe the self-consistent (field-particle) motion of the collisionless cold ions by using the following system of equations in rationalized MKS units:

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial r} = -\frac{e}{m_i} \frac{\partial \phi}{\partial r} \tag{1}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial r}(n_i v_i) + \frac{n_i v_i}{r} = 0 \tag{2}$$

$$\epsilon_0 \left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} \right) = e(n_e - n_i) \tag{3}$$

$$n_e = n_0 \exp(e\phi/kT_e) \tag{4}$$

where v_i is the ion velocity. The initial conditions $(t=0)$ consist of the undisturbed neutral plasma (i.e., $n_e = n_i = n_0$) at $r \geq R$ and $n_i = 0$ at $r < R$. The boundary conditions at $t > 0$ include an outer condition of a neutral plasma $n_i = n_e = n_0$ at $r = \infty$, and an inner condition at the position $(r = r_w)$ of the imploding ion wavefront which advances into and matches the field potential of the electron (core) cloud $[\phi_c(r)]$ governed by an ion-void equation (3):

$$\epsilon_0 \left(\frac{\partial \phi_c}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_c}{\partial r} \right) = e n_0 \exp(e\phi_c/kT_e) \tag{5}$$

The solution to Eq. (5), with boundary conditions $\phi_c = \phi_0$ and $d\phi_c/dr = 0$ at $r = 0$, can be readily obtained[9]; thus, the inner conditions at the ion wavefront $(r = r_w)$ become

$$\phi(r_w) = \phi_c(r_w) = (kT_e/e)\ln[8C^2(1 - C^2\lambda_D^{-2}r_w^2)^{-2}] \tag{6}$$

$$\left. \frac{\partial \phi}{\partial r} \right|_{r_w} = \left. \frac{\partial \phi_c}{\partial r} \right|_{r_w} = (kT_e/e)4C^2\lambda_D^{-1}r_w(1 - C^2\lambda_D^{-2}r_w^2)^{-1} \tag{7}$$

where $\lambda_D = (\epsilon_0 kT_e/n_0 e^2)^{1/2}$ and $8C^2 = \exp(e\phi_0/kT_e)$.

It is noted that with the substitution of $t = z/V_b$ in Eqs. (1) and (2), the system of Eqs. (1-4) becomes the governing

equations for the axisymmetric steady-state electrohydrodynamic flows. These, together with the boundary conditions at $z = 0$, $n_e = n_i = n_0$ at $r \geq R$; $n_i = 0$ at $z = 0$ and $r < R$; also, at $z > 0$, $n_i = n_e = n_0$ at $r = \infty$, plus the boundary conditions on ϕ as prescribed in Eqs. (6) and (7), give the mathematical boundary-value problem of a plasma near-wake having the coordinate fixed to the body with $z = 0$ at its base. It should be noted that the present similitude is contingent on the neglect of the perturbation of the axial ion velocity from V_b. This departure is expected to be of the order of the ion acoustic speed $(\sqrt{KT_e/m_i})$, which is small in comparison with V_b under the condition of mesothermal flows. The radial ion velocity of this order must be retained because it is at right angles to the axial direction; hence, not to be in direct competition against the axial velocity V_b.

Numerical studies of the solutions to the above-stated initial value problem of plasma implosion and their related near-wakes have been made by the use of a special numerical scheme[8] for the case $R = 10\lambda_D$, $V_b = 10(kT_e/m_i)^{1/2}$. The results of computations are shown in Fig. 1.

IV. Discussions and Conclusions

The existence of an ion wavefront as a result of charge separation is clearly seen from the ion density distributions at consecutive times after the start of implosion as shown in Fig. 1. The ion wavespeed can be determined from the advance of the ion front in a unit time. In contrast to the paradoxically high speed of ion front predicted in some theories of one-dimensional plasma expansion,[7] the magnitude of the ion wavespeed herein remains of the order of the ion acoustic speed. This is due to the cushion effect on the advance of the implosive ion wave as exerted by the stagnant electron cloud, at the core of the wake, which is caused by the inclusion of the convergence effects in the continuity equation (2) and Poisson equation (3).

It is of interest to compare the present calculations of mesothermal near-wakes with the experimental results[5,6] under the conditions of cold ions $(T_i/T_e \ll 1)$. They appear to agree qualitatively, particularly in the wave angles at comparable speed ratios (V_b/C_s). Precise comparison is made difficult because of the differences in boundary and freestream conditions of the experiments as compared to those of the present theory. In the case where $T_i \approx T_e$, the pressure gradient term in Eq. (1) must be retained. The nature of the pressure gradient effect on the near-wake structure should be qualitatively similar to that on the ion wave propagation,[10] bearing in mind that the essence of the present
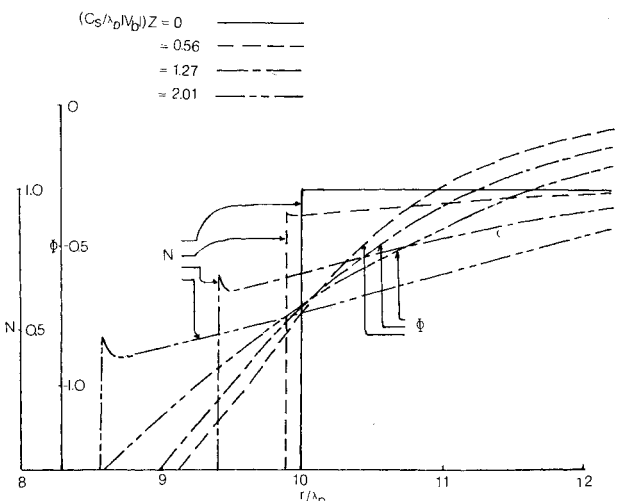

Fig. 1 Ion density $N[=n_i(r,z)/n_0]$ and electrostatic potential $\Phi[e\phi(r,z)/kT_e]$ in a plasma near-wake of a projectile $(R = 10\lambda_D)$ moving at a body speed $(V_b = 10C_s)$; $\lambda_D = (\epsilon_0 kT_e/n_0 e^2)^{1/2}$, $C_s = (kT_e/m_i)^{1/2}$; $\omega_{pi} = (n_0 e^2/\epsilon_0 m_i)^{1/2}$, $\omega_{pi}t = (C_s/\lambda_D)|V_b|)z$.

modeling of a near wake is to draw the mathematical analogy between a steady near-wake and an implosion wave generator. This analog remains valid when the pressure gradient term is included in Eq. (1). It is expected that the density and potential peaks in the wake would be less sharp with the pressure gradient taken into account, which is appropriate when $T_i \approx T_e$.

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Marker Particle Velocity Perturbations in Compressible Flows over a Wavy Wall

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Introduction

A THEORETICAL investigation using a perturbation technique of the compressible flow of a contaminated gas over a wavy wall is made. Different behaviors of the marking particles are noted for subsonic and supersonic outer streams.

In a previous report[1] an analytical investigation of the steady motion of a dusty gas impinging at a right angle to a flat wall was made. The gas flow was assumed to be inviscid and incompressible. Also, the dust particles were assumed to be spherically shaped and not interact with one another. A perturbation technique was used to solve for the motion of the dust particles and the gas flow. The main conclusion reached was that the trajectories of the contaminant particles do not

coincide with the streamlines of the gas flow to even first order. The dust particles are slow to respond to any change in the direction of flow. In the simple case of the impinging uniform stream, the dust particles tended to settle out of the flow.

The purpose of this Note is twofold. Initially, the general equations will be extended to the case when the flow is steady, isentropic, and compressible. The equations will then be applied to both subsonic and supersonic gas flows over a wavy wall. A perturbation technique will again be used.

Liepman and Roshko[2] have shown that the scalar form of the momentum equation for an isentropic flow is

$$u_i u_j \frac{\partial u_i}{\partial x_j} = a^2 \frac{\partial u_k}{\partial x_k}$$

where u_i and a are the velocity of the flow and the speed of sound, respectively. Next, consider the effects of the presence of dust particles on the flowfield. Saffman[3] used the following equations to represent the motion of an inviscid dusty gas:

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + KN(v_{pi} - u_i) \quad (1)$$

$$m \frac{\partial v_{pi}}{\partial t} + mv_{pj} \frac{\partial v_{pi}}{\partial x_j} = K(u_i - v_{pi}) \quad (2)$$

where v_{pi} is the particle velocity and N is the number density of the dust particles, each of mass m; K is the Stokes' coefficient of resistance; and ρ is the density of the gas. Combining Eqs. (1) and (2) and transforming into scalar form yields

$$a^2 \frac{\partial u_k}{\partial x_k} = u_i u_j \frac{\partial u_i}{\partial x_j} + \frac{f}{\tau} u_i (u_i - v_{pi}) \quad (3)$$

where $f = Nm/\rho$, and τ, the relaxation parameter, equals m/K.

Application

Consider a slightly disturbed uniform flow such that

$$u_1 = U_\infty + u_{c_1}, \quad u_2 = u_{c_2}, \quad u_3 = u_{c_3} \quad (4)$$

and the particle velocity is given by

$$v_{p_1} = u_1 + v_{d_1}, \quad v_{p_2} = u_2 + v_{d_2}, \quad v_{p_3} = u_3 + v_{d_3} \quad (5)$$

where u_{c_i} are perturbations due to a small disturbance and v_{d_i} are perturbations between the flow and particle velocities.

Neglecting terms containing squares of perturbation powers and all terms on the right-hand side in comparison with those on the left which contain no perturbation velocity yields

$$(1 - M_\infty^2) \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = \frac{-f M_\infty^2}{\tau U_\infty} [v_{d_i}] \quad (6)$$

The equations of motion for the compressible flow of a dusty gas have been developed. The flow over a wavy wall in both the subsonic and supersonic regimes will now be analyzed.[2]

Consider the flow past a boundary of sinusoidal shape with the boundary specified by

$$x_2 - \epsilon \sin \alpha x_1 = 0 \quad (7)$$

where ϵ denotes the amplitude of the waves of the wall and $\ell = 2\pi/\alpha$, the wavelength.